Math 241 Quiz #5 version: 1

Instructor: Wenqiang Feng Name: ______________________

(1) (5 points) Describe the set $E = \{(x, y, z) | 1 \leq x^2 + y^2 \leq 4, 0 \leq z \leq 3\}$ in cylindrical coordinates.

The Cylindrical coordinate (r, θ, z) in $\mathbb{R}^3$ is

$$\begin{cases} x &= r\cos\theta, \\ y &= r\sin\theta, \\ z &= z, \\ r &= \sqrt{x^2+y^2} \end{cases}$$

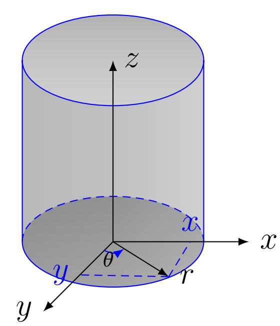

Figure 1. Cylindrical coordinate in $\mathbb{R}^3$

① Since $E = \{(x, y, z) | 1 \leq x^2 + y^2 \leq 4, 0 \leq z \leq 3\}$ and

$$\begin{cases} z &= z, \\ r &= \sqrt{x^2+y^2}, \end{cases}$$

then we have

$$\begin{cases} 0 \leq & z \leq 3, \\ 1 \leq & r^2 \leq 4 \end{cases} \Rightarrow \begin{cases} 0 \leq & z \leq 3, \\ 1 \leq & r \leq 2 \end{cases}$$

② For θ, we project the cylinder to the xy-plane Since there is no constrains for θ,

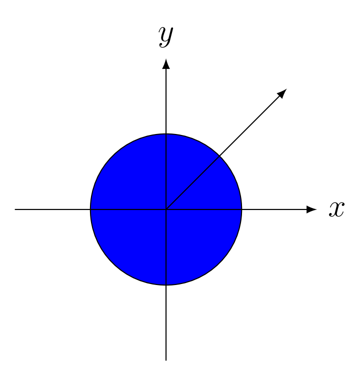

Figure 2. The projection region of cylinder onto xy-plane.

so $0 \leq \theta \leq 2\pi$.
Hence

$$E = \{(r, \theta, z) | 1 \leq r \leq 2, 0 \leq \theta \leq 2\pi, 0 \leq z \leq 3\}$$

(2) (5 points) Describe the set $E = \{(x, y, z) | x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ in spherical coordinates.

The Spherical coordinate (r, θ, ϕ) in $\mathbb{R}^3$ is

$$\begin{cases} x &= \rho \cos\theta \sin\phi, \\ y &= \rho \sin\theta \sin\phi, \\ z &= \rho \cos\phi \\ \rho &= \sqrt{x^2 + y^2 + z^2}. \end{cases}$$

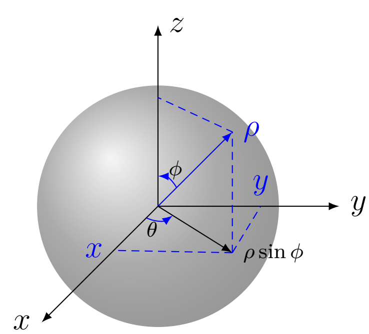

FIGURE 3. Spherical coordinate in $\mathbb{R}^3$

1 Since $E = \{(x, y, z) | x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ and

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

, then

$$\rho^2 \leq 4,$$

i.e.

$$0 \leq \rho \leq 2,$$

2 For θ. Since θ only involves in xy-plane, so we project the sphere to xy-plane.

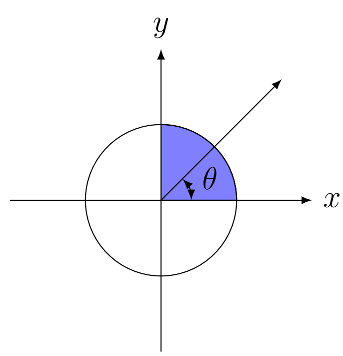

FIGURE 4. The projection region of sphere onto xy-plane.

Since $x \geq 0$ and $y \geq 0$, so θ should sit in the blue sector. That is to say

$$0 \leq \theta \leq \frac{\pi}{2}$$

3 For ϕ

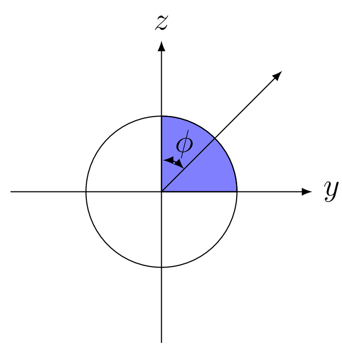

FIGURE 5. The projection region of sphere onto yz-plane.

Since $y \geq 0$ and $z \geq 0$, so θ should sit in the blue sector. That is to say

$$0 \leq \phi \leq \frac{\pi}{2}$$

Hence $E = \left\{(\rho, \theta, \phi) | 0 \leq \rho \leq 2, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq \frac{\pi}{2}\right\}$

Math 241 Quiz #5 version: 2

Instructor: Wenqiang Feng Name: ______________________

(1) (5 points) Describe the set $E = \{(x, y, z)|x^2 + y^2 \leq 1, x \geq 0, y \geq 0, 0 \leq z \leq 2\}$ in cylindrical coordinates.

The Cylindrical coordinate (r, θ, z) in $\mathbb{R}^3$ is

$$\begin{cases} x &= r\cos\theta, \\ y &= r\sin\theta, \\ z &= z, \\ r &= \sqrt{x^2+y^2} \end{cases}$$

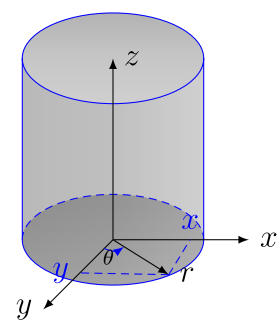

FIGURE 6. Cylindrical coordinate in $\mathbb{R}^3$

1 Since $E = \{(x, y, z)|x^2 + y^2 \leq 1, x \geq 0, y \geq 0, 0 \leq z \leq 2\}$ and

$$\begin{cases} z &= z, \\ r &= \sqrt{x^2+y^2}, \end{cases}$$

then we have

$$\begin{cases} 0 \leq & z \leq 2, \\ 0 \leq & r^2 \leq 1 \end{cases} \Rightarrow \begin{cases} 0 \leq & z \leq 3, \\ 0 \leq & r \leq 1 \end{cases}$$

2 For θ, we project the cylinder to the xy-plane Since $x \geq 0, y \geq 0$, then θ should

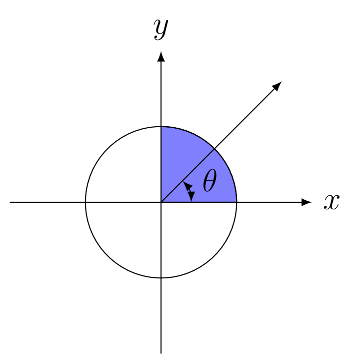

FIGURE 7. The projection region of sphere onto xy-plane.

sit in the blue sector, i.e.

$$0 \leq \theta \leq \frac{\pi}{2}$$

Hence

$$E = \left\{(r, \theta, z) | 0 \leq r \leq 1, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq z \leq 2\right\}$$

(2) (5 points) Describe the set $E = \{(x, y, z)|1 \leq x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ in spherical coordinates.

The Spherical coordinate (r, θ, ϕ) in $\mathbb{R}^3$ is

$$\begin{cases} x &= \rho \cos\theta \sin\phi, \\ y &= \rho \sin\theta \sin\phi, \\ z &= \rho \cos\phi \\ \rho &= \sqrt{x^2 + y^2 + z^2}. \end{cases}$$

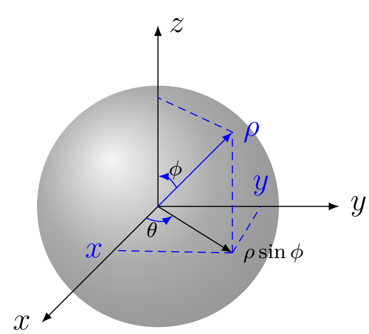

FIGURE 8. Spherical coordinate in $\mathbb{R}^3$

1 Since $E = \{(x, y, z)|1 \leq x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ and

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

, then

$$1 \leq \rho^2 \leq 4,$$

i.e.

$$1 \leq \rho \leq 2,$$

2 For θ. Since θ only involves in xy-plane, so we project the sphere to xy-plane. Since $x \geq 0$ and $y \geq 0$, so θ should sit in the blue sector. That is to say

$$0 \leq \theta \leq \frac{\pi}{2}$$

3 For ϕ
Since $y \geq 0$ and $z \geq 0$, so θ should sit in the blue sector. That is to say

$$0 \leq \phi \leq \frac{\pi}{2}$$

Hence $E = \left\{(\rho, \theta, \phi)|1 \leq \rho \leq 2, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq \frac{\pi}{2}\right\}$

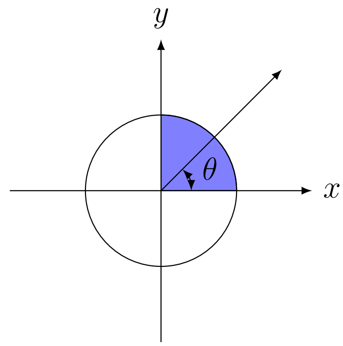

FIGURE 9. The projection region of sphere onto xy-plane.

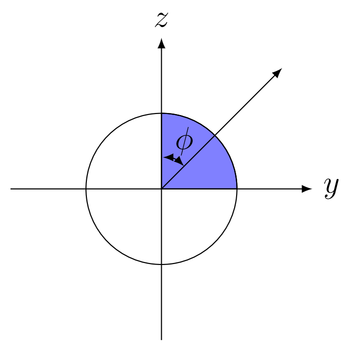

FIGURE 10. The projection region of sphere onto yz-plane.