Extended Kalman Filter

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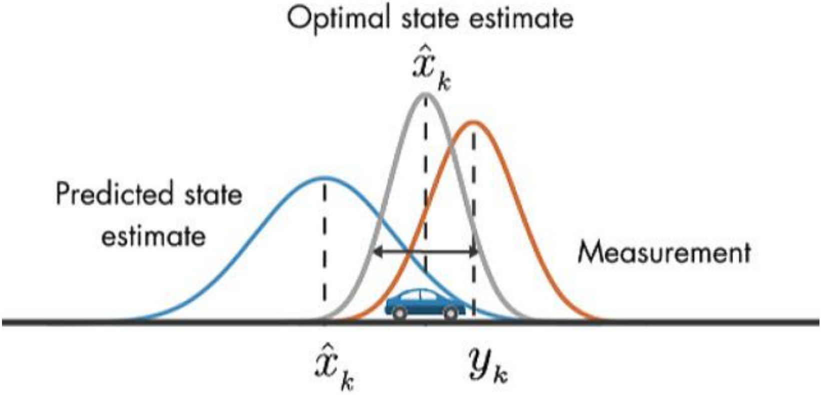
History

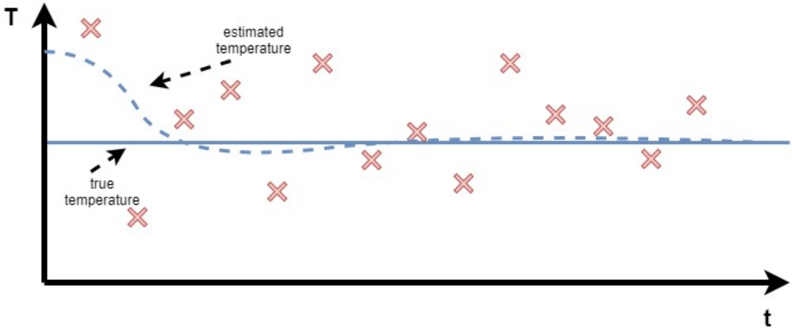
- Rudolf E. Kálmán (May 19, 1930 – July 2, 2016).
- "A new approach to linear filtering and prediction problems," Transactions of the ASME—Journal of Basic Engineering, 82 (D), 35–45, 1960.
- It was originally designed for aerospace guidance applications (Apollo Project).
- While it is the optimal observer for system with noise, this only true for the linear case.
- A non-linear Kalman Filter can not be proven to be optimal.

Kalman Filter

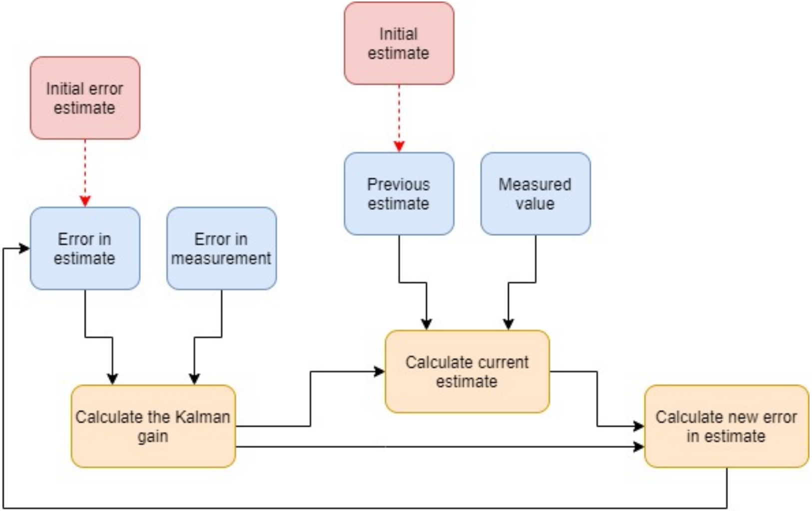


Kalman Filter

Kalman filter is an iterative mathematical process that uses a set of equations and consecutive data inputs to quickly estimate the true state and parameter of dynamical systems under uncertainties.

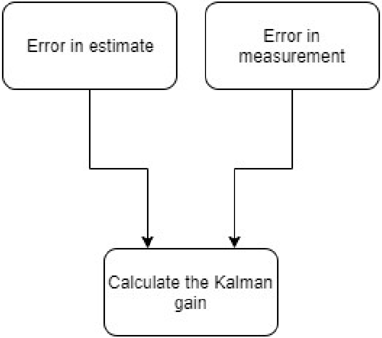


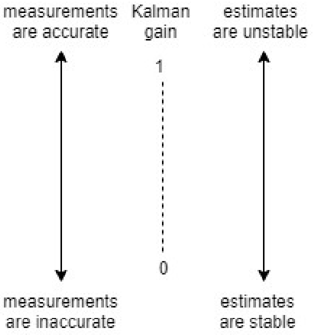
Kalman Filter



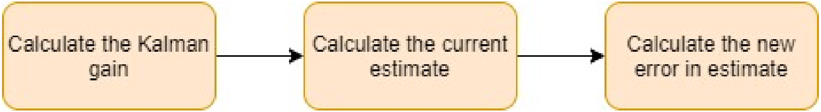
Kalman Filter

$$
\begin{aligned}
\mathrm{KG} &= \frac{\mathrm{E}_{\mathrm{EST}}}{\mathrm{E}_{\mathrm{EST}} + \mathrm{E}_{\mathrm{MEA}}}, \quad 0 \leq \mathrm{KG} \leq 1 \\
\mathrm{EST}_k &= \mathrm{EST}_{k-1} + \mathrm{KG}(\mathrm{MEA} - \mathrm{EST}_{k-1})
\end{aligned}
$$





Kalman Filter



$$
\begin{aligned}
\mathrm{KG} &= \frac{\mathrm{E}_{\mathrm{EST}}}{\mathrm{E}_{\mathrm{EST}} + \mathrm{E}_{\mathrm{MEA}}} \\
\mathrm{EST}_k &= \mathrm{EST}_{k-1} + \mathrm{KG}(\mathrm{MEA} - \mathrm{EST}_{k-1}) \\
\mathrm{E}_{\mathrm{EST}_k} &= (1 - \mathrm{KG})\mathrm{E}_{\mathrm{EST}_{k-1}}
\end{aligned}
$$

Kalman Filter

Consider dynamical systems that can be written as follow:

$$
\begin{aligned}
\boldsymbol{x}_k &= \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1} + \boldsymbol{w}_{k-1} \\
\boldsymbol{y}_k &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \\
\boldsymbol{E}(\boldsymbol{w}_k\boldsymbol{w}_j^{\mathsf{T}}) &= \boldsymbol{Q}_k\delta_{k-j} \\
\boldsymbol{E}(\boldsymbol{v}_k\boldsymbol{v}_j^{\mathsf{T}}) &= \boldsymbol{R}_k\delta_{k-j} \\
\boldsymbol{E}(\boldsymbol{w}_k\boldsymbol{v}_j^{\mathsf{T}}) &= 0
\end{aligned}
$$

The Kalman filter is initialized as follows:

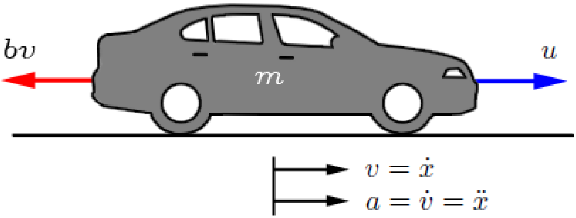
$$
\begin{aligned}
\hat{\boldsymbol{x}}_0^+ &= \boldsymbol{E}(\boldsymbol{x}_0) \\
\hat{\boldsymbol{P}}_0^+ &= \boldsymbol{E}[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0^+)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0^+)^{\mathsf{T}}]
\end{aligned}
$$

Kalman Filter

The Kalman filter algorithm is given by

$$
\begin{aligned}
&\text{predict} \\
\hat{\boldsymbol{x}}_k^- &= \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1}^+ + \boldsymbol{G}_{k-1}\boldsymbol{u}_{k-1} = \text{a priori estimate} \\
\boldsymbol{P}_0^- &= \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^\intercal + \boldsymbol{Q}_{k-1} \\
&\text{update} \\
\boldsymbol{K}_k &= \boldsymbol{P}_k^-\boldsymbol{H}_k^\intercal\left(\boldsymbol{H}_k\boldsymbol{P}_k^-\boldsymbol{H}_k^\intercal + \boldsymbol{R}_k\right)^{-1} \\
\hat{\boldsymbol{x}}_k^+ &= \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_k(\boldsymbol{y}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_k^-) = \text{a posteriori estimate} \\
\boldsymbol{P}_k^+ &= \left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{P}_k^-
\end{aligned}
$$

Cruise Control System



$$
\begin{aligned}
\dot{x}(t) &= -\frac{b}{m}x(t) + \frac{1}{m}u(t) \\
y(t) &= x(t)
\end{aligned}
$$

Cruise Control System

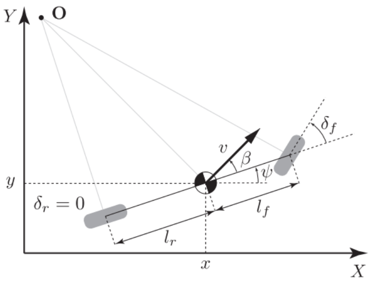
Forward Euler method

$$\dot{x}(t) = \frac{x_{k+1} - x_k}{\Delta t}$$

Thus, we have

$$\begin{aligned} x_{k+1} &= \left(1 - \frac{b\Delta t}{m}\right) x_k + \frac{\Delta t}{m} u_k \\ y_k &= x_k \end{aligned}$$

Autonomous Car



$$
\begin{aligned}
x_{k+1} &= x_k + v_k \cos(\Psi_k)\Delta t \\
y_{k+1} &= y_k + v_k \sin(\Psi_k)\Delta t \\
\Psi_{k+1} &= \Psi_k + \frac{v_k}{l_f}\delta_k \Delta t \\
v_{k+1} &= v_k + a_k \Delta t
\end{aligned}
$$