Sensor Fusion

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Outline

- Implementing EKF for robot systems.
- Sensor fusion using GPS and accelerometer.
- Indoor sensor fusion using UWB.
- Complementary and Madgwick filter.

A Single Link Flexible Joint Robot

$$
\begin{aligned}
\dot{\theta}_m(t) &= \omega_m(t) \\
\dot{\omega}_m(t) &= \frac{k}{J_m}(\theta_l(t) - \theta_m(t)) - \frac{B}{J_m}\omega_m(t) + \frac{K_\tau}{J_m}u(t) \\
\dot{\theta}_l(t) &= \omega_l(t) \\
\dot{\omega}_m(t) &= -\frac{k}{J_l}(\theta_l(t) - \theta_m(t)) - \frac{mgb}{J_l}\sin(\theta_l(t))
\end{aligned}
$$

System parameters	Values
Motor inertia, J_m ($\mathrm{kg\,m^2}$)	3.7×10^{-3}
Link inertia, J_ℓ ($\mathrm{kg\,m^2}$)	9.3×10^{-3}
Pointer mass, m (kg)	2.1×10^{-1}
Link length, $2b$ (m)	3.1×10^{-1}
Torsional spring constant, k ($\mathrm{Nm\,rad^{-1}}$)	1.8×10^{-1}
Viscous friction coefficient, B ($\mathrm{Nm\,V^{-1}}$)	4.6×10^{-2}
Amplifier gain, K_τ ($\mathrm{Nm\,V^{-1}}$)	8×10^{-2}

A Single Link Flexible Joint Robot

Assignment

1. Les us assume we can only measure the first two states. Write the system into the following nonlinear state space representation:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{D}u(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \end{aligned}$$

2. Use Euler discretization to transform the continuous time system in question (1) into the discretize time system:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{f}(\boldsymbol{x}_k) + \boldsymbol{B}u_k \\ \boldsymbol{y}_k &= \boldsymbol{C}\boldsymbol{x}_k \end{aligned}$$

3. Let us assume the error covariance matrices for model and measurement are given by $\boldsymbol{Q}_F = I_{4\times4}$ and $\boldsymbol{R}_F = 4I_{2\times2}$, respectively, and the initial conditions for the system and estimate are given by $\boldsymbol{x}_0 = \begin{pmatrix} 0 & 1 & 2 & 0 \end{pmatrix}^{\mathsf{T}}$ and $\hat{\boldsymbol{x}}_0 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}^{\mathsf{T}}$, respectively. Implement EKF for system in question (2).

GPS+Accelerometer

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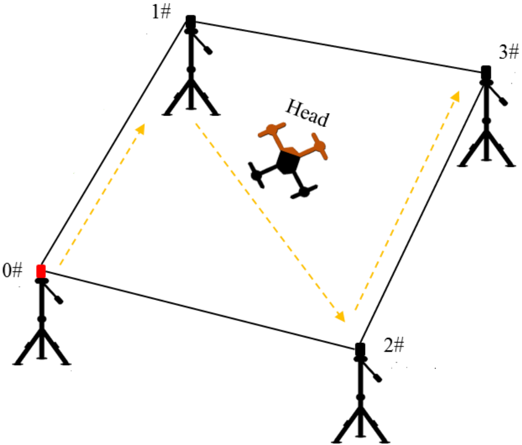
- GPS: low frequency, low accuracy, absolute position.
- Kalman filter can be used to estimate a position using an accelerometer (predict) and a GPS (update).
- Consider the following kinematic equation:

$$\begin{aligned} p_{k+1} &= p_k + \Delta t v_k + \frac{\Delta t^2}{2} a_k \\ v_{k+1} &= v_k + \Delta t a_k \end{aligned}$$

Using KF, we can estimate position and velocity of an object.

Indoor sensor fusion using UWB

How to determine position of an object using 4 ultra-wideband sensors?



Filters

- Kalman filter
- Complementary filter
- Madgwick filter

Inertial Measurement Unit

IMU can measure:

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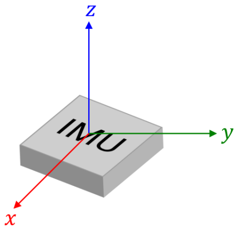
A combination of Gyroscope and Accelerometer is called an Inertial Measurement Unit or IMU or a 6-DoF IMU.

A combination an IMU and a Compass is called a Magnetic, Angular rate, and Gravity (MARG) sensor or Attitude and Heading Reference System (AHRS) or a 9-DoF IMU

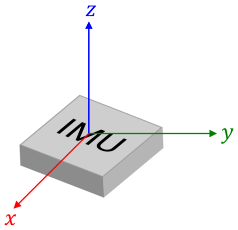
Inertial Measurement Unit

- Estimate the orientation in World frame!



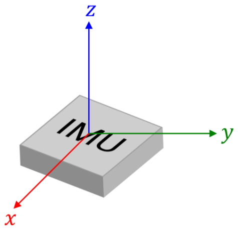
Inertial Measurement Unit

- Estimate the orientation in World frame!
- Orientation is also called Attitude.



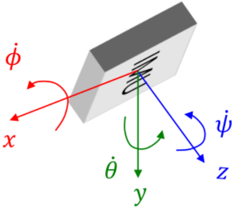
Inertial Measurement Unit

- Estimate the orientation in World frame!
- Orientation is also called Attitude.
- Orientation can be parametrized as either Euler angles (roll, pitch and yaw) or Quaternions.



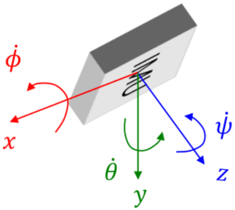
Attitude Estimation from Gyroscope

- We want to estimate $\begin{pmatrix} \phi & \theta & \psi \end{pmatrix}_{t+1}^{\mathsf{T}}$



Attitude Estimation from Gyroscope

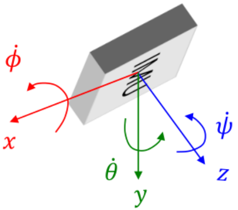
- We want to estimate $\begin{pmatrix}\phi & \theta & \psi\end{pmatrix}^{\mathsf{T}}_{t+1}$
- What we have is $\begin{pmatrix}\dot{\phi} & \dot{\theta} & \dot{\psi}\end{pmatrix}^{\mathsf{T}}_{t}$



Attitude Estimation from Gyroscope

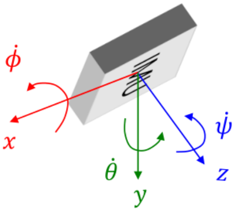
- We want to estimate $\begin{pmatrix}\phi & \theta & \psi\end{pmatrix}^{\mathsf{T}}_{t+1}$
- What we have is $\begin{pmatrix}\dot\phi & \dot\theta & \dot\psi\end{pmatrix}^{\mathsf{T}}_{t}$
- We estimate by numerical integration

$$\begin{pmatrix}\phi\\ \theta\\ \psi\end{pmatrix}_{t+1} = \begin{pmatrix}\phi\\ \theta\\ \psi\end{pmatrix}_{t+1} + \begin{pmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{pmatrix}_{t} \Delta t$$



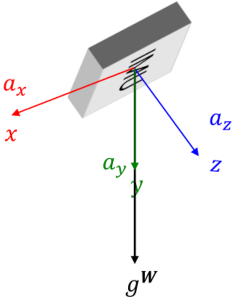
Attitude Estimation from Gyroscope

- We want to estimate $(\phi \quad \theta \quad \psi)^{\mathsf{T}}_{t+1}$
- What we have is $(\dot\phi \quad \dot\theta \quad \dot\psi)^{\mathsf{T}}_{t}$
- We estimate by numerical integration
$$\begin{pmatrix}\phi\\ \theta\\ \psi\end{pmatrix}_{t+1} = \begin{pmatrix}\phi\\ \theta\\ \psi\end{pmatrix}_{t+1} + \begin{pmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{pmatrix}_{t} \Delta t$$
- However! It is impossible to obtain attitude without knowing initial conditions!



Attitude Estimation from Accelerometer

- Let us assume that our IMU/accelerometer is only rotating.



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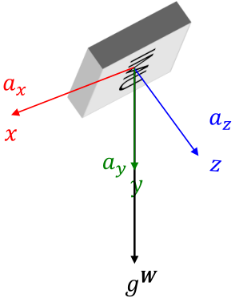


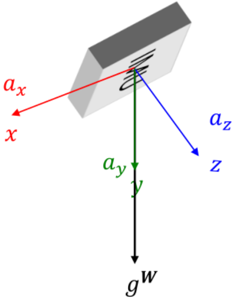
Attitude Estimation from Accelerometer

- Let us assume that our IMU/accelerometer is only rotating.
- We want to estimate $\begin{pmatrix}\phi & \theta & \psi\end{pmatrix}^\mathsf{T}_{t+1}$
- We have $\begin{pmatrix}a_x & a_y & a_z\end{pmatrix}^\mathsf{T}_{t}$
- How to estimate the attitude?

$$\phi = \tan^{-1}\left(\frac{a_y}{\sqrt{a_x^2 + a_z^2}}\right)$$

$$\theta = \tan^{-1}\left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right)$$

$$\psi = \tan^{-1}\left(\frac{\sqrt{a_x^2 + a_y^2}}{a_z}\right)$$



Real World Situations

Both Gyroscope and Accelerometer have noise and bias!

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Bias: They don't read zero at rest!

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Both Gyroscope and Accelerometer have noise and bias!

Bias: They don't read zero at rest!

Noise: The values don't remain constant when the sensor is not moving!

Gyroscope and Accelerometer models

Gyroscope can be modelled by

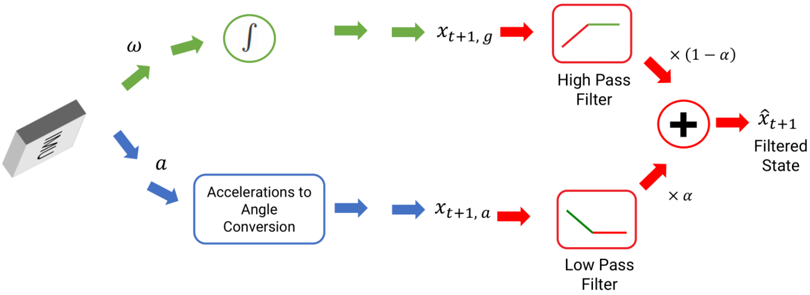
$$\omega = \hat{\omega} + b_g + n_g, \quad \dot{b}_g \sim \mathcal{N}(0, \boldsymbol{Q}_g)$$

Accelerometer can be modelled by

$$a = \boldsymbol{R}^\top(\hat{a} - g) + b_a + n_a, \quad \dot{b}_a \sim \mathcal{N}(0, \boldsymbol{Q}_a)$$

- Bias will overshadow real values when integrating if not compensated for!
- Noise will make values drift from real value during integration!

Complementary Filter



$$\hat{x}_{t+1} = (1-\alpha)\hat{x}_{t+1,g} + \alpha\hat{x}_{t+1,a}$$

Madgwick Filter

We want to model attitude estimation from the accelerometer measurements by modelling it as an optimization problem instead of direct angle computation

$$\min_{{}^{I}_{W}\hat{\mathbf{q}} \in \mathbb{R}^{4\times 1}} f\left({}^{I}_{W}\hat{\mathbf{q}}, {}^{W}\hat{\mathbf{g}}, {}^{I}\hat{\mathbf{a}}\right)$$

$$f\left({}^{I}_{W}\hat{\mathbf{q}}, {}^{W}\hat{\mathbf{g}}, {}^{I}\hat{\mathbf{a}}\right) = {}^{I}_{W}\hat{\mathbf{q}}^{*} \otimes {}^{W}\hat{\mathbf{g}} \otimes {}^{I}_{W}\hat{\mathbf{q}} - {}^{I}\hat{\mathbf{a}}$$

We can achieve this using Gradient Descent!

ref: S. Madgwick, et al., Estimation of IMU and MARG orientation using a gradient descent algorithm, 2011 IEEE international conference on rehabilitation robotics, Zurich, Switzerland, 2011.