Causal Inference - Part A

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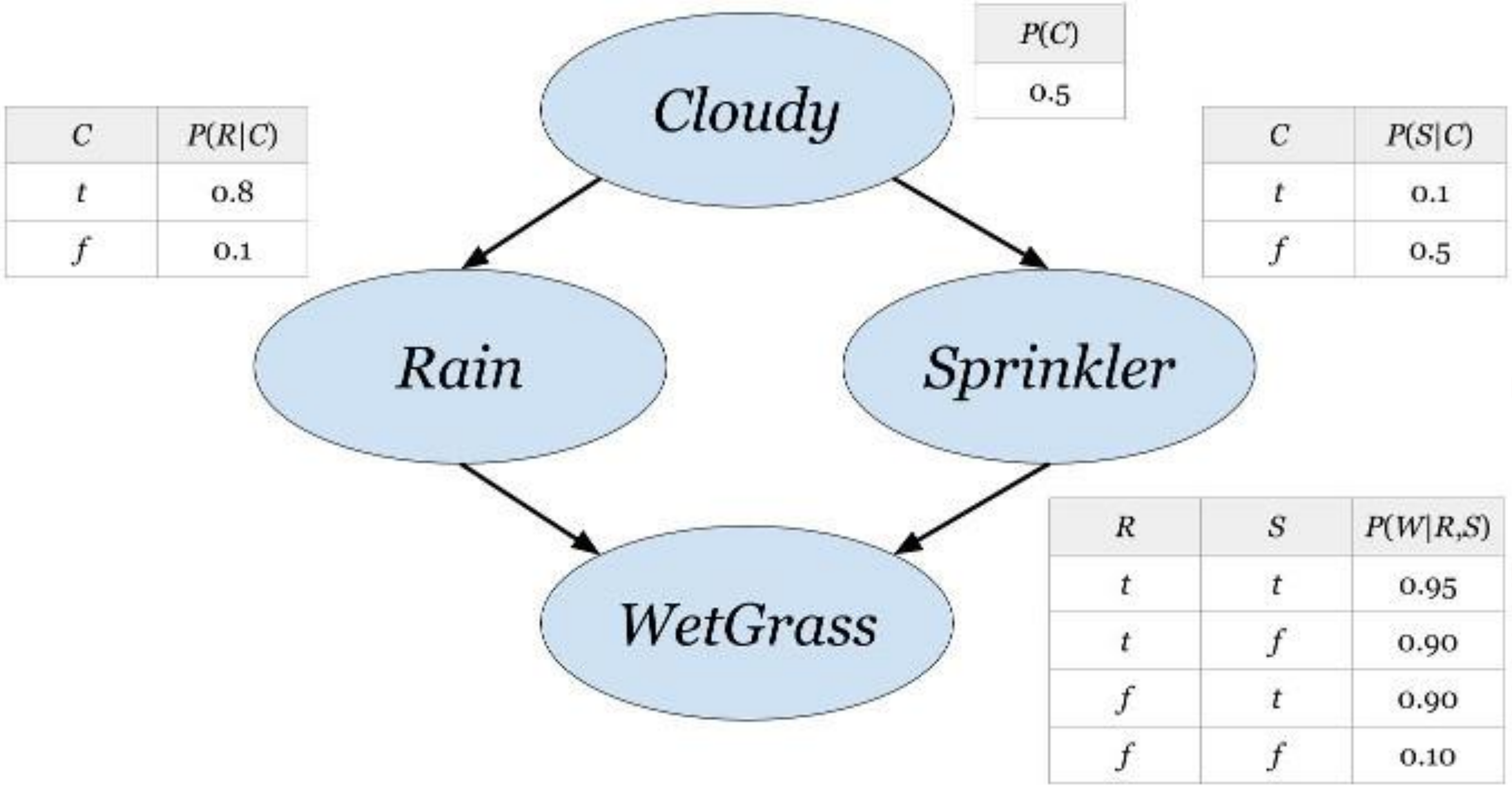
Topics:

- Causal effect of rain on wet grass: Sprinkler example
- Recap on Interventions
- Recap on adjustment formula
- pyAgrum
- Simpson’s paradox
- Example Simpson's paradox via pyAgrum

DIPARTIMENTO
DI INGEGNERIA
DELL’INFORMAZIONE

Causal effect of rain on wet grass: Sprinkler example

Let's consider again our Sprinkler network, assuming this is a reliable description of the causal relationships between its four variables:



P(C)
0.5

C	P(R|C)
t	0.8
f	0.1

C	P(S|C)
t	0.1
f	0.5

R	S	P(W|R,S)
t	t	0.95
t	f	0.90
f	t	0.90
f	f	0.10

We want to estimate
the **causal effects of the rain on the "wetness" of the grass**.

Causal effect of rain on wet grass: Sprinkler example

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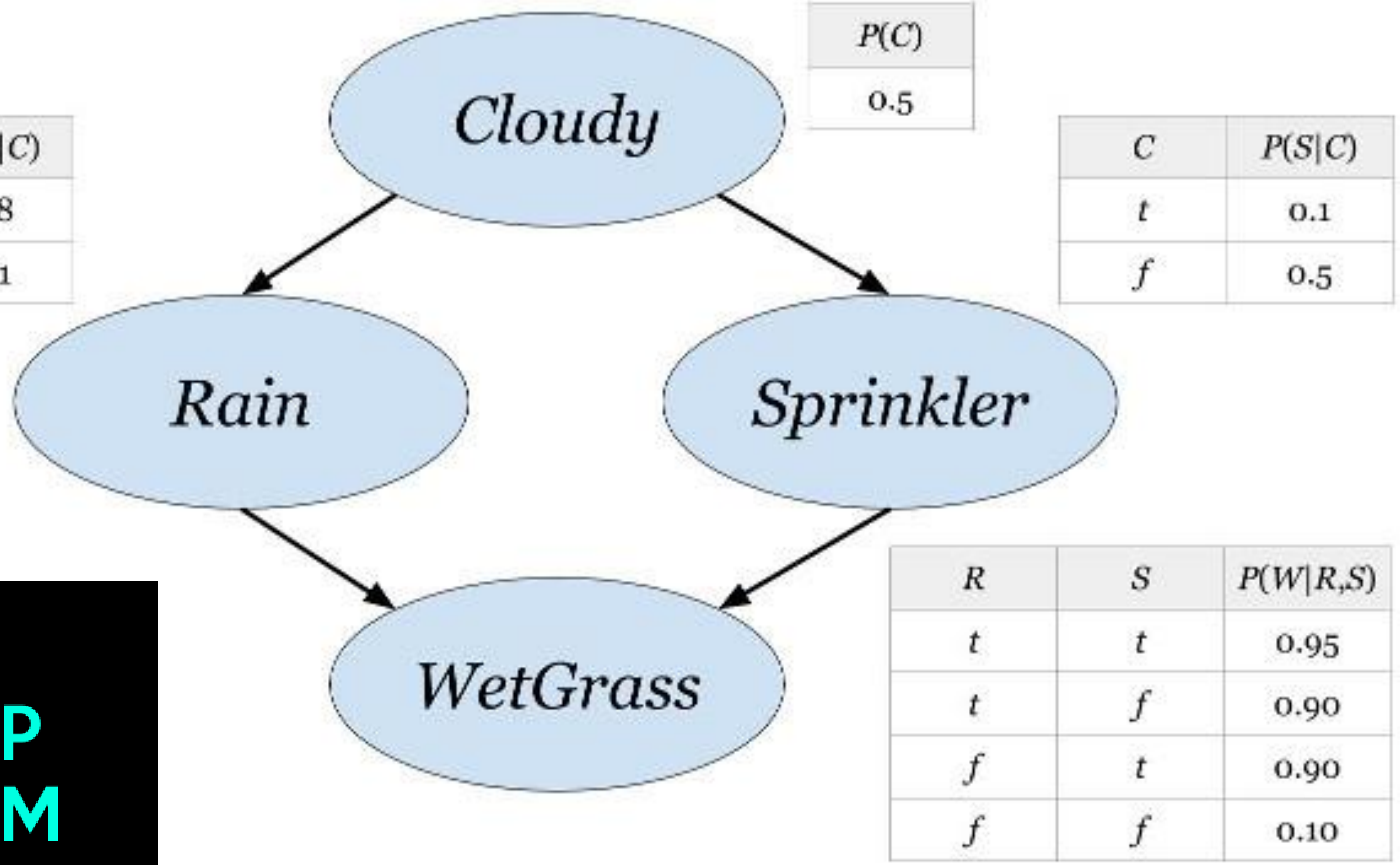
Note that it wouldn't be physically possible to modify the rain variable R.

Yet, we can **use probabilities from observational data** of the weather to compute its causal effect "as if" we were able to intervene on it.



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C	*P(S|C)*
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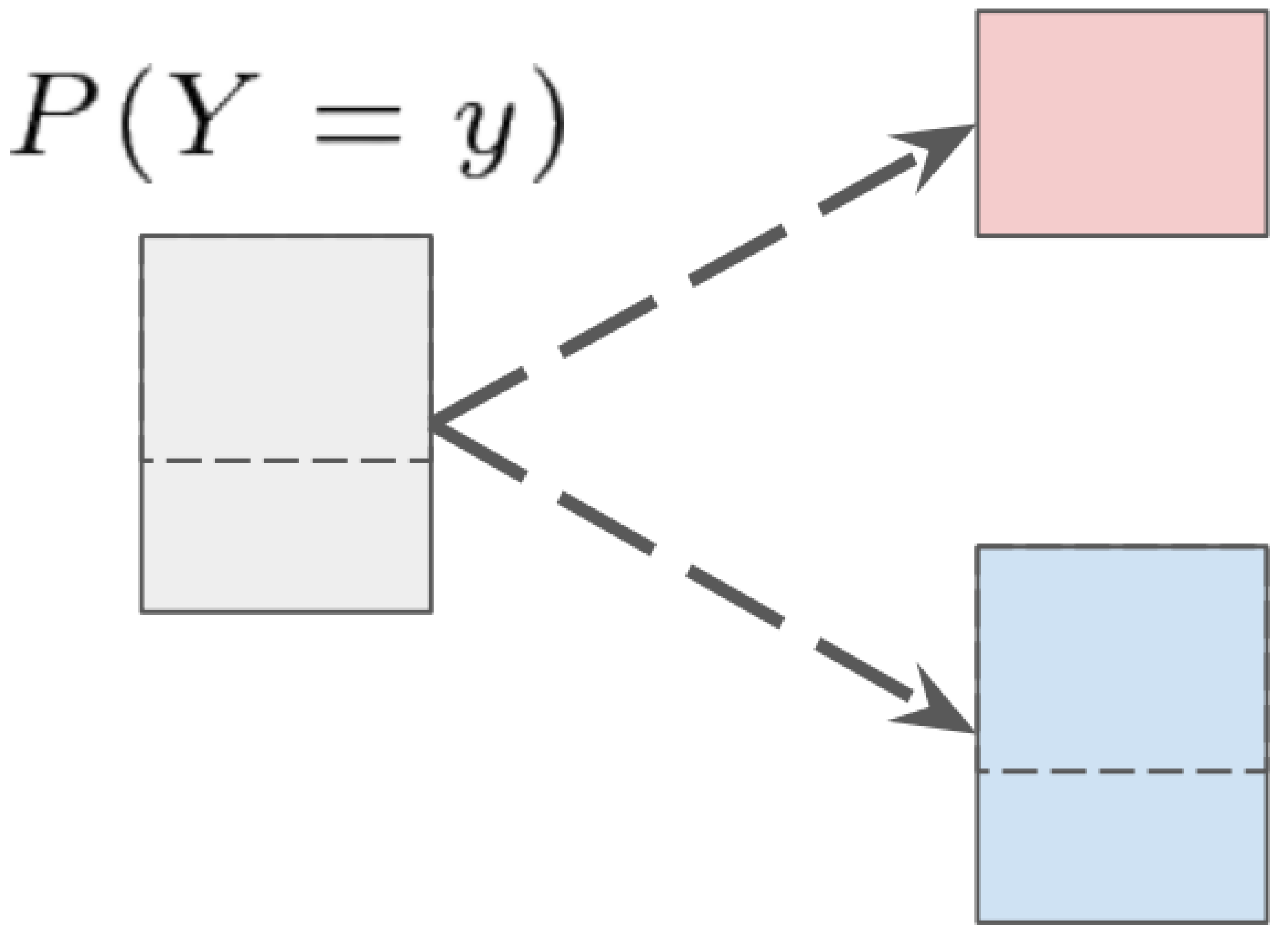
f	0.5

R	*S*	*P(W|R,S)*
t	*t*	0.95
t	*f*	0.90
f	*t*	0.90
f	*f*	0.10

Causal effect of rain on wet grass: Sprinkler example

Interventions

- In a dataset, when we condition the outcome $Y = y$ on an **observation** $X = x$, we simply consider the subset of Y where we observe that X is equal to x
- But when we condition $Y = y$ on an **intervention** $do(X = x)$, we force the value of X for the entire set Y



$$P(Y = y|X = x) = \frac{P(Y = y, X = x)}{P(X = x)}$$

$$P(Y = y|do(X = x)) = \ ?$$

Slide 28 by prof. Bellotto

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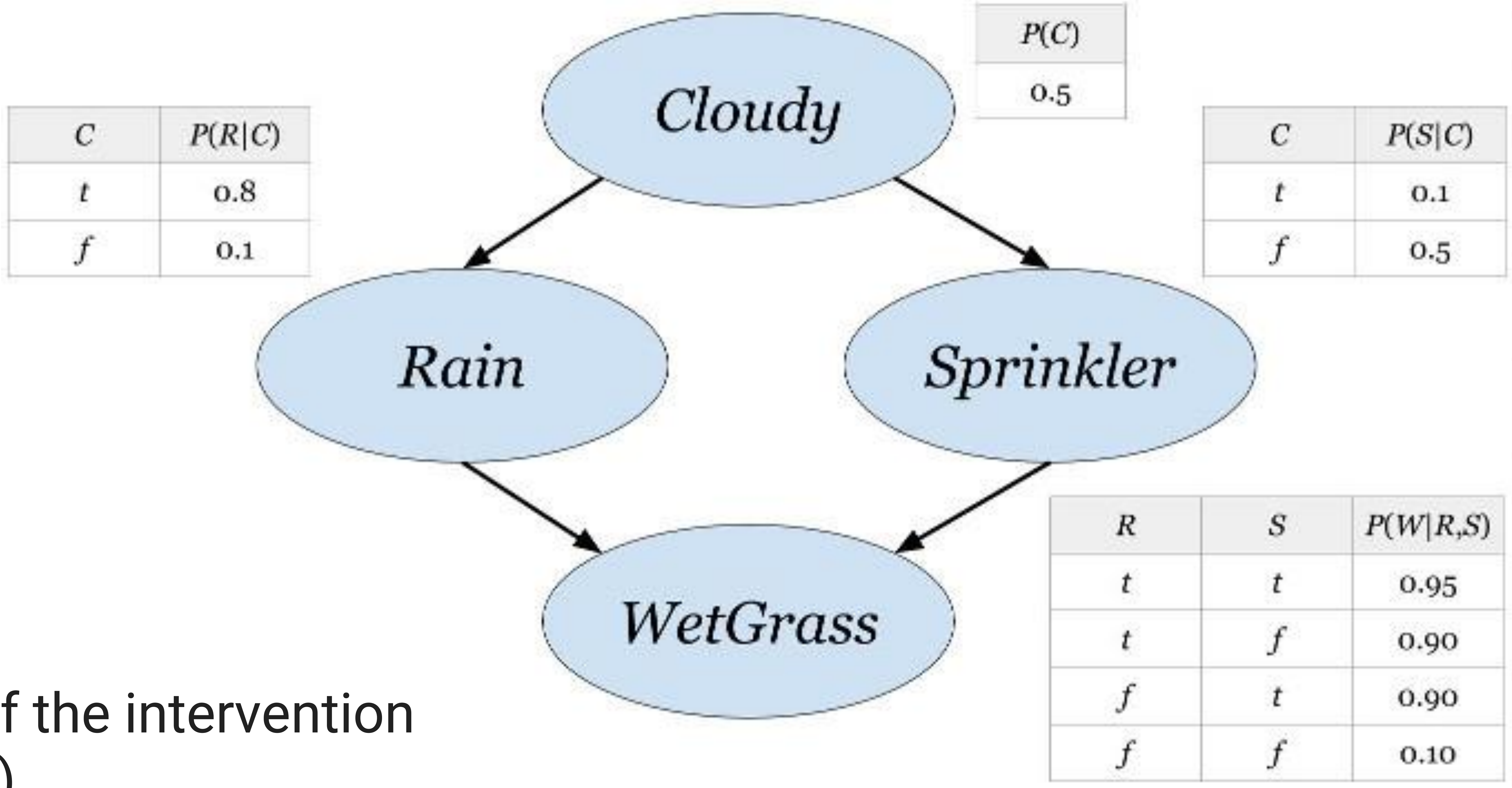

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To this end, we can compute the effect of the intervention P(G=true|do(R=true)), or simply P(g|do(r)),

by using the adjustment formula for the only parent of R, which is C

Causal effect of rain on wet grass: Sprinkler example

Adjustment formula

$$\begin{aligned} P(y|do(x)) &= P_m(y|x) && \text{from definition of intervention} \\ &= \sum_z P_m(y|x,z)P_m(z|x) && \text{from Law of Total Probability} \\ &= \sum_z P_m(y|x,z)P_m(z) && \text{from independence of } X \text{ and } Z \\ &= \sum_z P(y|x,z)P(z) && \text{from previous slide's equalities} \end{aligned}$$

- More in general, we can write the **adjustment formula**, or **causal effect rule**:

$$P(y|do(x)) = \sum_{z \in \Lambda} P(y|x,z)P(z)$$

where Λ is the set of parents of X

Slide 33 by prof. Bellotto

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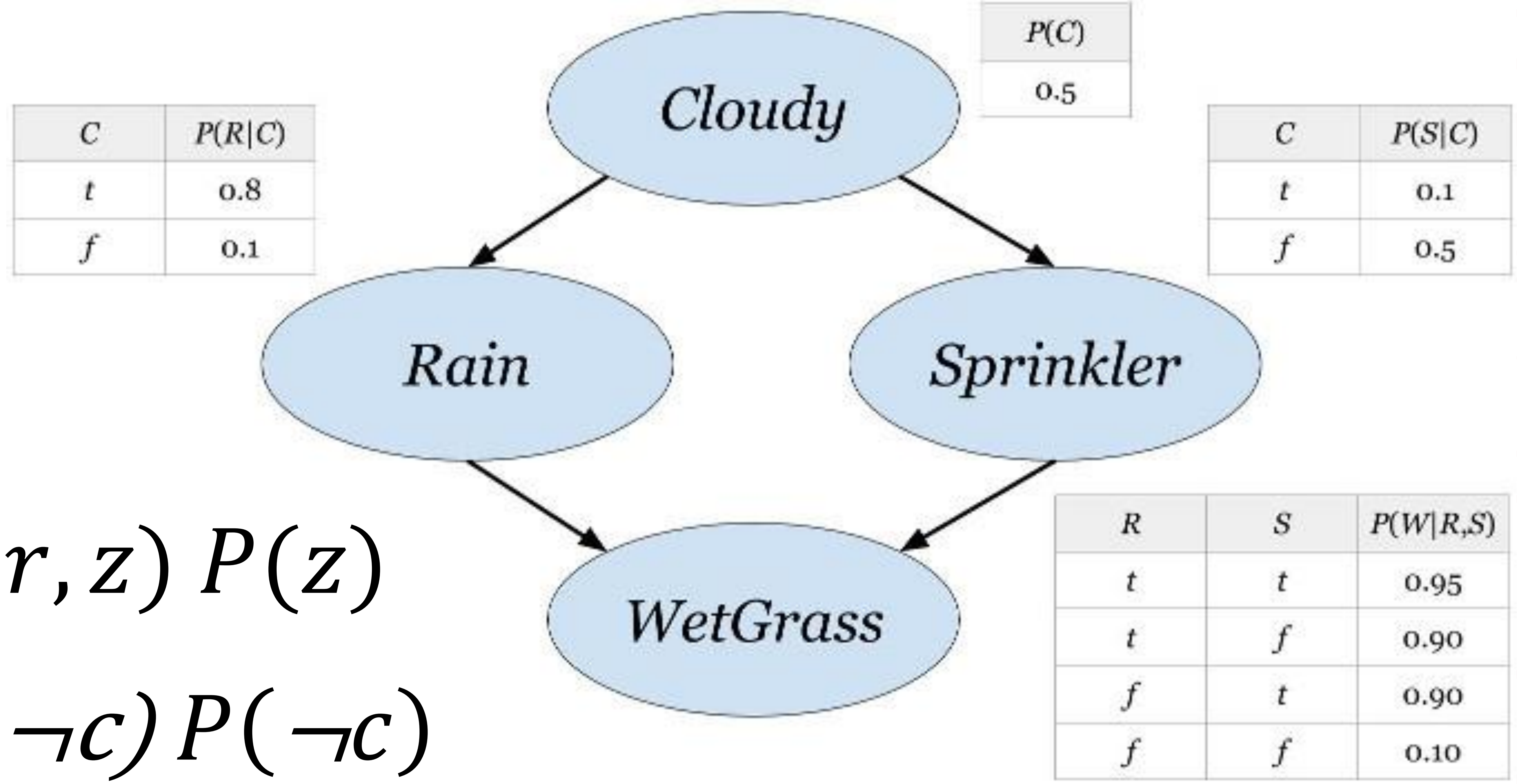

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$$P(g|do(r)) = \sum_{z \in C} P(g|r,z)\,P(z)$$

$$= P(g|r,c)\,P(c) + P(g|r,\neg c)\,P(\neg c)$$

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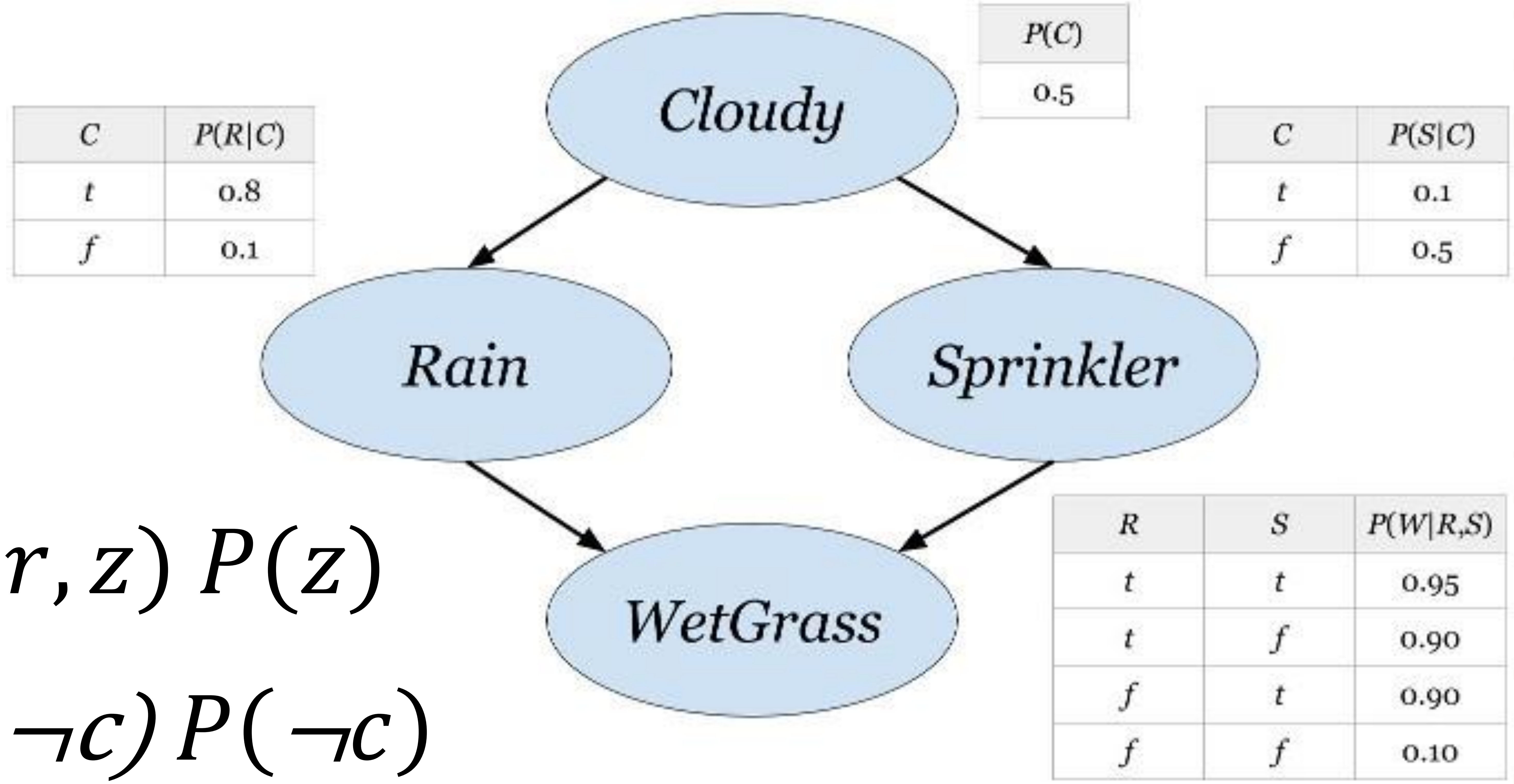

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$$P(g/do(r)) = \sum_{z\in C} P(g|r,z)\,P(z)$$

$$= P(g/r,c)\,P(c) + P(g/r,\neg c)\,P(\neg c)$$

The probability distribution $\mathbf{P}(C) = \langle P(c), P(\neg c)\rangle$ is already given by the network.

Causal effect of rain on wet grass: Sprinkler example

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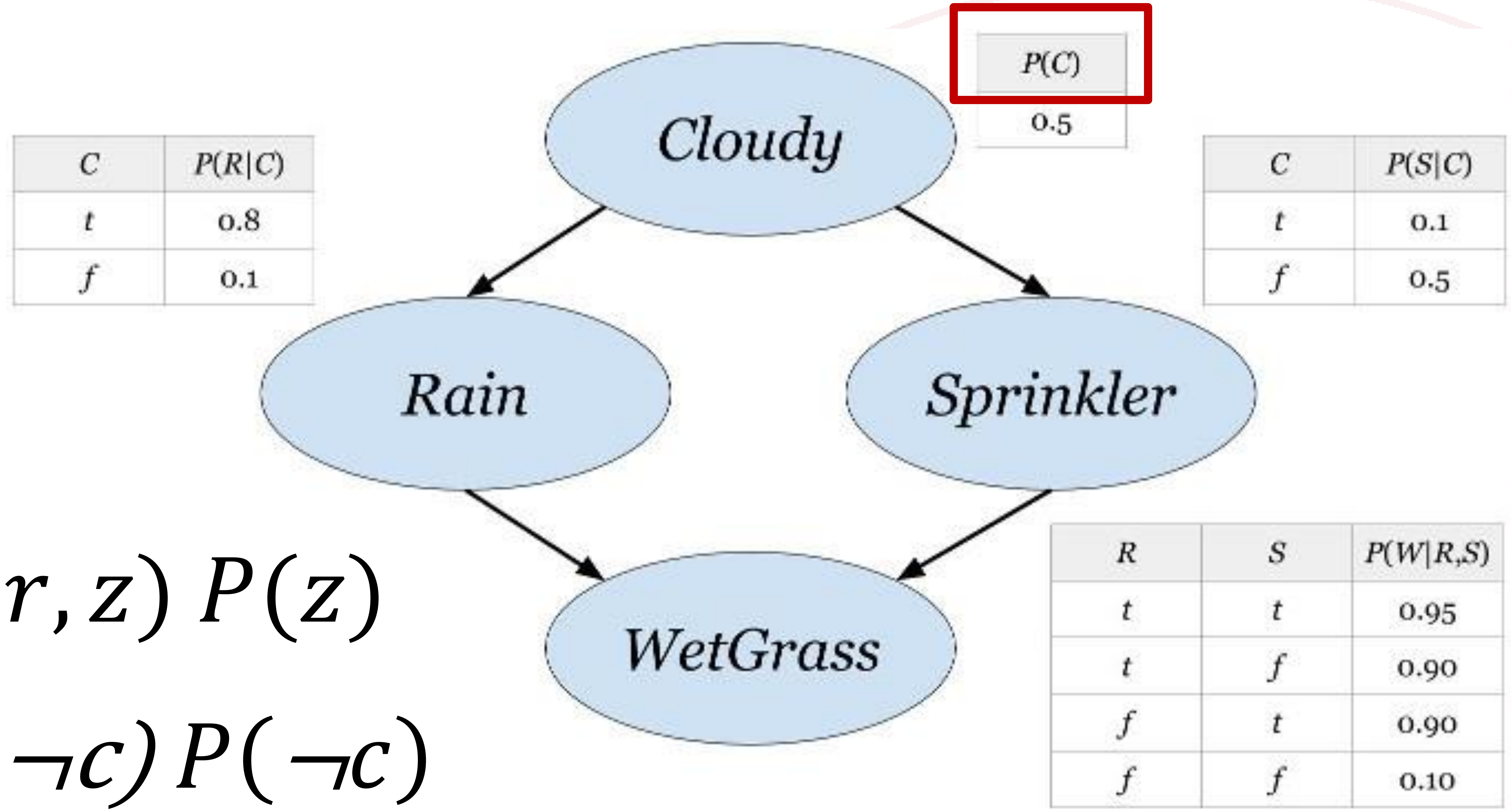

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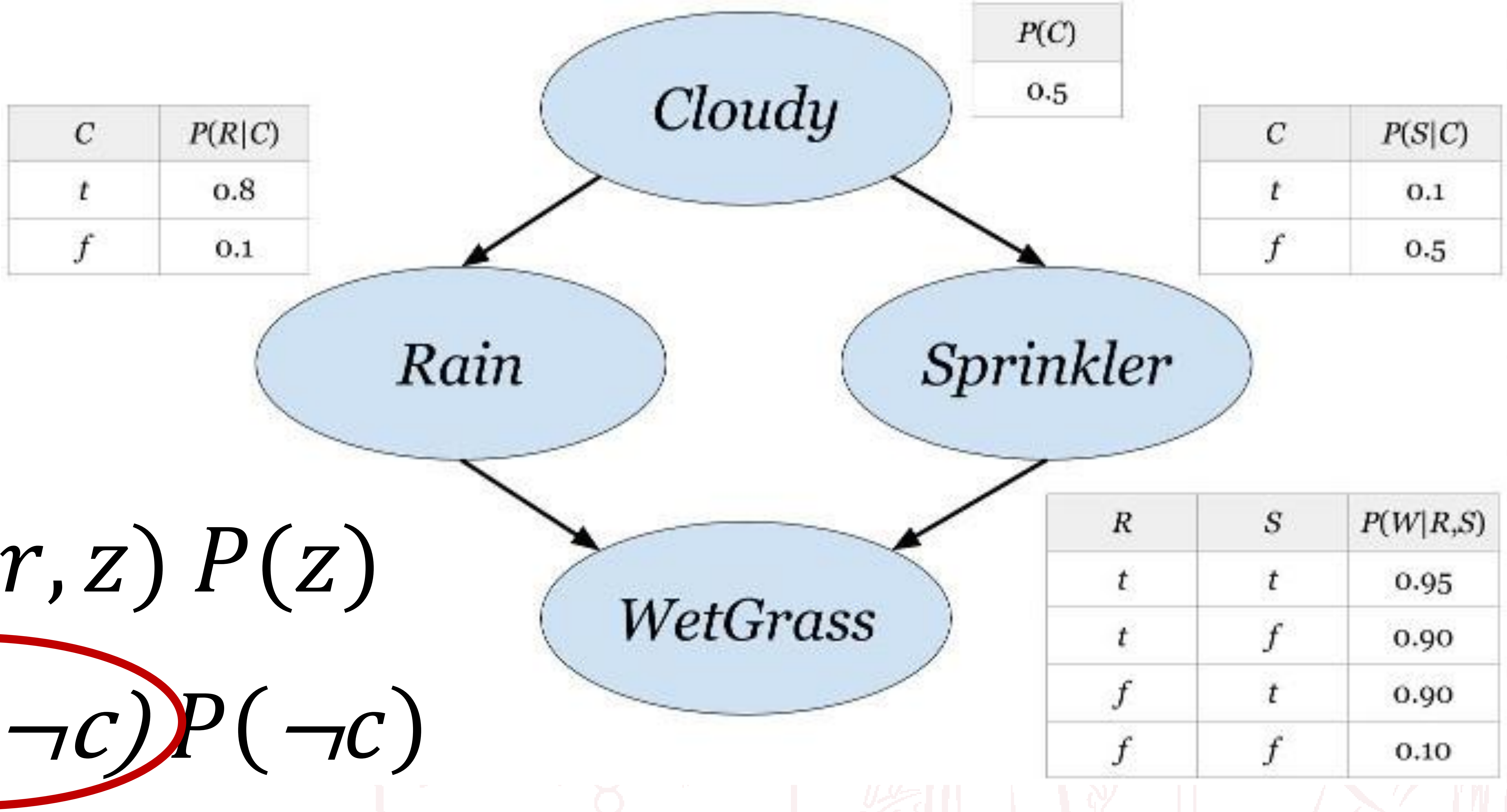

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$$= \boxed{P(g/r,c)}\,P(c) + \boxed{P(g/r,\neg c)}\,P(\neg c)$$

We need to compute $\boldsymbol{P}(G|r,c) = \langle P(g|r,c), P(\neg g|r,c)\rangle$

Causal effect of rain on wet grass: Sprinkler example

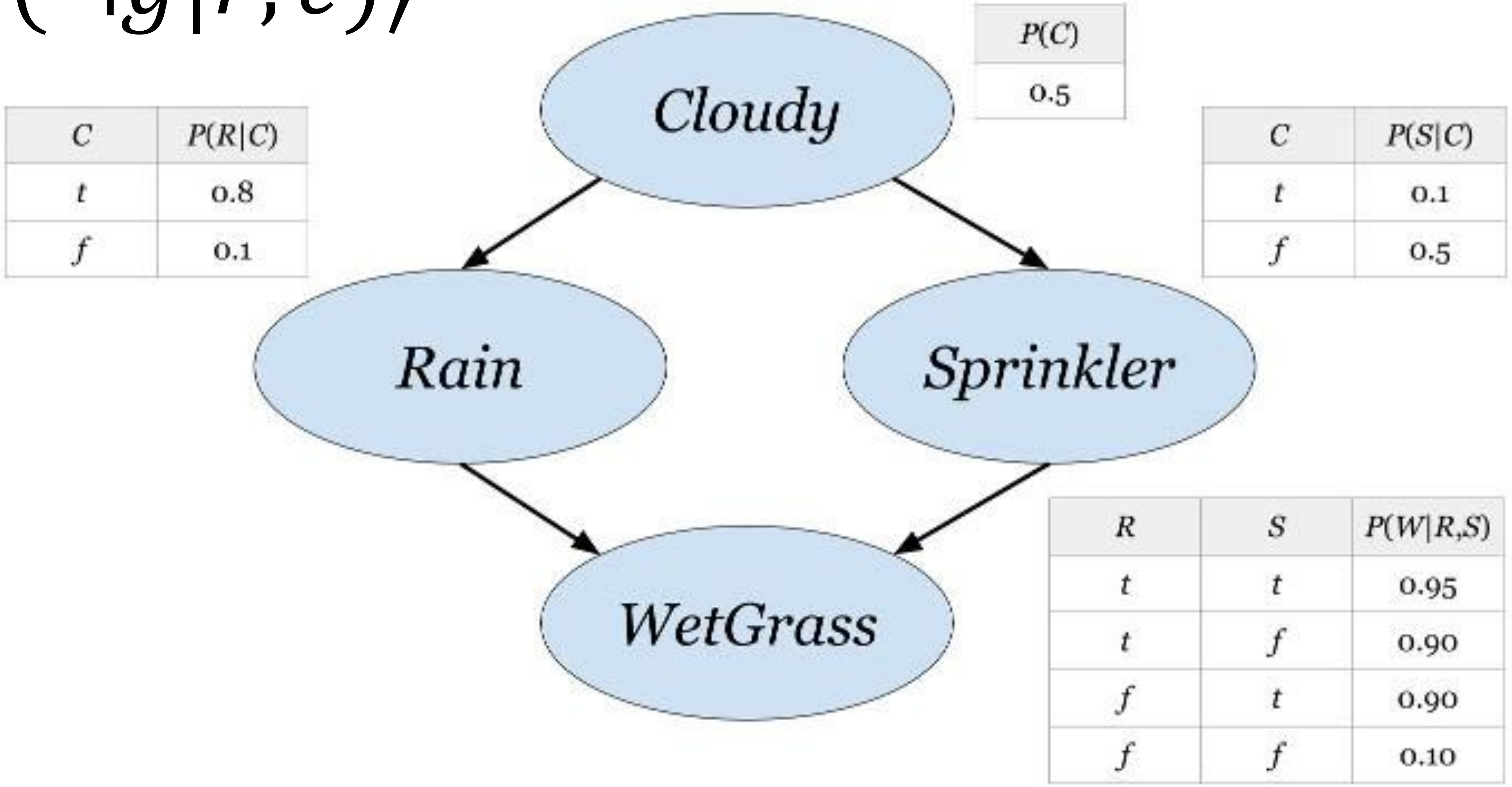
The conditional distribution

$$\boldsymbol{P}(G|r,c) = \langle P(g|r,c), P(\neg g|r,c)\rangle$$

can be computed as follows:

$$\boldsymbol{P}(G|r,c) = \frac{\boldsymbol{P}(G,r,c)}{P(r,c)}$$

$$= \alpha\, \boldsymbol{P}(G,r,c)$$



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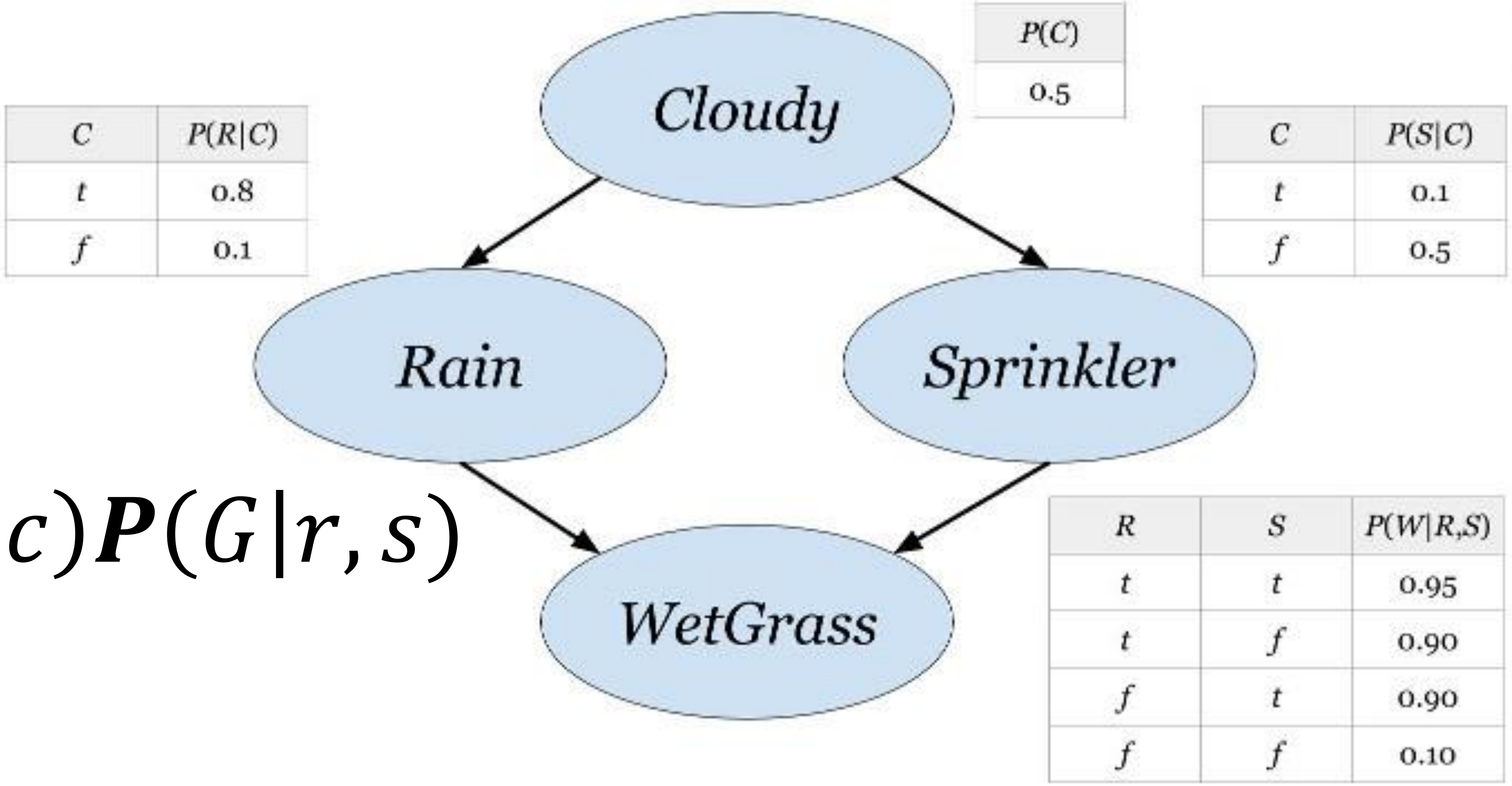
Causal effect of rain on wet grass: Sprinkler example

$$= \alpha\, \boldsymbol{P}(G, r, c)$$

$$= \alpha \sum_{s} \boldsymbol{P}(G, r, c, s)$$

$$= \alpha \sum_{s} P(c)P(r|c)P(s|c)\boldsymbol{P}(G|r, s)$$



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As we did in Lab 5

Causal effect of rain on wet grass: Sprinkler example

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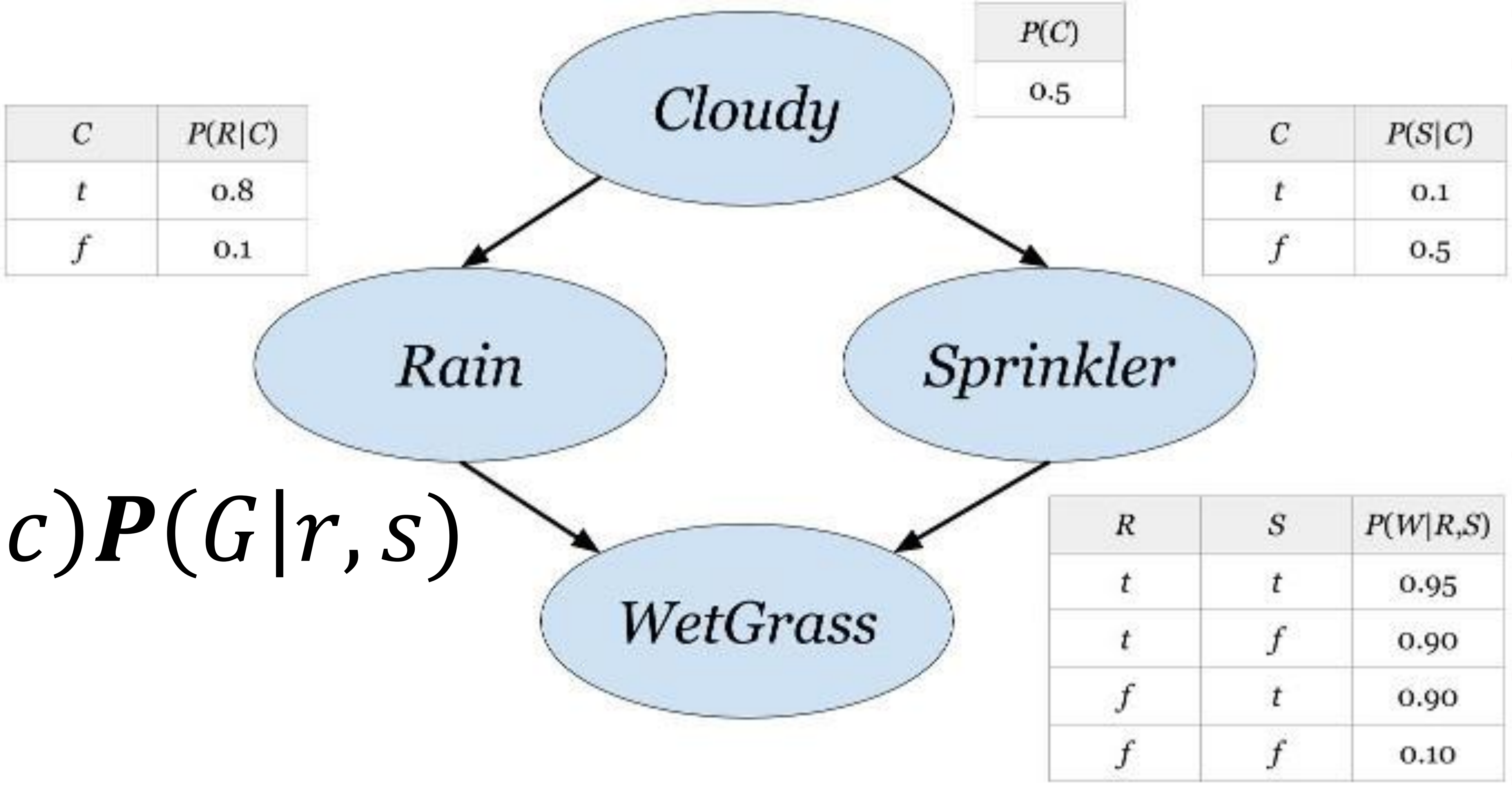
$$= \alpha \sum_{s} \boldsymbol{P}(G, r, c, s)$$

$$= \alpha \sum_{s} P(c)P(r|c)P(s|c)\boldsymbol{P}(G|r, s)$$

$$= \alpha P(c)\boxed{P(r|c)}\sum_{s} P(s|c)\boldsymbol{P}(G|r, s)$$



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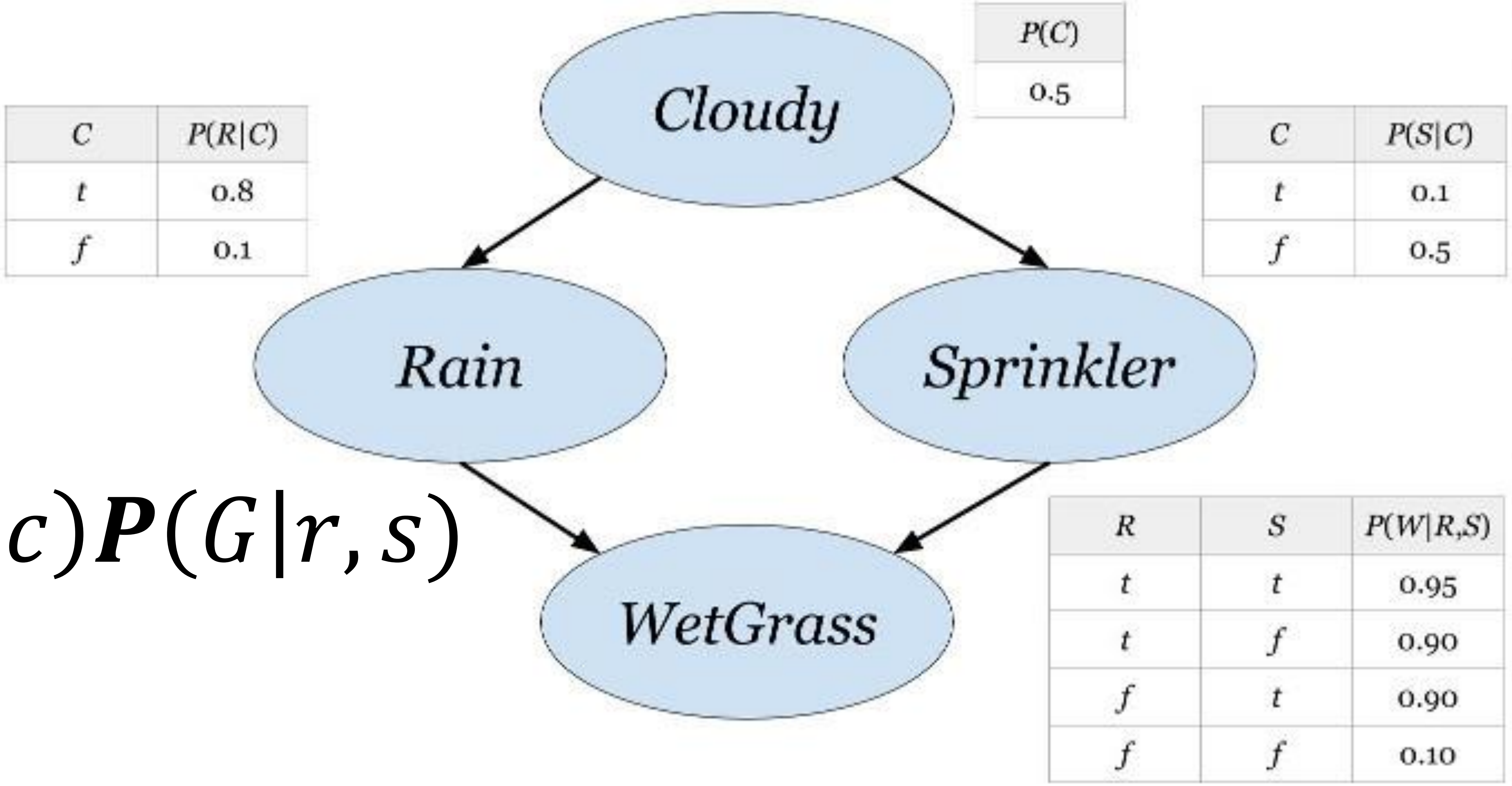
$$= \alpha \sum_{s} P(c)P(r|c)P(s|c)\mathbf{P}(G|r, s)$$

$$= \underbrace{\alpha P(c)P(r|c)}_{\alpha'} \sum_{s} P(s|c)\mathbf{P}(G|r, s)$$



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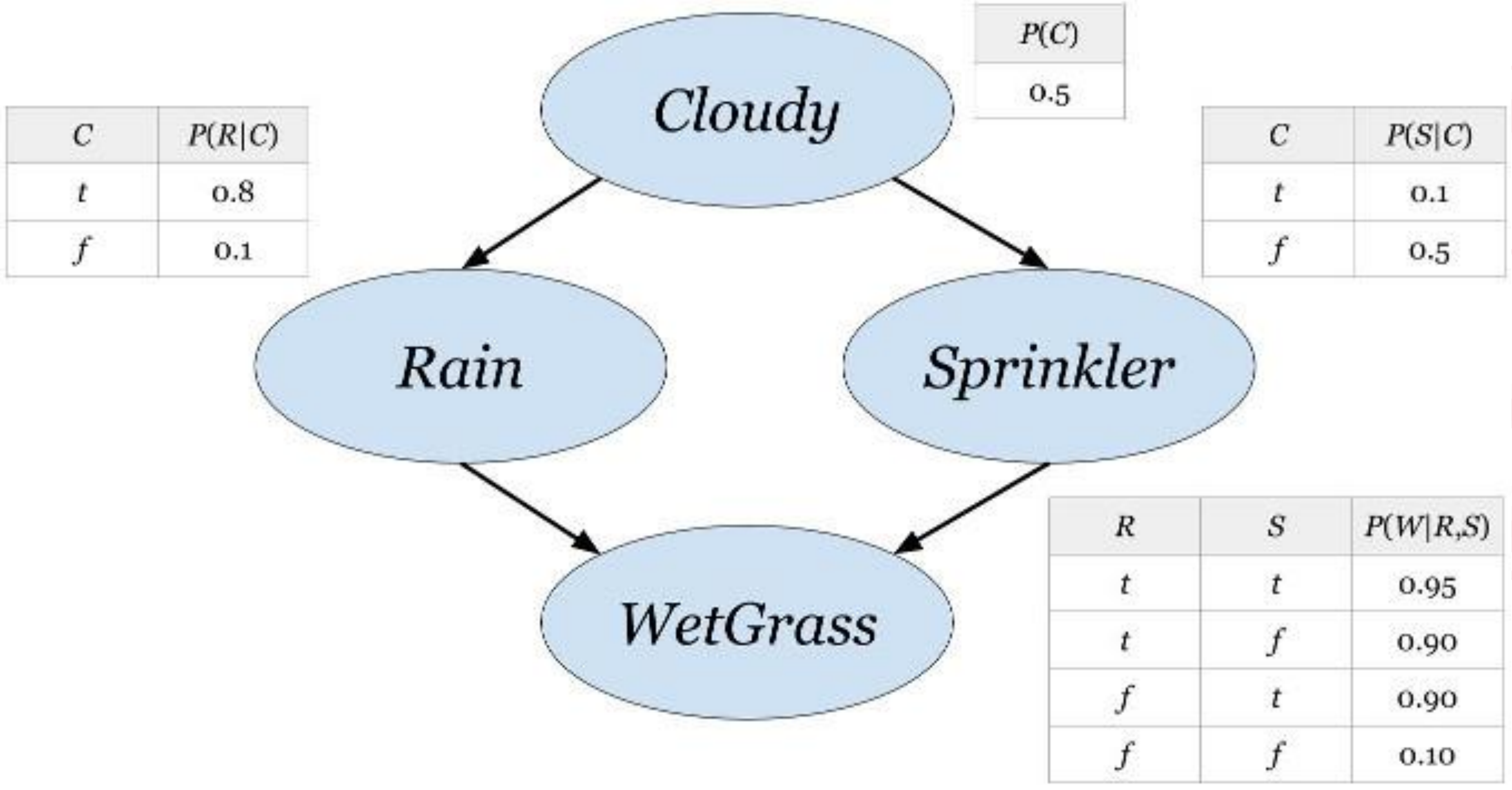
α’ is a new normalization factor

Causal effect of rain on wet grass: Sprinkler example

$$= \alpha' \sum_{s} P(s|c)\mathbf{P}(G|r,s)$$

$$= \alpha' [P(s/c)\mathbf{P}(G/r,s)+P(\neg s/c)\mathbf{P}(G/r,\neg s)]$$

Substituting the values from the network's CPTs, we get the following:



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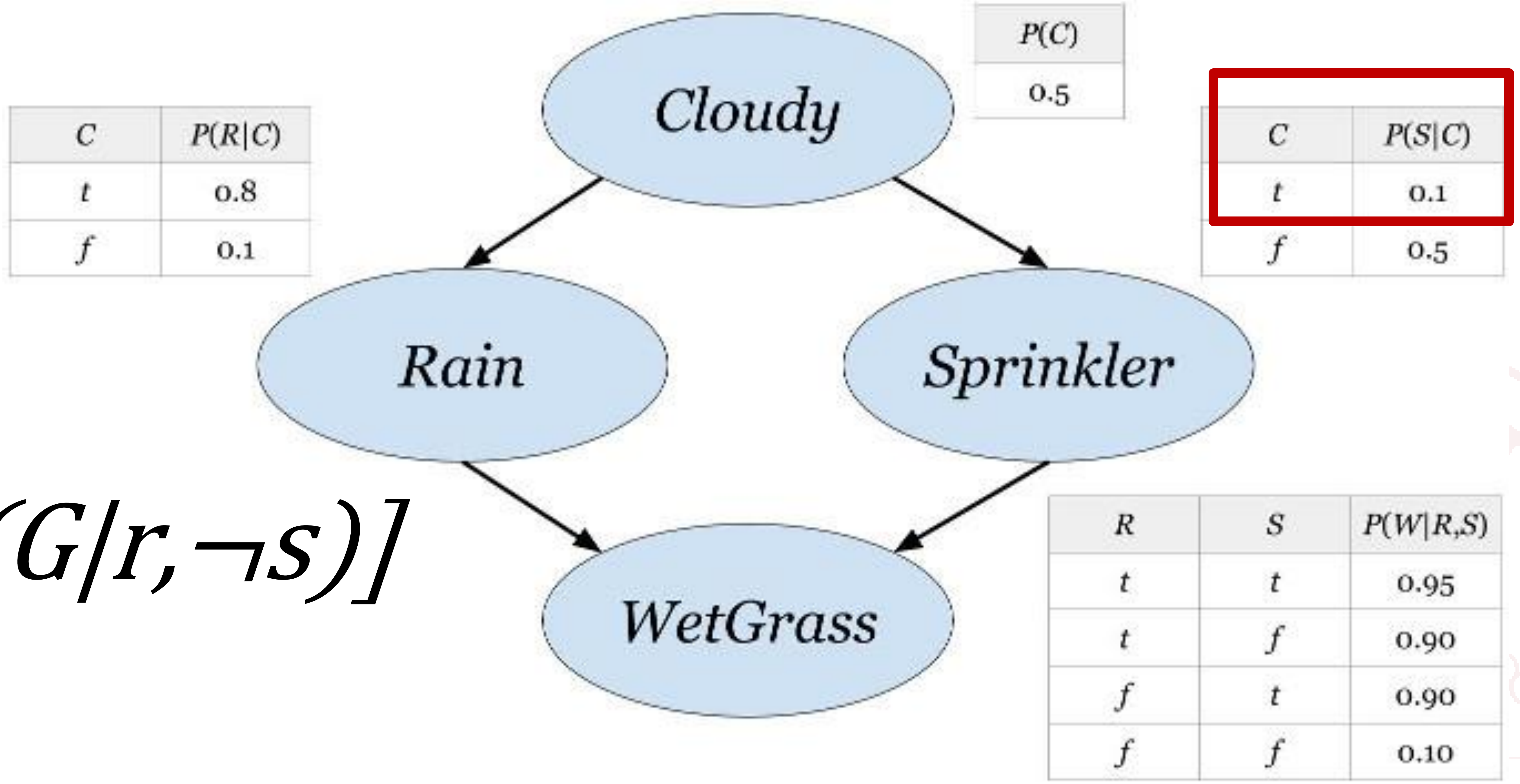

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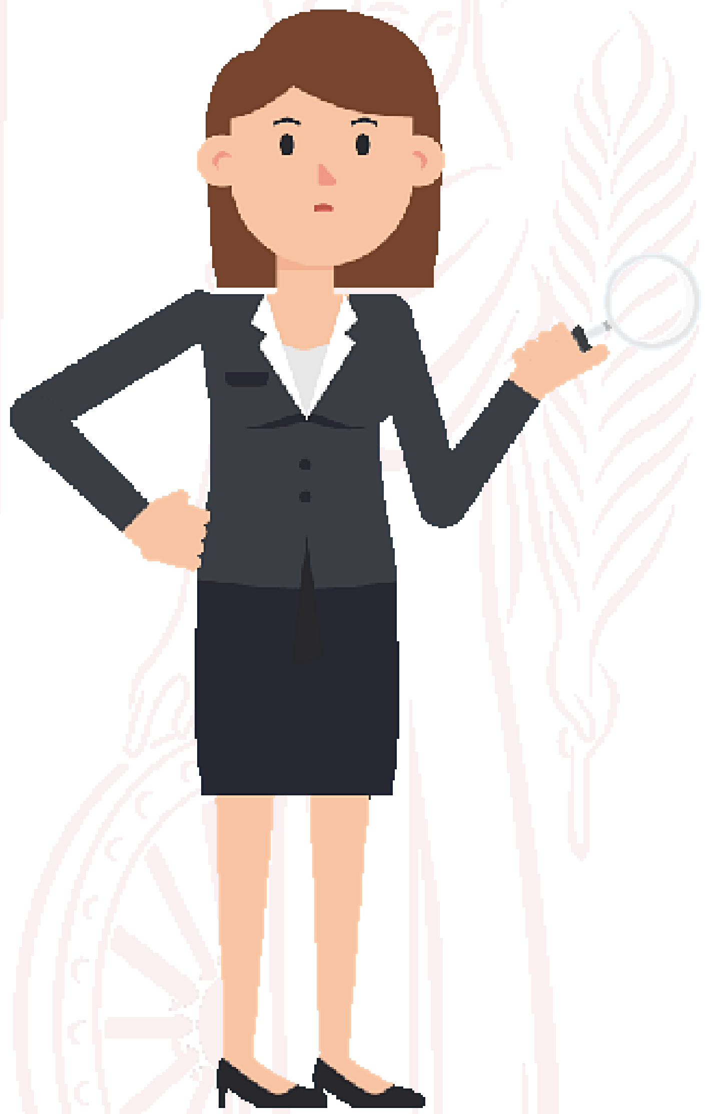
$$= \alpha' [\boxed{P(s/c)}\mathbf{P}(G/r,s)+P(\neg s/c)\mathbf{P}(G/r,\neg s)]$$

$$P(G|r,c) =$$
$$\alpha'[0.1 \times \langle 0.95,0.05\rangle + 0.9 \times \langle 0.90,0.10\rangle]$$
$$= \langle 0.905,0.095\rangle$$



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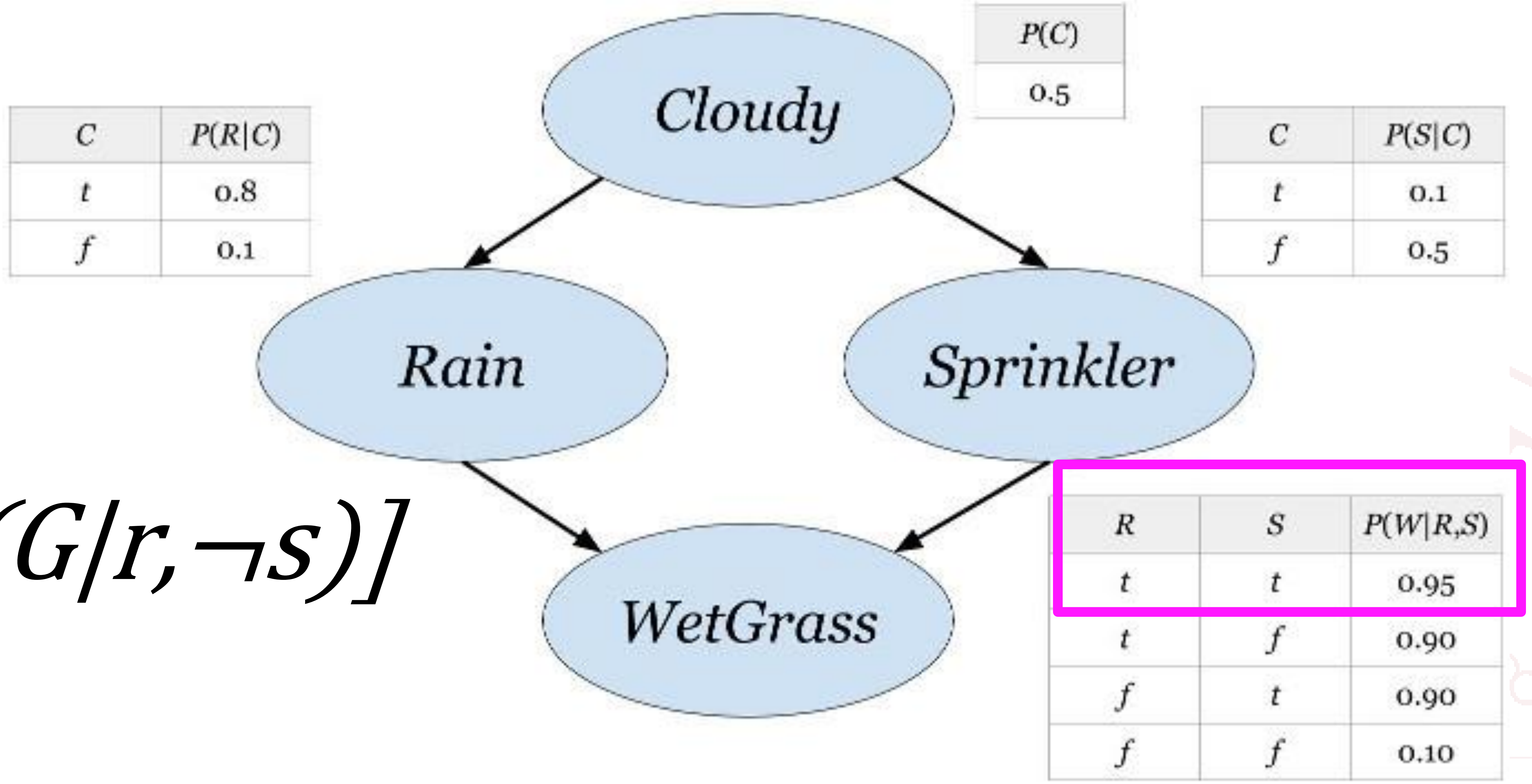

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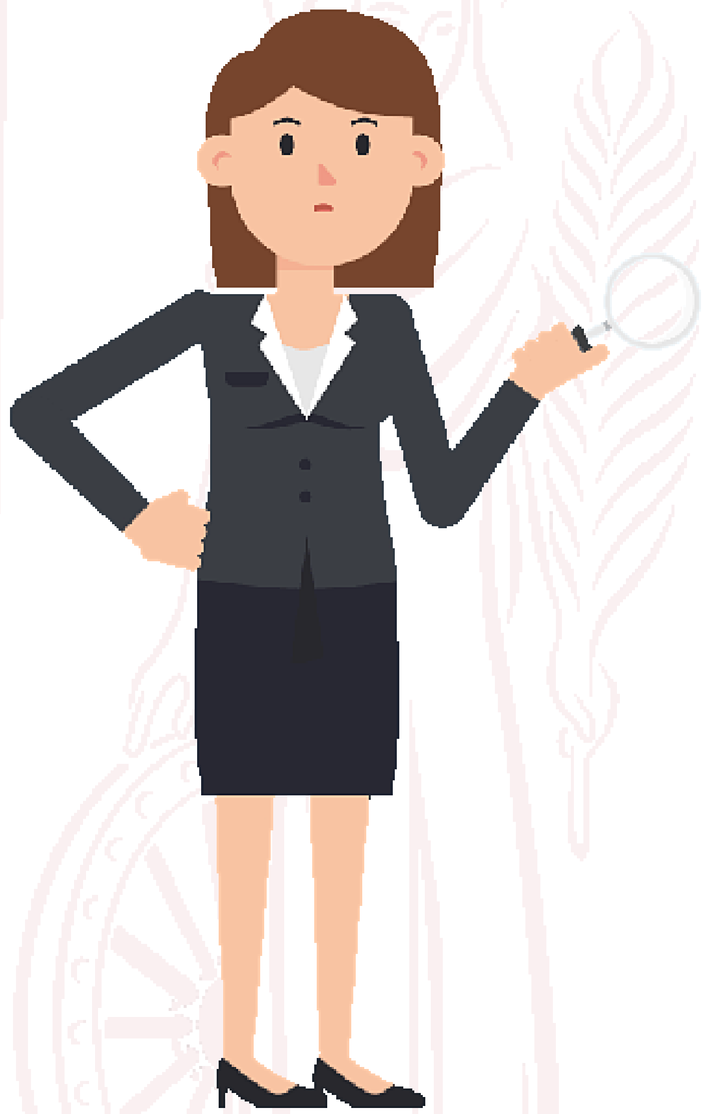

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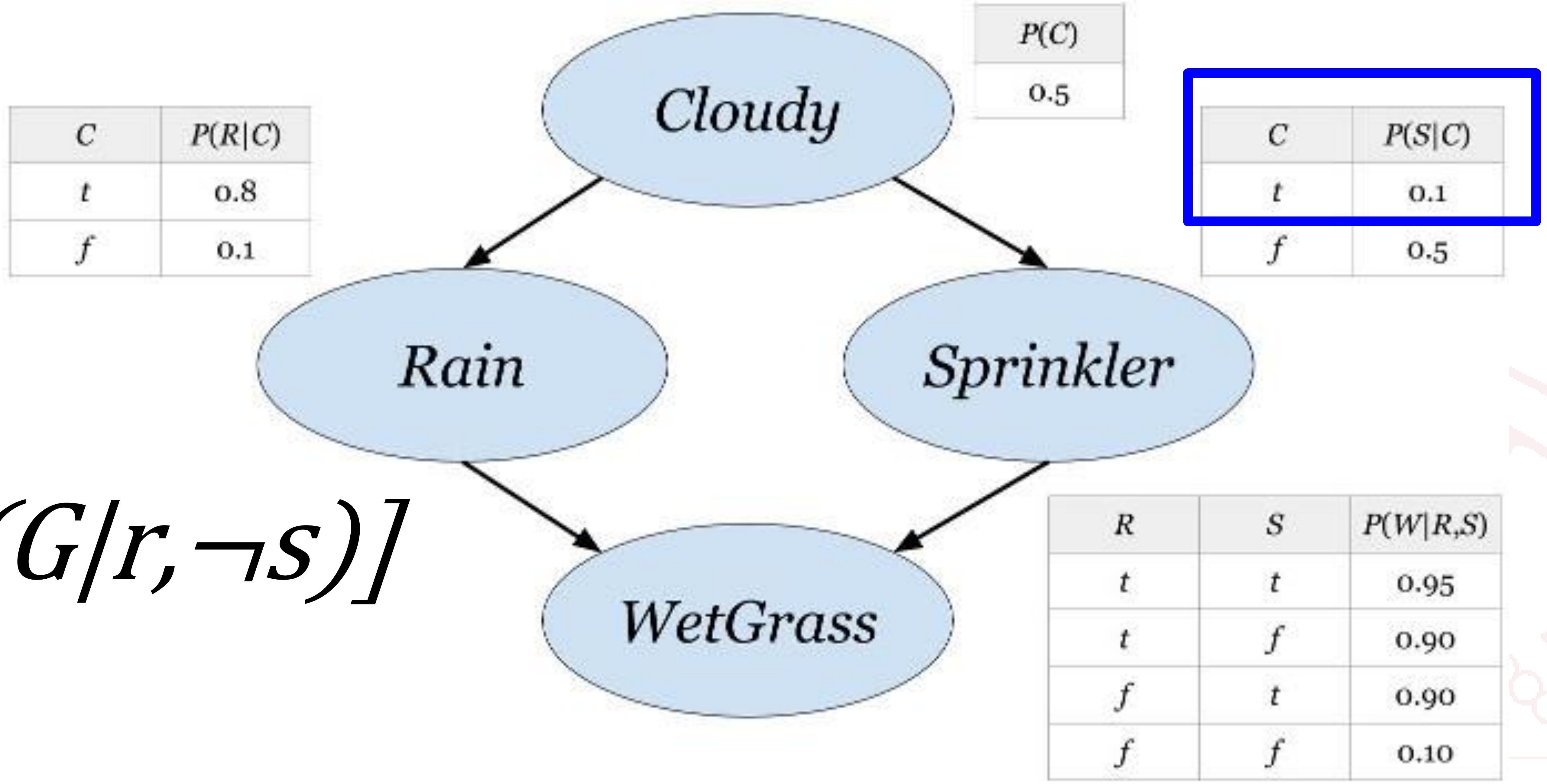

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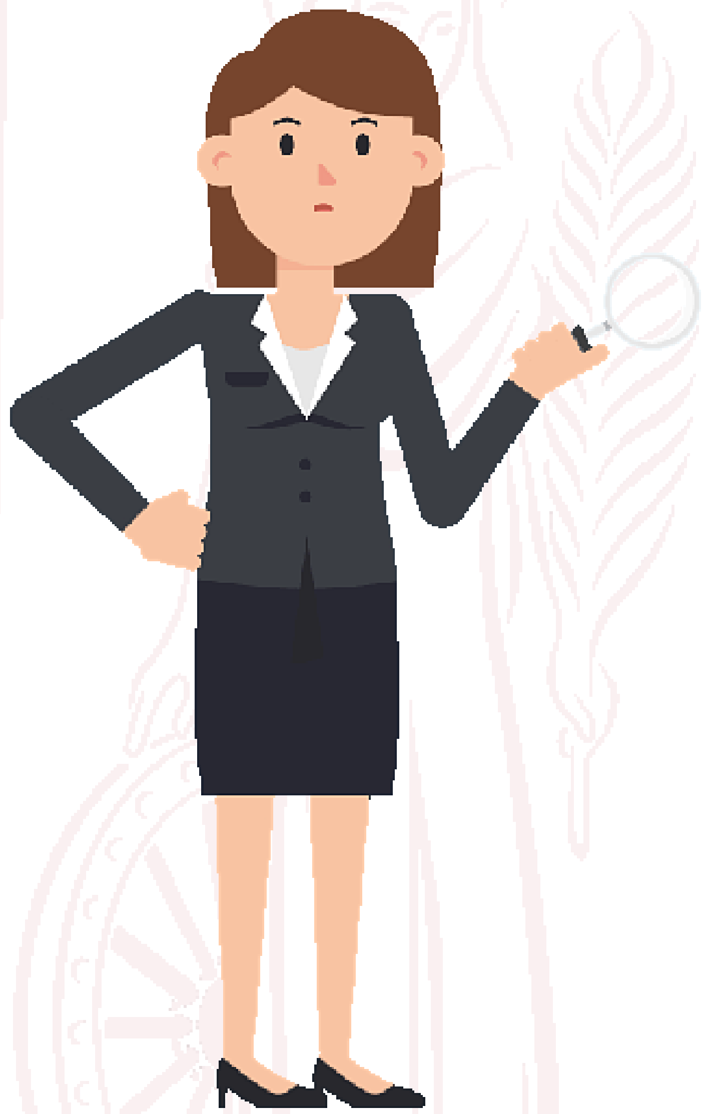

Causal effect of rain on wet grass: Sprinkler example

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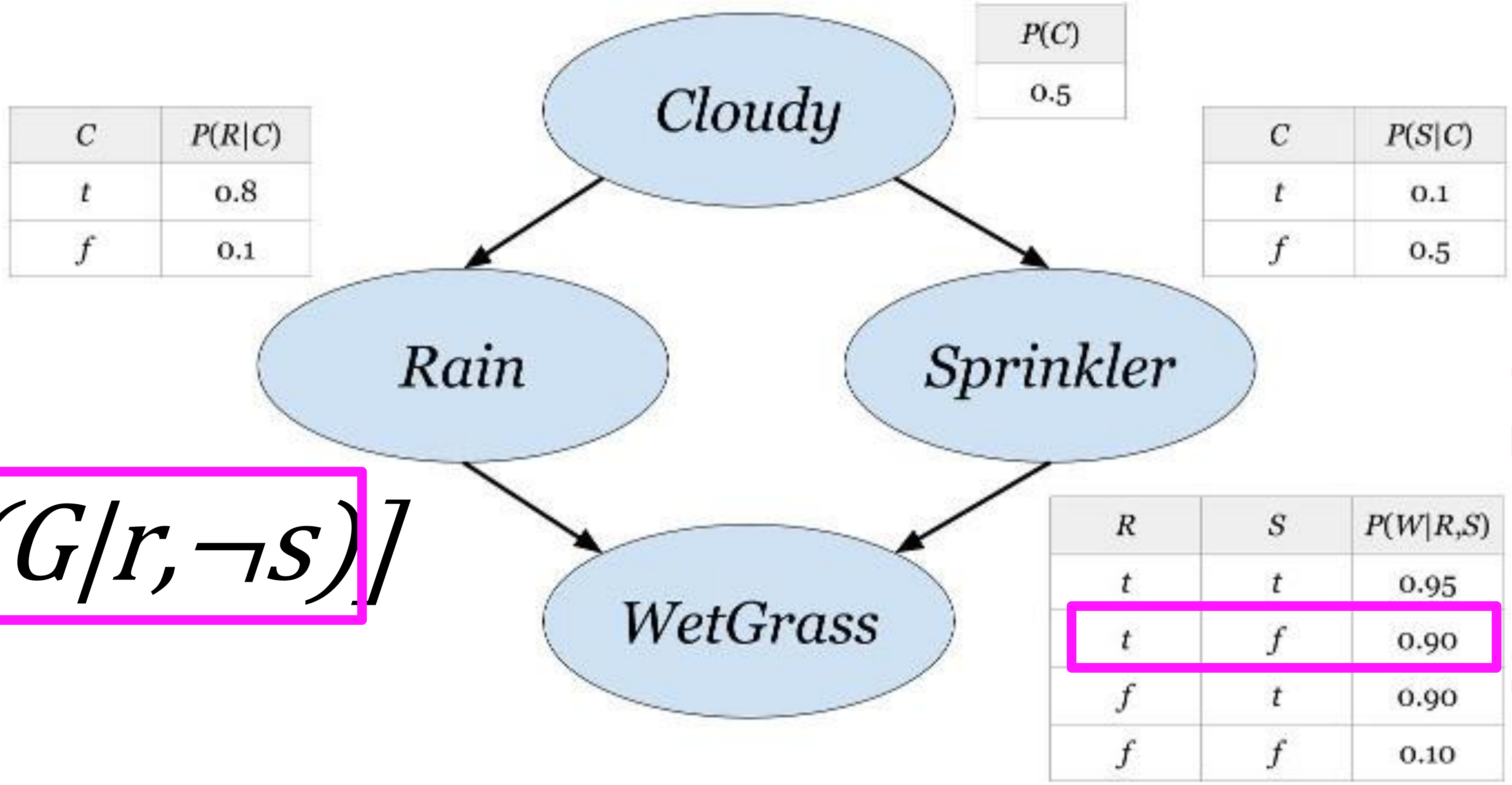

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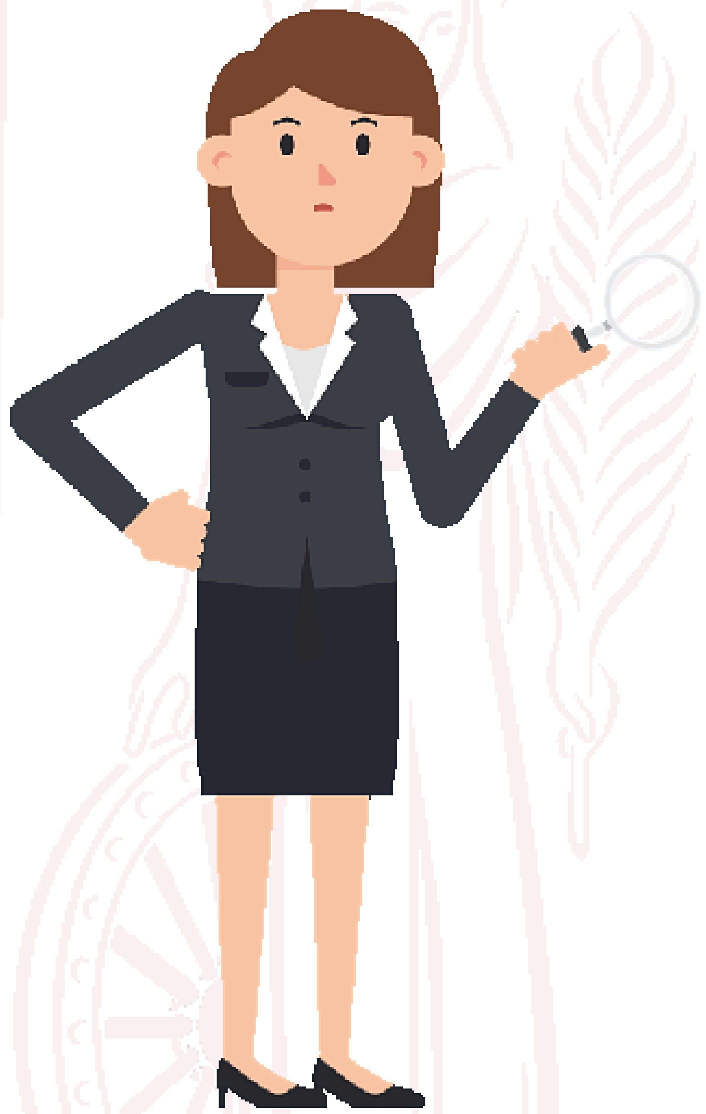

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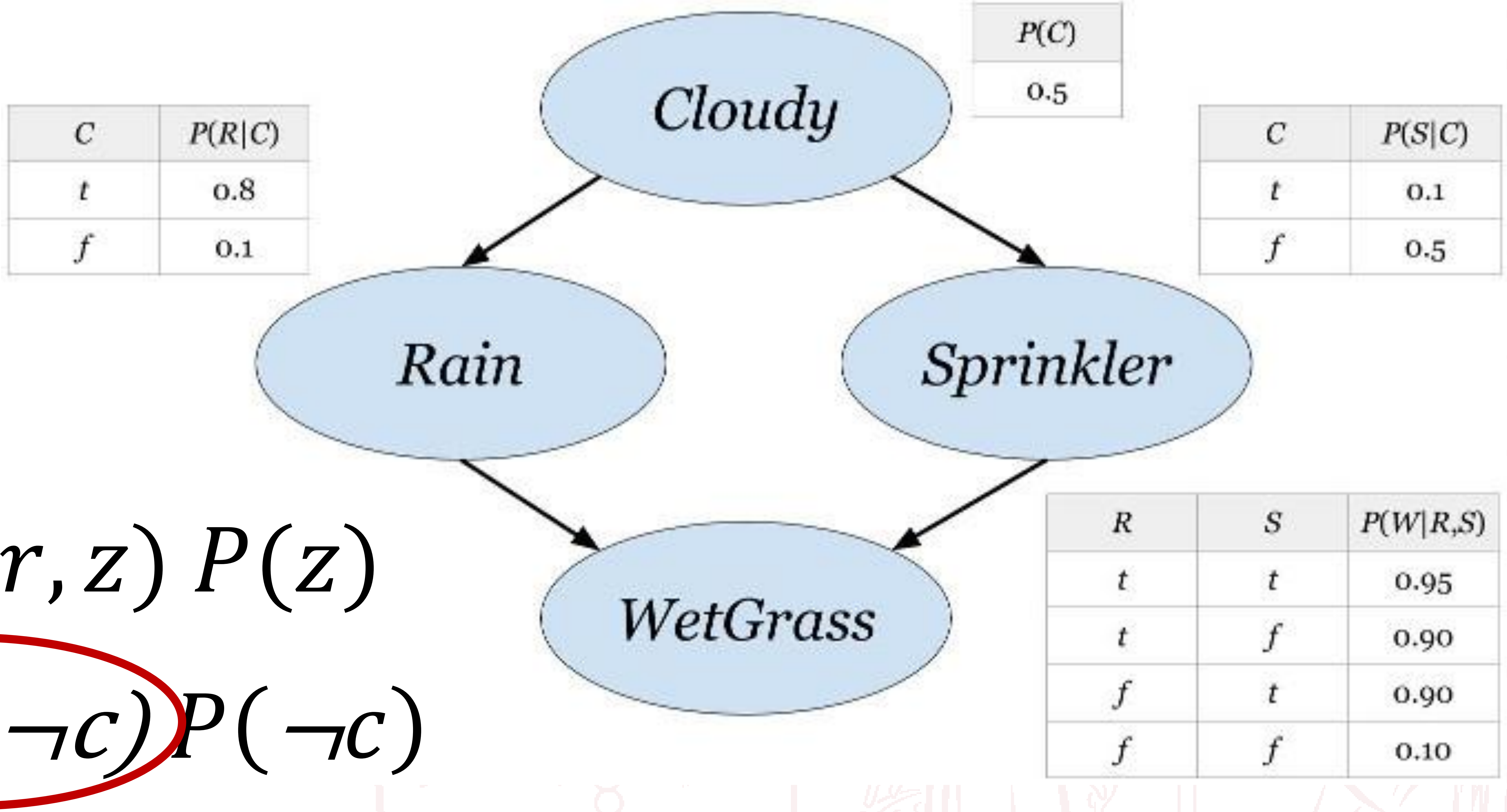

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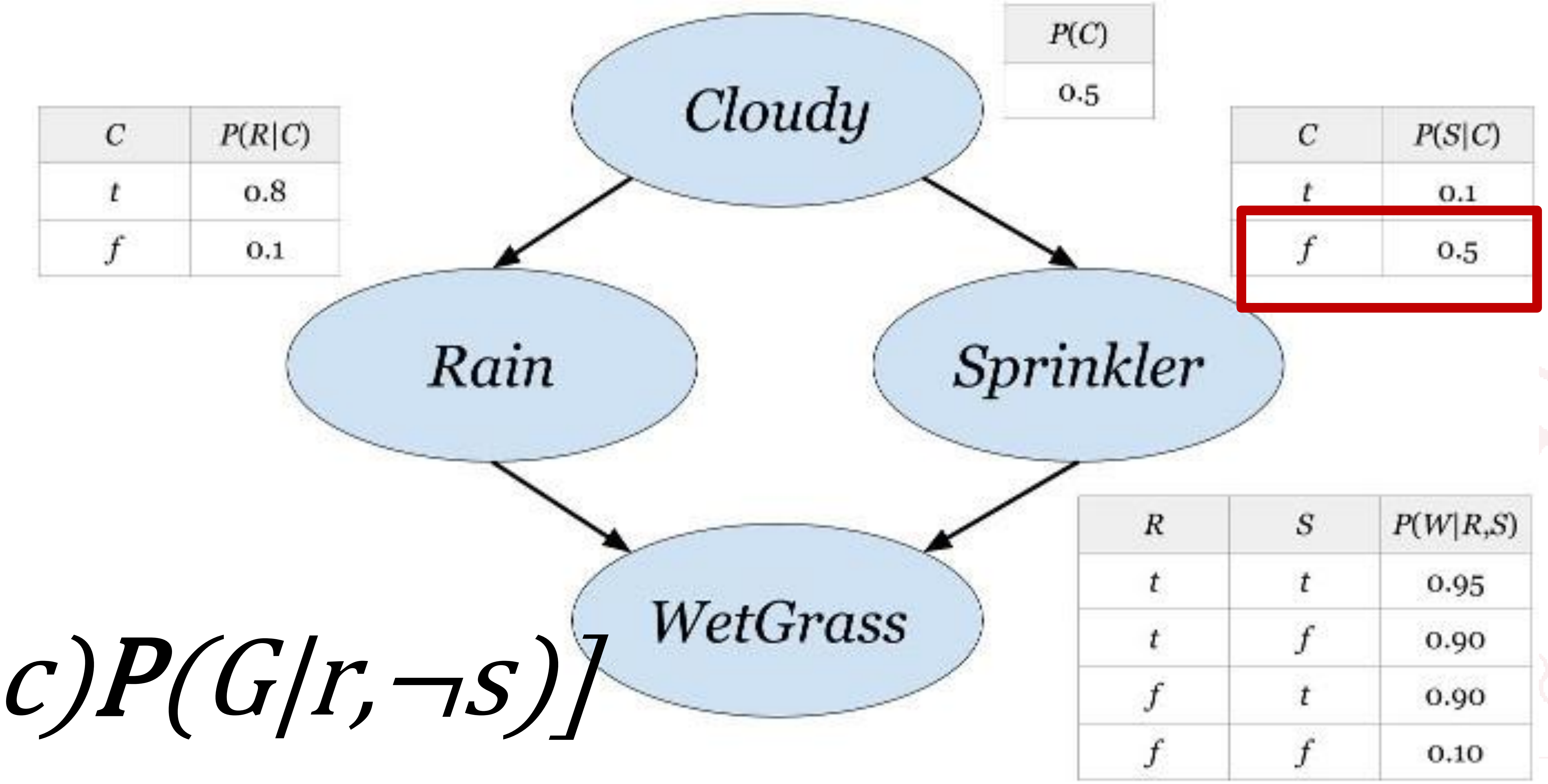
✓

We need to compute $\boldsymbol{P}(G|r,\neg c) = \langle P(g|r,\neg c), P(\neg g|r,\neg c)\rangle$

Causal effect of rain on wet grass: Sprinkler example

If we do the same for the condition ¬c,
we obtain the following distribution



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$$P(G|r,\neg c) =$$

$$= \alpha' [\boxed{P(s/\neg} c)P(G/r,s)+P(\neg s/\neg c)P(G/r,\neg s)]$$

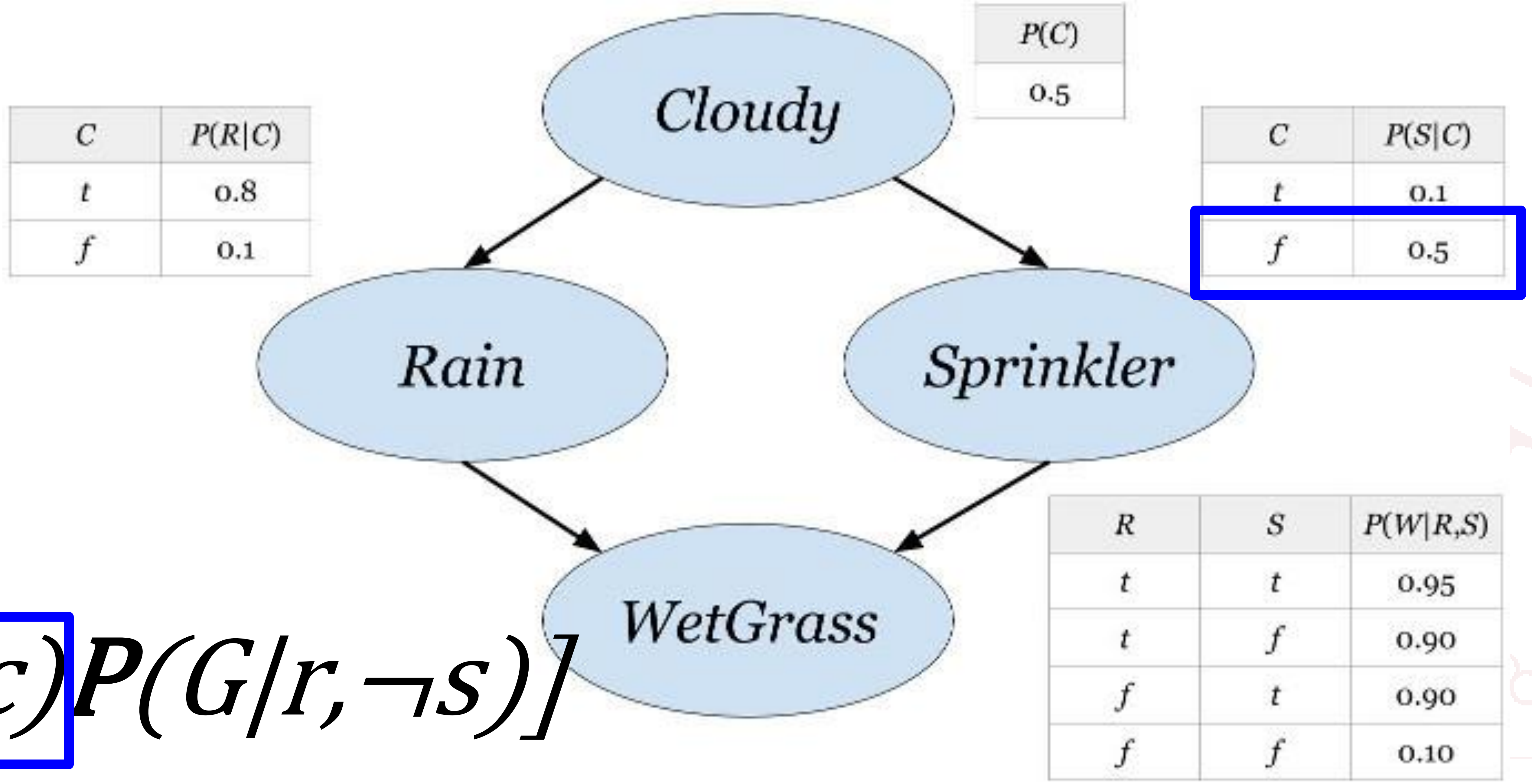
$$P(G|r,\neg c) =$$

$$\alpha' [0.5 \times \langle 0.95, 0.05 \rangle + 0.5 \times \langle 0.90, 0.10 \rangle]$$

$$= \langle 0.925, 0.075 \rangle$$

Causal effect of rain on wet grass: Sprinkler example

If we do the same for the condition ¬c, we obtain the following distribution



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$$P(G|r,\neg c) =$$

$$= \alpha' [P(s/\neg c)\mathbf{P}(G/r,s) + \boxed{P(\neg s/\neg c)}\mathbf{P}(G/r,\neg s)]$$

$$P(G|r,\neg c) =$$

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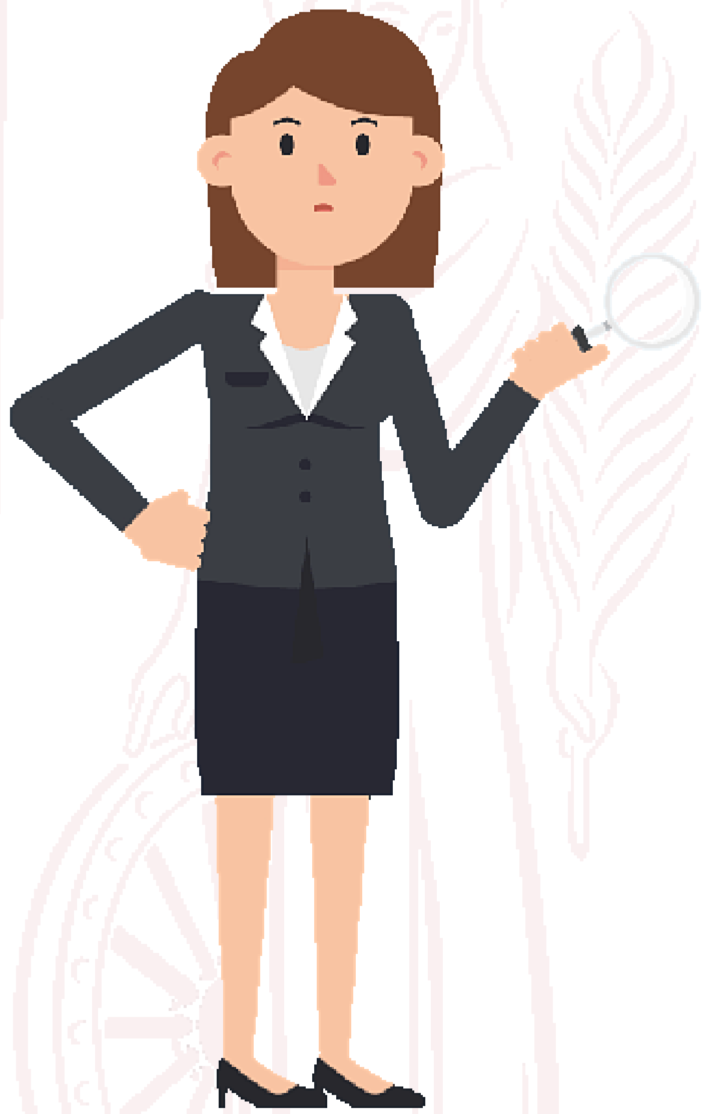
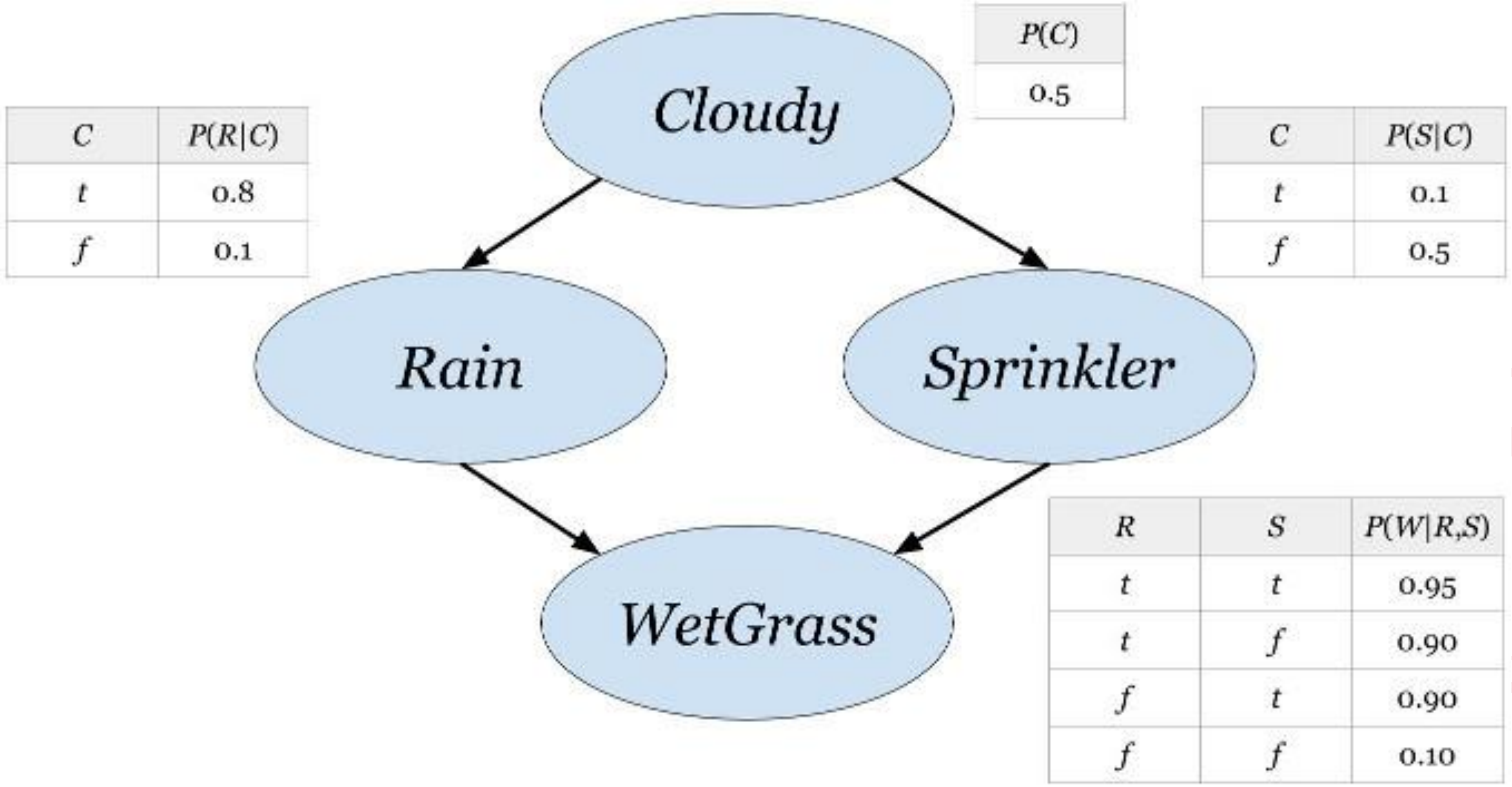


Causal effect of rain on wet grass: Sprinkler example

Finally, we use the calculated values,

$$P(g|r,c) = 0.905$$

$$P(g|r,\neg c) = 0.925$$



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in the previous adjustment formula
 and obtain the following:

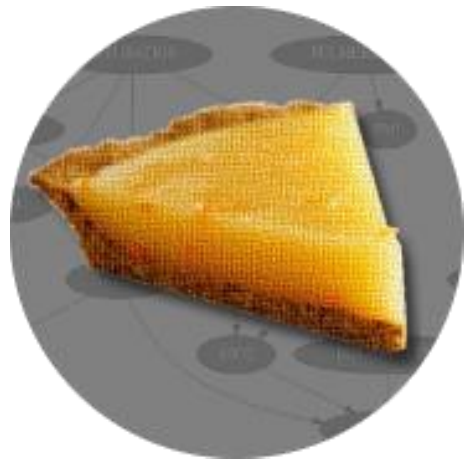
$$P(g/do(r)) \;= \textstyle\sum_{z\in C} P(g|r,z)\,P(z)$$

$$= P(g/r,\,c)\,P(c) + P(g/r,\neg c)\,P(\neg c)$$

$$=0.905 \times 0.5 + 0.925 \times 0.5 = 0.915$$

which is our causal effect of the intervention R=true on the wetness G=true.

pyAgrum



pyAgrum is a scientific C++ and Python library dedicated to Bayesian networks (BN) and other Probabilistic Graphical Models.

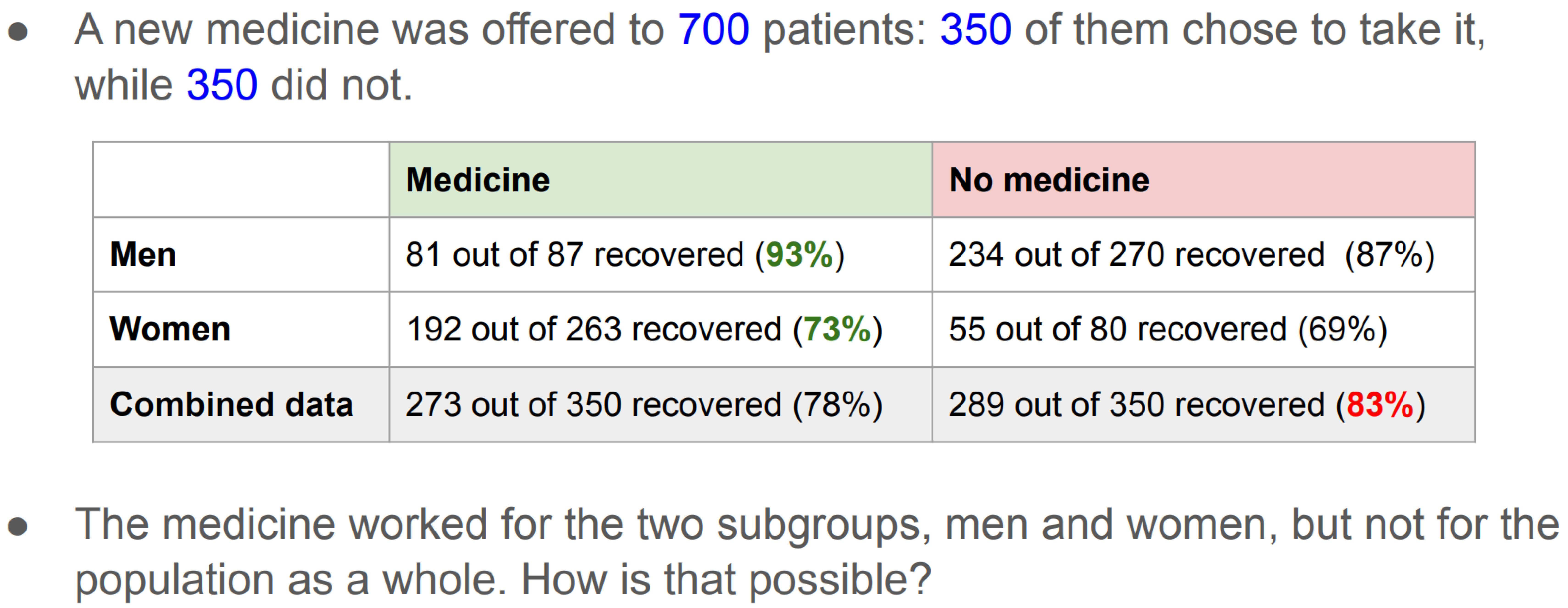
Based on the C++ aGrUM library, it provides a high-level interface to the C++ part of aGrUM allowing to create, manage and perform efficient computations with Bayesian networks and others probabilistic graphical models:

Markov random fields (MRF),
influence diagrams (ID) and LIMIDs,
credal networks (CN),
dynamic BN (dBN),
probabilistic relational models (PRM).

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox

Simpson's paradox is a phenomenon in probability and statistics in which a trend appears in several groups of data but disappears or reverses when the groups are combined.



- A new medicine was offered to 700 patients: 350 of them chose to take it, while 350 did not.

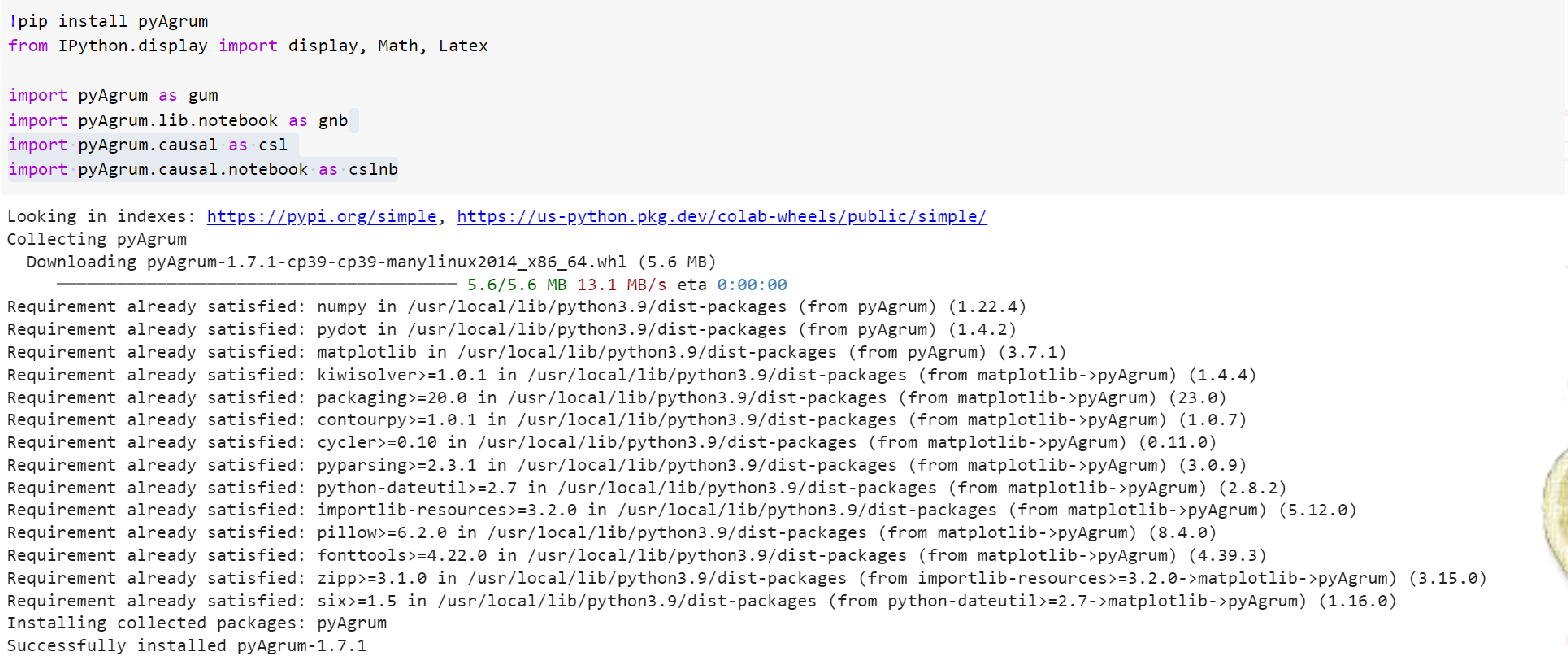
	Medicine	No medicine
Men	81 out of 87 recovered (**93%**)	234 out of 270 recovered (87%)
Women	192 out of 263 recovered (**73%**)	55 out of 80 recovered (69%)
Combined data	273 out of 350 recovered (78%)	289 out of 350 recovered (**83%**)

- The medicine worked for the two subgroups, men and women, but not for the population as a whole. How is that possible?

See slide 8 – Causality1 by prof. Bellotto

Simpson's paradox via pyAgrum

Let’s apply pyAgrum to the Simpson’s paradox with the following example



```
!pip install pyAgrum
from IPython.display import display, Math, Latex

import pyAgrum as gum
import pyAgrum.lib.notebook as gnb
import pyAgrum.causal as csl
import pyAgrum.causal.notebook as cslnb
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting pyAgrum
  Downloading pyAgrum-1.7.1-cp39-cp39-manylinux2014_x86_64.whl (5.6 MB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 5.6/5.6 MB 13.1 MB/s eta 0:00:00
Requirement already satisfied: numpy in /usr/local/lib/python3.9/dist-packages (from pyAgrum) (1.22.4)
Requirement already satisfied: pydot in /usr/local/lib/python3.9/dist-packages (from pyAgrum) (1.4.2)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.9/dist-packages (from pyAgrum) (3.7.1)
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (1.4.4)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (23.0)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (1.0.7)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (0.11.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (3.0.9)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (2.8.2)
Requirement already satisfied: importlib-resources>=3.2.0 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (5.12.0)
Requirement already satisfied: pillow>=6.2.0 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (8.4.0)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.9/dist-packages (from matplotlib->pyAgrum) (4.39.3)
Requirement already satisfied: zipp>=3.1.0 in /usr/local/lib/python3.9/dist-packages (from importlib-resources>=3.2.0->matplotlib->pyAgrum) (3.15.0)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.9/dist-packages (from python-dateutil>=2.7->matplotlib->pyAgrum) (1.16.0)
Installing collected packages: pyAgrum
Successfully installed pyAgrum-1.7.1
```

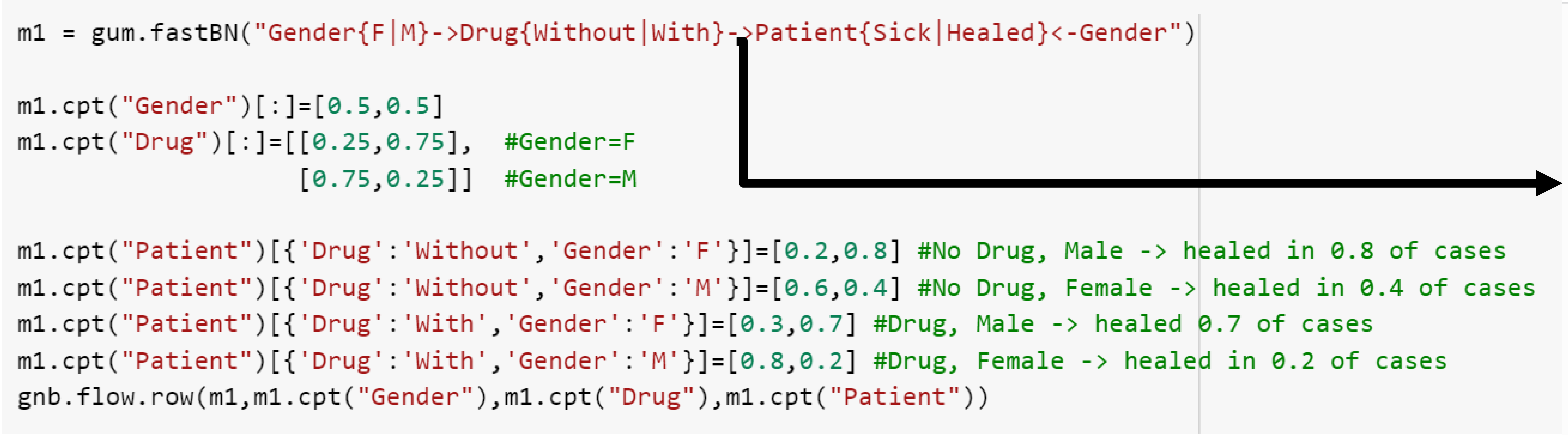
https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum

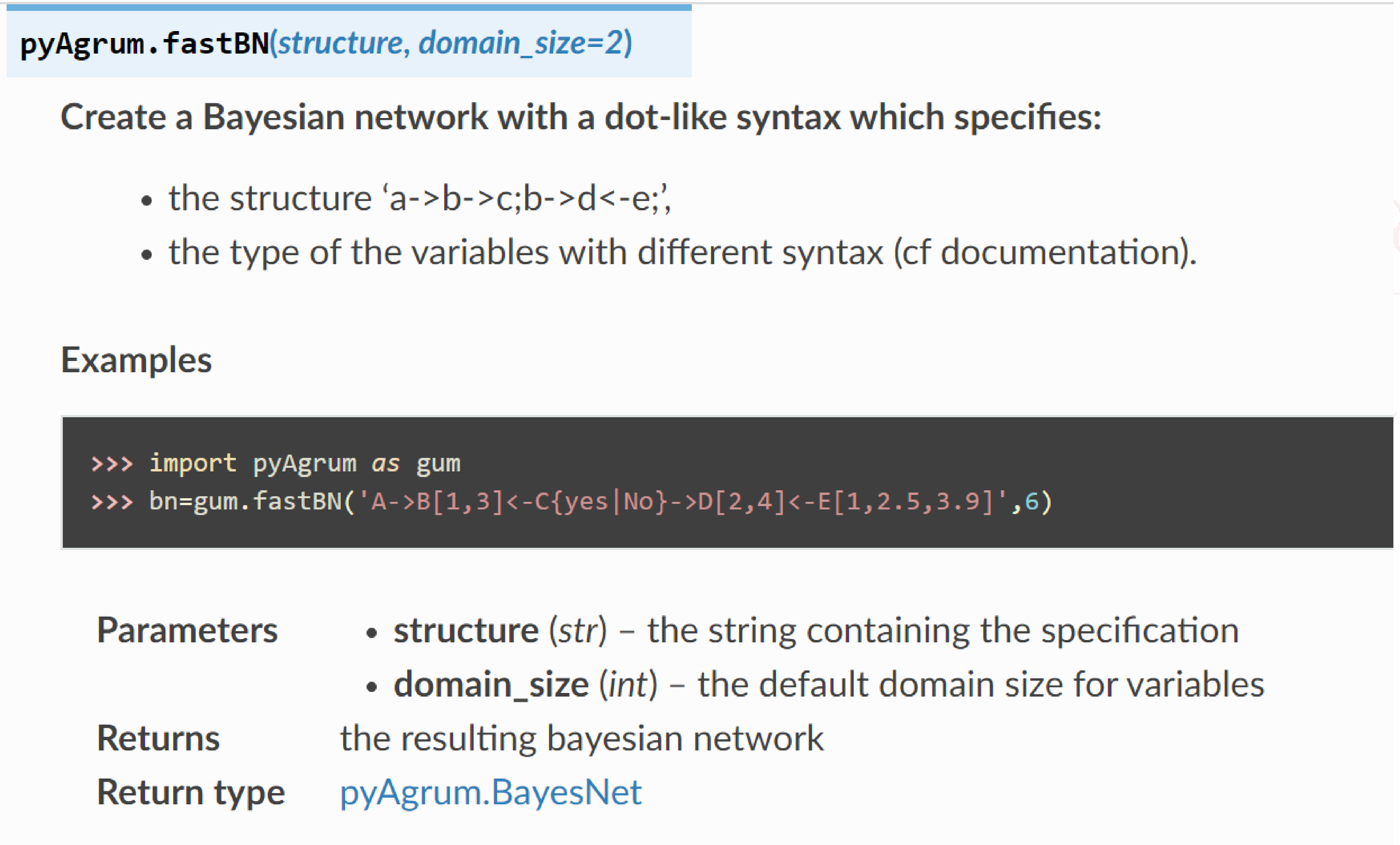
In a statistical study about a drug, we try to evaluate the latter’s efficiency among a population of men and women.

Let’s note: - Drug : drug taking - Patient : cured patient - Gender : patient’s gender

The model from the observed date is as follow :



```
m1 = gum.fastBN("Gender{F|M}->Drug{Without|With}->Patient{Sick|Healed}<-Gender")

m1.cpt("Gender")[:]=[0.5,0.5]
m1.cpt("Drug")[:]=[[0.25,0.75],  #Gender=F
                   [0.75,0.25]]  #Gender=M

m1.cpt("Patient")[{'Drug':'Without','Gender':'F'}]=[0.2,0.8] #No Drug, Male -> healed in 0.8 of cases
m1.cpt("Patient")[{'Drug':'Without','Gender':'M'}]=[0.6,0.4] #No Drug, Female -> healed in 0.4 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'F'}]=[0.3,0.7] #Drug, Male -> healed 0.7 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'M'}]=[0.8,0.2] #Drug, Female -> healed in 0.2 of cases
gnb.flow.row(m1,m1.cpt("Gender"),m1.cpt("Drug"),m1.cpt("Patient"))
```



pyAgrum.fastBN(*structure, domain_size=2*)

Create a Bayesian network with a dot-like syntax which specifies:

- the structure ‘a->b->c;b->d<-e;’,
- the type of the variables with different syntax (cf documentation).

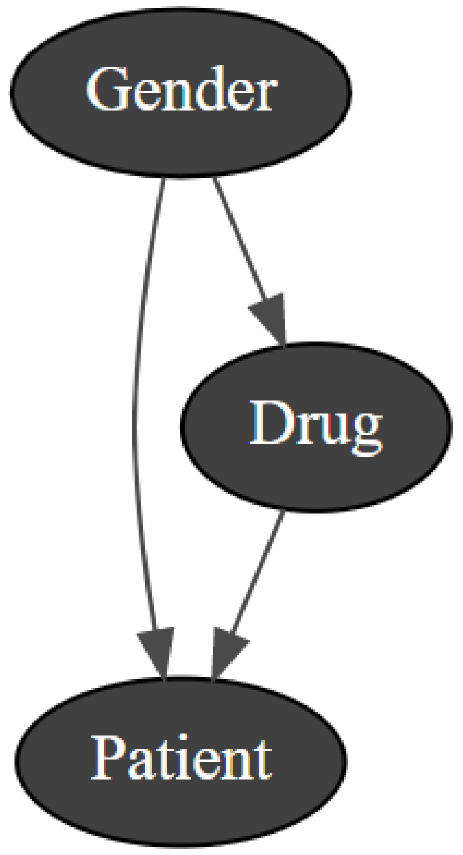
Examples

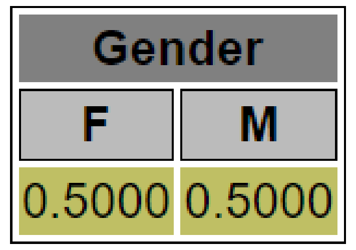
```
>>> import pyAgrum as gum
>>> bn=gum.fastBN('A->B[1,3]<-C{yes|No}->D[2,4]<-E[1,2.5,3.9]',6)
```

Parameters
- **structure** (*str*) – the string containing the specification
- **domain_size** (*int*) – the default domain size for variables

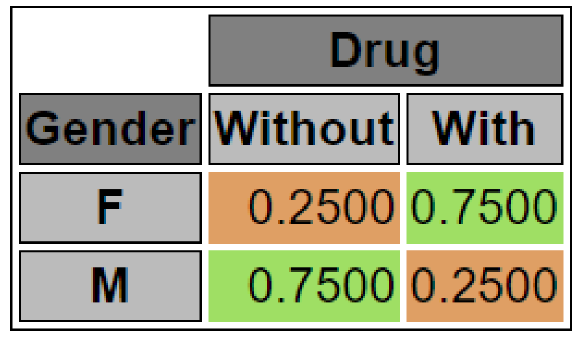
Returns the resulting bayesian network

Return type pyAgrum.BayesNet

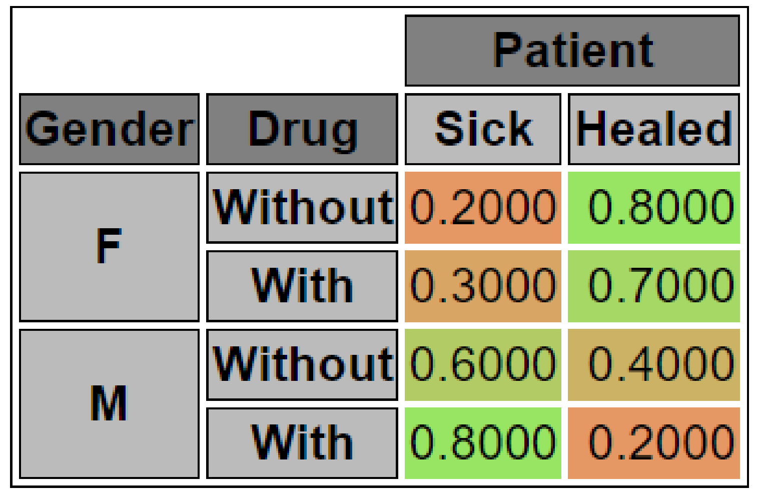




Gender	
F	M
0.5000	0.5000



	Drug	
Gender	Without	With
F	0.2500	0.7500
M	0.7500	0.2500



		Patient	
Gender	Drug	Sick	Healed
F	Without	0.2000	0.8000
	With	0.3000	0.7000
M	Without	0.6000	0.4000
	With	0.8000	0.2000

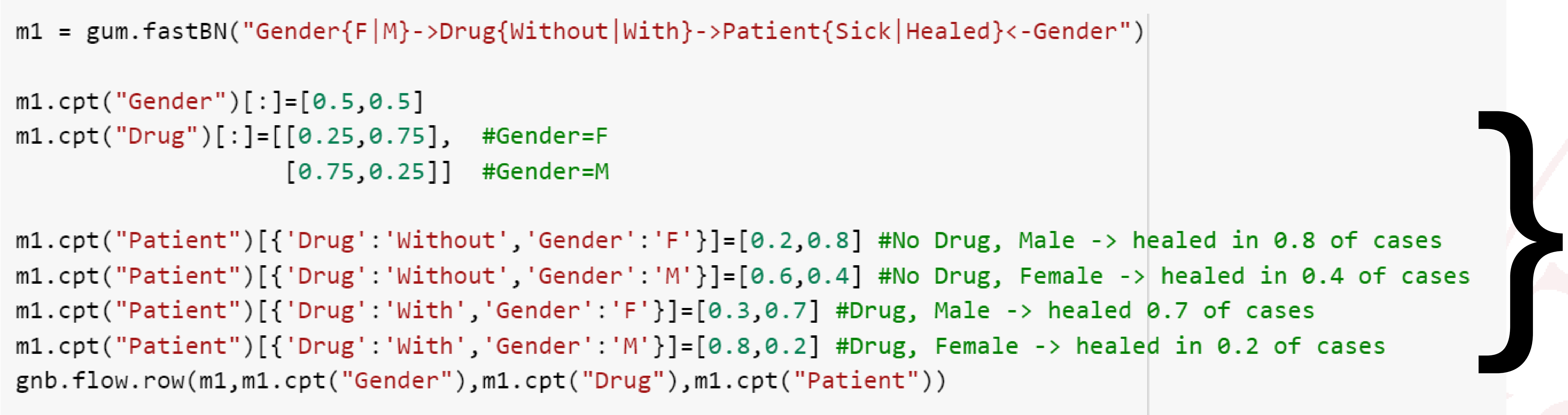
https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum

In a statistical study about a drug, we try to evaluate the latter’s efficiency among a population of men and women.

Let’s note: - Drug : drug taking - Patient : cured patient - Gender : patient’s gender

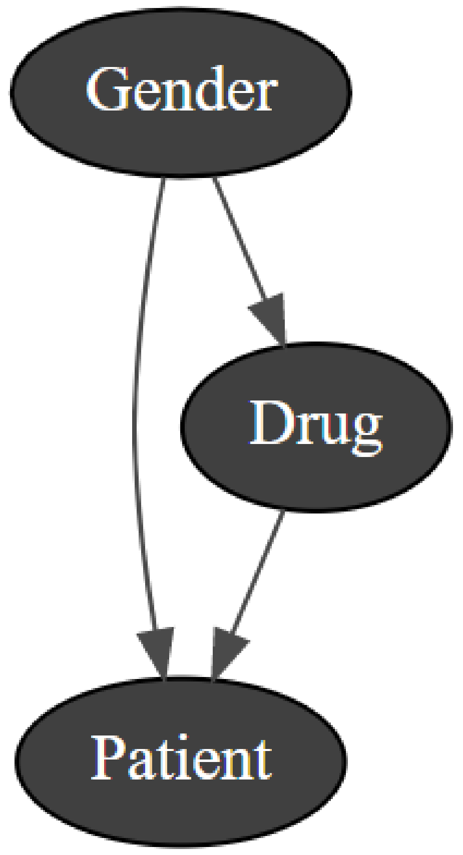
The model from the observed date is as follow :

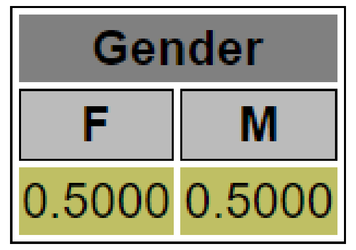


```
m1 = gum.fastBN("Gender{F|M}->Drug{Without|With}->Patient{Sick|Healed}<-Gender")

m1.cpt("Gender")[:]=[0.5,0.5]
m1.cpt("Drug")[:]=[[0.25,0.75],  #Gender=F
                   [0.75,0.25]]  #Gender=M

m1.cpt("Patient")[{'Drug':'Without','Gender':'F'}]=[0.2,0.8] #No Drug, Male -> healed in 0.8 of cases
m1.cpt("Patient")[{'Drug':'Without','Gender':'M'}]=[0.6,0.4] #No Drug, Female -> healed in 0.4 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'F'}]=[0.3,0.7] #Drug, Male -> healed 0.7 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'M'}]=[0.8,0.2] #Drug, Female -> healed in 0.2 of cases
gnb.flow.row(m1,m1.cpt("Gender"),m1.cpt("Drug"),m1.cpt("Patient"))
```

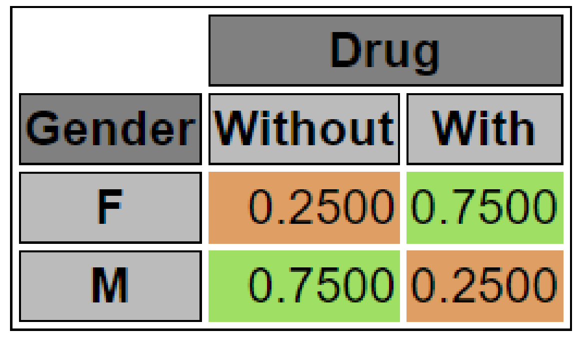
}

Prepare the Conditional Probability Table

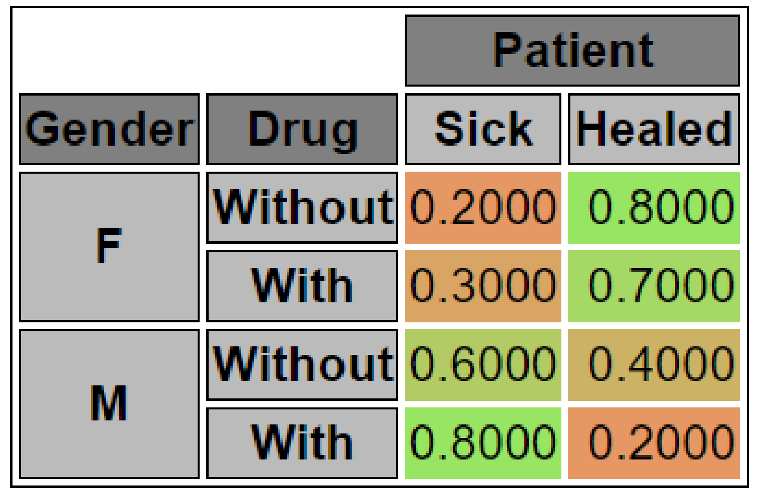




Gender	
F	M
0.5000	0.5000



	Drug	
Gender	Without	With
F	0.2500	0.7500
M	0.7500	0.2500



		Patient	
Gender	Drug	Sick	Healed
F	Without	0.2000	0.8000
	With	0.3000	0.7000
M	Without	0.6000	0.4000
	With	0.8000	0.2000

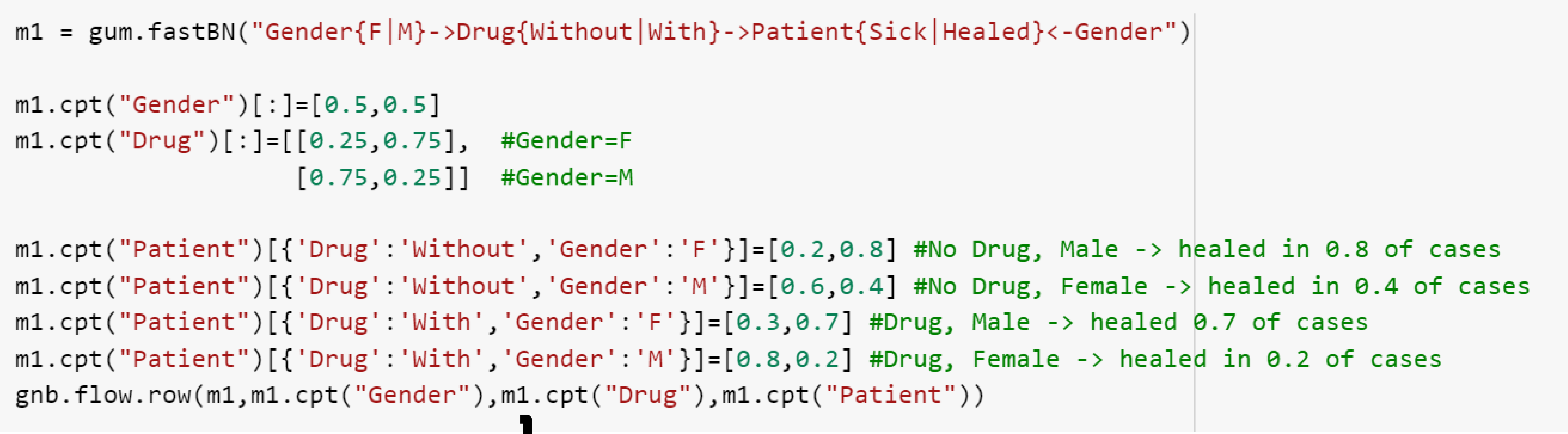
https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum

In a statistical study about a drug, we try to evaluate the latter’s efficiency among a population of men and women.

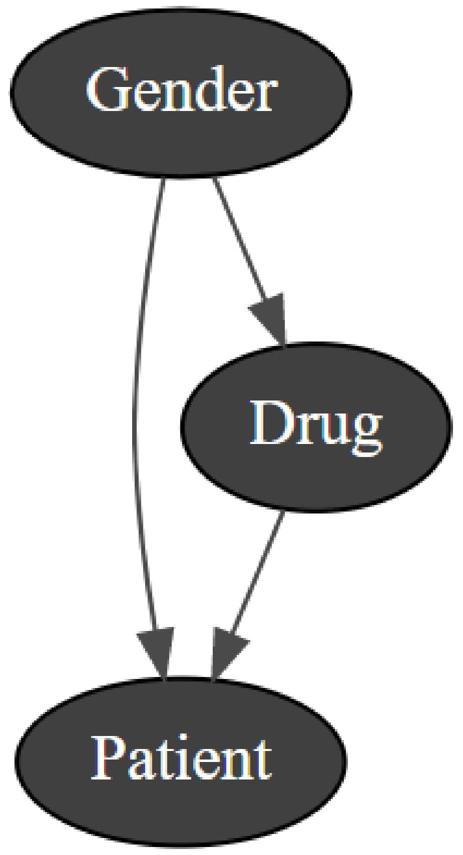
Let’s note: - Drug : drug taking - Patient : cured patient - Gender : patient’s gender

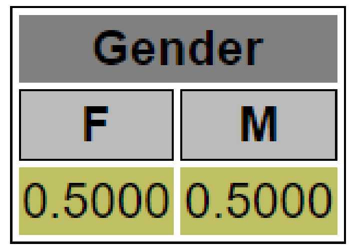
The model from the observed date is as follow :



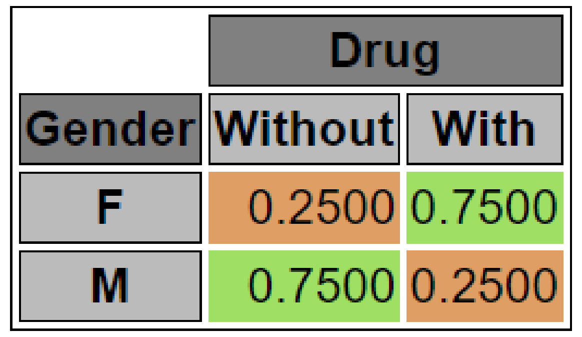
```
m1 = gum.fastBN("Gender{F|M}->Drug{Without|With}->Patient{Sick|Healed}<-Gender")

m1.cpt("Gender")[:]=[0.5,0.5]
m1.cpt("Drug")[:]=[[0.25,0.75],  #Gender=F
                   [0.75,0.25]]  #Gender=M

m1.cpt("Patient")[{'Drug':'Without','Gender':'F'}]=[0.2,0.8] #No Drug, Male -> healed in 0.8 of cases
m1.cpt("Patient")[{'Drug':'Without','Gender':'M'}]=[0.6,0.4] #No Drug, Female -> healed in 0.4 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'F'}]=[0.3,0.7] #Drug, Male -> healed 0.7 of cases
m1.cpt("Patient")[{'Drug':'With','Gender':'M'}]=[0.8,0.2] #Drug, Female -> healed in 0.2 of cases
gnb.flow.row(m1,m1.cpt("Gender"),m1.cpt("Drug"),m1.cpt("Patient"))
```

To display the CPT

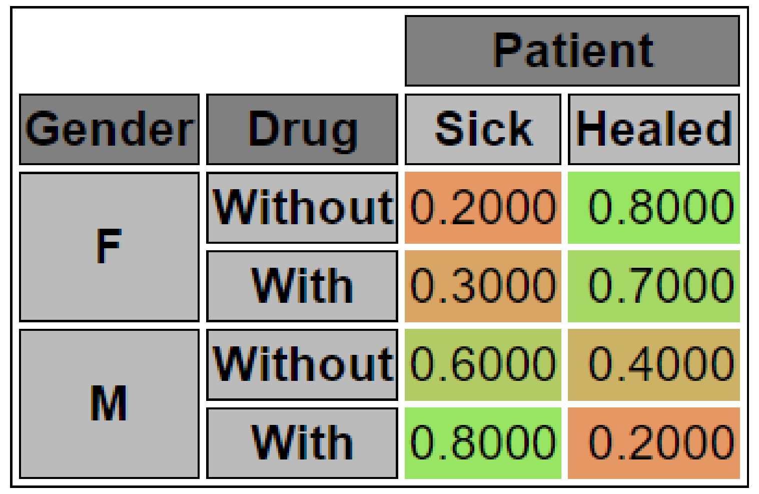




Gender	
F	M
0.5000	0.5000



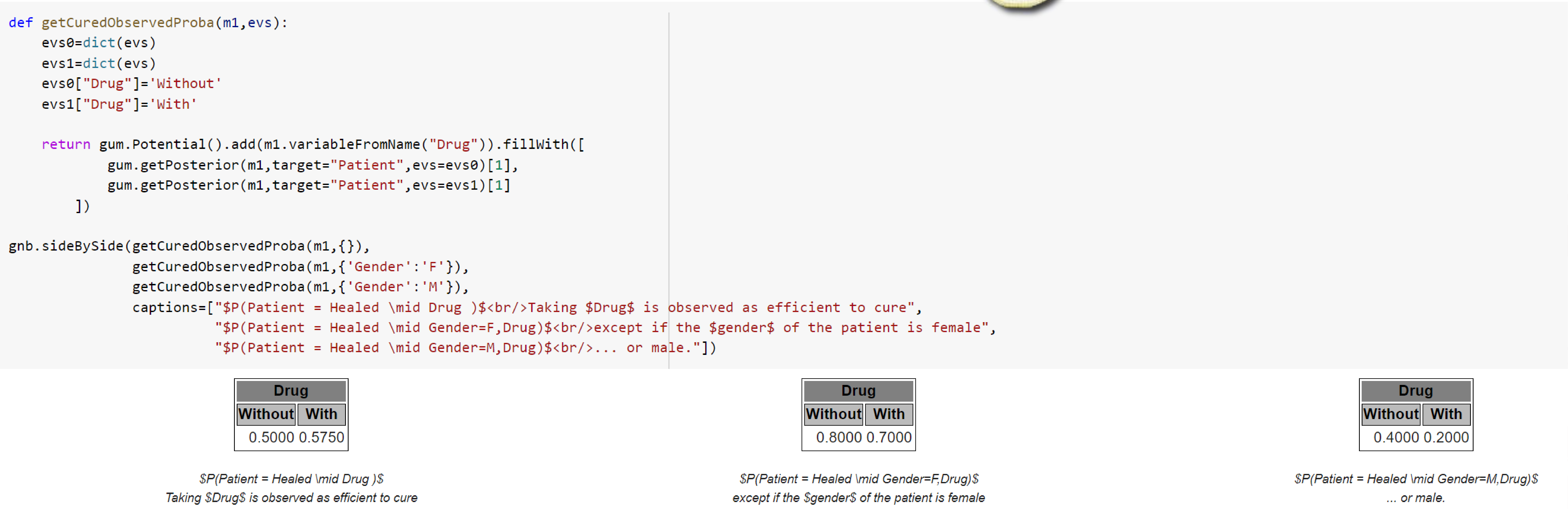
	Drug	
Gender	Without	With
F	0.2500	0.7500
M	0.7500	0.2500



		Patient	
Gender	Drug	Sick	Healed
F	Without	0.2000	0.8000
	With	0.3000	0.7000
M	Without	0.6000	0.4000
	With	0.8000	0.2000

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum



```
def getCuredObservedProba(m1,evs):
    evs0=dict(evs)
    evs1=dict(evs)
    evs0["Drug"]='Without'
    evs1["Drug"]='With'

    return gum.Potential().add(m1.variableFromName("Drug")).fillWith([
            gum.getPosterior(m1,target="Patient",evs=evs0)[1],
            gum.getPosterior(m1,target="Patient",evs=evs1)[1]
        ])

gnb.sideBySide(getCuredObservedProba(m1,{}),
               getCuredObservedProba(m1,{'Gender':'F'}),
               getCuredObservedProba(m1,{'Gender':'M'}),
               captions=["$P(Patient = Healed \mid Drug )$<br/>Taking $Drug$ is observed as efficient to cure",
                         "$P(Patient = Healed \mid Gender=F,Drug)$<br/>except if the $gender$ of the patient is female",
                         "$P(Patient = Healed \mid Gender=M,Drug)$<br/>... or male."])
```

Drug	
Without	With
0.5000	0.5750

$P(Patient = Healed \mid Drug)$
Taking $Drug$ is observed as efficient to cure

Drug	
Without	With
0.8000	0.7000

$P(Patient = Healed \mid Gender=F,Drug)$
except if the $gender$ of the patient is female

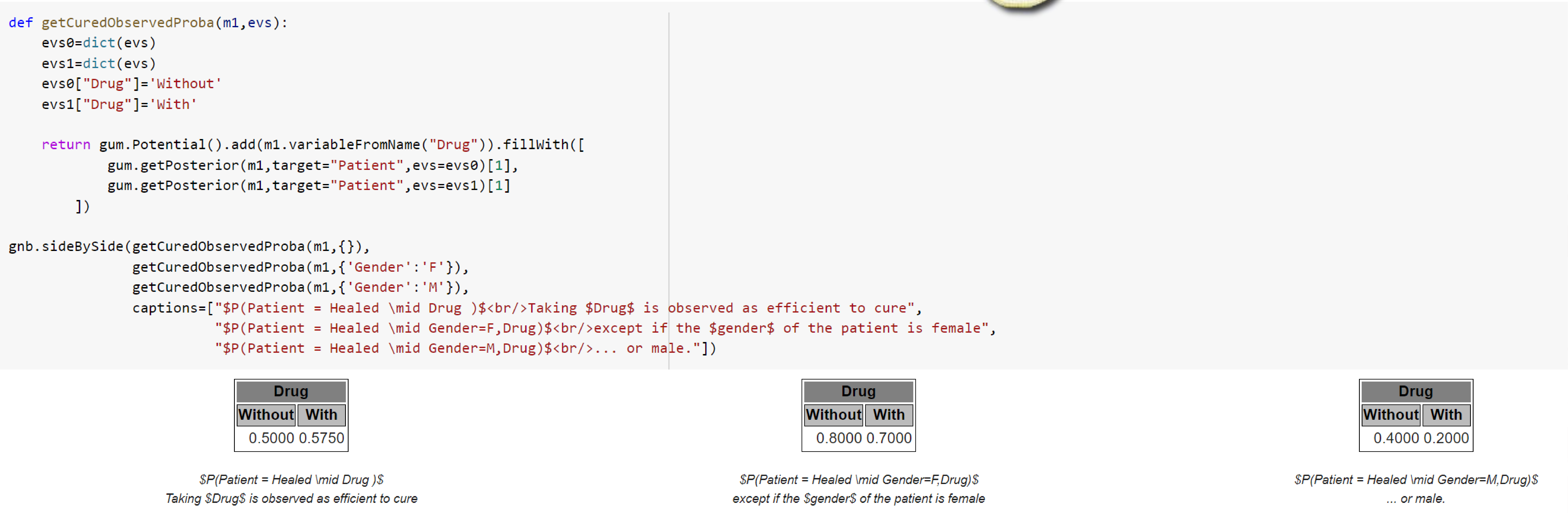
Drug	
Without	With
0.4000	0.2000

$P(Patient = Healed \mid Gender=M,Drug)$
... or male.

A Potential function is a function that associates a non-negative value (or probability) with each possible assignment of values to a set of random variables. Potential functions are used to represent the local relationships between random variables in a graphical model. Specifically, a potential function is associated with each factor node in the graph, which typically corresponds to a set of random variables in the model.

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum



```
def getCuredObservedProba(m1,evs):
    evs0=dict(evs)
    evs1=dict(evs)
    evs0["Drug"]='Without'
    evs1["Drug"]='With'

    return gum.Potential().add(m1.variableFromName("Drug")).fillWith([
            gum.getPosterior(m1,target="Patient",evs=evs0)[1],
            gum.getPosterior(m1,target="Patient",evs=evs1)[1]
        ])

gnb.sideBySide(getCuredObservedProba(m1,{}),
               getCuredObservedProba(m1,{'Gender':'F'}),
               getCuredObservedProba(m1,{'Gender':'M'}),
               captions=["$P(Patient = Healed \mid Drug )$<br/>Taking $Drug$ is observed as efficient to cure",
                         "$P(Patient = Healed \mid Gender=F,Drug)$<br/>except if the $gender$ of the patient is female",
                         "$P(Patient = Healed \mid Gender=M,Drug)$<br/>... or male."])
```

Drug	
Without	With
0.5000	0.5750

$P(Patient = Healed \mid Drug)$
Taking $Drug$ is observed as efficient to cure

Drug	
Without	With
0.8000	0.7000

$P(Patient = Healed \mid Gender=F,Drug)$
except if the $gender$ of the patient is female

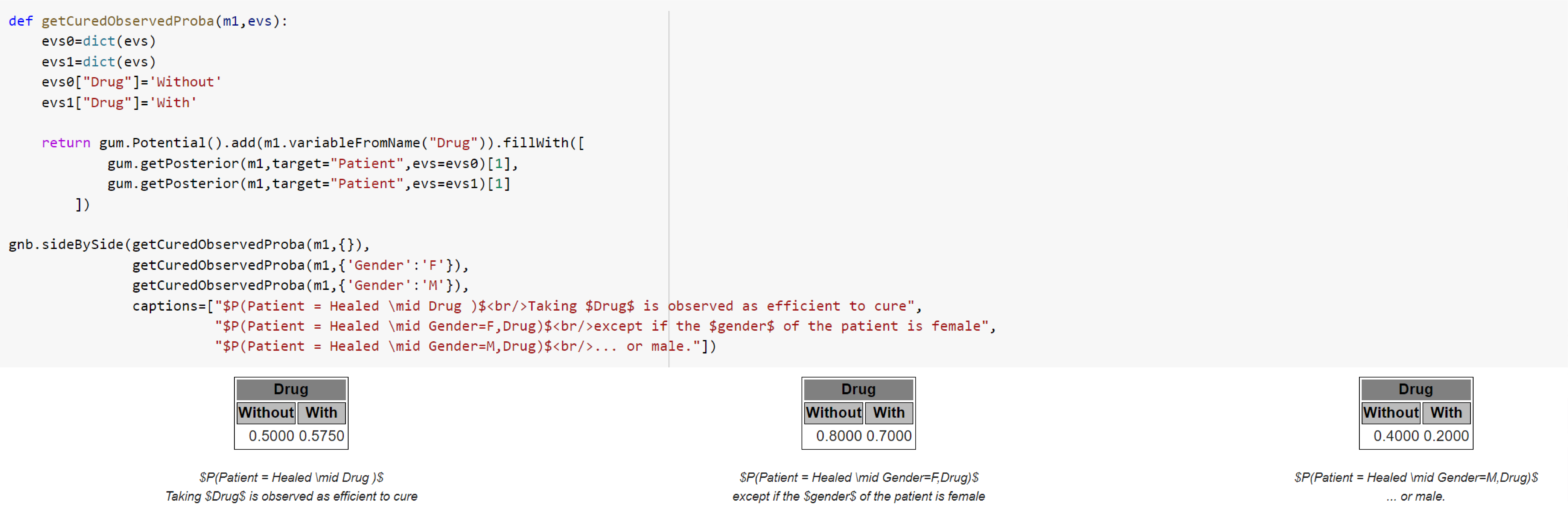
Drug	
Without	With
0.4000	0.2000

$P(Patient = Healed \mid Gender=M,Drug)$
... or male.

pyAgrum.getPosterior() is a function from the Python package pyAgrum which is used to compute the posterior probabilities of a set of variables given some evidence. The function returns an array of values because it is designed to compute the posterior probability distribution of the variables, which is a probability distribution over all possible values of the variables.

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum



```
def getCuredObservedProba(m1,evs):
    evs0=dict(evs)
    evs1=dict(evs)
    evs0["Drug"]='Without'
    evs1["Drug"]='With'

    return gum.Potential().add(m1.variableFromName("Drug")).fillWith([
            gum.getPosterior(m1,target="Patient",evs=evs0)[1],
            gum.getPosterior(m1,target="Patient",evs=evs1)[1]
        ])

gnb.sideBySide(getCuredObservedProba(m1,{}),
               getCuredObservedProba(m1,{'Gender':'F'}),
               getCuredObservedProba(m1,{'Gender':'M'}),
               captions=["$P(Patient = Healed \mid Drug )$<br/>Taking $Drug$ is observed as efficient to cure",
                         "$P(Patient = Healed \mid Gender=F,Drug)$<br/>except if the $gender$ of the patient is female",
                         "$P(Patient = Healed \mid Gender=M,Drug)$<br/>... or male."])
```

Drug	
Without	With
0.5000	0.5750

$P(Patient = Healed \mid Drug)$
Taking $Drug$ is observed as efficient to cure

Drug	
Without	With
0.8000	0.7000

$P(Patient = Healed \mid Gender=F,Drug)$
except if the $gender$ of the patient is female

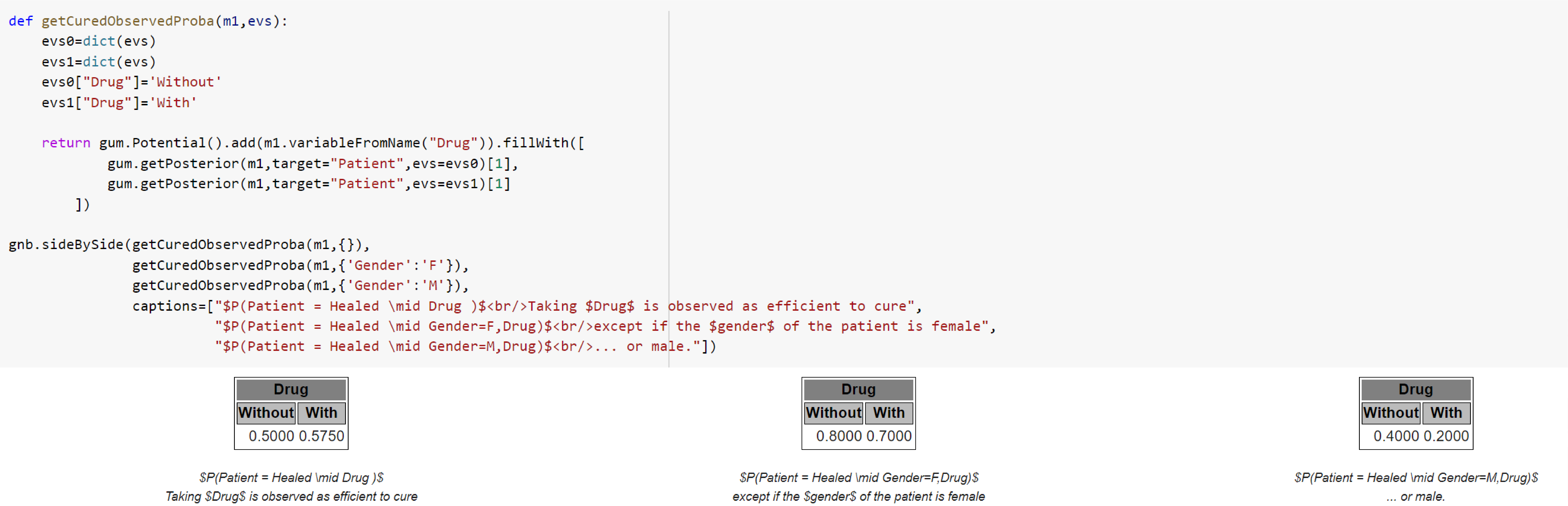
Drug	
Without	With
0.4000	0.2000

$P(Patient = Healed \mid Gender=M,Drug)$
... or male.

Those results form a paradox called Simpson paradox :
P(C|¬D) = 0.5 < P(C|D) = 0.575
P(C|¬D,G=Male) = 0.8 > P(C|D,G=Male)=0.7
P(C|¬D,G=Female) = 0.4 > P(C|D,G=Female)=0.2

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum



```
def getCuredObservedProba(m1,evs):
    evs0=dict(evs)
    evs1=dict(evs)
    evs0["Drug"]='Without'
    evs1["Drug"]='With'

    return gum.Potential().add(m1.variableFromName("Drug")).fillWith([
            gum.getPosterior(m1,target="Patient",evs=evs0)[1],
            gum.getPosterior(m1,target="Patient",evs=evs1)[1]
        ])

gnb.sideBySide(getCuredObservedProba(m1,{}),
               getCuredObservedProba(m1,{'Gender':'F'}),
               getCuredObservedProba(m1,{'Gender':'M'}),
               captions=["$P(Patient = Healed \mid Drug )$<br/>Taking $Drug$ is observed as efficient to cure",
                         "$P(Patient = Healed \mid Gender=F,Drug)$<br/>except if the $gender$ of the patient is female",
                         "$P(Patient = Healed \mid Gender=M,Drug)$<br/>... or male."])
```

Drug	
Without	With
0.5000	0.5750

$P(Patient = Healed \mid Drug)$
Taking $Drug$ is observed as efficient to cure

Drug	
Without	With
0.8000	0.7000

$P(Patient = Healed \mid Gender=F,Drug)$
except if the $gender$ of the patient is female

Drug	
Without	With
0.4000	0.2000

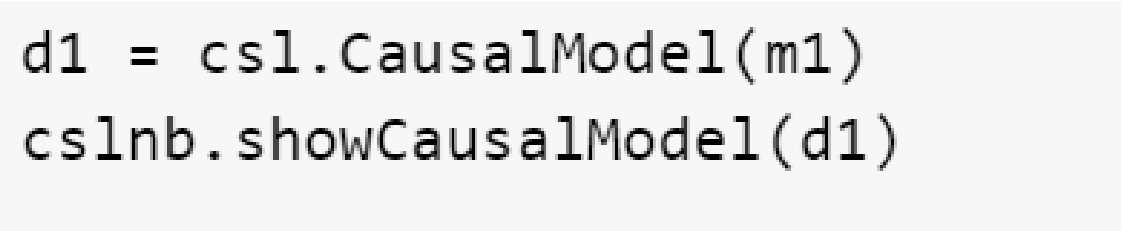
$P(Patient = Healed \mid Gender=M,Drug)$
... or male.

Actually, giving a drug is not an **observation in our model but rather an intervention**.
What if we use intervention instead of observation ?

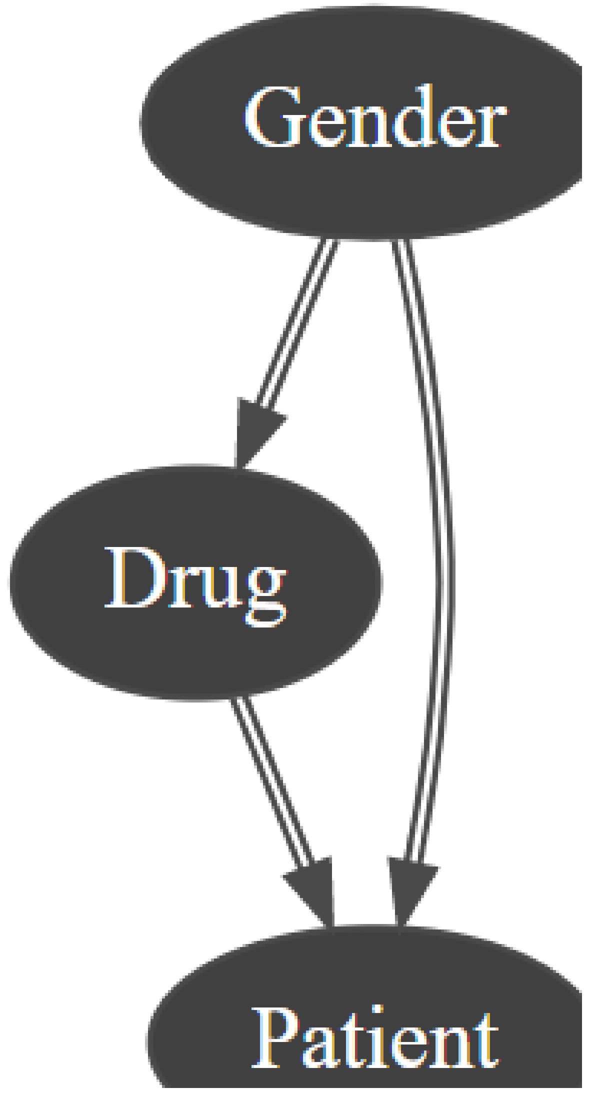
https://pyagrum.readthedocs.io/en/latest/index.html

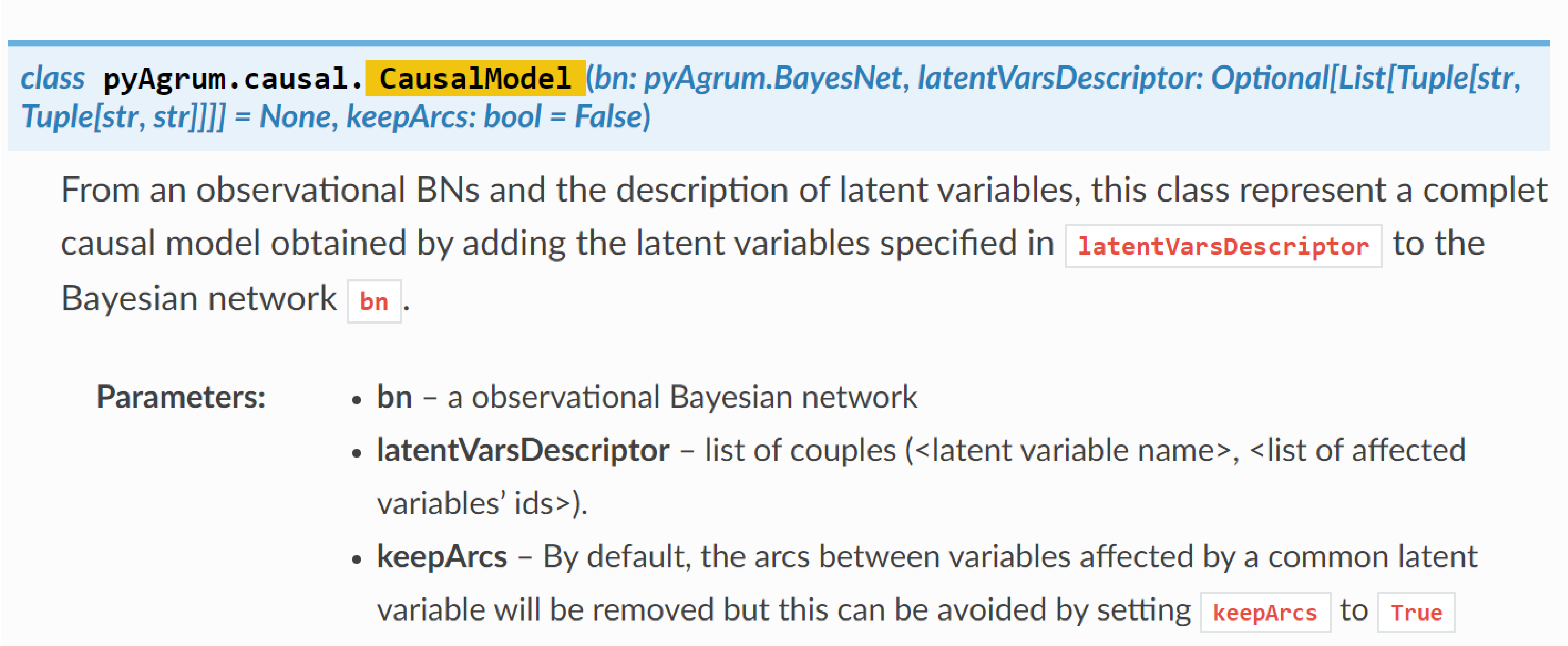
Simpson's paradox via pyAgrum

How to compute causal impacts on the patient’s health ?
We propose this causal model.



```
d1 = csl.CausalModel(m1)
cslnb.showCausalModel(d1)
```





class `pyAgrum.causal.CausalModel`*(bn: pyAgrum.BayesNet, latentVarsDescriptor: Optional[List[Tuple[str, Tuple[str, str]]]] = None, keepArcs: bool = False)*

From an observational BNs and the description of latent variables, this class represent a complet causal model obtained by adding the latent variables specified in `latentVarsDescriptor` to the Bayesian network `bn`.

Parameters:

- **bn** – a observational Bayesian network
- **latentVarsDescriptor** – list of couples (<latent variable name>, <list of affected variables’ ids>).
- **keepArcs** – By default, the arcs between variables affected by a common latent variable will be removed but this can be avoided by setting `keepArcs` to `True`

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum

How to compute causal impacts on the patient’s health ?

Computing P(Patient=Healed|↪Drug=Without)

```
cslnb.showCausalImpact(d1, "Patient", doing="Drug",values={"Drug" : "Without"})
```

Gender
Drug
Patient

pyAgrum.causal.notebook. showCausalImpact *(model: pyAgrum.causal._CausalModel.CausalModel, on: Union[str, Set[str]], doing: Union[str, Set[str]], knowing: Optional[Set[str]] = None, values: Optional[Dict[str, int]] = None)*

display a HTML representing of the three values defining a causal impact : formula, value, explanation :param model: the causal model :param on: the impacted variable(s) :param doing: the variable(s) of intervention :param knowing: the variable(s) of evidence :param values : values for certain variables

Patient	
Sick	Healed
0.4000	0.6000

$$\begin{equation*}P(Patient \mid \hookrightarrow\mkern-6.5muDrug) = \sum_{Gender}{P\left(Patient\mid Drug,Gender\right) \cdot P\left(Gender\right)}\end{equation*}$$

Causal Model

Explanation : backdoor ['Gender'] found.

Impact

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum

How to compute causal impacts on the patient’s health ?

Computing P(Patient=Healed|↪Drug=With)

```
d1 = csl.CausalModel(m1)
cslnb.showCausalImpact(d1, "Patient", "Drug",values={"Drug" : "With"})
```

Gender
Drug
Patient

Causal Model

$$\begin{equation*}P(Patient \mid \hookrightarrow\mkern-6.5muDrug) = \sum_{Gender}{P\left(Patient\mid Drug,Gender\right) \cdot P\left(Gender\right)}\end{equation*}$$

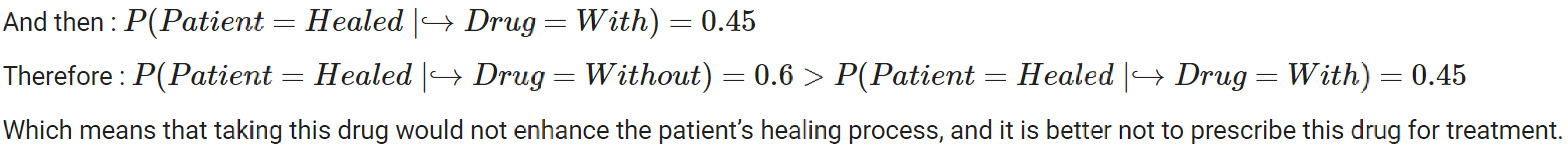
Explanation : backdoor ['Gender'] found.

Patient	
Sick	Healed
0.5500	0.4500

Impact

https://pyagrum.readthedocs.io/en/latest/index.html

Simpson's paradox via pyAgrum



And then : $P(Patient = Healed \mid \hookrightarrow Drug = With) = 0.45$

Therefore : $P(Patient = Healed \mid \hookrightarrow Drug = Without) = 0.6 > P(Patient = Healed \mid \hookrightarrow Drug = With) = 0.45$

Which means that taking this drug would not enhance the patient’s healing process, and it is better not to prescribe this drug for treatment.

https://pyagrum.readthedocs.io/en/latest/index.html

Repeat the computation with this problem using pyAgrum

- A new medicine was offered to 700 patients: 350 of them chose to take it, while 350 did not.

	Medicine	No medicine
Men	81 out of 87 recovered (**93%**)	234 out of 270 recovered (87%)
Women	192 out of 263 recovered (**73%**)	55 out of 80 recovered (69%)
Combined data	273 out of 350 recovered (78%)	289 out of 350 recovered (**83%**)

- The medicine worked for the two subgroups, men and women, but not for the population as a whole. How is that possible?

Compare the solution with computation in slide 35 – Causality1 by prof. Bellotto

Questions