Homework 7

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1 Exercieses 8.1 - 2

$F[0] = 0, F[1] = c_1 = 5$
$F[2] = max\{1 + 0, 5\} = 5$
$F[3] = max\{2 + 5, 5\} = 7$
$F[4] = max\{10 + 5, 7\} = 15$
$F[5] = max\{7 + 6, 15\} = 15$

2 Exercieses 8.1 - 4

$F[0] = 0$
$F[1] = min\{F[1 - 1]\} + 1 = 1$
$F[2] = min[F[2 - 1]] + 1 = 2$
$F[3] = min\{F[3 - 1], F[3 - 3]\} + 1 = 1$
$F[4] = min\{F[4 - 1], F[4 - 3]\} + 1 = 2$
$F[5] = min\{F[5 - 1], F[5 - 3], F[5 - 5]\} + 1 = 1$
$F[6] = min\{F[6 - 1], F[6 - 3], F[6 - 5]\} + 1 = 2$
$F[7] = min\{F[7 - 1], F[7 - 3], F[7 - 5]\} + 1 = 3$
$F[8] = min\{F[8 - 1], F[8 - 3], F[8 - 5]\} + 1 = 2$
$F[9] = min\{F[9 - 1], F[9 - 3], F[9 - 5]\} + 1 = 3$

3 Exercieses 8.1 - 9

//**Input:** nonnegative integers n and k.
//**Output:** The value of $C(n, k)$
function BionomialCoef(n, k)
  if $k = 0$ or $k = n$ **then**
    return 1
  else
    return $BionomialCoef(n - 1, k - 1) + BionomialCoef(n - 1, k)$
  end if
end function

Time efficiency:
$\frac{(k-1)k}{2} + k(n-k) = nk - \frac{1}{2}k^2 - 1/2 \in \theta(nk)$
Space efficiency:
$S(n,k) = \sum_{i=0}^{k}(i+1) + \sum_{i=k+1}^{n}(k+1) = \frac{(k+1)(k+2)}{2} + (k+1)(n-1) \in \theta(nk)$

4 Exercieses 8.2 - 2

a.

//**Input:** Arrays $w[1..n]$ and $v[1..n]$. Integer W
//**Output:** A table $T[0..n, 0..W]$

```
function KNAPSACK(w[1..n], v[1..n], W)
    for i ← 0 to n do
        T[i, 0] ← 0
    end for
    for j ← 1 to W do
        T[0, j] ← 0
    end for
    for i ← 1 to n do
        for j ← 1 to W do
            if j − w[i] ≥ 0 then
                T[i, j] ← max{V[i − 1, j], v[i] + T[i − 1, j − w[i]]}
            else
                T[i, j] ← T[i − 1, j]
            end if
        end for
    end for
    return T
end function
```

b.

//**Input:** Arrays $w[1..n]$ and $v[1..n]$. An integer W. A table $T[0..n, 0..W]$
//**Output:** A list $L[1..k]$

```
function OPTICALSUBSET(w[1..n], v[1..n], W)
    temp ← 0
    j ← W
    for i ← n decrease 1 do
        if T[i, j] > T[i − 1, j] then
            k ← k + 1
            L[k] ← i
            j ← j − w[i]
        end if
    end for
    return L
```

end function

5 Exercieses 8.3 - 7

a.

Let k be the number of nodes in the left tree, then the right tree must have n - 1 - k nodes. Thus the number of trees is $b(k)b(n-1-k)$. $b(n) = \sum_{k=0}^{n-1} b(k)b(n-1-k)$

b.

$b(1) = 1$
$b(2) = 2$
$b(3) = 5$
$b(4) = 14$
$b(5) = 42$

c.

With the help of Stirling's approximation:

$$\begin{aligned}
b(n) &= \frac{(2n)!}{(n!)^2}\frac{1}{n+1} \\
&\approx \frac{\sqrt{2\pi 2n}(\frac{2n}{e})^{2n}}{[\sqrt{2\pi n}(\frac{n}{e})^n]^2(n+1)} \\
&= \frac{\sqrt{4\pi n}(\frac{2n}{e})^{2n}}{2\pi n(\frac{n}{e})^{2n}(n+1)} \\
&= \frac{2^{2n}}{\sqrt{\pi n}(n+1)} \\
&\in \theta(4^n n^{-\frac{3}{2}})
\end{aligned}$$

It will be very expensive for running this algorithm with a large n.

6 Exercieses 8.4 - 1

$$R^{(0)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(2)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(3)} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(4)} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = T$$

7 Exercieses 8.4 - 7

$$D^{(0)} = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

$$D^{(1)} = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 8 & 4 & 0 \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 8 & 4 & 0 \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ \infty & \infty & 0 & 4 & 7 \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix}$$

$$D^{(5)} = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ 10 & 12 & 0 & 4 & 7 \\ 6 & 8 & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix} = D$$