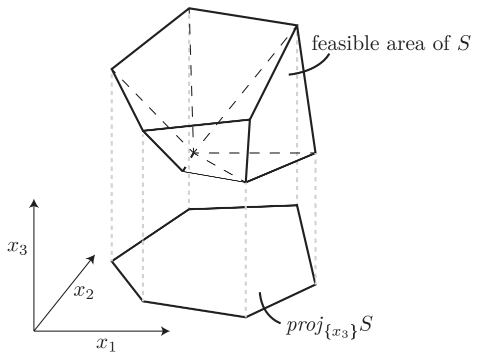

Figure 1: The projection of the S with respect to $\{x_3\}$.

1 Definitions and notation

(In)equality systems In the following, an (in)equality system S is a set of equalities and inequalities over the same set of variables, $VAR(S) = \{x_1, \ldots, x_n\}$. Each (in)equality c is written as either $a_1x_1 + \ldots + a_nx_n = b$ or $a'_1x_1 + \ldots + a'_nx_n \leq b'$, though the left-hand-side is also written using a dot-product. We let $var(c)$ denote the variables whose coefficient in c is nonzero and say that c *uses* x if $x \in var(c)$.

The set of points in $\mathbb{R}^{|VAR(S)|}$ that satisfies all (in)equalities in S is called S's *feasible area*. An (in)equality $c \in S$ is *redundant* if it does not influence the feasible area for S. In other words the inequality $c : \mathbf{a} \cdot \mathbf{x} \leq b$ is redundant iff $\max \mathbf{a} \cdot \mathbf{x}$ w.r.t. $S \setminus \{c\}$ is less or equal to b. An equality is redundant iff both corresponding inequalities are redundant. If the (in)equality c is *not* redundant, it is called *non-redundant*.

Projection As described, the feasible area of S describes the combination of values for the variables in $VAR(S)$ that satisfy all (in)equalities in S. However, there are some variables $Y \subseteq VAR(S)$ whose value in a feasible point we are not interested in; we just want to know that a satisfying value exists. This information is captured by the *projection* of the feasible area of S w.r.t. Y, $proj_Y S \in \mathbb{R}^{|VAR(S) \setminus Y|}$. This is the largest set consisting of values for $VAR(S) \setminus Y$ that can be extended with values for Y such that all (in)equalities in S are satisfied (see Figure **??**).

The projection of a system S is a set of points in Euclidian space, and it is the feasible region of (another) inequality system S' (see e.g. [**?**]). However, many (in)equality systems determine the same feasible area, and when we say e.g. "S' is the projection of S w.r.t. Y" we mean that "S' is one of the (in)equality systems whose feasible area equals the projection of the feasible area of S w.r.t Y".

A note on $VAR(S)$ In this paper, we are mainly interested in the original system S and its projection S', while the associated feasible areas of S and S' are the "mediator" between the two systems. However, since the feasible area of a system is just a set of points in a multi-dimensional Euclidian space, the number and the order of the variables are important and needs to be given (explicitly or implicitly). Though, the inequality $c : a_1x_1 \leq b$ can be considered both as an inequality over the set $\{x_1\}$ as well as over any set X where $x_1 \in X$, and we will not specify $VAR(S')$ formally for every considered (in)equality system S. Intuitively, we just make sure that the dimensions (and order of variables) "match". For example, when two systems S and S' are joined to form another system S'', we consider S and S' as (in)equalities over the same variable set, namely the variables used in either S or S', i.e. $VAR(S'') = var(S) \cup var(S')$. A more stringent exposition keeping track of the variable sets and ordering can be found in [**?**].