Aerodynamics Computational Assignment #3: Flow Over Finite Wings

Assigned Date: October 12/15, 2021
Due Date: November 1/4, 2021

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script along with a pdf of your code and its execution (i.e. printed comments and figures) should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Practice using Prandtl Lifting Line Theory to calculate lift and drag on a finite airfoil.
2. Understand how the number of terms in PLLT affect the resulting error in the solution.
3. Understand how design factors, like the taper and aspect ratio, affect aerodynamic efficiency and induced drag.

Problem #1:

Write a MATLAB function which solves the fundamental equation of Prandtl Lifting Line Theory for finite wings with thick airfoils:

$$\alpha(\theta) = \frac{4b}{a_0(\theta)c(\theta)} \sum_{n=1}^{\infty} A_n \sin(n\theta) + \alpha_{L=0}(\theta) + \sum_{n=1}^{\infty} nA_n \frac{\sin(n\theta)}{\sin(\theta)}$$

by satisfying the equation at the N prescribed locations:

$$\theta_i = \frac{i\pi}{2N}, \quad i = 1, \ldots, N$$

and truncating the series expansion for circulation using N odd terms:

$$\Gamma(\theta) = 2bV_\infty \sum_{j=1}^{N} A_{(2j-1)} \sin((2j-1)\theta)$$

Your function should be general enough to work for an arbitrary number of terms in the series expansion for circulation and should allow for a linear spanwise variation of the cross-sectional lift slope, the local chord length, the aerodynamic twist, and the geometric twist. Your function should return as output the span efficiency factor as well as the coefficient of lift and coefficient of induced drag. Consequently, your function should take the form:

```
function [e,c_L,c_Di] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N)
```

where `e` is the span efficiency factor (to be computed and returned), `c_L` is the coefficient of lift (to be computed and returned), `c_Di` is the induced coefficient of drag (to be computed and returned), `b` is the span (in feet), `a0_t` is the cross-sectional lift slope at the tips (per radian), `a0_r` is the cross-sectional lift slope at the root (per radian), `c_t` is the chord at the tips (in feet), `c_r` is the chord at the root (in feet), `aero_t` is the zero-lift angle of attack at the tips (in degrees), `aero_r` is the zero-lift angle of attack at the root (in degrees), `geo_t` is the geometric angle of attack at the tips (in degrees), `geo_r` is the geometric angle of attack at the root (in degrees), and N is the number of odd terms to include in the series expansion for circulation.

Problem #2:

Consider a wing with a span of 80 ft and a straight taper from 12 ft root chord to 4 ft tip chord. The root airfoil is chosen to be a NACA 2412 while the tip airfoil is chosen to be a NACA 0012. This results in a linear spanwise variation of cross-sectional lift slope and zero-lift angle of attack. The wing is also twisted such that the geometric angle of attack varies linearly from $6°$ at the root to $1°$ at the tips.

Using the MATLAB function you wrote for Problem #1, determine the lift and induced drag for the wing at a sea level airspeed of 130 miles per hour. Moreover, complete the following tasks:

- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with five percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with two percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with 2/10 percent relative error. Print this value to the command window.

Reflection: In this lab only the odd terms were utilized in the PLLT series expansion, why? When would both the odd and even terms be required?

Note: To compute the cross-sectional lift slope and zero-lift angle of attack at the root and tips, use the vortex panel code you developed in Computational Assignment 2.

Problem #3:

Using the MATLAB function you wrote for Problem #1, make a plot of the span efficiency factor e versus taper ratio c_t/c_r for a *thin* wing with no aerodynamic or geometric twist and aspect ratios $AR = 4, 6, 8, 10$ where c_t is the tip chord and c_r is the root chord. Use at least twenty odd terms in your series expansion for circulation in generating your plot. Your resulting plot should look similar in style to Fig. 5.20 in Anderson's *Fundamental's of Aerodynamics*.

Reflection: Consider the dependence of the span efficiency factor on both the taper ratio and aspect ratio. Under what conditions is the wing the most aerodynamically efficient or under what conditions is the induced drag minimized? How does this compare to the theoretical wing planform with the minimum induced drag?

Note: The aerodynamic twist is defined as the difference in zero-lift angle of attack between a given wing section and the wing section at the root, and the geometric twist is defined as the difference in geometric angle of attack between a given wing section and the wing section at the root. Therefore, a wing with no aerodynamic or geometric twist may still be at some geometric angle of attack, but it is uniform across the spanwise direction.

Hint: For a wing with no aerodynamic or geometric twist, the span efficiency factor is independent of the geometric and zero-lift angles of attack.