Homework 2

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1 Exercises 2.3 - 1

1.1 a.

$$
\begin{aligned}
1 + 3 + 5 + 7 + ... + 999 &= \frac{(1 + 999)500}{2} \\
&= 250000
\end{aligned}
$$

1.2 b.

$$
\begin{aligned}
2 + 4 + 8 + 16 + ... + 1024 &= \frac{2^{10+1} - 1}{2 - 1} - 1 \\
&= 2047 - 1 \\
&= 2046
\end{aligned}
$$

1.3 c.

$$
\begin{aligned}
\sum_{i=3}^{n+1} 1 &= (n + 1) - 3 + 1 \\
&= n - 1
\end{aligned}
$$

1.4 d.

$$\begin{aligned}\sum_{i=3}^{n+1} i &= 3 + 4 + ...n + 1 \\ &= (\sum_{i=1}^{n+1} i) - 1 - 2 \\ &= \frac{(n+1)(n+2)}{2} - 3 \\ &= \frac{n^2 + 3n - 4}{2}\end{aligned}$$

1.5 e.

$$\begin{aligned}\sum_{i=0}^{n-1} i(i+1) &= \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i \\ &= \frac{(n-1)n(2n-1)}{6} + \frac{(n-1)n}{2} \\ &= \frac{(n^2-1)n}{3}\end{aligned}$$

2 Exercises 2.3 - 2

2.1 a.

$$\begin{aligned}\sum_{0}^{n-1} (i^2+1)^2 &= \sum_{0}^{n-1} i^4 + 2\sum_{0}^{n-1} i^2 + \sum_{0}^{n-1} 1 \\ &= \frac{1}{5}(n-1)^5 + \frac{1}{3}(n-1)^3 + n - 1 \\ &\approx \frac{1}{5}n^5 \\ &\in \Theta(n^5)\end{aligned}$$

2.2 b.

$$\begin{aligned}\sum_{i=2}^{n-1} \ln i^2 &= 2\sum_{i=1}^{n-1} \ln i - 2\ln 1 \\ &= 2(n-1)\ln(n-1) \\ &\approx 2n\ln n \\ &\in \Theta(n \ln n)\end{aligned}$$

2.3 c.

$$\begin{aligned}\sum_{i=1}^{n}(i+1)2^{i-1} &= \frac{1}{4}\sum_{i=1}^{n}(i+1)2^{i+1} \\ &= \frac{1}{4}\sum_{k=0}^{n+1} k2^k \\ &= \frac{1}{4}\sum_{k=1}^{n+1} k2^k \\ &= \frac{n}{4}2^{n+2} + 2 \\ &\approx \frac{n}{16}2^n \\ &\in \Theta(n2^n)\end{aligned}$$

2.4 d.

$$\begin{aligned}
\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) &= \sum_{i=0}^{n-1}[i(i-1)+\sum_{j=0}^{i-1}j] \\
&= \sum_{i=0}^{n-1}[i^2-i+\frac{(i-1)i}{2}] \\
&= \frac{3}{2}\sum_{i=0}^{n-1}(i^2-i) \\
&= \frac{3}{2}\sum_{i=0}^{n-1}i^2-\frac{3}{2}\sum_{i=0}^{n-1}i \\
&= \frac{3}{2}\frac{(n-1)n(2n-1)}{6}-\frac{3}{2}\frac{n(n-1)}{2} \\
&\approx \frac{n^3}{2} \\
&\in \Theta(n^3)
\end{aligned}$$

3 Exercises 2.3 - 3

Let us denote $D(n)$ the number of divisions, $M(n)$ the number of multiplications, $A(n)$ the number of additions and $S(n)$ the number ofsubtractions.

For $\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}$ where $\overline{x}=\frac{\sum_{i=1}^{n}x_i}{n}$, $D(n)=2, M(n)=n, A(n)+S(n)=[(n-1)+(n-1)]+(n+1)=3n-1.$

For $\frac{\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2/n)}{n-1}$, $D(n)=2, M(n)=n+1, A(n)+S(n)=[(n-1)+(n-1)]+2=2n$

4 Exercises 2.3 - 6

a. If the input is a symmetric matrix, the algorithm returns true. If the matrix is not symmetric, the algorthm returns false.

b. The comparisons: $if A[i,j]\neq A[j,i]$

c.

$$
\begin{aligned}
C(n) &= \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 \\
&= \sum_{i=0}^{n-2} [(n-1) - (i+1) + 1] \\
&= \sum_{i=1}^{n-2} (n-1-i) \\
&= (n-1)(n-1) - \sum_{i=0}^{n-2} i \\
&= (n-1)^2 - \frac{(n-2)(n-1)}{2} \\
&= \frac{n(n-1)}{2}
\end{aligned}
$$

d. $C(n) \in \Theta(n^2)$

e. There are no better algorithms because any algorithms have to compare $\frac{n(n-1)}{2}$ times for this problem.

5 Exercises 2.3 - 9

Basis: For n = 1, the left-hand side of the equation is 1, and the right-hand side of the equation is $\frac{1(1+1)}{2} = 1$. Thus the statement is true for n = 1.

Inductive step: Assume $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ holds for some unspecified value of k. Then $\sum_{i=1}^{n+1} i = (n+1) + \sum_{i=1}^{n} i = (n+1) + \frac{n(n+1)}{2} = \frac{(n+1)(n+2)}{2}$

Since both the basis and the inductive step have been performed, by mathematical induction, the statement $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ holds for all natural numbers n. Q.E.D.