Über Pulley Calculus Problem

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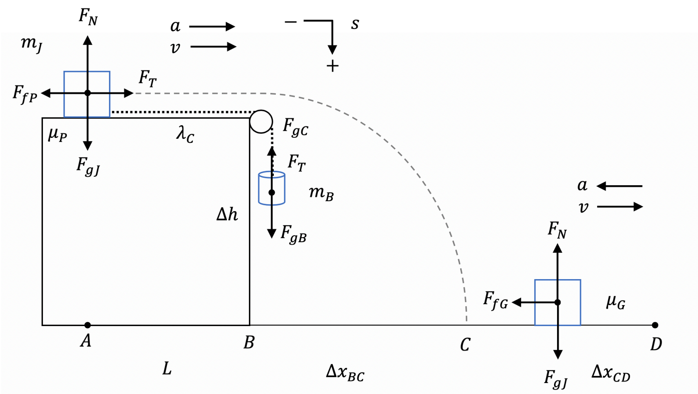
Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

Diagram



Givens

$m_J = 64\ kg$	$\mu_P = 0.12$
$m_B = 152\ kg$	$\Delta x_{BD} = 52\ m$
$m_C = 45\ kg$	$\lambda_c = \frac{m_C}{L}$
$L_C = 11\ m$	$m_{tot} = 261\ kg$
$\Delta h = -20\ m$	$\mu_G = ???$

Strategy

To find the coefficient of friction between the jumper and the ground during segment CD, the problem will be divided into 3 stages: AB, BC, and CD. In Stage AB, Newton's Second Law will be used to calculate the acceleration equation for the jumper. The velocity equation will be found using the acceleration equation, which will allow the launch velocity v_B to be calculated. In Stage BC, the landing velocity v_C and the range Δx_{BC} will be obtained using the kinematics equations and v_B. The landing speed will be multiplied by 0.75 as only 75% of the landing speed transitions into horizontal speed. In Stage CD, μ_G is found from the initial horizontal speed when the jumper hits the ground as well as the acceleration from C to D (found using Newton's Second Law) which values are substituted into a final kinematics equation.

Stage AB

The force equation for the entire system which includes the jumper, chain, and barrel is found using Newton's Second Law. Let x be the length of the chain hanging from the pulley system:

$$\sum F_S: F_{gB} + F_{gC} - F_{fP} = m_{tot}a$$
$$m_B g + \lambda_c x g - \mu_P m_J g = m_{tot}a$$
$$g * (m_B + \frac{m_C}{L}x - \mu_P m_J) = m_{tot}a$$
$$a[x] = {g * (m_B + \frac{m_C}{L}x - \mu_P m_J)}\Big/{m_{tot}}$$

Substituting the given values, the acceleration equation is found:

$$a[x] = {9.8 * (152 + \frac{45}{11}x - 0.12 * 64)}\Big/{261}$$
$$\underline{a[x] = 0.15361x + 5.4189}$$

The velocity equation can now be derived:

$$\int_0^v v dv = \int_0^v a[x]\, dx$$
$$\int_0^v v dv = \int_0^v (0.15361x + 5.4189)\, dx$$
$$v^2/2 = (0.15361/2)x^2 + 5.4189\text{x}$$
$$v = \sqrt{0.15361x^2 + (2 * 5.4189)\text{x}}$$
$$\underline{v[x] = \sqrt{0.15361x^2 + 10.838\text{x}}}$$

The largest x can be is L, the length of the chain. Substituting this value, the horizontal velocity at which Jerry is launched off at the end of the platform can be found:

$$v[x] = \sqrt{0.15361(11)^2 + 10.838(11)}$$
$$v[x] = \sqrt{18.586 + 119.22}$$
$$\underline{v_B = 11.739\ m/s}$$

Stage BC

In order to calculate the landing speed, the final vertical velocity must be calculated first:

$$v_y{}^2 = v_{y0}{}^2 + 2g\Delta h$$
$$v_y{}^2 = 2(-9.8)(-20)$$
$$v_y = \sqrt{392}$$
$$\underline{v_y = -19.799\ m/s}$$

The speed at which Jerry hits the ground at C is obtained:

$$v_C = \sqrt{v_B{}^2 + v_y{}^2}$$
$$v_C = \sqrt{(11.739)^2 + (19.799)^2}$$
$$v_C = \sqrt{137.80 + 392.0}$$
$$v_C = \sqrt{529.80}$$
$$\underline{v_C = 23.017\ m/s}$$

The initial horizontal speed at which the jerry continues on the ground is 75% of the landing speed:

$$v_{x0} = 0.75v_C$$
$$v_{x0} = 0.75 * 23.017$$
$$\underline{v_{x0} = 17.263\ m/s}$$

The total time of Jerry's flight in the air will be calculated:

$$\Delta h = v_{y0}t + \frac{1}{2}gt^2$$
$$-20 = \frac{1}{2}(-9.80)t^2$$
$$20/4.9 = t^2$$
$$t^2 = 4.0816$$
$$\cancel{t_{tot} = -2.0203\text{ s}}\ and\ \underline{t_{tot} = 2.0203\text{ s}}$$

Using this total time, the range Jerry traveled can be found:

$$\Delta x_{BC} = v_x t$$
$$\Delta x_{BC} = 11.739 * 2.0203$$
$$\underline{\Delta x_{BC} = 23.716\ m}$$

Stage CD

In order to find Δx_{CD}, the difference will be taken between Δx_{BD} and Δx_{BC}:

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$
$$\Delta x_{CD} = 52 - 23.716$$
$$\underline{\Delta x_{CD} = 28.284\ m}$$

The force equations in the x and y directions are calculated for the Jumper using Newton's Second Law:

$$\sum F_y: F_N - m_J g = m_J a_y$$
$$\underline{F_N = m_J g}$$

$$\sum F_x: F_{fG} = m_J a_x$$
$$\mu_G F_N = m_J a_x$$
$$\mu_G m_J g = m_J a_x$$
$$\underline{a_x = \mu_G g}$$

Substituting a_x, Δx_{CD}, and v_{x0} into the kinematics equation below, the coefficient of friction between the ground and the jumper is found:

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x_{CD}$$
$$v_x^2 = v_{x0}^2 + 2\mu_G g \Delta x_{CD}$$
$$0 = (17.263)^2 + 2\mu_G(-9.8)(28.284)$$
$$-298.01 = (-554.36) * \mu_G$$

$$\boxed{\mu_G = 0.5376}$$