### 3.3 Geometric

Four equations of spheres, where x y z defines the position of the UAV.

$$r_0^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \tag{20}$$
$$r_1^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 \tag{21}$$
$$r_2^2 = (x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2 \tag{22}$$
$$r_3^2 = (x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2 \tag{23}$$

The only unknowns are x y z, which means this can be converted to 3 linear equations. Removing the parenthesis for the four equations will look like the following example for $x_0$ $y_0$ $z_0$:

$$x^2 + x_0^2 - 2xx_0 + y^2 + y_0^2 - 2yy_0 + z^2 + z_0^2 - 2zz_0 = r_0^2 \tag{24}$$

The the non-linear parts $x^2$ $y^2$ $z^2$, can the be isolated:

$$x^2 + y^2 + z^2 - 2(xx_0 + yy_0 + zz_0) = r_0^2 - x_0^2 - y_0^2 - z_0^2 \tag{25}$$

This can then be written in matrix form, AX + B:

$$\begin{bmatrix} x_0 - x_1 & y_0 - y_1 & z_0 - z_1 \\ x_0 - x_2 & y_0 - y_2 & z_0 - z_2 \\ x_0 - x_3 & y_0 - y_3 & z_0 - z_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} q_1 - q_0 \\ q_1 - q_0 \\ q_1 - q_0 \end{bmatrix} \tag{26}$$

Where q is:

$$q_i = \frac{r_i^2 - x_i^2 - y_i^2 - z_i^2}{2} \tag{27}$$