Hanford Washington Problem

Description:

In an article taken from the Journal of Environmental Health, May-June 1965, Volume 27, Number 6, pages 883- 897, author Robert Fadely explains that the Atomic Energy Plant in Hanford, Washington has been a plutonium production facility since the Second World War. Some of the waste have been stored underground in the same area. Radioactive waste has been seeping into the Columbia River, and eight Oregon counties and the city of Portland have been exposed to radioactive contamination. The table below lists the number of cancer deaths per 100,000 residents for Portland and these counties. The table also includes an index of exposure that measures the proximity of the residents to the contamination. The index is based on the assumption that city or county exposure is directly proportional to river frontage and inversely proportional both to the distance from Hanford, WA site and to the square of the county's or city's average distance from the river.

Problem:

Display a "Two-Point" line, a median-median line, and a Least Squares line.

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1. Original Table

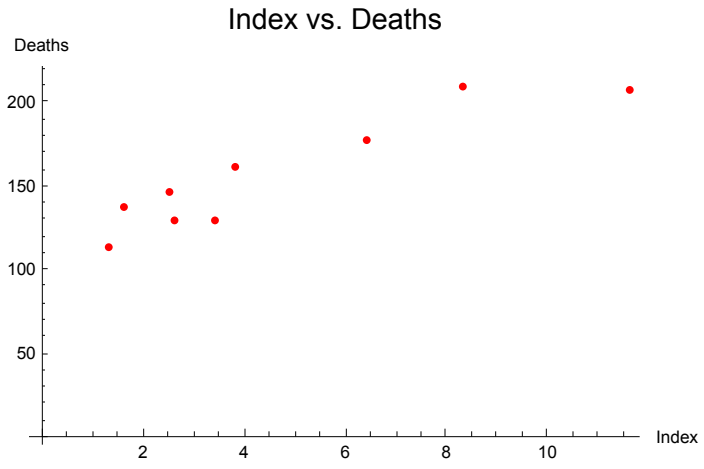
```
In[•]:= index = {2.5, 2.6, 3.4, 1.3, 1.6, 3.8, 11.6, 6.4, 8.3};

        deaths = {147, 130, 130, 114, 138, 162, 208, 178, 210};

In[•]:= list1 = Sort[Transpose[{index, deaths}]];

        Text[Grid[Prepend[list1, {"Index", "Deaths"}],
          Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

Out[•]=

Index	Deaths
1.3	114
1.6	138
2.5	147
2.6	130
3.4	130
3.8	162
6.4	178
8.3	210
11.6	208

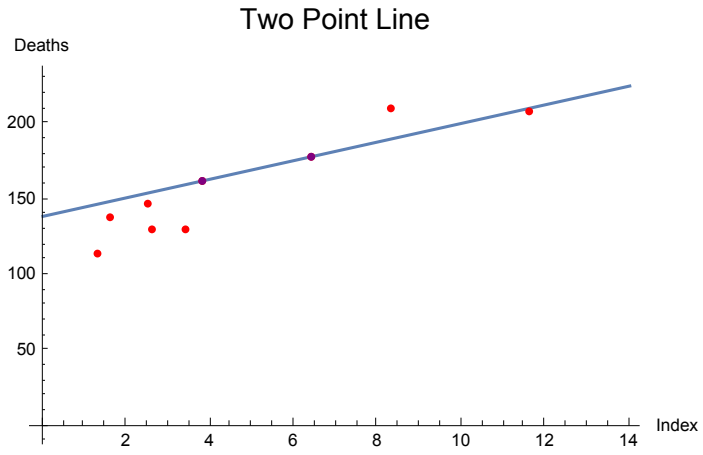
2. Original Graph

```
In[•]:= listplot1 = ListPlot[list1, PlotStyle → Red,
          AxesLabel → {"Index", "Deaths"}, PlotLabel → Style["Index vs. Deaths", 15, Black],
          ImageSize → Medium, AxesOrigin → {0, 0}]
```



3. Two-Point Line

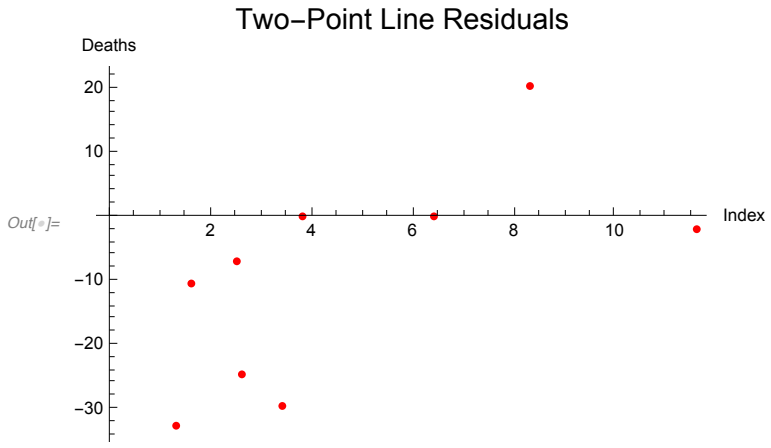
```
In[•]:= Solve[y - 162 == (178 - 162)/(6.4 - 3.8) (x - 3.8), y]
Out[•]= {{y → 162. + 6.15385 (-3.8 + x)}}

In[•]:= f[x_] = 162.` + 6.153846153846152` (-3.8` + x)

In[•]:= Show[Plot[f[x], {x, 0, 14}], listplot1,
  ListPlot[{{3.8, 162}, {6.4, 178}}, PlotStyle → Purple],
  PlotRange → All, PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
  PlotLabel → Style["Two Point Line", 15, Black],
  ImageSize → Medium, AxesOrigin → {0, 0}]
```



4. Two-Point Line Residuals

```
In[•]:= xcoords = list1[[All, 1]];

In[•]:= ycoords = list1[[All, 2]];

In[•]:= predictedy = f[xcoords];

In[•]:= resy = ycoords - predictedy;

In[•]:= res1 = ListPlot[Transpose[{xcoords, resy}],
          PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
          PlotLabel → Style["Two-Point Line Residuals", 15, Black],
          ImageSize → Medium, AxesOrigin → {0, 0}]
```

Out[•]=



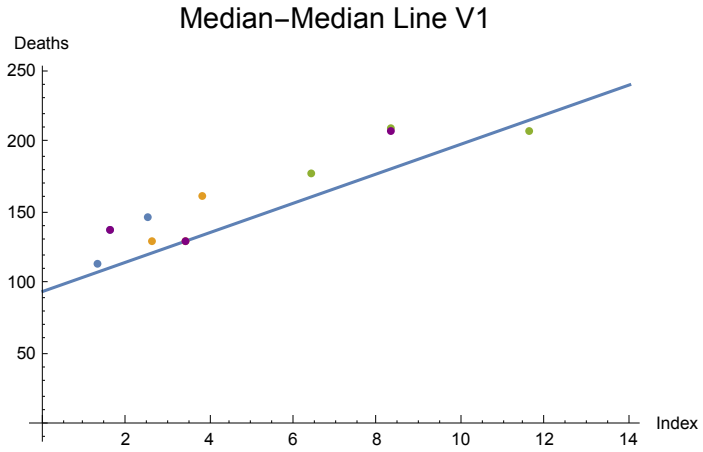

```
(* Below is the value for the sum of residuals *)

In[•]:= ressum = Total[resy]
```

Out[•]= −85.9231

5. Median-Median Line V1

```
In[•]:= parts = Partition[list1, 3]
Out[•]= {{{1.3, 114}, {1.6, 138}, {2.5, 147}},
  {{2.6, 130}, {3.4, 130}, {3.8, 162}}, {{6.4, 178}, {8.3, 210}, {11.6, 208}}}

In[•]:= median1 = Median[Part[parts, 1]];

In[•]:= median2 = Median[Part[parts, 2]];

In[•]:= median3 = Median[Part[parts, 3]];

In[•]:= medians = List[median1, median2, median3]
Out[•]= {{1.6, 138}, {3.4, 130}, {8.3, 208}}

    slope = (208 - 138) / (8.3 - 1.6);

In[•]:= Solve[130 == slope * 3.4 + b2, b2]

In[•]:= b2 = 94.47761194029852;

In[•]:= f1[x_] = slope * x + b2;

In[•]:= Show[Plot[f1[x], {x, 0, 14}], ListPlot[parts],
   ListPlot[medians, PlotStyle → Purple], PlotRange → All,
   PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
   PlotLabel → Style["Median-Median Line V1", 15, Black],
   ImageSize → Medium, AxesOrigin → {0, 0}]
```

Out[•]=

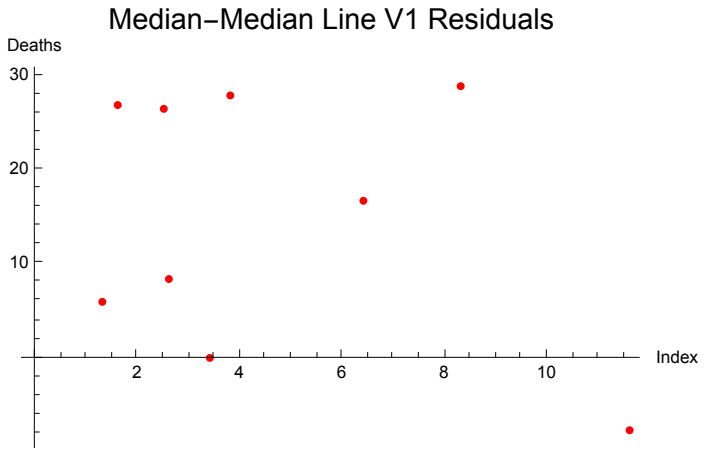



```
(*The blue points mean the first part,
yellow points mean the second part, and green points mean the third
 part. The purple points means the summary points for each part. *)
```

6. Median-Median Line V1 Residuals

```
In[•]:= predictedy1 = f1[xcoords];

In[•]:= resy1 = ycoords - predictedy1;

In[•]:= res2 = ListPlot[Transpose[{xcoords, resy1}],
          PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
          PlotLabel → Style["Median-Median Line V1 Residuals", 15, Black],
          ImageSize → Medium, AxesOrigin → {0, 0}]
```

Out[•]=

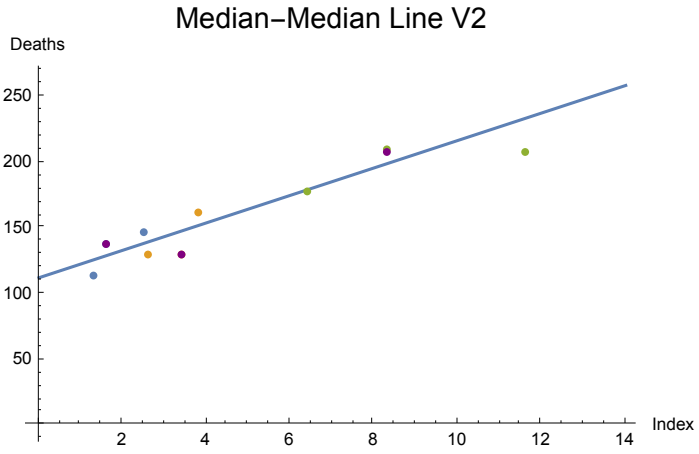



```
(* Below is the value for the sum of residuals *)

In[•]:= ressum1 = Total[resy1]
```

133.119

7. Median-Median Line V2

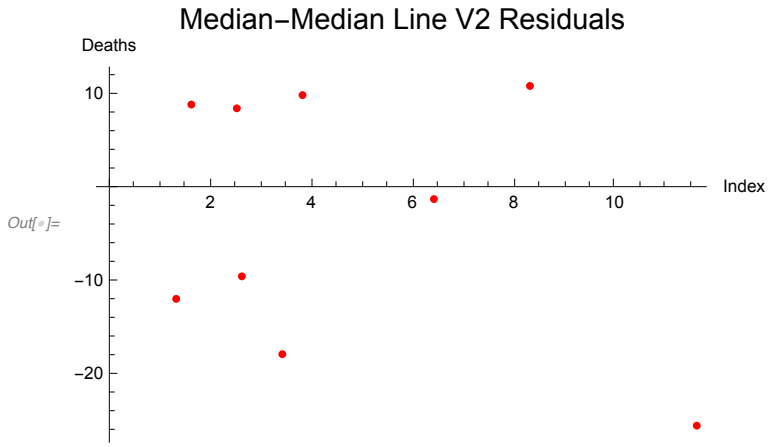
```
In[•]:= Solve[138 == slope * 1.6 + b1, b1]
Out[•]= {{b1 → 121.284}}

In[•]:= b1 = 121.28358208955224`

In[•]:= Solve[208 == slope * 8.3 + b3, b3]
Out[•]= {{b3 → 121.284}}

In[•]:= b3 = 121.28358208955224`

In[•]:= avgb = (b1 + b2 + b3) / 3
Out[•]= 112.348

In[•]:= f2[x_] = slope * x + avgb;

In[•]:= Show[Plot[f2[x], {x, 0, 14}], ListPlot[parts],
         ListPlot[medians, PlotStyle → Purple], PlotRange → All,
         PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
         PlotLabel → Style["Median-Median Line V2", 15, Black],
         ImageSize → Medium, AxesOrigin → {0, 0}]
```



8. Median-Median Line V2 Residuals

```
In[•]:= predictedy2 = f2[xcoords];

In[•]:= resy2 = ycoords - predictedy2;

In[•]:= res3 = ListPlot[Transpose[{xcoords, resy2}],
          PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
          PlotLabel → Style["Median-Median Line V2 Residuals", 15, Black],
          ImageSize → Medium, AxesOrigin → {0, 0}]
```

Out[•]=




```
(* Below is the value for the sum of residuals *)

In[•]:= ressum2 = Total[resy2]
Out[•]= -27.7164
```

9. Least-Squares Regression Line

```
B1 = Total[index^2];
C1 = Total[index];
D1 = Total[deaths * index];
E1 = Total[deaths];
```

In[•]:=
```
n = Length[index];
```

$$m = \frac{D1 * n - E1 * C1}{B1 * n - C1^2};$$

In[•]:=
```
Clear[b]
```

$$\text{Solve}\left[b == \frac{E1 - \frac{(D1 - b*C1)*C1}{B1}}{n}, b\right]$$

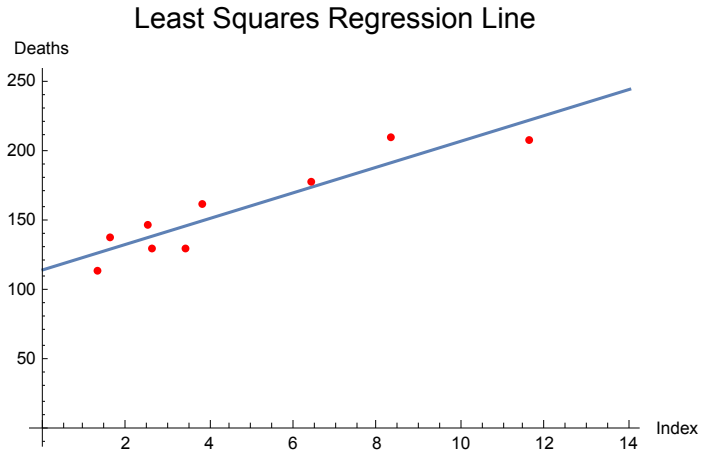
Out[•]= $\{\{b \to 114.682\}\}$

In[•]:=
```
b = 114.6816262488752`
```

In[•]:=
```
f3[x_] = m * x + b
```

Out[•]= $114.682 + 9.27386\ x$

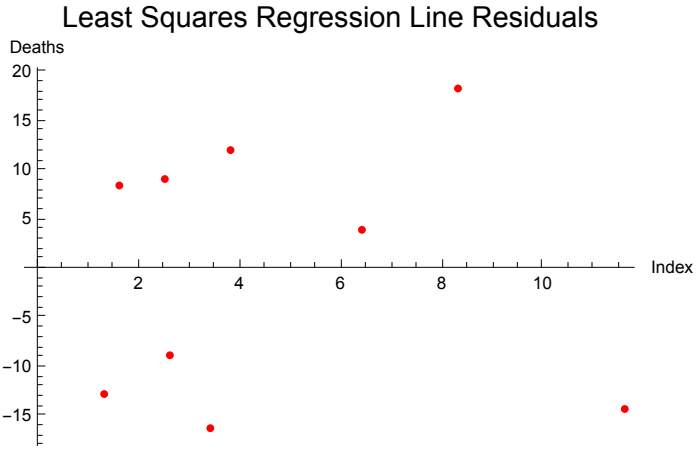
In[•]:=
```
Show[Plot[f3[x], {x, 0, 14}], listplot1, PlotRange → All,
 PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
 PlotLabel → Style["Least Squares Regression Line", 15, Black],
 ImageSize → Medium, AxesOrigin → {0, 0}]
```



10. Least-Squares Regression Line Residuals

```
In[•]:= predictedy3 = f3[xcoords];

In[•]:= resy3 = ycoords - predictedy3;

In[•]:= res4 = ListPlot[Transpose[{xcoords, resy3}],
          PlotStyle → Red, AxesLabel → {"Index", "Deaths"},
          PlotLabel → Style["Least Squares Regression Line Residuals", 15, Black],
          ImageSize → Medium, AxesOrigin → {0, 0}]
```

Out[•]=



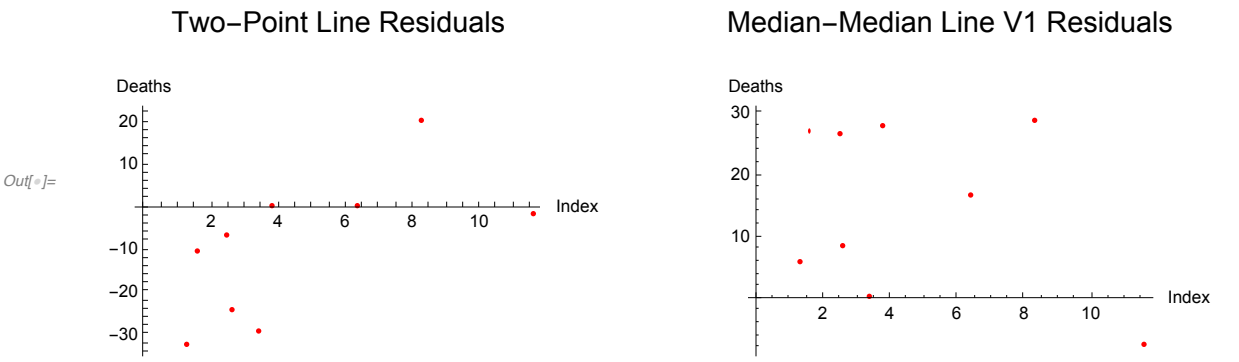

```
(* Below is the value for the sum of residuals *)

In[•]:= ressum3 = Total[resy3]
```

-8.10019×10^{-13}

11. Side By Side Comparison of Residuals

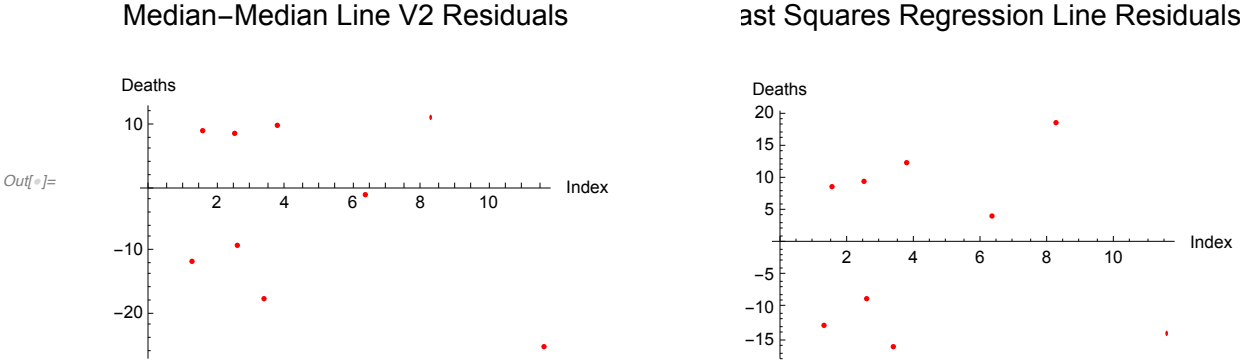
In[•]:=
```
GraphicsRow[{res1, res2}]
```

Out[•]=



In[•]:=
```
GraphicsRow[{res3, res4}]
```

Out[•]=



In[•]:=
```
reslist = {{"Two-Point", ressum},
    {"MM V1", ressum1}, {"MM V2", ressum2}, {"Regression", ressum3}};
```

In[•]:=
```
Text[Grid[Prepend[reslist, {"Type", "Residual Sum"}],
  Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

Out[•]=

Type	Residual Sum
Two−Point	−85.9231
MM V1	133.119
MM V2	−27.7164
Regression	-8.10019×10^{-13}

Thank you