Assignment 1

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Classical Autonome Systems
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1 Task 1

The purpose of task 1 is to learn how to use the Kalman filter to estimate unknown system parameters. The system is a mass spring damper system where the spring coefficient [k] and the damper coefficient [b] is unknown.
The first step in the process is to develop an appropriate mathematical model of the mass-spring-damper system. To do this the free-body diagram, figure 1.1, of the system is inspected.

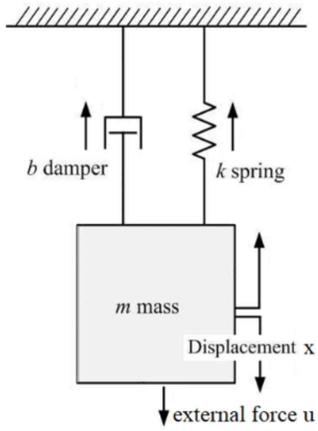

Figure 1.1: Free-body diagram of mass-spring-damper system

From figure 1.1 and by using Newtons second law of motion, Hooks law and the dampening ratio, a second order differential equation can be outlined.
We start with the sum of forces.

$$\sum F = m \cdot a \tag{1}$$

Expanding equation 1 with the external force F_u Hooks law F_k and the force from the dampening ratio F_b we get.

$$m \cdot a = F_u + F_k + F_d \tag{2}$$

Inserting the definitions for the different parameters in equation 2 we get.

$$m \cdot a = F_u - k \cdot x(t) - b \cdot v(t) \tag{3}$$

Writing equation 3 in terms of position and dividing with the mass [m] we get the system model.

$$\ddot{x}(t) = \frac{F_u}{m} - \frac{k}{m} \cdot x(t) - \frac{b}{m} \cdot \dot{x}(t) \tag{4}$$

Since the model is a second order equation we create a system of first order equations and express the model in terms of these equations.
$x_1(t) = x(t)$

$x_2(t) = \dot{x}_1(t) = \frac{d \cdot x(t)}{dt}$

$\dot{x}_1(t) = x_2(t) = \frac{d \cdot x(t)}{dt}$

$\dot{x}_2(t) = \frac{d^2 \cdot x_1(t)}{dt^2} = \frac{F_u}{m} - \frac{k}{m} \cdot x(t) - \frac{b}{m} \cdot \dot{x}(t)$
Since the unknown parameters are constant values, but we need them to be functions, we create functions of time from the constants and take the derivative of these functions.
$\dot{k}(t) = k(t)$
$\dot{b}(t) = b(t)$
From the set of first order differential equations we create the state space form of the model.

2 Task 2

The purpose of task 2 is to use the Kalman filter to merge sensor data from different sensors, what is also known as sensor fusion, in order to find the position of a moving object. The sensors are GNSS, accelerometer and speedometer with update frequencies of 1Hz for the GNSS, 10Hz for the speedometer and 100Hz for the accelerometer.
First part of the task is to determine the position by only using the GNSS data. Since this is the only sensor there is no sensor fusion needed here. The position is determined simply by plotting the data form the GNSS sensor and the result can be seen in figure 2.1. Second part of the task is to fuse the GNSS data

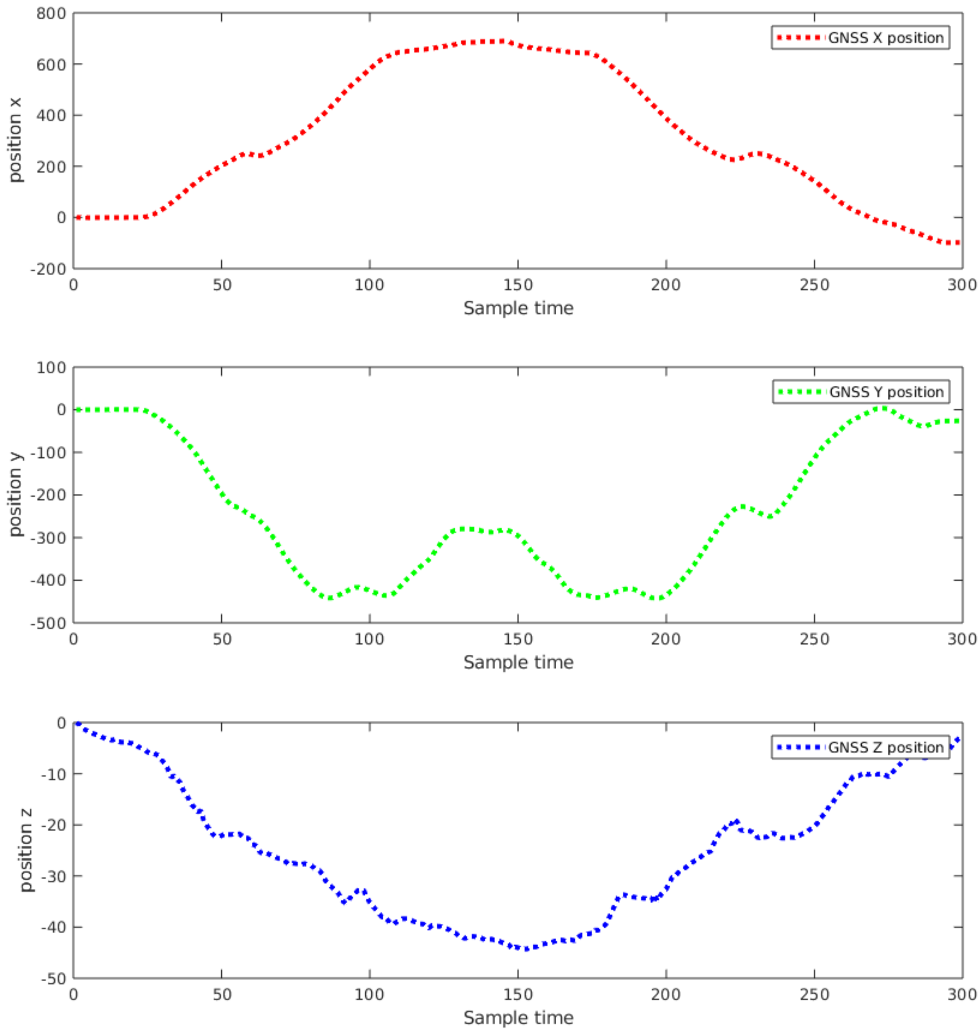

Figure 2.1: Position from GNSS only

with the accelerometer data. To do this the accelerometer is used in the prediction step of the Kalman filter, as the input [u], and the GNSS is used in the update step. Since the accelerometer has a 100 times faster update rate the prediction step of the Kalman filter is computed 100 times before the update step is computed.

3 Discussion

http://ctms.engin.umich.edu/CTMS/index.php?example=Introductionsection=SystemModeling

4 Conclusion

References

Appendices