1. Consider a generalised linear model with log-link for (ungrouped) data with exponentially distributed response. The model can be summarised as

$$\begin{aligned} y_i|x_i \sim \text{Exp}(\lambda_i) &= \text{Exp}\left(\frac{1}{\mu_i}\right) \\ \frac{1}{\lambda_i} = \mu_i &= E(y_i|x_i) = \exp(\beta^T x_i). \end{aligned}$$

   (a) Starting from the generic expression for the score-function in a GLM,

$$S(\beta) = \frac{1}{\phi}\sum_{i=1}^{n}(y_i - \mu_i)\frac{1}{\mathcal{V}(\mu_i)}\frac{\partial h(\eta_i)}{\partial \eta_i}x_i \, ,$$

   find the score function for this particular situation.

   (b) By differentiating the expression derived in part (a), find the observed Fisher Information $F_{\text{obs}}(\beta)$.

   (c) By taking an expectation, find the Fisher-Information $F(\beta)$.

2. Suppose that we are given observations for 17 leukemia patients on the variables `wbc` (white blood cell count) and `time` (survival time between diagnosis and deaths in weeks).

`wbc`	3.36	2.88	3.63	3.41	3.78	4.02	4.00	4.23	3.73
`time`	65	156	100	134	16	108	121	4	39
`wbc`	3.85	3.97	4.51	4.54	5.00	5.00	4.72	5.00	
`time`	143	56	26	22	1	1	5	65	

   (a) A generalised linear model of the type described above is fitted to these data, with linear predictor $\eta_i = \beta_1 + \beta_2 \texttt{wbc}_i$. Compute the expected Fisher Information matrix and its inverse (you can use your calculations from Question 1).

   (b) The (edited) summary of the model fitted in R is as follows:

```
> linm = glm(time~wbc, family = Gamma(link=log))
> summary(linm, dispersion = 1)

Call:
glm(formula = time ~ wbc, family = Gamma(link = log))

Coefficients   Estimate
(Intercept)     8.4775
wbc            -1.1093
---

(Dispersion parameter for Gamma family taken to be 1)
```

   Use the above code and your answer to Question 1 to calculate an estimate of the expected value of `time` as well as an approximate 95% confidence interval for this expected value, when $\texttt{wbc} = 3$.

   (c) The estimate for β_2 is $\hat{\beta_2} = -1.109$. Interpret this value, and suggest and evaluate a test of $H_0 : \beta_2 = 0$ at the 5% level of significance.