

Spam filter

Week1-ex1, solution

This exercise, with many alternative stories and numerical values, is a classical example of the importance of prior information and of taking into account the baseline prevalence.

Erik knows that 65% of his emails are spam. Spam filter classifies 75% of spam to be spam. Furthermore, filter classifies 6% of emails as spam even if they are not spam. What is the probability that an email classified as spam is not spam?

Answer

prior: $p(\text{spam} = 1) = 0.65$

likelihood: $p(\text{test} = 1|\text{spam} = 1) = 0.75$ and $p(\text{test} = 1|\text{spam} = 0) = 0.06$

posterior probability that an email classified as spam is actually spam is:

$$\begin{aligned} p(\text{spam} = 1|\text{test} = 1) &= \frac{p(\text{test} = 1|\text{spam} = 1)p(\text{spam} = 1)}{p(\text{test} = 1|\text{spam} = 1)p(\text{spam} = 1) + p(\text{test} = 1|\text{spam} = 0)p(\text{spam} = 0)} \\ &= \frac{0.75 \times 0.65}{0.75 \times 0.65 + 0.06 \times 0.35} \end{aligned}$$

Hence, the final answer is

$$p(\text{spam} = 0|\text{test} = 1) = 1 - p(\text{spam} = 1|\text{test} = 1)$$

$$1 - (0.75 * 0.65) / (0.75 * 0.65 + 0.06 * 0.35)$$

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## [1] 0.04129794
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Grading

Total 10 points: 3 points if prior and likelihood have been defined correctly. Extra 4 points if the Bayes Theorem has been formed correctly. Extra 3 points (total 10) if the result is correct.