

Conditional probability

Week1-ex3, solution

Suppose that if $\theta = 1$, then y has a Binomial distribution with success probability 0.2, and if $\theta = 2$, then y has a Binomial distribution with success probability 0.6. Also, suppose that the sample size is $n = 10$ and $Pr(\theta = 1) = 0.5$ and $Pr(\theta = 2) = 0.5$.

- Write the formula for the marginal probability density for y and sketch it.
- What is $Pr(\theta = 1|y = 3)$
- sketch the posterior predictive distribution for a new observation \tilde{y} with sample size $\tilde{n} = 10$. What is the posterior predictive probability that $\tilde{y} = 3$?

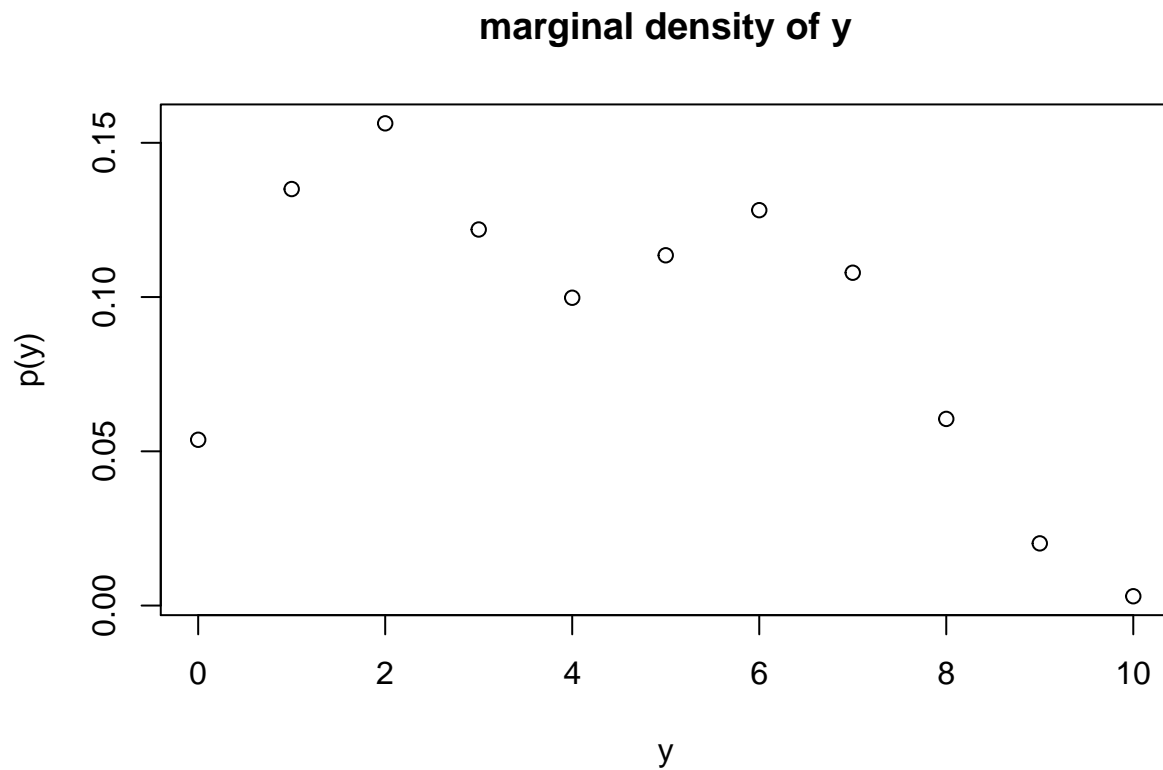
Solution

Part a)

The marginal probability density for y is

$$p(y) = 0.5 * \text{Bin}(y|10, 0.2) + 0.5 * \text{Bin}(y|10, 0.6)$$

```
n=10
y = seq(0, n, length=n+1)
# Calculate and draw the marginal density of y
ypdf = 0.5*dbinom(y,size = n,prob = 0.2) + 0.5*dbinom(y,size = n,prob = 0.6)
plot(y,ypdf, main="marginal density of y", xlab="y", ylab="p(y)")
```



Part b)

posterior probability that $\theta = 1$

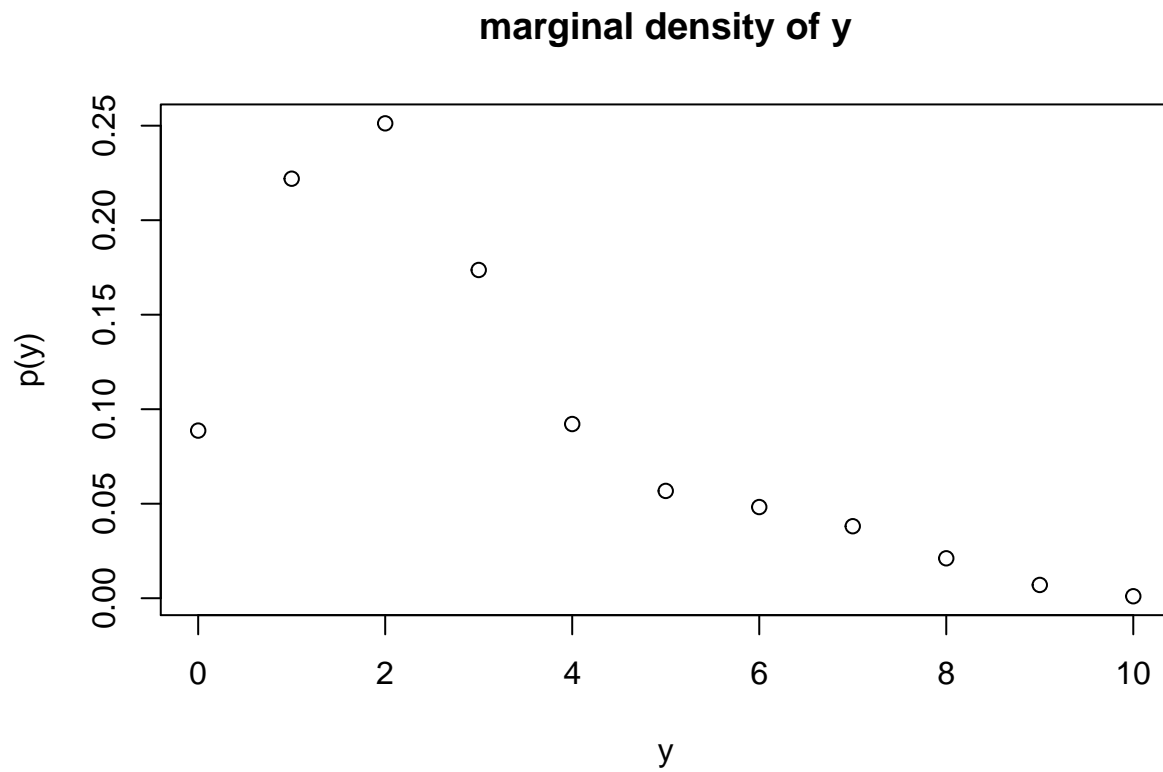
```
Pr_post = dbinom(3,size=n,prob=0.2)/(dbinom(3,size=n,prob=0.2) + dbinom(3,size=n,prob=0.6))
print(Pr_post)
```

```
## [1] 0.8258065
```

Part c)

Let's sketch the posterior predictive distribution of new observation

```
n=10
y = seq(0, n, length=n+1)
# Calculate and draw the marginal density of y
ypdf = Pr_post*dbinom(y,size = n,prob = 0.2) + (1-Pr_post)*dbinom(y,size = n,prob = 0.6)
plot(y,ypdf, main="marginal density of y", xlab="y", ylab="p(y)")
```



The posterior predictive probability that $\tilde{y} = 3$ is

```
Pr_post*dbinom(3,size = n,prob = 0.2) + (1-Pr_post)*dbinom(3,size = n,prob = 0.6)
```

```
## [1] 0.1736543
```

Grading

Total 10 points: 3 points for getting a) correct. 4 points for getting b) correct. 3 points for getting c) correct.