Lecture 5: Block Ciphers

TTM4135

Relates to Stallings Chapter 4 and 6

Spring Semester, 2025

Motivation

- Block ciphers are the main bulk encryption algorithms used in commercial applications.
- Standardised block cipher AES and legacy cipher DES are widely deployed in real applications.
- NIST's AES algorithm validation list includes over 13500 validated implementations including examples such as USB drives, door controllers, media server encryption, disk encryption, bluetooth devices, iPhone and hundreds more.

Outline

Block ciphers

- Block ciphers are symmetric key ciphers in which each block of plaintext is encrypted with the same key.
- A *block* is a set of plaintext symbols of a fixed size. Typical block sizes for modern block ciphers are between 64 and 256 bits.
- In practice block ciphers are used in certain configurations called *modes of operation*. We look at modes in a later lecture.

Notation in this lecture

- P: Plaintext block (length n bits)
- C: Ciphertext block (length n bits)
- K: Key (length k bits)
- $C = E(P, K)$: Encryption function
- $P = D(C, K)$: Decryption function

Criteria for block cipher design

In the 1940s Claude Shannon discussed two important encryption techniques.

- **Confusion**: This involves substitution to make the relationship between the key and ciphertext as complex as possible.
- **Diffusion**: This involves transformations that dissipate the statistical properties of the plaintext across the ciphertext.

Shannon proposed to use these techniques repeatedly using the concept of *product cipher*.

Outline

Product cipher

- A product cipher is a cryptosystem in which the encryption function is formed by applying (or *composing*) several sub-encryption functions.
- Most block ciphers are the composition of simple functions f_i for $i = 1, \ldots, r$ where each f_i has a different key K_i.
- Thus we can write

$$C = E(P, K) = f_r(\ldots(f_2(f_1(P, K_1), K_2)\ldots), K_r)$$

Iterated ciphers

Most modern block ciphers are in a special class of product ciphers known as *iterated ciphers*.

- The encryption process is divided into r similar *rounds*
- The sub-encryption functions are all the same function, g, called the *round function*
- Each key K_i is derived from the overall master key K. The keys K_i are called *round keys* or *subkeys* and are derived from K using a process called the *key schedule*

Encryption in iterated ciphers

Given a plaintext block, P, a round function g and round keys $K_1, K_2, \ldots, K_r$, the ciphertext block, C, is derived through r rounds as follows.

$$\begin{aligned} W_0 &= P \\ W_1 &= g(W_0, K_1) \\ W_2 &= g(W_1, K_2) \\ &\vdots \quad \vdots \quad \vdots \\ W_r &= g(W_{r-1}, K_r) \\ C &= W_r \end{aligned}$$

Decrypting iterated ciphers

- The round function g must have an inverse function g^{-1} with $g^{-1}(g(W, K_i), K_i) = W$ for all keys K_i and blocks W.
- Decryption is then the reverse of encryption.

$$\begin{aligned} W_r &= C \\ W_{r-1} &= g^{-1}(W_r, K_r) \\ W_{r-2} &= g^{-1}(W_{r-1}, K_{r-1}) \\ \vdots &\ \vdots \ \vdots \\ W_0 &= g^{-1}(W_1, K_1) \\ P &= W_0 \end{aligned}$$

Types of iterated cipher

Two widely used general block cipher designs are:

1. **Feistel ciphers:** an example is the Data Encryption Standard (DES)
2. **Substitution-Permutation Networks (SPNs):** an example is the Advanced Encryption Standard (AES)

Outline

Feistel cipher

- Named after Horst Feistel, a cryptographer working for IBM who influenced the design of DES
- A Feistel cipher is an iterated cipher in which the round function swaps the two halves of the block and forms a new right hand half
- The process is sometimes called a *Feistel network* since the process can be seen as a network which the two halves of the plaintext block travel through.

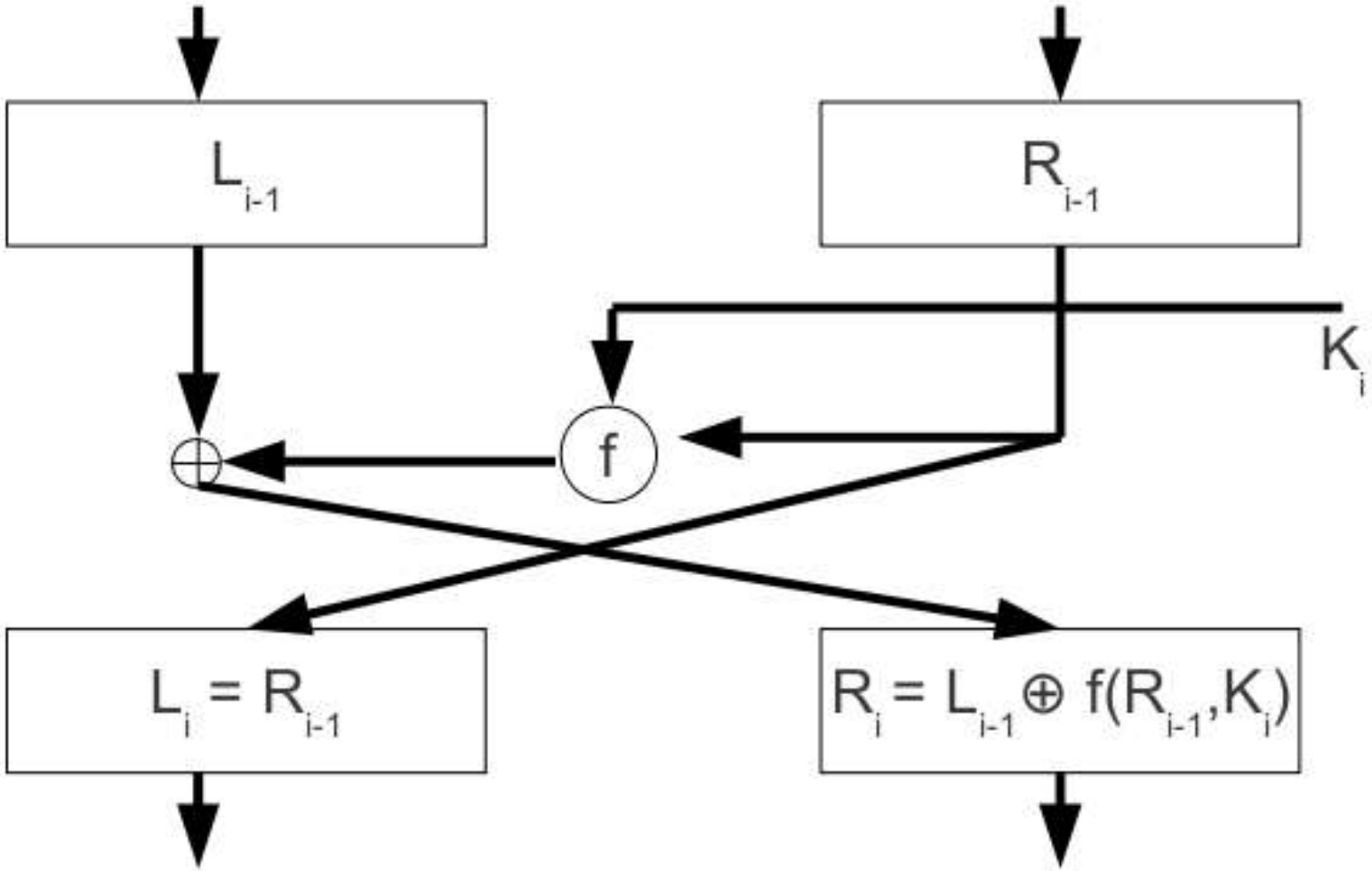
Feistel encryption

1. Split plaintext block $P = W_0$ into two halves: $W_0 = (L_0, R_0)$.
2. For each of the r rounds perform the following:

$$\begin{aligned} L_i &= R_{i-1} \\ R_i &= L_{i-1} \oplus f(R_{i-1}, K_i) \end{aligned}$$

3. Ciphertext $C = W_r$ is defined by $C = (L_r, R_r)$.

Feistel ladder diagram



Feistel ladder diagram

Source: Introduction to Modern Cryptography, Jonathan Katz and Yehuda Lindell, third edition

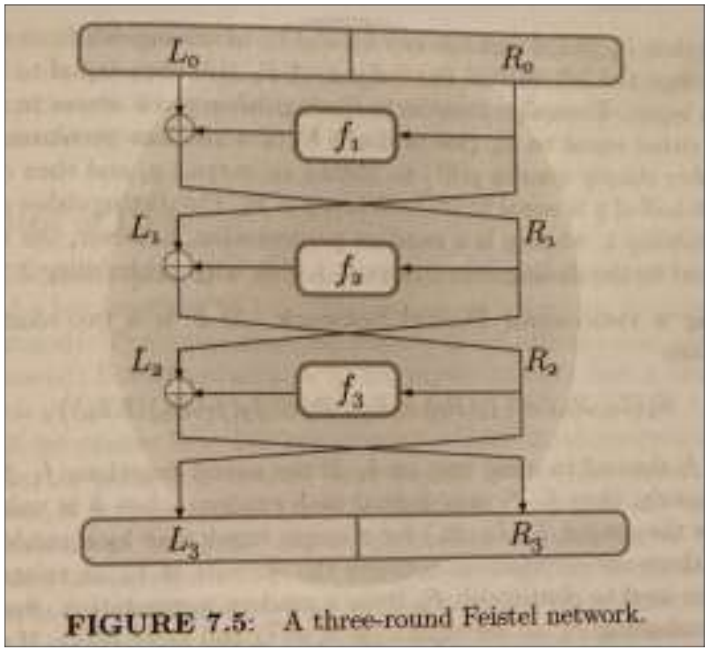


FIGURE 7.5: A three-round Feistel network.

Feistel decryption

1. Write the ciphertext block C as $C = (L_r, R_r)$.
2. For each of the r rounds perform the following:

$$\begin{aligned} L_{i-1} &= R_i \oplus f(L_i, K_i) \\ R_{i-1} &= L_i \end{aligned}$$

3. Finally the plaintext is $P = (L_0, R_0)$

- We never have to invert the function f so we can always decrypt for *any* function f.
- However, choice of f is critical for security as it is the only **non-linear** part of the encryption function.

Outline

SPNs

- A substitution-permutation network is an iterated cipher.
- The block length n must allow each block to be split into m sub-blocks of length l so that $n = lm$. Two permutations are defined.
- Permutation π_S operates on sub-blocks of size l bits:

$$\pi_S : \{0,1\}^l \rightarrow \{0,1\}^l$$

  The permutation π_S is normally called an S-box (substitution box).
- Permutation π_P swaps the inputs from $\{1, \ldots, n\}$. This is similar to the transposition cipher.

$$\pi_P : \{1,2,\ldots,n\} \rightarrow \{1,2,\ldots,n\}$$

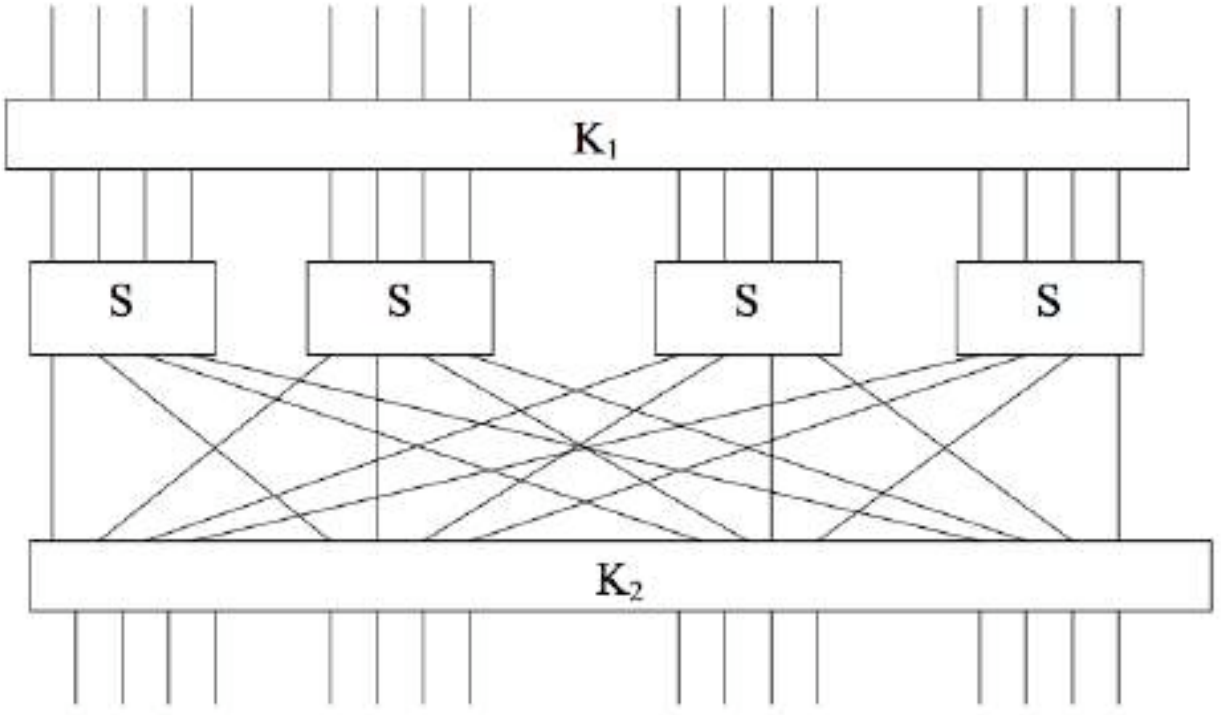
Steps in SPN round function

The round function is defined by three steps

1. The round key K_i is XORd with the current state block W_i
2. Each sub-block is replaced (substituted) by application of π_S
3. The whole block is permuted using π_P.

In the following picture the boxes marked S implement the permutation π_S. One complete round is shown with the start of a second one.

SPN network



- The round key K_i is added (XOR) into the current block
- The same substitution, S, is applied to each sub-block
- The whole block is permuted at the bit level (transposition)

One-round SPN

Source: Introduction to Modern Cryptography, Jonathan Katz and Yehuda Lindell, third edition

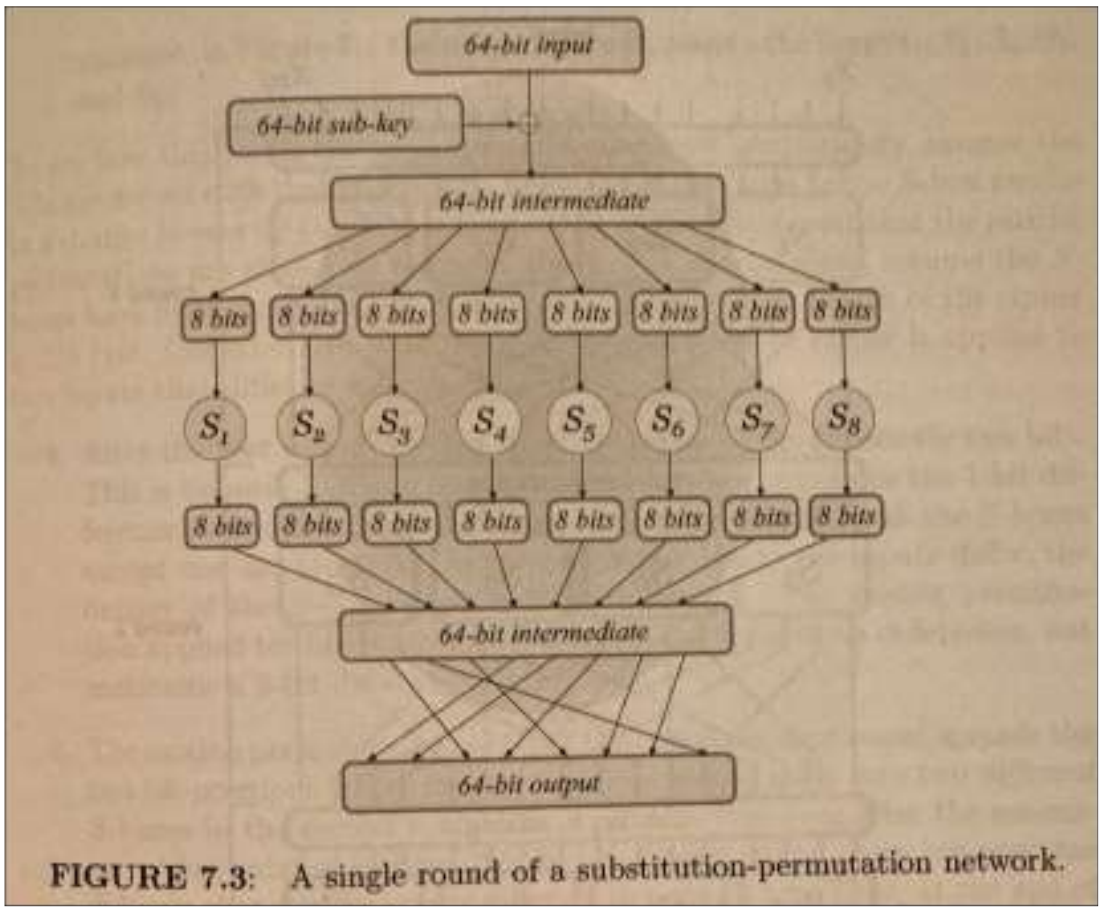


FIGURE 7.3: A single round of a substitution-permutation network.

Three-round SPN

Source: Introduction to Modern Cryptography, Jonathan Katz and Yehuda Lindell, third edition

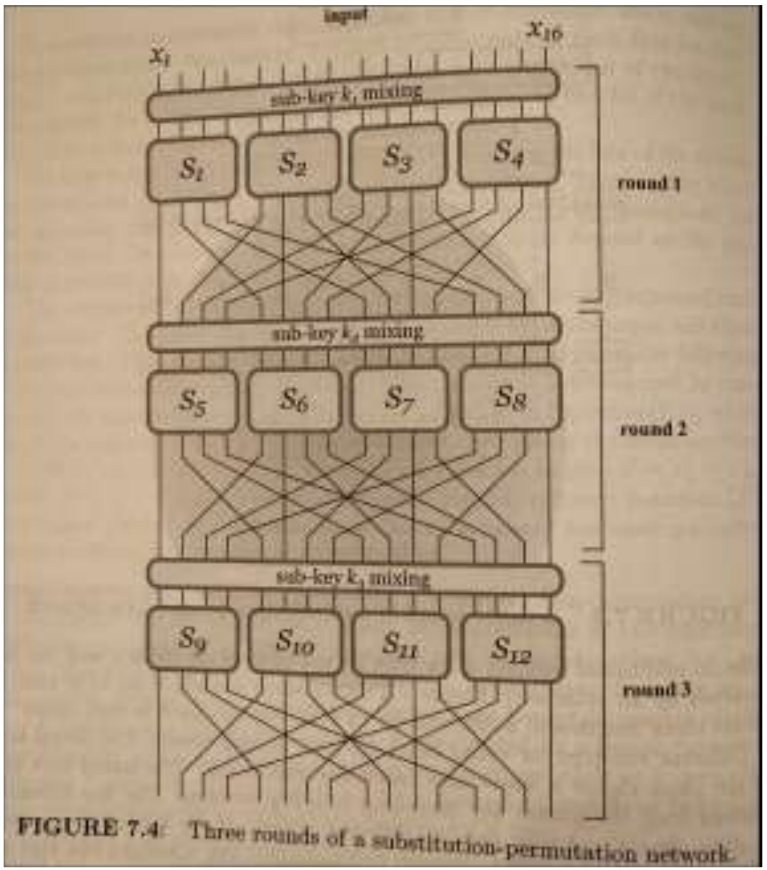


FIGURE 7.4: Three rounds of a substitution-permutation network

Outline

Avalanche effects

- Good block ciphers typically exhibit *avalanche effects* with respect to both key and plaintext.
- **Plaintext avalanche**: a small change in the plaintext should result in a large change in the resulting ciphertext. Ideally, changing one bit of the plaintext changes each of the bits in the output block with probability 1/2.
- We can relate the plaintext avalanche effect to Shannon's notion of diffusion.
- **Key avalanche**: a small change in the key (with the same plaintext) should result in a large change in the resulting ciphertext.
- We can relate the key avalanche effect to Shannon's notion of confusion.

Differential and Linear Cryptanalysis

- Differential cryptanalysis is a powerful technique first published in 1992. It is a chosen plaintext attack.
- Based on the idea that the difference between two input plaintexts can be correlated to the difference between two output ciphertexts.
- Linear cryptanalysis is a known plaintext attack first published in 1993. It can be theoretically used to break DES.
- Modern block ciphers are normally designed to be immune to both differential and linear cryptanalysis.

Outline

Data Encryption Standard (DES)

- Designed by researchers from IBM and submitted to the NBS (National Bureau of Standards) in US in a call for a publicly available cipher.
- Approved in 1977 as the US standard for encryption.
- The encryption and decryption definitions are public property. The security of the DES algorithm resides in the difficulty of decryption without knowledge of the key.
- DES is a 16-round Feistel cipher with key length of 56 bits and data block length of 64 bits.

Outline

Encryption operation

An input block of 64 bits denoted by P.

- **Step 1** The 64 bits of P are permuted according to an initial fixed permutation, denoted by IP.
- **Step 2** After the permutation, 16 rounds of a Feistel operation are applied, denoted by function f. A different 48 bit subkey is used for each round of the f function.
- **Step 3** After the 16 round operations, a final fixed inverse permutation, denoted by IP^{-1}, is applied.

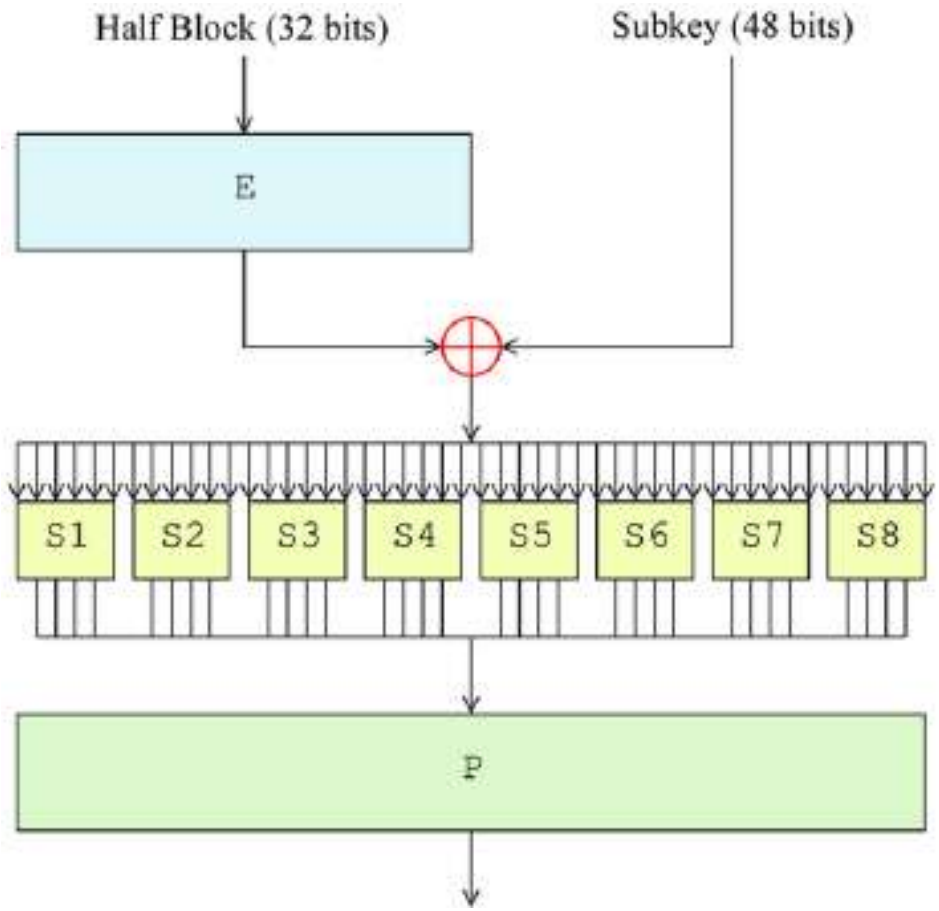
After Step 3, the output ciphertext block, denoted by C, has been formed.

DES Feistel operation

For each round the following steps are followed (see picture on next slide)

- **Step 1** Expand 32 bits to 48 bits
- **Step 2** Bitwise modulo two add (XOR) 48 bits to 48 bit subkey for round
- **Step 3** Break 48 bits into eight blocks of six bits each
- **Step 4** Put block *i* into substitution table *i* resulting in block of length four.
- **Step 5** Apply permutation to resulting 32 bits.

Feistel f function used in DES



Picture courtesy of Wikimedia commons

S-box example I

Row No.	Column Number															
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

- Suppose input block B is $x_1x_2x_3x_4x_5x_6$
- Digits x_1 and x_6 define row number between 0 and 3
- Digits $x_2x_3x_4x_5$ define column number between 0 and 15

S-box example II

One good example of a fixed table is the S-box from DES (S_5), mapping 6-bit input into a 4-bit output:

S_5		Middle 4 bits of input															
		0000	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**
Outer bits	**00**	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

https://en.wikipedia.org/wiki/S-box

Key schedule

- Each of the sixteen rounds involves 48 bits of the 56 bit key.
- Each 48-bit subkey is defined by a series of permutations and shifts on the full 56-bit key

Outline

Brute force attack

- A brute force attack on a block cipher consists of testing all possible 2^k keys in order to find the key K.
- The right key can be identified by using a small number of ciphertext blocks, or by looking for low entropy in the decrypted plaintext.
- In the case of DES there are 2^{56} keys to test so that, on the average, it would take 2^{55} trial samples to find the right key.
- Right from its first publication the short size of the DES key was criticised (by academics).
- As technology evolved (i.e. computational power), this became insecure.

Real world attacks

1997	• $10,000 DES Challenge in February 1997 • Solved in June 1997 • Linked together thousands of computers over the Internet (parallel processing)
1998	• EFF DES Cracker built • cost less than $250 000 • less than three days to find 56-bit DES key • searched 88 billion keys per second
1999	• EFF DES Cracker plus distributed search • 22 hours 15 minutes to find 56-bit DES key • searched 245 billion keys per second
2007	• Parallel FPGA-based machine COPACOBANA • cost $10,000 to build • less than 1 week to find 56-bit DES key

Outline

Double encryption

- Let K_1 and K_2 denote two keys of the block cipher. Then double encryption is defined by:

$$C = E(E(P, K_1), K_2) \tag{1}$$

- If the key length of the original block cipher is k then exhaustive key attack requires 2^{2k-1} trials on average.
- In fact there is a time-memory tradeoff which reduces this using a meet-in-the-middle method.

Meet-in-the-middle attack on double encryption

Suppose we have a single ciphertext/plaintext pair (P, C) satisfying equation (1).

Step 1. For each key, store $C' = E(P, K)$ in memory.

Step 2. Check whether $D(C, K') = C'$ for any key K'.

Step 3. K from Step 1 is K_1 and K' from Step 2 is K_2.

Step 4. Check whether key values in Step 3 work for other (P, C) pairs.

This attack requires storage of one plaintext block for every possible key.

Attack applied to double DES

- The attack requires:
  - storage of one plaintext block for every key
  - a single encryption for every key
  - a single decryption for every key
- For DES algorithm this would require storage of 2^{56} 64-bit blocks, 2^{56} encryption operations and 2^{56} decryption operations.
- Expensive, but much easier than brute force search through 2^{111} keys.

Triple encryption

- Much better security can be provided by using triple encryption.
- In general three keys K_1, K_2 and K_3 are used. Then encryption is defined by:

$$C = E(D(E(P, K_1), K_2), K_3)$$

- This is secure from the above meet-in-the-middle attack.
  - *EDE* for backward compatibility (if the keys are the same, i.e. $K_1 = K_2 = K_3 = K$ this is the same as E)
  - If E has strong pseudorandomn properties, so does $D = E^{-1}$.

Standardised options

- The 1999 version of the DES standard specified three options.
  1. Use three independent keys K_1, K_2, K_3. The most secure.
  2. Use two keys with $K_1 = K_3$. Still secure enough.
  3. Use one key with $K_1 = K_2 = K_3$. Backward compatible with single key DES (vulnerable to brute-force key search).
- NIST SP 800-131A, March 2019 states:
  - Two-key triple DES is allowed only for *legacy use* (decryption only).
  - Three-key triple DES remains allowed in existing applications only, and after 2023 only for legacy use.

Outline

Advanced Encryption Standard (AES)

- Due to controversy over DES design, AES was designed in an open competition
- Process took several years with much public debate
- From 15 original submissions, 5 finalists were all widely believed secure
- Winner was Rijndael, designed by two Belgian cryptographers, Vincent Rijmen and Joan Daeman

Outline

AES overview

- Symmetric key block cipher
- 128-bit data block; 128-, 192- or 256-bit master key
- Number of rounds, NR, is 10, 12 or 14 (for 128-, 192-, 256-bit keys)
- Byte-based design
- Structure is essentially a substitution-permutation network:
  - initial round key addition
  - NR-1 rounds
  - final round

State - matrix of bytes

Data block size = 16 bytes

a_{00}	a_{01}	a_{02}	a_{03}
a_{10}	a_{11}	a_{12}	a_{13}
a_{20}	a_{21}	a_{22}	a_{23}
a_{30}	a_{31}	a_{32}	a_{33}

- byte-based
- mixture of finite field operations in $GF(2^8)$ and bit string operations.

Round transformation

Four basic operations:

1. ByteSub (non-linear substitution)
2. ShiftRow (permutation)
3. MixColumn (diffusion)
4. AddRoundKey

- Essentially a substitution-permutation network with $n = 128$ and $l = 8$
- S-box is look-up table but mathematically defined in $GF(2^8)$
- CrypTool allows step-by-step computation of the encryption and decryption process

Example

Reminder on blackboard.

GF(8) example

The polynomial $x^3 + x + 1$ is irreducible in $\mathbb{Z}_2$.

product mod $p(x)$	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	0	0	0	0	0	0	0	0
1	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
x	0	x	x^2	x^2+x	$x+1$	1	x^2+x+1	x^2+1
$x+1$	0	$x+1$	x^2+x	x^2+1	x^2+x+1	x^2	1	x
x^2	0	x^2	$x+1$	x^2+x+1	x^2+x	x	x^2+1	1
x^2+1	0	x^2+1	1	x^2	x	x^2+x+1	$x+1$	x^2+x
x^2+x	0	x^2+x	x^2+x+1	1	x^2+1	$x+1$	x	x^2
x^2+x+1	0	x^2+x+1	x^2+1	x	1	x^2+x	x^2	$x+1$

http://homepages.math.uic.edu/~leon/mcs425-s08/handouts/field.pdf

Key schedule

- The master key input is 128 bits (or 192 bits or 256 bits).
- Each of the 10 (or 12 or 14 respectively) encryption/decryption rounds uses a 128-bit subkey.
- The number of subkeys required is one for each round (10 or 12 or 14) plus an initial subkey. Therefore, for a 128-bit key 11 subkeys are required.
- The key schedule derives the eleven 128-bit subkeys from the 128-bit master key.

AES security

- Some cracks have appeared but not significant breaks
- Attacks exist on reduced-round versions
- *Related key attacks* exist. Such attacks require the attacker to obtain ciphertext encrypted with a key related to the actual key in a specified way.
- Most serious real attack so far reduces effective key size by around 2 bits.

Outline

DES/AES comparison

- **Data block size**: DES - 64 bits; AES - 128 bits
- **Key size**: DES - 56 bits; AES - 128, 192 or 256 bits
- **Design structure**:
  - both are iterated ciphers
  - DES has a Feistel structure; AES is a SPN;
  - DES is bit-based; AES is byte-based
  - AES substantially faster in both hardware and software

Conclusion

- Block ciphers are the workhorses of secure communications
- AES is the choice of today but triple-DES is still in use in older applications
- Designing good block ciphers is a difficult and time-consuming process and requires years of validation by experts
- In future lectures we will see how to use block ciphers as a building block for confidentiality and authentication