

MATH113: Mathematical Reasoning- Exam 2

Spring 2015

Name (Print): _____ Solutions _____

Time Limit: 50 Minutes

Instructor(Sec:002): Wenqiang Feng

This exam contains **6** pages (including this cover page) and **9** problems, for a total of **100** points and **5** bonus points. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes on this exam.

You are required to show your work on each problem on this exam. The following Instructions apply:

- **You must have your own calculator if you want to use one. No cell phones, no sharing, and no trading allowed.** There are 100 points possible on this exam.
- **Do not ask questions during the exam.** I will not answer questions similar to the following: "Am I doing this right?" "Can you give me a hint?" "What does this mean?" If you have a photocopy issue and are unable to read something, then ask me. Otherwise, do not cause a disturbance during the exam.
- **Show all work whenever possible!** Full credit will be awarded for supporting work and/or justified answers on all short answer questions, so do not just give an answer. Please be neat and thorough with all your responses and proofs.
- **No scrap paper allowed.** If more room is needed to show work for a problem, use the back of the page.

Problem	Points	Score
1	10	
2	10	
3	10	
4	15	
5	15	
6	15	
7	15	
8	10	
9	0	
Total:	100	

Read and Sign Below.

The Honor Statement

An essential feature of the University of Tennessee is a commitment to maintaining an atmosphere of intellectual integrity and academic honesty. As a student of the University, I pledge that I will neither knowingly give nor receive any inappropriate assistance in academic work, thus affirming my own personal commitment to honor and integrity.

Pledged _____

1. (10 points) **TRUE** or **FALSE**.
 - (a) (2 points) $\sqrt{4}$ is a rational number. ☒ **TRUE** ☐ **FALSE**
 - (b) (2 points) The multiplication of two irrational numbers is always irrational.
☐ **TRUE** ☒ **FALSE**
 - (c) (2 points) 1 is a prime number. ☐ **TRUE** ☒ **FALSE**
 - (d) (2 points) Both rational and irrational numbers are real numbers. ☒ **TRUE** ☐ **FALSE**
 - (e) (2 points) The cardinality of Rational Numbers is \aleph_0 . ☒ **TRUE** ☐ **FALSE**

2. (10 points) Multiple choice. Circle the correct answer choice.
 - (a) (2 points) Find a rational number that is bigger than 2.7189 but smaller than 2.718901:
A. 2.7189005 B. 2.718900123456... C. 2.718900999999... D. 2.7
 - (b) (2 points) Which of the following sets S_1 and S_2 have a one-to-one correspondence?
 A. $\begin{cases} S_1=\{1,2,3,4\} \\ S_2=\{4,5,6\} \end{cases}$ B. $\begin{cases} S_1=\{5,6,7\} \\ S_2=\{1,2,3,4\} \end{cases}$ C. $\begin{cases} S_1=\{\#, \$, *\} \\ S_2=\{!, ?, \%\} \end{cases}$ D. $\begin{cases} S_1=\{\&, !, \#, *\} \\ S_2=\{1, 2, 3\} \end{cases}$
 - (c) (2 points) Which cardinality of the following set is different from the others?
A. Real Number B. Natural Number C. Whole Number D. Integers
 - (d) (2 points) Which of the following is **WRONG** for ∞ ?
A. $\frac{\infty}{\infty} = 1$ B. $1 + \infty = \infty$ C. $\infty + \infty = \infty$ D. $\infty * \infty = \infty$
 - (e) (2 points) Who proved the set of real numbers has more elements than the set of natural numbers?
 A. Fibonacci B. Fermat C. Pythagoras **D. Cantor**

3. (10 points) Let \mathbb{T} be the set of all real numbers between 0 and 1 with the property that their decimal expansions only have 1's and 7's.
 - (a) (5 points) Show that there exists a rational number in \mathbb{T} .

Solution. eg: 0.17, 0.1771, $0.\overline{17}\dots$



- (b) (5 points) Show that there exists an irrational number in \mathbb{T} .

Solution. eg: 0.1717777771111... which is following are randomly chose from 1 or 7.



4. (15 points) Put the following numbers into the proper categories (remember that some numbers might belong to several categories).

$$\sqrt{3}, -1, 0, 1, 5, \frac{1}{3}, \sqrt{4}, -\frac{14}{7}, 0.23, \frac{\sqrt{3}}{2}, 0.3\overline{17}, \infty$$

<p>Natural numbers</p> <p>1, 5, $\sqrt{4}$</p>	<p>Whole numbers</p> <p>0, 1, 5, $\sqrt{4}$</p>
<p>Rational numbers</p> <p>$-1, 0, 1, 5, \frac{1}{3}, \sqrt{4}, -\frac{14}{7}, 0.23, 0.3\overline{17}$</p>	<p>Irrational numbers</p> <p>$\sqrt{3}, \frac{\sqrt{3}}{2}$</p>
<p>Integers</p> <p>$-1, 0, 1, 5, \sqrt{4}, -\frac{14}{7}$</p>	<p>Real numbers</p> <p>all except ∞</p>

5. (15 points) The UPC((Universal Product Code) bar codes on my package was unfortunately destroyed and one number was missing. Please help me figure out the value of this missing number.



Solution. From the bar code format, we know that the value of **fourth digit number** was missing. let the value of **sixth digit number** be x . Then according to the algorithm, we need to compute the following component:

1. The sum of the odd digit number:

$$8 + 1 + 0 + 0 + 2 + 4 = 15$$

and the $3 \times$ (sum of the odd digit)

$$3 \times 15 = 45$$

2. The sum of the even digit number: $1 + x + 4 + 1 + 3 + 4(\text{check digit}) = 13 + x$

3. modulo 10 part:

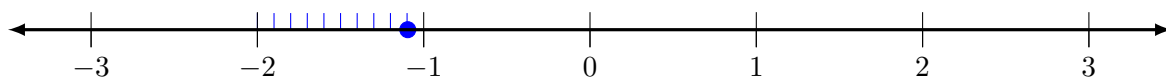
$$(45 + 13 + x) \mod 10 = 0$$

$$\Rightarrow x = 2$$

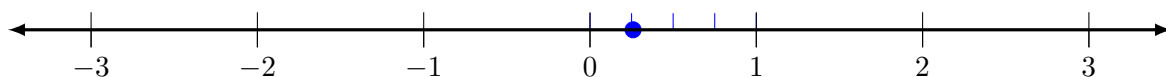


6. (15 points) Answer the following using the given number lines.

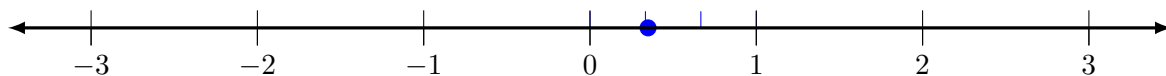
(a) (2 points) Mark the exact location for $-\frac{11}{10}$.



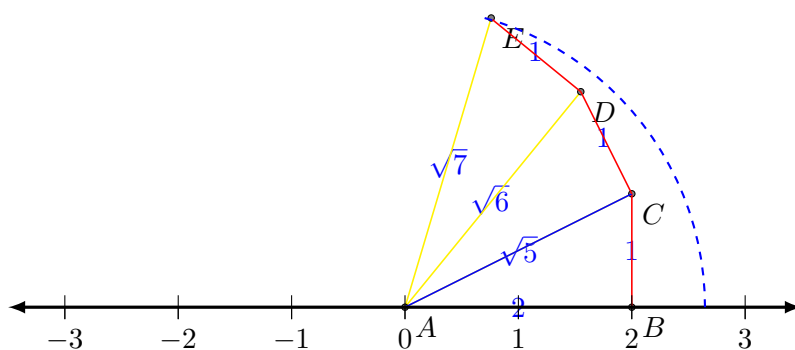
(b) (3 points) Mark the exact location for 0.25.



(c) (5 points) Mark the exact location for $0.\bar{3}$.



(d) (5 points) Mark the exact location for $\sqrt{7}$.



7. (15 points) The following problems are short answer. Fully justify all answers .
- (a) (5 points) Write the 3.14 into fraction.

Solution.

$$3.14 = \frac{3.14}{1} = \frac{314}{100} = \frac{157}{50}$$



- (b) (10 points) Prove that $0.3\overline{17}$ is a rational number.

Solution. Let $x = 0.3\overline{17}$, then we have

$$\begin{aligned} 100 * x &= 31.717171717171717 \dots \\ x &= 0.317171717171717 \dots \end{aligned}$$

So,

$$99x = 31.4.$$

Hence,

$$x = \frac{31.4}{99} = \frac{314}{990} = \frac{157}{495} = 0.3\overline{17}.$$

Therefore, $0.3\overline{17}$ is a rational number.



8. (10 points) Setting up a one-to-one correspondence between two sets.

(a) (2 points) Finite case: $\mathcal{S}_1 = \{1, 3, 4\}$ and $\mathcal{S}_2 = \{5, 7, 8\}$.

Solution.

$$\begin{array}{lcl} 1 & \rightarrow & 5 \\ 3 & \rightarrow & 7 \\ 4 & \rightarrow & 8 \end{array}$$



(b) (8 points) Infinite case: Set of Integers: $\mathcal{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ and the set of natural numbers $\mathcal{N} = \{1, 2, 3, \dots\}$.

Solution.

$$\begin{array}{lcl} 1 & \rightarrow & 0 \\ 2 & \rightarrow & 1 \\ 3 & \rightarrow & -1 \\ 4 & \rightarrow & 2 \\ 5 & \rightarrow & -2 \\ \vdots & & \vdots \end{array}$$



9. (5 points (bonus)) Your friend, a spy, gives you a list of four 5 digit numbers that have the power to set off a series of catastrophic events unless you can help. Can you write a number that you know for sure will **NOT** be on the list?

4					
	3				
		6			
			7		
				2	

Your New Number: 5 4 7 8 3

