DSP LAB Experiment 7

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GITHUB LINK: https://github.com/ramyashri1887/DSP-LAB

Problem 4. Differentiate between the time-domain windowing and the window-based FIR filter design. Can you give a descriptive comparison of the same?

(a) Time-domain windowing is carried out to eliminate spectral leakage. Time-domain windowing is usually discussed along with concepts like frequency resolution and spectral leakage.
(b)Window-based filter design: Different windows offer different stopband attenuation and spectral selectivity.

Descriptive comparison (Magnitude response compared)
The spectrum of the different windows

1) Rectangular window:
   Time-domain Analysis
   The main lobe of $X_r(\Omega)$ has a width $2\pi/\tau$
   The narrowest main lobe possible for any window: Best frequency resolution of all windows.
   Disadvantage: Large side lobes
   The normalized first side lobe magnitude is -13dB, i.e only 13dB less than the peak magnitude. This leads to the issue of spectral leakage.
   So in conclusion, Has a high-frequency response but high spectral leakage as well.
   Window-based filter design
   Has high-frequency selectivity (narrowest main lobe) but will with a low stopband attenuation(-13dB) as the side lobes are equally high for a rectangular window

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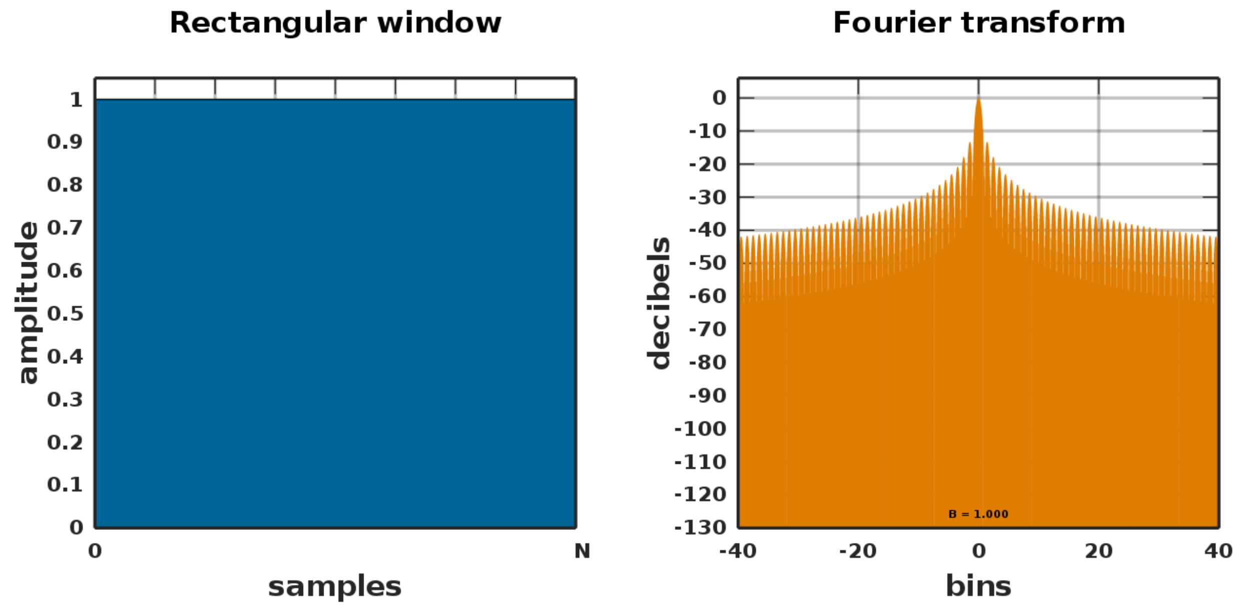


2) Hann window

Time-domain Analysis

The width of its main lobe is exactly double that of the rectangular window of the same width, i.e 4π/τ .

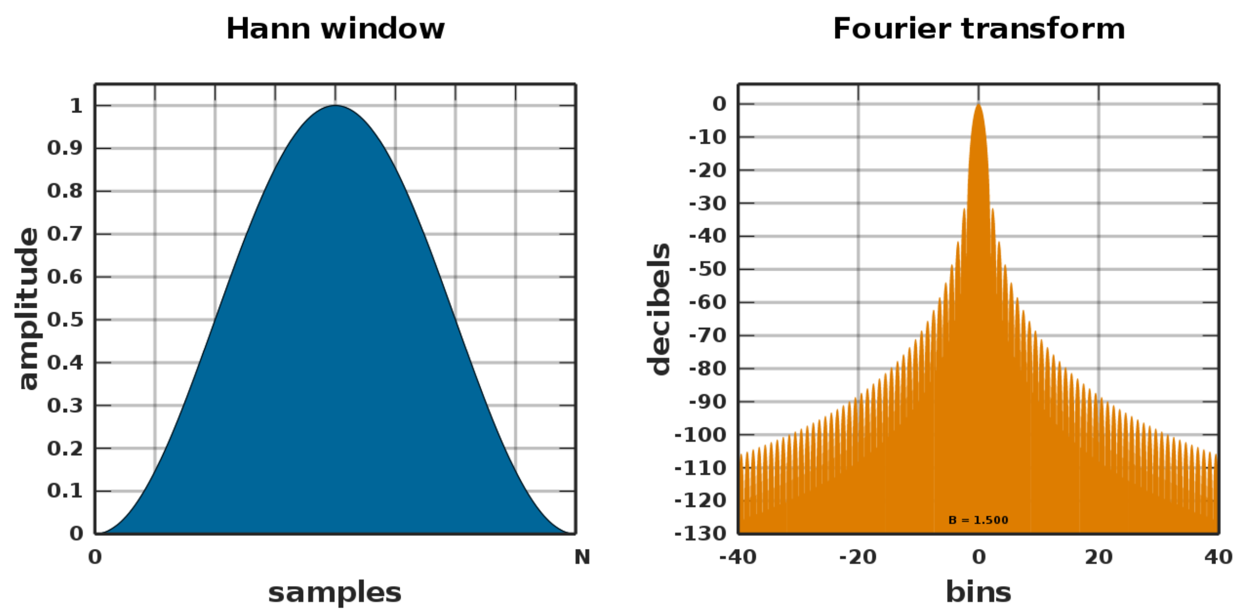
Bad frequency resolution compared to the rectangular window.

The normalized peak sidelobe magnitude is -31dB

The rate of fall-off of sidelobe levels is higher.

Window-based filter design

Has poorer frequency selectivity compared to the rectangular windows but with higher stopband attenuation(-31).



3) Hamming window

Time-domain Analysis

The width of its main lobe is exactly double that of the rectangular window of the same width, i.e 4π/τ.

Identical to that of a Hann window of the same size.

Advantage: Lowest possible first sidelobe level.

Due to this, the first side lobe's peak magnitude is -44dB (normalized).

Disadvantage: Slower side-lobe fall-off.

Window-based filter design

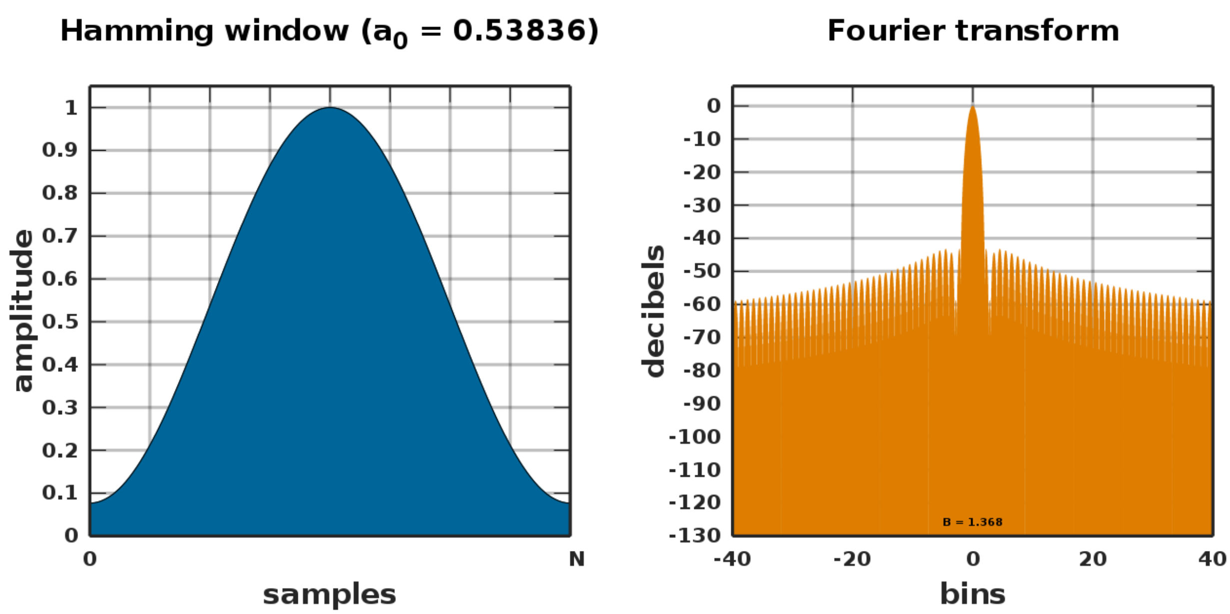
Hamming window has the same frequency selectivity as the hann window but higher stopband attenuation (-45dB) for the first side lobe.

Hann window equation :

$$h\left(n\right) = \alpha + \left(1.0 - \alpha\right)\cos\left[\left(\frac{2\pi}{N}\right)n\right]$$

The parameter α permits the optimization of the destructive sidelobe cancellation of the Hann window. In particular, when α is adjusted to 25/46 (0.543478261 …), the first sidelobe (Hann window) is canceled. The common approximation to this value of α is 0.54, for which the window is called the Hamming window and is of the form

$$H\left(\theta\right) = 0.54 + 0.46\ \cos\left[\left(\frac{2\pi}{N}\right)n\right]$$



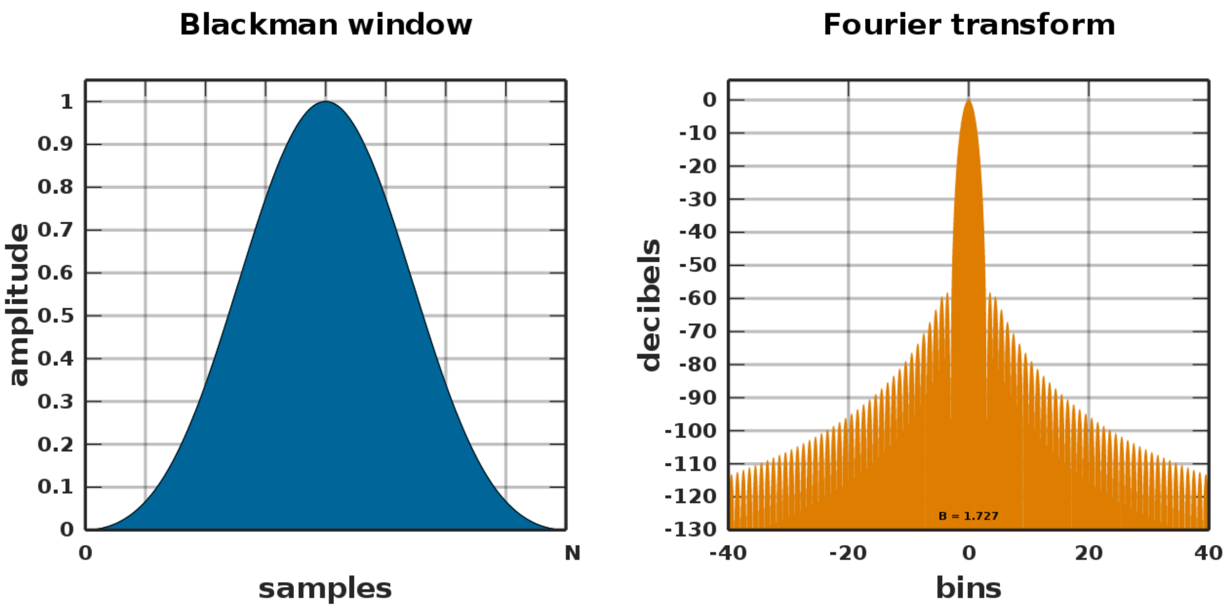
4) Blackman window

Time-domain Analysis

The width of the main lobe in the magnitude response is about 50% wider than that of the Hamming window ie 12π/τ.

Window-based filter design

The Blackman window exhibits an even lower maximum stopband ripple (about 74 dB down) in the resulting FIR filter than the Hamming window but the frequency selectivity is poorer as the width of the passband ripple is double the hanning and hamming window which in turn is double the rectangular window.



Problem 1. (Window Functions)

(a) Rectangular

(b) Hamming

(c) Hanning

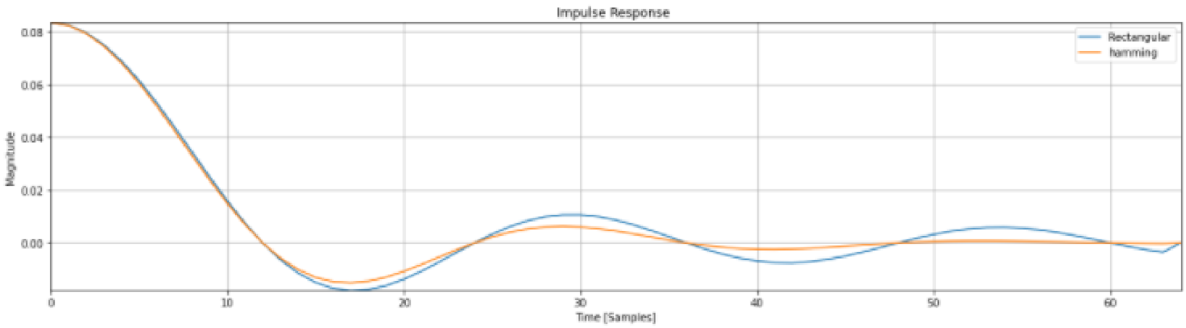
Plot the spectrum of the window signal for different lengths. Use a 1024 point DFT and normalize the magnitude by the actual lengths (Say N = 100, 200, 300).

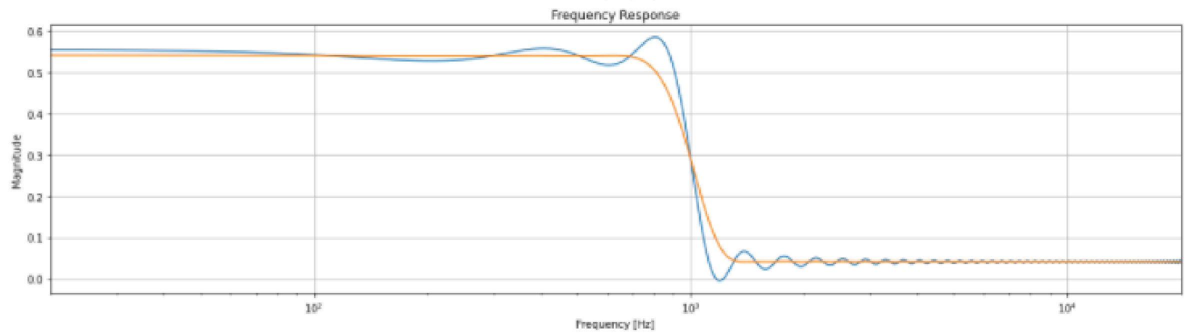
Plotting for different lengths: for lengths of the form 1024/n where n is an integer, frequency response is without spectral leakage. As we have taken a 1024 point DFT, the signal length should be below 1024 to avoid errors. Small-length signals will be zero-padded to get the results.

Problem 2. (FIR Filter Design)

Design digital low pass FIR filter using rectangular and Hamming window functions for a cut-off frequency of $\omega c = \pi + 1$ rad/sample. Select the window length for all the window functions to be 21.

1. Plot the impulse response of the two filters.


Impulse Response
Rectangular
hamming
Magnitude
Time [Samples]
0.08
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60


Frequency Response
Magnitude
Frequency [Hz]
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^2
10^3
10^4