



Figure 1: A vessel with structures [More reefer plugs] [-skal laves om]

## 1 Standard Vessel Model

In this section we will describe the data given for a stowage model, and we will explain how this information is translated into our proposed *Standard Vessel Model* (SVM), whose main purpose is to abstract away much of the unnecessary complexity relating to the physical layout of the vessel.

The data comes from...

### Vessel data

**Vessel structure** The cargo space of a container vessel is divided into parts called *bays* that each consists of a grid of *cells*. Each cell is divided into two *slots* and can accordingly hold one standard 40' container or two 20' containers. Some cells have power plugs allowing for *reefer* containers to be refrigerated as required.

The cargo itself is described by a container *type*, which is defined by a length (20' or 40'<sup>1</sup>), whether it is a reefer-container or not, and a weight class (in a discrete set of weights). Besides containers, vessels also carry *ballast tanks* in fixed positions along the vessel that can be filled with water to improve the stability of the vessel.

### Vessel data

**Capacities** Though containers physically are placed in specific slots, the considered vessel data specifies certain capacities in subsections of each bay, called *locations*. These capacities are given in *TEU* (Twenty-foot Equivalent Units), i.e. a standard 20' container takes up one TEU, while a 40' container takes up two TEU. For each location, the vessel data includes upper bounds for: the total number of TEUs, 20' containers, 40' containers, reefer slots, reefer cells, plus separate total weight limit for 20' and 40' containers, respectively. The latter exist, since only 20' containers rest on the middle support posts of the stack it is in, while the end posts hold weight of both 20' and 40' containers. This may lead to different weight capacities of 20' and 40' containers.

Limits for each ballast tank are likewise given in the input data, as well as a limit for the total displacement, i.e. the weight of the vessel, including ballast water, cargo and the ship itself. The weight of the (empty) vessel (called *lightship*) is given in the data by a set of "blocks" placed along the vessel. Each block has a given weight, which is assumed equally distributed along the block.

**Stress forces** On a vessel, stress forces arise as a result of gravitation acting downwards and buoyancy acting upwards. This results in shear forces and bending moments along the longitudinal axis of the vessel, and limits on these are given for a set of reference points along the vessel called *frames*. The buoyancy force comes from the vessel's displacement of water and hence depends on the varying (and irregular) shape of the hull and the displacement of the vessel. The area submerged in water is given at another set of reference points called *stations* for a discrete set of displacement values.

Further requirements to ensure the stability of the vessel are imposed in real life and considered eg. in [?], but is not considered here.

**Sections** The goal of our Standard Vessel Model is to describe the various capacity- and hydrostatic constraints reasonably accurate, while abstracting away from unnecessary details relating to the irregularity of the vessel's shape and the non-coinciding reference points of tanks, bays, stations and frames. Instead, we will use bigger

<sup>1</sup>45' long containers also exists but are more rare and is not considered in this paper.

sections of the vessel and the endpoints of these as the common reference for capacities and hydrostatic constraints. Each of these sections are defined to either span a number of succeeding bays, or a part of the ship containing no bays at all.

**Input Data** The vessel structure is described using the sets  $L$  (locations),  $BT$  (ballast tanks),  $B$  (blocks with a constant weight  $W_b$  for all  $b \in B$ ),  $ST$  (stations) and  $F$  (frames). All elements in these sets have a fixed longitudinal position fore and aft ( $P_e^f$  and  $P_e^a$  for all  $e \in L \cup BT \cup B \cup ST \cup F$ ), and for the latter two sets, the fore and aft position coincide. The division of the vessel into sections are described by the sets  $S$  (sections), which is divided into sections fore and aft ( $S^f$  and  $S^a$ ), as well as a set  $L_s$  for all  $s \in S$  (locations in section  $s$ ). A fore and aft position of each  $s$  ( $P_s^f$  and  $P_s^a$ ) is implicitly given by the locations in  $L_s$ .

The cargo is described by the set  $T$  (container types) with subsets  $T^{20}$  (20' container types –  $T \setminus T^{20}$  hence indicates the set of 40' container types) and  $T^R$  (reefer container types). Further, each container type has a weight,  $W^\tau$ , associated.

The capacity-limits are given by the constant  $C_l^{TEU}$ ,  $C_l^{20}$ ,  $C_l^{40}$ ,  $C_l^{RS}$ ,  $C_l^{RC}$ ,  $C_l^{W20}$ ,  $C_l^{W40}$  for each  $l \in L$ , and by  $Max_b^w$  for all  $b \in BT$ , plus  $Max^w$ . The limits on the stress forces are given by  $Max_f^{sf}$ ,  $Min_f^{sf}$ ,  $Max_f^{bm}$ ,  $Min_f^{bm}$  for all  $f \in F$ . At each station  $\sigma \in ST$ , the submerged area of the ship at  $\sigma$  is given for a discrete set of displacements in the table  $A_\sigma$ .

## Standard Vessel Model

The input data outlined above is used to construct the SVM described in the following.

**Variables** As decision variables we use  $x_{s,\tau}$  for all  $s \in S$  and  $\tau \in T$ , which denotes the number of containers of type  $\tau$  stowed in sections  $s$ , and  $t_s$  for all  $s \in S$ , which denotes the amount of ballast water in ballast tanks in section  $s$ .

As important auxiliary variables, we define  $x_\tau = \sum_{s \in S} x_{s,\tau}$  for all  $\tau \in T$ , the number of containers of type  $\tau$  on the vessel in total, disregarding their placements. It is the relationship among these variables that we are actually interested in. Another important auxiliary variable is  $w_s$ , denoting the total weight of section  $s$ , everything included. Likewise the total weight,  $w = \sum_{s \in S} w_s$  is used in hydrostatics constraints (see further below). Other auxiliary variables used to ease notation are mentioned below as necessary.

**Capacity constraints** For each section  $s$  we define the capacity of a certain type, e.g.  $C_s^{20}$ , by summing the corresponding capacities for all locations  $l \in L_s$ . We then require that the number/weight of containers of the given type in  $s$  is within this limit, e.g.  $\sum_{\tau \in T^{20}} x_{s,\tau} \leq C_s^{20}$ . This principle is applied to all the aforementioned location-based capacities.

The amount of (ballast) water in any section  $s$  should not exceed the combined amount of water in the portion of each tank that lies within that section; multiplying the latter amount of water with the density for ballast water, we get an upper bound for  $t_s$ , which is then imposed as a constraint. We also define constraints to ensure that all variables  $x_{s,\tau}$  and  $t_s$  are positive and that  $w \leq Max^w$ .

**Hydrostatic constraints** From the table  $A_\sigma$  we make an approximation of the area at  $\sigma$  for any positive  $d$  by linearizing it between the maximal  $d$  value given in the table and a value  $d_{min}$  at the point where the hull does not “curve” too much anymore; for the cross-section given in Figure 1 this would correspond to the displacement giving the marked water line. **We note that since our SVM will mainly be used for some sort of maximization of loaded cargo it is a fair assumption to make that the displacement will be above the found  $d_{min}$ .** The submerged area at a point  $p$  between two consecutive stations  $\sigma$  and  $\sigma'$  for a displacement  $d$  can then also be linearized as a function of the longitudinal position of  $p$ . From this we then calculate an approximation of the buoyancy of section  $s$  for a given  $d$  by averaging the areas between the two endpoints of  $s$  and multiplying with the distance between the points; if there are stations lying within  $s$ , several volumes are calculated and added. On the other hand, the downward-acting force at section  $s$  is given by the weight  $w_s$ . For  $s$ , the resulting force is thus the weight of section  $s$  minus the (positive) buoyancy stemming from  $s$ .

Given a displacement  $d$ , the shear forces and bending moment at each section  $s$ 's aft endpoint is then calculated. For sections in  $S^f$  ( $S^a$ ) the shear force equals the sum of resulting forces fore (aft) of the aft endpoint  $P_s^a$ , while the bending moment equals the sum of resulting forces of sections  $s'$  lying fore (aft)  $P_s^a$  times the distance from  $P_s^a$  to  $s'$ 's (longitudinal) midpoint.

To obtain upper (lower) bounds for the shear force at the sections' aft endpoints, we linearize the upper (lower) bound for the shear force between two consecutive frames as a function of their position. To obtain the bounds for the shear force at the aft endpoint of  $s$  we then use this linearization given by the point's two closest surrounding frames. Similarly for the upper and lower bounds for the bending moment.

Hydrostatic constraints are then added to our model to ensure that the shear force and bending moment at each section's aft endpoint is within the calculated upper and lower bounds.