TDT4171 Artificial Intelligence Methods

Lecture 5 – Rational Agents

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Outline

1. Summary from last time
2. Chapter 15: Rational Agents
   - Rational preferences
   - Utilities
   - Decision networks
   - Value of information
3. Summary

Summary from last time

- **Temporal models** — variables replicated over time
- **Markov assumptions** and **stationarity assumption**, so we need
  - Transition model $\mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1})$
  - Sensor model $\mathbf{P}(\mathbf{E}_t|\mathbf{X}_t)$
- Tasks are filtering, prediction, smoothing, most likely sequence; **all done recursively with constant cost per time step**
- Classes of models we consider:
  - **Hidden Markov models** have a single discrete state variable; used for speech recognition
  - **Dynamic Bayes nets** subsume HMMs – exact update intractable; approximations exist

Chapter 15 – Learning goals

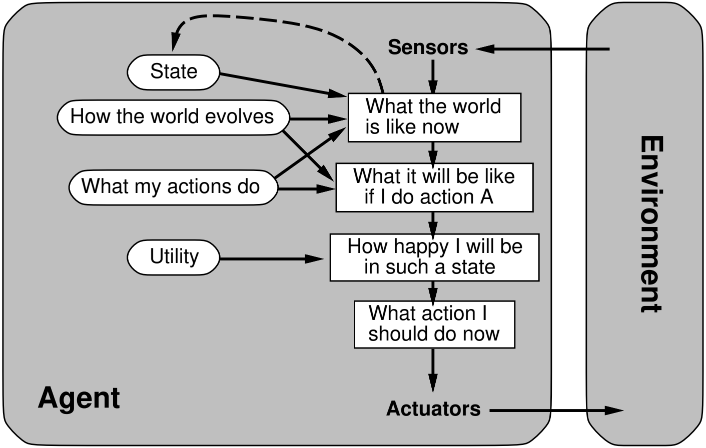
Understanding the relationship between

1. **Rational behaviour** – *"doing what is expected to maximize goal achievement, given the available information"*
2. **Preference structures**
3. **Utilities**

Being familiar with:

- **Utility functions** – Their foundation and definition
- **Utility elicitation**
- **Influence diagrams**

The utility-based agent



Preferences

An agent chooses among **prizes** ("world states") A, B, etc.

Notation:

- $A \succ B$ A preferred to B
- $A \sim B$ indifference between A and B
- $A \succsim B$ A preferred to B or indifference between A and B

Assumption:

An agent will always be able to compare two prizes A and B.

$\Rightarrow$ **No indecisiveness**.

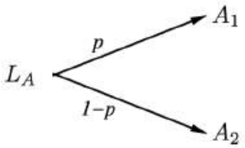
Preferences

An agent chooses among **prizes** ("world states") A, B, etc., and **lotteries**, i.e., situations with uncertain prizes.

Notation:

- $L_A \succ L_B$ — L_A preferred to L_B
- $L_A \sim L_B$ — indifference between L_A and L_B
- $L_A \succsim L_B$ — L_A preferred to L_B or indifference

Lotteries: $L_A = [p, A_1;\ (1-p), A_2]$
$L_B = [p, B_1;\ (1-p), B_2]$



Again:
It is **not an option** to "chicken out"; a relation between L_A and L_B can always be established.

A small "quiz"

Which of the following two lotteries would you prefer?

- Lottery A: $[1, \$10\text{mill}]$,
- Lottery B: $[0.1, \$50\text{mill}; 0.89, \$10\text{mill}; 0.01, \$0]$.

What about these two:

- Lottery C: $[0.11, \$10\text{mill}; 0.89, \$0]$,
- Lottery D: $[0.1, \$50\text{mill}; 0.9, \$0]$.

Do you make **rational** choices if you follow your "gut-feeling"?
. . . **and what does rationality even mean?**

Discuss with your neighbour for a couple of minutes.

Rational preferences

Idea: Preferences of a rational agent must obey constraints.

Constraints:

- **Orderability:** $(A \succ B) \vee (B \succ A) \vee (A \sim B)$
- **Transitivity:** $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
- **Continuity:** $A \succ B \succ C \Rightarrow \exists p \; [p, A; \; 1-p, C] \sim B$
- **Substitutability:** $A \sim B \Rightarrow [p, A; \; 1-p, C] \sim [p, B; 1-p, C]$
- **Monotonicity:**
  $A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; \; 1-p, B] \succsim [q, A; \; 1-q, B])$

We hope:

Since rational preferences follow some "rules", then ...

$\Rightarrow$ Behavior can be described using a mathematical formulation.

$\Rightarrow$ Behavior can be implemented in an intelligent agent.

Rational preferences contd.

Violating the constraints leads to self-evident irrationality

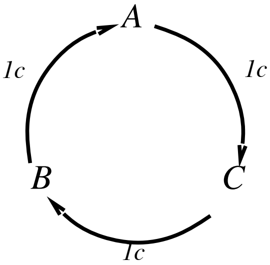
Example: An agent with intransitive preferences can be induced to give away all its money!

Assume he has preferences $A \succ B \succ C \succ A$ and see what happens.

Since $B \succ C$, then an agent who has C would pay (say) 1 cent to get B.

Since $A \succ B$, then an agent who has B would pay (say) 1 cent to get A.

Since $C \succ A$, then an agent who has A would pay (say) 1 cent to get C.



Violating transitivity (or any of the other constraints) is **irrational**!

Maximizing expected utility

Theorem: The foundation of the *Utility function*

Given preferences satisfying the constraints there exists a real-valued function U such that

1. $U(A) \geq U(B) \quad \Leftrightarrow \quad A \succsim B$
2. $U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i\ p_i \cdot U(S_i) = \mathbb{E}_S[U(S)]$

This gives rise to the MEU principle:
To be rational, the agent must choose the action that maximizes expected utility!

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Action selection – Choosing from the set $\{A_1, \ldots, A_m\}$. Effect of A_j uncertain; futures are $\{\texttt{Result}_1(A_j), \ldots, \texttt{Result}_n(A_j)\}$.

$$\mathbb{E}U(A_j \mid \mathbf{e}) = \sum_i P\left(\texttt{Result}_i(A_j) \mid \mathbf{do}(A_j), \mathbf{e}\right) \cdot U\left(\texttt{Result}_i(A_j)\right)$$

Then, the best choice is $\alpha = \arg\max_{A_j} \mathbb{E}U(A_j \mid \mathbf{e})$.

Maximizing expected utility

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This gives rise to the MEU principle:
To be rational, the agent must choose the action that maximizes expected utility!

Note:
An agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities!
(For example, a lookup table for perfect tic-tac-toe)

Poker: Rational decision making under uncertainty

Poker-movie: `./../../poker.mp4`

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Poker: Rational decision making under uncertainty

Being rational under uncertainty is **not** the same as . . .

- Being "all-seeing":
  - Knowing the probabilities the TV viewers are presented with
  - Knowing the hidden cards
- Being lucky

Rationality is doing the best out of **what we know**.

Extra twist: Players do not equate utilities to the no. poker chips they have, but rather to their pay-out (position in tournament).

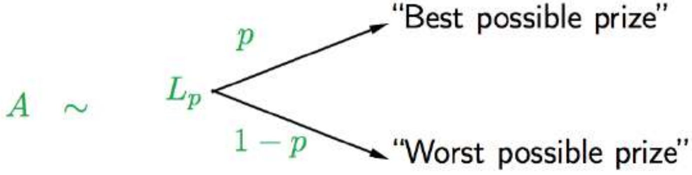
Utilities – and how to quantify them. . .

Utilities map states to real numbers – but which numbers?

Standard approach to assessment of human utilities:

- Compare a given state A to a **standard lottery** L_p that has
  - "best possible prize" $u_\top$ with probability p
  - "worst possible catastrophe" $u_\perp$ with probability $(1-p)$
- Adjust lottery probability p until $A \sim L_p$;
  $U(A) \leftarrow p \cdot u_\top + (1-p) \cdot u_\perp$

(This makes sense, as we already think of probabilities in terms of accepting bets. . .)



Utilities – and how to quantify them...

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Note: Behavior is **invariant** w.r.t. linear transformation

$$U^*(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

Therefore, it is natural use **normalized utilities**; $u_\top = 1.0$, $u_\perp = 0.0$, and we get $U(A) = p$ in the procedure above.

Are you rational? – Example continued

Recall:

- Lottery A: $[1, \$10\text{mill}]$,
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Let $U(\$50\text{mill}) = u_\top = 1$, $U(\$0) = u_\perp = 0$, $U(\$10\text{mill}) = u$.

Questions:

1. Is there a u that fits with my pref's ($\{L_A \succ L_B \wedge L_D \succ L_C\}$)?
2. What does it mean if there is no such u?

Are you rational? – Example continued

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If you prefer Lottery A over Lottery B it means that

$$\mathbb{E}U(A) > \mathbb{E}U(B) \Leftrightarrow u > 0.1 + 0.89u \qquad \Leftrightarrow \qquad u > \frac{10}{11} \approx .91.$$

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Since $\mathbb{E}U(D) = 0.1$, we have that $L_A \succ L_B \Rightarrow L_C \succ L_D$.

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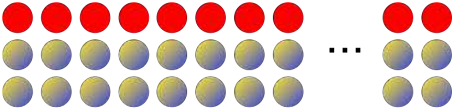
The preference-combo $\{L_A \succ L_B \wedge L_C \prec L_D\}$ is **not** rational.

Are you rational? – New example

- 60 balls in an urn
- 20 are RED, 40 are BLUE **or** YELLOW
- We don't know how many are BLUE or YELLOW



- A: You receive \$100 if you draw a RED ball,
- B: You receive \$100 if you draw a BLUE ball
- C: You receive \$100 if you draw a RED or YELLOW ball
- D: You receive \$100 if you draw a BLUE or YELLOW ball

Which is better?

- Lottery A or Lottery B?
- Lottery C or Lottery D?

Are you rational? – New example

- 60 balls in an urn
- 20 are **RED**, 40 are **BLUE or YELLOW**
- We don't know how many are **BLUE** or **YELLOW**
- A: You receive \$100 if you draw a **RED** ball,
- B: You receive \$100 if you draw a **BLUE** ball

Let $R = P(\text{Red ball})$, $B = P(\text{Blue ball})$ $Y = P(\text{Yellow ball})$.
Let $U(\$0) \equiv 0$ and $U(\$100) \equiv 1$.

If Lottery A $\succ$ Lottery B, it means that

$$R \cdot 1 + (B + Y) \cdot 0 > B \cdot 1 + (R + Y) \cdot 0 \;\Rightarrow\; R > B$$

Are you rational? – New example

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If you are like me, you suffer from **ambiguity diversion**.

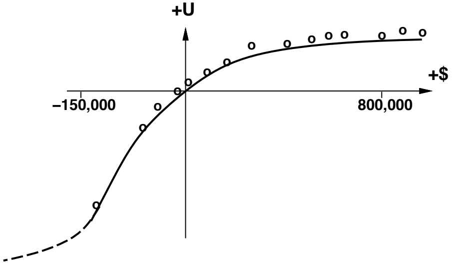
Money

Money does **not** behave as a utility function in itself

Given a lottery L with **expected monetary value** $\mathtt{EMV}(L)$, usually $U(L) < U(\mathtt{EMV}(L))$, i.e., people are **risk-averse**.

This is known as the **certainty-effect**.

Typical empirical data:



Money and utilities – An example

- You pay a fixed fee M to enter a game.
- A fair coin is tossed repeatedly until a "`tail`" appears, ending the game. You win 2^k, where k is the number of "`heads`" you have seen prior to the "`tail`".

What would be a fair entry-cost for entering the game?

- How can a fair entry-cost be found?
- What is it in this case?
- Are you willing to pay the fair entry-cost to take part?

Discuss with your neighbour for a couple of minutes.

Money and utilities – An example

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$$\begin{aligned}
\mathbb{E}[\text{Winnings}] &= -M + \sum_{i=0}^{\infty} P(k=i) \cdot \text{Payout}(i) \\
&= -M + \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^{i+1} \cdot 2^i \\
&= -M + \sum_{i=0}^{\infty} \frac{1}{2} = \infty
\end{aligned}$$

Fair: $\mathbb{E}[\text{Winnings}] = 0$ (or maybe ≥ 0), so **any finite M is OK!**
If not willing to pay, e.g., $M = 10.000.000$, you are risk averse.

Money and utilities – An example

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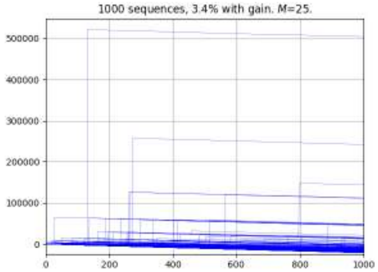
Example: Pay 25 to take part: $P(\text{Gains from one game}) = 3.1\%$.
Only 3.4% of sequences have positive gain after 1000 games.
Yet, the simulated average is always positive!



Human are sometimes irrational – Consequences?

Why are people (sometimes) irrational?

- Debunking the MEU principle? (If so: Which constraints are unreasonable??)
- Lacking the computational power to do this correctly?
- Focusing on "the lottery itself", e.g., thinking about **regret**?
- Other things (like ambiguity diversion, certainty effect, . . .)?

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- Other things (like ambiguity diversion, certainty effect, ...)?

Consequences for AI:

- The choices a rational agent can make are **only as good** as the preferences they are based on.
- If the agent is given conflicting preference judgements, it is **not possible** for the rational agent to understand (or mimic) them.
- Acting rationally (the point of the agents this course) is **not** the same as acting like a human!

Decision networks (a.k.a. "influence diagrams")

Add **decision nodes** and **utility nodes** to belief networks to enable rational decision making.

- **Decision nodes** ("action nodes") define decision points.
  - Shown as rectangular nodes
  - States define decision alternatives
  - Incoming arcs: Things known before decision is made.
  - Outgoing arcs: Quantities *directly* influenced by decision
- **Utility nodes** give utility of potential outcomes.
  - Diamond-shaped nodes
  - Defines utilities in terms of real numbers (utilities)
  - Incoming arcs: Definition of the situation giving a utility (can be both decision nodes and "standard" nodes)
  - Outgoing arcs: Illegal

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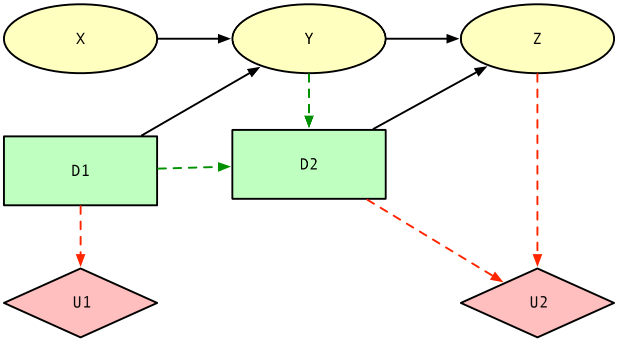

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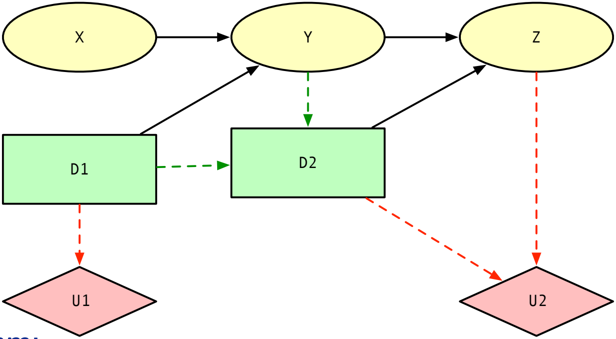

Algorithm:

- **For each value of action node(s):**
  - Compute expected value of sum of utility nodes given action(s), evidence
- **Return MEU action(s)**

Decision networks – An example

Peter wakes up feeling bad. He either has common cold (gives mild fever) or angina (fever and spots in throat). He checks both symptoms. He wants to decide if he should go to work. Staying home is nice and relaxing, but his coworkers will hate him for it if he only has the cold.

How can you model this using a decision graph?

- What are the random variabels?
- What decision(s) are to be made?
- How do we model the utilities?

Discuss with your neighbour for a couple of minutes.

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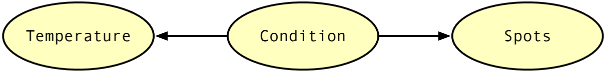

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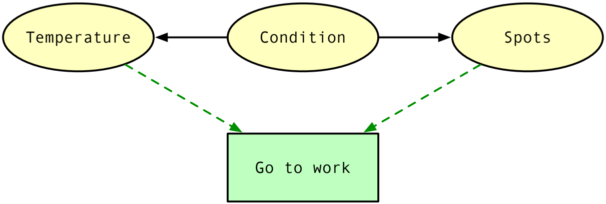

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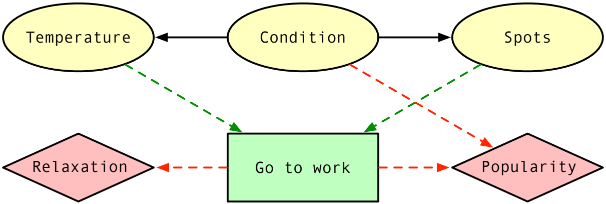

Decision networks – in GeNIe

Must define the utility of each combination of consequences

- `Relaxation`
- `Popularity`

GeNIe-demo: `Peter.xdsl`

Check effect of evidences on decision. What if we change utilities?

Value of information

Idea: Compute value of acquiring each possible piece of evidence.

Example: Buying oil drilling rights:

- Two blocks A and B, exactly one has oil, worth 1
- Prior probabilities 0.5 each, mutually exclusive.
- Price of each block is $1/2$, so we may gain $1/2$ or loose $1/2$.
  Picking randomly we would gain $0.5 \cdot 1/2 + 0.5 \cdot (-1/2) = 0$.
- A consultant offers accurate survey of A. **Fair price?**

Solution: compute expected value of information (VOI)

VOI = expected value of best action **given** the information − expected value of best action **without** information

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Solution: compute expected value of information (VOI)

- Survey may say "oil in A" or "no oil in A", **prob. 0.5 each**

$$\begin{aligned} \text{VOI} &= [0.5 \times \text{value of ``buy A'' given ``oil in A''} + \\ &\quad\; 0.5 \times \text{value of ``buy B'' given ``no oil in A''}] - 0 \\ \text{VOI} &= (0.5 \times 1/2) + (0.5 \times 1/2) - 0 = 1/2 \end{aligned}$$

General formula for VPI: Value of Perfect Information

- Current evidence $\boldsymbol{E} = \boldsymbol{e}$, current best action α. Possible outcomes when choosing an action $A = a$ are the states of a variable S_i, so

$$\mathbb{E}U(\alpha \mid \boldsymbol{e}) = \max_a \sum_i P(S_i \mid a, \boldsymbol{E} = \boldsymbol{e}) \cdot U(S_i)$$

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$$\mathbb{E}U(\alpha \mid \boldsymbol{e}) = \max_a \sum_i P(S_i \mid a, \boldsymbol{E} = \boldsymbol{e}) \cdot U(S_i)$$

- Consider potential new evidence E_j. Suppose we knew $E_j = e_{jk}$, then we would choose $\alpha_{e_{jk}}$ s.t.

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- E_j is a random variable whose value is *currently* unknown
  $\Rightarrow$ must compute expected gain over all possible values:

$$\mathtt{VPI}_{\boldsymbol{E}}(E_j) = \overbrace{\sum_k \underbrace{P(E_j = e_{jk} \mid \boldsymbol{e})}_{\text{Prob. for } E_j = e_{jk}.} \cdot \underbrace{\mathbb{E}U(\alpha_{e_{jk}} \mid \boldsymbol{e}, E_j = e_{jk})}_{\text{Expected utility when } E_j = e_{jk}.}}^{\text{Expected utility over uncertain but to-be-observed } E_j.} - \mathbb{E}U(\alpha \mid \boldsymbol{e})$$

Luckily, we can do this in GeNIe instead!

Value Of Information – in GeNIe

GeNIe-demo: `VOI.xdsl`

GeNIe-demo: `oil.xdsl`

Properties of VPI

Let $\text{VPI}_E(E_j)$ denote the value of getting perfect info about E_j in a a setting when E is already known. Then the following holds:

Nonnegative in expectation:

$$\forall\, j, E \ \ \text{VPI}_E(E_j) \geq 0$$

Order-independent:

$$\begin{aligned} \text{VPI}_E(E_j, E_k) &= \text{VPI}_E(E_j) + \text{VPI}_{\{E,E_j\}}(E_k) \\ &= \text{VPI}_E(E_k) + \text{VPI}_{\{E,E_k\}}(E_j) \end{aligned}$$

Nonadditive; consider, e.g., obtaining E_j twice

$$\text{VPI}_E(E_j, E_k) \neq \text{VPI}_E(E_j) + \text{VPI}_E(E_k)$$

Note: when more than one piece of evidence can be gathered, evidence-gathering becomes a **sequential** decision problem. It is **NP-complete** in general.

Summary

- Rational agents can always use **utilities** to make decisions
- The **MEU principle** tells us how to behave
- It can be quite laborious to elicit preference structures from domain experts
  - $\Rightarrow$ **structured approaches** are available
- **Value of Information** helps focus information gathering for rational agents
- **Influence diagrams** are extensions to BNs that let us make rational decisions.