# biostatistical methods homework 4

library(tidyverse)
library(knitr)
library(patchwork)
library(readxl)

# Problem1

(a)

$$b_{1} = \frac{n \sum X_{i}Y_{i} - \sum X_{i} \sum Y_{i}}{n \sum X_{i}^{2} - (\sum X_{i})^{2}}$$

$$= \frac{\sum X_{i}Y_{i} - n\bar{Y}\bar{X}}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$b_{0} = \bar{Y} - b_{1}\bar{X}$$

$$\sum X_{i}Y_{i} - n\bar{Y}\bar{X} = \sum X_{i}Y_{i} - \bar{X} \sum Y_{i}$$

$$= \sum (X_{i} - \bar{X})Y_{i}$$

$$E\left\{\sum (X_{i} - \bar{X})Y_{i}\right\} = \sum (X_{i} - \bar{X})E(Y_{i})$$

$$= \sum (X_{i} - \bar{X})(\beta_{0} + \beta_{1}X_{i})$$

$$= \beta_{0} \sum X_{i} - n\bar{X}\beta_{0} + \beta_{1} \sum X_{i}^{2} - n\bar{X}^{2}\beta_{1}$$

$$= \beta_{1}(\sum X_{i}^{2} - n\bar{X}^{2})$$

$$E(b_{1}) = \frac{E\left\{\sum (X_{i} - \bar{X})Y_{i}\right\}}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$= \frac{\beta_{1}(\sum X_{i}^{2} - n\bar{X}^{2})}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$= \beta_{1}$$

$$E(b_{0}) = E(\bar{Y} - b_{1}\bar{X})$$

$$= \frac{1}{n} \sum E(Y_{i}) - E(b_{1})\bar{X}$$

$$= \frac{1}{n} \sum [\beta_{0} + \beta_{1}X_{i}] - \beta_{1}\bar{X}$$

$$= \frac{1}{n} [n\beta_{0} + n\beta_{1}\bar{X}] - \beta_{1}\bar{X}$$

(b)

$$Y_i = \hat{\beta}_1 X_i + \hat{\beta}_0$$
$$= \hat{\beta}_1 X_i + \bar{Y} - \hat{\beta}_1 \bar{X}$$
$$X_i = \bar{X}$$

$$X_i = X$$

$$Y_i = \hat{\beta}_1 \bar{X} + \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= \bar{Y}$$

So the Least Square line equation always goes through the point  $(\bar{X}, \bar{Y})$ 

(c)

$$log_{e}L = -\frac{n}{2}log_{e}2\pi - \frac{n}{2}log_{e}\sigma^{2} - \frac{1}{2\sigma^{2}}\sum(Y_{i} - \beta_{0} - \beta_{1}X_{i})^{2}$$

$$\frac{\partial(log_{e}L)}{\partial\sigma^{2}} = -\frac{n}{2\sigma^{2}} + \frac{1}{2\sigma^{4}}\sum(Y_{i} - \beta_{0} - \beta_{1}X_{i})^{2}$$

$$\hat{\sigma}^{2} = \frac{\sum(Y_{i} - \hat{\beta}_{0} - \hat{\beta}_{1}X_{i})^{2}}{n}$$

$$= \frac{\sum(Y_{i} - \hat{Y}_{i})^{2}}{n}$$

Find its expected value

$$E(\hat{\sigma}^2) = E\left(\frac{SSE}{n}\right)$$

$$= E\left(\frac{SSE}{n-2} \times \frac{n-2}{n}\right)$$

$$= \frac{n-2}{n} \times E\left(\frac{SSE}{n-2}\right)$$

$$= \frac{n-2}{n}\sigma^2$$

#### Comment on the unbiasness property

As the result shown above,  $\hat{\sigma}^2$  is a biasd estimator of  $\sigma^2$  as the unbiased estimator of  $\sigma^2$  is MSE:

$$s^{2} = MSE = \frac{SSE}{n-2} = \frac{\sum (Y_{i} - \hat{Y}_{i})^{2}}{n-2} = \frac{\sum e_{i}^{2}}{n-2}$$

$$E\{MSE\} = \sigma^{2}$$

# Problem 2

For this problem, you will be using data 'HeartDisease.csv'.

```
heart_data = read_csv("./data/HeartDisease.csv")
```

The investigator is mainly interested if there is an association between 'total cost' (in dollars) of patients diagnosed with heart disease and the 'number of emergency room (ER) visits'.

Further, the model will need to be adjusted for other factors, including 'age', 'gender', 'number of complications' that arose during treatment, and 'duration of treatment condition'.

# a)

Provide a short description of the data set: what is the main outcome, main predictor and other important covariates.

This dataset include 10 variables and 788 observations. The main outcome is totalcost which represents the total cost (in dollars) of heart-diseased patients. The main predictor is the ERvisits which represents the number of emergency room (ER) visits. Other important covariates are age, gender, complications and duration.

Also, generate appropriate descriptive statistics for all variables of interest (continuous and categorical) – no test required.

```
mean_and_sd = function(x) {

if (!is.numeric(x)) {
    stop("Argument x should be numeric")
} else if (length(x) == 1) {
    stop("Cannot be computed for length 1 vectors")
}

mean_x = mean(x)
    sd_x = sd(x)

tibble(
    mean = mean_x,
    sd = sd_x
)
}
```

### totalcost

## 1 2800. 6690.

```
mean_and_sd(heart_data$totalcost)

## # A tibble: 1 x 2

## mean sd

## <dbl> <dbl>
```

The mean of the total cost is about 2800 with a standard deviation of 6690.26.

#### **ERvisits**

```
summary(heart_data$ERvisits)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.000 2.000 3.000 3.425 5.000 20.000
```

The minimum number of emergency room (ER) visits is 0 and the maximum is 20. The median is 3 with 1st Qu. of 2 and 3rd Qu. of 5.

#### age

```
mean_and_sd(heart_data$age)
```

```
## # A tibble: 1 x 2
## mean sd
## <dbl> <dbl>
## 1 58.7 6.75
```

The distribution of age is centered at about 59 with a standard deviation of 6.75.

#### gender

```
(summary(as.factor(heart_data$gender)))
## 0 1
## 608 180
```

As 0 represents female and 1 represents male, there are 608 female and 180 male in the dataset.

#### complications

```
(summary(as.factor(heart_data$complications)))
## 0 1 3
## 745 42 1
```

As we observed from the dataset, there number of complications existing in this dataset is simply 0, 1 and 3. Using summary function, we can conclude that there are 745 patients have zero complications and 42 patients have one complications, and there is only 1 patient has 3 complication.

#### duration

## 1 164. 121.

```
mean_and_sd(heart_data$duration)

## # A tibble: 1 x 2

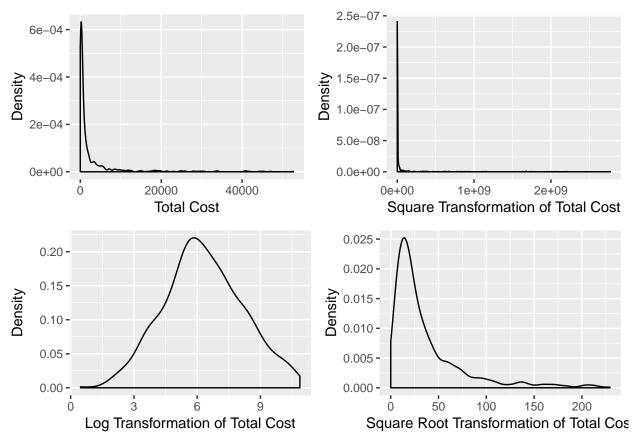
## mean sd
## <dbl> <dbl>
```

The average duration of treatment condition is 164 with a standard deviation of 121.

b)

```
totalcost_non = heart_data %>%
  ggplot(aes(x = totalcost)) +
  geom_density() +
  labs(
       x = 'Total Cost',
       y = 'Density'
totalcost_sq = heart_data %>%
  ggplot(aes(x = (totalcost)^2)) +
  geom_density() +
  labs(
       x = 'Square Transformation of Total Cost',
       y = 'Density'
totalcost_log = heart_data %>%
  ggplot(aes(x = log(totalcost))) +
  geom_density() +
  labs(
       x = 'Log Transformation of Total Cost',
       y = 'Density'
       )
totalcost_sqrt = heart_data %>%
  ggplot(aes(x = sqrt(totalcost))) +
  geom_density() +
  labs(
      x = 'Square Root Transformation of Total Cost',
      y = 'Density'
(totalcost_non + totalcost_sq) / (totalcost_log + totalcost_sqrt)
```

## Warning: Removed 3 rows containing non-finite values (stat\_density).



The shape of the distribution for totalcost is right skewed. After trying different transformation we find that the log transformation makes the plot approximate to normal distribution.

# **c**)

Create a new variable called 'comp\_bin' by dichotomizing 'complications': 0 if no complications, and 1 otherwise.

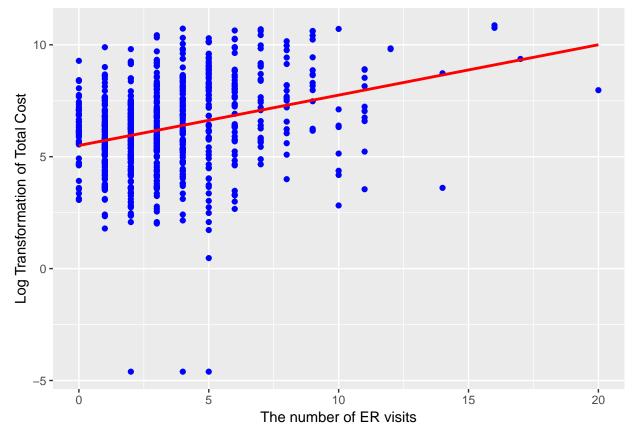
```
heart_data = heart_data %>%
  mutate(comp_bin = ifelse(complications == 0, 0, 1)) %>%
  mutate(comp_bin = as.character(comp_bin))
```

# d)

Based on our decision in part b), fit a simple linear regression (SLR) between the original or transformed 'total cost' and predictor 'ERvisits'. This includes a scatterplot and results of the regression, with appropriate comments on significance and interpretation of the slope.

```
heart_data_trans = heart_data
heart_data_trans$totalcost[heart_data_trans$totalcost==0]=0.01
heart_data_trans = heart_data_trans %>%
    mutate(totalcost = log(totalcost))
```

```
heart_data_trans %>%
    ggplot(aes(x = ERvisits, y = totalcost)) +
    geom_point(color = 'blue') +
    geom_smooth(method = "lm", color = 'red', se = FALSE) +
    labs(
        x = 'The number of ER visits',
        y = 'Log Transformation of Total Cost'
        )
```



```
fit_SLR = lm(totalcost ~ ERvisits, data = heart_data_trans)
summary(fit_SLR)
```

```
##
## Call:
## lm(formula = totalcost ~ ERvisits, data = heart_data_trans)
## Residuals:
##
        Min
                  1Q
                       Median
                                    ЗQ
                                            Max
## -11.2321 -1.1013
                       0.0529
                                1.3055
                                          4.3224
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 5.5016
                            0.1106 49.725
                                             <2e-16 ***
## ERvisits
                 0.2251
                            0.0256
                                     8.792
                                             <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 1.894 on 786 degrees of freedom
## Multiple R-squared: 0.08954, Adjusted R-squared: 0.08838
## F-statistic: 77.3 on 1 and 786 DF, p-value: < 2.2e-16</pre>
```

The plot above shows the scatterplot and results of the regression. Using summary function, we can see that the estimate slope is 0.2251 with a p-value <2e-16, which strongly indicates that the slope is not equal to 0 and there is significant relationship with ERvisits and totalcost. The estimate of slope means that when the number of ER visits increase 1, total cost will increase 25%.

$$\log\left(\frac{Y_2}{Y_1}\right) = \beta_1 = 0.22672$$

$$\frac{Y_2}{Y_1} = \exp^{0.2251} = 1.25$$

$$Y_2 = 1.25Y_1$$

**e**)

Fit a multiple linear regression (MLR) with 'comp\_bin' and 'ERvisits' as predictors.

i)

Test if 'comp\_bin' is an effect modifier of the relationship between 'total cost' and 'ERvisits'. Comment.

```
fit_MLR_interaction = lm(totalcost ~ comp_bin * ERvisits, data = heart_data_trans)
summary(fit_MLR_interaction)
```

```
##
## Call:
## lm(formula = totalcost ~ comp_bin * ERvisits, data = heart_data_trans)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                     3Q
                                             Max
##
  -11.1126 -1.0605
                       0.0257
                                 1.2181
                                          4.4258
##
## Coefficients:
##
                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                       5.46319
                                   0.11072
                                            49.343 < 2e-16 ***
                                   0.58466
## comp_bin1
                       2.21549
                                             3.789 0.000163 ***
                       0.20884
                                   0.02626
                                             7.954 6.32e-15 ***
## ERvisits
## comp_bin1:ERvisits -0.09686
                                   0.10154
                                           -0.954 0.340430
##
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Residual standard error: 1.855 on 784 degrees of freedom
## Multiple R-squared: 0.1291, Adjusted R-squared: 0.1258
## F-statistic: 38.73 on 3 and 784 DF, p-value: < 2.2e-16
```

The definition of modifier is when the magnitude of association differs at different levels of another variable (in this case comp\_bin), it suggests that effect modification is present. From the result shown above, comp\_bin is not a modifier according to the p-value of comp\_bin1:ERvisits is larger than 0.05.

ii)

Test if 'comp\_bin' is a confounder of the relationship between 'total cost' and 'ERvisits'. Comment.

```
lm(totalcost ~ comp_bin + ERvisits, data = heart_data_trans) %>%
   summary()
##
## Call:
```

```
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits, data = heart_data_trans)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                    3Q
                                            Max
##
  -11.1017 -1.0561
                       0.0165
                                1.2104
                                         4.4301
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.48475
                           0.10838
                                    50.607 < 2e-16 ***
                1.73361
                           0.29432
                                     5.890 5.72e-09 ***
## comp_bin1
## ERvisits
                0.20236
                           0.02536
                                     7.979 5.23e-15 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.855 on 785 degrees of freedom
## Multiple R-squared: 0.1281, Adjusted R-squared: 0.1259
## F-statistic: 57.65 on 2 and 785 DF, p-value: < 2.2e-16
```

Using the summary function we can observe that after adding comp\_bin as predictor the adjusted R-squared is increasing comparing with only using ERvisits as predictor. So comp\_bin is a confounder.

#### iii)

Decide if 'comp bin' should be included along with 'ERvisits'. Why or why not?

comp\_bin should be included along with ERvisits according to the test in ii). The p-value of comp\_bin coefficient shows significance. Besides, judging from the adjusted R-squared, when including comp\_bin the value increases comparing with only using ERvisits as predictor. So, comp\_bin should be included along with ERvisits

# f)

Use your choice of model in part e) and add additional covariates (age, gender, and duration of treatment).

#### i)

Fit a MLR, show the regression results and comment.

1Q

Median

Regression model in e):

## Residuals:

Min

##

```
lm(totalcost ~ comp_bin + ERvisits, data = heart_data_trans) %>%
    summary()

##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits, data = heart_data_trans)
##
```

Max

3Q

```
## -11.1017 -1.0561
                      0.0165
                              1.2104
                                        4.4301
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 5.48475
                          0.10838 50.607 < 2e-16 ***
## comp bin1
               1.73361
                          0.29432
                                    5.890 5.72e-09 ***
## ERvisits
               0.20236
                          0.02536
                                    7.979 5.23e-15 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.855 on 785 degrees of freedom
## Multiple R-squared: 0.1281, Adjusted R-squared: 0.1259
## F-statistic: 57.65 on 2 and 785 DF, p-value: < 2.2e-16
lm(totalcost ~ comp_bin + ERvisits + age + gender + duration, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits + age + gender +
       duration, data = heart_data_trans)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -9.9436 -1.0080 -0.0886 0.9771 4.3492
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.8469672 0.5387065 10.854 < 2e-16 ***
## comp_bin1
               1.5258252 0.2728204
                                      5.593 3.09e-08 ***
## ERvisits
                                      7.290 7.58e-13 ***
               0.1736943 0.0238252
              -0.0198581
                          0.0091556 -2.169
                                              0.0304 *
## age
## gender
              -0.2848042 0.1463906 -1.946
                                              0.0521 .
               0.0059649 0.0005159 11.561 < 2e-16 ***
## duration
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.714 on 782 degrees of freedom
## Multiple R-squared: 0.2583, Adjusted R-squared: 0.2536
## F-statistic: 54.47 on 5 and 782 DF, p-value: < 2.2e-16
```

Comment: The result shows that almost all covariates contributed to the model besides gender's coefficient is less significance.

#### ii)

Compare the SLR and MLR models. Which model would you use to address the investigator's objective and why?

```
fit_SLR = lm(totalcost ~ ERvisits, data = heart_data_trans)
fit_MLR = lm(totalcost ~ comp_bin + ERvisits + age + gender + duration, data = heart_data_trans)
anova(fit_SLR, fit_MLR)

## Analysis of Variance Table
##
```

```
## Model 1: totalcost ~ ERvisits
## Model 2: totalcost ~ comp_bin + ERvisits + age + gender + duration
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 786 2820.0
## 2 782 2297.2 4 522.78 44.49 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1</pre>
```

Given the result of anova test, it's obviously that the MLR is prefered. So, I would choose MLR to address the investigator's objective.

# Problem 3 (15p)

A hospital administrator wishes to test the relationship between 'patient's satisfaction' (Y) and 'age', 'severity of illness', and 'anxiety level' (data 'PatSatisfaction.xlsx'). The administrator randomly selected 46 patients, collected the data, and asked for your help with the analysis.

```
sat_data = read_excel("./data/PatSatisfaction.xlsx")

colnames(sat_data)[1] <- "satisfaction"

sat_data = sat_data %>%
    janitor::clean_names()
```

### **a**)

Create a correlation matrix and interpret your initial findings.

```
cor(sat_data, method = "pearson")
##
                satisfaction
                                           severity
                                                       anxiety
                                     age
                   1.0000000 -0.7867555 -0.6029417 -0.6445910
## satisfaction
                  -0.7867555
                              1.0000000
                                         0.5679505
                                                     0.5696775
## age
                                          1.0000000
## severity
                  -0.6029417
                              0.5679505
                                                     0.6705287
## anxiety
                  -0.6445910
                             0.5696775 0.6705287
                                                     1.0000000
```

The result is a table containing the correlation coefficients between each variable and the others. We can observe that age, severity and axiety have negative relationship with satisfaction. Among those three variables, age has the strongest negative relationship with satisfaction.

# b)

Fit a multiple regression model and test whether there is a regression relation. State the hypotheses, decision rule and conclusion.

To build a multiple regression model, we add age, severity and anxiety as predictors:

$$Y_i = \beta_0 + \beta_1 X_i \ _{age} + \beta_2 X_i \ _{anxiety} + \beta_3 X_i \ _{severity} + \varepsilon_i$$

Hypotheses:

$$H_0: \ \beta_1 = \beta_2 = \beta_3 = 0$$
  
 $H_1: \ at \ least \ one \ \beta \ is \ not \ zero$ 

Decision rule:

 $Test\ statistic:$ 

$$F^* = \frac{MSR}{MSE} > F(1-\alpha; p, n-p-1), \ reject \ H_0.$$

 $The \ null \ model \ contains \ only \ the \ intercept:$ 

$$F^* = \frac{MSR}{MSE} \leqslant F(1-\alpha; p, n-p-1), \text{ fail to reject } H_0$$

```
sat_fit = lm(satisfaction ~ age + severity + anxiety, data = sat_data)
summary(sat_fit)
```

```
##
## Call:
## lm(formula = satisfaction ~ age + severity + anxiety, data = sat_data)
##
## Residuals:
       Min
                      Median
                                           Max
                 1Q
                                   3Q
## -18.3524 -6.4230
                      0.5196
                               8.3715 17.1601
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 158.4913
                          18.1259
                                   8.744 5.26e-11 ***
## age
               -1.1416
                           0.2148 -5.315 3.81e-06 ***
## severity
               -0.4420
                           0.4920 - 0.898
                                            0.3741
## anxiety
              -13.4702
                           7.0997 -1.897
                                            0.0647 .
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 10.06 on 42 degrees of freedom
## Multiple R-squared: 0.6822, Adjusted R-squared: 0.6595
## F-statistic: 30.05 on 3 and 42 DF, p-value: 1.542e-10
qf(0.975, 3, 42)
```

## [1] 3.445689

$$F^* = 30.05$$
  
 $F(0.975, 3, 42) = 3.445689$   
 $F^* > F(0.975, 3, 42)$ 

Judging from the p-value and F\*, we reject the null.

 $\mathbf{c}$ 

Show the regression results for all estimated coefficients with 95% CIs. Interpret the coefficient and 95% CI associated with 'severity of illness'.

CI:

```
sat_fit = lm(satisfaction ~ age + severity + anxiety, data = sat_data)
summary(sat_fit)
```

##

```
## Call:
## lm(formula = satisfaction ~ age + severity + anxiety, data = sat_data)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
##
                       0.5196
                                        17.1601
  -18.3524
            -6.4230
                                8.3715
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 158.4913
                           18.1259
                                     8.744 5.26e-11 ***
                -1.1416
                            0.2148
                                    -5.315 3.81e-06 ***
                            0.4920
                                    -0.898
## severity
                -0.4420
                                              0.3741
               -13.4702
                            7.0997
                                    -1.897
                                              0.0647
## anxiety
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.06 on 42 degrees of freedom
## Multiple R-squared: 0.6822, Adjusted R-squared: 0.6595
## F-statistic: 30.05 on 3 and 42 DF, p-value: 1.542e-10
confint(sat_fit)
                    2.5 %
                               97.5 %
## (Intercept) 121.911727 195.0707761
## age
                -1.575093
                           -0.7081303
## severity
                -1.434831
                            0.5508228
## anxiety
               -27.797859
                            0.8575324
```

Interpretation:

From the summary we see that the estimated coefficients of age, severity and anxiety is -1.1416, -0.4420 and -13.4702 respectively. Judging from the p-value, only the coefficients of age shows significance.

The function of confint shows the CIs for each estimated coefficients. From the result we can conclude that we are 95% confidence that the true coefficient of severity will fall within the interval of (-1.434831, 0.5508228).

# d)

Obtain an interval estimate for a new patient's satisfaction when Age=35, Severity=42, Anxiety=2.1. Interpret the interval.

For a given value of x, the interval estimate of the dependent variable y is called the prediction interval.

```
pi_data = data.frame(age = 35, severity = 42, anxiety = 2.1)

predict(sat_fit, pi_data, interval="predict")

## fit lwr upr
## 1 71.68332 50.06237 93.30426
```

The 95% prediction interval of the satisfaction for the age is 35, severity is 42 and anxiety is 2.1 is between 50.06237 and 93.30426. The result means that we are 95% confidence that a future observation of satisfaction for age is 35, severity is 42 and anxiety is 2.1 will be contained within the interval between 50.06237 and 93.30426.

**e**)

Test whether 'anxiety level' can be dropped from the regression model, given the other two covariates are retained. State the hypotheses, decision rule and conclusion. (3p)

The hypothese are as below:

```
\begin{aligned} Model \ 1: \ Y_i &= \beta_0 \ + \ \beta_1 X_i \ _{age} \ + \ \beta_2 X_i \ _{severity} \ + \ \varepsilon_i \\ Model \ 2: \ Y_i &= \beta_0 \ + \ \beta_1 X_i \ _{age} \ + \ \beta_2 X_i \ _{severity} \ + \ \beta_3 X_i \ _{anxiety} \ + \ \varepsilon_i \\ H_0: \beta_3 &= 0 \\ H_1: \beta_3 \neq 0 \end{aligned}
```

```
anova(lm(satisfaction ~ age + severity, data = sat_data),
    lm(satisfaction ~ age + severity + anxiety, data = sat_data))
```

```
## Analysis of Variance Table
##
## Model 1: satisfaction ~ age + severity
## Model 2: satisfaction ~ age + severity + anxiety
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 43 4613.0
## 2 42 4248.8 1 364.16 3.5997 0.06468 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The result shows that the anxiety should NOT be included in the model.