Uber Rocket Calculus Problem

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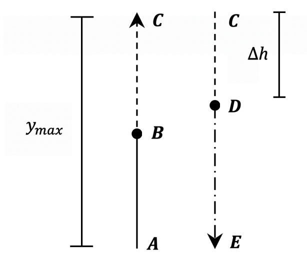
Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram



Givens

$t_{AB} = 5.70\ s$

$v_A = 0\ m/s$

$y_A = 0\ m$

$v_C = 0\ m/s$

$v_D = 0\ m/s$

$\Delta h = -89.0\ m$

$a_{BD} = -9.80\ m/s^2$

$a_{AB}[t] = -0.8t^2 + 21\ m/s^2$

$v_p[t] = -20 + 20e^{-\frac{t}{8}}\ m/s$

Strategy

To find the total time the rocket is in the air, the problem will be divided into 3 stages: AB, BD, and DE. In Stage AB, the height and velocity of the rocket will be found at t_{AB}, the time at which the engine burns out. In Stage BD, the max height of the rocket, y_{max}, will be found between points B and C. Using this result, time elapsed between B and D will be found. In Stage DE, the time between D and E will be calculated by finding the vertical displacement equation for v_p. By adding the time found in Stages AB, BD, and DE, the total time will be known.

Stage AB

First, the vertical velocity equation v_{AB} is found by taking the indefinite integral of the given acceleration equation, a_{AB}. The constant c is v_A.

$$a_{AB}[t] = -0.8t^2 + 21\ m/s^2\ \ , v_A = 0\ m/s$$
$$v_{AB}[t] = \int(a_{AB}[t])\ dt + c$$
$$v_{AB}[t] = \int(-0.8t^2 + 21)\ dt + v_A$$
$$v_{AB}[t] = \frac{-0.8}{3}t^3 + 21t$$

Substitute the value of t_{AB} for t in the v_{AB} equation to find the rocket's velocity when the engine burns out:

$$v_{AB}[5.70] = \frac{-0.8}{3}(5.70)^3 + 21(5.70)$$
$$\underline{v_B = 70.315\ m/s}$$

Similarly, the vertical displacement equation y_{AB} is found by taking the indefinite integral of the v_{AB} equation. The constant c is y_A.

$$v_{AB}[t] = \frac{-0.8}{3}t^3 + 21t\ \ , y_A = 0\ m$$
$$y_{AB}[t] = \int(v_{AB}[t])\ dt + c$$
$$y_{AB}[t] = \int(\frac{-0.8}{3}t^3 + 21t)\ dt + y_A$$
$$y_{AB}[t] = \frac{-0.8}{12}t^4 + \frac{21}{2}t^2$$

Substitute the value of t_{AB} for t in the y_{AB} equation to find the rocket's height when the engine burns out:

$$y_{AB}[5.70] = \frac{-0.8}{12}(5.70)^4 + \frac{21}{2}(5.70)^2$$
$$\underline{y_{AB}[5.70] = 270.77\ m/s}$$

Stage BC

The max height of the rocket is found using the equation below:

$$v_C = v_B{}^2 + 2a_{BD}\Delta y_{Bc}$$
$$0 = (70.315)^2 + 2(-9.80)(y_{max} - 270.77)$$
$$y_{max} = \frac{-(70.315)^2}{-2(-9.80)} + 270.77$$
$$\underline{y_{max} = 523.03\ m}$$

The height of the rocket at point D is found below using y_{max} and Δh:

$$\Delta h = y_B - y_{max}$$
$$y_B = \Delta h + y_{max}$$
$$y_B = -89.0 + 523.03$$
$$\underline{y_B = 434.03\ m}$$

The time elapsed between B and D is found using the equation below:

$$\Delta y_{BD} = v_B t + \frac{1}{2}a_{BD}t^2$$
$$434.03 - 270.77 = (70.315)t + \frac{1}{2}(-9.80)t^2$$
$$434.03 - 270.77 = (70.315)t - 4.9t^2$$
$$163.26 = (70.315)t - 4.9t^2$$
$$-4.9t^2 + (70.315)t - 163.26 = 0\ ,\ solver$$
$$\cancel{t_{BD} = 2.9132s}\ \ and\ \ \underline{t_{BD} = 11.437s}$$

Note that the first time is invalid as it does not account for the rocket reaching the max height and then going down.

Stage DE

Begin by taking the indefinite integral of the v_p equation:

$$v_p[t] = -20 + 20e^{-\frac{t}{8}}\ m/s$$
$$y_p[t] = \int(v_p[t])\ dt + c$$
$$y_p[t] = \int(-20 + 20e^{-\frac{t}{8}})\ dt + c$$
$$y_p[t] = -20t - 160e^{-\frac{t}{8}} + c$$

Substitute $t = 0\ s$ into the y_p equation and set $y_p = y_B$ to find c:

$$434.03 = -20(0) - 160e^{-\frac{0}{8}} + c$$
$$c = 594.03\ m$$
$$y_p[t] = -20t - 160e^{-\frac{t}{8}} + 594.03$$

The time elapsed between D and E is found by setting $y_p = 0$:

$$-20t - 160e^{-\frac{t}{8}} + 594.03 = 0\ ,\ solver$$
$$\cancel{t_{DE} = -13.490s}\ \ and\ \ \underline{t_{DE} = 29.501s}$$

Final Step

The total time can be found by adding the times from each stage:

$$t_{total} = t_{AB} + t_{BD} + t_{DE}$$
$$t_{total} = 5.70 + 11.437 + 29.501$$
$$\boxed{t_{total} = 46.638s}$$

Table

t	y	v	a
(s)	(m)	(m/s)	(m/s²)
0.00	0.00	0.00	21.00
2.00	40.93	39.87	24.20
4.00	150.93	66.93	33.80
5.70	270.77	70.32	46.99
7.70	391.80	50.72	-9.80
9.70	473.63	31.12	-9.80
11.70	516.26	11.52	-9.80
13.70	519.69	-8.08	-9.80
15.70	483.92	-27.68	-9.80
17.14	434.03	-41.77	-9.80
19.14	429.42	-4.42	-1.95
21.14	416.98	-7.87	-1.52
23.14	398.45	-10.55	-1.18
25.14	375.17	-12.64	-0.92
27.14	348.19	-14.27	-0.72
29.14	318.33	-15.54	-0.56
31.14	286.22	-16.52	-0.43
33.14	252.37	-17.29	-0.34
35.14	217.16	-17.89	-0.26
37.14	180.89	-18.36	-0.21
39.14	143.80	-18.72	-0.16
41.14	106.06	-19.00	-0.12
43.14	67.82	-19.22	-0.10
45.14	29.20	-19.40	-0.08
46.64	0.00	-19.50	-0.06

Graphs