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$$U = \text{LIN} \left\{ \begin{pmatrix} -2 \\ -3 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -2 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ 9 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ -6 \\ -1 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

gesucht: Basis von  $U$

$$(A, 0) = \begin{pmatrix} -2 & 4 & -7 & 9 & 0 & 0 \\ -3 & 6 & 9 & -6 & 0 & 0 \\ 1 & -2 & 0 & -1 & 0 & 0 \\ 4 & -8 & 1 & -5 & 0 & 0 \end{pmatrix}$$

$$I \leftrightarrow III$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 0 & -1 & 0 & 0 \\ -3 & 6 & 9 & -6 & 0 & 0 \\ -2 & 4 & -7 & 9 & 0 & 0 \\ 4 & -8 & 1 & -5 & 0 & 0 \end{pmatrix}$$

$$3 \cdot I + II = II$$

$$2 \cdot I + III = III$$

$$-4 \cdot I + IV = IV$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 9 & -9 & 0 & 0 \\ 0 & 0 & -7 & 7 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix}$$

$$II_S \leftrightarrow IV_S$$

$$\longrightarrow \begin{pmatrix} 1 & -1 & 0 & -2 & 0 & 0 \\ 0 & -9 & 9 & 0 & 0 & 0 \\ 0 & 7 & -7 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$-\frac{1}{9} \cdot II = II$$

$$\longrightarrow \begin{pmatrix} 1 & -1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 7 & -7 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$-7 \cdot II + III = III$$

$$II + IV = IV$$

$$\longrightarrow \begin{pmatrix} 1 & -1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$II + I = I$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & -1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n = 4$$

$$rg(A) = rg(A, 0) = 2$$

$$\Rightarrow \left( \begin{pmatrix} -2 \\ -3 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 9 \\ -6 \\ -1 \\ -5 \end{pmatrix} \right) \text{ ist Basis von } U$$