

$$y = \sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\begin{aligned} y' &= \left(\frac{1}{1 + e^{-x}} \right)' \\ &= (-1) * \frac{1}{(1 + e^{-x})^2} * e^{-x} * (-1) \\ &= \frac{e^{-x}}{(1 + e^{-x})^2} \\ &= \frac{1 + e^{-x} - 1}{(1 + e^{-x})^2} \\ &= \frac{1}{1 + e^{-x}} - \frac{1}{(1 + e^{-x})^2} \\ &= y - y^2 \end{aligned}$$