Markov Decision Processes in Python

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Topics:

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- Example: Time for coffee?- Maximax – Maximin
- MDP toolbox
- Exercise: Solving a simple MDP using the MDP toolbox
- Value iteration algorithm recap
- Policy iteration algorithm recap
- Value iteration algorithm via MDP toolbox
- Policy iteration algorithm via MDP toolbox
- Q-Learning via MDP toolbox

DIPARTIMENTO
DI INGEGNERIA
DELL'INFORMAZIONE

Expected utility: Recap

Expected utility

- Let's call $U(s)$ the **utility** of being in a certain state s
- $U(s)$ is a sort of numerical "score" assigned to s – i.e. how desirable is to reach that particular state (e.g. money earned in the quiz game)

- The **expected utility** of an action, $EU(a)$, is then the average utility value of the outcomes, weighted by the probability that the outcome occurs:

$$EU(a) = \sum_{s'} P(\mathrm{RESULT}(a) = s')\, U(s')$$

- **MEU principle**: choose the action that maximizes expected utility

Slide 8 by prof. Bellotto – Part1

Expected utility: Example Time for coffee?

I have half an hour to spare in my busy schedule, and I have a choice between working quietly in my office and going out for a coffee.

If I stay in my office, three things can happen:

- I can get some **work done** (**Utility = 8**),
- I can **get distracted** looking at the latest news (**Utility = 1**),
- A **colleague might stop** by to talk about some work we are doing (**Utility = 5**).

If I go out for coffee, I will most likely enjoy a good cup of **smooth caffeination** (**Utility = 10**), but there is also a chance I will end up **spilling coffee** all over myself (**Utility = −20**).

The probability of getting work done if I choose to **stay in the office is 0.5**, while the probabilities of **getting distracted**, and a **colleague stopping** by are **0.3** and **0.2** respectively.

If I go out for a coffee, my chance of **enjoying my beverage is 0.95**, and the chance of **spilling my drink is 0.05**.

QUESTION 1(a): What is the expected utility of staying in my office?
QUESTION 1(b): What is the expected utility of going out for a coffee?
QUESTION 1(c): By the principle of maximum expected utility, what should I do?

Expected utility: Example Time for coffee?

QUESTION 1(a): What is the expected utility of staying in my office?

SOLUTION 1(a):

Staying in the *office* means that I will either *work*, get *distracted*, or talk with a *colleague*.
These states have the following utilities:

$$U(work) = 8$$
$$U(distracted) = 1$$
$$U(colleague) = 5$$

and the probabilities of these happening, given I stay in the *office* are:

$$P(work|office) = 0.5$$
$$P(distracted|office) = 0.3$$
$$P(colleague|office) = 0.2$$

Let's declare these as Python arrays

Expected utility: Example Time for coffee?

Let's declare these as Python arrays

```
import numpy as np
# Setup arrays with: symbolic names for outcomes (not currently used), utilities of outcomes, and
# probabililites of those outcomes
office_outcomes = ["work", "distracted", "colleague"]
print('office_outcomes = ', office_outcomes)
u_office_outcomes = np.array([8, 1, 5])
print('U(office_outcomes) = ', u_office_outcomes)
p_office_outcomes_office = np.array([0.5, 0.3, 0.2])
print('P(office_outcomes|office) =', p_office_outcomes_office)
```

label

utilities

probabilities

```
office_outcomes =  ['work', 'distracted', 'colleague']
U(office_outcomes) =  [8 1 5]
P(office_outcomes|office) = [0.5 0.3 0.2]
```

Expected utility: Example Time for coffee?

The **expected utility** of an action, $EU(a)$, is then the average utility value of the outcomes, weighted by the probability that the outcome occurs:

$$EU(a) = \sum_{s'} P(\mathrm{RESULT}(a) = s')\, U(s')$$

SOLUTION 1(a):

$$\begin{aligned} EU(office) = \ & P(work|office) * U(work) \ + \\ & P(distracted|office) * U(distracted) + \\ & P(colleague|office) * U(colleague) \end{aligned}$$

$$EU(office) = 0.5 * 8 + 0.3 * 1 + 0.2 * 5 = 5.3$$

Expected utility: Example Time for coffee?

SOLUTION 1(a):

Now let's implement this in Python:

```
# The weighted utility ofeach outcome is each to compute by pairwise multiplication
eu_office_outcomes = u_office_outcomes * p_office_outcomes_office
print('EU by outcome =', eu_office_outcomes)
# Summing the weighted utilities gets us the expected utility
eu_office = np.sum(eu_office_outcomes)
print('EU(office) = ', eu_office)
```

```
EU by outcome = [4.  0.3 1. ]
EU(office) =  5.3
```

So the expected utility of staying in the office is **5.3**

Expected utility: Example Time for coffee?

QUESTION 1(b): What is the expected utility of going out for a coffee?

SOLUTION 1(b):

Going out for a coffee means that I will either *enjoying* my beverage, or *spilling* my drink. These states have the following utilities:

$$U(caffeination) = 10$$
$$U(spillage) = \text{-}20$$

and the probabilities of these happening, given I stay in the *office* are:

$$P(caffeination|coffee) = 0.95$$
$$P(spillage|coffee) = 0.05$$

Let's implement these in Python using the same notation as before

Expected utility: Example Time for coffee?

SOLUTION 1(b):

```
# The coffee calculation is the same as the office calculation, first set up arrays
coffee_outcomes = ["caffeination", "spillage"]
print('coffee_outcomes = ', coffee_outcomes)
u_coffee_outcomes = np.array([10, -20])
print('U(coffee_outcomes) = ', u_coffee_outcomes)
p_coffee_outcomes_coffee = np.array([0.95, 0.05])
print('P(coffee_outcomes|coffee) =', p_coffee_outcomes_coffee)
print('\n')
# Then compute the expected utility
eu_coffee_outcomes = u_coffee_outcomes * p_coffee_outcomes_coffee
print('EU by outcome =', eu_coffee_outcomes)
eu_coffee = np.sum(eu_coffee_outcomes)
print('EU(coffee) = ', eu_coffee)
```

```
coffee_outcomes =  ['caffeination', 'spillage']
U(coffee_outcomes) =  [ 10 -20]
P(coffee_outcomes|coffee) = [0.95 0.05]


EU by outcome = [ 9.5 -1. ]
EU(coffee) =  8.5
```

So the expected utility of going out for coffee is **8.5**

Expected utility: Example Time for coffee?

QUESTION 1(c): By the principle of maximum expected utility, what should I do?

SOLUTION 1(c):

MEU principle: choose the action that maximizes expected utility

Clearly in the case of numbers in the example, the option of *going out for coffee* is the one with the maximum expected utility.

However, we will also program it in Python so that we can see what happens as the probabilities of the outcomes vary:

```
if eu_office > eu_coffee:
    print('Office is the MEU choice')
else:
    print('Coffee is the MEU choice')
```

```
Coffee is the MEU choice
```

Expected utility: Example Time for coffee? - Maximax

Revisit the decision for the coffee example using the **maximax decision criterion**

SOLUTION:

The **maximax decision criterion** rates each choice by the **utility** of its **best outcome**, and then picks the choice with best utility.

In Python we would do this calculation as follows

```
# The utility of each choice is the max utility of their outcomes
max_u_office = np.max(u_office_outcomes)
print('MaxU(office) =', max_u_office)
max_u_coffee = np.max(u_coffee_outcomes)
print('MaxU(coffee) =', max_u_coffee)
print('\n')
# The decision criterion is then to pick the outcome with the highest utility:
if max_u_office > max_u_coffee:
    print('Office is the Maximax choice')
else:
    print('Coffee is the Maximax choice')
```

```
MaxU(office) = 8
MaxU(coffee) = 10


Coffee is the Maximax choice
```

Expected utility: Example Time for coffee? - Maximin

Revisit the decision for the coffee example using the **maximin decision criterion**

SOLUTION:

The **maximin decision criterion** rates each choice by the **utility** of its **worst outcome**, and then picks the choice with the best utility.

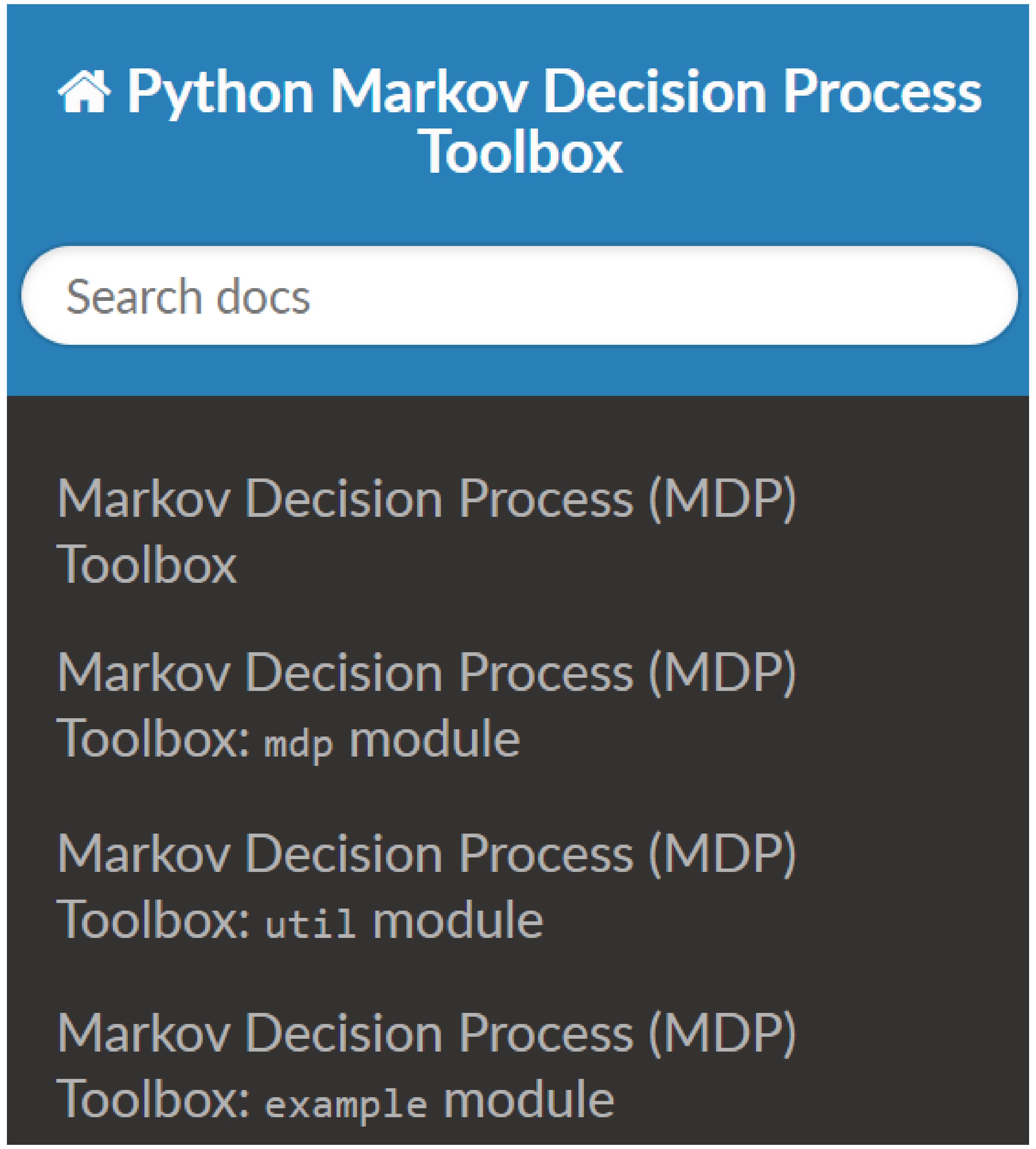
In Python we would do this calculation as follows

```
# The utility of each choice is the max utility of their outcomes
min_u_office = np.min(u_office_outcomes)
print('MinU(office) =', min_u_office)
min_u_coffee = np.min(u_coffee_outcomes)
print('MinU(coffee) =', min_u_coffee)
print('\n')
# The decision criterion is then to pick the outcome with the highest utility:
if min_u_office > min_u_coffee:
    print('Office is the Maximin choice')
else:
    print('Coffee is the Maximin choice')
```

```
MinU(office) = 1
MinU(coffee) = -20


Office is the Maximin choice
```

MDP toolbox



The MDP toolbox provides classes and functions for the resolution of **discrete-time Markov Decision Processes**.

The list of algorithms that have been implemented includes backwards induction, linear programming, policy iteration, q-learning and value iteration along with several variations.

The documentation is available at:

https://pymdptoolbox.readthedocs.io/en/latest/index.html

We need to install it using:

```
pip install pymdptoolbox
```

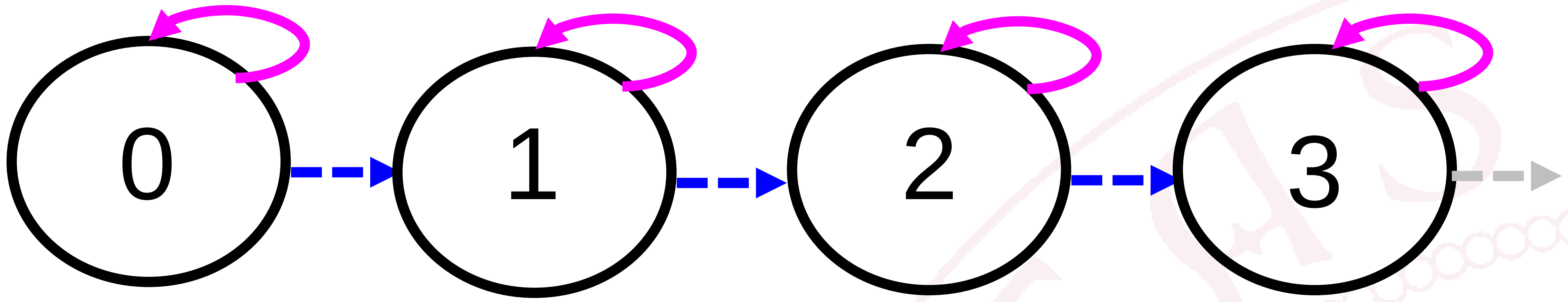
Exercise: Solving a simple MDP using the MDP toolbox

Let's start with a really simple problem.

We have 4 states and two actions.

There are two actions:

- 0 is "Stay“,
- 1 is "Right".



0 **always succeeds** and leaves the agent in the same state.
1 moves the agent right with **probability 0.8**, stays in place with **probability 0.2**.

The states are 0, 1, 2, 3.
0 is left of 1, which is left of 2 and so on.
(Thus the states are in a line which runs 0, 1, 2, 3 from left to right.)

The agent remains in **state 3** with **probability 1**.

State 3 has a **reward of 1**, and the **cost** of any action is **-0.04**.

Exercise: Solving a simple MDP using the MDP toolbox

The MDP Toolbox defines MDPs through a **probability array** and a **reward array**.

The probability array has shape **(A, S, S)**, where **A are actions** and **S are states**.

For **each action** specify the **transitions probabilities** of reaching the **second state** by applying that action in the **first state**.

```
!pip install pymdptoolbox
import mdptoolbox
import numpy as np

# The MDP Toolbox defines MDPs through a probability array and a reward array.

# The probability array has shape (A, S, S), where A are actions and S
# are states. For each action specify the transitions probabilities of reaching
# the second state by applying that action in the first state.

# So, to implement the action model described above, we need:
P1 = np.array([[[1, 0, 0, 0],
                [0, 1, 0, 0],
                [0, 0, 1, 0],
                [0, 0, 0, 1]],
               [[0.2, 0.8, 0,   0],
                [0,   0.2, 0.8, 0],
                [0,   0,   0.2, 0.8],
                [0,   0,   0,   1]]])
```

Exercise: Solving a simple MDP using the MDP toolbox

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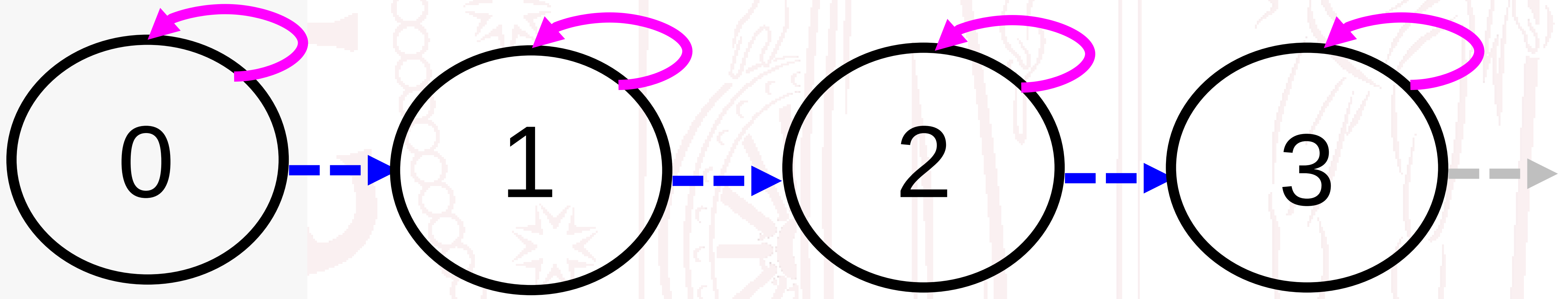
P1 = np.array([[[1, 0, 0, 0],
                [0, 1, 0, 0],
                [0, 0, 1, 0],
                [0, 0, 0, 1]],
               [[0.2, 0.8, 0,   0],
                [0,   0.2, 0.8, 0],
                [0,   0,   0.2, 0.8],
                [0,   0,   0,   1]]])
```

Stay

Right



Exercise: Solving a simple MDP using the MDP toolbox

The MDP Toolbox defines MDPs through a **probability array** and a **reward array**.

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For **each action** specify the **transitions probabilities** of reaching the **second state** by applying that action in the **first state**.

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!pip install pymdptoolbox
import mdptoolbox
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# The probability array has shape (A, S, S), where A are actions and S
# are states. For each action specify the transitions probabilities of reaching
# the second state by applying that action in the first state.

# So, to implement the action model described above, we need:
P1 = np.array([[[1, 0, 0, 0],
                [0, 1, 0, 0],
                [0, 0, 1, 0],
                [0, 0, 0, 1]],
               [[0.2, 0.8, 0,   0],
                [0,   0.2, 0.8, 0],
                [0,   0,   0.2, 0.8],
                [0,   0,   0,   1]]])
```

Stay

Right

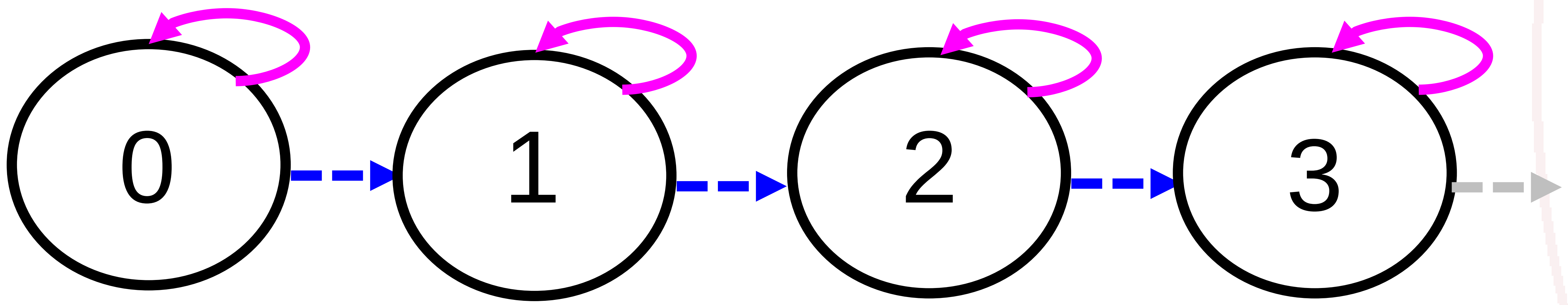
The first matrix is that for the action "Stay" (when executed in a given state the agent stays there) and the second is for the action "Right" (which shifts the agent right with probability 0.8 except in state 3 when the agent remains in state 3 with probability 1).

Exercise: Solving a simple MDP using the MDP toolbox

The reward array has **shape (S, A)**, so there is a set of S vectors, one for each state, and each is a vector with one element for each the actions --- **each element is the reward for executing the relevant action in the state** (so this is really modelling cost of the action).

```
R1 = np.array([[-0.04, -0.04], [-0.04, -0.04], [-0.04, -0.04], [1, 1]])
# R1 says that executing either action in states 0, 1, or 2 has a reward
# of -0.04, and executing either action in state 3 has reward 1.
```

R1 says that executing either action in states 0, 1, or 2 has a **reward of -0.04**, and executing either action in **state 3 has reward 1**.



Exercise: Solving a simple MDP using the MDP toolbox

The **util.check()** function checks that the reward and probability matrices are well-formed, and match.

Success is silent, failure provides somewhat useful error messages.

```
# The util.check() function checks that the reward and probability matrices
# are well-formed, and match.
#
# Success is silent, failure provides somewhat useful error messages.
mdptoolbox.util.check(P1, R1)
```

`mdptoolbox.util.check`(*P*, *R*) [source]

Check if `P` and `R` define a valid Markov Decision Process (MDP).

Let `S` = number of states, `A` = number of actions.

Parameters:

- **P** (*array*) – The transition matrices. It can be a three dimensional array with a shape of (A, S, S). It can also be a one dimensional arraye with a shape of (A,), where each element contains a matrix of shape (S, S) which can possibly be sparse.
- **R** (*array*) – The reward matrix. It can be a three dimensional array with a shape of (S, A, A). It can also be a one dimensional array with a shape of (A,), where each element contains matrix with a shape of (S, S) which can possibly be sparse. It can also be an array with a shape of (S, A) which can possibly be sparse.

Notes

Raises an error if `P` and `R` do not define a MDP.

https://pymdptoolbox.readthedocs.io/en/latest/api/util.html

Value iteration algorithm recap

Value iteration algorithm

- The iteration step, called a Bellman update, looks like this

$$U_{i+1}(s) \leftarrow \max_{a \in A(s)} \sum_{s'} P(s' \mid s,a)[R(s,a,s') + \gamma U_i(s')]\,,$$

- Update is assumed to be applied simultaneously to all the states at each iteration, where

function Q-VALUE(mdp, s, a, U) **returns** a utility value
  return $\sum_{s'} P(s' \mid s,a)[R(s,a,s') + \gamma U[s']]$

- The algorithm stops when the difference between old and updated utilities is below a certain threshold

```
function VALUE-ITERATION(mdp, ε) returns a utility function
  inputs: mdp, an MDP with states S, actions A(s), transition model P(s' | s,a),
            rewards R(s,a,s'), discount γ
          ε, the maximum error allowed in the utility of any state
  local variables: U, U', vectors of utilities for states in S, initially zero
                   δ, the maximum relative change in the utility of any state

  repeat
     U ← U'; δ ← 0
     for each state s in S do
        U'[s] ← max_{a∈A(s)} Q-VALUE(mdp, s, a, U)
        if |U'[s] − U[s]| > δ then δ ← |U'[s] − U[s]|
  until δ ≤ ε(1 − γ)/γ
  return U
```

Value iteration algorithm via MDP toolbox

class `mdptoolbox.mdp.ValueIteration`(*transitions, reward, discount, epsilon=0.01, max_iter=1000, initial_value=0, skip_check=False*) [source]

Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the value iteration algorithm.

ValueIteration applies the value iteration algorithm to solve a discounted MDP. The algorithm consists of solving Bellman's equation iteratively. Iteration is stopped when an epsilon-optimal policy is found or after a specified number (`max_iter`) of iterations. This function uses verbose and silent modes. In verbose mode, the function displays the variation of `V` (the value function) for each iteration and the condition which stopped the iteration: epsilon-policy found or maximum number of iterations reached.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **epsilon** (*float, optional*) – Stopping criterion. See the documentation for the `MDP` class for details. Default: 0.01.
- **max_iter** (*int, optional*) – Maximum number of iterations. If the value given is greater than a computed bound, a warning informs that the computed bound will be used instead. By default, if `discount` is not equal to 1, a bound for `max_iter` is computed, otherwise `max_iter` = 1000. See the documentation for the `MDP` class for further details.
- **initial_value** (*array, optional*) – The starting value function. Default: a vector of zeros.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- –––––– –
- **V** (*tuple*) – The optimal value function.
- **policy** (*tuple*) – The optimal policy function. Each element is an integer corresponding to an action which maximises the value function in that state.
- **iter** (*int*) – The number of iterations taken to complete the computation.
- **time** (*float*) – The amount of CPU time used to run the algorithm.

run() [source]

Do the algorithm iteration.

To run value iteration we create a value iteration object, and run it.

Note that the discount value is 0.9

```
# To run value iteration we create a value iteration object, and run it. Note that
# discount value is 0.9
vi1 = mdptoolbox.mdp.ValueIteration(P1, R1, 0.9)
vi1.run()
```

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Exercise: Solving a simple MDP using the MDP toolbox

class `mdptoolbox.mdp.ValueIteration`(*transitions, reward, discount, epsilon=0.01, max_iter=1000, initial_value=0, skip_check=False*) [source]

Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the value iteration algorithm.

ValueIteration applies the value iteration algorithm to solve a discounted MDP. The algorithm consists of solving Bellman's equation iteratively. Iteration is stopped when an epsilon-optimal policy is found or after a specified number (`max_iter`) of iterations. This function uses verbose and silent modes. In verbose mode, the function displays the variation of `V` (the value function) for each iteration and the condition which stopped the iteration: epsilon-policy found or maximum number of iterations reached.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **epsilon** (*float, optional*) – Stopping criterion. See the documentation for the `MDP` class for details. Default: 0.01.
- **max_iter** (*int, optional*) – Maximum number of iterations. If the value given is greater than a computed bound, a warning informs that the computed bound will be used instead. By default, if `discount` is not equal to 1, a bound for `max_iter` is computed, otherwise `max_iter` = 1000. See the documentation for the `MDP` class for further details.
- **initial_value** (*array, optional*) – The starting value function. Default: a vector of zeros.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- ------ –
- **V** (*tuple*) – The optimal value function.
- **policy** (*tuple*) – The optimal policy function. Each element is an integer corresponding to an action which maximises the value function in that state.
- **iter** (*int*) – The number of iterations taken to complete the computation.
- **time** (*float*) – The amount of CPU time used to run the algorithm.

We can then display the values (utilities) computed, and look at the policy:

```python
# We can then display the values (utilities) computed, and look at the policy:
print('Values:\n', vi1.V)
print('Policy:\n', vi1.policy)
```

```
Values:
 (2.766226988084275, 3.7438891127976524, 4.857502678650809, 6.12579511)
Policy:
 (1, 1, 1, 0)
```

This says that the optimum policy is to go Right in every state until reaching state 3, then Stay.

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Policy iteration algorithm recap

Policy iteration algorithm

- How to do POLICY-EVALUATION?
- Action in each state is fixed by the policy
  - At the i^{th} iteration, the policy π_i specifies the action $\pi_i(s)$ in state s

$$U_i(s) = \sum_{s'} P(s' \mid s, \pi_i(s))[R(s, \pi_i(s), s') + \gamma U_i(s')] .$$

- Basically a simplified version of Bellman eq. relating the utility of s (with π_i) to those of its neighbors
  - No "max" operator ⇒ linear equations

function POLICY-ITERATION(*mdp*) **returns** a policy
  inputs: *mdp*, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$
  local variables: U, a vector of utilities for states in S, initially zero
        π, a policy vector indexed by state, initially random

  repeat
    $U \leftarrow$ POLICY-EVALUATION(π, U, mdp)
    unchanged? $\leftarrow$ true
    for each state s **in** S **do**
      $a^* \leftarrow \underset{a \in A(s)}{\operatorname{argmax}}$ Q-VALUE(mdp, s, a, U)
      if Q-VALUE(mdp, s, a^*, U) $>$ Q-VALUE($mdp, s, \pi[s], U$) **then**
        $\pi[s] \leftarrow a^*$; *unchanged?* $\leftarrow$ false
  until *unchanged?*
  return π

Policy iteration algorithm via MDP toolbox

class `mdptoolbox.mdp.PolicyIteration`(*transitions, reward, discount, policy0=None, max_iter=1000, eval_type=0, skip_check=False*) [source]

Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the policy iteration algorithm.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **policy0** (*array, optional*) – Starting policy.
- **max_iter** (*int, optional*) – Maximum number of iterations. See the documentation for the `MDP` class for details. Default is 1000.
- **eval_type** (*int or string, optional*) – Type of function used to evaluate policy. 0 or "matrix" to solve as a set of linear equations. 1 or "iterative" to solve iteratively. Default: 0.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- ––––– –
- **V** (*tuple*) – value function
- **policy** (*tuple*) – optimal policy
- **iter** (*int*) – number of done iterations
- **time** (*float*) – used CPU time

Although we have been looking at the policy, we go it through value iteration.

Solving the same problem using policy iteration is easy with the MDP Toolbox:

```
# To run policy iteration we create a policy iteration object, and run it. Note that
# discount value is 0.9
pi1 = mdptoolbox.mdp.PolicyIteration(P1, R1, 0.9)
pi1.run()
```

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Exercise: Solving a simple MDP using the MDP toolbox

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Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the policy iteration algorithm.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **policy0** (*array, optional*) – Starting policy.
- **max_iter** (*int, optional*) – Maximum number of iterations. See the documentation for the `MDP` class for details. Default is 1000.
- **eval_type** (*int or string, optional*) – Type of function used to evaluate policy. 0 or "matrix" to solve as a set of linear equations. 1 or "iterative" to solve iteratively. Default: 0.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- ––––– –
- **V** (*tuple*) – value function
- **policy** (*tuple*) – optimal policy
- **iter** (*int*) – number of done iterations
- **time** (*float*) – used CPU time

Although we have been looking at the policy, we go it through value iteration.

Solving the same problem using policy iteration is easy with the MDP Toolbox:

```python
# We can then display the values (utilities) computed, and look at the policy:
print('Values:\n', pi1.V)
print('Policy:\n', pi1.policy)
```

```
Values:
 (6.6402692938291725, 7.6180844735276665, 8.731707317073173, 10.000000000000002)
Policy:
 (1, 1, 1, 0)
```

Note that the methods disagree on the value while agreeing on the policy.

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Q-Learning algorithm via MDP toolbox

class `mdptoolbox.mdp.QLearning`(*transitions, reward, discount, n_iter=10000, skip_check=False*) [source]

Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the Q learning algorithm.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **n_iter** (*int, optional*) – Number of iterations to execute. This is ignored unless it is an integer greater than the default value. Defaut: 10,000.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- ------ –
- **Q** (*array*) – learned Q matrix (SxA)
- **V** (*tuple*) – learned value function (S).
- **policy** (*tuple*) – learned optimal policy (S).
- **mean_discrepancy** (*array*) – Vector of V discrepancy mean over 100 iterations. Then the length of this vector for the default value of N is 100 (N/100).

Solving a problem using reinforcement learning (well, the **Q-learning** kind of Reinforcement Learning)

Action-utility function, or **Q-function**: $Q(s, a)$

- expected utility of taking a given action in a given state
- related to utilities in the obvious way →

is also easy using the MDP Toolbox:

```
# To run q-learning we create a q-learning object, and run it. Note that
# discount value is 0.9
ql1 = mdptoolbox.mdp.QLearning(P1, R1, 0.9)
ql1.run()
```

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Exercise: Solving a simple MDP using the MDP toolbox

class `mdptoolbox.mdp.QLearning`(*transitions, reward, discount, n_iter=10000, skip_check=False*) [source]

Bases: `mdptoolbox.mdp.MDP`

A discounted MDP solved using the Q learning algorithm.

Parameters:

- **transitions** (*array*) – Transition probability matrices. See the documentation for the `MDP` class for details.
- **reward** (*array*) – Reward matrices or vectors. See the documentation for the `MDP` class for details.
- **discount** (*float*) – Discount factor. See the documentation for the `MDP` class for details.
- **n_iter** (*int, optional*) – Number of iterations to execute. This is ignored unless it is an integer greater than the default value. Defaut: 10,000.
- **skip_check** (*bool*) – By default we run a check on the `transitions` and `rewards` arguments to make sure they describe a valid MDP. You can set this argument to True in order to skip this check.
- **Attributes** (*Data*) –
- ―――――― –
- **Q** (*array*) – learned Q matrix (SxA)
- **V** (*tuple*) – learned value function (S).
- **policy** (*tuple*) – learned optimal policy (S).
- **mean_discrepancy** (*array*) – Vector of V discrepancy mean over 100 iterations. Then the length of this vector for the default value of N is 100 (N/100).

Solving a problem using reinforcement learning (well, the **Q-learning** kind of Reinforcement Learning) is also easy using the MDP Toolbox:

```python
# We can then display the values (utilities) computed, and look at the policy:
print('Values:\n', ql1.V)
print('Policy:\n', ql1.policy)
```

```
Values:
 (0.26457661943006405, 2.0695643950241327, 6.421037194032352, 9.999999762119485)
Policy:
 (1, 1, 1, 0)
```

Note that the methods disagree on the value while agreeing on the policy.

https://pymdptoolbox.readthedocs.io/en/latest/api/mdp.html

Questions