# biostatistical methods homework 4

library(tidyverse)
library(knitr)
library(patchwork)
library(readxl)

## Problem1

(a)

$$b_{1} = \frac{n \sum X_{i}Y_{i} - \sum X_{i} \sum Y_{i}}{n \sum X_{i}^{2} - (\sum X_{i})^{2}}$$

$$= \frac{\sum X_{i}Y_{i} - n\bar{Y}\bar{X}}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$b_{0} = \bar{Y} - b_{1}\bar{X}$$

$$\sum X_{i}Y_{i} - n\bar{Y}\bar{X} = \sum X_{i}Y_{i} - \bar{X} \sum Y_{i}$$

$$= \sum (X_{i} - \bar{X})Y_{i}$$

$$E\left\{\sum (X_{i} - \bar{X})Y_{i}\right\} = \sum (X_{i} - \bar{X})E(Y_{i})$$

$$= \sum (X_{i} - \bar{X})(\beta_{0} + \beta_{1}X_{i})$$

$$= \beta_{0} \sum X_{i} - n\bar{X}\beta_{0} + \beta_{1} \sum X_{i}^{2} - n\bar{X}^{2}\beta_{1}$$

$$= \beta_{1}(\sum X_{i}^{2} - n\bar{X}^{2})$$

$$E(b_{1}) = \frac{E\left\{\sum (X_{i} - \bar{X})Y_{i}\right\}}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$= \frac{\beta_{1}(\sum X_{i}^{2} - n\bar{X}^{2})}{\sum X_{i}^{2} - n\bar{X}^{2}}$$

$$= \beta_{1}$$

$$E(b_{0}) = E(\bar{Y} - b_{1}\bar{X})$$

$$= \frac{1}{n} \sum E(Y_{i}) - E(b_{1})\bar{X}$$

$$= \frac{1}{n} \sum [\beta_{0} + \beta_{1}X_{i}] - \beta_{1}\bar{X}$$

$$= \frac{1}{n} [n\beta_{0} + n\beta_{1}\bar{X}] - \beta_{1}\bar{X}$$

(b)

$$Y_i = \hat{\beta}_1 X_i + \hat{\beta}_0$$
$$= \hat{\beta}_1 X_i + \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\begin{split} X_i &= \bar{X} \\ Y_i &= \hat{\beta}_1 \bar{X} + \bar{Y} - \hat{\beta}_1 \bar{X} \\ &= \bar{Y} \end{split}$$

So the Least Square line equation always goes through the point  $(\bar{X}, \bar{Y})$ 

(c)

$$log_e L = -\frac{n}{2}log_e 2\pi - \frac{n}{2}log_e \sigma^2 - \frac{1}{2\sigma^2} \sum (Y_i - \beta_0 - \beta_1 X_i)^2$$

$$\frac{\partial(log_e L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum (Y_i - \beta_0 - \beta_1 X_i)^2$$

$$\hat{\sigma}^2 = \frac{\sum (Y_i - \hat{\beta_0} - \hat{\beta_1} X_i)^2}{n}$$

$$= \frac{\sum (Y_i - \hat{Y_i})^2}{n}$$

Find its expected value

$$E(\hat{\sigma}^2) = E\left(\frac{SSE}{n}\right)$$

$$= E\left(\frac{SSE}{n-2} \times \frac{n-2}{n}\right)$$

$$= \frac{n-2}{n} \times E\left(\frac{SSE}{n-2}\right)$$

$$= \frac{n-2}{n} \sigma^2$$

#### Comment on the unbiasness property

As the result shown above,  $\hat{\sigma}^2$  is a biasd estimator of  $\sigma^2$  as the unbiased estimator of  $\sigma^2$  is MSE:

$$s^{2} = MSE = \frac{SSE}{n-2} = \frac{\sum (Y_{i} - \hat{Y}_{i})^{2}}{n-2} = \frac{\sum e_{i}^{2}}{n-2}$$

$$E\{MSE\} = \sigma^{2}$$

## Problem 2

For this problem, you will be using data 'HeartDisease.csv'.

```
heart_data = read_csv("./data/HeartDisease.csv")
```

The investigator is mainly interested if there is an association between 'total cost' (in dollars) of patients diagnosed with heart disease and the 'number of emergency room (ER) visits'.

Further, the model will need to be adjusted for other factors, including 'age', 'gender', 'number of complications' that arose during treatment, and 'duration of treatment condition'.

## **a**)

Provide a short description of the data set: what is the main outcome, main predictor and other important covariates.

This dataset include 10 variables and 788 observations. The main outcome is totalcost which represents the total cost (in dollars) of heart-diseased patients. The main predictor is the ERvisits which represents the number of emergency room (ER) visits. Other important covariates are age, gender, complications and duration.

Also, generate appropriate descriptive statistics for all variables of interest (continuous and categorical) – no test required.

```
mean_and_sd = function(x) {
   if (!is.numeric(x)) {
      stop("Argument x should be numeric")
   } else if (length(x) == 1) {
      stop("Cannot be computed for length 1 vectors")
   }
   mean_x = mean(x)
   sd_x = sd(x)

   tibble(
      mean = mean_x,
      sd = sd_x
   )
}
```

#### totalcost

```
mean_and_sd(heart_data$totalcost)
```

```
## # A tibble: 1 x 2

## mean sd

## <dbl> <dbl>

## 1 2800. 6690.
```

The mean of the total cost is about 2800 with a standard deviation of 6690.26.

#### **ERvisits**

```
summary(heart_data$ERvisits)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.000 2.000 3.000 3.425 5.000 20.000
```

The minimum number of emergency room (ER) visits is 0 and the maximum is 20. The median is 3 with 1st Qu. of 2 and 3rd Qu. of 5.

age

```
mean_and_sd(heart_data$age)

## # A tibble: 1 x 2

## mean sd

## <dbl> <dbl>
## 1 58.7 6.75
```

The distribution of age is centered at about 59 with a standard deviation of 6.75.

#### gender

```
(summary(as.factor(heart_data$gender)))
## 0 1
## 608 180
```

As 0 represents female and 1 represents male, there are 608 female and 180 male in the dataset.

#### complications

```
(summary(as.factor(heart_data$complications)))
## 0 1 3
## 745 42 1
```

As we observed from the dataset, there number of complications existing in this dataset is simply 0, 1 and 3. Using summary function, we can conclude that there are 745 patients have zero complications and 42 patients have one complications, and there is only 1 patient has 3 complication.

#### duration

```
mean_and_sd(heart_data$duration)

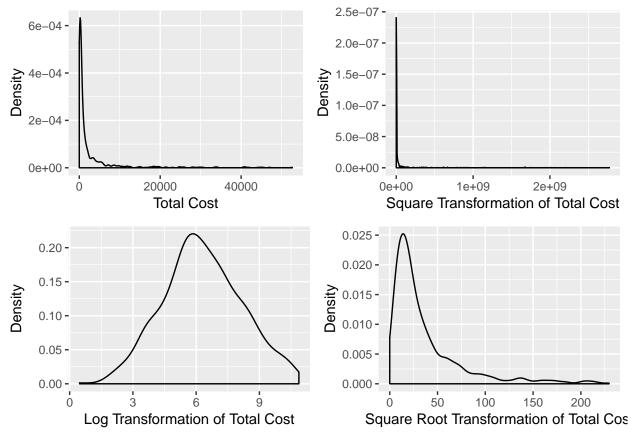
## # A tibble: 1 x 2
## mean sd
## <dbl> <dbl>
## 1 164. 121.
```

The average duration of treatment condition is 164 with a standard deviation of 121.

b)

```
totalcost_non = heart_data %>%
  ggplot(aes(x = totalcost)) +
  geom_density() +
  labs(
      x = 'Total Cost',
       y = 'Density'
       )
totalcost_sq = heart_data %>%
  ggplot(aes(x = (totalcost)^2)) +
  geom_density() +
  labs(
       x = 'Square Transformation of Total Cost',
       y = 'Density'
totalcost_log = heart_data %>%
  ggplot(aes(x = log(totalcost))) +
  geom_density() +
  labs(
       x = 'Log Transformation of Total Cost',
       y = 'Density'
totalcost_sqrt = heart_data %>%
  ggplot(aes(x = sqrt(totalcost))) +
  geom_density() +
  labs(
     x = 'Square Root Transformation of Total Cost',
      y = 'Density'
      )
(totalcost_non + totalcost_sq) / (totalcost_log + totalcost_sqrt)
```

## Warning: Removed 3 rows containing non-finite values (stat\_density).



The shape of the distribution for totalcost is right skewed. After trying different transformation we find that the log transformation makes the plot approximate to normal distribution.

## **c**)

Create a new variable called 'comp\_bin' by dichotomizing 'complications': 0 if no complications, and 1 otherwise.

```
heart_data = heart_data %>%
  mutate(comp_bin = ifelse(complications == 0, 0, 1)) %>%
  mutate(comp_bin = as.character(comp_bin))
```

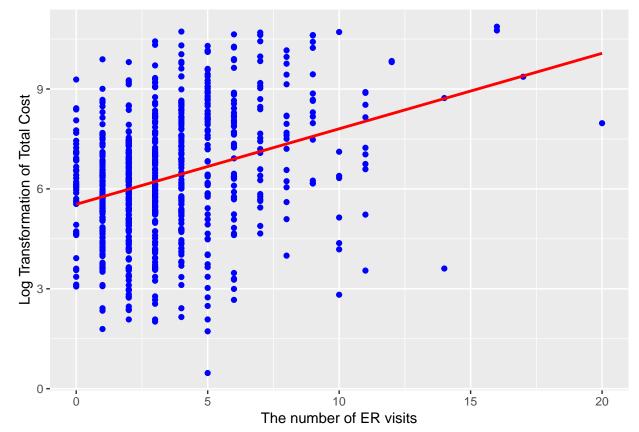
### d)

Based on our decision in part b), fit a simple linear regression (SLR) between the original or transformed 'total cost' and predictor 'ERvisits'. This includes a scatterplot and results of the regression, with appropriate comments on significance and interpretation of the slope.

```
heart_data_trans = heart_data %>%
  mutate(totalcost = log(totalcost)) %>%
  filter(totalcost != -Inf)

heart_data_trans %>%
  ggplot(aes(x = ERvisits, y = totalcost)) +
  geom_point(color = 'blue') +
  geom_smooth(method = "lm", color = 'red', se = FALSE) +
```

```
labs(
    x = 'The number of ER visits',
    y = 'Log Transformation of Total Cost'
)
```



```
fit_SLR = lm(totalcost ~ ERvisits, data = heart_data_trans)
summary(fit_SLR)
```

```
##
## lm(formula = totalcost ~ ERvisits, data = heart_data_trans)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                      Max
   -6.2013 -1.1265 0.0191
                           1.2668
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.53771
                           0.10362
                                     53.44
                                             <2e-16 ***
                0.22672
                           0.02397
                                     9.46
                                             <2e-16 ***
## ERvisits
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.772 on 783 degrees of freedom
## Multiple R-squared: 0.1026, Adjusted R-squared: 0.1014
## F-statistic: 89.5 on 1 and 783 DF, p-value: < 2.2e-16
```

The plot above shows the scatterplot and results of the regression. Using summary function, we can see that

the estimate slope is 0.22672 with a p-value <2e-16, which strongly indicates that the slope is not equal to 0 and there is significant relationship with ERvisits and totalcost. The estimate of slope means that when the number of ER visits increase 1, total cost will increase 25%.

$$log\left(\frac{Y_2}{Y_1}\right) = \beta_1 = 0.22672$$

$$\frac{Y_2}{Y_1} = e^{0.22672} = 1.25$$

$$Y_2 = 1.25Y_1$$

e)

Fit a multiple linear regression (MLR) with 'comp\_bin' and 'ERvisits' as predictors.

i)

Test if 'comp\_bin' is an effect modifier of the relationship between 'total cost' and 'ERvisits'. Comment.

```
fit_MLR_interaction = lm(totalcost ~ comp_bin * ERvisits, data = heart_data_trans)
summary(fit_MLR_interaction)
```

```
##
## Call:
## lm(formula = totalcost ~ comp_bin * ERvisits, data = heart_data_trans)
##
## Residuals:
##
                1Q Median
      Min
                                3Q
                                       Max
   -6.0852 -1.0802 -0.0078 1.1898
##
                                    4.3803
##
## Coefficients:
##
                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                       5.49899
                                  0.10349
                                           53.138 < 2e-16 ***
                       2.17969
                                            3.992 7.17e-05 ***
## comp_bin1
                                  0.54604
## ERvisits
                       0.21125
                                  0.02453
                                            8.610
                                                   < 2e-16 ***
## comp_bin1:ERvisits -0.09927
                                  0.09483
                                           -1.047
                                                     0.296
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.732 on 781 degrees of freedom
## Multiple R-squared: 0.1449, Adjusted R-squared: 0.1417
## F-statistic: 44.13 on 3 and 781 DF, p-value: < 2.2e-16
```

The definition of modifier is when the magnitude of association differs at different levels of another variable (in this case comp\_bin), it suggests that effect modification is present. From the result shown above, comp\_bin is not a modifier according to the p-value of comp\_bin1:ERvisits is larger than 0.05.

ii)

Test if 'comp bin' is a confounder of the relationship between 'total cost' and 'ERvisits'. Comment.

```
lm(totalcost ~ comp_bin + ERvisits, data = heart_data_trans) %>%
summary()
```

```
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits, data = heart_data_trans)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
  -6.0741 -1.0737 -0.0181 1.1810
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 5.5211
                            0.1013
                                    54.495 < 2e-16 ***
                 1.6859
                            0.2749
                                     6.132 1.38e-09 ***
## comp_bin1
## ERvisits
                 0.2046
                            0.0237
                                     8.633 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.732 on 782 degrees of freedom
## Multiple R-squared: 0.1437, Adjusted R-squared: 0.1416
## F-statistic: 65.64 on 2 and 782 DF, p-value: < 2.2e-16
```

Using the summary function we can observe that after adding comp\_bin as predictor the adjusted R-squared is increasing comparing with only using ERvisits as predictor. So comp\_bin is not a confounder.

#### iii)

Decide if 'comp\_bin' should be included along with 'ERvisits'. Why or why not?

comp\_bin should be included along with ERvisits according to the test in ii). The p-value of comp\_bin coefficient shows significance. Besides, judging from the adjusted R-squared, when including comp\_bin the value increases comparing with only using ERvisits as predictor. So, comp\_bin should be included along with ERvisits

### f)

Use your choice of model in part e) and add additional covariates (age, gender, and duration of treatment).

#### i)

Fit a MLR, show the regression results and comment. (5p)

Regression model in e):

```
lm(totalcost ~ comp_bin + ERvisits, data = heart_data_trans) %>%
summary()
```

```
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits, data = heart_data_trans)
##
## Residuals:
## Min    1Q Median    3Q Max
## -6.0741 -1.0737 -0.0181    1.1810    4.3848
##
## Coefficients:
```

```
Estimate Std. Error t value Pr(>|t|)
                            0.1013 54.495 < 2e-16 ***
                 5.5211
## (Intercept)
## comp bin1
                 1.6859
                            0.2749
                                     6.132 1.38e-09 ***
                 0.2046
                            0.0237
## ERvisits
                                     8.633 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.732 on 782 degrees of freedom
## Multiple R-squared: 0.1437, Adjusted R-squared: 0.1416
## F-statistic: 65.64 on 2 and 782 DF, p-value: < 2.2e-16
Add age as a covariate:
lm(totalcost ~ comp_bin + ERvisits + age, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits + age, data = heart_data_trans)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
## -6.0191 -1.0649 0.0098 1.2091 4.3112
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                           0.545373 11.116 < 2e-16 ***
## (Intercept) 6.062580
## comp bin1
                           0.275253
                                    6.076 1.92e-09 ***
               1.672570
## ERvisits
               0.206229
                           0.023755
                                     8.682 < 2e-16 ***
               -0.009302
                           0.009205 -1.010
                                               0.313
## age
## -
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.732 on 781 degrees of freedom
## Multiple R-squared: 0.1449, Adjusted R-squared: 0.1416
## F-statistic: 44.1 on 3 and 781 DF, p-value: < 2.2e-16
Judging from p-value and adjusted R-squared, age should NOT be included in the model.
Add gender as a covariate:
heart_data_trans = heart_data_trans %>%
  mutate(gender = as.character(gender))
lm(totalcost ~ comp_bin + ERvisits + gender, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits + gender, data = heart_data_trans)
## Residuals:
##
       Min
                1Q Median
## -6.0127 -1.0579 -0.0132 1.2074 4.3654
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
```

```
## (Intercept) 5.53470
                           0.10431 53.060 < 2e-16 ***
## comp_bin1
                1.68596
                           0.27506
                                     6.129 1.4e-09 ***
## ERvisits
                0.20604
                           0.02385
                                     8.638 < 2e-16 ***
               -0.08222
                           0.14894
                                    -0.552
                                              0.581
## gender1
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.733 on 781 degrees of freedom
## Multiple R-squared: 0.1441, Adjusted R-squared: 0.1408
## F-statistic: 43.82 on 3 and 781 DF, p-value: < 2.2e-16
Judging from p-value and adjusted R-squared, gender should NOT be included in the model.
Add duration as a covariate:
lm(totalcost ~ comp_bin + ERvisits + duration, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits + duration, data = heart_data_trans)
## Residuals:
##
      Min
                1Q Median
                                30
## -5.1450 -1.1008 -0.1479 0.9593 4.6166
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.7605975 0.1163153 40.928 < 2e-16 ***
                                     5.972 3.56e-09 ***
## comp_bin1
               1.5285357 0.2559535
## ERvisits
               0.1708778
                         0.0222372
                                      7.684 4.62e-14 ***
## duration
              0.0053724 0.0004823 11.140 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.61 on 781 degrees of freedom
## Multiple R-squared: 0.2611, Adjusted R-squared: 0.2583
## F-statistic: 92.01 on 3 and 781 DF, p-value: < 2.2e-16
Judging from p-value and adjusted R-squared, duration should be included in the model.
Test whether duration is an effect modifier:
lm(totalcost ~ comp_bin * duration + ERvisits, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin * duration + ERvisits, data = heart_data_trans)
## Residuals:
                1Q Median
                                3Q
                                       Max
## -5.1300 -1.0881 -0.1405 0.9573 4.6307
##
## Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
```

4.7380358 0.1184976 39.984 < 2e-16 \*\*\*

## (Intercept)

```
## comp bin1
                      1.9552348 0.4987652
                                            3.920 9.63e-05 ***
                      0.0054804 0.0004943 11.087 < 2e-16 ***
## duration
## ERvisits
                       0.1723849 0.0222886
                                             7.734 3.21e-14 ***
## comp_bin1:duration -0.0021385 0.0021454
                                           -0.997
                                                      0.319
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.61 on 780 degrees of freedom
## Multiple R-squared: 0.2621, Adjusted R-squared: 0.2583
## F-statistic: 69.26 on 4 and 780 DF, p-value: < 2.2e-16
lm(totalcost ~ comp_bin + ERvisits * duration, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp_bin + ERvisits * duration, data = heart_data_trans)
## Residuals:
      Min
                1Q Median
                               3Q
                                      Max
## -5.1592 -1.0697 -0.1308 0.9520 4.5454
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                    4.9284866 0.1584830 31.098 < 2e-16 ***
## comp_bin1
                    1.4945266 0.2566500
                                          5.823 8.43e-09 ***
## ERvisits
                    0.1157961 0.0417556
                                          2.773 0.00568 **
## duration
                    0.0044308 0.0007729
                                           5.732 1.42e-08 ***
## ERvisits:duration 0.0002911 0.0001868
                                           1.558 0.11964
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.609 on 780 degrees of freedom
## Multiple R-squared: 0.2634, Adjusted R-squared: 0.2597
## F-statistic: 69.74 on 4 and 780 DF, p-value: < 2.2e-16
lm(totalcost ~ comp_bin * duration + ERvisits * duration, data = heart_data_trans) %>%
  summary()
##
## Call:
## lm(formula = totalcost ~ comp bin * duration + ERvisits * duration,
       data = heart_data_trans)
##
##
## Residuals:
      Min
                1Q Median
                               3Q
## -5.2021 -1.0579 -0.1353 0.9294 4.5541
## Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
                       4.9218489 0.1585081 31.051 < 2e-16 ***
## (Intercept)
## comp_bin1
                       2.0389283 0.5004234
                                             4.074 5.09e-05 ***
                                             5.751 1.27e-08 ***
## duration
                      0.0044442 0.0007727
## ERvisits
                      0.1103900 0.0419569
                                             2.631 0.00868 **
## comp_bin1:duration -0.0027511 0.0021713 -1.267 0.20551
```

```
## duration:ERvisits 0.0003299 0.0001893 1.743 0.08171 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.608 on 779 degrees of freedom
## Multiple R-squared: 0.2649, Adjusted R-squared: 0.2602
## F-statistic: 56.16 on 5 and 779 DF, p-value: < 2.2e-16</pre>
```

Judging from the p-value and adjusted R-squared, duration is NOT an effective modifier.

In conclusion, the final MLR will include comp\_bin, duration and ERvisits:

$$\hat{Y}_{i} = 1.53 X_{i~comp\_bin} ~+~ 0.17 X_{i~ERvisits} ~+~ 0.005 X_{i~duration} ~+~ 4.76$$

ii)

Compare the SLR and MLR models. Which model would you use to address the investigator's objective and why? (2p)

```
fit_SLR = lm(totalcost ~ ERvisits, data = heart_data_trans)
fit_MLR = lm(totalcost ~ comp_bin + ERvisits + duration, data = heart_data_trans)
anova(fit_SLR, fit_MLR)
```

```
## Analysis of Variance Table
##
## Model 1: totalcost ~ ERvisits
## Model 2: totalcost ~ comp_bin + ERvisits + duration
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 783 2459.8
## 2 781 2025.2 2 434.62 83.803 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1</pre>
```

Given the result of anova test, it's obviously that the MLR is prefered. So, I would choose MLR to address the investigator's objective.

# Problem 3 (15p)

A hospital administrator wishes to test the relationship between 'patient's satisfaction' (Y) and 'age', 'severity of illness', and 'anxiety level' (data 'PatSatisfaction.xlsx'). The administrator randomly selected 46 patients, collected the data, and asked for your help with the analysis.

```
sat_data = read_excel("./data/PatSatisfaction.xlsx")

colnames(sat_data)[1] <- "satisfaction"

sat_data = sat_data %>%
    janitor::clean_names()
```

**a**)

Create a correlation matrix and interpret your initial findings. (2p)

## b)

Fit a multiple regression model and test whether there is a regression relation. State the hypotheses, decision rule and conclusion. (3p)

## **c**)

Show the regression results for all estimated coefficients with 95% CIs. Interpret the coefficient and 95% CI associated with 'severity of illness'. (5p)

## d)

Obtain an interval estimate for a new patient's satisfaction when Age=35, Severity=42, Anxiety=2.1. Interpret the interval. (2p)

## **e**)

Test whether 'anxiety level' can be dropped from the regression model, given the other two covariates are retained. State the hypotheses, decision rule and conclusion. (3p)