EXAM 2 NAME: ______________________

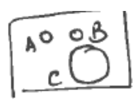
Engineering Analysis Course No: **ECE 5371** *Instructor: Ranadip Pal*
Total Points: 100

<u>Question 1</u> (15 points)

Consider a probabilistic model with a sample space Ω, a collection of events that are subsets of Ω, and a probability law P() defined on the collection of events—all exactly as usual. Let A, B and C be events. (provide short explanations for your selections)

(a) If P(A) + P(B) ≥ P(C) then (A U B) ⊇ C. True **False**

False.



(b) Assuming P(B) >0, P(A|B) is at least as large as P(A) or P(B). True **False**

False. if $A = B^c$ $P(A|B) = 0$

But $P(A) = 1 - P(B) > 0$ for $0 < B < 1$

Now let X and Y be random variables defined on the same probability space Ω as above.
(c) If E[X] > 0 and E[Y] > 0 then Covariance(X,Y) > 0 True **False**
X = 5 -4
Y=-4 5
E[X]=1/2, E[Y]=1/2 but cov(XY) = -40 – 1/4

(d) If for some constant *c* we have P({X>*c*})= 1/ 2 and P({X≤ 0}) = 0, then E[X]>= *c*/2 **True** False

$E[x] \geq c/2$ as $P(X > c) = 1/2$

and thus $E(x) = \int_c^{\infty} f_x(x) x\,dx + \int_0^c f_x(x) x\,dx$

$\geq c \int_c f_x(x)dx = c/2$ non negative

Let X and Y be uniformly distributed continuous random variables over interval [0,1].
(e) The random variable Z=max(X,Y) will be distributed over the interval [0 1] as 2Z **True** False

FZ(z) = P(max(X,Y)<=z)=P(X<=z,Y<=z)=z^2, fZ(z)= derivative of FZ(z)=2z

Question 2 (15 points)

Let X denote the voltage across a resistor with distribution $f_X(x)$ with X ranging from 0 to 1 volt. You design a new modulation scheme that generates a signal Y that is related to X using the following function $Y = e^X + 1$. Derive the distribution of Y in terms of the distribution of X?

$$Y = e^x + 1 \qquad e^x = Y - 1$$

$$F_Y(y) = P(e^x + 1 \le y)$$

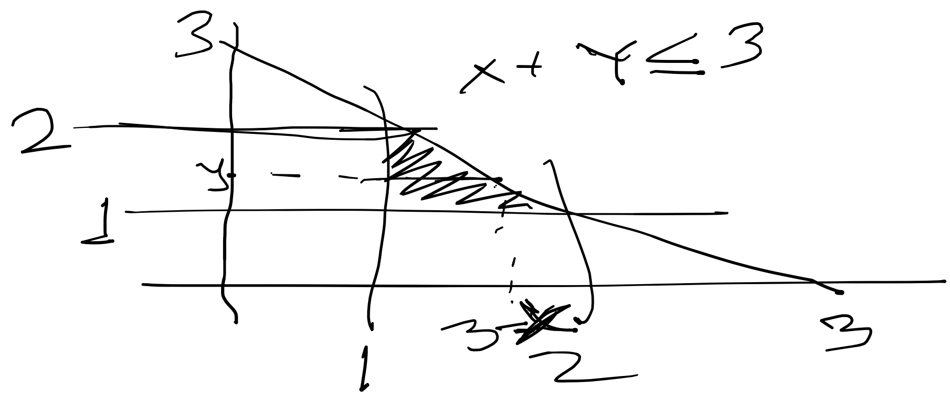
$$\Rightarrow \quad P(x \le \log(y-1))$$

$$f_Y(y) = \frac{\partial F_Y(y)}{dy} = f_X(\log(y-1)) \times \frac{1}{y-1}$$

for $1+e \ge y \ge 2$

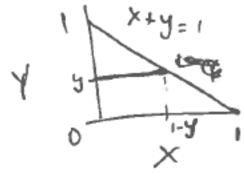
Question 3 (15 points)

Let (X; Y) be a point picked uniformly in the triangle f(x; y) |x >= 1; y>= 1; x + y <= 3|. Find E[X | Y].



$$E[X|Y] = \frac{3 - Y + 1}{2} = 2 - \frac{Y}{2}$$

for a given y,
x is Uniform in the line 0 to 1-y



$$\boxed{E[X\mid Y] = \frac{1-Y}{2}}$$

Question 4 (15 points)

Let $X_1; X_2; X_3$ be i.i.d. Uniform [0; 1] and $Y = \max\{X_1;X_2, X_3\}$. Calculate $E[X_1 \mid Y]$.

X1 is Y with probability 1/3 or between 0 and Y with probability 2/3.
*Thus E[X1|Y]= Y/3 +2/3*Y/2= 2Y/3*

Question 5 (5+5 +5 = 15 points)

In analyzing switching between different engineering degree programs for freshman, survey data has been used to estimate the following transition matrix for the probability of moving between departments each month:

To Department

From D

	D1	D2	D3
D1	0.8	0.1	0.1
D2	0.1	0.8	0.1
D3	0.1	0.3	0.6

The current (month 1) share of students for the department are 50%, 30% and 20% for departments D1, D2, and D3 respectively.

(1) What will be the expected share of students for the departments after two months have elapsed (i.e. in month 3)?

P =

	D1	D2	D3
D1	0.8	0.1	0.1
D2	0.1	0.8	0.1
D3	0.1	0.3	0.6

$\pi_1 = [.5 \quad .3 \quad .2]$ $\pi_1 P \cdot P = [.415 \quad .385 \quad .2]$

$\pi_1 P = [.45 \quad .35 \quad .2]$

(2) What is the long-run prediction for the expected share for each of the three departments? Would you expect the actual share to approach the long-run prediction or not (and why)?

$[.3333 \quad .4667 \quad .2]$

$\pi P = \pi$

(3) Department D3 wants to improve their chances by providing scholarship incentive. They can change the transition probabilities only for the month for which they give the incentive and the transition probabilities revert back to the original for the next month. If the incentive changes the probabilities as follows and D3 can provide the incentive in only one month, should they provide the incentive in the first month or the second month so as to increase the expected number of students in D3 after two months have elapsed (and why)?

P2

	D1	D2	D3
D1	0.8	0.1	0.1
D2	0.1	0.8	0.1
D3	0.1	0.1	0.8

$$\pi_1 P_1 P_2 = \cancel{[0.3333 \quad .4267]} = [.415 \quad .345 \quad .24]$$

$$\pi_1 P_2 P_1 = [.415 \quad .365 \quad .22]$$

Thus P_1 then P_2 is better

(**Question 6**) (15 points)

Suppose you are playing roulette with a biased wheel, such that your chance of winning on each spin is p < 1/2. Suppose that you start out with x dollars, and that on each play, you bet 1-2p of what you have. Assuming that when you win a round you win what you bet, and that otherwise you lose what you bet, find your expected holdings after n rounds of play.

Solution:

If you start with X, then after that round you have

You bet (1-2p)*X and keep the remaining 2p*X. After the game,
you have (1-2p)*X*2*p + 0 +(2p)X = 4p(1-p))X.

Thus, after n rounds, if you started with X, you have $(4p(1-p))^n X$