Mercurial Machine Filter

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1 Introduction

Analysis of the Mercurial Machine filter. Heavily based on the Serge VCFQ, but also with influence from the Mutable Instruments Blades. Much of this is pulled verbatim from the SSM2164 SVF Analysis by Emilie Gillet and then just somewhat modified.

1.1 Notations

R_i is the value of the resistor through which the input signal is fed to the circuit.

R_g is the value of the resistor through which the HP and LP outputs are fed back into the input.

R_q is the value of the resistor through which the attenuated BP output is fed back into the input.

R and R_s are the values of the resistor at the voltage divider input of the OTAs.

R_f is the value of the resistor through which the current at the output of the Q attenuator is converted back into a voltage.

C is the value of the integrators' capacitors.

v_{cv} is the cutoff frequency control voltage.

v_q is the resonance control voltage.

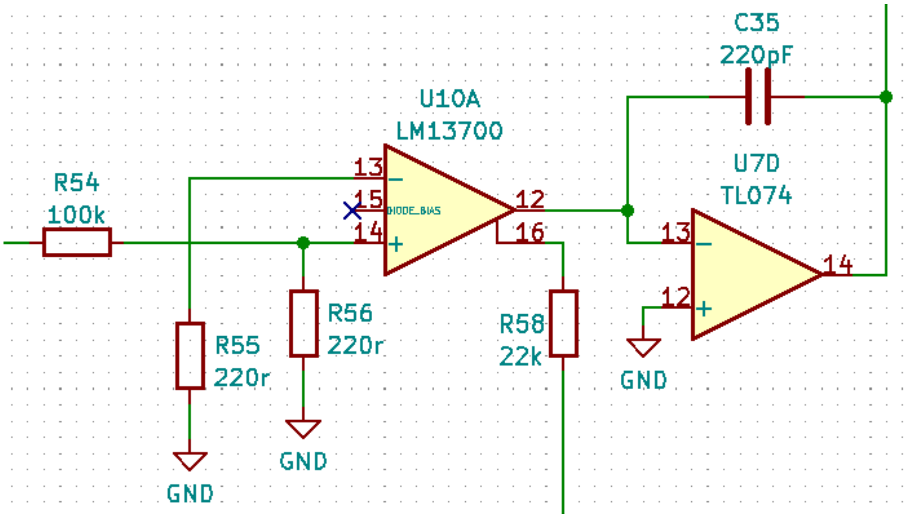
$v_i(s)$ is the input voltage.

$v_{hp}(s)$, $v_{bp}(s)$ and $v_{lp}(s)$ are the high-pass, band-pass and low-pass output voltages.

For reference, the standard second-order filter transfer functions are:

$$H_{lp}(s) = \frac{1}{s^2T^2 + \frac{1}{q}sT + 1}$$
$$H_{bp}(s) = \frac{1}{\frac{q}{sT} + qsT + 1}$$
$$H_{hp}(s) = \frac{1}{\frac{1}{s^2T^2} + \frac{1}{qsT} + 1}$$

1.2 OTA Integrator Cell



The transfer function of an OTA is:

$$i_o = g_m(v_+ - v_-)$$
$$i_o = 19.2 i_{cv}(v_+ - v_-)$$

If R_f is equal to R_i Kirchoff in v_+ gives:

$$\begin{aligned}
\frac{1}{R}(v_i(s) - v_+(s)) &= \frac{1}{R_s}v_+(s) \\
R_s v_i(s) - R_s v_+(s) &= R v_+(s) \\
R_s v_i(s) &= R v_+(s) + R_s v_+(s) \\
R_s v_i(s) &= v_+(s)(R + R_s) \\
v_+(s)(R + R_s) &= R_s v_i(s) \\
v_+(s) &= \frac{R_s}{R + R_s} v_i(s)
\end{aligned}$$

The current i_c at the output if the OTA is:

$$\begin{aligned}
i_c(s) &= g_m(v_+(s) - v_-(s)) \\
&= 19.2 i_{cv}(\frac{R_s}{R + 2R_s} v_i(s) - 0) \\
&= 19.2 i_{cv} \frac{R_s}{R + 2R_s} v_i(s)
\end{aligned}$$

The voltage out of the OpAmp:

$$v_o(s) = \frac{-i_c(s)}{Cs}$$

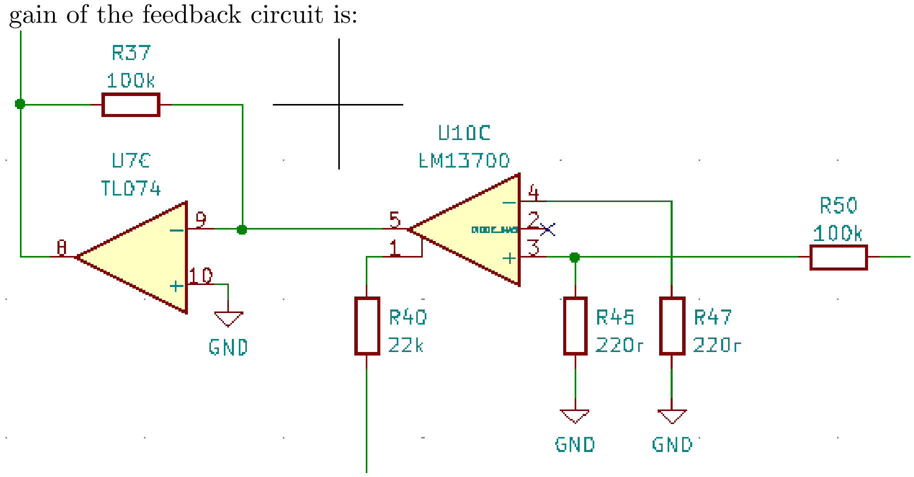
$$v_o(s) = -19.2 i_{cv} \frac{R_s}{Cs(R + 2R_s)} v_i(s)$$

So the transfer function is:

$$\alpha(s) = -19.2 i_{cv} \frac{R_s}{Cs(R + 2R_s)} \tag{1}$$

1.3 Feedback Control

The gain of the feedback circuit is:



$$v_+ = v_i \frac{R_s}{R + R_s}$$

$$i_o = 19.2 i_q (v_+ - v_-)$$

$$i_o = 19.2 i_q (v_+ - 0)$$

$$i_o = 19.2 i_q v_i \frac{R_s}{R + R_s}$$

$$i_o = R_f (v_- - v_o)$$

$$i_o = -R_f v_o$$

$$-R_f v_o = 19.2 i_q v_i \frac{R_s}{R + R_s}$$

$$-R_f v_o = 19.2 i_q v_i \frac{R_s}{R + R_s}$$

$$v_o = -19.2 i_q v_i \frac{R_f R_s}{R_i + R_s}$$

$$\beta = -19.2 i_q \frac{R_f R_s}{R_i + R_s}$$

1.4 Filter

$$\frac{v_i - v_-}{R_i} + \frac{v_{hp} - v_-}{R_g} + \frac{v_{lp} - v_-}{R_g} + \frac{v_{bp}\beta - v_-}{R_q} = i_- = 0$$

$$\frac{v_i}{R_i} + \frac{v_{hp}}{R_g} + \frac{v_{lp}}{R_g} + \frac{v_{bp}\beta}{R_q} = 0$$

It is important to note that $v_{bp}(s) = v_{hp}(s)\alpha(s)$, and $v_{lp}(s) = v_{hp}(s)\alpha(s^2)$.

1.4.1 High-pass

$$\begin{aligned}
\frac{v_{hp}(s)}{R_g} &= -\frac{v_i(s)}{R_i} - \frac{v_{lp}(s)}{R_g} - \frac{v_{bp}(s)\beta}{R_q} \\
\frac{v_{hp}(s)}{R_g} &= -\frac{v_i(s)}{R_i} - \frac{v_{hp}(s)\alpha(s^2)}{R_g} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
\frac{v_i(s)}{R_i} &= -\frac{v_{hp}(s)}{R_g} - \frac{v_{hp}(s)\alpha(s^2)}{R_g} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
H_{hp}(s) &= \frac{v_{hp}(s)}{v_i(s)} \\
\frac{v_i(s)}{R_i} &= -v_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{1}{R_i} &= -H_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{-1}{R_i} &= H_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
H_{hp}(s) &= \frac{-1/R_i}{\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}} \\
&= \frac{-R_g/R_i}{1 + \frac{R_g\alpha(s)\beta}{R_q} + \alpha(s^2)}
\end{aligned}$$

1.4.2 Low-Pass

$$
\begin{aligned}
H_{lp}(s) &= \frac{v_{lp}(s)}{v_i(s)} \\
&= \frac{v_{hp}(s)}{v_i(s)}\alpha^2(s) \\
&= \frac{-R_g/R_i}{\frac{1}{\alpha^2(s)} + \frac{R_g\beta}{R_q\alpha(s)} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{\alpha^2(s)} + \beta\frac{R_g}{R_q}\frac{1}{\alpha(s)} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{(-19.2i_{cv}\frac{R_s}{Cs(R+2R_s)})^2} + (-19.2i_q\frac{R_fR_s}{R_i+R_s})\frac{R_g}{R_q}\frac{1}{-19.2i_{cv}\frac{R_s}{Cs(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{(19.2i_{cv}\frac{R_s}{Cs(R+2R_s)})^2} + 19.2i_q\frac{R_fR_s}{R_i+R_s}\frac{R_g}{R_q}\frac{1}{19.2i_{cv}\frac{R_s}{Cs(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{\frac{1}{s^2}(19.2i_{cv}\frac{R_s}{C(R+2R_s)})^2} + 19.2i_q\frac{R_fR_s}{R_i+R_s}\frac{R_g}{R_q}\frac{1}{19.2i_{cv}\frac{1}{s}\frac{R_s}{C(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{s^2}{(19.2i_{cv}\frac{R_s}{C(R+2R_s)})^2} + 19.2i_q\frac{R_fR_s}{R_i+R_s}\frac{R_g}{R_q}\frac{s}{19.2i_{cv}\frac{R_s}{C(R+2R_s)}} + 1}
\end{aligned}
$$

One simplification worth making is to say that $R_q = R_g$.

By comparing with the standard form of a second order filter transfer function we can work out the following.

Pass-band gain, $-\frac{R_g}{R_i}$

Cutoff frequency, $19.2i_{cv}\frac{R_s}{C2\pi(R+2R_s)}$

Quality factor, $\frac{1}{19.2i_q\frac{R_fR_s}{R_i+R_s}}$

1.5 Calculating cutoff frequencies

Given the calculation for frequency, and picking some standard values we can calculate cutoff for different i_{cv} values.

R is 100k

R_s is 220r

C is 220pF

$$
f = 19.2i_{cv}\frac{220}{220 * 10^{-12} * 2\pi(100000 + 2(220))} \tag{2}
$$

for i_{cv} of 0.5ma $f = 15212Hz$

for i_{cv} of 0.3ma $f = 9127Hz$

for i_{cv} of 0.1ma $f = 3042Hz$

for i_{cv} of 0.05ma $f = 1521Hz$

for i_{cv} of 0.01ma $f = 304Hz$

1.6 Calculating resonance

A quality factor of 1/2 gives no resonance, whilst the resonance (and likelihood of self oscillating) increases as Q goes to infinity.

R_f is 100k

R_i is 100k

R_s is 220r

C is 220pF

$$q = \frac{1}{19.2 i_q \frac{100000*220}{100000+220}} \tag{3}$$

for i_q of 0.5ma $q = 0.47$

for i_q of 0.3ma $q = 0.79$

for i_q of 0.1ma $q = 2.37$

for i_q of 0.05ma $q = 4.75$

for i_q of 0.01ma $q = 23.7$