

Directed acyclic graphs

Week4-ex1, solution

a)-c) 1) Write down the joint posterior probability distribution of all the parameters in the directed acyclic graphs (DAG) shown in Figure 1 and 2) Write down a Stan pseudo code that tells how a model corresponding to that DAG would be written. You can assume that all variables get values in real numbers. An example of a model answer is provided on the right

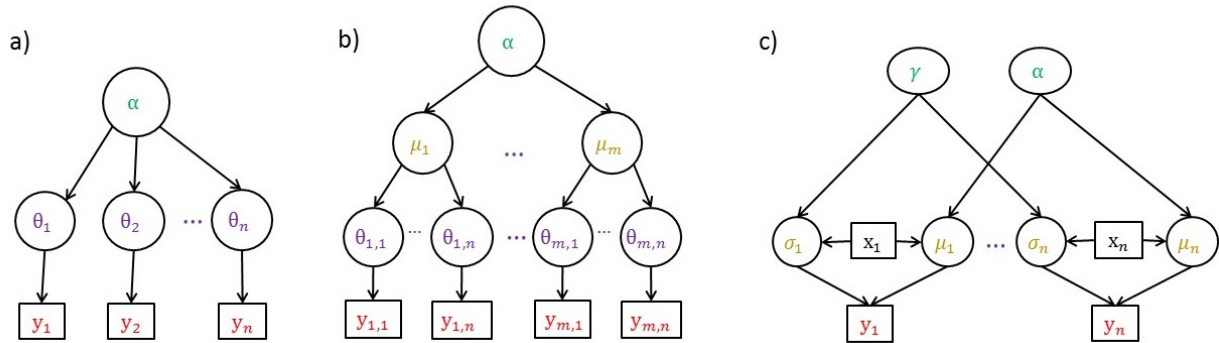
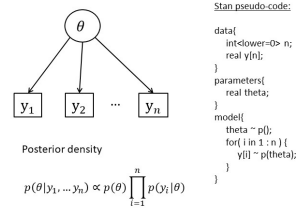


Figure 1: The DAGs for which the joint distribution and pseudo code have to be defined. Note variables denoted by x should be treated as covariates.

d) Draw a Directed acyclic graph (DAG) and write a Stan pseudo code of the following model

$$\begin{aligned}
 y_{i,j} &\sim N(\mu_j, \sigma_j^2), i = 1, \dots, n, j = 1, \dots, J \\
 \mu_j &\sim N(\mu_0, \phi) \\
 \mu_0 &\sim N(0, 10^6) \\
 \phi &\sim \text{Inv-}\chi^2(\nu_1, s_1^2) \\
 \sigma_j^2 &\sim \text{Inv-}\chi^2(\nu_2, s_2^2)
 \end{aligned}$$

See the previous problem for an example on the needed accuracy for the pseudo code.

Solution

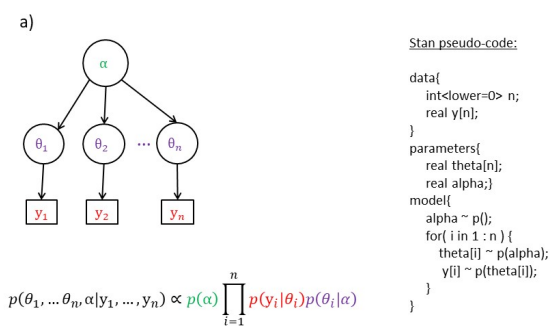


Figure 2: Model answer a)

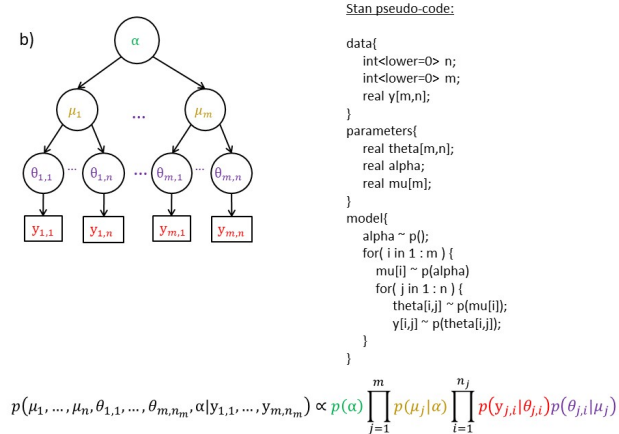


Figure 3: Model answer b)

Grading

Total 20 points. a)-c) Two points for correct joint density function and 3 points for correct pseudo-code.
d) 3 points for correct DAG and 2 points for correct pseudo-code.

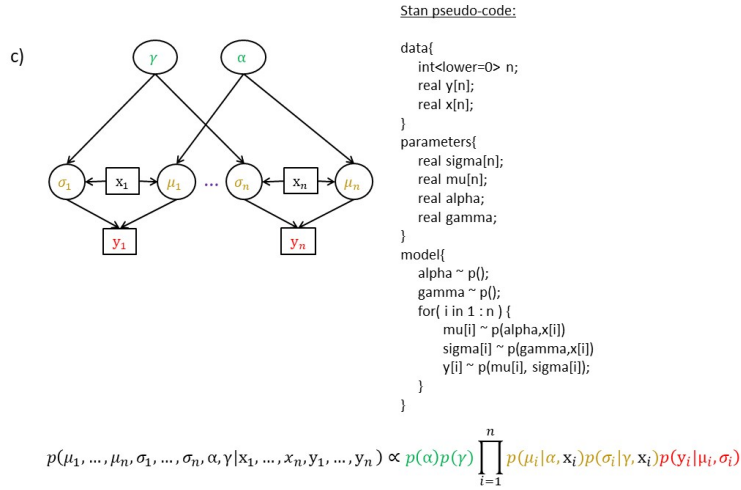


Figure 4: Model answer c)

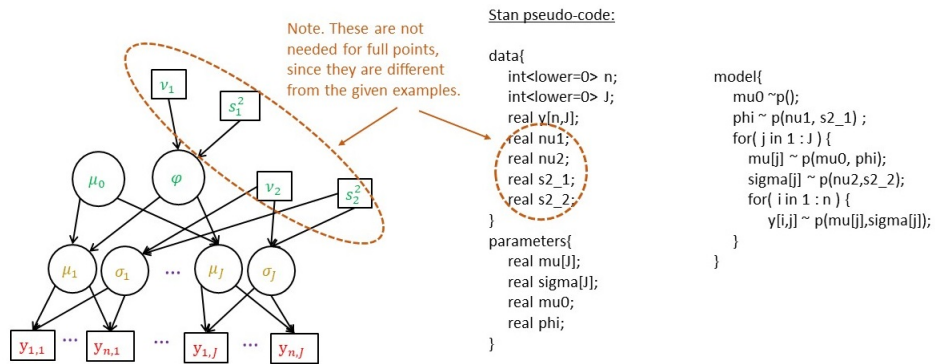


Figure 5: Model answer d)