MATH113: Mathematical Reasoning- Exam 1

Spring 2015 **Name (Print):** Solution

Time Limit: 50 Minutes **Instructor(Sec:002):** Wenqiang Feng

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This exam contains 5 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes on this exam.

You are required to show your work on each problem on this exam. The following Instructions apply:

- **You must have your own calculator if you want to use one. No cell phones, no sharing, and no trading allowed.** There are 100 points possible on this exam.
- **Do not ask questions during the exam.** I will not answer questions similar to the following: "Am I doing this right?" "Can you give me a hint?" "What does this mean?" If you have a photocopy issue and are unable to read something, then ask me. Otherwise, do not cause a disturbance during the exam.
- **Show all work whenever possible!** Full credit will be awarded for supporting work and/or justified answers on all short answer questions, so do not just give an answer. Please be neat and thorough with all your responses and proofs.
- **No scrap paper allowed.** If more room is needed to show work for a problem, use the back of the page.

Problem	Points	Score
1	10	
2	10	
3	10	
4	15	
5	15	
6	20	
7	20	
Total:	100	

Read and Sign Below.

The Honor Statement

An essential feature of the University of Tennessee is a commitment to maintaining an atmosphere of intellectual integrity and academic honesty. As a student of the University, I pledge that I will neither knowingly give nor receive any inappropriate assistance in academic work, thus affirming my own personal commitment to honor and integrity.

Pledged _____

1. (10 points) Circle one: **TRUE** or **FALSE** for quantitative describing sentence.

   (a) (2 points) We have 24 people in our class.
   ✓TRUE FALSE

   (b) (2 points) Many people spend much of their time playing computer games.
   TRUE ✓FALSE

   (c) (2 points) The orchard is 30 acres in size.
   ✓TRUE FALSE

   (d) (2 points) Nearly 1.3 million people die in road crashes each year.
   ✓TRUE FALSE

   (e) (2 points) About 71 percent of the Earth's surface is water-covered.
   ✓TRUE FALSE

2. (10 points) Multiple choice. Circle the correct answer choice.

   (a) (2 points) Let $\varphi = 1 + \frac{1}{1+\frac{1}{1+\cdots}}$, the Golden ratio Identity that comes from this is:?

   a. $\varphi = \frac{1}{\varphi}$ b. $\varphi = \varphi + \frac{1}{\varphi}$ (c.) $\varphi = 1 + \frac{1}{\varphi}$ d. $\varphi = 1 + \varphi$

   (b) (2 points) The Golden Ratio (φ) is equivalent to:
   (a.) $\frac{1+\sqrt{5}}{2}$ b. $1 + \frac{1}{3}$ c. 1.516 d. $\frac{8}{5}$

   (c) (2 points) It is now spring, what season will it be 63 seasons from now?
   a. spring b. summer c. fall (d.) winter

   (d) (2 points) 70 can be rewrite as a sum of Fibonacci numbers:
   a. 55+15 (b.) 55+13+2 c. 60+8+2 d. 21+49

   (e) (2 points) What are the prime factors of 100?
   (a.) 2, 5 b. 1, 2, 5 c. 2, 50 d. 100 is prime

3. (10 points) Using what you have learned answer the following questions.

   (a) (5 points) Describe in your own words the pigeonhole principle.

   Solution. (This answer from Eric Davis): if there are 10 birds and 9 nests, then there one nest will have more than 1 birds. ◀

   (b) (5 points) A famer has a apple orchard. The orchard is 30 acres in size and has about 35 trees per acre with each tree producing about 200 apples. Estimate the total number of apples.

   Solution. Trees: $30 \times 35 = 1050$
   Apples: $1050 \times 200 = 210,000$. ◀

4. (15 points) Fill in the table below with the desired Fibonacci Numbers.

n	F_{n+1}	F_n	$(2 \cdot F_{n+1}) + F_n$
1	1	1	3
2	2	1	5
3	3	2	8
4	5	3	13
5	8	5	21

5. (15 points) The following problems are short answer. Fully justify all answers .

   (a) (7 points) Take this division problem $59 \div 4$ and write an equation using the division algorithm. Using your equation, explain how to know if 4 is a factor of 59?

   Solution. By the long division algorithm, we have

$$\begin{array}{r} 14 \\ 4\overline{)59} \\ \underline{4} \\ 19 \\ \underline{16} \\ 3 \end{array}$$

   So, we can rewrite $59 \div 4$ in the division algorithm as

$$59 = 4 \times 14 + 3.$$

   Since the reminder $r = 3 \neq 0$, then 4 does not divide 59 evenly. Therefore, 4 is not a factor of 59.

   ◀

   (b) (8 points) Suppose P is a prime number greater than or equal to 3. Can P + 1 be prime? (CLEARLY prove your answer.)

   Solution. Since P is a prime number greater than or equal to 3, then we know that P should be odd number. (Since if P is a even number, then 2 divides P evenly, and then 2 must be a factor of P. Therefore P should not be prime number which contradicts to P is a prime number.) Since P is a also a odd number, then

$$P + 1 = \text{even} \geq 2.$$

   Therefore $P + 1$ can not be prime, since the even number greater than 2 has factor 2. ◀

6. (20 points) The following problems are short answer. Fully justify all answers .

(a) (5 points) $34 \equiv$ _____ mod 13

Solution. By the definition of congruence, we need to find some number which satisfies

$$34 \mod 13 = \_\_\_\_\_ \mod 13$$

Then by long division algorithm, we have

$$\begin{array}{r} 2 \\ 13\overline{)34} \\ \underline{26} \\ 8 \end{array}$$

So, we have

$$\_\_\_\_\_ \mod 13 = 8.$$

Then, by division algorithm, we have

$$\_\_\_\_\_ = Int * 13 + 8,$$

where Int is an arbitrary integer.
E.g :$-5, 8, 21, \cdots$ ◀

(b) (5 points) $-55 \equiv$ _____ mod 13

Solution. By the definition of congruence, we need to find some number which satisfies

$$-55 \mod 13 = \_\_\_\_\_ \mod 13$$

Then by long division algorithm, we have

$$\begin{array}{r} -5 \\ 13\overline{)-55} \\ \underline{-65} \\ 10 \end{array}$$

So, we have

$$\_\_\_\_\_ \mod 13 = 10.$$

Then, by division algorithm, we have

$$\_\_\_\_\_ = Int * 13 + 10,$$

where Int is an arbitrary integer.
E.g :$3, 10, 36, \cdots$ ◀

(c) (5 points) Verify the identity: $(14 + 17) \mod 5 = (14 \mod 5 + 17 \mod 5) \mod 5$.

Solution. Firstly, we compute the Left Hand Side (LHS).

$$LHS = (14 + 17) \mod 5 = 31 \mod 5 = 1.$$

Then, we compute the Right Hand Side (RHS).

$$LHS = (14 \mod 5 + 17 \mod 5) \mod 5 = (4 + 2) \mod 5 = 6 \mod 5 = 1.$$

Therefore: $LHS = RHS$. ◀

(d) (5 points) Verify the identity: $(4 * 7) \mod 6 = (4 \mod 6 * 7 \mod 6) \mod 6$.

Solution. Firstly, we compute the Left Hand Side (LHS).

$$LHS = 28 \mod 6 = 4.$$

Then, we compute the Right Hand Side (RHS).

$$LHS = (4 * 1) \mod 6 = 4 \mod 6 = 4.$$

Therefore: $LHS = RHS$. ◀

7. (20 points) Fill out the following table.

   (a) (10 points) Fill out the ADDITION table for modular arithmetic (mod 6) as shown:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	1
3	3	4	5	0	1
4	4	5	0	1	2

   (b) (10 points) Fill out the MULTIPLICATION table for modular arithmetic (mod 6) as shown:

*	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	0	2
3	0	3	0	3	0
4	0	4	2	0	4