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CogSci Methods 2 – F24

Solutions to class 6 exercises

Exercises

A. Calculating Determinants

A1: Determinants of 2x2 matrices

1. $A = \begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$

$$\det\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$
$$= 4 * 6 - 7 * 2$$
$$= 24 - 14$$
$$= 10$$

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2. $B = \begin{pmatrix} 1 & -5 \\ 3 & 2 \end{pmatrix}$

$$\det\begin{pmatrix} 1 & -5 \\ 3 & 2 \end{pmatrix}$$
$$= 1 * 2 - (-5) * 3$$
$$= 2 - (-15)$$
$$= 2 + 15$$
$$= 17$$

RULES USED

(Approximately on the line where it has been applied)

Using the formula for determinant for 2x2 matrices:

$$det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Same as above

3. $$C = \begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}$$

$$\det\begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}$$

$$= (-3) * (-2) - 4 * 5$$

$$= 6 - 20$$

$$= -14$$

Same as above

A2: Determinants of 3x3 matrices

Help: https://www.youtube.com/watch?v=ZMuWKZya2E0

1. $$E = \begin{pmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{pmatrix}$$

$$\det\begin{pmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{pmatrix}$$

$$= 6 * \det\begin{pmatrix} -2 & 5 \\ 8 & 7 \end{pmatrix} - 1 * \det\begin{pmatrix} 4 & 5 \\ 2 & 7 \end{pmatrix} + 1 * \det\begin{pmatrix} 4 & -2 \\ 2 & 8 \end{pmatrix}$$

$$= 6 * \big((-2) * 7 - 5 * 8\big) - 1 * (4 * 7 - 5 * 2)$$
$$+1 * (4 * 8 - (-2) * 2)$$

$$= 6 * (-14 - 40) - (28 - 10) + (32 + 4)$$

$$= 6(-54) - 18 + 36$$

$$= -324 + 18$$

$$= -306$$

Using the expansion formula for determinants with co-factoring:

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$= a * \det\begin{pmatrix} e & f \\ h & i \end{pmatrix} - b * \det\begin{pmatrix} d & f \\ g & i \end{pmatrix} +$$
$$c * \det\begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

After that, using the determinant formula for 2x2 matrices:

$$det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$F = \begin{pmatrix} 0 & 9 & -4 \\ 5 & 3 & 2 \\ 1 & 0 & 8 \end{pmatrix}$$

2.

Same as above

$$\det\begin{pmatrix} 0 & 9 & -4 \\ 5 & 3 & 2 \\ 1 & 0 & 8 \end{pmatrix}$$

(The first term is gonna cancel out w 0, so not writing it out):

$$= 0 * \det(\quad) - 9 * \det\begin{pmatrix} 5 & 2 \\ 1 & 8 \end{pmatrix} + (-4) * \det\begin{pmatrix} 5 & 3 \\ 1 & 0 \end{pmatrix}$$

$$= -9 * \det\begin{pmatrix} 5 & 2 \\ 1 & 8 \end{pmatrix} + (-4) * \det\begin{pmatrix} 5 & 3 \\ 1 & 0 \end{pmatrix}$$

$$= -9 * (5 * 8 - 2 * 1) + (-4) * (5 * 0 - 3 * 1)$$

$$= -9 * 38 + (-4) * (-3)$$

$$= -342 + 12$$

$$= -330$$

$$G = \begin{pmatrix} 7 & 3 & 1 \\ 2 & 6 & 4 \\ 0 & 2 & 8 \end{pmatrix}$$

3.

Same as above

$$\det\begin{pmatrix} 7 & 3 & 1 \\ 2 & 6 & 4 \\ 0 & 2 & 8 \end{pmatrix}$$

$$= 7 * \det\begin{pmatrix} 6 & 4 \\ 2 & 8 \end{pmatrix} - 3 * \det\begin{pmatrix} 2 & 4 \\ 0 & 8 \end{pmatrix} + 1 * \det\begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$$

$$= 7 * (6 * 8 - 4 * 2) - 3 * (2 * 8 - 4 * 0)$$

$$+1 * (2 * 2 - 6 * 0)$$

$$= 7 * 40 - 3 * 16 + 1 * 4$$

$$= 236$$

B. Finding the orthogonal vector of each the following vectors

1. $$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

To find the orthogonal vector we need to find a vector that has a dot product of zero with the given vector.

So let's start with v1. We can find the orthogonal vector by swapping the positions of x and y and changing the sign of one. This gives us:

$$v_{1_{ortho}} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

We can check its dot product with v1:

$$v_{1_{ortho}} * v_1$$

$$= \begin{pmatrix} -3 \\ 2 \end{pmatrix} * \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= -3 * 2 + 2 * 3$$

$$= -6 + 6$$

$$= 0$$

Similarly, for v2, we can do the exact same: switch positions of x and y and changing the sign of one. This gives us:

$$v_{2_{ortho}} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

We can check its dot product with v2:

$$v_{2_{ortho}} * v_2$$

$$= \begin{pmatrix} 4 \\ 1 \end{pmatrix} * \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$= 4 * (-1) + 1 * 4$$

$$= -4 + 4$$

$$= 0$$

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Voilà

C. Matrix inversion

C1: Invert the following 2x2 matrices

1. $$M = \begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}^{-1}$$

$$= \frac{1}{det\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}}\begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$

$$= \frac{1}{4*6-7*2}\begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$

$$= \frac{1}{10}\begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{10} & -\frac{7}{10} \\ -\frac{2}{10} & \frac{4}{10} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{5} & -\frac{7}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix}$$

2. $$N = \begin{pmatrix} 1 & -5 \\ 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -5 \\ 3 & 2 \end{pmatrix}^{-1}$$

$$= \frac{1}{det\begin{pmatrix} 1 & -5 \\ 3 & 2 \end{pmatrix}}\begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$$

RULES USED

(Approximately on the line where it has been applied)

Using the formula 2x2 matrix inverse (the most random formula known to man):

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{det\begin{pmatrix} a & b \\ c & d \end{pmatrix}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(☺ pure FUN and GOOD vibes)

Also using the formula for finding determinants for 2x2 matrices.

Again using the fun and good vibes formula above + formula for determinants of 2x2 matrices.

$$= \frac{1}{1 * 2 - (-5) * 3} \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$$

$$= \frac{1}{17} \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{17} & \frac{5}{17} \\ -\frac{3}{17} & \frac{1}{17} \end{pmatrix}$$

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3. $$O = \begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}$$

Again using the fun and good vibes formula above + formula for determinants of 2x2 matrices.

$$\begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}^{-1}$$

$$= \frac{1}{det \begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}} \begin{pmatrix} -2 & -4 \\ -5 & -3 \end{pmatrix}$$

$$= \frac{1}{(-3) * (-2) - 4 * 5} \begin{pmatrix} -2 & -4 \\ -5 & -3 \end{pmatrix}$$

$$= \frac{1}{-14} \begin{pmatrix} -2 & -4 \\ -5 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2}{-14} & -\frac{4}{-14} \\ -\frac{5}{-14} & -\frac{3}{-14} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{7} & \frac{2}{7} \\ \frac{5}{14} & \frac{3}{14} \end{pmatrix}$$

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