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$$f : \mathbb{R}^4 \rightarrow \mathbb{R}^4 : \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \rightarrow \begin{pmatrix} d \\ 2c - d \\ 3a - 4c + 3d \\ 6a + 4b - 6c + 3d \end{pmatrix}$$

$$v = \begin{pmatrix} 4 \\ 2 \\ 3 \\ 8 \end{pmatrix}$$

$$A_f = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 3 & 0 & -4 & 3 \\ 6 & 4 & -6 & 3 \end{pmatrix}$$

$$(A_f, b) = \begin{pmatrix} 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & -1 & 2 \\ 3 & 0 & -4 & 3 & 3 \\ 6 & 4 & -6 & 3 & 8 \end{pmatrix}$$

$$I \leftrightarrow III$$

$$\rightarrow \begin{pmatrix} 3 & 0 & -4 & 3 & 3 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 6 & 4 & -6 & 3 & 8 \end{pmatrix}$$

$$\frac{1}{3} \cdot I = I$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 6 & 4 & -6 & 3 & 8 \end{pmatrix}$$

$$-6 \cdot I + IV = IV$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 4 & 2 & -3 & 2 \end{pmatrix}$$

$$II \leftrightarrow IV$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 4 & 2 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & -1 & 2 \end{pmatrix}$$

$$\frac{1}{4} \cdot II = II$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4} & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & -1 & 2 \end{pmatrix}$$

$$III \leftrightarrow IV$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4} & \frac{1}{2} \\ 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\frac{1}{2} \cdot III = III$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 1 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

(Halbdiagonalform: $rg(A_f) = 4 = n \Rightarrow$ eine einzige Lösung)

$$\frac{4}{3} \cdot III + I = I$$

$$-\frac{1}{2} \cdot III + II = II$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & \frac{7}{3} \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

$$-\frac{1}{3} \cdot IV + I = I$$

$$\frac{1}{2} \cdot IV + II = II$$

$$\frac{1}{2} \cdot IV + III = III$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

(Gaussnormalform)

$$L\ddot{O}S(A_f, v) = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right\}$$