1 分块解卷积中的数学公式推导

优化公式：

$$L(f)=\min_f \frac{\lambda}{2}\|Mask\odot(f-g)\|_2^2+\frac{\alpha}{4}\left(\|Mask_{xx}\odot f_{xx}\|_2^2+\|Mask_{yy}\odot f_{yy}\|_2^2+2\|Mask_{xy}\odot f_{xy}\|_2^2\right)$$

$\frac{\partial\|Mask\odot(f-g)\|_2^2}{\partial f}$

$$\frac{\partial\|Mask\odot(f-g)\|_2^2}{\partial f_{i,j}}=\frac{\sum_{m,n}Mask_{m,n}^2(f_{m,n}-g_{m,n})^2}{f_{i,j}}=2Mask_{m,n}^2\left(f_{m,n}-g_{m,n}\right)$$

于是

$$\frac{\partial\|Mask\odot(f-g)\|_2^2}{\partial f}=2Mask\odot Mask\odot(f-g)$$

$\frac{\partial\|Mask_{xx}\odot f_{xx}\|_2^2}{\partial f}$

$$\begin{aligned}\frac{\partial\|Mask\odot f_{xx}\|_2^2}{\partial f_{i,j}}&=\frac{\sum_{m,n}Mask_{m,n}^2\left(f_{m,n}^{(xx)}\right)^2}{\partial f_{i,j}}\\&=\frac{\sum_{m,n}Mask_{m,n}^2(f_{i,j+1}+f_{\mathrm{i,j-1}}-2f_{\mathrm{i,j}})^2}{\partial f_{i,j}}\\&=-4Mask_{i,j}^2\left(f_{i,j+1}+f_{\mathrm{i,j-1}}-2f_{\mathrm{i,j}}\right)\end{aligned}$$

于是

$$\frac{\partial\|Mask_{xx}\odot f_{xx}\|_2^2}{\partial f}=-4Mask_{xx}\odot Mask_{xx}\nabla_{xx}f$$

其中，$\nabla_{xx}=[1,-2,1]$，为二阶差分算子。

同理，

$$\frac{\partial\|Mask_{yy}\odot f_{yy}\|_2^2}{\partial f}=-4Mask_{yy}\odot Mask_{yy}\nabla_{yy}f$$

其中，$\nabla_{yy}=[1;-2;1]$。

$$\begin{aligned}\frac{\partial\|Mask_{xy}\odot f_{xx}\|_2^2}{\partial f_{i,j}}&=\frac{\sum_{m,n}Mask_{m,n}^2\left(f_{m,n}^{(xy)}\right)^2}{\partial f_{i,j}}\\&=\frac{\sum_{m,n}Mask_{m,n}^2((f_{m,n}-f_{m,n+1})-(f_{m+1,n}-f_{m+1,n+1}))^2}{\partial f_{i,j}}\\&=2Mask_{i,j}^2\left(f_{i,j}+f_{\mathrm{i+1,j+1}}-f_{\mathrm{i,j+1}}-f_{i+1,j}\right)\end{aligned}$$

于是

$$\frac{\partial\|Mask_{xy}\odot f_{xy}\|_2^2}{\partial f}=2Mask\odot Mask\nabla_{xy}f$$

其中，$\nabla_{xy}=[1,-1;-1,1]$

:

$$\frac{\partial L\left(f\right)}{\partial f} = \lambda Mask \odot Mask \odot \left(f - g\right) + \alpha\left(-Mask_{xx} \odot Mask_{xx} \nabla_{xx} f - Mask_{yy} \odot Mask_{yy} \nabla_{yy} f + Mask \odot Mask \nabla_{xy} f\right)$$

g:原图

f0:alpha_blending