Feedback — Problem Set-2

Help Center

You submitted this quiz on **Sun 1 Nov 2015 8:14 AM PST**. You got a score of **5.00** out of **5.00**.

Question 1

This question will give you further practice with the Master Method. Suppose the running time of an algorithm is governed by the recurrence $T(n) = 7 * T(n/3) + n^2$. What's the overall asymptotic running time (i.e., the value of T(n))? Note: If you take this quiz multiple times, you may see different variations of this question.

Your Answer		Score	Explanation
$\theta(n \log n)$			
$\theta(n^2 \log n)$			
$\theta(n^2)$	✔	1.00	a=7, b=3, d=2. Since b^d > a, this is case 2 of the Master Method.
$\theta(n^{2.81})$			
Total		1.00 / 1.00	

Question 2

Consider the following pseudocode for calculating a^b (where a and b are positive integers)

```
FastPower(a,b) :

    if b = 1

        return a
```

```
            otherwise

                c := a*a

                ans := FastPower(c,[b/2])

            if b is odd

                return a*ans

            otherwise return ans

end
```

Here [x] denotes the floor function, that is, the largest integer less than or equal to x.

Now assuming that you use a calculator that supports multiplication and division (i.e., you can do multiplications and divisions in constant time), what would be the overall asymptotic running time of the above algorithm (as a function of b)?

Your Answer		Score	Explanation
$\Theta(b)$			
$\Theta(\sqrt{b})$			
$\Theta(\log(b))$	✔	1.00	Constant work per digit in the binary expansion of b.
$\Theta(b\log(b))$			
Total		1.00 / 1.00	

Question Explanation

This gives you a nice way of raising a number to the power in multiplications much less than b. You can get the answer by looking at the binary expression for b.

Question 3

Let $0 < \alpha < .5$ be some constant (independent of the input array length n). Recall the Partition subroutine employed by the QuickSort algorithm, as explained in lecture. What is the probability that, with a randomly chosen pivot element, the Partition subroutine produces a split in which the size of the smaller of the two subarrays is $\geq \alpha$ times the size of the original array?

Your Answer		Score	Explanation
$2 - 2 * \alpha$			
α			
$1 - \alpha$			
$1 - 2 * \alpha$	✔	1.00	That's correct!
Total		1.00 / 1.00	

Question 4

Now assume that you achieve the approximately balanced splits above in every recursive call --- that is, assume that whenever a recursive call is given an array of length k, then each of its two recursive calls is passed a subarray with length between αk and $(1 - \alpha)k$ (where $0 < \alpha < .5$). How many recursive calls can occur before you hit the base case, as a function of α and the length n of the original input? Equivalently, which levels of the recursion tree can contain leaves? Express your answer as a range of possible numbers d, from the minimum to the maximum number of recursive calls that might be needed. [The minimum occurs when you always recurse on the smaller side; the maximum when you always recurse on the bigger side.]

Your Answer		Score	Explanation
$-\frac{\log(n)}{\log(\alpha)} \leq d \leq -\frac{\log(n)}{\log(1-\alpha)}$	✔	1.00	That's correct!
$-\frac{\log(n)}{\log(1-2*\alpha)} \leq d \leq -\frac{\log(n)}{\log(1-\alpha)}$			
$0 \leq d \leq -\frac{\log(n)}{\log(\alpha)}$			
$-\frac{\log(n)}{\log(1-\alpha)} \leq d \leq -\frac{\log(n)}{\log(\alpha)}$			
Total		1.00 / 1.00	

Question 5

Define the recursion depth of QuickSort to be the maximum number of successive recursive calls before it hits the base case --- equivalently, the number of the last level of the corresponding recursion tree. Note that the recursion depth is a random variable, which depends on which pivots get chosen. What is the minimum-possible and maximum-possible recursion depth of QuickSort, respectively?

Your Answer		Score	Explanation
Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n)$	✔	1.00	The best case is when the algorithm always picks the median as a pivot, in which case the recursion is essentially identical to that in MergeSort. In the worst case the min or the max is always chosen as the pivot, resulting in linear depth.
Minimum: $\Theta(\log(n))$; Maximum: $\Theta(n\log(n))$			
Minimum: $\Theta(1)$; Maximum: $\Theta(n)$			
Minimum: $\Theta(\sqrt{n})$; Maximum: $\Theta(n)$			
Total		1.00 / 1.00	