DSP386 Epidemic modeling -Lab 8

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Link to codes: https://github.com/rum1887

Problem 1) First order model

1. a) Transfer function (Solution)

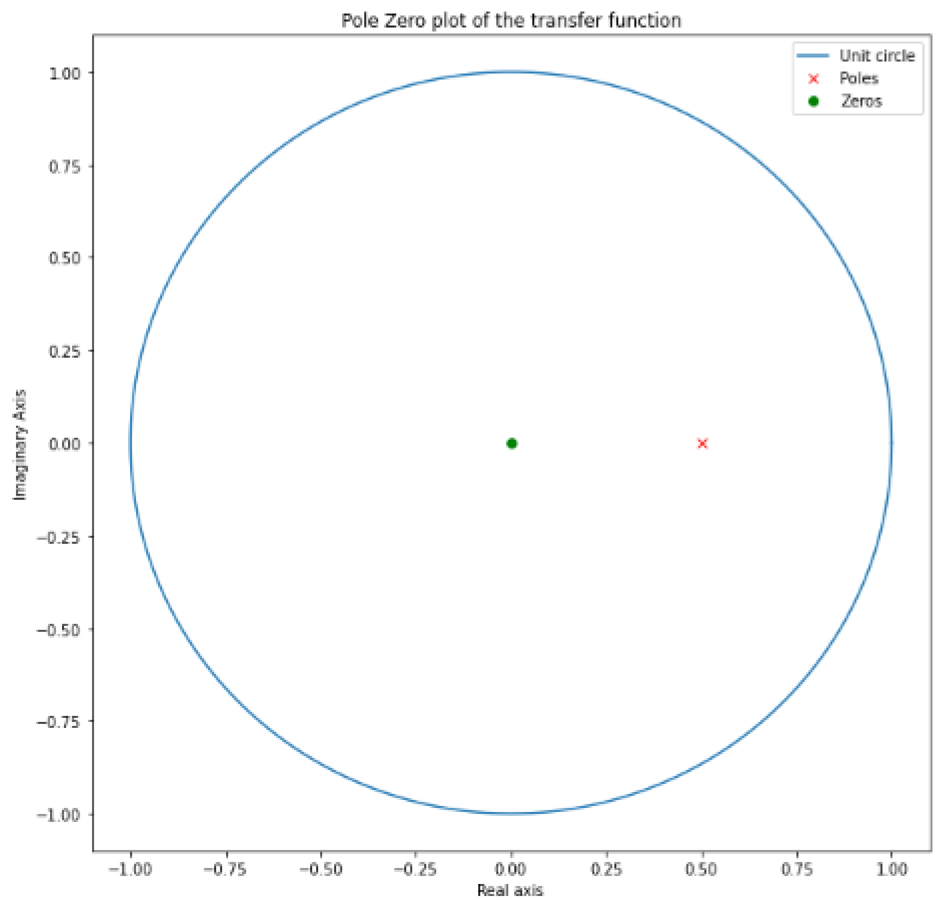
   R: Reproduction number
   $H(z) = 1/(1 - Rz^{-1)})$

   Recursive difference equation
   $y(n) = \delta(n) + R\,y(n-1)$
   Taking z transform
   $Y(z) = 1 + R * Y(z) * z^{-1}$
   $Y(z) = 1/(1 - Rz^{-1)})$
   Taking input as $\delta(n)$ function.
   $X(z) = 1$

   The transfer function for the system is $H(z) = Y(z) / X(z)$
   $H(z) = 1/(1 - Rz^{-1)})$

   b) Pole zero plot
   Location of poles and zeroes
   Poles: R
   Zero: 0

   Plot 1.1: Pole zero plot of the first-order transfer function for R=0.5



Remarks: Effects of parameter R

Since R (Reproduction number)is the pole of the system.

$R < 1$ *System stable*

$R = 1$ *System marginally stable*

$R > 1$ *System unstable*

2. Time-domain equation

   $y(n) = \delta(n) + R\, y(n - 1)$

   $y(0) = 1$ Assume that an initial patient zero appears at day n = 0.

   $\delta(n)$ is zero for all value of $n \neq 0$

   $y(1) = R$

   Time-domain equation of no of newly infected people

   $y(n) = R^{n}$, Exponential function with base R (basic reproduction number).

   Remarks: Effects of parameter R

   $R < 1$, *Exponential function will decay to zero at* $n =$ *infinity making it stable*

$R = 1,\ Exponential\ function\ has\ a\ constant\ value\ of\ 1\ irrespective\ of\ n,\ hence\ mariginally\ stable$
$R > 1, Exponential\ function\ reaches\ \infty\ as\ n\ tends\ to\ \infty\ making\ it\ unstable,\ or\ unpredictibale$

Different methods same conclusion, the same conclusions were made using pole-zero plots as well.

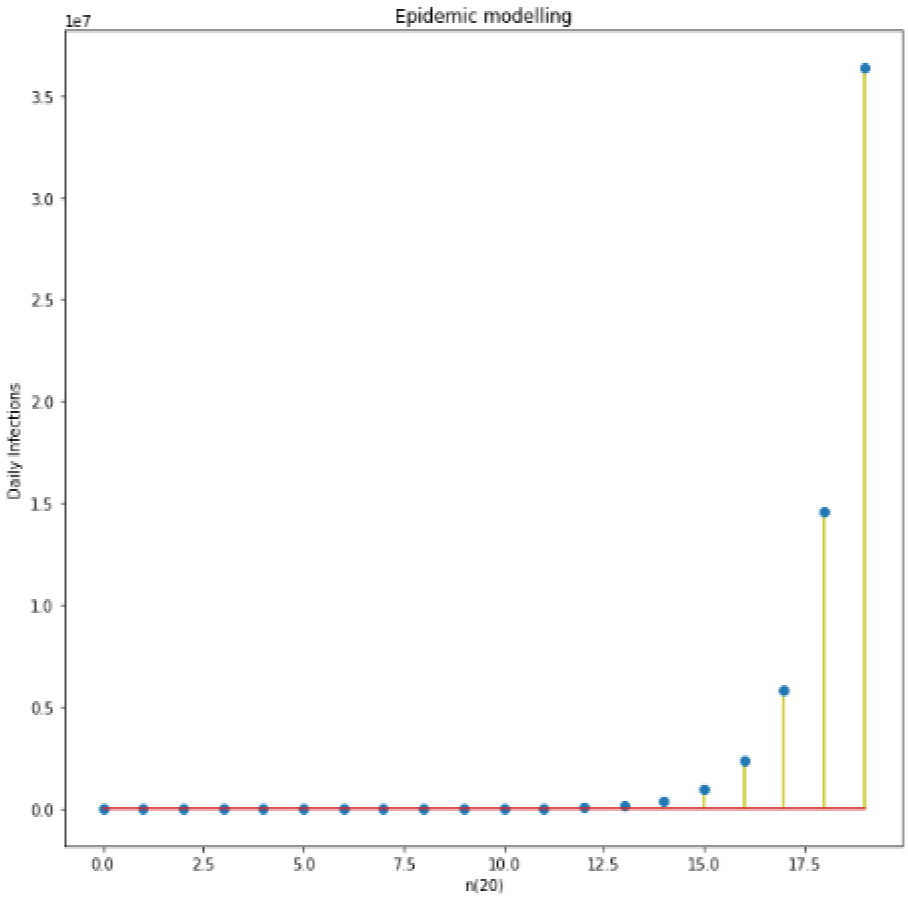
3. $y(n) = R^n$
   $y(n) = 10^6$(1 million new infections)
   $R = 2.5$
   $n = log(2.5,\ 10^6)$
   $n = 15.077649568392$
   It will take 15 days to reach 1 million new daily infections.

4. The initial phase of the first wave of the equation:
   Time period: Assume the first person got infected on this day Apr 10, 2021 to Apr 24, 2021 .
   $n = 15$
   $y(15) = 348941$as of Apr 24, 2021
   $R = y(n)^{1/n}$
   $R = 348941^{1/15}$
   $R = 2.341$

   Similarly, R as of today, assuming cases began on Nov 18, 2021 to Nov 11, 2021
   $n = 7$
   $y(7) = 152606$
   $R = y(n)^{1/n}$
   $R = 152606^{1/7}$
   $R = 5.50186880089$

   One point method is not the most reliable, because we might not exactly know how much time is elapsed from day 1, to accurately use n.
   Alternative methods to estimate R: Maybe taking multiple values of y(n), precisely 2 different values are sufficient to calculate R for first-order systems.

5. Plot 2.1: Plot of new daily infections for the first n = 20 days for R=2.5.



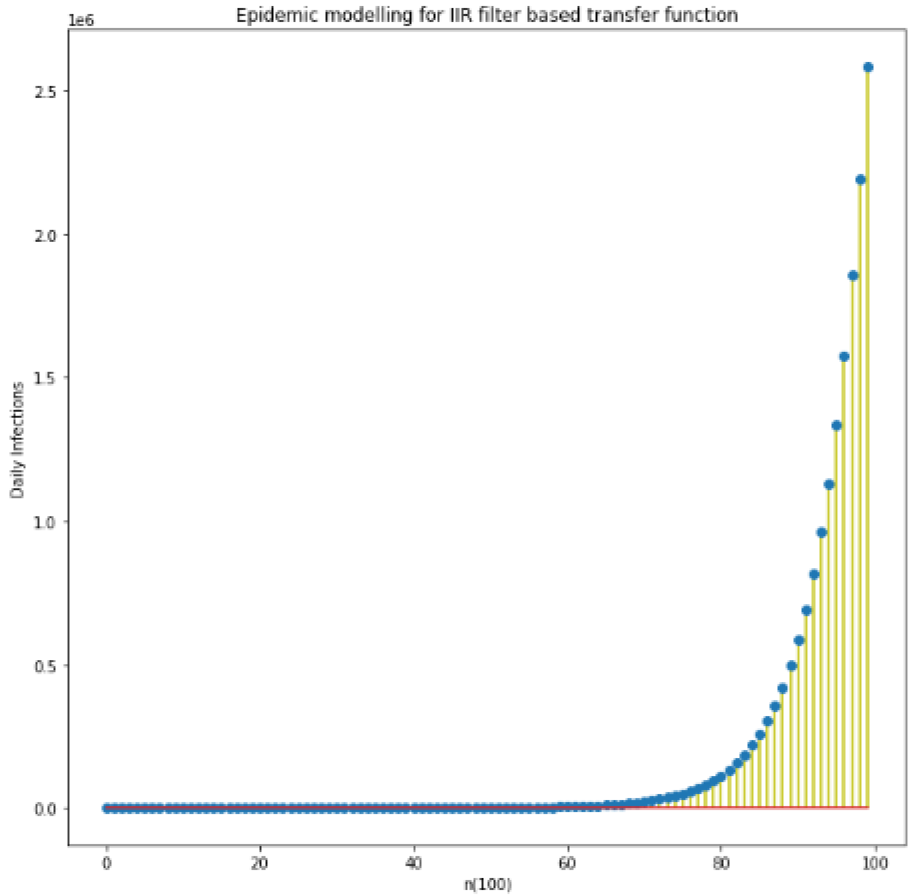
Integrator filter to find the total number of infections for a period of 20 days :
Using np.sum() method, since daily infections are discrete in time with LSB of 1 day.

Total number of infected people: 60632979

Problem 2) Increasing the complexity

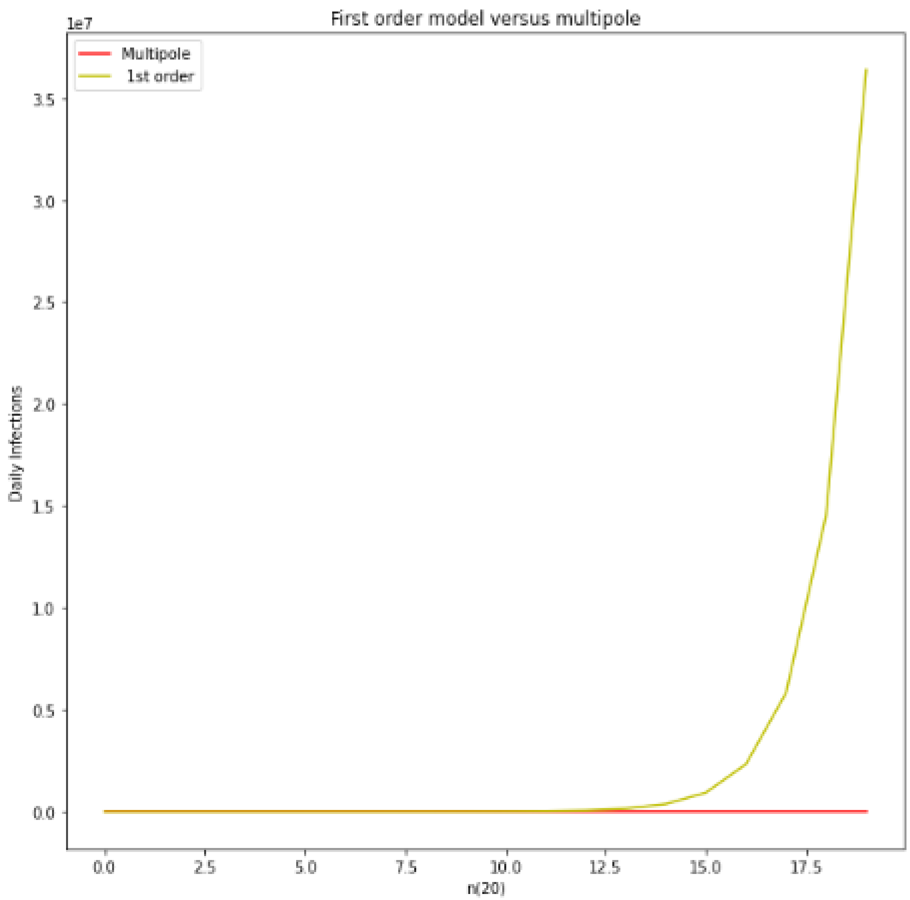
1. Multiplying input with the transfer function
   $H(z) = 1/1 - \sum a_K z^{-k}$
   $y(n) = \delta(n) + a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3)...$
   $y(0) = 1$
   Coded the function
   Using integrator:
   The total number of infected patients for 100 days: 16995618
   Observation: Significant decrease in the number of infected patients.

   Plot 2.1: Plotting the new daily infections for the first n = 100 days by implementing the filter with the Kronecker delta as the input.



2. Comment on the differences between the trends that are obtained with the first-order model.
   Differences in trend:
   The rise in the function is slower in multipole compared to first-order. High-order filters are used because they have the ability to roll off gain at a sharper rate than low-order filters. The attenuation of a filter above the \ grows proportionally to the number of poles.

   Plot 2.2: First-order model vs Multipole model



3. A reliable technique to estimate the coefficients. :(

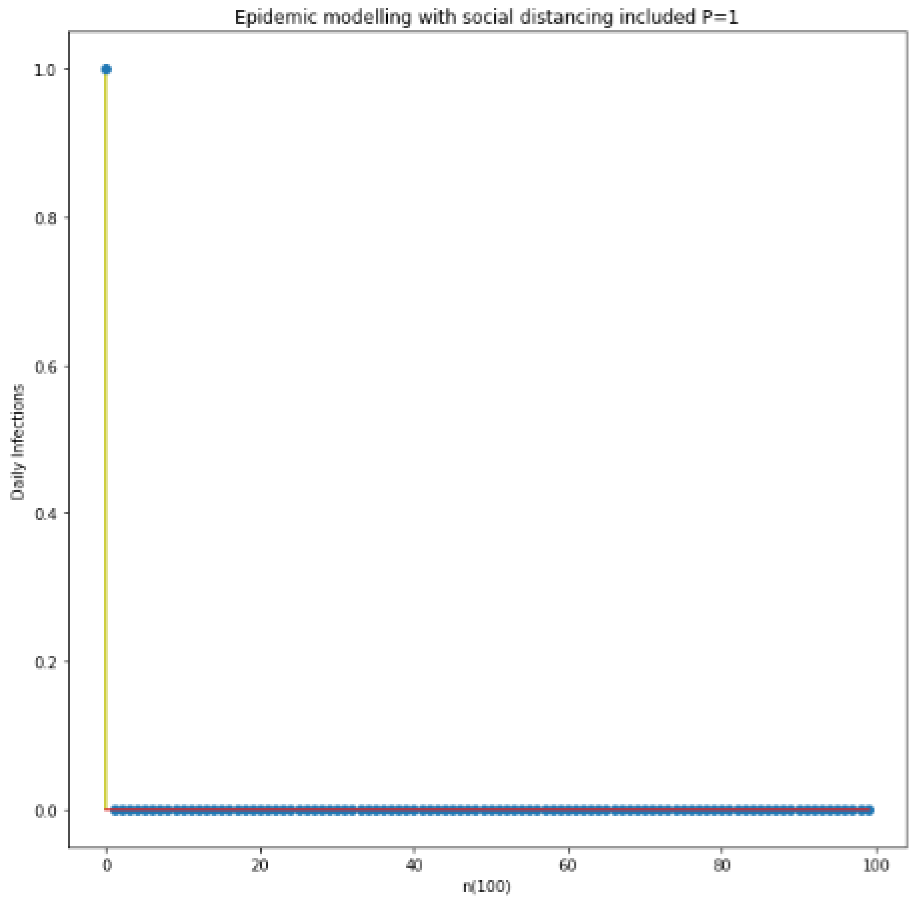
Problem 3) Effects of social distancing

1. <u>Role of ρ</u>

   Intuitively, ρ =1 indicates that contact is reduced 100% (since ρ is the factor by which social interaction is reduced, 1 means 100 percent reduction or complete isolation), ie the human is completely isolated and hence will not transmit the virus further to anybody.

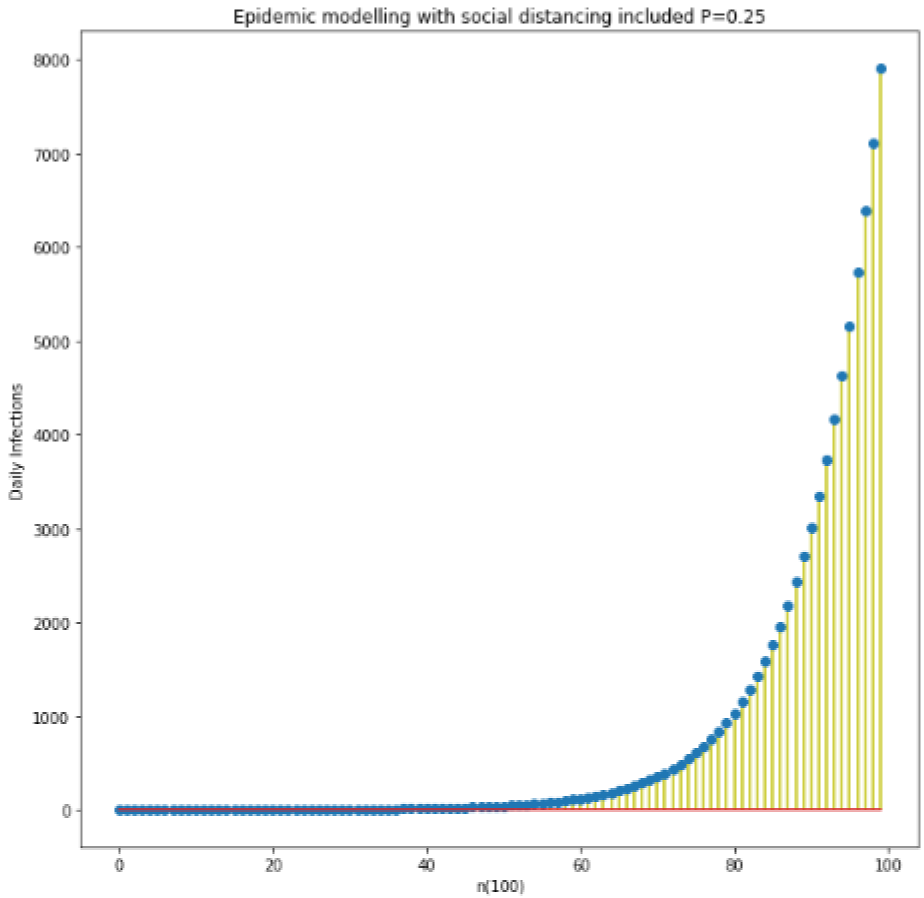
   Mathematically, the Transfer function is 1.
   Which, means
   $y(n) = \delta(n)$
   There will only be one person infected(at n=0 or day 0) and no transmission.

   Plot 3.1: Epidemic modeling with social distancing parameter included for 1.

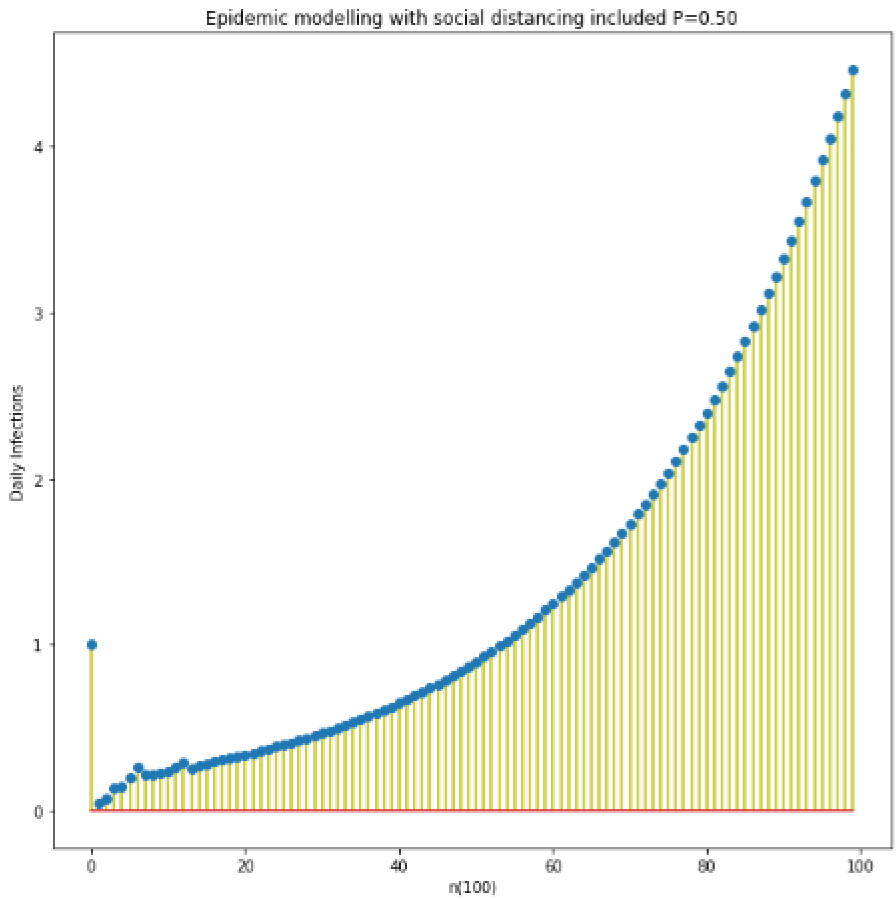


For values ρ other than 1, since we are reducing the coefficients by some factor, we are essentially reducing the coefficients of the impulse response, hence a decrease in the effective value of the output.

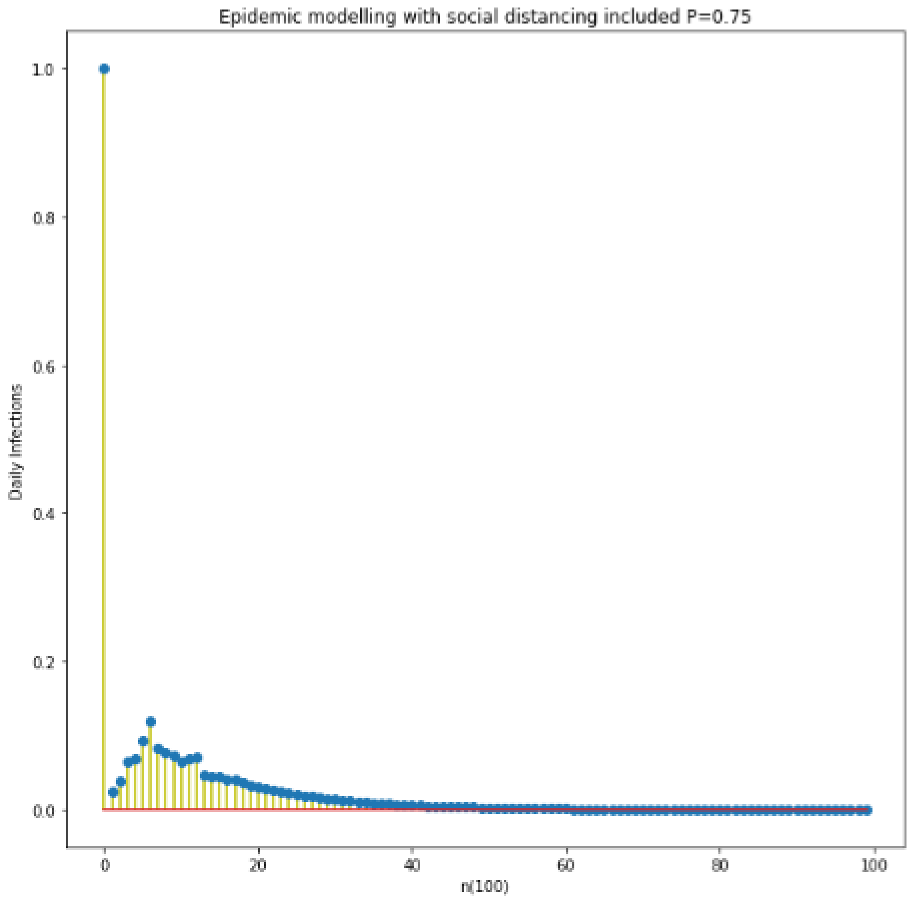
2. Plot 3.2: Plot of new daily infections for the first n = 100 days, ρ = 0.25. Total No of infections: 77745



Plot 3.3: Plot for ρ = 0.50. Total No of infections: 134



Plot 3.4: Plot for ρ = 0.75. Total No of infections: 2.5309944507906685



3) <u>Observations</u>

Tabulated total number of infections for n=100 days for different values of ρ.

ρ (social interaction parameter)	**Total number of infections over the period of 100 days using integrator function**
0.25	77745
0.50	134
0.75	3
1	1

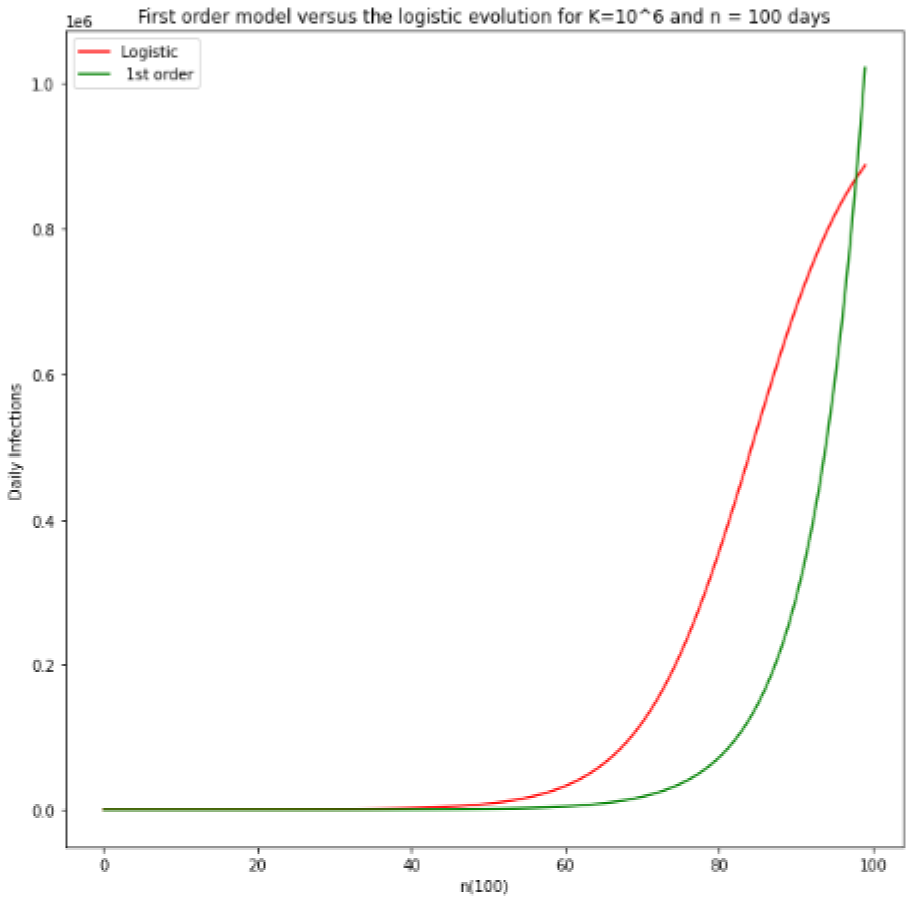
<u>Results</u>

Clearly, with an increase in ρ (social interaction factor) there is a decrease in the number of infected patients. Social distancing reduces the number of total infections. As ρ is the factor by which the interaction is reduced, it's logical to guess that a higher reduction in contact will lead to a lower number of people infected.

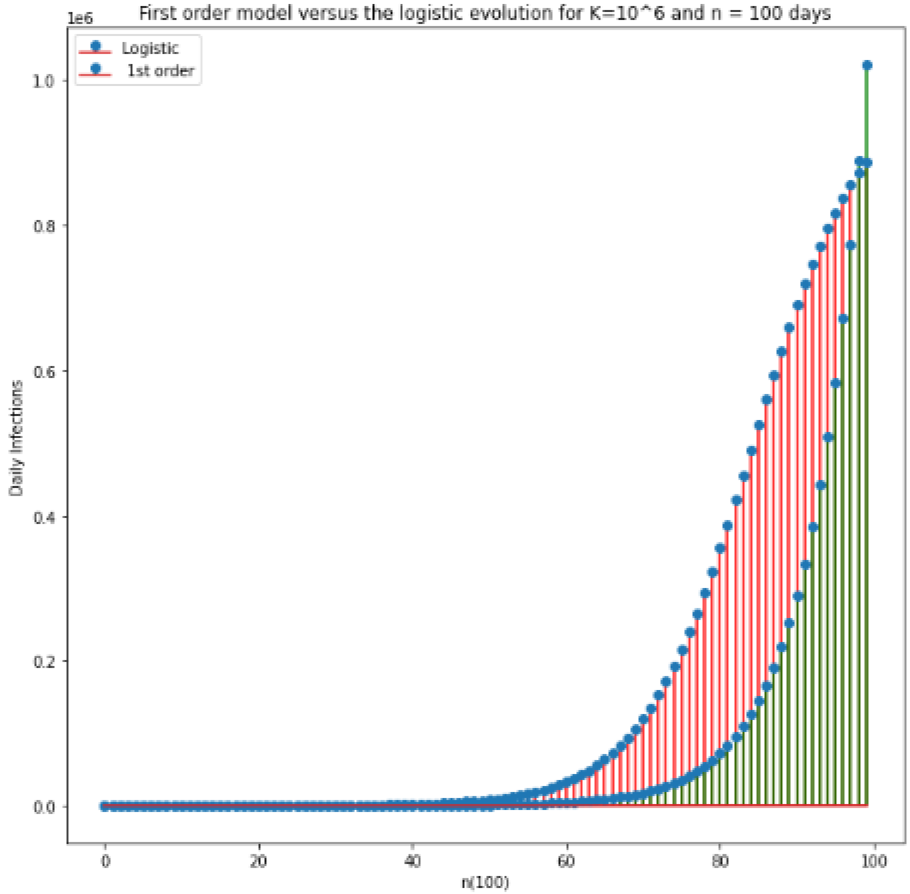
Problem 4) Saturation and towards normality

The first-order function is an exponential function and hence does not follow a logistic hypothesis that it will level out when n is equal to infinity.

Plot 4.1: First order model Vs Logistic evolution plot



Plot 4.2: First order model Vs Logistic evolution stem



1. $D_1(z) = 1 - z^{-1}$

   Finding a point of inflection using the filter functions.

   The filter function is implemented as a direct II transposed structure.

   .

   $y(n) = x(n) - x(n - 1) = 0$

   Global maximunt: n = 340

   $D_2(z) = 1 - 2z^{-1} + z^{-2}$

   $y(n) = x(n) - 2x(n - 1) + x(n - 2) = 0$

   Zero crossing: n=340

   Inflection point n=340

   After 340 days , epidemic starts to level out.

   Thank you!