math behind Assignment 2

Task 2 (math): Trying to find the intersection of 2 lines

Lines are of the form $ax + by + c = 0$

let's call line 1: $ax + by + c = 0 \iff y = \frac{-(c+ax)}{b}$

& line 2 is: $dx + ey + f = 0 \iff y = \frac{-(f+dx)}{e}$

at the point of intersection, the x & y values of line 1 & line 2 are the same, ∴

$$\frac{-(f+dx)}{e} = \frac{-(c+ax)}{b}$$

By setting the ~~x~~ y values equal, we assume they exist, ∴ $b \neq 0$ & $e \neq 0$.

$$\therefore\ b(f+dx) = e(c+ax) \iff bf + bdx = ce + aex$$

$x(ae - bd) = bf - ce$ now, find y, both eqns. should

$x = \frac{bf - ce}{ae - bd}$ result in the same y-value, ∴ it doesn't matter which eqn. we use.

$$\therefore\ y = \frac{-(c+ax)}{b} = \frac{-c - a\left(\frac{bf-ce}{ae-bd}\right)}{b}$$

$$\iff y = \frac{-c(ae-bd) - a(bf-ce)}{b(ae-bd)}$$

numerator of y: $\cancel{-ace} + bcd - abf \cancel{+ace} = bcd - abf$

$$\therefore\ y = \frac{bcd - abf}{b(ae-bd)} = \frac{b(cd-af)}{b(ae-bd)}$$

$b \neq 0$, ∴ we can cancel

$y = \frac{cd - af}{ae - bd}$, ∴ point of intersection is $\left(\frac{bf-ce}{ae-bd}, \frac{cd-af}{ae-bd}\right)$

∴ the point of intersection won't exist if $ae = bd$, to consider floating point values, do $|ae - bd| < \varepsilon$ where ε is a small chosen number.

Task 3 (math) want to find the intersection between 2 line segments

Task 3a: find the line to which a segment belongs

call the segment $[[a,b],[c,d]]$

gradient of segment $= \dfrac{d-b}{c-a} = m$

"$y - y_1 = m(x - x_1)$"

$$y - b = \frac{(d-b)(x-a)}{c-a}$$

numerator of right-hand side: $dx - bx - ad + ab$

$$= (d-b)x - (d-b)a$$

$$\therefore \quad y - b = \frac{(d-b)x}{(c-a)} - \frac{(d-b)a}{c-a}$$

$$y = \frac{(d-b)x}{c-a} + \frac{b(c-a) - (d-b)a}{c-a}$$

$\therefore$ in desired form: $\left[\dfrac{d-b}{c-a}, -1, b - \dfrac{(d-b)a}{c-a}\right]$

$\therefore \quad \dfrac{d-b}{c-a} = m$

$\therefore \quad [m, -1, b - ma]$

$\therefore$ if $c = a$, this fails, $\therefore$ for this x values are the same

$\therefore$ $x = a$ or $x = c$ [illegible] line is: $[1, 0, a]$. In the program

$|c - a| \le \varepsilon$ is done to consider floating [illegible] errors [illegible]

ε is a small chosen number.

Task 3b: checking if a point lies on a segment, assuming the point lies on the line to which the segment belongs. The segment is of the form [[a, b], [c, d]] & ~~the line~~ let's call the point (x, y).

~~To do this~~

To do this, think about the negation, what happens when the point does not lie on the segment.

When this happens, x is either greater than the ~~max~~ larger value of a & c or x is smaller than the smaller value of a & c. ~~we have to work out which is larger & which is smaller~~

~~we~~ ∴, say we have a vertical line segment, ~~all the~~ & we are testing if a point lies on it, ~~bah~~ a = c = x, ∴ it will always be true. ∴, we have to test y. The point will also not lie on the segment if y is either greater than the larger value of b & d or y is smaller than the smaller value of ~~a & c~~ b & d.

we always have to compare to the larger / smaller value of the x/y - co-ordinates of the segment as the order of the segment does not matter. ~~& it considers segments with decreasing~~

~~∴ if $a \leq x \leq c$ & $b \leq y \leq d$, the point lies in the segment.~~ So, for the program, if $x >$ max. of a & c, do $|x -$ max. of a or c$| < \varepsilon$ & if $x <$ min. of a & c, do $|$min. of a or c $- x| < \varepsilon$, where ε is a chosen small no. Then do the same for y compared to the max. and min. of d & f, & ensure it's smaller than δ, where δ is another chosen small no.

Task 3c: finding the intersection of 2 segments.

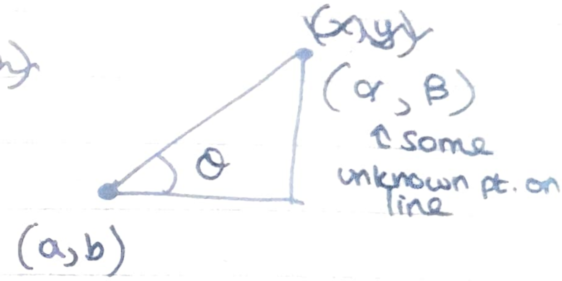
First, change the segments into 2 lines ~~as done~~ via the ~~method shown in~~ Task 3a. Then, find the intersection of the 2 lines via Task 2.

Assuming they intersect, check if the point of intersection lies in both segments via Task 3b. If the point of intersection lies in both segments, that's your answer. If not, then there's no intersection.

Task 4 (math): finding the intersection between a ray & a line segment

This is like Task 3, ∵ we need to find out ways to ~~work~~ find the line to which a ray belongs & to check if a point lies on a ray. Call the point (x, y) & the ray $= [[a, b], \theta]$

4a. The gradient of the line is : ~~tan(θ)~~



No matter the angle, gradient $= \tan\theta = m$, ∴ to find the line: "$y - y_1 = m(x - x_1)$":

$$y - b = \tan\theta\,(x - a)$$

$$y = x\tan\theta + (b - a\tan\theta)$$

in form:

$$\therefore [\tan\theta, -1, b - a\tan\theta]$$

$$= [m, -1, b - ma]$$ — line form of Ray

4b. To check if a point lies on the ray, it depends on the angle (initial assumption: the point lies on the line the ray belongs to). Use the point (x, y) & call the ray $[[a,b], \theta]$

The potential angles can be split into 4 quadrants, which are intervals $(0, \pi/2)$, $(\pi/2, \pi)$, $(-\pi, -\pi/2)$ & $(-\pi/2, 0)$. For the point to lie on the ray, any of the following must be true.

~~For $(0, \pi/2)$, $x > a$ & $y > b$~~

For $\theta \in (0, \pi/2)$, $x > a$ & $y > b$
For $\theta \in (\pi/2, \pi)$, $x < a$ & $y > b$
For $\theta \in (-\pi, -\pi/2)$, $x < a$ & $y < b$
For $\theta \in (-\pi/2, 0)$, $x > a$ & $y < b$

$\therefore$, this will not work for $\theta = 0$ & ~~$\theta = \pm\pi/2$~~, $\theta = \pm\pi$ as all ~~y values~~ ~~x~~ y values on their ~~respective line~~ ~~form~~ ray are the same

This also would not work for $\theta = \pm \pi/2$ as all x-values on the ray are the same

$\therefore$ For $\theta = 0$, $x > a$ & $y = b$
~~For $\theta = \pm\pi$, $x < a$ & $y = b$~~
For $\theta = \pi/2$, $x = a$ & $y > b$
~~For $\theta = \pi/2$,~~
For $\theta = \pm\pi$, $x < a$ & $y = b$
For $\theta = -\pi/2$, $x = a$ & $y < b$

If none of these are true, the point does not lie on the ray.

4c. finding the intersection between a ray & a segment.

~~change ray into a line via task 4a change~~

First convert the ray & segment to lines. Convert the ray to a line by task 4a. Convert the segment to a line by task 3a. Use Task 2 to find the point of intersections of these lines.

Assuming the point of intersection exists, check if the point lies in the ray & segment via task 3b for the segment & task 4b for the line. If the point does lies in both, the ray & segment have a point of intersection. If not, theres ~~point does not exist.~~ is no intersection.

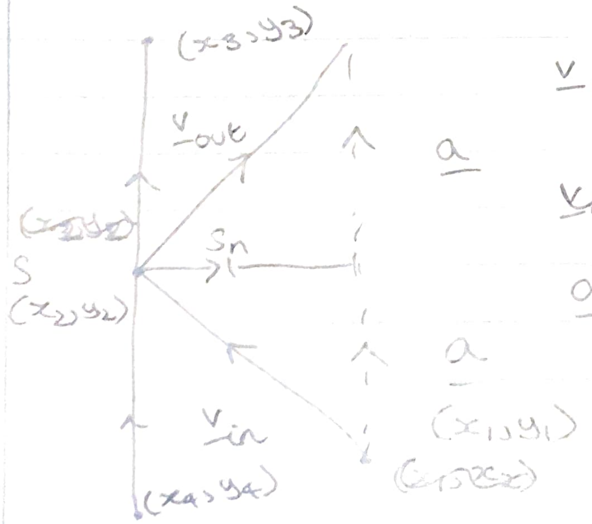
Task 8 (math):

First, make sure the ray angle is between $-\pi$ & π by adding/subtracting 2π till you get an angle within that range. Then put this new angle in the ray.

Work out the ray & segment intersection using task 4c*. If this intersection doesn't intersect, the value where the ray leaves the window is from ray-window-intersect.

For ray-window-intersect: write the window in terms of 4 segments & then check ~~if / when~~ if & where ~~& where~~ the ray intersects these segments. It has to intersect at least 1. Only 1 value is returned though. Then the ray-window-intersect is that value.

If the initial ray-segment-intersect from * exists, this is the general case:



$\underline{v}_{out} = -\underline{v}_{in} + 2\underline{a}$

~~$\underline{v}_{in}$~~

$\underline{a} = \underline{v}_{in}$ − projection of $-\underline{v}_{in}$ onto S_n

To work out the projection of $-\underline{v}_{in}$ onto S_n:

$$= \frac{-\underline{v}_{in} \cdot S_n}{|S_n|}$$

$|S_n| = 1$, $\therefore = -\underline{v}_{in} \cdot S_n$, this is the scalar we multiply to S_n to extend it.

$$\therefore \underline{a} = \cancel{\$}\, s_n(-\underline{v}_{in} \cdot s_n) + \underline{v}_{in}$$

$$\underline{v}_{out} = -\underline{v}_{in} \overset{(+2a)}{+ 2}\left[s_n(-\underline{v}_{in} \cdot s_n) + \underline{v}_{in} \right]$$

$$\underline{v}_{out} = \underline{v}_{in} + 2\left[s_n(-\underline{v}_{in} \cdot s_n) \right]$$

$$\underline{v}_{out} = \underline{v}_{in} - 2 s_n(\underline{v}_{in} \cdot s_n)$$

This is the vector of the reflected ray.

From diagram:

$$\underline{v}_{in} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} \rightarrow \frac{1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}} \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$ as $\underline{v}_{in}$ is a unit vector

$s_n \cdot s = 0$, let's call $s_n = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$

$$s = \begin{pmatrix} x_3 - x_4 \\ y_3 - y_4 \end{pmatrix} \rightarrow \frac{1}{\sqrt{(x_3-x_4)^2 + (y_3-y_4)^2}} \begin{pmatrix} x_3 - x_4 \\ y_3 - y_4 \end{pmatrix}$$ as s is a unit vector

$$s_1(x_3 - x_4) + s_2(y_3 - y_4) = 0$$

$$s_2 = \frac{\cancel{y_3 - y_4} - s_1(x_3 - x_4)}{y_3 - y_4}$$

$$s_n = \begin{pmatrix} s_1 \\ \dfrac{-s_1(x_3 - x_4)}{y_3 - y_4} \end{pmatrix}$$

Sub in $x2$ for $s1$ as it was on s_n

$$\therefore s_n = \begin{pmatrix} x_2 \\ \dfrac{-x_2(x_3 - x_4)}{y_3 - y_4} \end{pmatrix}$$

Get the magnitude of S_n to make it a unit vector

(Like for $\underline{v}_{in}$ & S.

use $\underline{v}_{out} = \underline{v}_{in} - 2S_n(\underline{v}_{in} \cdot S_n)$

~~For 2D, can just~~

Then use atan2 to find $\underline{v}_{out}$'s angle with the positive x-axis & the new-ray is where the ray & segment intersect with the angle just worked out.

To find where this leaves the window use the ~~ray-window~~ ray-window-intersect function from earlier.