Math 241 Quiz #5 version: 1

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(1) (5 points) Find the volume of the solid under the plane $z = x + y$ and above the region $R = [1,2] \times [3,4]$.

1 Find out the integral function and the corresponding domain of the integral

$$\text{integral function} : z = x + y$$
$$\text{the domain of integral} : [1,2] \times [3,4]$$

2 set up the integral

(i) Method I

$$\begin{aligned}
\int_1^2 \left[\int_3^4 x + y dy\right] dx &= \int_1^2 \left(xy + \frac{1}{2}y^2\right)\Big|_3^4 dx \\
&= \int_1^2 \left(4x + \frac{1}{2}4^2\right) - \left(3x + \frac{1}{2}3^2\right) dx \\
&= \int_1^2 x + \frac{7}{2} dx \\
&= \left(\frac{1}{2}x^2 + \frac{7}{2}x\right)\Big|_1^2 \\
&= \left(\frac{1}{2}2^2 + \frac{7}{2}2\right) - \left(\frac{1}{2}1^2 + \frac{7}{2}1\right) \\
&= 2 + 7 - 4 = 5.
\end{aligned}$$

(ii) Method II

$$\begin{aligned}
\int_1^2 \left[\int_3^4 x + y dy\right] dx &= \int_1^2 \int_3^4 x dy dx + \int_1^2 \int_3^4 y dy dx \\
&= \int_1^2 xy\Big|_3^4 dx + \int_1^2 \frac{1}{2}y^2\Big|_3^4 dx \\
&= \int_1^2 x(4-3) dx + \int_1^2 \frac{1}{2}\left(4^2 - 3^2\right) dx \\
&= \int_1^2 x dx + \int_1^2 \frac{7}{2} dx \\
&= \frac{1}{2}x^2\Big|_1^2 + \frac{7}{2}x\Big|_1^2 \\
&= \frac{1}{2}2^2 - \frac{1}{2}1^2 + \frac{7}{2}(2-1) \\
&= 2 + 3 = 5.
\end{aligned}$$

(2) (5 points) Evaluate the double integral $\int\int_D y e^{y^2} dA$, where $D = [0,1] \times [0,2]$.

(1) Method I

$$\begin{aligned}\int\int_D ye^{y^2}dA &= \int_0^2\int_0^1 ye^{y^2}dxdy\\ &= \int_0^2 ye^{y^2}\int_0^1 dxdy\\ &= \int_0^2 ye^{y^2}(1-0)dy\\ &= \int_0^2 ye^{y^2}dy\\ &= \frac{1}{2}\int_0^2 e^{y^2}dy^2\\ &= \frac{1}{2}\int_0^4 e^u du\\ &= \frac{1}{2}e^u\Big|_0^4\\ &= \frac{1}{2}\left(e^4-e^0\right)\\ &= \frac{1}{2}\left(e^4-1\right).\end{aligned}$$

(2) Method II

$$\begin{aligned}\int\int_D ye^{y^2}dA &= \int_0^2\int_0^1 ye^{y^2}dxdy\\ &= \int_0^2 ye^{y^2}\int_0^1 dxdy\\ &= \int_0^2 ye^{y^2}(1-0)dy\\ &= \int_0^2 ye^{y^2}dy\\ &= \frac{1}{2}e^{y^2}\Big|_0^2\\ &= \frac{1}{2}\left(e^{2^2}-e^{0^2}\right)\\ &= \frac{1}{2}\left(e^4-1\right).\end{aligned}$$