Machine Learning
(機器學習)

Lecture 12: Bagging and Boosting

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Roadmap

1. When Can Machines Learn?
2. Why Can Machines Learn?
3. How Can Machines Learn?
4. How Can Machines Learn Better?
5. Embedding Numerous Features: Kernel Models
6. Combining Predictive Features: Aggregation Models

Lecture 12: Bagging and Boosting

- Uniform Blending
- Linear and Any Blending
- Bagging (Bootstrap Aggregation)
- Motivation of Boosting
- Diversity by Re-weighting
- Adaptive Boosting Algorithm
- Adaptive Boosting in Action

An Aggregation Story

Your T friends $g_1, \cdots, g_T$ predicts whether stock will go up as $g_t(\mathbf{x})$.

You can . . .

- **select** the most trust-worthy friend from their **usual performance**
  —**validation!**
- **mix** the predictions from all your friends **uniformly**
  —let them **vote!**
- **mix** the predictions from all your friends **non-uniformly**
  —let them vote, but **give some more ballots**
- **combine** the predictions **conditionally**
  —if **[t satisfies some condition]** give some ballots to friend t
- ...

aggregation models: **mix** or **combine**
hypotheses (for better performance)

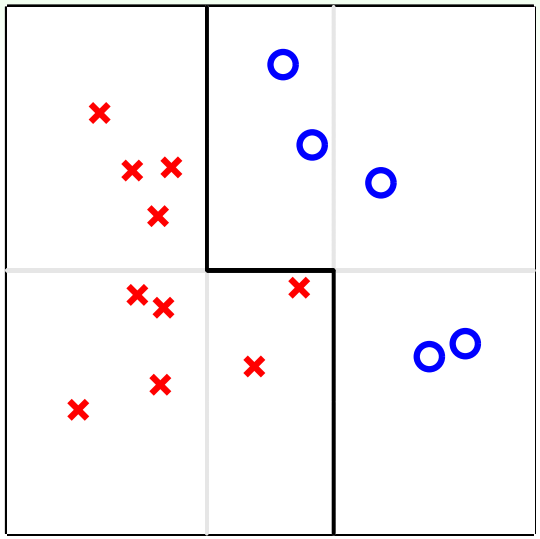
Aggregation with Math Notations

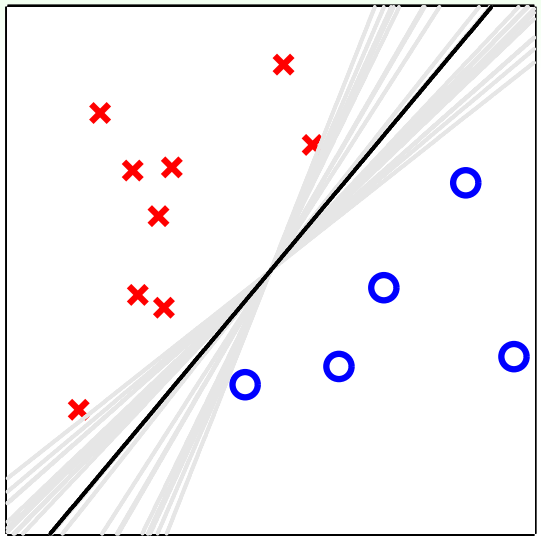
Your T friends $g_1, \cdots, g_T$ predicts whether stock will go up as $g_t(\mathbf{x})$.

- **select** the most trust-worthy friend from their **usual performance**
$$G(\mathbf{x}) = g_{t_*}(\mathbf{x}) \text{ with } t_* = \text{argmin}_{t \in \{1,2,\cdots,T\}} E_{\text{val}}(g_t^-)$$
- **mix** the predictions from all your friends **uniformly**
$$G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} 1 \cdot g_t(\mathbf{x})\right)$$
- **mix** the predictions from all your friends **non-uniformly**
$$G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t \cdot g_t(\mathbf{x})\right) \text{ with } \alpha_t \geq 0$$
  - include **select**: $\alpha_t = [\![E_{\text{val}}(g_t^-) \text{ smallest}]\!]$
  - include **uniformly**: $\alpha_t = 1$
- **combine** the predictions **conditionally**
$$G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} q_t(\mathbf{x}) \cdot g_t(\mathbf{x})\right) \text{ with } q_t(\mathbf{x}) \geq 0$$
  - include **non-uniformly**: $q_t(\mathbf{x}) = \alpha_t$

aggregation models: a **rich family**

Why Might Aggregation Work?





- mix **different weak hypotheses** uniformly —$G(\mathbf{x})$ 'strong'
- aggregation $\Longrightarrow$ **feature transform (?)**

- mix **different random-PLA hypotheses** uniformly —$G(\mathbf{x})$ 'moderate'
- aggregation $\Longrightarrow$ **regularization (?)**

proper aggregation $\Longrightarrow$ **better performance**

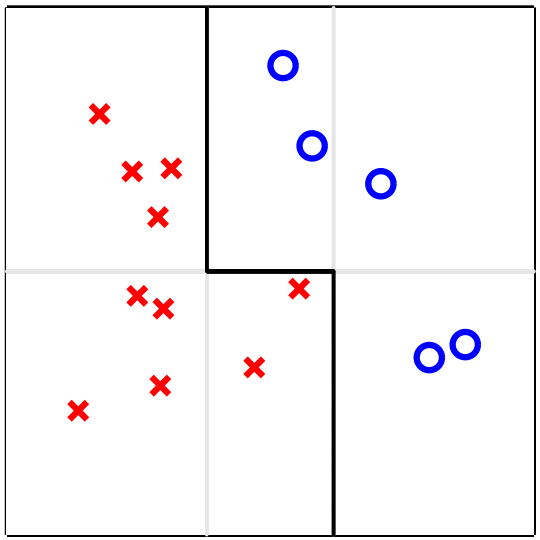
Uniform Blending (Voting) for Classification

uniform blending: known g_t, each with 1 ballot

$$G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} 1 \cdot g_t(\mathbf{x})\right)$$

- same g_t (autocracy): as good as one single g_t
- very different g_t (**diversity** + **democracy**): majority can **correct** minority
- similar results with uniform voting for multiclass

$$G(\mathbf{x}) = \underset{1 \le k \le K}{\text{argmax}} \sum_{t=1}^{T} [\![g_t(\mathbf{x}) = k]\!]$$



how about **regression**?

Uniform Blending for Regression

$$G(\mathbf{x}) = \frac{1}{T}\sum_{t=1}^{T} g_t(\mathbf{x})$$

- same g_t (autocracy):
  as good as one single g_t
- very different g_t (**diversity** + **democracy**):
  some $g_t(\mathbf{x}) > f(\mathbf{x})$, some $g_t(\mathbf{x}) < f(\mathbf{x})$
  $\Longrightarrow$ average **could be** more accurate than individual

diverse hypotheses:

even simple uniform blending
can be better than any single hypothesis

Theoretical Analysis of Uniform Blending

$$G(\mathbf{x}) = \frac{1}{T}\sum_{t=1}^{T} g_t(\mathbf{x})$$

$$\begin{aligned}
\text{avg}\left((g_t(\mathbf{x}) - f(\mathbf{x}))^2\right) &= \text{avg}\left(g_t^2 - 2g_t f + f^2\right) \\
&= \text{avg}\left(g_t^2\right) - 2Gf + f^2 \\
&= \text{avg}\left(g_t^2\right) - G^2 + (G - f)^2 \\
&= \text{avg}\left(g_t^2\right) - 2G^2 + G^2 + (G - f)^2 \\
&= \text{avg}\left(g_t^2 - 2g_t G + G^2\right) + (G - f)^2 \\
&= \text{avg}\left((g_t - G)^2\right) + (G - f)^2
\end{aligned}$$

$$\begin{aligned}
\text{avg}\left(E_{\text{out}}(g_t)\right) &= \text{avg}\left(\mathcal{E}(g_t - G)^2\right) + E_{\text{out}}(G) \\
&\geq \qquad\qquad\qquad + E_{\text{out}}(G)
\end{aligned}$$

Questions?

Linear Blending

linear blending: known g_t, each to be given α_t ballot

$$G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t \cdot g_t(\mathbf{x})\right) \text{ with } \alpha_t \geq 0$$

computing 'good' α_t : $\min_{\alpha_t \geq 0} E_{\text{in}}(\boldsymbol{\alpha})$

linear blending for regression

$$\min_{\alpha_t \geq 0} \frac{1}{N} \sum_{n=1}^{N} \left(y_n - \sum_{t=1}^{T} \alpha_t g_t(\mathbf{x}_n)\right)^2$$

LinReg + transformation

$$\min_{w_i} \frac{1}{N} \sum_{n=1}^{N} \left(y_n - \sum_{i=1}^{\tilde{d}} w_i \phi_i(\mathbf{x}_n)\right)^2$$

linear blending = LinModel + hypotheses as transform + constraints

Constraint on α_t

linear blending = LinModel + hypotheses as transform + constraints:

$$\min_{\alpha_t \geq 0} \quad \frac{1}{N} \sum_{n=1}^{N} \text{err}\left(y_n, \sum_{t=1}^{T} \alpha_t g_t(\mathbf{x}_n)\right)$$

linear blending for binary classification

$$\text{if } \alpha_t < 0 \implies \alpha_t g_t(\mathbf{x}) = |\alpha_t| \left(-g_t(\mathbf{x})\right)$$

- negative α_t for $g_t \equiv$ positive $|\alpha_t|$ for $-g_t$
- **if you have a stock up/down classifier with 99% error, tell me! :-)**

in practice, often
linear blending = LinModel + hypotheses as transform ~~+ constraints~~

Linear Blending versus Selection

in practice, often

$$g_1 \in \mathcal{H}_1, g_2 \in \mathcal{H}_2, \ldots, g_T \in \mathcal{H}_T$$

by minimum E_{in}

- recall: **selection by minimum** E_{in}
  —best of best, paying $d_{\text{VC}}\left(\bigcup_{t=1}^{T} \mathcal{H}_t\right)$
- recall: linear blending includes selection as special case
  —by setting $\alpha_t = [\![E_{\text{val}}(g_t^-) \text{ smallest}]\!]$
- complexity price of linear blending with E_{in} (aggregation of best):
  $\geq d_{\text{VC}}\left(\bigcup_{t=1}^{T} \mathcal{H}_t\right)$

like selection, blending practically done with
(E_{val} instead of E_{in}) + (g_t^- from minimum E_{train})

Any Blending

Given $g_1^-, g_2^-, \ldots, g_T^-$ from $\mathcal{D}_{\text{train}}$, transform $(\mathbf{x}_n, y_n)$ in $\mathcal{D}_{\text{val}}$ to $(\mathbf{z}_n = \Phi^-(\mathbf{x}_n), y_n)$, where $\Phi^-(\mathbf{x}) = (g_1^-(\mathbf{x}), \ldots, g_T^-(\mathbf{x}))$

Linear Blending

1. compute α
   $= \text{LinearModel}\Big(\{(\mathbf{z}_n, y_n)\}\Big)$
2. return $G_{\text{LINB}}(\mathbf{x}) =$
   $\text{LinearHypothesis}_\alpha\,(\Phi(\mathbf{x}))$,

Any Blending (Stacking)

1. compute $\tilde{g}$
   $= \textbf{AnyModel}\Big(\{(\mathbf{z}_n, y_n)\}\Big)$
2. return $G_{\text{ANYB}}(\mathbf{x}) = \tilde{g}(\Phi(\mathbf{x}))$,

where $\Phi(\mathbf{x}) = (g_1(\mathbf{x}), \ldots, g_T(\mathbf{x}))$

any blending:

- **powerful**, achieves conditional blending
- but **danger of overfitting**, as always **:-(**

Blending in Practice

(Chen et al., A linear ensemble of individual and blended models for music rating prediction, 2012)

KDDCup 2011 Track 1: World Champion Solution by NTU

- validation set blending: a special any blending model

  E_{test} (squared): 519.45 $\Longrightarrow$ 456.24

  —helped **secure the lead** in last two weeks

- test set blending: linear blending using $\tilde{E}_{\text{test}}$

  E_{test} (squared): 456.24 $\Longrightarrow$ 442.06

  —helped **turn the tables** in last hour

blending 'useful' in practice,
despite the computational burden

Questions?

What We Have Done

blending: aggregate after getting g_t;
learning: aggregate as well as getting g_t

aggregation type	blending	learning
uniform	voting/averaging	?
non-uniform	linear	?
conditional	stacking	?

learning g_t for uniform aggregation: diversity important

- diversity by different models: $g_1 \in \mathcal{H}_1, g_2 \in \mathcal{H}_2, \ldots, g_T \in \mathcal{H}_T$
- diversity by different parameters: GD with $\eta = 0.001, 0.01, \ldots, 10$
- diversity by algorithmic randomness:
  random PLA with different random seeds
- diversity by data randomness:
  within-cross-validation hypotheses g_v^-

next: diversity by data randomness without g^-

Bootstrap Aggregation

bootstrapping

bootstrap sample $\tilde{\mathcal{D}}_t$: re-sample N examples from $\mathcal{D}$ **uniformly with replacement**—can also use arbitrary N' instead of original N

- $\tilde{\mathcal{D}}_t \neq \mathcal{D}$ in general (diverse)
- $\tilde{\mathcal{D}}_t$ 'acts like' $\mathcal{D}$ statistically (g_t 'acts like' good g)
- each g_t similarly good/bad (uniform)

bootstrap aggregation

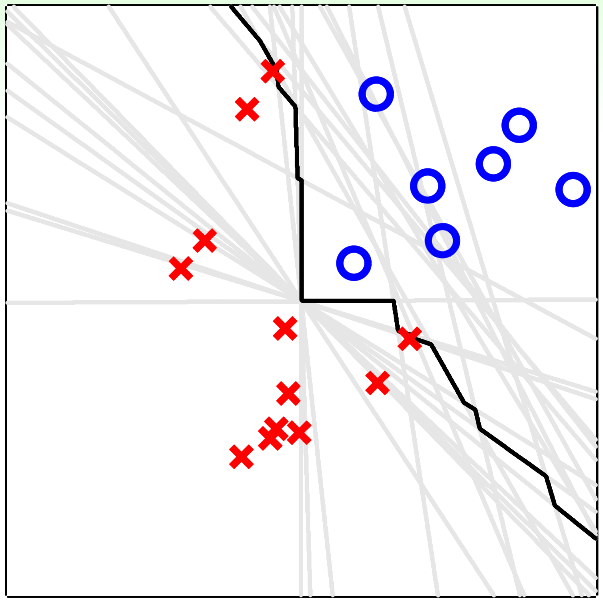
iterative process that for $t = 1, 2, \ldots, T$

1. request size-N' data $\tilde{\mathcal{D}}_t$ from bootstrapping
2. obtain g_t by $\mathcal{A}(\tilde{\mathcal{D}}_t)$

$$G = \text{Uniform}(\{g_t\})$$

bootstrap aggregation (BAGging):
a simple **meta algorithm**

Bagging on Top of (Modified) PLA in Action



$T_{\text{BAG}} = 25$

- very diverse g_t from bagging
- proper **non-linear** boundary after aggregating binary classifiers

bagging works reasonably well **if base algorithm sensitive to data randomness**

Questions?

Apple Recognition Problem

- is this a picture of an apple?
- say, want to teach a class of **6 year olds**
- gather photos under CC-BY-2.0 license on Flicker (**thanks to the authors below!**)

(APAL stands for Apple and Pear Australia Ltd)

Dan Foy
https://flic.kr/p/jNQ55

APAL
https://flic.kr/p/jzP1VB

adrianbartel
https://flic.kr/p/bdy2hZ

ANdrzej cH.
https://flic.kr/p/51DKA8

Stuart Webster
https://flic.kr/p/9C3Ybd

nachans
https://flic.kr/p/9XD7Ag

APAL
https://flic.kr/p/jzRe4u

Jo Jakeman
https://flic.kr/p/7jwtGp

APAL
https://flic.kr/p/jzPYNr

APAL
https://flic.kr/p/jzScif

Apple Recognition Problem

- is this a picture of an apple?
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Mr. Roboto.

https://flic.kr/p/i5BN85

Richard North

https://flic.kr/p/bHhPkB

Richard North

https://flic.kr/p/d8tGou

Emilian Robert Vicol

https://flic.kr/p/bpmGXW

Nathaniel McQueen

https://flic.kr/p/pZv1Mf

Crystal

https://flic.kr/p/kaPYp

jfh686

https://flic.kr/p/6vjRFH

skyseeker

https://flic.kr/p/2MynV

Janet Hudson

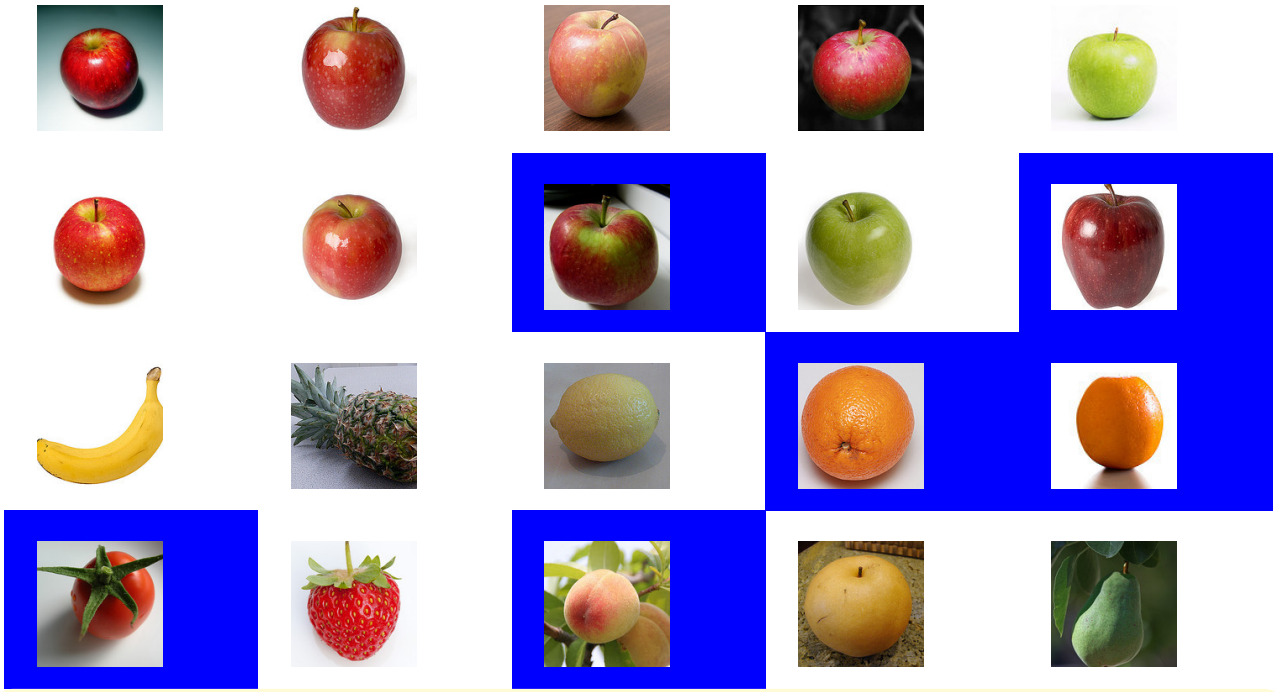
https://flic.kr/p/7QDBbm

Rennett Stowe

https://flic.kr/p/agmnrk

Our Fruit Class Begins

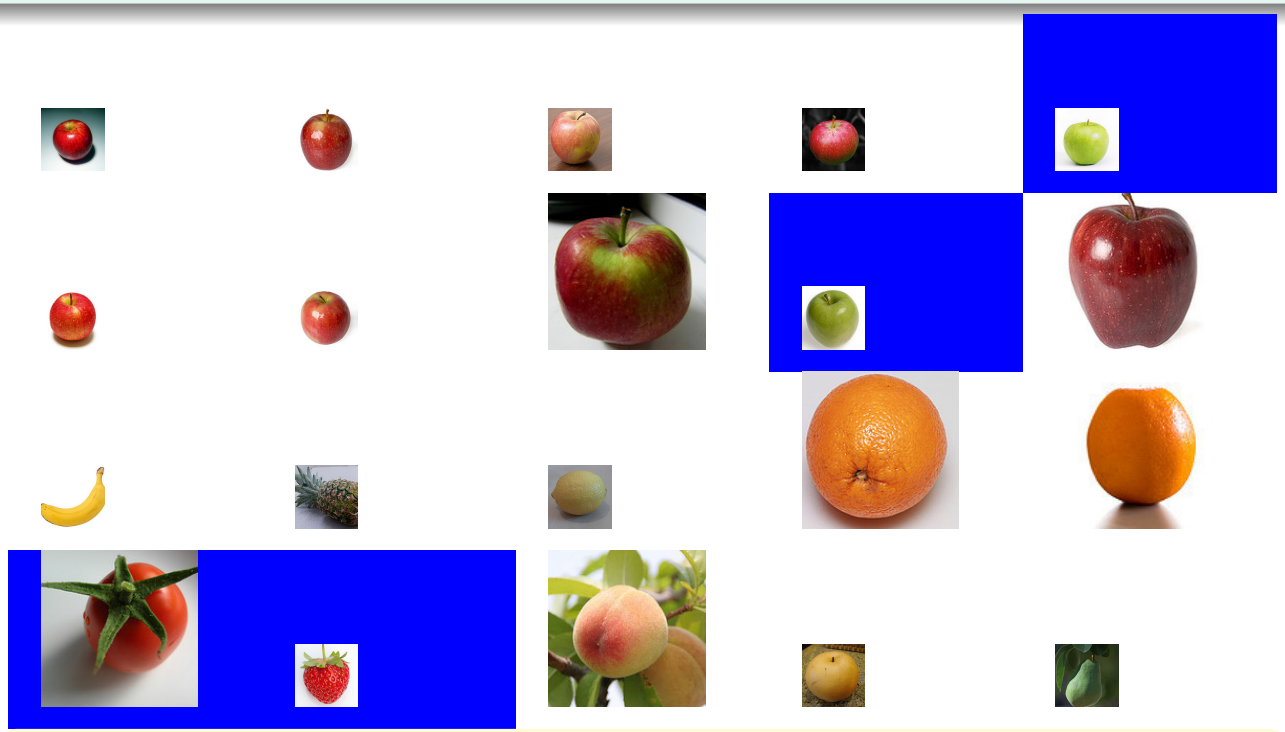
- Teacher: Please look at the pictures of apples and non-apples below. Based on those pictures, how would you describe an apple? Michael?
- Michael: I think apples are **circular**.



(Class): Apples are **circular**.

Our Fruit Class Continues

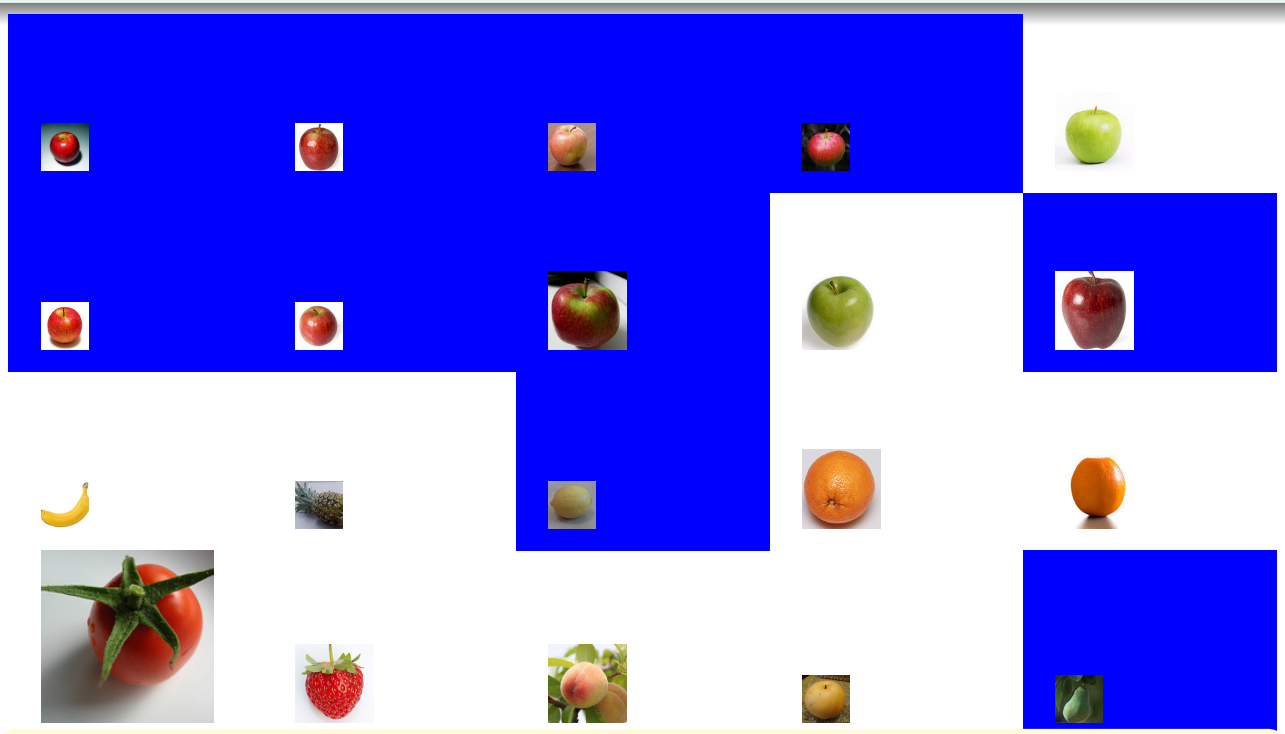
- Teacher: Being circular is a good feature for the apples. However, if you only say circular, you could make several mistakes. What else can we say for an apple? Tina?
- Tina: It looks like apples are **red**.



(Class): Apples are somewhat **circular** and somewhat **red**.

Our Fruit Class Continues More

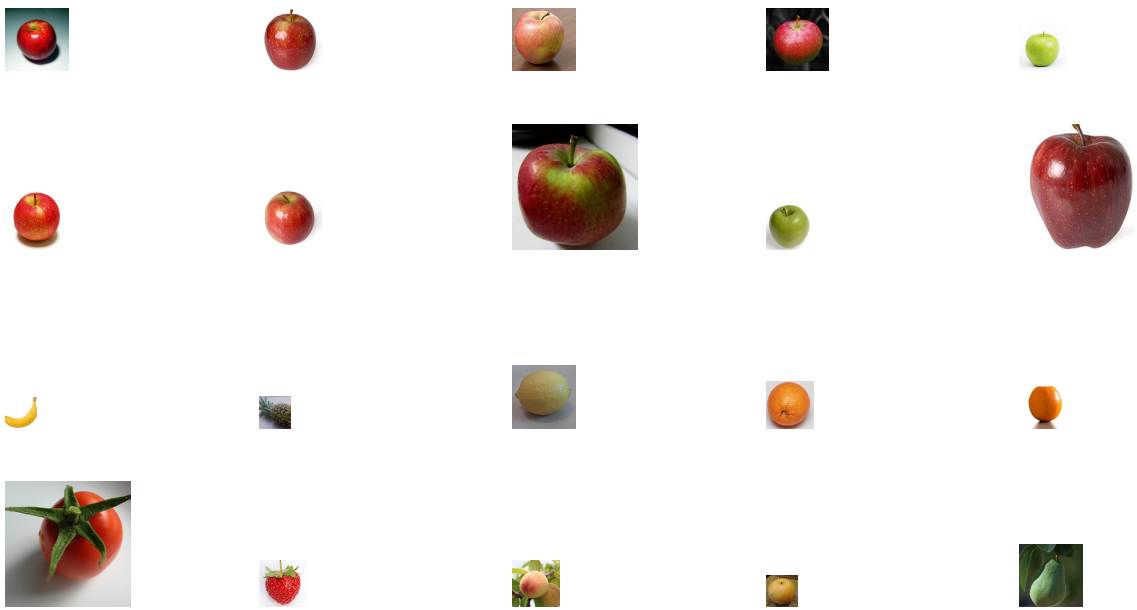
- Teacher: Yes. Many apples are red. However, you could still make mistakes based on circular and red. Do you have any other suggestions, Joey?
- Joey: Apples could also be **green**.



(Class): Apples are somewhat **circular** and somewhat **red** and possibly **green**.

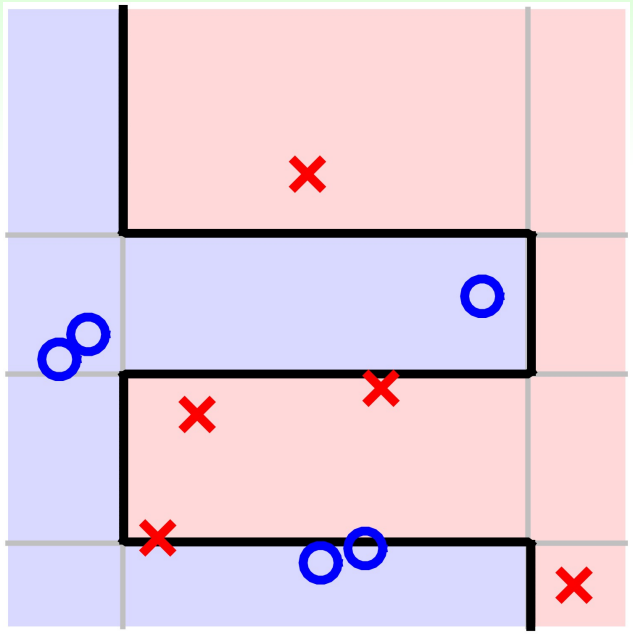
Our Fruit Class Ends

- Teacher: Yes. It seems that apples might be circular, red, green. But you may confuse them with tomatoes or peaches, right? Any more suggestions, Jessica?
- Jessica: Apples have **stems** at the top.



(Class): Apples are somewhat **circular**, somewhat **red**, possibly **green**, and may have **stems** at the top.

Motivation



- students: simple hypotheses g_t (like vertical/horizontal lines)
- (Class): sophisticated hypothesis G (like black curve)
- Teacher: a tactic learning algorithm that **directs the students to focus on key examples**

next: the '**math**' of such an algorithm

Questions?

Bootstrapping as Re-weighting Process

$$\mathcal{D} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), (\mathbf{x}_3, y_3), (\mathbf{x}_4, y_4)\}$$

$$\xrightarrow{\text{bootstrap}} \tilde{\mathcal{D}}_t = \{(\mathbf{x}_1, y_1), (\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), (\mathbf{x}_4, y_4)\}$$

weighted E_{in} on $\mathcal{D}$

$$E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{4}\sum_{n=1}^{4} u_n^{(t)} \cdot [\![y_n \neq h(\mathbf{x}_n)]\!]$$

$(\mathbf{x}_1, y_1)$, $u_1 = 2$
$(\mathbf{x}_2, y_2)$, $u_2 = 1$
$(\mathbf{x}_3, y_3)$, $u_3 = 0$
$(\mathbf{x}_4, y_4)$, $u_4 = 1$

E_{in} on $\tilde{D}_t$

$$E_{\text{in}}^{0/1}(h) = \frac{1}{4}\sum_{(\mathbf{x},y)\in\tilde{D}_t} [\![y \neq h(\mathbf{x})]\!]$$

$(\mathbf{x}_1, y_1)$, $(\mathbf{x}_1, y_1)$
$(\mathbf{x}_2, y_2)$

$(\mathbf{x}_4, y_4)$

each diverse g_t in bagging:
by minimizing bootstrap-weighted error

Weighted Base Algorithm

minimize (regularized)

$$E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{N}\sum_{n=1}^{N} u_n \cdot \text{err}(y_n, h(\mathbf{x}_n))$$

e.g. logistic regression

$E_{\text{in}}^{\mathbf{u}} \propto \sum_{n=1}^{N} u_n \text{err}_{\text{CE}}$ by SGD

$\Leftrightarrow$ sample $(\mathbf{x}_n, y_n)$ with probability proportional to u_n

example-weighted learning:
modified error function, optimized by **sampling**/re-derivation/...

Re-weighting for More Diverse Hypothesis

'improving' bagging for binary classification:
how to re-weight for **more diverse hypotheses**?

$$g_t \leftarrow \operatorname*{argmin}_{h\in\mathcal{H}} \left(\sum_{n=1}^{N} u_n^{(t)} [\![y_n \neq h(\mathbf{x}_n)]\!] \right)$$

$$g_{t+1} \leftarrow \operatorname*{argmin}_{h\in\mathcal{H}} \left(\sum_{n=1}^{N} u_n^{(t+1)} [\![y_n \neq h(\mathbf{x}_n)]\!] \right)$$

if g_t '**not good**' for $\mathbf{u}^{(t+1)} \implies g_t$-like hypotheses not returned as g_{t+1}
$\implies g_{t+1}$ diverse from g_t

idea: **construct $\mathbf{u}^{(t+1)}$** to make g_t **random-like**

$$\frac{\sum_{n=1}^{N} u_n^{(t+1)} [\![y_n \neq g_t(\mathbf{x}_n)]\!]}{\sum_{n=1}^{N} u_n^{(t+1)}} = \frac{1}{2}$$

'Optimal' Re-weighting

want: $\frac{\sum_{n=1}^{N} u_n^{(t+1)} [\![y_n \neq g_t(\mathbf{x}_n)]\!]}{\sum_{n=1}^{N} u_n^{(t+1)}} = \frac{\blacksquare_{t+1}}{\blacksquare_{t+1} + \bullet_{t+1}} = \frac{1}{2}$, where

$$\blacksquare_{t+1} = \sum_{n=1}^{N} u_n^{(t+1)} [\![y_n \neq g_t(\mathbf{x}_n)]\!], \bullet_{t+1} = \sum_{n=1}^{N} u_n^{(t+1)} [\![y_n = g_t(\mathbf{x}_n)]\!]$$

- need: $\underbrace{(\text{total } u_n^{(t+1)} \text{ of incorrect})}_{\blacksquare_{t+1}} = \underbrace{(\text{total } u_n^{(t+1)} \text{ of correct})}_{\bullet_{t+1}}$
- one possibility by **re-scaling (multiplying) weights**, if

(total $u_n^{(t)}$ of incorrect) = 1126 ; (weighted incorrect rate) = $\frac{1126}{7337}$	(total $u_n^{(t)}$ of correct) = 6211 ; (weighted correct rate) = $\frac{6211}{7337}$
incorrect: $u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot 6211$	correct: $u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot 1126$

'optimal' re-weighting under weighted incorrect rate ϵ_t:
multiply incorrect $\propto (1 - \epsilon_t)$; multiply correct $\propto \epsilon_t$

Questions?

Scaling Factor

'optimal' re-weighting: let $\epsilon_t = \frac{\sum_{n=1}^{N} u_n^{(t)} [\![y_n \neq g_t(\mathbf{x}_n)]\!]}{\sum_{n=1}^{N} u_n^{(t)}}$,

multiply incorrect $\propto (1 - \epsilon_t)$; multiply correct $\propto \epsilon_t$

define scaling factor $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$

$$\begin{array}{rcrcl} \text{incorrect} & \leftarrow & \text{incorrect} & \cdot & \blacklozenge_t \\ \text{correct} & \leftarrow & \text{correct} & / & \blacklozenge_t \end{array}$$

- **equivalent** to optimal re-weighting
- $\blacklozenge_t \geq 1$ iff $\epsilon_t \leq \frac{1}{2}$
  —physical meaning: **scale up incorrect**; **scale down correct**
  —like what Teacher does

scaling-up incorrect examples
leads to **diverse hypotheses**

A Preliminary Algorithm

$\mathbf{u}^{(1)} = ?$

for $t = 1, 2, \ldots, T$

1. obtain g_t by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$,
   where $\mathcal{A}$ tries to minimize $\mathbf{u}^{(t)}$-weighted 0/1 error
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$,
   where ϵ_t = weighted error (incorrect) rate of g_t

return $G(\mathbf{x}) = ?$

- want g_1 'best' for E_{in}: $u_n^{(1)} = \frac{1}{N}$
- $G(\mathbf{x})$:
  - uniform? but g_2 very bad for E_{in} (**why? :-)**)
  - linear, non-linear? **as you wish**

next: a special algorithm to aggregate
linearly on the fly with theoretical guarantee

Linear Aggregation on the Fly

$\mathbf{u}^{(1)} = [\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}]$
for $t = 1, 2, \ldots, T$

1. obtain g_t by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$, where ...
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$, where ...
3. compute $\alpha_t = \ln(\blacklozenge_t)$

return $G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t g_t(\mathbf{x})\right)$

- wish: large α_t for 'good' $g_t \Longleftarrow \alpha_t = \text{monotonic}(\blacklozenge_t)$
- will take $\alpha_t = \ln(\blacklozenge_t)$
  - $\epsilon_t = \frac{1}{2} \Longrightarrow \blacklozenge_t = 1 \Longrightarrow \alpha_t = 0$ (bad g_t zero weight)
  - $\epsilon_t = 0 \Longrightarrow \blacklozenge_t = \infty \Longrightarrow \alpha_t = \infty$ (super g_t superior weight)

Adaptive Boosting = weak base learning algorithm $\mathcal{A}$ (Student)
+ optimal re-weighting factor $\blacklozenge_t$ (Teacher)
+ 'magic' linear aggregation α_t (Class)

Adaptive Boosting (AdaBoost) Algorithm

$\mathbf{u}^{(1)} = [\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}]$
for $t = 1, 2, \ldots, T$

1. obtain g_t by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$,
   where $\mathcal{A}$ tries to minimize $\mathbf{u}^{(t)}$-weighted 0/1 error
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by

$$[\![y_n \neq g_t(\mathbf{x}_n)]\!] \text{ (incorrect examples)}: \quad u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot \blacklozenge_t$$
$$[\![y_n = g_t(\mathbf{x}_n)]\!] \text{ (correct examples)}: \quad u_n^{(t+1)} \leftarrow u_n^{(t)} / \blacklozenge_t$$

   where $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$ and $\epsilon_t = \frac{\sum_{n=1}^{N} u_n^{(t)} [\![y_n \neq g_t(\mathbf{x}_n)]\!]}{\sum_{n=1}^{N} u_n^{(t)}}$

3. compute $\alpha_t = \ln(\blacklozenge_t)$

return $G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t g_t(\mathbf{x})\right)$

AdaBoost: provable **boosting property**

Theoretical Guarantee of AdaBoost

- From VC bound

$$E_{\text{out}}(G) \leq E_{\text{in}}(G) + O\left(\sqrt{\underbrace{O(d_{\text{VC}}(\mathcal{H}) \cdot T \log T)}_{d_{\text{VC}} \text{ of all possible } G} \cdot \frac{\log N}{N}}\right)$$

- **first term can be small**:
  $E_{\text{in}}(G) = 0$ after $T = O(\log N)$ iterations if $\epsilon_t \leq \epsilon < \frac{1}{2}$ always
- **second term can be small**:
  overall d_{VC} grows "slowly" with T

boosting view of AdaBoost:

if $\mathcal{A}$ is weak but always **slightly better than random** $(\epsilon_t \leq \epsilon < \frac{1}{2})$,
then (AdaBoost+$\mathcal{A}$) can be strong ($E_{\text{in}} = 0$ and E_{out} small)

Questions?

Decision Stump

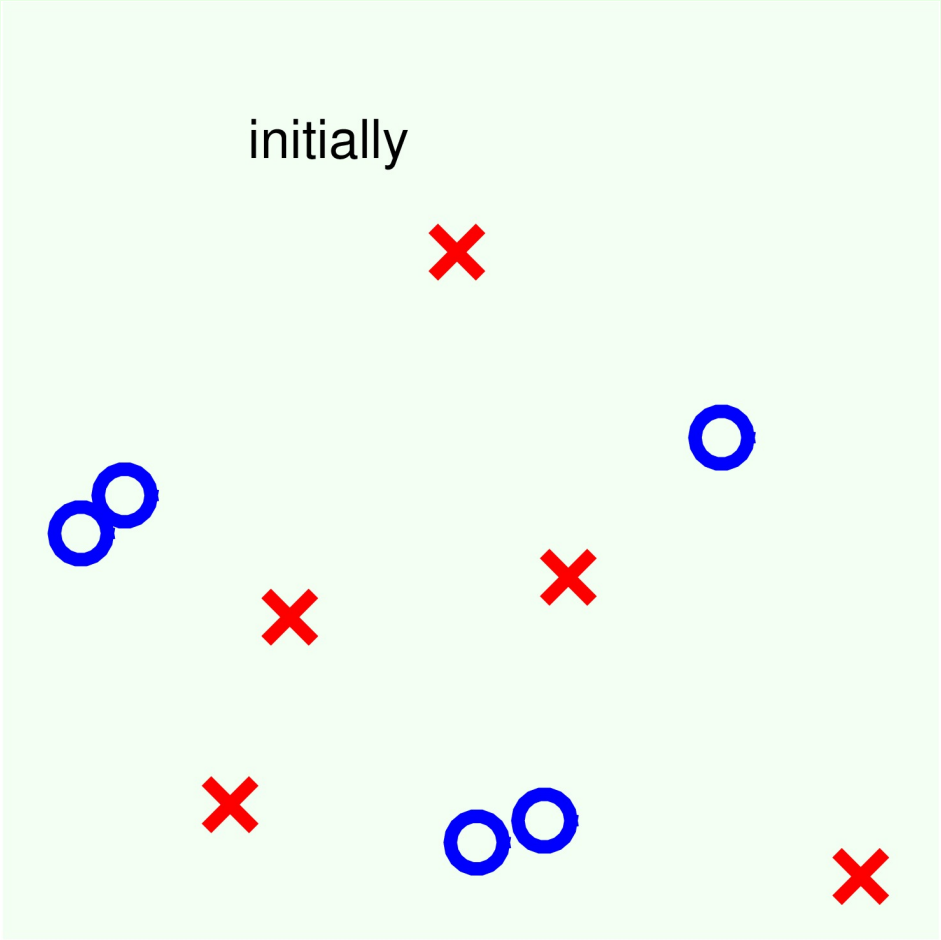
want: a '**weak**' base learning algorithm $\mathcal{A}$
that minimizes $E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{N}\sum_{n=1}^{N} u_n \cdot [\![y_n \neq h(\mathbf{x}_n)]\!]$ **a little bit**

a popular choice: decision stump

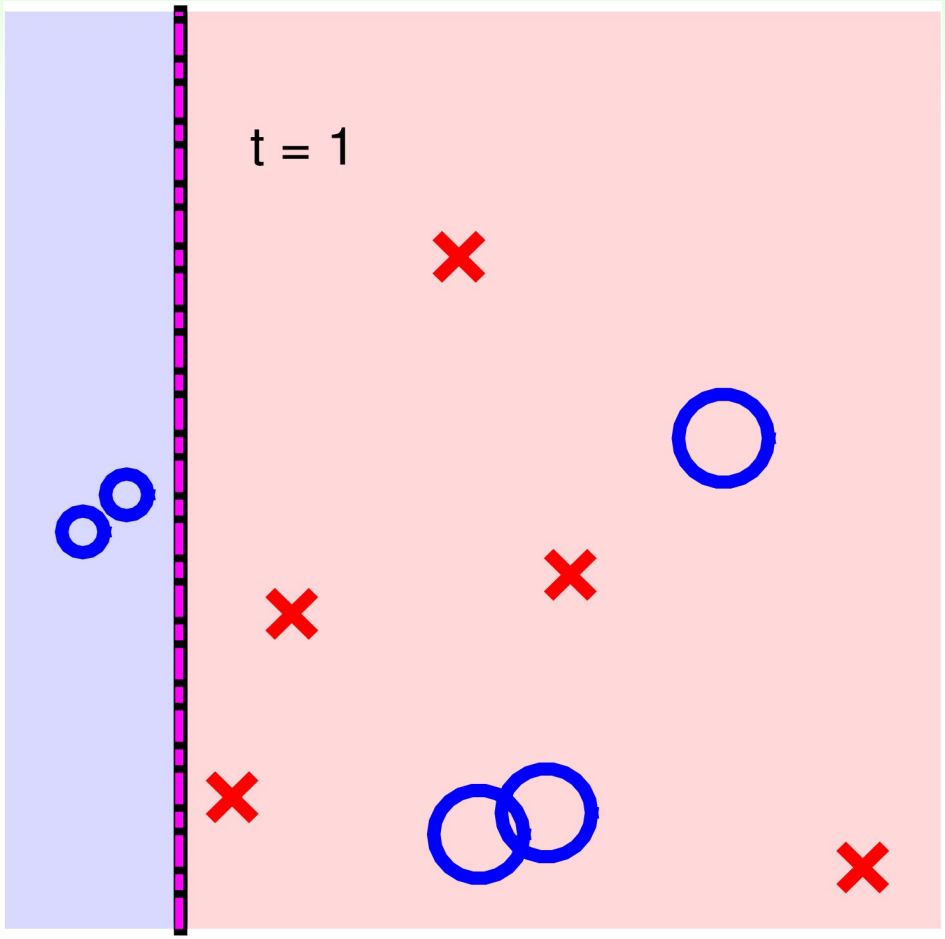
- $$h_{s,i,\theta}(\mathbf{x}) = s \cdot \text{sign}(x_i - \theta)$$
- positive and negative rays on some feature: three parameters (feature i, threshold θ, direction s)
- physical meaning: vertical/horizontal lines in 2D
- efficient to optimize: $O(d \cdot N \log N)$ time

decision stump model:
allows efficient minimization of $E_{\text{in}}^{\mathbf{u}}$
but perhaps **too weak to work by itself**

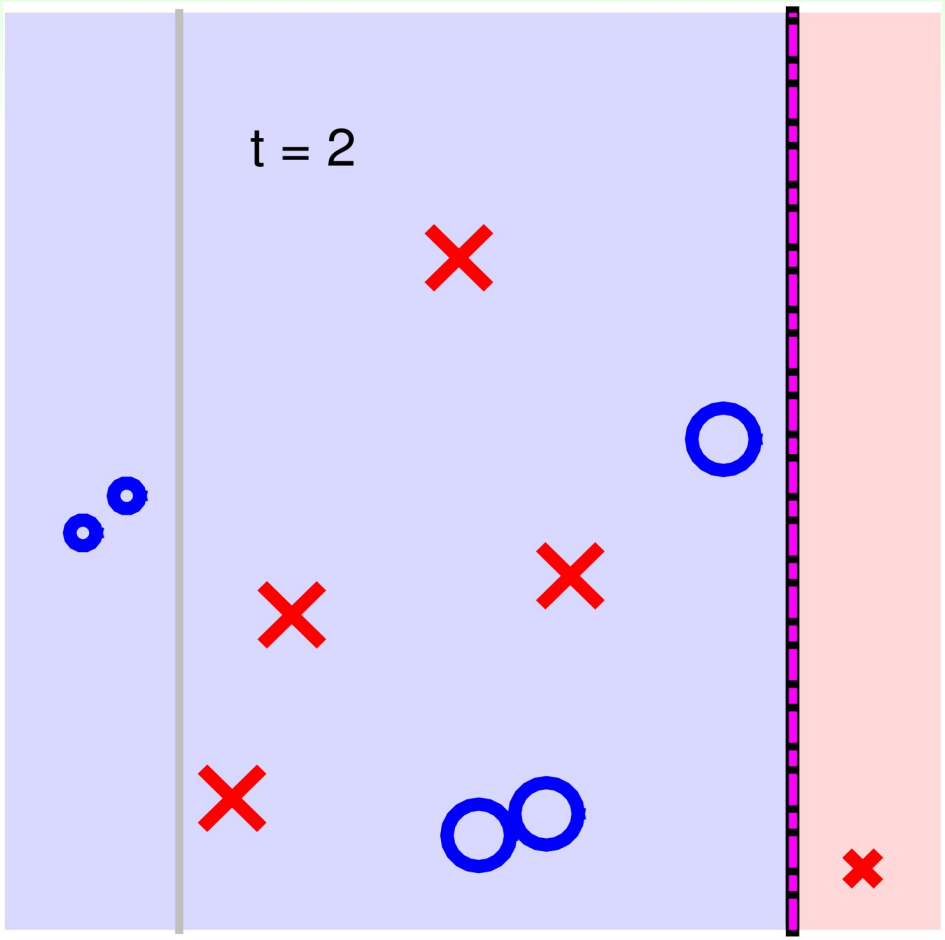
A Simple Data Set



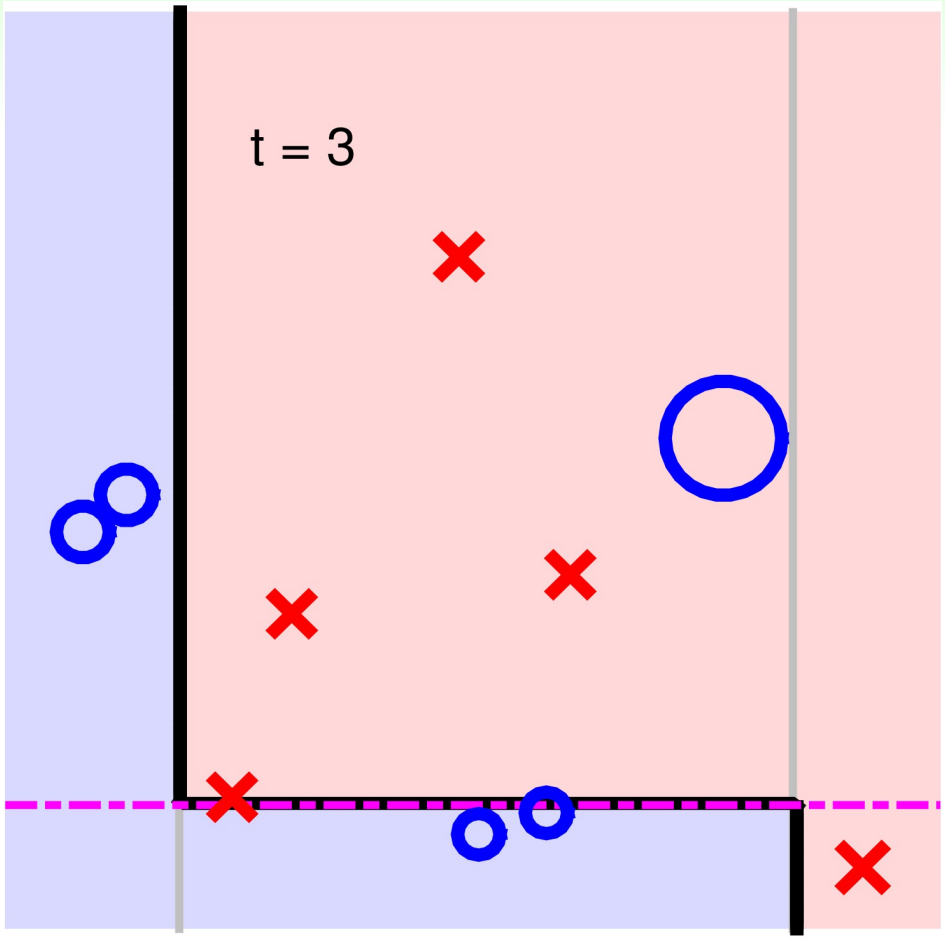
A Simple Data Set



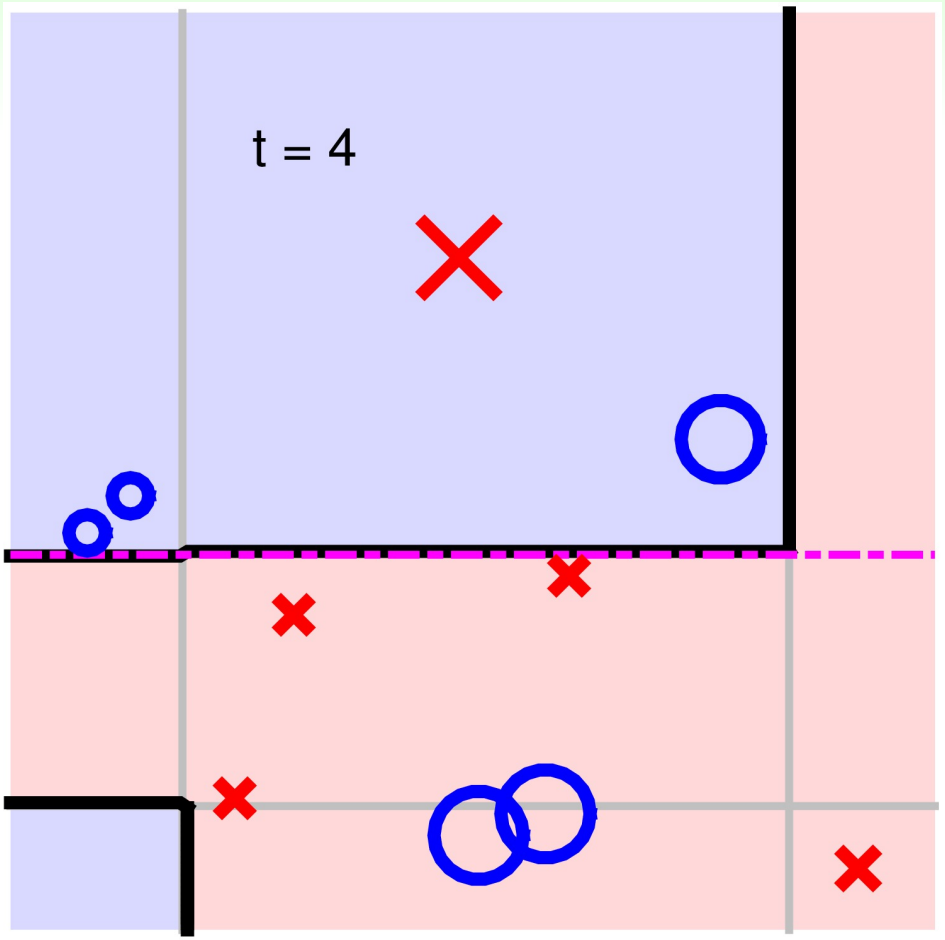
A Simple Data Set



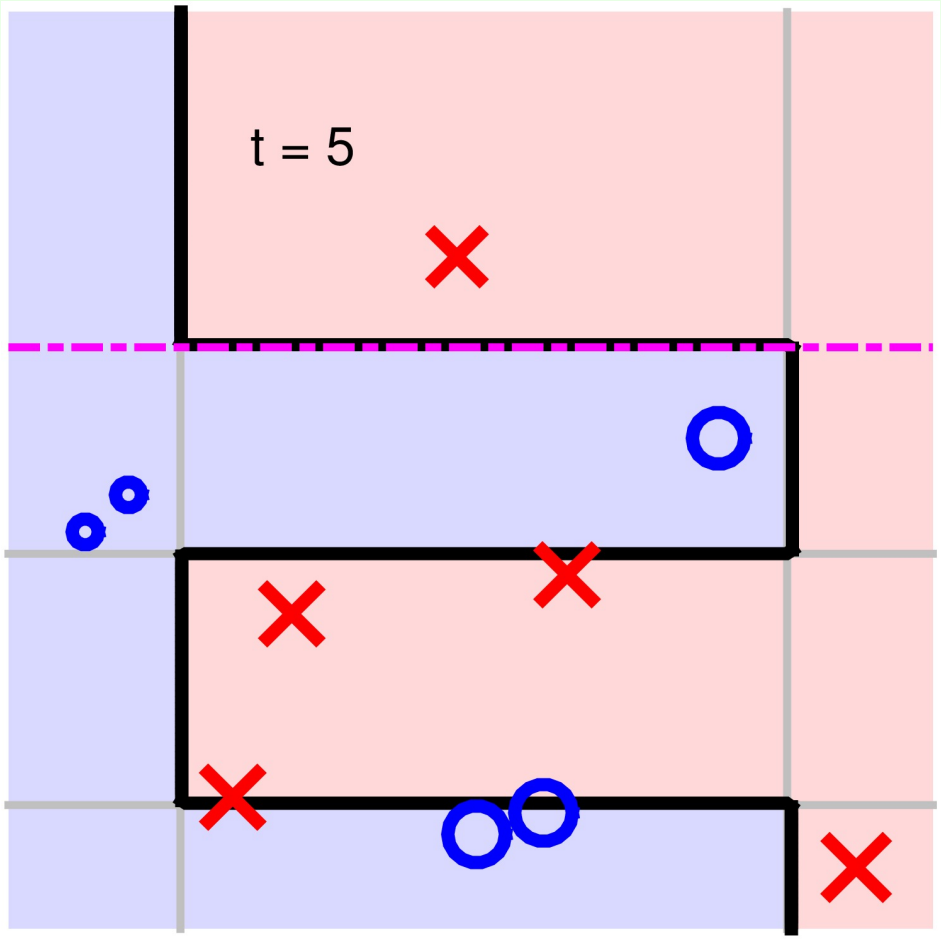
A Simple Data Set



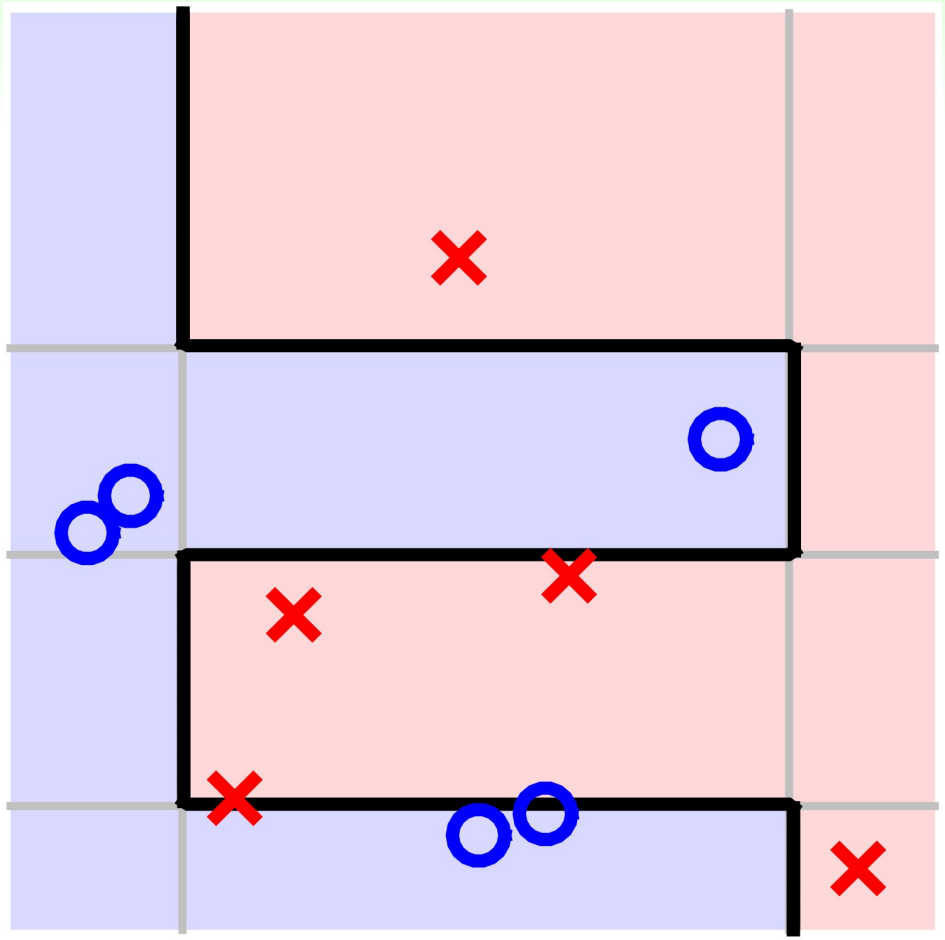
A Simple Data Set



A Simple Data Set



A Simple Data Set



'Teacher'-like algorithm works!

Putting Everything Together

Gradient Boosted Decision Tree (GBDT)

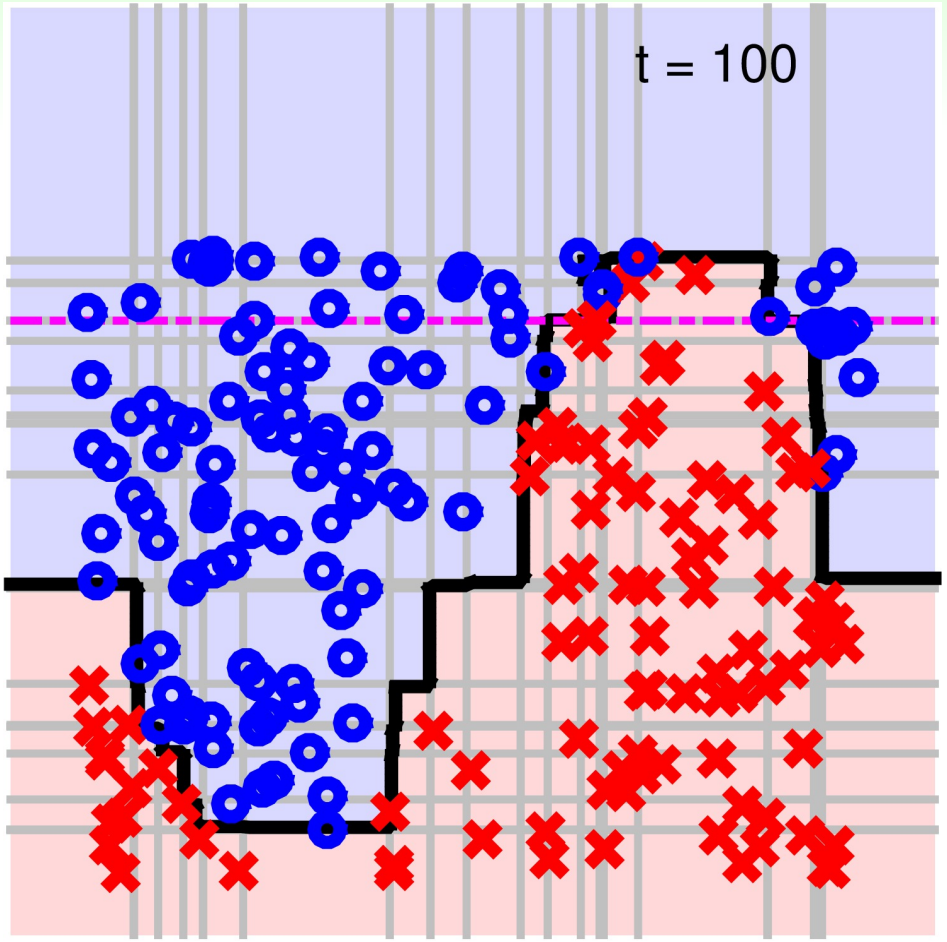
$s_1 = s_2 = \ldots = s_N = 0$
for $t = 1, 2, \ldots, T$

1. obtain g_t by $\mathcal{A}(\{(\mathbf{x}_n, y_n - s_n)\})$ where $\mathcal{A}$ is a (squared-error) regression algorithm
   —**how about sampled and pruned C&RT?**
2. compute $\alpha_t = \text{OneVarLinearRegression}(\{(g_t(\mathbf{x}_n), y_n - s_n)\})$
3. update $s_n \leftarrow s_n + \alpha_t g_t(\mathbf{x}_n)$

return $G(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t g_t(\mathbf{x})$

GBDT: 'regression sibling' of AdaBoost-DTree
—**popular in practice**

A Complicated Data Set



AdaBoost-Stump: **non-linear yet efficient**

AdaBoost-Stump in Application

original picture by F.U.S.I.A. assistant and derivative work by Sylenius via Wikimedia Commons

The World's First 'Real-Time' Face Detection Program

- AdaBoost-Stump as core model: linear aggregation of key patches selected out of 162,336 possibilities in 24x24 images
  —**feature selection** achieved through AdaBoost-Stump
- modified linear aggregation G to rule out non-face earlier
  —**efficiency** achieved through modified linear aggregation

AdaBoost-Stump:
efficient feature selection and aggregation

Questions?

Summary

1. Combining Predictive Features: Aggregation Models

Lecture 12: Bagging and Boosting

- Uniform Blending
  diverse hypotheses, 'one vote, one value'
- Linear and Any Blending

two-level learning with hypotheses as transform

- Bagging (Bootstrap Aggregation)
  bootstrapping for diverse hypotheses
- Motivation of Boosting
  aggregate weak hypotheses for strength
- Diversity by Re-weighting
  scale up incorrect, scale down correct
- Adaptive Boosting Algorithm
  two heads are better than one, theoretically
- Adaptive Boosting in Action
  AdaBoost-Stump useful and efficient