Math 241 Quiz #3 version: 1

Instructor: Wenqiang Feng Name: ______________________________

(1) (5 points) Find parametric equation for the tangent line to the curve $\mathbf{r}(t) = \langle t, 2t^2, t^3\rangle$ at the point (1,2,1).

Solution.

Step 1. Find out the specific t

$$\begin{cases} 1 = t \\ 2 = 2t^2 \\ 1 = t^3 \end{cases} \Rightarrow t = 1.$$

Step 2. Find out the direction vector of the tangent line

- direction vector for all t:

$$\mathbf{r}'(t) = \frac{d}{dt}\left\langle t, 2t^2, t^3\right\rangle = \left\langle 1, 4t, 3t^2\right\rangle$$

- direction vector for specific t=1:

$$\mathbf{r}'(1) = \langle 1, 4, 3\rangle .$$

Step 3. Write out the parametric equation for the tangent line

$$\begin{aligned} T(t) &= \mathbf{r}(1) + t\mathbf{r}'(1) \\ &= \langle 1, 2, 1\rangle + t\langle 1, 4, 3\rangle \end{aligned}$$

Hence, the parametric equation for the tangent line is as follows

$$\begin{cases} x = 1 + t, \\ y = 2 + 4t, \\ z = 1 + 3t. \end{cases}$$

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(2) (5 points) Find the arc-length of the curve $\mathbf{r} = \langle -\cos(2t), \sin(2t), t\rangle$ over the interval $0 \le t \le \pi$.

Solution.

Step 1. Find out $\mathbf{r}'(t)$

$$\mathbf{r}'(t) = \frac{d}{dt}\langle -\cos(2t), \sin(2t), t\rangle = \langle 2\sin(2t), 2\cos(2t), 1\rangle$$

Step 2. Find out the length of $\mathbf{r}'(t)$

$$\begin{aligned} \|\mathbf{r}'(t)\| &= \sqrt{4\sin^2(2t) + 4\cos^2(2t) + 1^2} \\ &= \sqrt{4\left(\sin^2(2t) + \cos^2(2t)\right) + 1} \\ &= \sqrt{4+1} \\ &= \sqrt{5}. \end{aligned}$$

Step 3. Compute the integral for $0 \le t \le \pi$

$$\int_0^\pi \|\mathbf{r}'(t)\|\, dt = \int_0^\pi \sqrt{5}dt = \sqrt{5}\int_0^\pi 1dt = \sqrt{5}(\pi - 0) = \sqrt{5}\pi.$$

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Math 241 Quiz #3 version: 2

Instructor: Wenqiang Feng Name: ____________________

(1) (5 points) Find parametric equation for the tangent line to the curve $\mathbf{r}(t) = \langle 2t^2, t^3, 4t\rangle$ at the point where $t = 1$.

Solution.

Step 1. Find out the specific point at t=1

$$\mathbf{r}(1) = \left\langle 2 \cdot 1^2, 1^3, 4 \cdot 1 \right\rangle = \langle 2, 1, 4\rangle .$$

Step 2. Find out the direction vector of the tangent line

- direction vector for all t:

$$\mathbf{r}'(t) = \frac{d}{dt}\left\langle 2t^2, t^3, 4t \right\rangle = \left\langle 4t, 3t^2, 4 \right\rangle$$

- direction vector for specific t=1:

$$\mathbf{r}'(1) = \langle 4, 3, 4\rangle .$$

Step 3. Write out the parametric equation for the tangent line

$$\begin{aligned} T(t) &= \mathbf{r}(1) + t\mathbf{r}'(1) \\ &= \langle 2, 1, 4\rangle + t\,\langle 4, 3, 4\rangle \end{aligned}$$

Hence, the parametric equation for the tangent line is as follows

$$\begin{cases} x = 2 + 4t, \\ y = 1 + 3t, \\ z = 4 + 4t. \end{cases}$$

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(2) (5 points) Find the arc-length of the curve $\mathbf{r} = \langle 4t, -\cos(t), \sin(t)\rangle$ from the point $(0, -1, 0)$ to the point $(2\pi, 0, 1)$.

Solution.

Step 1. Find out $\mathbf{r}'(t)$

$$\mathbf{r}'(t) = \frac{d}{dt}\langle 4t, -\cos(2t), \sin(t)\rangle = \langle 4, \sin(t), cos(t)\rangle .$$

Step 2. Find out the length of $\mathbf{r}'(t)$

$$\begin{aligned} \|\mathbf{r}'(t)\| &= \sqrt{4^2 + \sin^2(t) + \cos^2(t)} \\ &= \sqrt{16 + (\sin^2(t) + cos^2(t))} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17}. \end{aligned}$$

Step 3. compute the domain of the integral

$$\begin{cases} 0 = 4t, \\ -1 = -\cos(t), \\ 0 = \sin(t). \end{cases} \Rightarrow t = 0, \quad \begin{cases} 2\pi = 4t, \\ 0 = -\cos(t), \\ 1 = \sin(t). \end{cases} \Rightarrow t = \frac{\pi}{2}.$$

Step 4. Compute the integral for $0 \leq t \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \|\mathbf{r}'(t)\| \, dt = \int_0^{\pi/2} \sqrt{17} dt = \sqrt{17} \int_0^{\pi/2} 1 dt = \sqrt{17}(\frac{\pi}{2} - 0) = \frac{\sqrt{17}}{2}\pi.$$

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