Approximate inference & Bayesian Networks (Part B)

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Topics:

- Approximate inference
- Sampling from a probability distribution
- Sampling from a probability distribution: Sprinkler example
- Prior-Sampling
- Prior-Sampling: Sprinkler example
- Filtering: umbrella world example
- Pomegranate
- Pomegranate for umbrella world

DIPARTIMENTO
DI INGEGNERIA
DELL'INFORMAZIONE

Sampling from a probability distribution

The methods seen last time (e.g. inference by enumeration) allow for exact inference

Approximate inference methods are therefore a viable alternative to give reasonable answers in case of large models

All the algorithms for **approximate inference** with Bayesian Networks require a **method for sampling** from a known probability distribution.

Sampling from a probability distribution

Let's see a possible implementation of such sampling method in case of **Boolean variables** and known **Conditional Probability Tables** (CPTs):

```
import numpy as np
import random as rnd

t, f = 0, 1

def samplegen(Pdist, Parents = []):
  assert len(Parents) < len(Pdist.shape)
  if rnd.random() < Pdist[t][tuple(Parents)]:
    return t
  return f
```

probability distribution with the content of the CPT

Sampling from a probability distribution

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  assert len(Parents) < len(Pdist.shape)
  if rnd.random() < Pdist[t][tuple(Parents)]:
    return t
  return f
```

if available, the values of the Parents events (i.e., t or f)

Sampling from a probability distribution

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import random as rnd

t, f = 0, 1

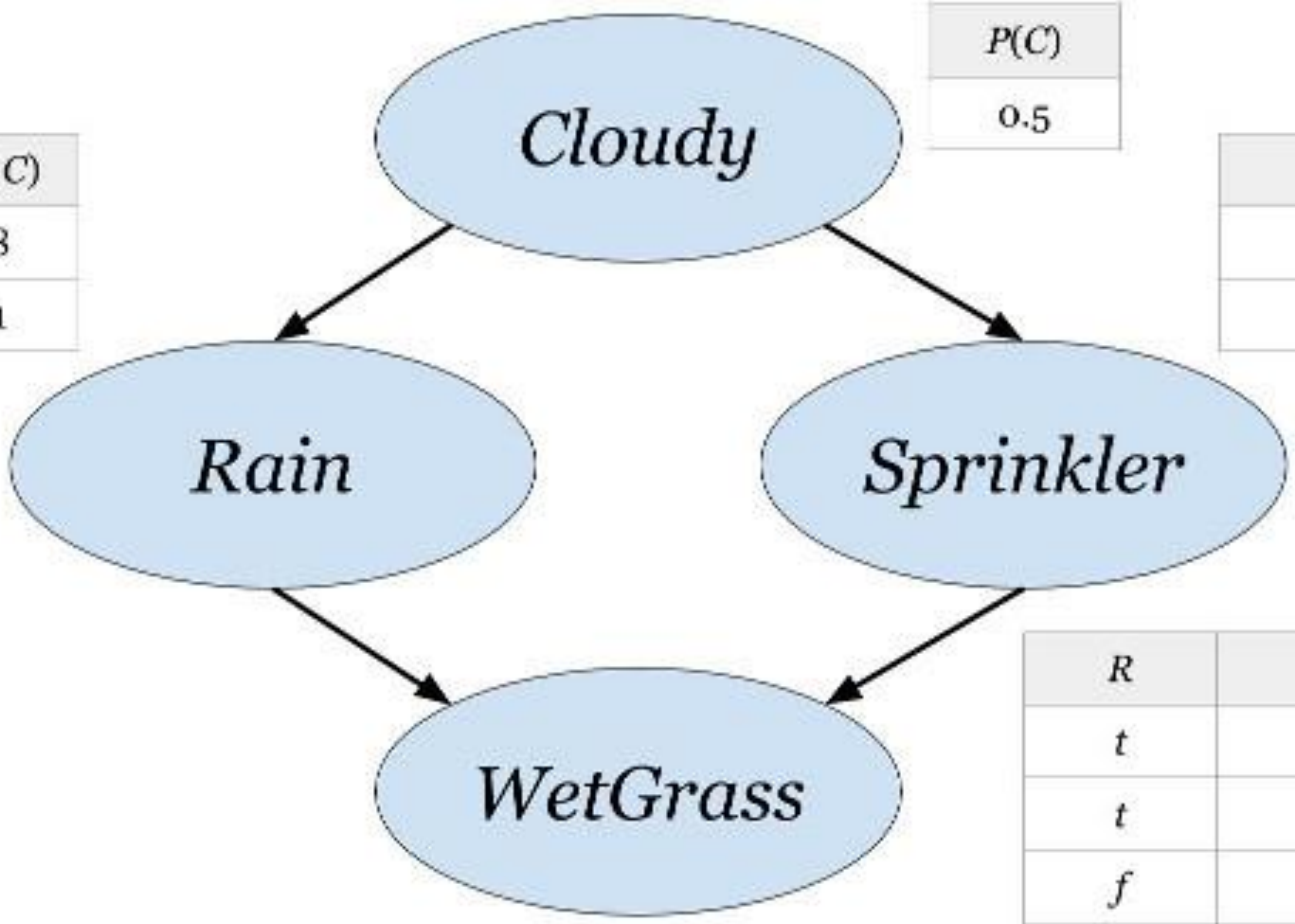
def samplegen(Pdist, Parents = []):
  assert len(Parents) < len(Pdist.shape)
  if rnd.random() < Pdist[t][tuple(Parents)]:
    return t
  return f
```

https://docs.python.org/3/library/random.html

random.random()
Return the next random floating point number in the range 0.0 <= X < 1.0

Sampling from a probability distribution: Sprinkler example

Let's consider the scenario in the figure, where the WetGrass can be caused a Sprinkler or the Rain, both of which depend on the Cloudy weather:



$P(C)$
0.5

C	$P(R\|C)$
t	0.8
f	0.1

C	$P(S\|C)$
t	0.1
f	0.5

R	S	$P(W\|R,S)$
t	t	0.95
t	f	0.90
f	t	0.90
f	f	0.10

Sampling from a probability distribution: Sprinkler example

In particular, let's say we want to generate 20 samples from the Cloudy distribution $P(C)$ which has no parents:

```
P_C = np.array([0.5, 0.5])

for i in range(20):
    s = samplegen(P_C)
    print("C = ", 'TRUE' if s == t else 'FALSE')
```

```
C =  FALSE
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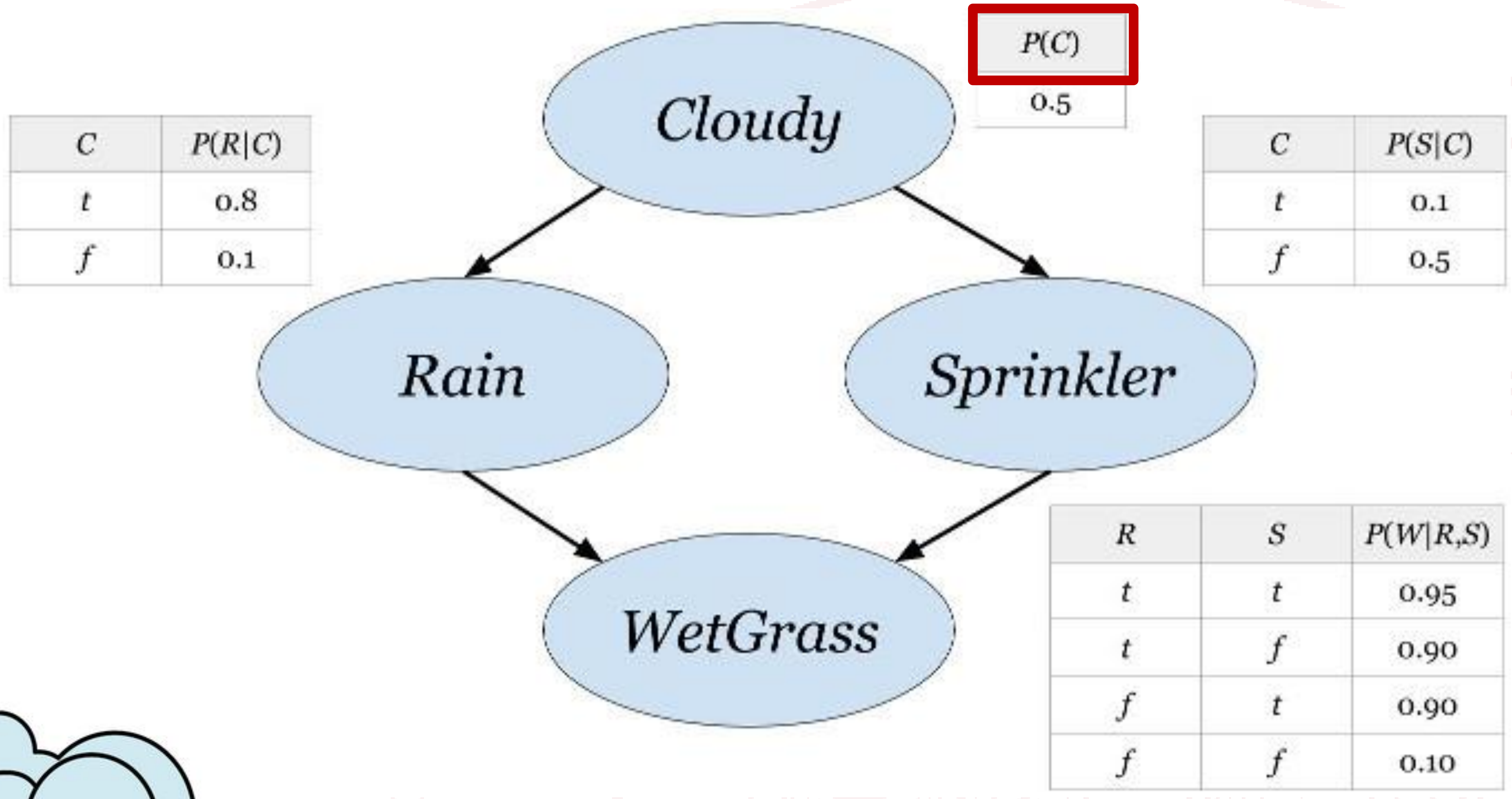
C =  TRUE
```



Sampling from a probability distribution: Sprinkler example

In particular, let's say we want to generate 20 samples from the Cloudy distribution $P(C)$ which has **no parents**:

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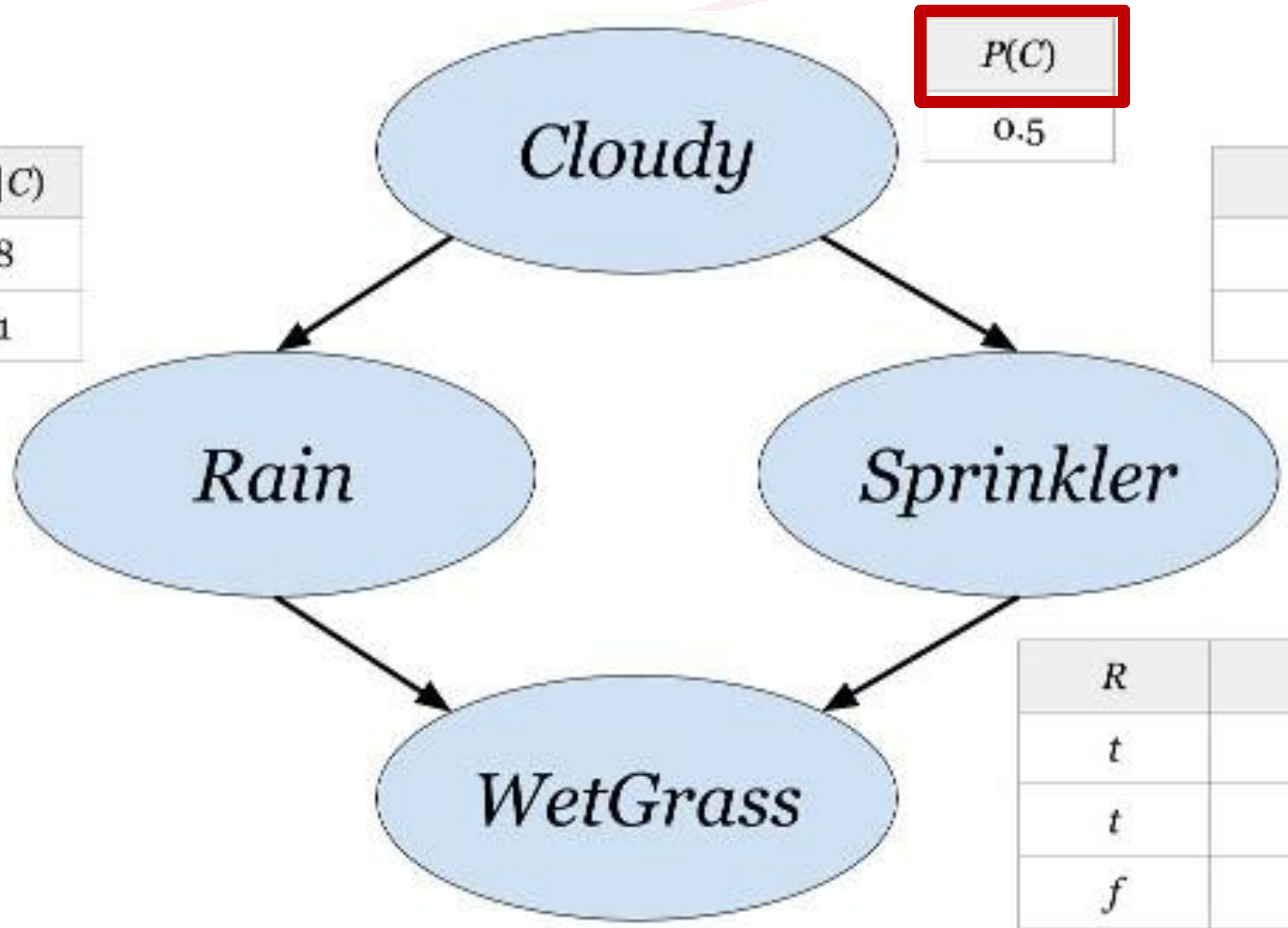
C =  TRUE
C =  FALSE
C =  TRUE
C =  FALSE
C =  FALSE
C =  FALSE
C =  FALSE
C =  TRUE
C =  TRUE
```



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C	$P(R\|C)$
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C	$P(S\|C)$
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f	0.5

R	S	$P(W\|R,S)$
t	t	0.95
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Sampling from a probability distribution: Sprinkler example

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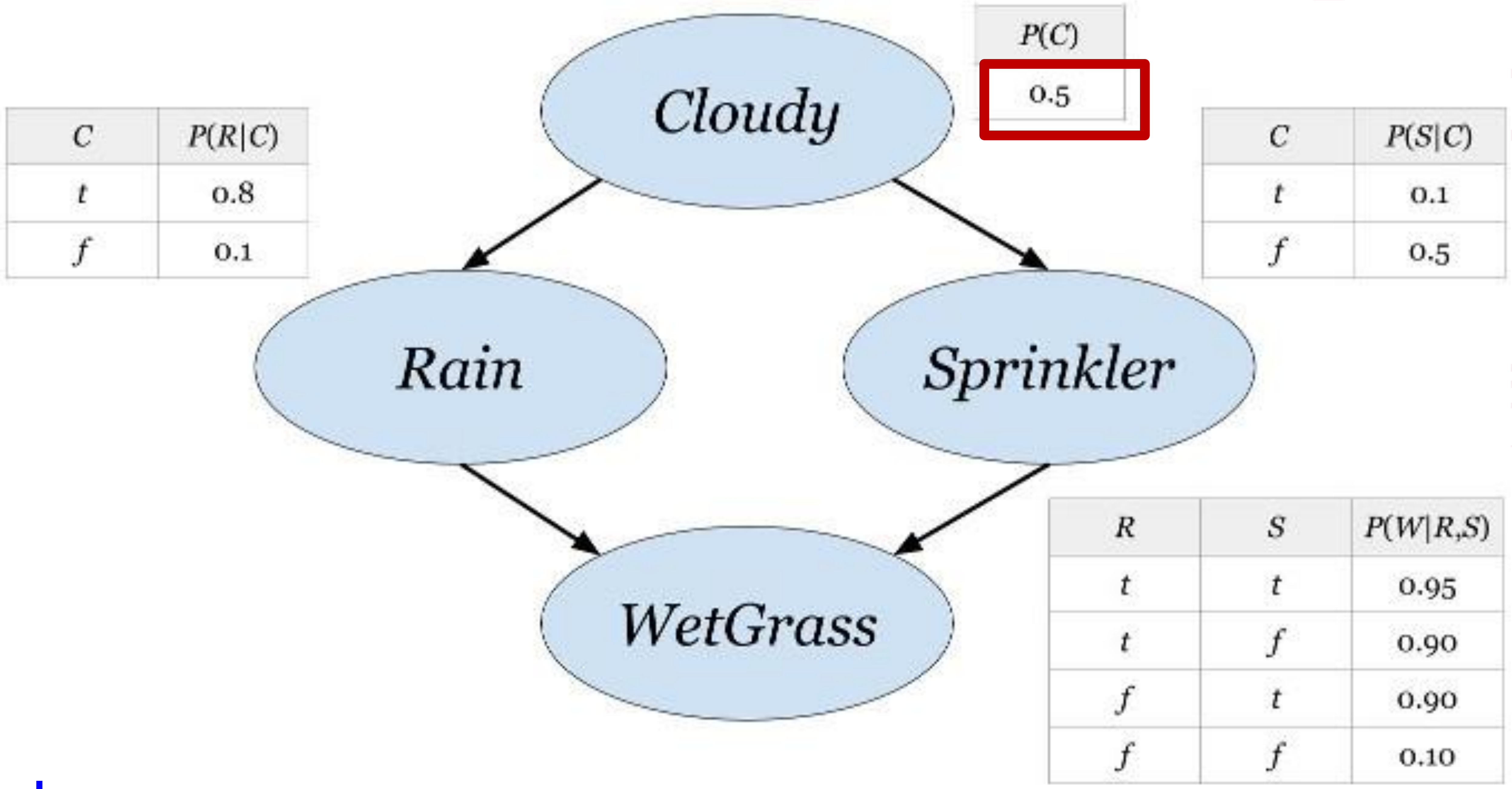
C =  FALSE
C =  TRUE
C =  FALSE
C =  TRUE
C =  FALSE
C =  FALSE
C =  FALSE
C =  FALSE
C =  TRUE
C =  TRUE
```

As expected, there are approximately 50% true and 50% false outcomes.



Sampling from a probability distribution: Sprinkler example

Let's sample from a **conditional distribution**, for example $P(R \mid \neg c)$

```
P_R_C = np.array([[0.8, 0.1],[0.2, 0.9]])

for i in range(20):
    s = samplegen(P_R_C, [f])
    print("R = ", 'TRUE' if s == t else 'FALSE')
```

```
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  TRUE
R =  FALSE
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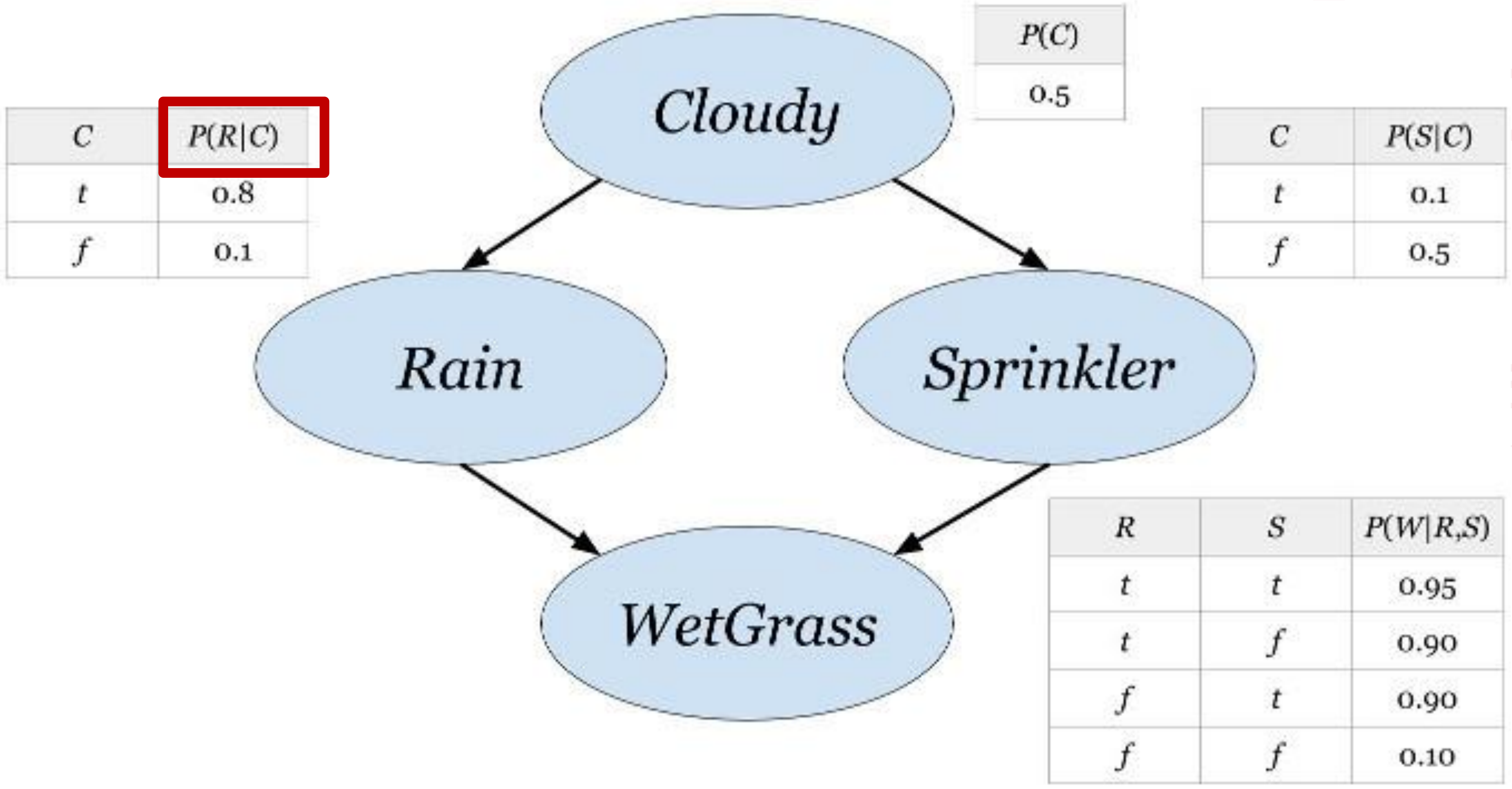
R =  FALSE
R =  FALSE
R =  FALSE
```



Sampling from a probability distribution: Sprinkler example

Let's sample from a **conditional distribution**, for example $P(R\,|\boxed{\neg c})$

```
P_R_C = np.array([[0.8, 0.1],[0.2, 0.9]])

for i in range(20):
    s = samplegen(P_R_C, [f])
    print("R = ", 'TRUE' if s == t else 'FALSE')
```

```
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  TRUE
R =  FALSE
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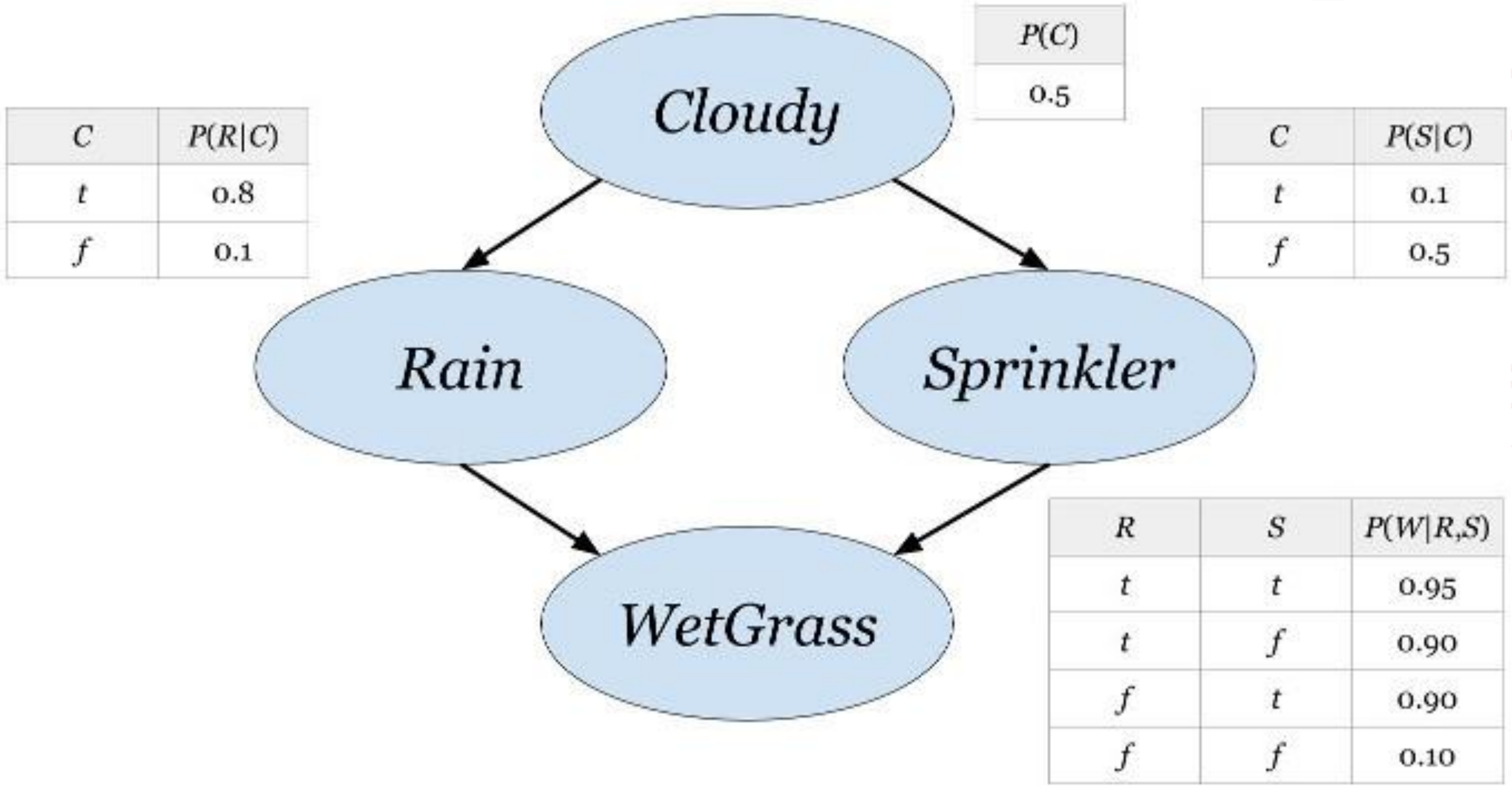
R =  FALSE
R =  FALSE
```



Sampling from a probability distribution: Sprinkler example

Let's sample from a **conditional distribution**, for example $P(R\,|\boxed{\neg c})$

```
P_R_C = np.array([[0.8, 0.1],[0.2, 0.9]])

for i in range(20):
    s = samplegen(P_R_C, [f])
    print("R = ", 'TRUE' if s == t else 'FALSE')
```

```
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  FALSE
R =  TRUE
R =  FALSE
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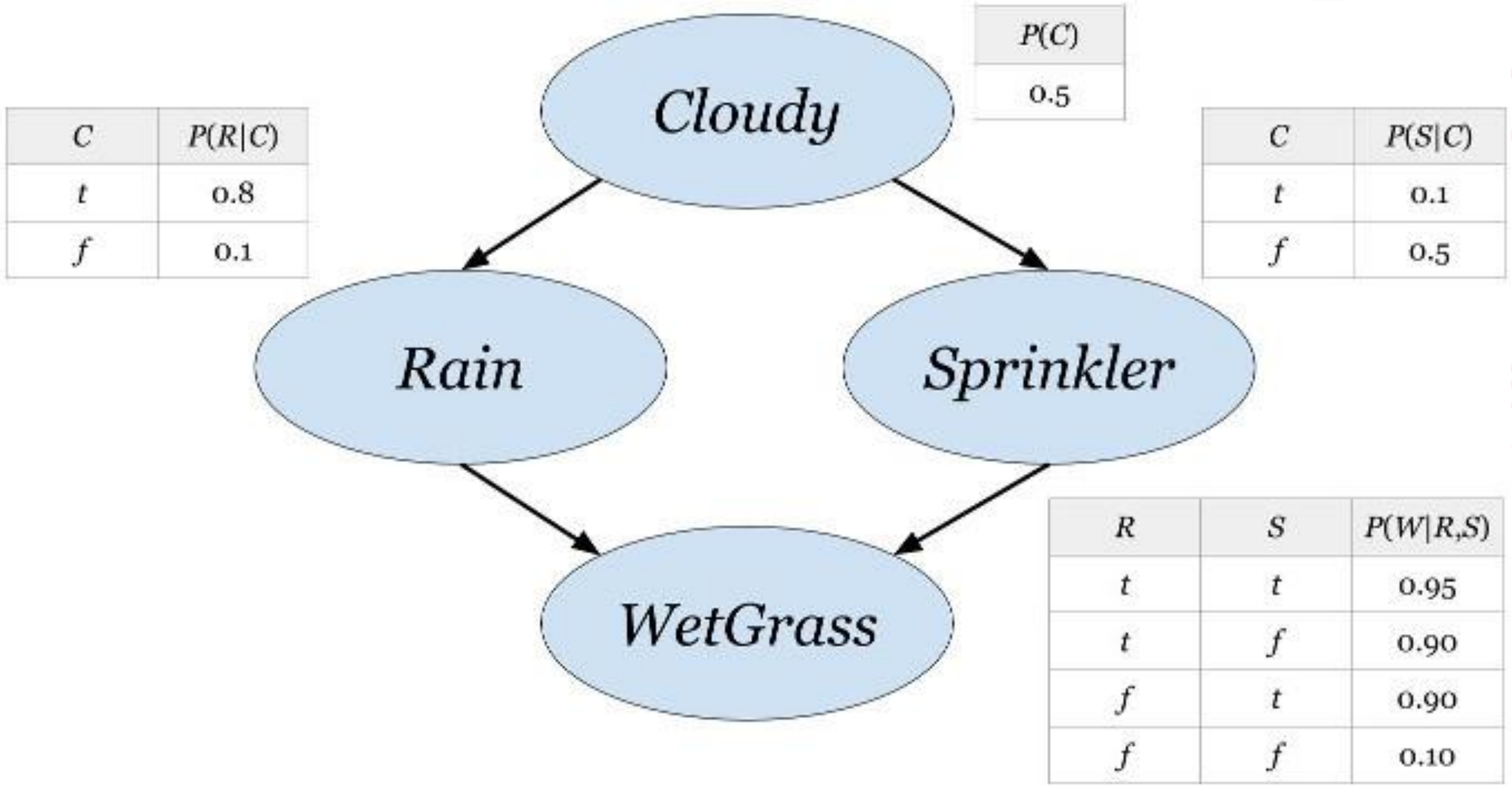
R =  FALSE
R =  FALSE
R =  FALSE
```

In this case there are many more false than true outcomes because

$$P(R\,|\neg c) = \langle 0.1,\, 0.9\rangle$$



Sampling from a probability distribution: Sprinkler example

Finally, let's sample $P(W|\neg s, r) = \langle 0.9, 0.1 \rangle$

```
P_W_SR = np.array([[[0.95, 0.9],[0.9, 0.1]],[[0.05, 0.1],[0.1, 0.9]]])

for i in range(20):
    s = samplegen(P_W_SR, [f, t])
    print("W = ", 'TRUE' if s == t else 'FALSE')
```

```
W =  FALSE
W =  TRUE
W =  TRUE
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W =  FALSE
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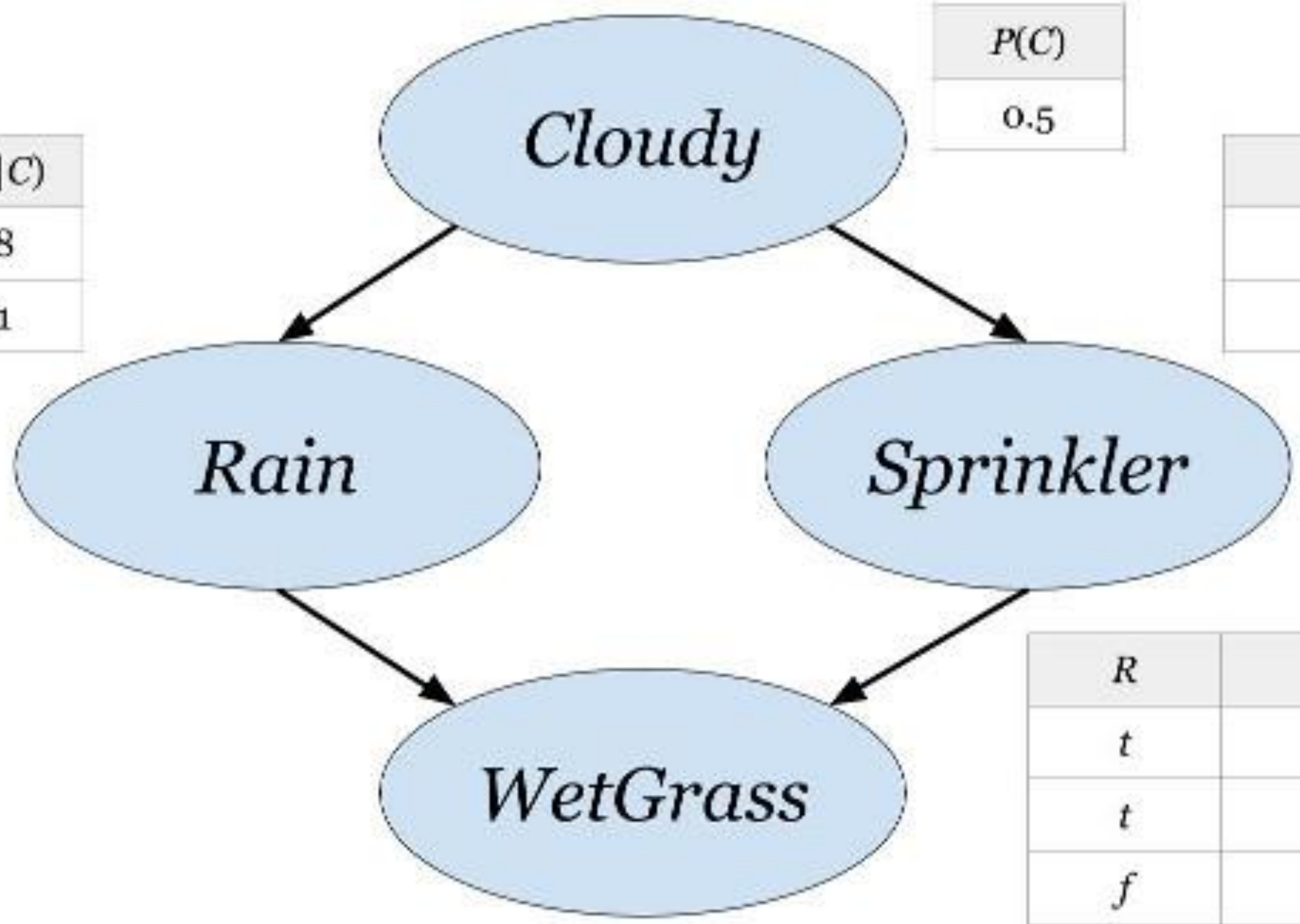
```



P(C)
0.5

C	P(R|C)
t	0.8
f	0.1

C	P(S|C)
t	0.1
f	0.5

R	S	$\boxed{P(W\|R,S)}$
t	t	0.95
t	f	0.90
f	t	0.90
f	f	0.10

Sampling from a probability distribution: Sprinkler example

Finally, let's sample $P(W|\boxed{\neg s, r}) = \langle 0.9, 0.1 \rangle$

```
P_W_SR = np.array([[[0.95, 0.9],[0.9, 0.1]],[[0.05, 0.1],[0.1, 0.9]]])

for i in range(20):
    s = samplegen(P_W_SR, [f, t])
    print("W = ", 'TRUE' if s == t else 'FALSE')
```

```
W =  FALSE
W =  TRUE
W =  TRUE
W =  TRUE
W =  TRUE
W =  FALSE
W =  TRUE
W =  TRUE
W =  FALSE
W =  TRUE
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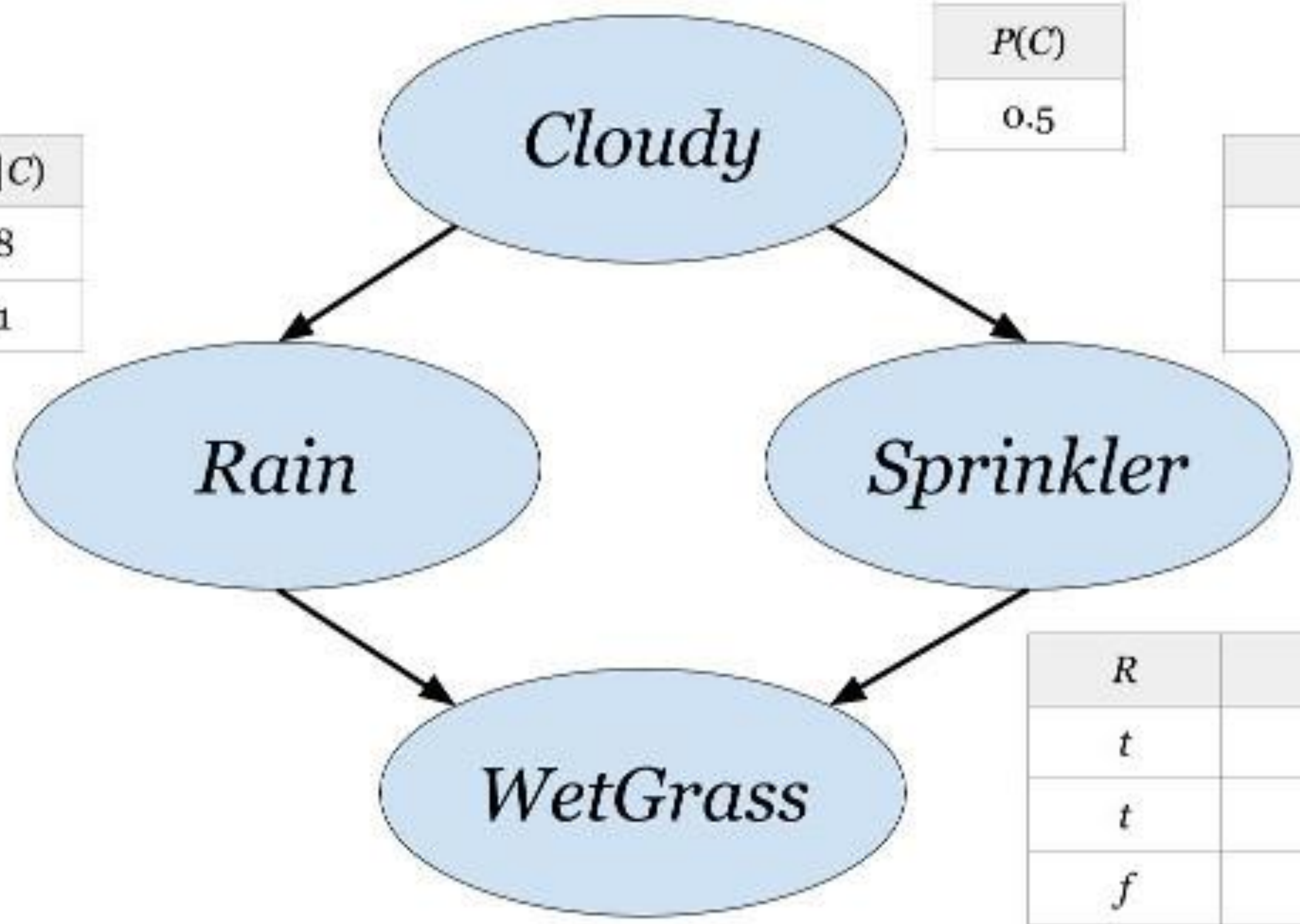
```



P(C)
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Sampling from a probability distribution: Sprinkler example

Finally, let's sample $P(W|\boxed{\neg s, r}) = \langle 0.9, 0.1 \rangle$

```
P_W_SR = np.array([[[0.95, 0.9],[0.9, 0.1]],[[0.05, 0.1],[0.1, 0.9]]])

for i in range(20):
    s = samplegen(P_W_SR, [f, t])
    print("W = ", 'TRUE' if s == t else 'FALSE')
```

```
W =  FALSE
W =  TRUE
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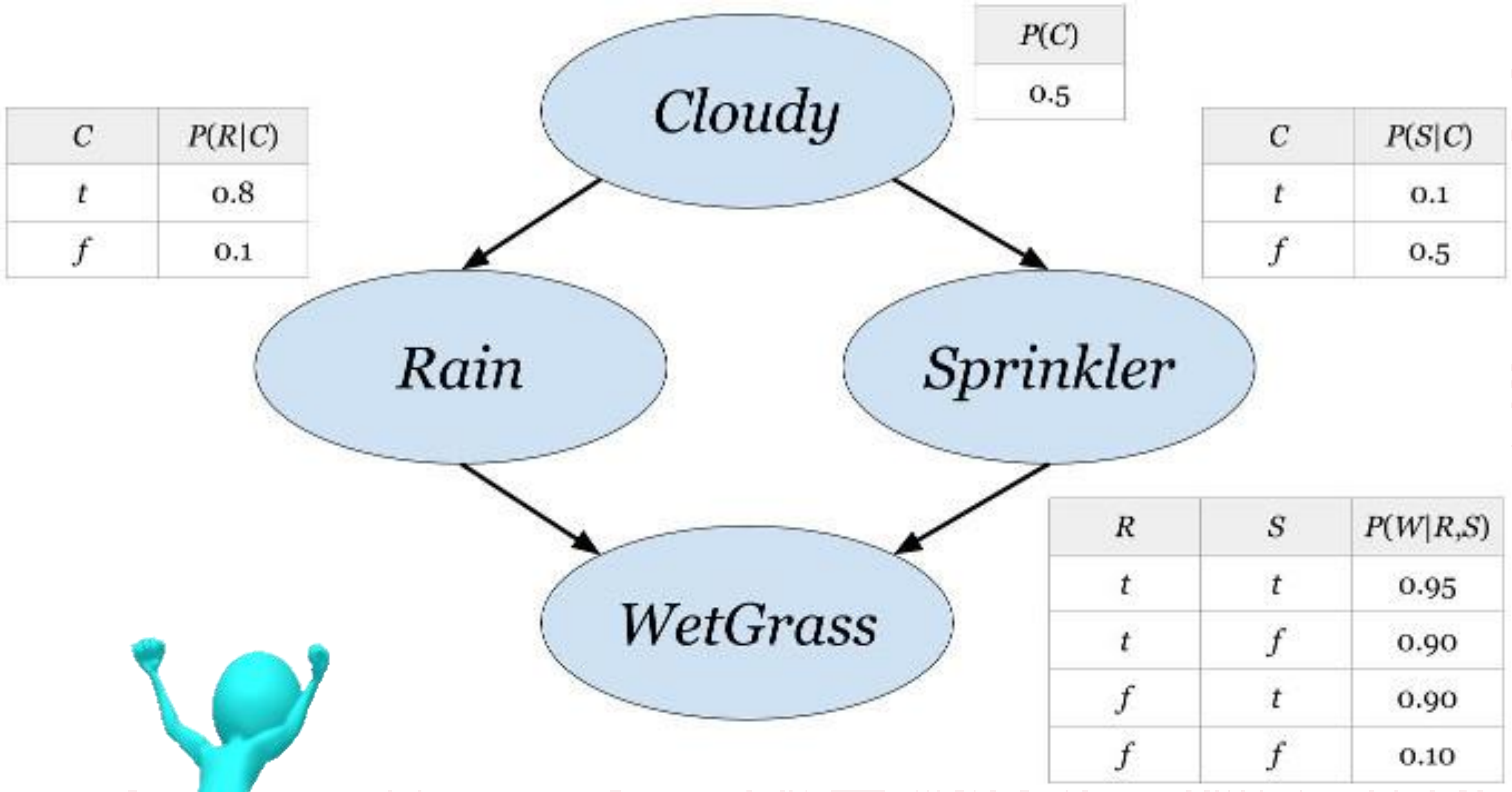
```

Even in this case, the samples are more or less as expected, about 90% true and 10% false.

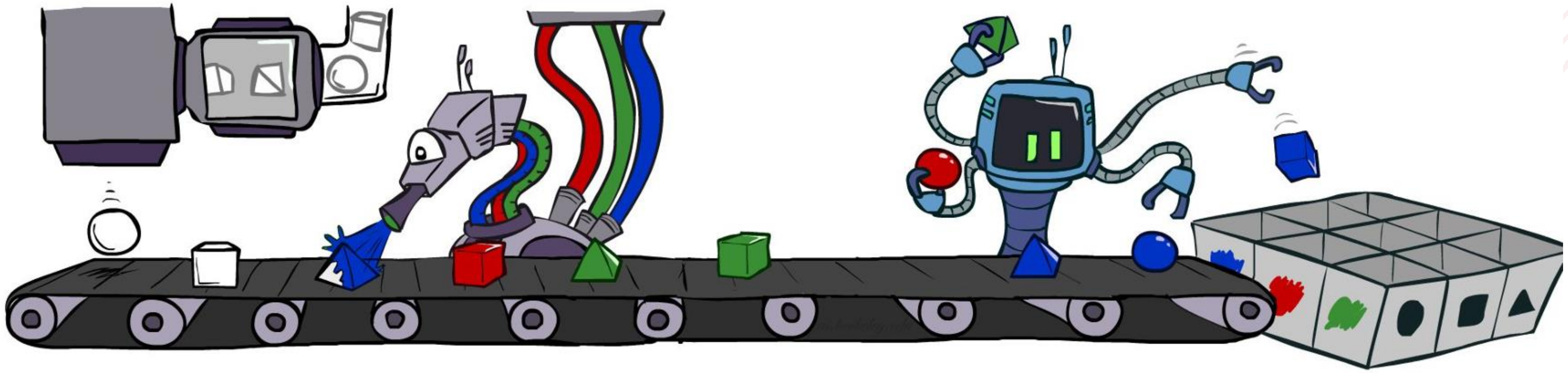


Prior-Sampling

function PRIOR-SAMPLE(bn) **returns** an event sampled from the prior specified by bn
  inputs: bn, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

  $\mathbf{x} \leftarrow$ an event with n elements
  for each variable X_i **in** $X_1, \ldots, X_n$ **do**
    $\mathbf{x}[i] \leftarrow$ a random sample from $\mathbf{P}(X_i \mid parents(X_i))$
  return $\mathbf{x}$

- For i=1, 2, …, n (in topological order)
  - Sample X_i from $P(X_i \mid parents(X_i))$
- Return $(x_1, x_2, \ldots, x_n)$



Prior-Sampling: Sprinkler example

Let's implement now the Prior Sampling algorithm starting with some data structures to represent the Sprinkler network:

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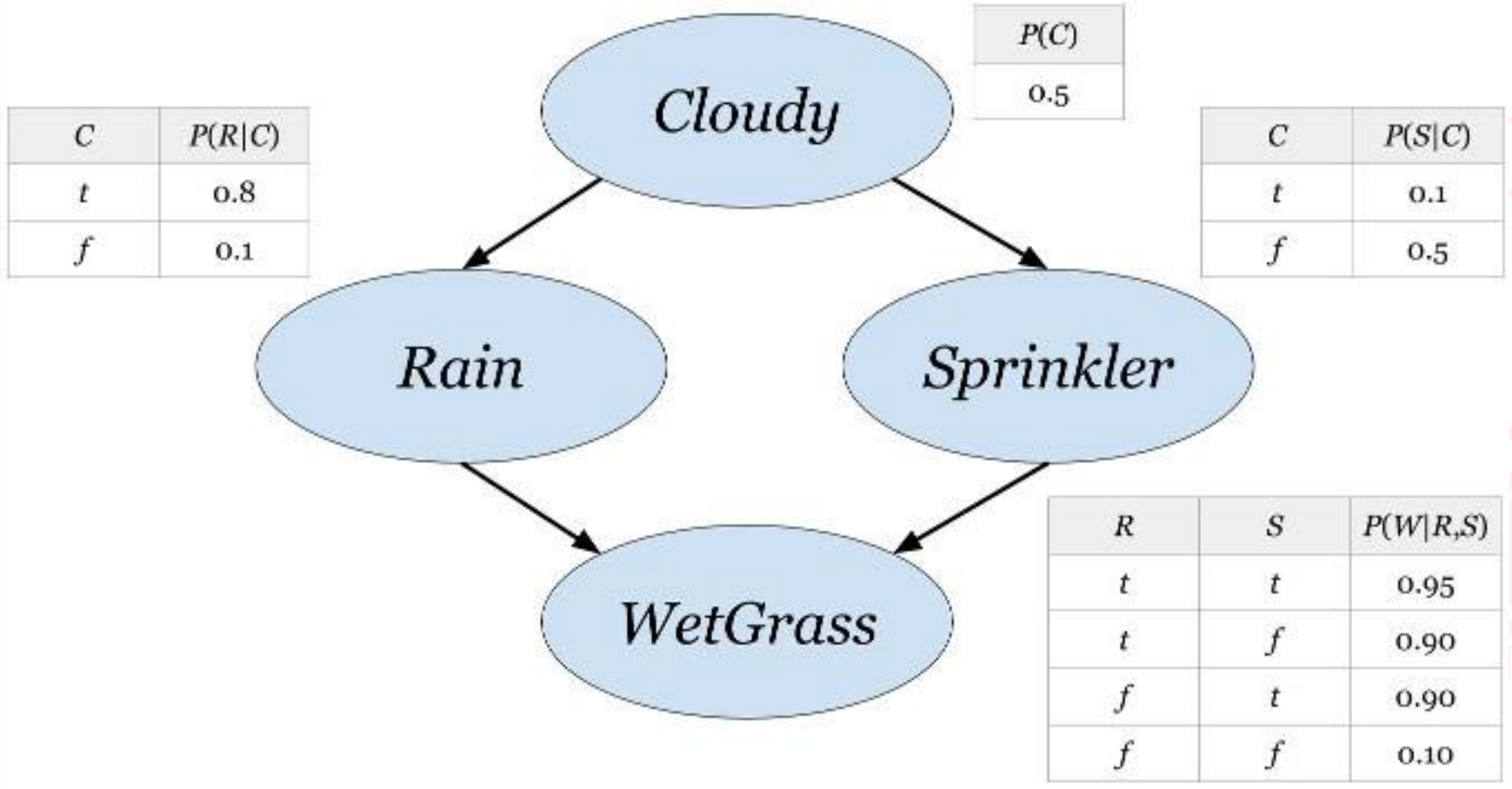
  $\mathbf{x} \leftarrow$ an event with n elements
  for each variable X_i **in** $X_1, \ldots, X_n$ **do**
    $\mathbf{x}[i] \leftarrow$ a random sample from $\mathbf{P}(X_i \mid parents(X_i))$
  return x

```
# some of these distributions were already defined before, but we repeate them just in case
P_C = np.array([0.5, 0.5])
P_S_C = np.array([[0.1, 0.5],[0.9, 0.5]])
P_R_C = np.array([[0.8, 0.1],[0.2, 0.9]])
P_W_SR = np.array([[[0.95, 0.9],[0.9, 0.1]],[[0.05, 0.1],[0.1, 0.9]]])

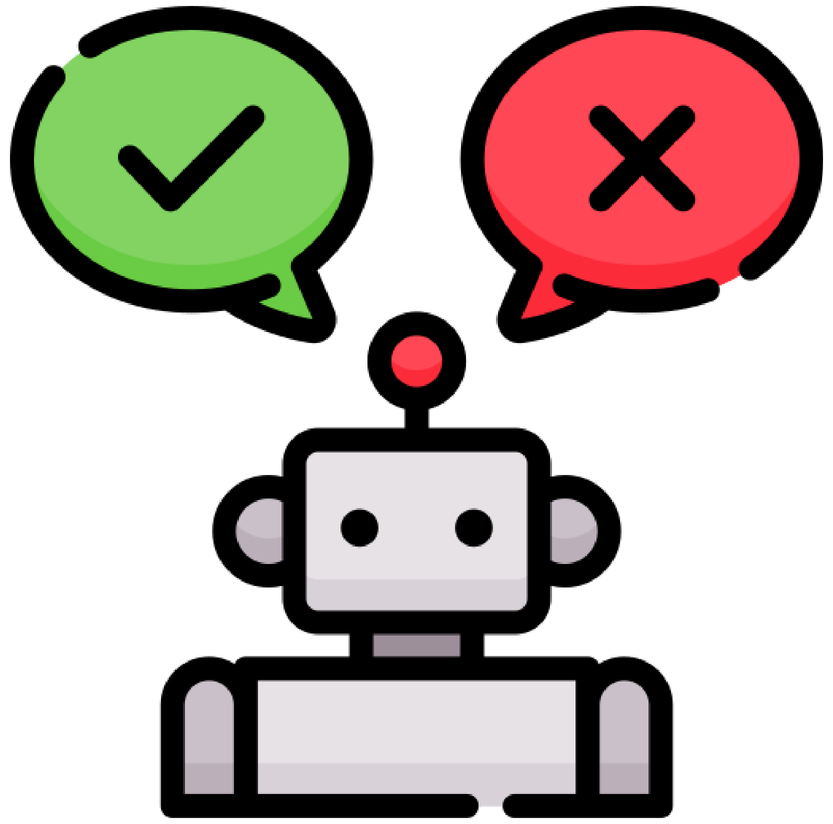
# network variables...
var = ['C','S','R','W']
# their distributions...
prd = {'C':P_C, 'S':P_S_C, 'R':P_R_C, 'W':P_W_SR}
# their parents...
par = {'C':[], 'S':['C'], 'R':['C'], 'W':['S','R']}
# and their initial values
val = {'C':f, 'S':f, 'R':f, 'W':f}
```



Prior-Sampling: Sprinkler example

Let's define also a function to **retrieve the values** of the parents of a variable:

```
def parents(X):
  return [val[i] for i in par[X]]
```



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P_W_SR = np.array([[[0.95, 0.9],[0.9, 0.1]],[[0.05, 0.1],[0.1, 0.9]]])

# network variables...
var = ['C','S','R','W']
# their distributions...
prd = {'C':P_C, 'S':P_S_C, 'R':P_R_C, 'W':P_W_SR}
# their parents...
par = {'C':[], 'S':['C'], 'R':['C'], 'W':['S','R']}
# and their initial values
val = {'C':f, 'S':f, 'R':f, 'W':f}
```

Prior-Sampling: Sprinkler example

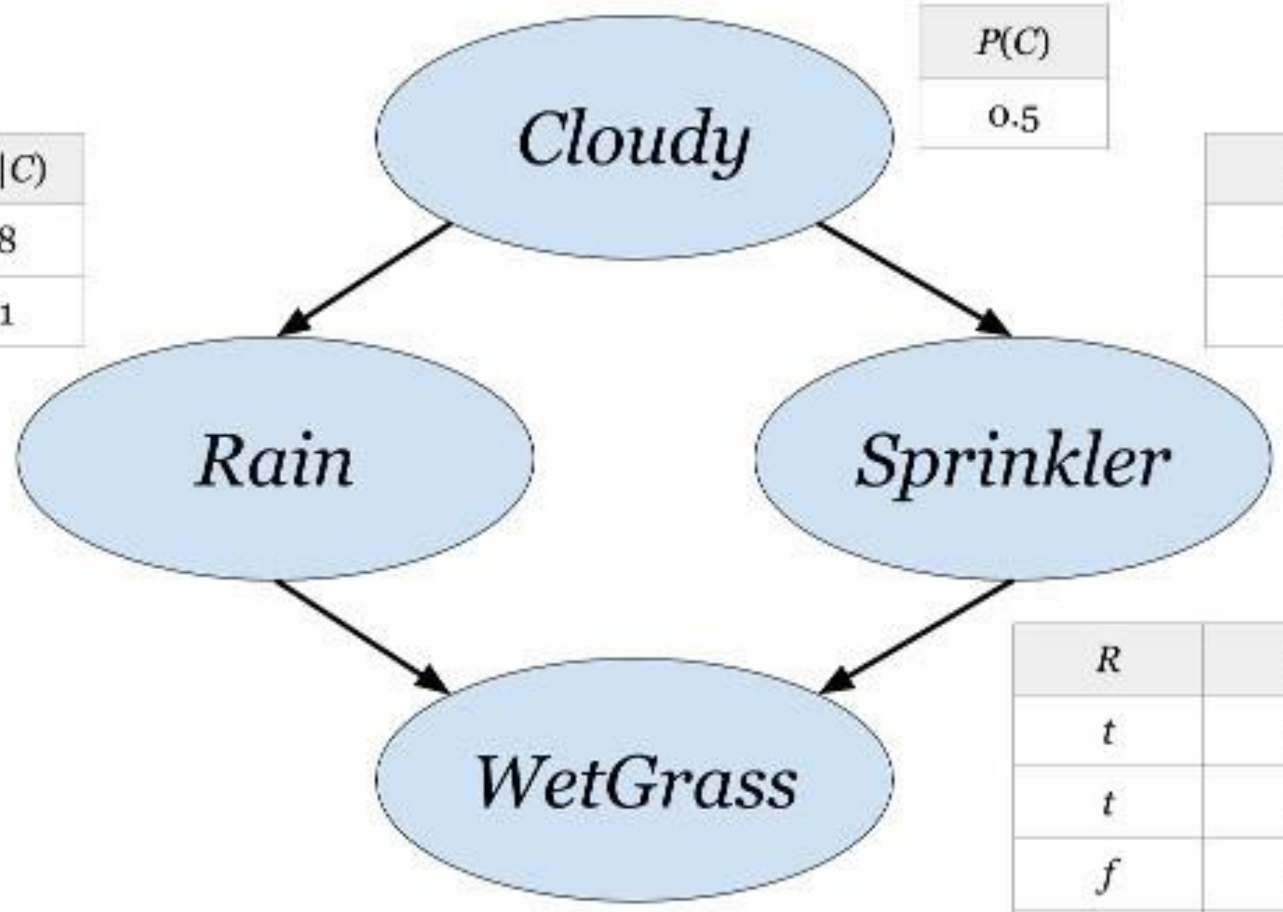
The following algorithm generates 1000 events from the Sprinkler network:

```
event = []

for n in range(1000):
  for x in var:
    val[x] = samplegen(prd[x], parents(x))
  event.append(['f' if val[x] else 't' for x in var])

print("First randomly generated event = ", event[0])
print("Number or randomly generated events = ", len(event))
```

```
First randomly generated event =  ['t', 'f', 't', 't']
Number or randomly generated events =  1000
```



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R	S	P(W|R,S)
t	t	0.95
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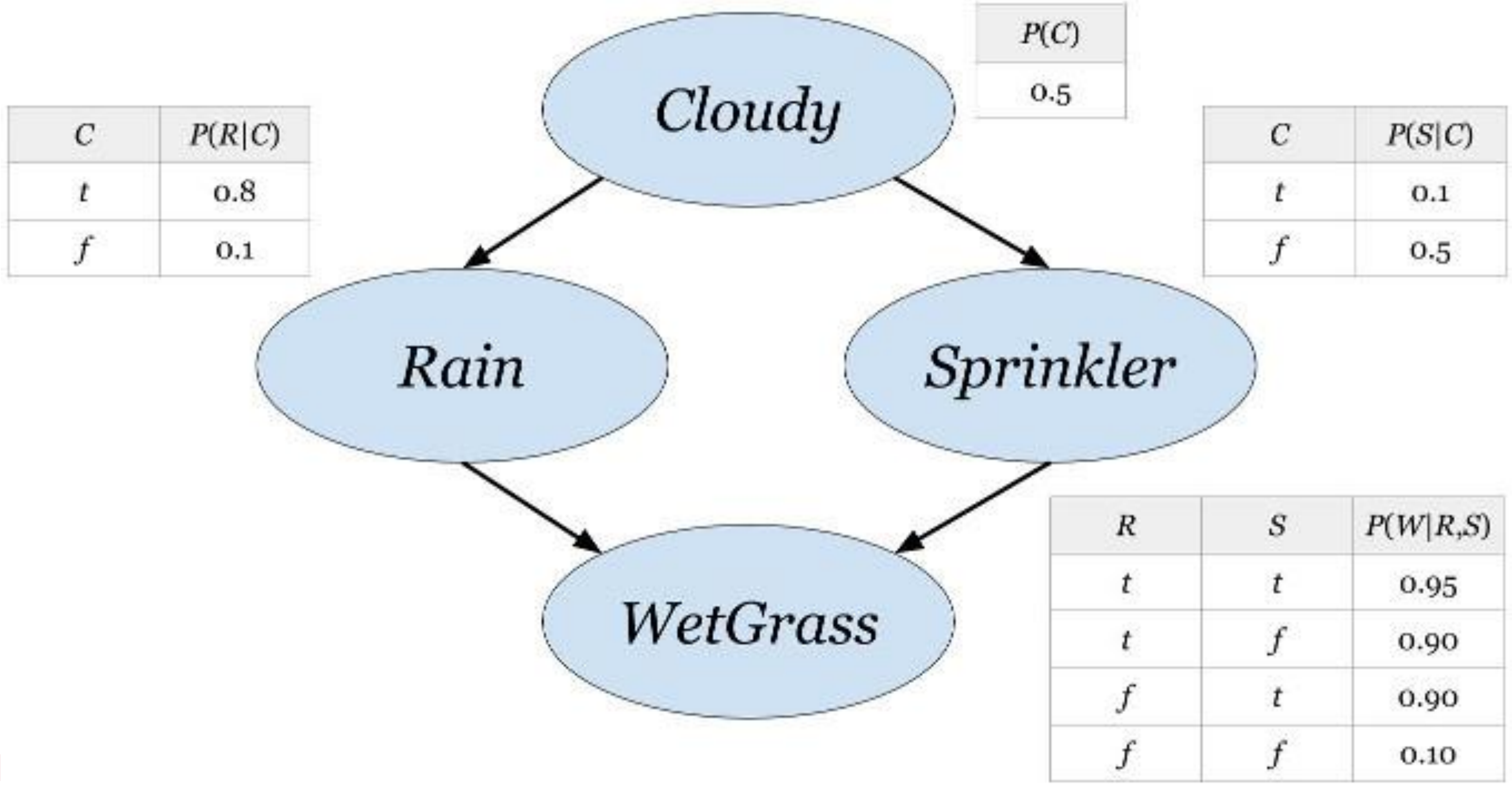
f	t	0.90
f	f	0.10

```
# network variables...
var = ['C','S','R','W']
# their distributions...
prd = {'C':P_C, 'S':P_S_C, 'R':P_R_C, 'W':P_W_SR}
# their parents...
par = {'C':[], 'S':['C'], 'R':['C'], 'W':['S','R']}
# and their initial values
val = {'C':f, 'S':f, 'R':f, 'W':f}
```

Prior-Sampling: Sprinkler example

Finally, we can compute the probability of any event by counting the number of times it was generated and normalising.

For example, we can verify



$$P(c, \neg s, r, w) = 0.328 \approx \frac{N_{PS}(c, \neg s, r, w)}{N}$$

```
P_query = event.count(['t', 'f', 't', 't']) / len(event)
print("P(c,¬s,r,w) = ", P_query)
```

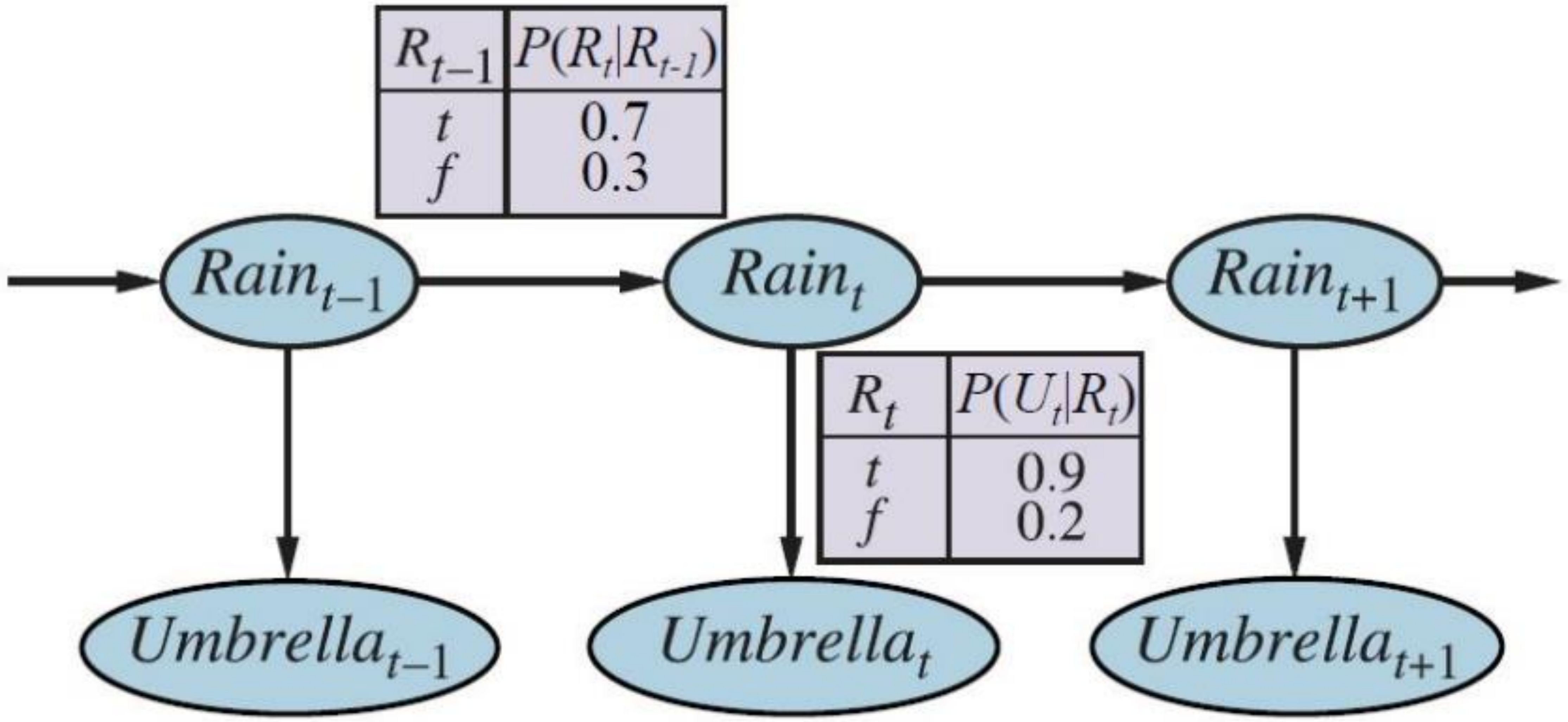
```
P(c,¬s,r,g) =  0.328
```

which is indeed very close to the exact probability!

Filtering: umbrella world example

Imagine to be imprisoned in a basement without windows, you only see whether the guard brings an umbrella or not

● First-order Markov process – the probability of rain is assumed to depend only on whether it rained the previous day



Bayesian network structure and conditional distributions describing the umbrella world: the transition model $P(Raint\,|Rain_{t-1})$ is

And the sensor model is $P(Umbrella_t\,|Rain_t)$

See the resolution in the excel

Pomegranate

pomegranate is a Python package that implements fast and **flexible probabilistic models** ranging from individual probability distributions to compositional models such as **Bayesian networks** and **hidden Markov models**.

The core philosophy behind pomegranate is that all **probabilistic models can be viewed as a probability distribution** in that they all yield probability estimates for samples and can be updated **given samples and their associated weights.**

We need to install it using: `pip install pomegranate`

https://pomegranate.readthedocs.io/en/latest/

Pomegranate for umbrella world

Then you can run the following version of the umbrella model.

pomegranate **can only solve Bayesian neworks (not Dynamic Bayesian Networks),** so we have to unroll the whole example to the depth that we want.

```
!pip install pomegranate
from pomegranate import *

# Variables are RainN and UmbrellaN+1 for N = 0, 1, ...
# We have a prior for Rain0, two values 'y'es and 'n'o:
Rain0   = DiscreteDistribution({'y': 0.5, 'n': 0.5})

# Transition model
#
# Conditional distribution relating RainN and RainN+1. Notation for
# the conditional probability table is:
#
# [ 'RainN', 'RainN+1', <probability>]
#
# for the conditional value P(Sprinkler|Cloudy). Note that we have to
# repeat the transition model for each pair of states
Rain1 = ConditionalProbabilityTable(
        [['y', 'y', 0.7],
         ['y', 'n', 0.3],
         ['n', 'y', 0.3],
         ['n', 'n', 0.7]], [Rain0])
```

Rain0 Rain1

```
Rain2 = ConditionalProbabilityTable(
        [['y', 'y', 0.7],
         ['y', 'n', 0.3],
         ['n', 'y', 0.3],
         ['n', 'n', 0.7]], [Rain1])
```

A discrete distribution, made up of characters and their probabilities, assuming that these probabilities will sum to 1.0.

https://pomegranate.readthedocs.io/en/stable/Distributions.html?highlight=DiscreteDistribution

Pomegranate for umbrella world

Then you can run the following version of the umbrella model.

pomegranate **can only solve Bayesian neworks (not Dynamic Bayesian Networks),** so we have to unroll the whole example to the depth that we want.

```
!pip install pomegranate
from pomegranate import *

# Variables are RainN and UmbrellaN+1 for N = 0, 1, ...
# We have a prior for Rain0, two values 'y'es and 'n'o:
Rain0   = DiscreteDistribution({'y': 0.5, 'n': 0.5})

# Transition model
#
# Conditional distribution relating RainN and RainN+1. Notation for
# the conditional probability table is:
#
# [ 'RainN', 'RainN+1', <probability>]
#
# for the conditional value P(Sprinkler|Cloudy). Note that we have to
# repeat the transition model for each pair of states
Rain1 = ConditionalProbabilityTable(
        [['y', 'y', 0.7],
         ['y', 'n', 0.3],
         ['n', 'y', 0.3],
         ['n', 'n', 0.7]], [Rain0])

Rain2 = ConditionalProbabilityTable(
        [['y', 'y', 0.7],
         ['y', 'n', 0.3],
         ['n', 'y', 0.3],
         ['n', 'n', 0.7]], [Rain1])
```

A conditional probability table, which is dependent on values from at least one previous distribution but up to as many as you want to encode for.

https://pomegranate.readthedocs.io/en/stable/MarkovChain.html?highlight=ConditionalProbabilityTable

Pomegranate for umbrella world

Then you can run the following version of the umbrella model.

pomegranate **can only solve Bayesian neworks (not Dynamic Bayesian Networks),** so we have to unroll the whole example to the depth that we want.

```
# Sensor model
#
# Conditional distribution relating Rain and Umbrella:
#
# [ 'Umbrella', 'Rain', <probability>]
#
# for the conditional value P(Sprinkler|Cloudy). Values for Umbrella are 'y'es and 'n'o.
# Again we have to enter the table for each day.
Umbrella1 = ConditionalProbabilityTable(
        [['y', 'y', 0.9],
         ['y', 'n', 0.1],
         ['n', 'y', 0.2],
         ['n', 'n', 0.8]], [Rain1])

Umbrella2 = ConditionalProbabilityTable(
        [['y', 'y', 0.9],
         ['y', 'n', 0.1],
         ['n', 'y', 0.2],
         ['n', 'n', 0.8]], [Rain2])
```

A conditional probability table, which is dependent on values from at least one previous distribution but up to as many as you want to encode for.

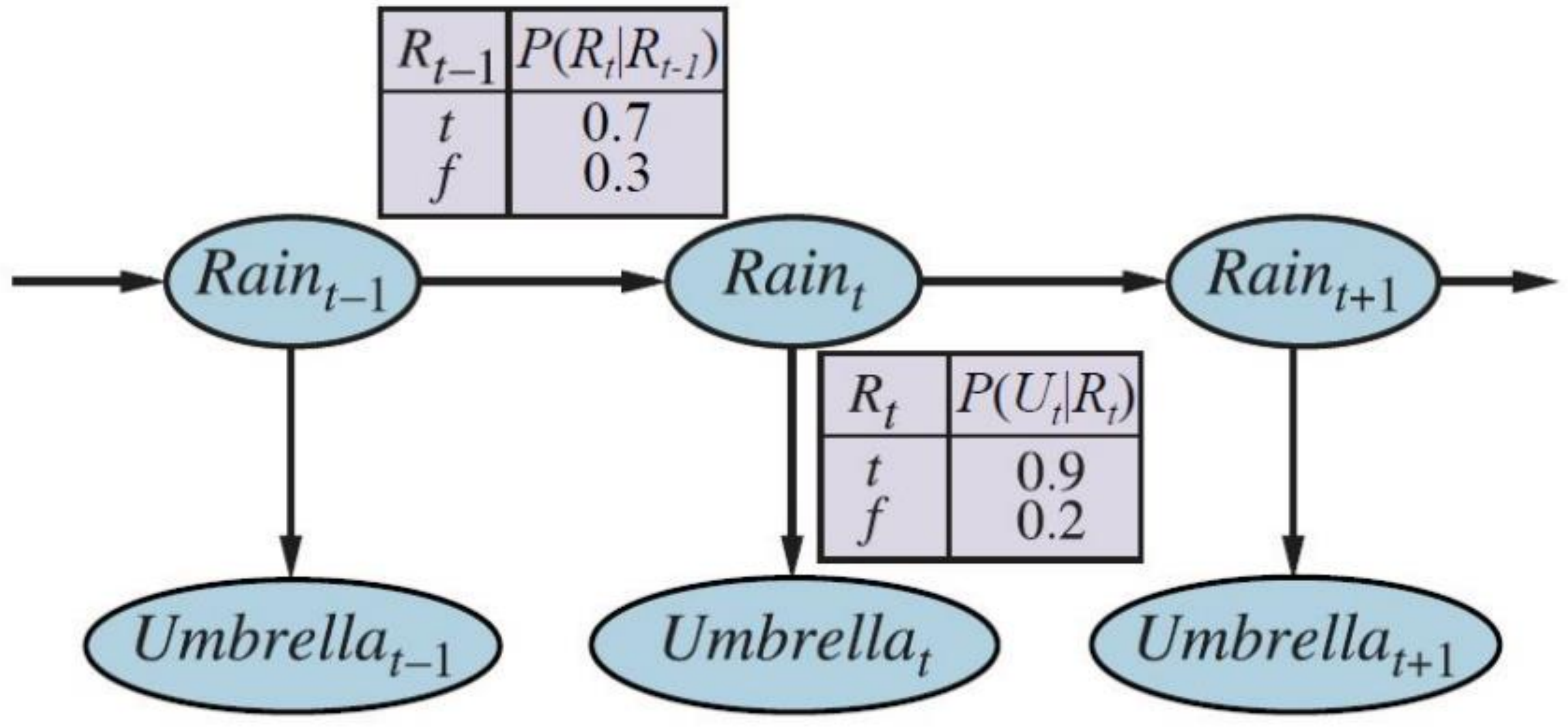
https://pomegranate.readthedocs.io/en/stable/MarkovChain.html?highlight=ConditionalProbabilityTable

Pomegranate for umbrella world

Then you can run the following version of the umbrella model.

pomegranate **can only solve Bayesian neworks (not Dynamic Bayesian Networks),** so we have to unroll the whole example to the depth that we want.

```
#
# The whole network has five nodes:
s1 = Node(Rain0, name="Rain0")
s2 = Node(Rain1, name="Rain1")
s3 = Node(Umbrella1, name="Umbrella1")
s4 = Node(Rain2, name="Rain2")
s5 = Node(Umbrella2, name="Umbrella2")
# Create a network that includes nodes and edges between them:
model = BayesianNetwork("Umbrella Network")
model.add_states(s1, s2, s3, s4, s5)
model.add_edge(s1, s2)
model.add_edge(s2, s3)
model.add_edge(s2, s4)
model.add_edge(s4, s5)
# Fix the model structure
model.bake()
```



https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Then you can run the following version of the umbrella model.

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s1 = Node(Rain0, name="Rain0")
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s5 = Node(Umbrella2, name="Umbrella2")
# Create a network that includes nodes and edges between them:
model = BayesianNetwork("Umbrella Network")
model.add_states(s1, s2, s3, s4, s5)
model.add_edge(s1, s2)
model.add_edge(s2, s3)
model.add_edge(s2, s4)
model.add_edge(s4, s5)
# Fix the model structure
model.bake()
```

Finalize the topology of the model. Assign a numerical index to every state and create the underlying arrays corresponding to the states and edges between the states. **This method must be called before any of the probability-calculating methods.** This includes converting conditional probability tables into joint probability tables and creating a list of both marginal and table nodes.

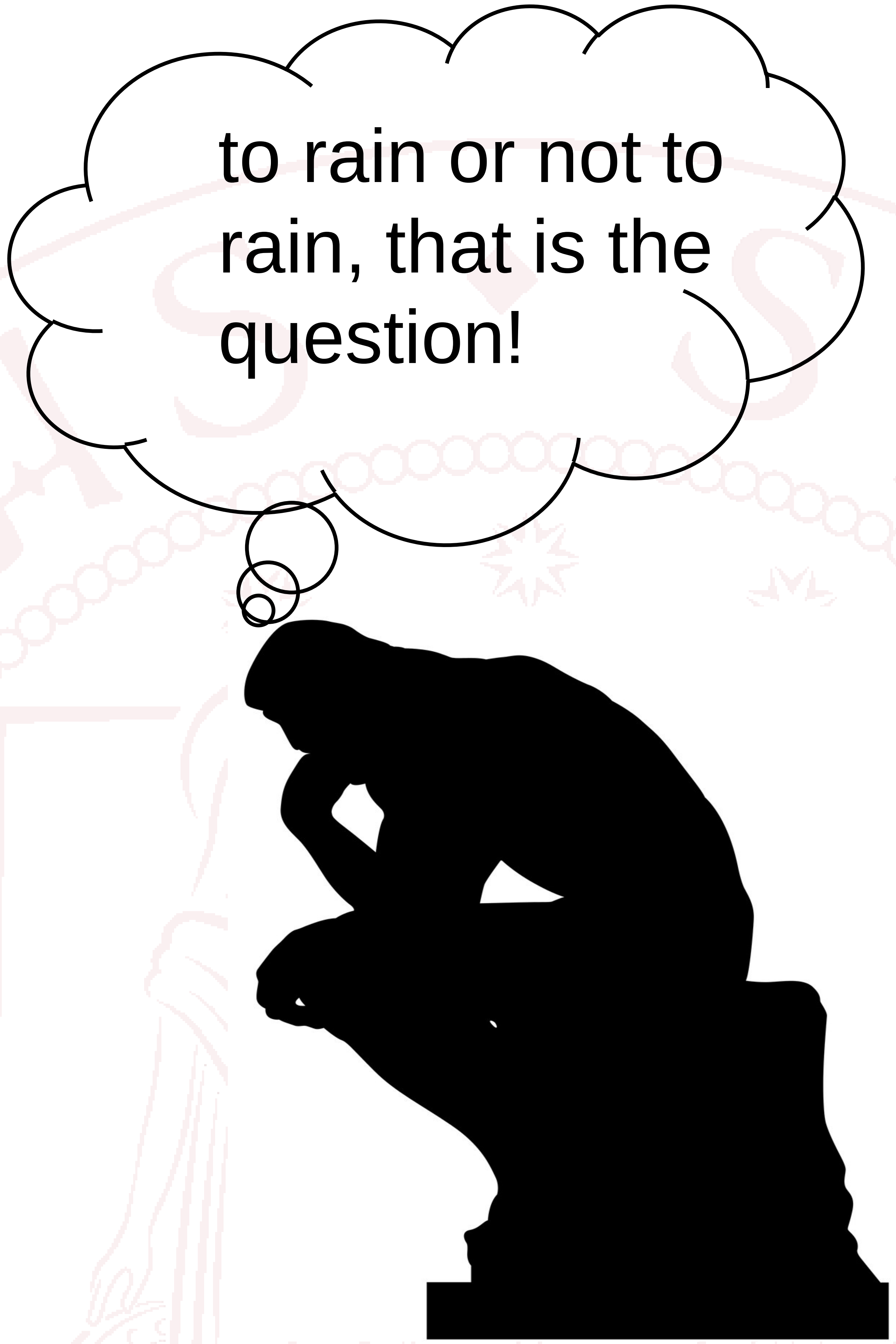
https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now that we have the model entered, we can ask it questions.
We can first ask it to predict the probability of rain on days 1 and 2:

```
# Do not instantiate any of the variables:
scenario = [[None, None, None, None, None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.5000000000000001), ('n', 0.49999999999999999))
(('y', 0.5000000000000001), ('n', 0.49999999999999999))
```



The reason that we ask for elements 1 and 3 of the datastructure results is because **they are elements 1 and 3 of model.add_states(s1, s2, s3, s4, s5)**

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now that we have the model entered, we can ask it questions.
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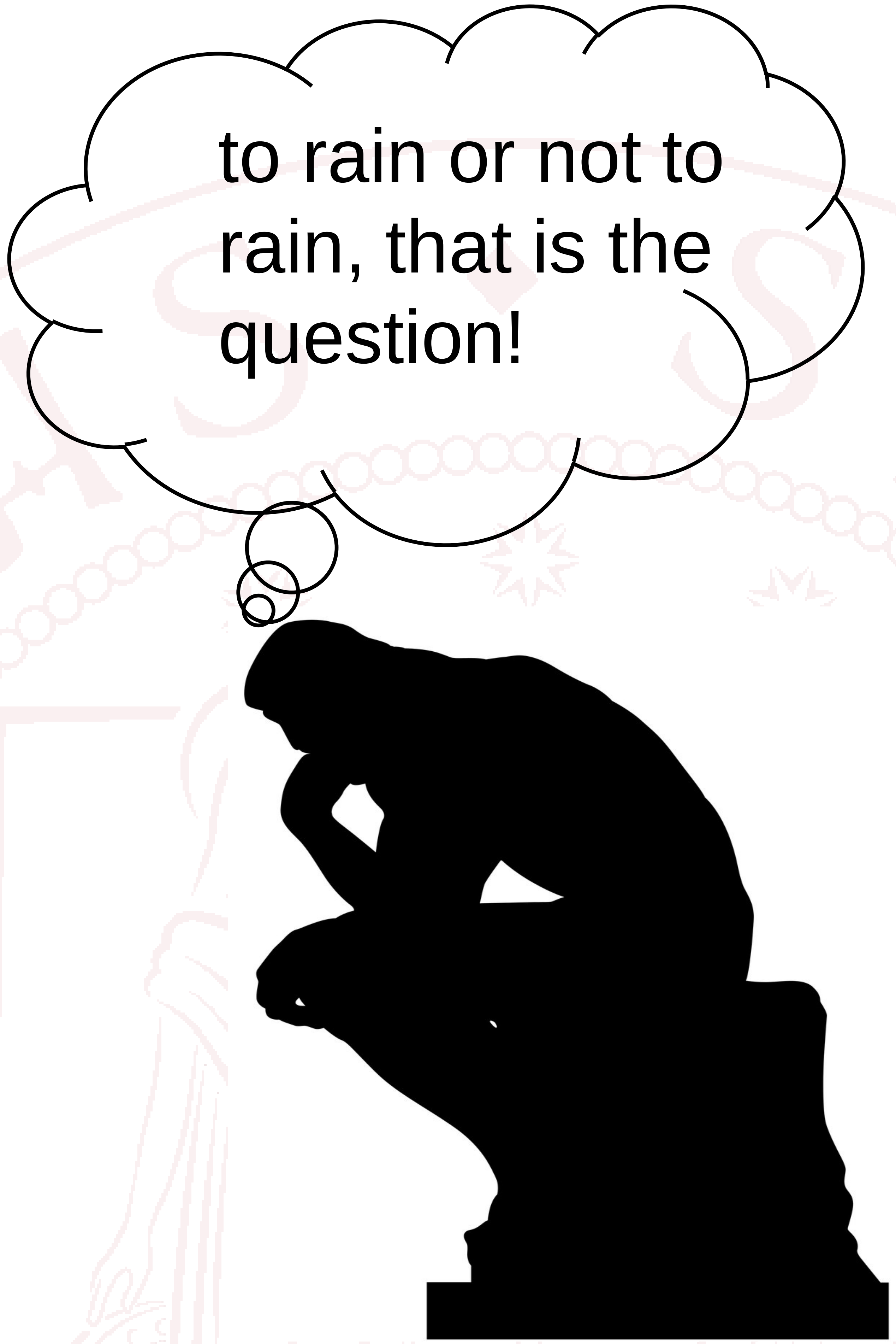
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.5000000000000001), ('n', 0.4999999999999999))
(('y', 0.5000000000000001), ('n', 0.4999999999999999))
```



So both Day 1 and Day 2 have a probability 0.5 of being rainy before we see any umbrellas.

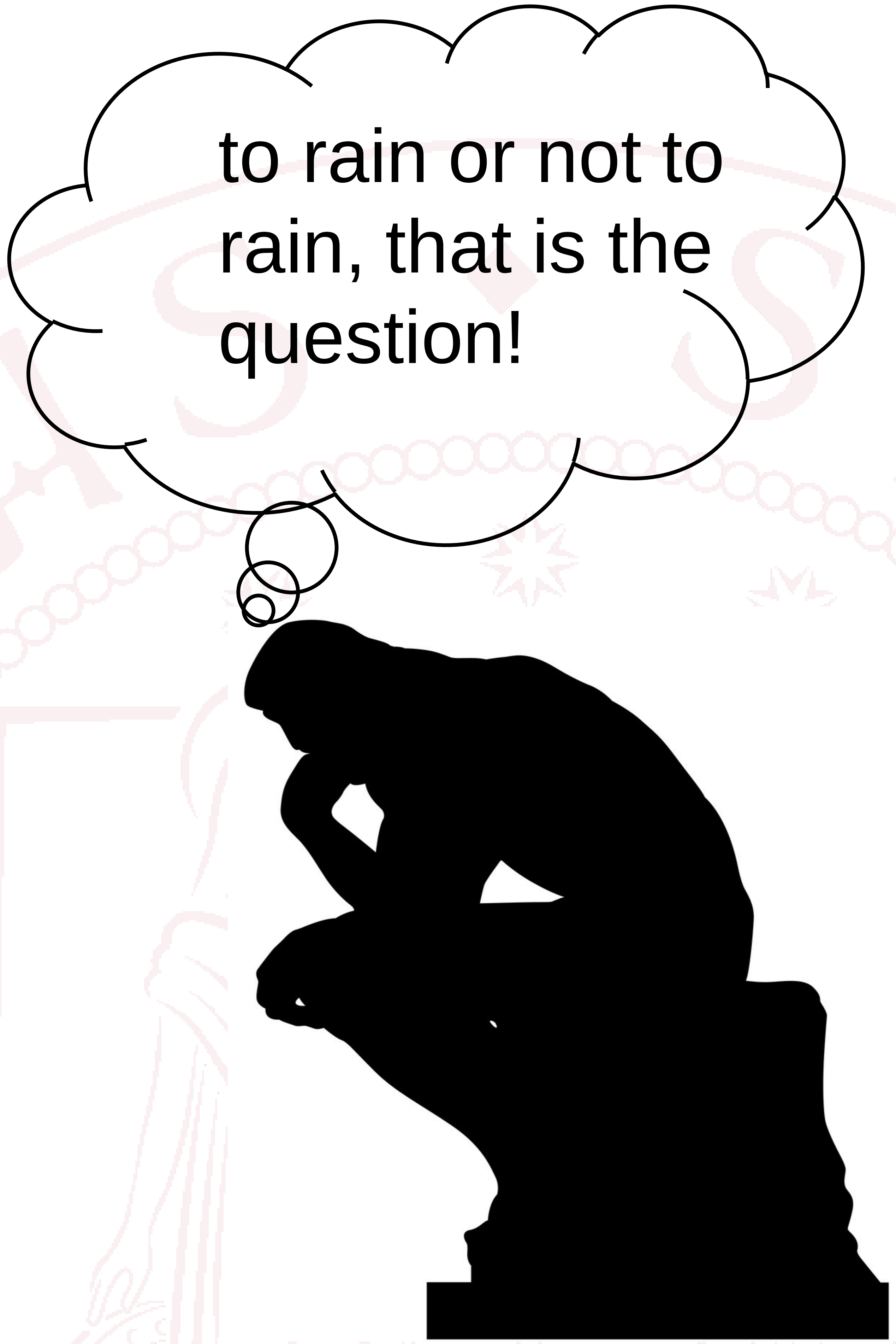
https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now that we have the model entered, we can ask it questions.
We can first ask it to predict the probability of rain on days 1 and 2:

```
# Do not instantiate any of the variables:
scenario = [[None, None, None, None, None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.5000000000000001), ('n', 0.4999999999999999))
(('y', 0.5000000000000001), ('n', 0.4999999999999999))
```



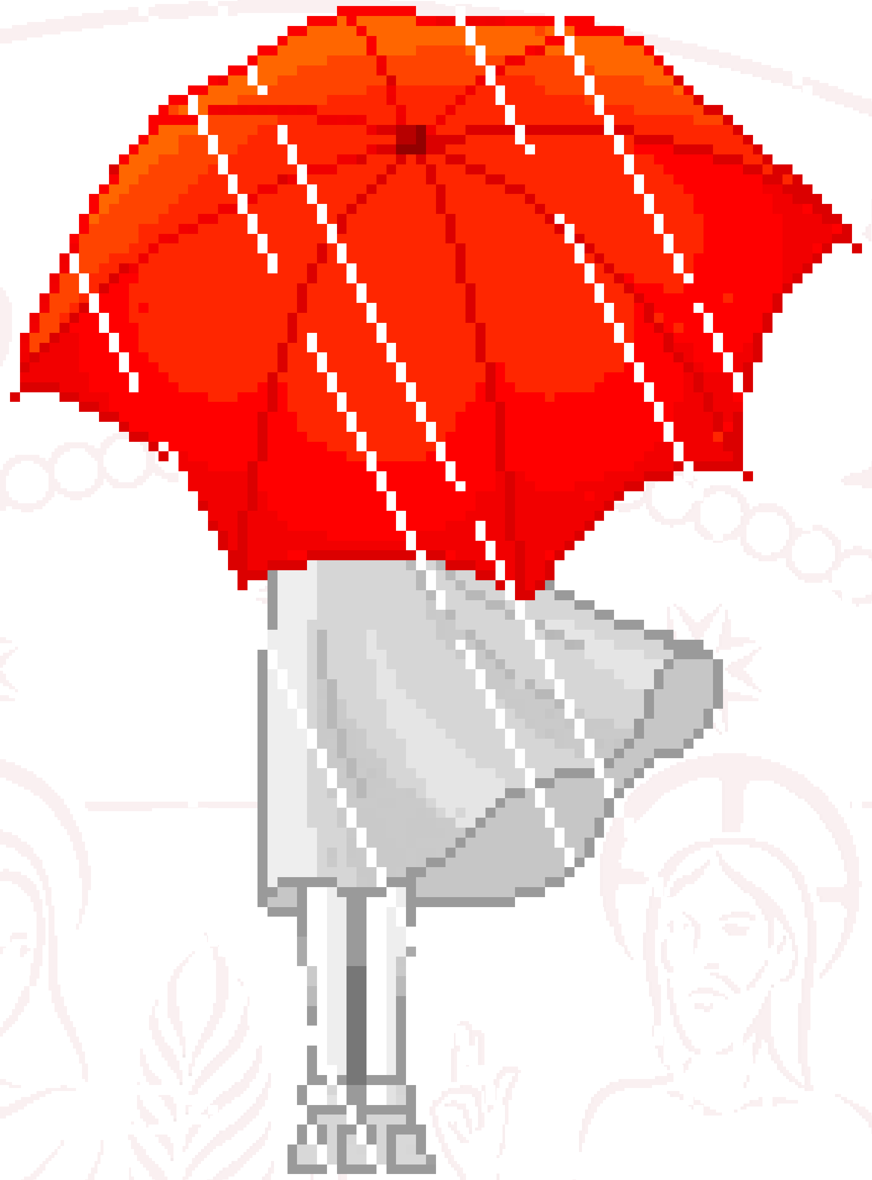
In Bayesian probability terms, this tells us that we can't say anything about how likely it is to rain.
A binary variable with probability of 0.5 for both values is how we represent

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now let's tell the model that we see an **umbrella on Day 1** and see what that gets us:

```
# Set Rain1 to 'y'
scenario = [[None, None, 'y', None, None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```



```
(('y', 0.8181818181818179), ('n', 0.18181818181818121))
(('y', 0.627272727272727271), ('n', 0.372727272727272729))
```

So it has filtered the probability of rain for Day 1,
and also predicted the probability for Day 2 as well.

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now let's tell the model that we see an **umbrella on Day 1** and see what that gets us:

```
# Set Rain1 to 'y'
scenario = [[None, None, 'y', None, None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.8181818181818179), ('n', 0.18181818181818821))
(('y', 0.6272727272727271), ('n', 0.3727272727272729))
```

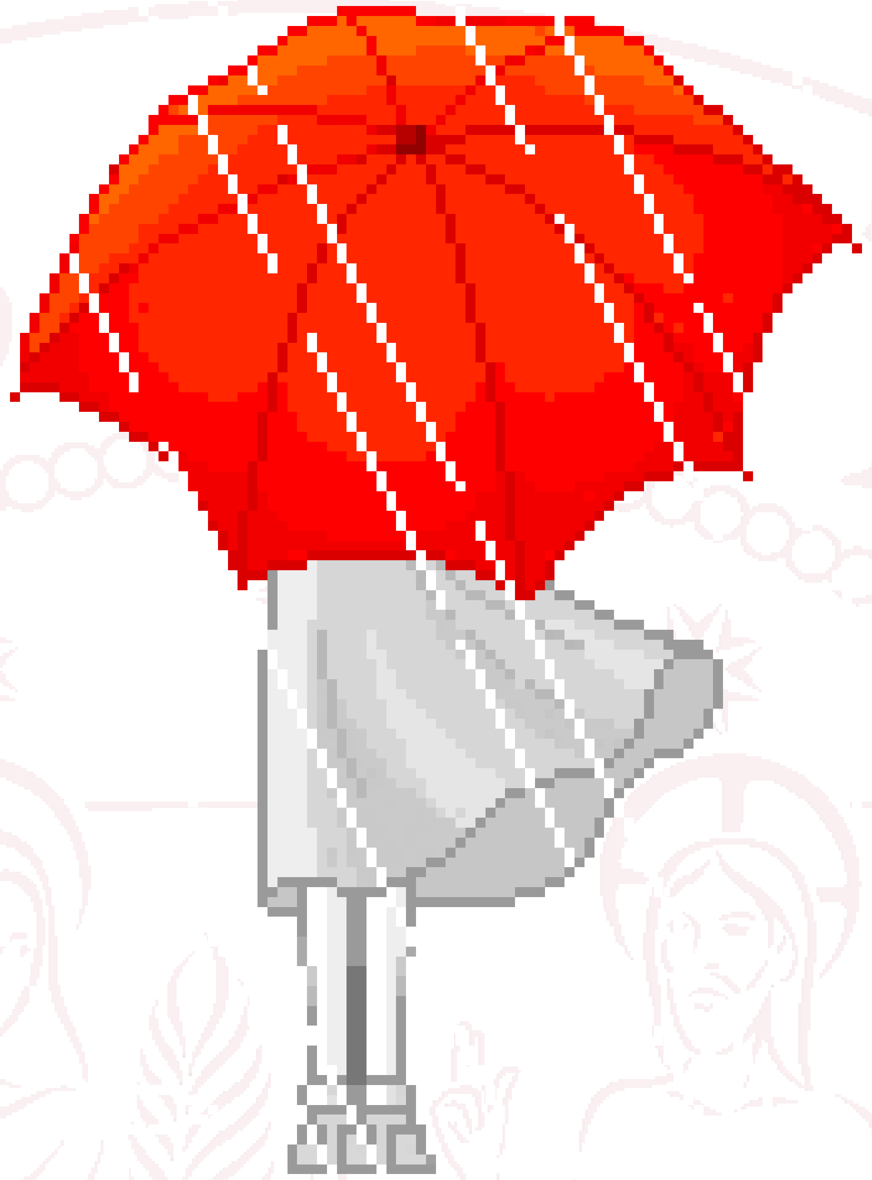
That is because pomegranate **propagates all updates** through the whole model/network.

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

That is because pomegranate **propagates all updates** through the whole model/network.

It has, for example also **computed the probability of rain on Day 0** (that it rained on Day 0 even though we said noting about the rain that day):



```
# Ask for the probability of rain on Day 0:
print(results[0][0].items())
```

```
(('y', 0.6272727272727271), ('n', 0.3727272727272727 3))
```

This is what we call the **smoothed probability** of rain on Day 0.

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now let's tell pomegranate about rain on Day 3, so we need to add information about Day3:

```
# Conditional probability table
Rain3 = ConditionalProbabilityTable(
        [['y', 'y', 0.7],
         ['y', 'n', 0.3],
         ['n', 'y', 0.3],
         ['n', 'n', 0.7]], [Rain2])
# Node
s6 = Node(Rain3, name="Rain3")
# State
model.add_states(s6)
# Edge
model.add_edge(s4, s6)
# Fix the model structure
model.bake()
```

Note that we only call model.bake() once the last elements are entered.

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

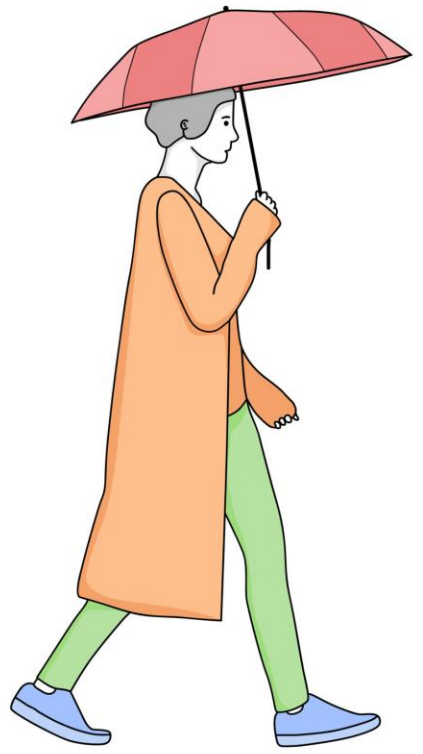
Pomegranate for umbrella world

Now that we have the model entered, we can ask it questions.
We tell the model that we saw Umbrellas on Days 1 and 2:

```
# Umbrellas on Day 1 and 2:
scenario = [[None, None, 'y', None, 'y', None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.8833570412517776), ('n', 0.11664295874822228))
(('y', 0.8833570412517776), ('n', 0.1166429587482225))
```

Day1



Day2

https://pomegranate.readthedocs.io/en/stable/BayesianNetwork.html

Pomegranate for umbrella world

Now that we have the model entered, we can ask it questions.
We tell the model that we saw Umbrellas on Days 1 and 2:

```
# Umbrellas on Day 1 and 2:
scenario = [[None, None, 'y', None, 'y', None]]
# Run the model
results =  model.predict_proba(scenario)
# Ask for the probability of rain on Day 1:
print(results[0][1].items())
# And the probability of rain on Day 2:
print(results[0][3].items())
```

```
(('y', 0.8833570412517776), ('n', 0.11664295874822228))
(('y', 0.8833570412517776), ('n', 0.1166429587482225))
```

Note that we **didn't tell pomegranate to do smoothing**. As we saw before with Day 0, it (in effect) **always runs the backwards propagation** and gives us smoothed probabilities for all days before the latest piece of evidence.

I said "in effect" because pomegranate doesn't do the computation the way we studied.

It just computes the probability of **every hidden variable** given the evidence.

Questions