Aerodynamics Computational Assignment #2: Flow Over Airfoils

Assigned Date: September 14/17, 2021
Due Date: October 12/15, 2021

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Part 1. Preliminary work on Thin Airfoils
Reflection Questions:

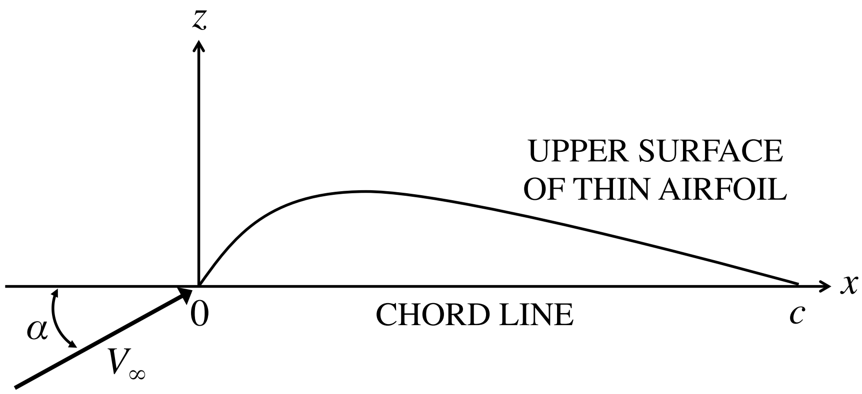
In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand how thin airfoil theory is used to approximate aerodynamic forces.
2. Practice using the superposition of elementary flows to complete analysis of an airfoil.
3. Understand the effect of flow parameters on streamlines, equipotential lines, and pressure contours.

Problem 1:

The flow about a thin symmetric airfoil can be approximated by potential flow theory, as elaborated in Chapter 4 of Anderson. Suppose that the chord of the airfoil extends along the x-axis from $x = 0$ to $x = c$ as illustrated in the below figure:

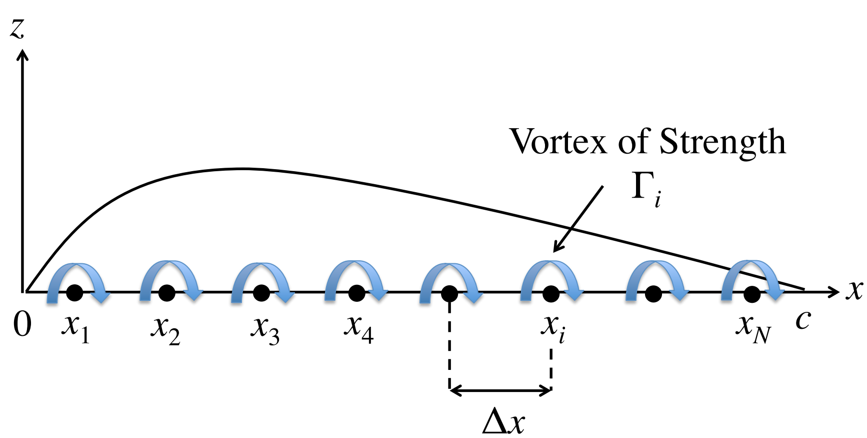


Then, the flow is represented by a vortex sheet whose strength $\gamma(x)$ is given by:

$$\gamma(x) = 2\alpha V_\infty \sqrt{\frac{1 - \frac{x}{c}}{\frac{x}{c}}}$$

where α is the angle of attack of the incoming flow relative to the x-axis and V_∞ is the free-stream flow speed. (This formula follows from Anderson 4.24 mapped back to x/c using 4.21.)

Numerically, one can approximate the vortex sheet by a set of N discrete vortices separated by a distance $\Delta x = c/N$ where the i^{th} vortex has strength $\Gamma_i = \gamma(x_i)\Delta x$. This is depicted in the below figure:



Write a MATLAB function which plots the stream lines, equipotential lines, and pressure contours for flow about a thin symmetric airfoil using the approximations detailed above.

Your function should take the form:

```
function Plot_Airfoil_Flow(c,alpha,V_inf,p_inf,rho_inf,N)
```

where `c` is the chord length c (in meters), `alpha` is the angle of attack α (in degrees), `V_inf` is the free-stream flow speed V_∞ (in meters per second), `p_inf` is the free-stream pressure p_∞ (in Pascals), `rho_inf` is the free-stream density ρ_∞ (in kilograms per meter cubed), and `N` is the number of discrete vortices N employed to approximate the vortex sheet.

Using your MATLAB function,

- Visualize or generate plots of the stream lines, equipotential lines, and pressure contours for flow about a thin symmetric airfoil with $c = 1.5$ m, $\alpha = 6°$, $V_\infty = 30$ m/s, $p_\infty = 101.3 \times 10^3$ Pa, and $\rho_\infty = 1.225$ kg/m^3.
- Conduct a study of the effect of the number of discrete vortices N on the resulting flow and pressure field accuracy for the aforementioned values. As we will be doing a more quantitative assessment of accuracy in part 2, this one can be simply qualitative.

Reflection: Evaluate the pressure contours, and identify the locations of minimum and maximum pressure. How do these change as the above conditions are altered? Consider the streamlines and equipotential lines; are the fields continuous? What does this imply?

Hint: You will need to use the principle of superposition within your MATLAB function. This principle is illustrated in the MATLAB file `Lifting_Cylinder.m` located on the course website, wherein the stream lines for the flow around a cylinder with circulation are plotted by superposing flow from a uniform flow, a dipole, and a vortex. It is recommended that you follow the logic of `Lifting_Cylinder.m` when building your function `Plot_Airfoil_Flow`.

Part 2. Thick airfoils
Reflection Questions:

In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Practice using the vortex panel method to calculate aerodynamic forces on a thick airfoil.
2. Understand the difference between the application and results of thin airfoil theory and the vortex panel method.
3. Understand how the changes in wing section camber and thickness alter the lift slope.

You will be provided (via Canvas) a MATLAB function to apply the vortex panel method for an arbitrary two-dimensional body defined by a set of (x, y) coordinates that define its surface. The provided function will include the following steps:

1. Read in the list of points.
2. Read in necessary flow conditions (e.g. the free-stream flow speed and angle of attack).
3. Form the system of equations.
4. Solve the system of equations.
5. Plot the coefficient of pressure (if flag is set).
6. Return the sectional coefficient of lift.

The provided function will take the form:

```
function c_l = Vortex_Panel(x,y,V_inf,alpha, flag)
```

where `c_l` is the section coefficient of lift (to be computed and returned), `x` is a vector containing the x-location of your coordinates (i.e. the boundary points in the panel method), `y` is a vector containing the y-location of your coordinates, `V_inf` is the free-stream flow speed, and `alpha` is the angle of attack. The first and last entries of x and y correspond to the trailing edge and traverse clockwise.

Problem #2:

Apply the provided vortex panel method MATLAB function to lifting flow over a NACA 0012 airfoil at various angles of attack. For an angle of attack of $0°$, compute the flow for a couple of different resolutions and compare the results. From this study, choose a nominal number of panels required for a desired *quantitative* level of accuracy. It is up to the student to define a measure of error in this regard. Produce a plot of the change in error versus the number of total panels (e.g. upper and lower surfaces) and print the nominal number of panels needed to the command window. Then, using the nominal number of panels, compute lifting flow over a NACA 0012 airfoil and plot the results (i.e. coefficients of lift and pressure) for the following angles of attack: $\alpha = -6°, 0°, 6°, 9°$.

Reflection: How does the pressure coefficient field change with increasing angle of attack? How similar is it to thin airfoil theory and what differences exist?

Problem #3:

Using the provided vortex panel method MATLAB function, obtain plots of the sectional coefficient of lift versus angle of attack for the following airfoils:

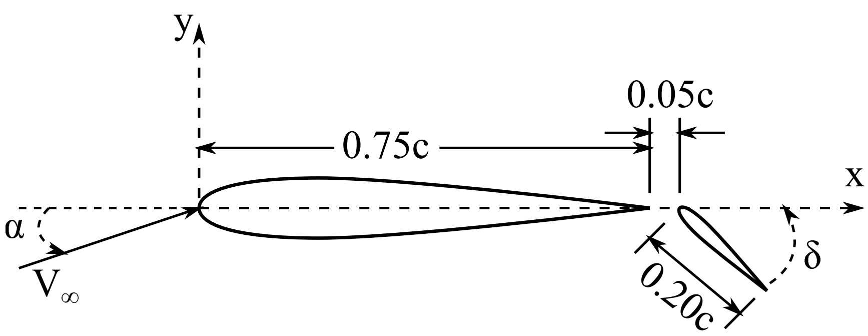
- NACA 0012
- NACA 2412
- NACA 4412
- NACA 2424

Using these plots, estimate the lift slope and zero-lift angle of attack for each of the airfoils, and compare these results with thin airfoil theory. It is recommended that you plot all of these together to provide a clearer comparison.

Reflection: How do changes in the wing section camber and thickness alter the lift slope and the zero lift angle of attack?

Bonus (Up to +10 **Points):**

Use the vortex panel method to compute lifting flow over the following multi-element wing: where both the airfoil and trailing edge flap are constructed from NACA 0012 airfoils. Obtain a plot of the section coefficient of lift versus angle of attack for a deflection angle of $\delta = 20°$, and compare this plot with the corresponding plot of sectional coefficient of lift versus angle of attack for a NACA 0012 airfoil without flap.



Suggested Approach:

You will need to build 4-digit NACA airfoils repeatedly throughout this lab. As such, it is suggested that you build a MATLAB function to construct panels for a given NACA airfoil. For instance, your MATLAB function may take the form:

```
[x,y] = function NACA_Airfoils(m,p,t,c,N)
```

where `x` is a vector containing the x-location of the boundary points, `y` is a vector containing the y-location of the boundary points, `m` is the maximum camber, `p` is the location of maximum camber, `t` is the thickness, `c` is the chord length, and `N` is the number of employed panels. Note, if you use points generated from a cosine map (fig 5.24 of KC handout where equal spaced points in θ produce points packed near the leading and trailing edge) the number of required points is substantially reduced but this is not required.

Note: The formula for the shape of a NACA 4-digit series airfoil with camber is a bit involved. The first ingredient is the thickness distribution of the airfoil from the mean camber line, which is given by:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (mean camber line to surface), and t is the maximum thickness as a fraction of the chord. As with the case of a symmetric NACA airfoil, the last two digits in the NACA XXXX description gives $100t$. The second ingredient is the formula for the mean camber line, which is:

$$y_c = \begin{cases} m\frac{x}{p^2}\left(2p - \frac{x}{c}\right), & 0 \le x \le pc \\ m\frac{c-x}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right), & pc \le x \le c \end{cases}$$

where m is the maximum camber and p is the location of maximum camber. The first digit in the NACA XXXX description gives $100m$ while the second digit gives $10p$. Then, the coordinates (x_U, y_U) and (x_L, y_L) of the upper and lower airfoil surface, respectively, become:

$$x_U = x - y_t \sin\xi \qquad\qquad y_U = y_c + y_t \cos\xi$$
$$x_L = x + y_t \sin\xi \qquad\qquad y_L = y_c - y_t \cos\xi$$

where

$$\xi = \arctan\left(\frac{dy_c}{dx}\right).$$

Note that for the NACA 4415 airfoil, $m = 4/100$, $p = 4/10$, and $t = 15/100$.