Incorrect Filter

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May 13, 2023

1 Introduction

Analysis of the original, incorrect design of the Mercurial Machine filter. Heavily based on the Serge VCFQ, but also with influence from the Mutable Instruments Blades. Much of this is pulled verbatim from the SSM2164 SVF Analysis by Emilie Gillet and then just somewhat modified.

1.1 Notations

R_i is the value of the resistor through which the input signal is fed to the circuit.

R_f is the value of the resistor through which the output signal of the integrator is fed back to the input.

R_s is the value of the resistor from the OTA inputs to ground.

C is the value of the integrators' capacitors.

R_g is the value of the resistor through which the HP and LP outputs are fed back into the input.

R_q is the value of the resistor through which the attenuated BP output is fed back into the input.

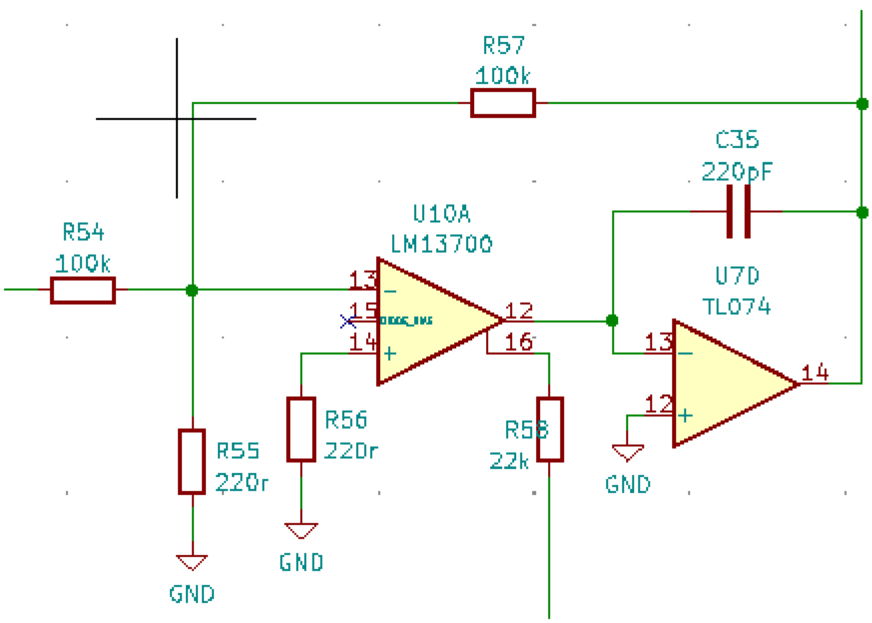
v_{cv} is the cutoff frequency control voltage.

v_q is the resonance control voltage.

$v_i(s)$ is the input voltage.

$v_{hp}(s)$, $v_{bp}(s)$ and $v_{lp}(s)$ are the high-pass, band-pass and low-pass output voltages.

1.2 OTA Integrator Cell



The transfer function of an OTA is:

$$
\begin{aligned}
i_o &= g_m(v_+ - v_-) \\
i_o &= 19.2 i_{cv}(v_+ - v_-)
\end{aligned}
$$

If R_f is equal to R_i Kirchoff in v_- gives:

$$
\begin{aligned}
\frac{1}{R_i}(v_i(s) - v_-(s)) + \frac{1}{R_i}(v_{out}(s) - v_-(s)) &= \frac{1}{R_s} v_-(s) \\
R_s v_i(s) - R_s v_-(s) + R_s v_{out}(s) - R_s v_-(s) &= R_i v_-(s) \\
R_s v_i(s) + R_s v_{out}(s) - R_s v_-(s) - R_s v_-(s) &= R_i v_-(s) \\
R_s v_i(s) + R_s v_{out}(s) - 2R_s v_-(s) &= R_i v_-(s) \\
R_s v_i(s) + R_s v_{out}(s) &= R_i v_-(s) + 2R_s v_-(s) \\
R_s(v_i(s) + v_{out}(s)) &= v_-(s)(R_i + 2R_s) \\
\frac{R_s}{R_i + 2R_s}(v_i(s) + v_{out}(s)) &= v_-(s)
\end{aligned}
$$

The current i_c at the output if the OTA is:

$$
\begin{aligned}
i_c(s) &= g_m(v_+(s) - v_-(s)) \\
&= 19.2 i_{cv}(0 - \frac{R_s}{R_i + 2R_s}(v_i(s) + v_{out}(s))) \\
&= -19.2 i_{cv} \frac{R_s}{R_i + 2R_s}(v_i(s) + v_{out}(s))
\end{aligned}
$$

The voltage out of the OpAmp:

$$
\begin{aligned}
v_{out}(s) &= \frac{-i_c(s)}{Cs} \\
v_{out}(s) &= -(-19.2 i_{cv} \frac{R_s}{Cs(R_i + 2R_s)}(v_i(s) + v_{out}(s))) \\
v_{out}(s) &= 19.2 i_{cv} \frac{R_s}{Cs(R_i + 2R_s)}(v_i(s) + v_{out}(s)) \\
v_{out}(s) Cs(R_i + 2R_s) &= 19.2 i_{cv} R_s (v_i(s) + v_{out}(s)) \\
v_{out}(s) Cs(R_i + 2R_s) &= v_i(s) 19.2 i_{cv} R_s + v_{out}(s) 19.2 i_{cv} R_s \\
v_{out}(s) Cs(R_i + 2R_s) - v_{out}(s) 19.2 i_{cv} R_s &= v_i(s) 19.2 i_{cv} R_s \\
v_{out}(s)(Cs(R_i + 2R_s) - 19.2 \imath_{cv} R_s) &= v_i(s) 19.2 i_{cv} R_s \\
v_{out}(s) &= v_i(s) \frac{19.2 i_{cv} R_s}{Cs(R_i + 2R_s) - 19.2 i_{cv} R_s} \\
v_{out}(s) &= v_i(s) \frac{1}{\frac{(R_i + 2R_s)}{R_s} Cs \frac{1}{19.2 i_{cv}} - 1}
\end{aligned}
$$

So the transfer function is:

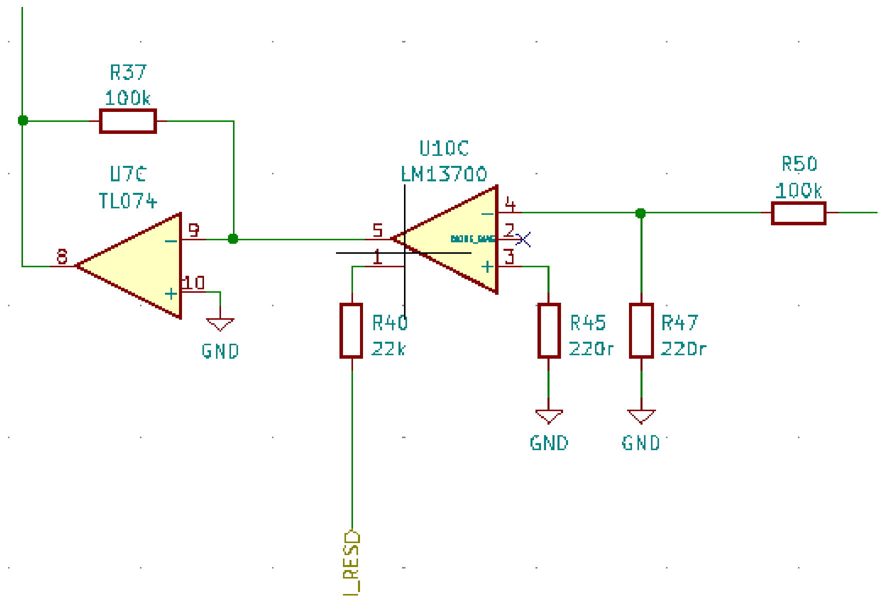
$$\alpha(s) = \frac{1}{\frac{(R_i+2R_s)}{R_s}Cs\frac{1}{19.2i_{cv}} - 1} \tag{1}$$

This should work as a single filter stage in itself, with the cutoff frequency being the point at which the ouput is 3dB lower than the input, or $\frac{1}{\sqrt{2}} \approx 0.707$

$$\begin{aligned}
0.707 &= \frac{1}{\frac{(R_i+2R_s)}{R_s}C2\pi f\frac{1}{19.2i_{cv}} - 1} \\
0.707\frac{(R_i+2R_s)}{R_s}C2\pi f\frac{1}{19.2i_{cv}} - 0.707 &= 1 \\
0.707\frac{(R_i+2R_s)}{R_s}C2\pi f\frac{1}{19.2i_{cv}} &= 1.707 \\
\frac{(R_i+2R_s)}{R_s}C2\pi f\frac{1}{19.2i_{cv}} &= \frac{19.2i_{cv}1.707}{0.707} \\
f\frac{(R_i+2R_s)}{R_s}C2\pi &= 19.2i_{cv}2.414 \\
f &= \frac{19.2i_{cv}2.414}{2\pi\frac{(R_i+2R_s)}{R_s}C} \\
f &= \frac{19.2i_{cv}1.207}{\pi\frac{(R_i+2R_s)}{R_s}C} \\
f &= \frac{23.18i_{cv}}{\pi\frac{(R_i+2R_s)}{R_s}C}
\end{aligned}$$

1.3 Feedback Control

The gain of the feedback circuit is:



$$v_- = v_i \frac{R_s}{R_i + R_s}$$

$$i_o = 19.2 i_{res}(0 - v_-)$$

$$i_o = -19.2 i_{res} v_i \frac{R_s}{R_i + R_s}$$

$$i_o + \frac{v_o}{R_f} = 0$$

$$i_o = -\frac{v_o}{R_f}$$

$$-i_o R_f = v_o$$

$$v_o = -i_o R_f$$

$$v_o = 19.2 i_{res} v_i \frac{R_f R_s}{R_i + R_s}$$

$$v_o = 19.2 i_{res} v_i \frac{R_f R_s}{R_i + R_s}$$

$$\beta = 19.2 i_{res} \frac{R_f R_s}{R_i + R_s}$$

1.4 Filter

$$\frac{v_i - v_-}{R_i} + \frac{v_{hp} - v_-}{R_f} + \frac{v_{lp} - v_-}{R_f} + \frac{v_{bp}\beta - v_-}{R_q} = i_- = 0$$
$$\frac{v_i}{R_i} + \frac{v_{hp}}{R_f} + \frac{v_{lp}}{R_f} + \frac{v_{bp}\beta}{R_q} = 0$$

It is important to note that $v_{bp}(s) = v_{hp}(s)\alpha(s)$, and $v_{lp}(s) = v_{hp}(s)\alpha(s^2)$.

1.4.1 High-pass

$$\begin{aligned}
\frac{v_{hp}(s)}{R_f} &= -\frac{v_i(s)}{R_i} - \frac{v_{lp}(s)}{R_f} - \frac{v_{bp}(s)\beta}{R_q} \\
\frac{v_{hp}(s)}{R_f} &= -\frac{v_i(s)}{R_i} - \frac{v_{hp}(s)\alpha(s^2)}{R_f} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
H_{hp}(s) &= \frac{v_{hp}(s)}{v_i(s)} \\
\frac{v_i(s)}{R_i} &= -\frac{v_{hp}(s)}{R_f} - \frac{v_{hp}(s)\alpha(s^2)}{R_f} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
\frac{v_i(s)}{R_i} &= -v_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{1}{R_i} &= -H_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{-1}{R_i} &= H_{hp}(s)(\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\alpha(s)\beta}{R_q}) \\
H_{hp}(s) &= \frac{-1/R_i}{\frac{1}{R_f} + \frac{\alpha(s^2)}{R_f} + \frac{\alpha(s)\beta}{R_q}} \\
&= \frac{-R_f/R_i}{1 + \frac{R_f\alpha(s)\beta}{R_q} + \alpha(s^2)}
\end{aligned}$$

1.4.2 Low-Pass

$$
\begin{aligned}
H_{lp}(s) &= \frac{v_{lp}(s)}{v_i(s)} \\
&= \frac{v_{hp}(s)}{v_i(s)}\alpha^2(s) \\
&= \frac{-}{\frac{1}{\alpha^2(s)} + \frac{R_f\beta}{R_q\alpha(s)} + 1} \\
&= \frac{-R_f/R_i}{\frac{1}{\alpha^2(s)} + \beta\frac{R_f}{R_q}\frac{1}{\alpha(s)} + 1} \\
&= \frac{-R_f/R_i}{\frac{1}{(\frac{1}{\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}}-1})^2} + (19.2i_{res}\frac{R_fR_s}{R_i+R_s})\frac{R_f}{R_q}\frac{1}{(\frac{1}{\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}}-1})} + 1} \\
&= \frac{-R_f/R_i}{(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}} - 1)^2 + (19.2i_{res}\frac{R_fR_s}{R_i+R_s})\frac{R_f}{R_q}(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}} - 1) + 1} \\
&= \frac{-R_f/R_i}{(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}})^2 - 2(19.2i_{res}\frac{R_fR_s}{R_i+R_s}) + 1 + (19.2i_{res}\frac{R_fR_s}{R_i+R_s}\frac{R_f}{R_q})(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}} - 1) + 1} \\
&= \frac{-R_f/R_i}{(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}})^2 - (2+\frac{R_f}{R_q})(19.2i_{res}\frac{R_fR_s}{R_i+R_s}) + 19.2i_{res}\frac{R_fR_s}{R_i+R_s}\frac{R_f}{R_q}\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}} + 2} \\
&= \frac{-R_f/R_i}{(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}})^2 + 19.2i_{res}\frac{R_fR_s}{R_i+R_s}\frac{R_f}{R_q}(\frac{R_i+2R_s}{R_s}Cs\frac{1}{19.2i_{cv}}) - (2+\frac{R_f}{R_q})(19.2i_{res}\frac{R_fR_s}{R_i+R_s}) + 2}
\end{aligned}
$$

At this point it seems clear that this doesn't fit the standard transfer function for a second order filter....