# Lecture 3
## $A^*$ Search and Search in Complex Environments

TDT4136: Introduction to Artificial Intelligence

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September 5, 2024

# Outline

# Recap on Uninformed Search

- Uninformed search strategies systematically navigate the search space blindly—not questioning where the goal may be in the space.
- The search space is often very large.

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- Uninformed search strategies systematically navigate the search space blindly—not questioning where the goal may be in the space.
- The search space is often very large.
- We can be smarter about it using a heuristic (guess estimate)
- We covered (Greedy) Best First, where you pick the option with the best estimate
- We also covered $A^*$, which uses both the cost and the estimate

# $A^*$ search

## Informed search strategies

What if we consider the cost *and* the heuristic?

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- $h(n)$ is the estimated cost of the node (to the goal)
- $f(n)$ is then the estimated cost of the cheapest solution through $n$ to the goal

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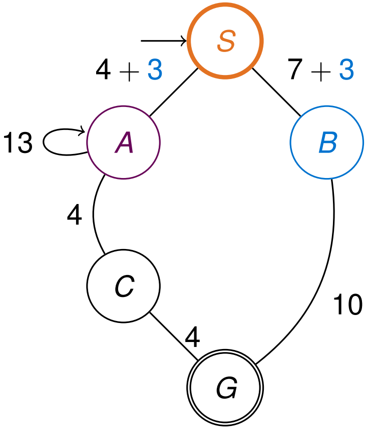
With the following estimated distances to the goal:

- $h(A) = 3$
- $h(B) = 3$
- $h(C) = 3$
- $h(G) = 0$



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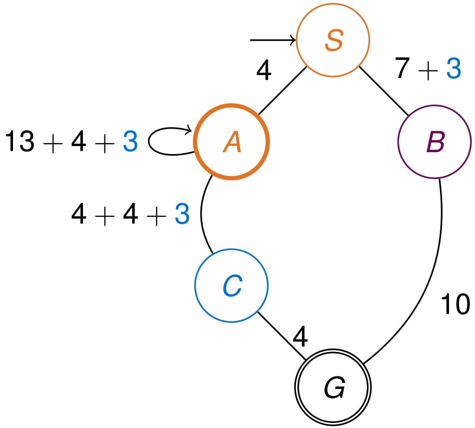

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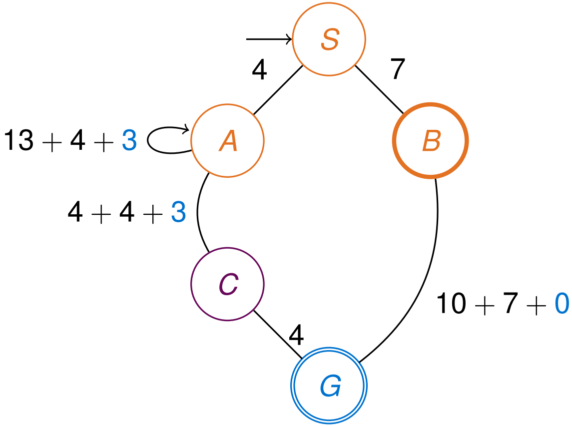

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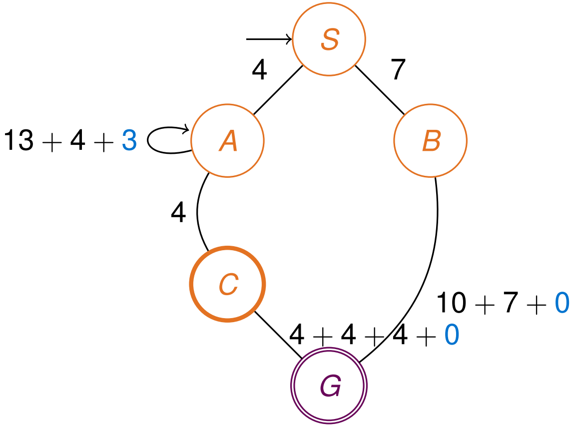

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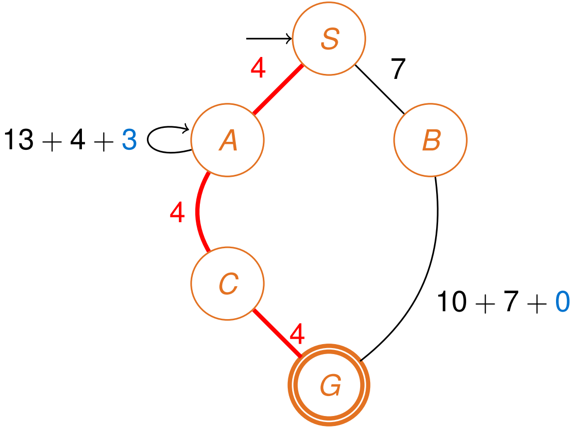

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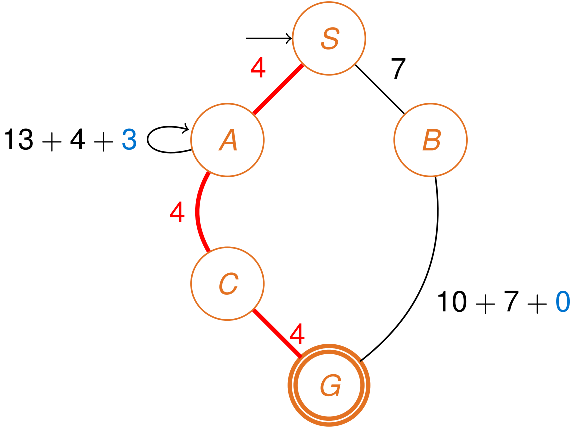
- It is **Complete** for positive costs, within a finite state space and an existing solution.



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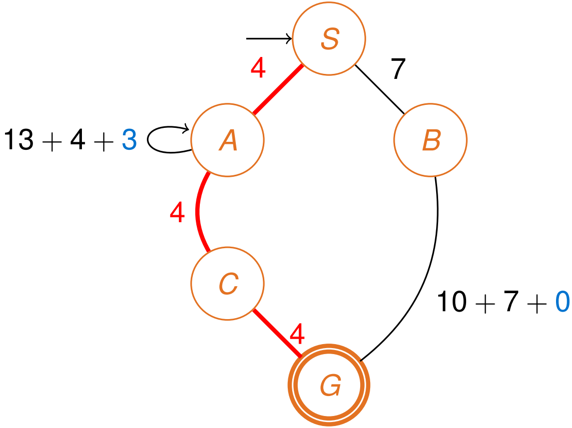
- We have found the goal!
- It is **Complete** for positive costs, within a finite state space and an existing solution.
- It is **Cost optimal** if **certain conditions are met**



# $A^*$ optimality

## More on $A^*$

$A^*$ is **cost optimal** if **certain conditions are met.** What are these conditions?

[1] They usually are.

# $A^*$ optimality
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$A^*$ is **cost optimal** if **certain conditions are met.** What are these conditions?

- Arc costs need to be positive[1]
- The heuristic function needs to be **admissible** and non-negative.

[1]They usually are.

# Admissibility

## More on $A^*$

### Admissibility of $h$

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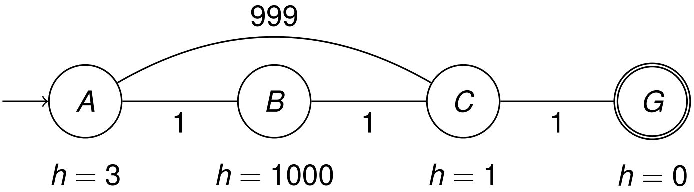
An admissible heuristic is *optimistic!*

# A crazy example

## More on $A^*$



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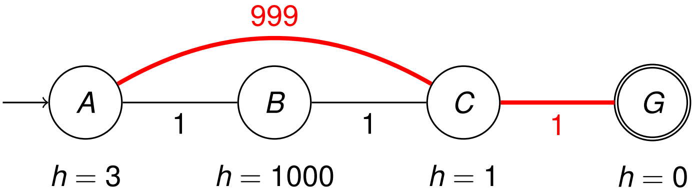

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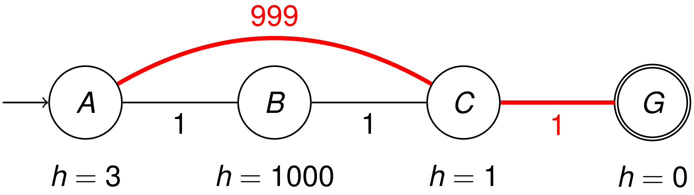


We would not choose the optimal path due to $h(B)$ being overestimated of the actual cost!

# Consistency

## More on $A^*$

Another important (and even stronger) property of a heuristic $h$ is **consistency**.

### Consistency of $h$

A heuristic $h$ is **consistent** if for every node $n$ and <u>all</u> of its successors $n'$ generated by an action $a$, we have

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In other words, the estimate of a node should be less or equal than the the estimate of a descendant plus the cost of reaching there.

# Consistency: an example

## More on $A^*$

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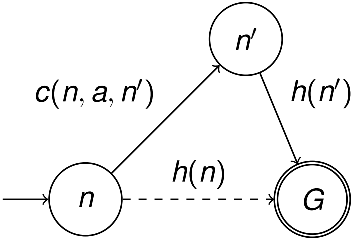
- A triangle inequality helps picturing it!
- Moving through $h(n)$ **has** to be cheaper than going to $G$ via the successor $n'$
- This **must** be true for every successor $n'$ of $n$
  - Think of an euclidean grid



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  - That means that $f(n)$ is non-decreasing along any path

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- Beam search – limit size of *frontier* to $k$-best candidates
- Iterative deepening $A^*$ – gradually increase the $f$-cost *cutoff*.
- Memory-bounded $A^*$ – expand until memory is ful, and then drop the worst candidate from *frontier*

# A generalised heuristic search

## More on $A^*$

### Generalised heuristic search

$$f(n) = g(n) + w \cdot h(n)$$

where $w$ is a *weight* defining how important the heuristic $h(n)$ is.

In most other applications, we usually have $w_1$ and $w_2$, one for $g(n)$ and one for $h(n)$. The book uses only $w$ for $h(n)$.

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Of course you can set $w$ to something else, depending for example if there is *uncertainty* on your heuristic (but this then becomes a whole other course :^))

# Building heuristics

## More on $A^*$


How far are we from solving this sliding puzzle?

- $h_1(n)$ will be the number of misplaced tiles
- $h_2(n)$ will be the **total** Manhatttan distance[a]

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Remember that each configuration is a state!

# Other ideas for building heuristics

## More on $A^*$

- Consider relaxations of the problem
- Consider creating the heuristic by looking **backwards from the goal**.
- Consider dividing into subproblems!
  - For example, instead of solving the whole sliding puzzle at once, consider getting in place four tiles only
  - Then store all these solutions in a DB. Create an admissible heuristic for this subproblem
  - Combine the subproblems to choose the best heuristic

The process of choosing the appropriate representation, data structures and heuristics for a problem is known as **modelling** and is crucial for AI developers and researchers!

# Dominance: comparing heuristics



**Which of the heuristics is better?**

# Dominance: comparing heuristics

## More on $A^*$

**Which of the heuristics is better?**
Admissible heuristics can be compared by looking at their values.

### Heuristic Domination

An admissible heuristic $h_2$ it is said to **dominate** another admissible heuristic $h_1$ if **for all** nodes $n$ if $h_2(n) \geq h_1(n)$.
This will reflect in $A^*$ expanding fewer nodes on $h_2$, and thus find an optimal solution, faster.

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A generalisation of this would then be

$$h_{best}(n) = \max(h_a(n), h_b(n), \ldots)$$

Section 3

# Search in Complex Environments

# Searching in complex environments

- Both informed and uninformed searching strategies are designed to explore search spaces systematically
- They keep one or more paths in memory, and record which alternatives have been explored at each point along the path
- The path to that goal constitutes a solution
- But in most problems in the real world, the path to a solution might be **irrelevant**

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- But in most problems in the real world, the path to a solution might be **irrelevant**

If we only care about finding a solution, then there are better ways to search the space!

# Local Search

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# Local Search

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- It eases up on the completeness and optimality in the interest of improving time and space complexity[2]
- Local Search algorithms use “little” memory (usually a constant amount)
- They can often find reasonable solutions in very large (or infinite) state spaces

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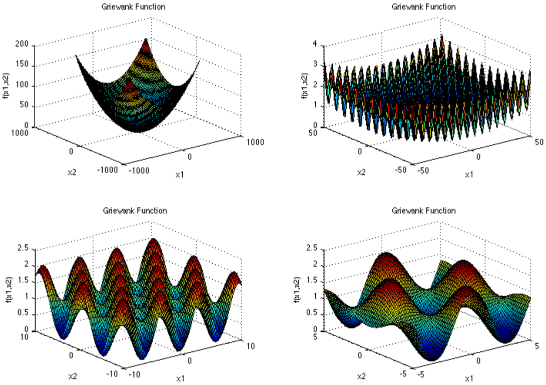
# The search landscape

## Search in complex environments

Usually, the **state space** is referred to as the **search space**. We can **visualise** this space by looking at the heuristic function!

$$f(x) = \sum_{i=1}^{d} \frac{x^2}{4000} - \prod_{i=1}^{d} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

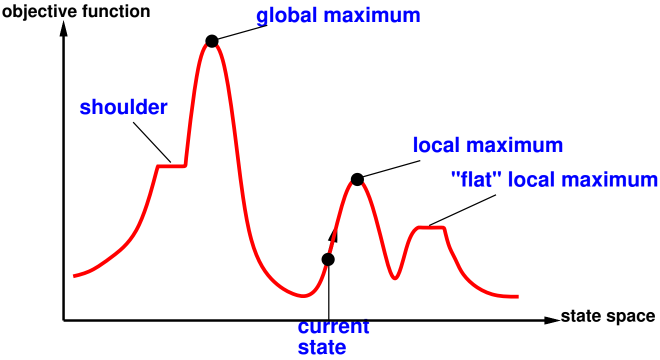



The Griewank function. Image from Surjanovic & Bingham

https://www.sfu.ca/~ssurjano/griewank.html

# The search landscape

## Search in complex environments

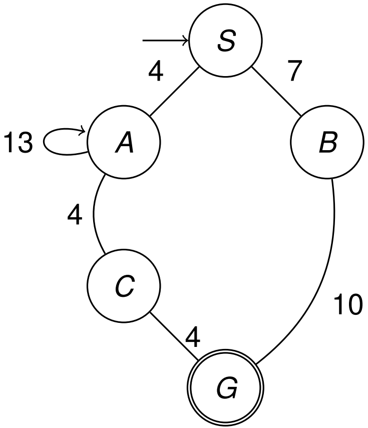



- Each point in the landscape represents a state in the search space and has “an elevation” (its $h(n)$)
- If the elevation corresponds to an objective function, then the aim is to find the highest peak (or **maximum**)
- If the elevation corresponds to a cost function, then we look for the lowest valley (or **minimum**)

# The search landscape

## Search in complex environments

Recall our search problems.




- $A$ is a neighbour of $S$, $C$ and itself because those are the states than can be reached from $A$.
- The **neighbourhood** of $A$ is then $\{A, C, S\}$.
- This concept of neighbourhood is very important for **local search**, as we decide *where to move next* by looking around us!

Section 4

# Local Search Algorithms

As the last time with algorithms, please check the full details on the book!

# Hill climbing and gradient descent

## Local search algorithms

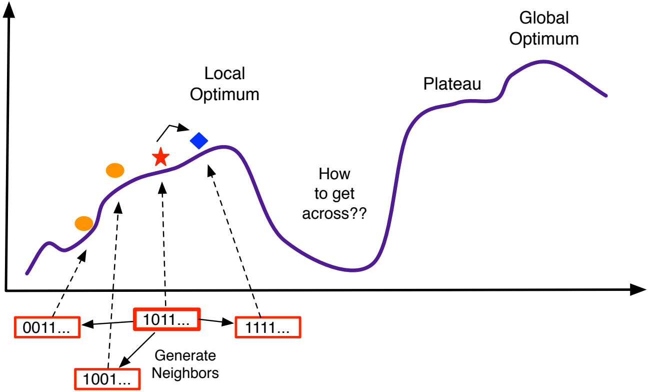
Idea: Go to the best spot you see now.

# Hill climbing and gradient descent

## Local search algorithms

Idea: Go to the best spot you see now.

- Assume you are doing **maximisation**
- You then want to **climb** the tallest peak
- This is called **hill-climbing**!




If you are **minimising** instead, then the procedure is called **gradient descent** as we want to move towards the direction where the difference in "height" is largest.

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Both **hill-climbing** and **gradient descent** get stuck in **local optima**. How do we get out of this mess?

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- Idea 2: Make it so that you gradually reduce the frequency of taking such “bad” decisions
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- Idea 3: Search multiple paths in batches

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  - Or doing *short* hops when you are in a promising state (you do not want to miss it)

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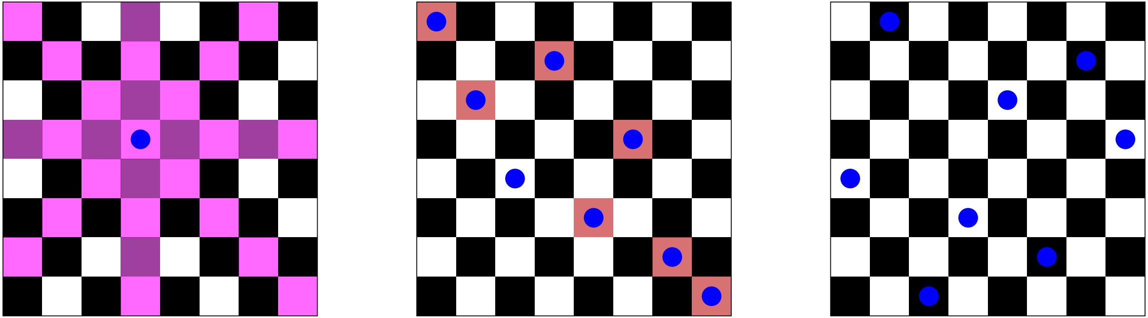
This process will be repeated until a solution has been found, or until enough *generations* have been replaced.

We have a whole course on evolutionary computation methods during the spring semester: IT3708 Bio-Inspired AI!

# The 8-queens problem

Place 8 queens in a chess board such that no queen checks each other.



(a) Queen constraints (b) Generated conflicts (c) Possible solution

Figure: The 8-queens problem. 1a shows the constraints (in pink) imposed by the placement of a single queen piece (in blue). 1b highlights the conflicts arising from a possible configuration of the board. 1c illustrates one possible solution with no conflicts.

See a worked example in `https://ntnu-ai-lab.github.io/EvoLP.jl/stable/tuto/8_queens.html`

Section 5

# Nondeterministic and partially observable environments

# Searching with Nondeterminism

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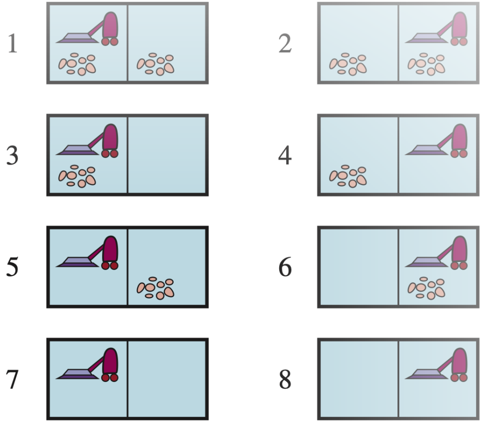
- So far, we have assumed that actions are deterministic
  - That our intended action will **always** yield the result we expect
- In the real-world, things do not always go as expected
- To account for different possible outcomes, we need to come up with a contingency plan instead of a single path of actions

# Example: The Erratic Vacuum World

## Searching with Nondeterminism

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  $suck(s_1) = \{s_5, s_7\}$

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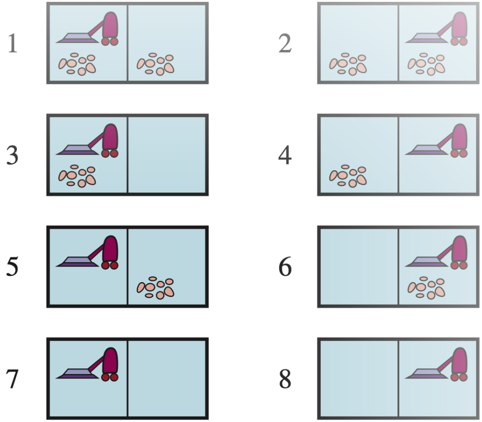


- Nondeterministic *suck* action:
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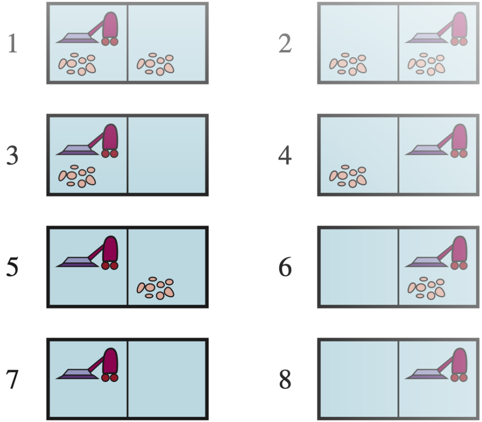


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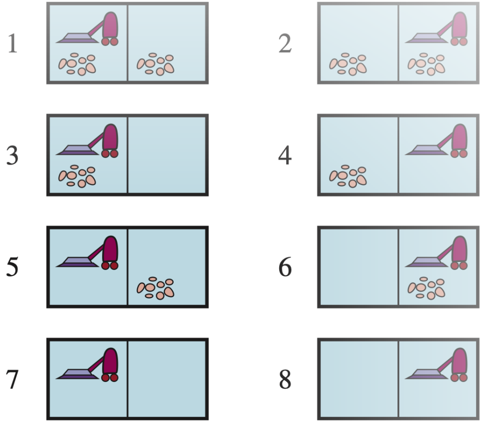


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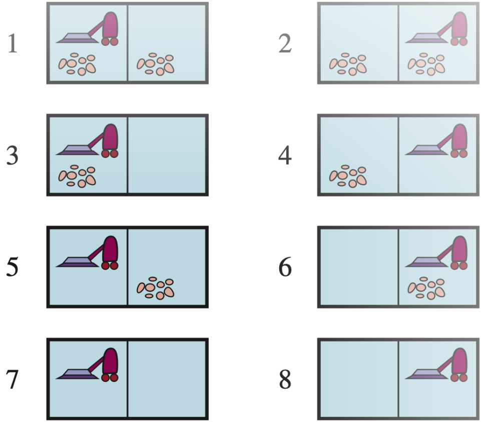


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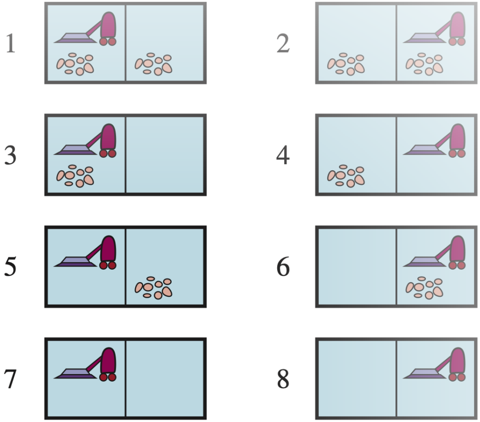


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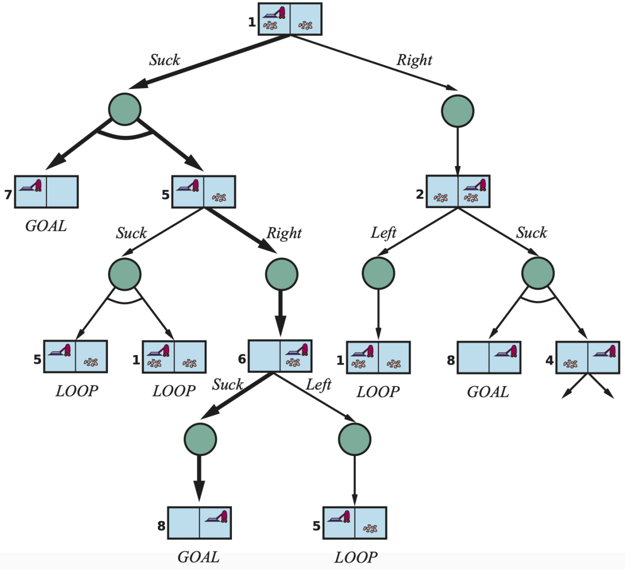
Nondeterminism can happen with other actions like *moveRight*! See the *slippery vacuum world* in the book!

# AND-OR search trees

## Searching with Nondeterminism

One way to handle these, is to consider *compound nodes*, made up of the possible states after a given action

- **OR** nodes represent actions
- **AND** nodes represent outcomes
- Since it is a **tree**, we can **search** in it
  - This is called **AND-OR search**
  - It is recursive, with a base case of either failure or an empty plan



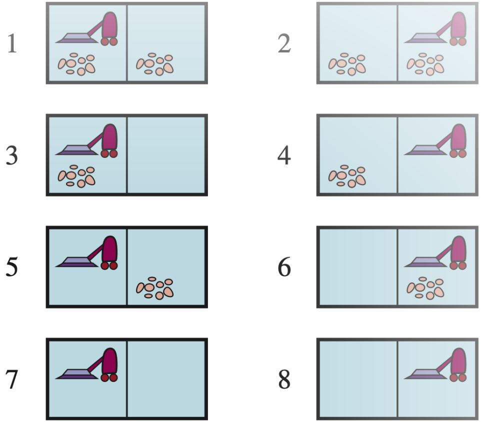
# Searching in Partially Observable Environments

- So far, we have assumed that the agent knows exactly the state of its environment
- In reality, an agent receives partial (and possibly noisy) observations
- Therefore, the state can only be *estimated* through a “belief”

# Sensorless deterministic vacuum world

## Searching in partially observable environments

- $$Result(\{1,2,3,4,5,6,7,8\}, moveRight) = \{2,4,6,8\}$$



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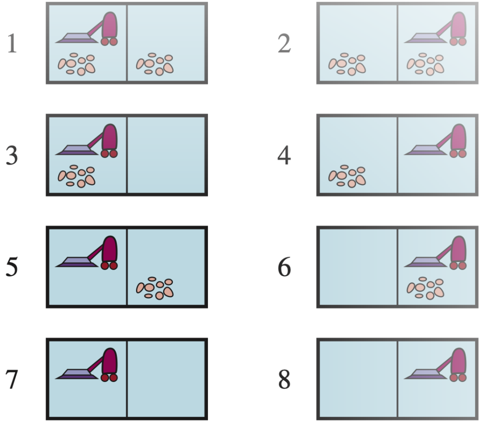

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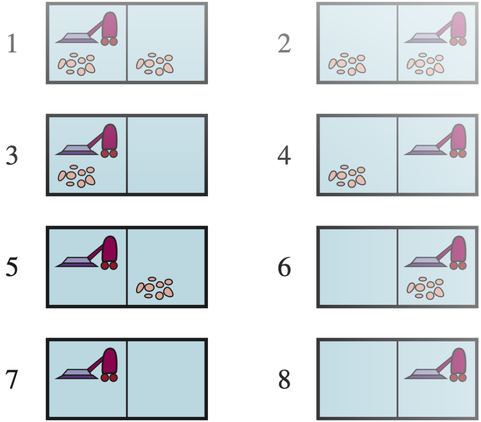

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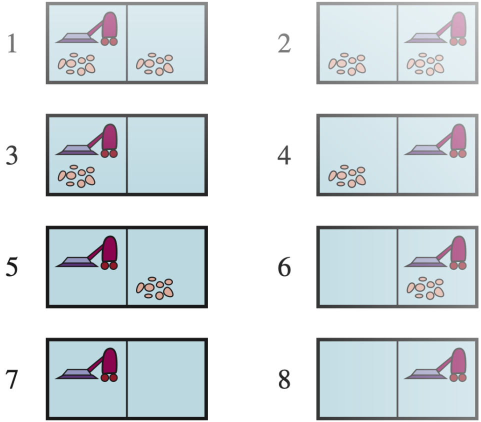

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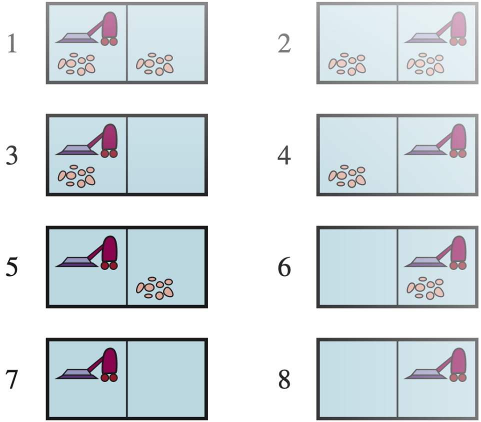

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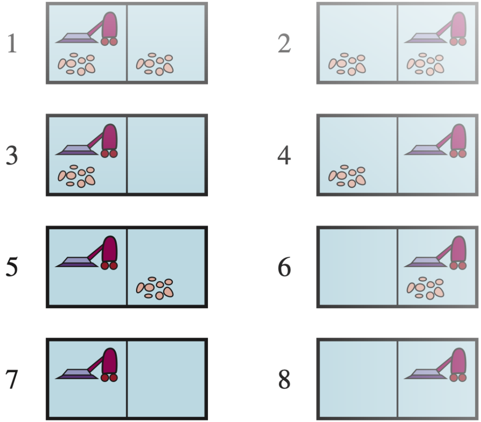
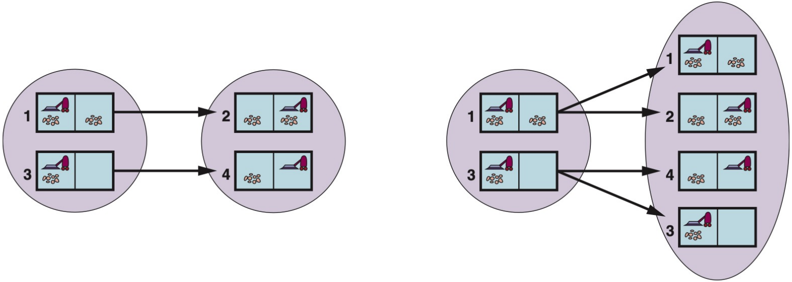



Think of 5D-chess: you solve the problem on multiple paths at the same time!

# Predicting the next state with sensorless agents

## Searching in partially observable environments

We are, in a way, making compound nodes with multiple outcomes in, where some of our actions lead to specific environment settings inside those belief states.



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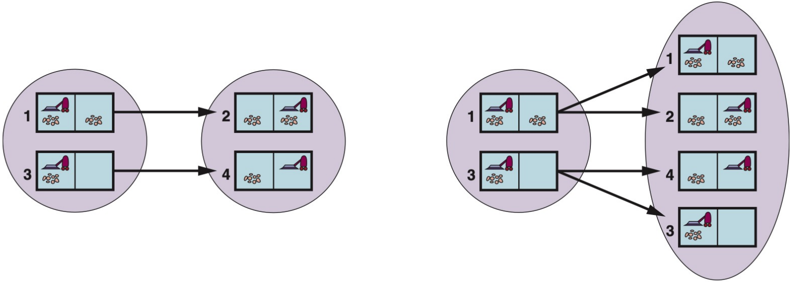
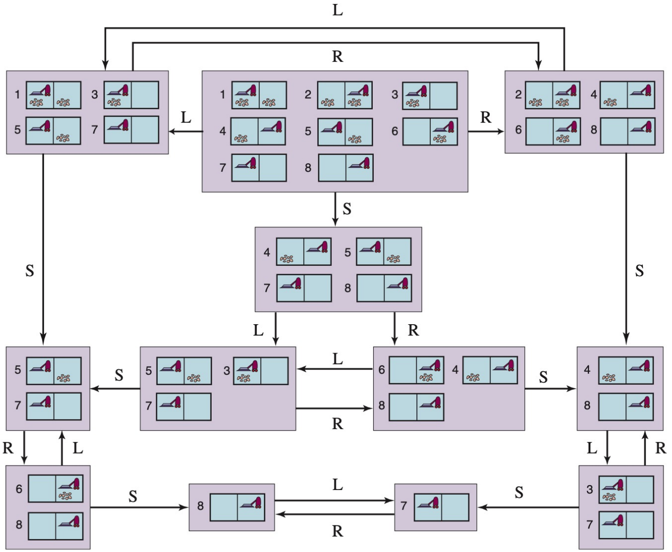



Of course it can be <u>**both**</u> nondeterministic and partially observable!

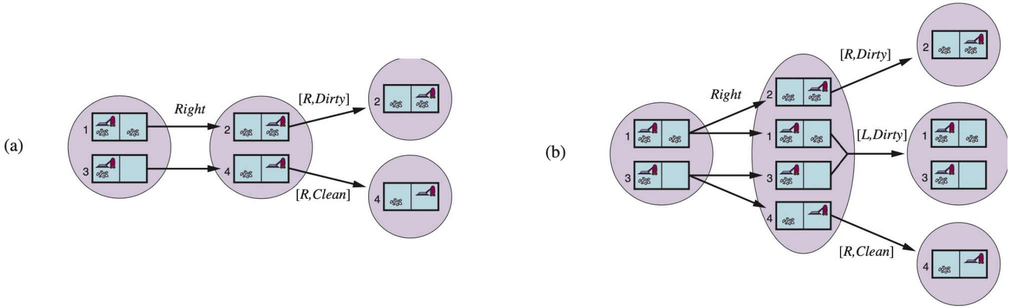
# Searching through the belief space in deterministic environments

If we have a deterministic setting, we can use an ordinary search algorithm.



# Searching through the belief space in partially observable environments

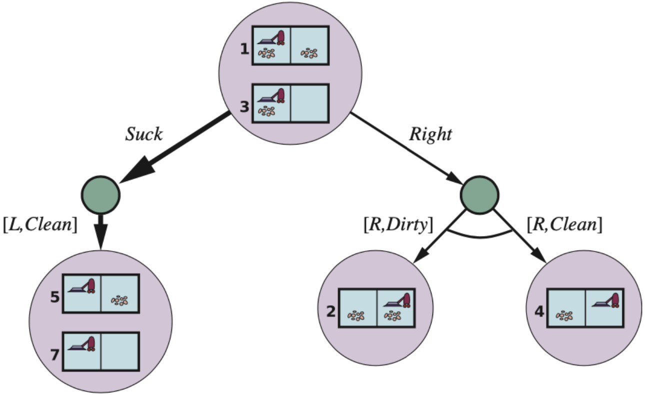
## With sensors




- The agent knows where it is and see the dirt (if any) on its spot
- The **transition model** becomes a **function** of a belief state, an action, and a **another** belief state
    - In case of nondeterminism (right), we do like Dr. Strange and consider possible outcomes on different universes. **How**?

# Seaching through the belief space in partially observable environments

## With sensors, in a nondeterministic world

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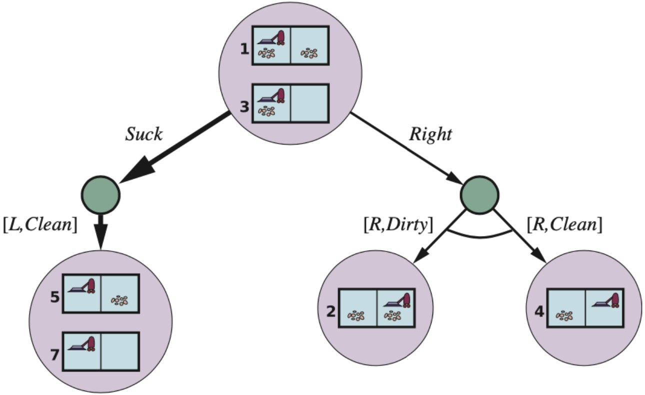


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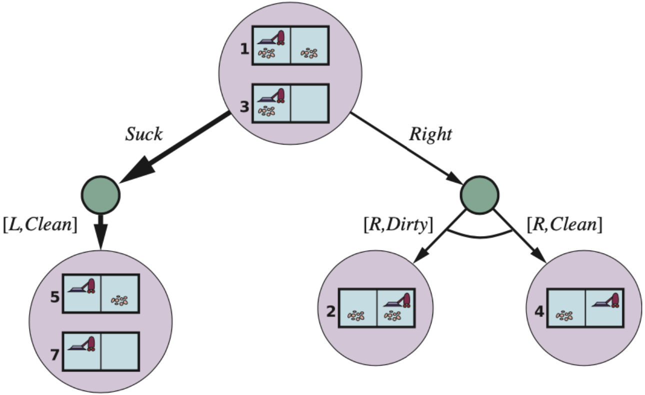


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