Homework 1 Solution

Question 1

Here is a one possible solution.

We first set up parameters and load relevant libraries.

```
rm(list=ls())
library(data.table)
library(kknn)

# Primitives
N_train=100
N_test=10000
```

Next, we write functions for generating data and plotting.

```
# Function to simulate data
Simulate <- function(func, N_train, N_test){
  set.seed(11)
  n=N_train+N_test
  x=runif(n, -1, 1)
  y=func(x)+rnorm(n, mean=0, sd=0.15)
  dt=data.table(y=y,x=x)
  tr_idx=sample(1:n, N_train)
  train=dt[tr_idx,]
  setkey(train,x)
  test=dt[-tr_idx,]
  setkey(test,x)
  return(list(model=func, full=dt, train=train, test=test))
}

# Function to create base scatter plot
Plot <- function(sim, method){
  # unpack
  true_model=sim$model
  train=sim$train
  if (method=="linear regression"){
    plot(train$x, train$y, cex=0.5, pch=19, col="grey")
    title(main="Scatter plot, true relationship, and linear fit")
    lines(train$x, true_model(train$x), col="black", lwd=2)
    linear=lm(y~x, data=train)
    yhat=predict(linear, train)
    lines(train$x, yhat, col="blue", lty=2, lwd=3)
    return(linear)
  } else if (method=="k-nn") {
    k_vec=2:15
    #near_list <- vector("list", length(k_vec))
    fit=matrix(0, nrow=dim(train)[1], ncol=length(k_vec))
    for (i in 1:length(k_vec)){
      k=k_vec[i]
      near <- kknn(y~x, train=train, test=train[,.(x)],k=k, kernel='rectangular')
```

```
      #near_list[[i]]=near
      fit[,i]=near$fitted
    }

    par(mfrow=c(1,2))
    plot(train$x, train$y, cex=0.5, pch=19, col="grey")
    title(main="k-nn fit: k=2")
    lines(train$x, true_model(train$x), col="black", lwd=2)
    lines(train$x, fit[,1], col="blue", lwd=2)
    plot(train$x, train$y, cex=0.5, pch=19, col="grey")
    title(main="k-nn fit: k=12")
    lines(train$x, true_model(train$x), col="black", lwd=2)
    lines(train$x, fit[,11], col="blue", lwd=2)
    return(k_vec)
  }
}


# Function to plot test set MSE
PlotTestError <- function(sim, base_plot_ls, base_plot_knn){
  linear=base_plot_ls
  k_vec=base_plot_knn
  train=sim$train
  test=sim$test
  testMSE_vec=numeric(length=length(k_vec))
  for (i in 1:length(k_vec)){
    near=kknn(y~x, train=train, test=test[,.(x)], k=k_vec[i], kernel='rectangular')
    testMSE_vec[i]=mean((near$fitted-test$y)^2)
  }
  lsMSE=mean((predict(linear, test)-test$y)^2)
  par(mfrow=c(1,1))
  plot(log(1/k_vec), testMSE_vec,
       ylim = c(0.95*min(c(testMSE_vec, lsMSE)),1.05*max(c(testMSE_vec, lsMSE))),
       type="n")
  title(main="Test set mean squared error")
  lines(log(1/k_vec), testMSE_vec, lty=6, lwd=3, col="forestgreen", type="o", cex=0.6 )
  abline(h=lsMSE, col="black", lty=2)
}
```

Linear model (part 1.1 - 1.5)

Define function

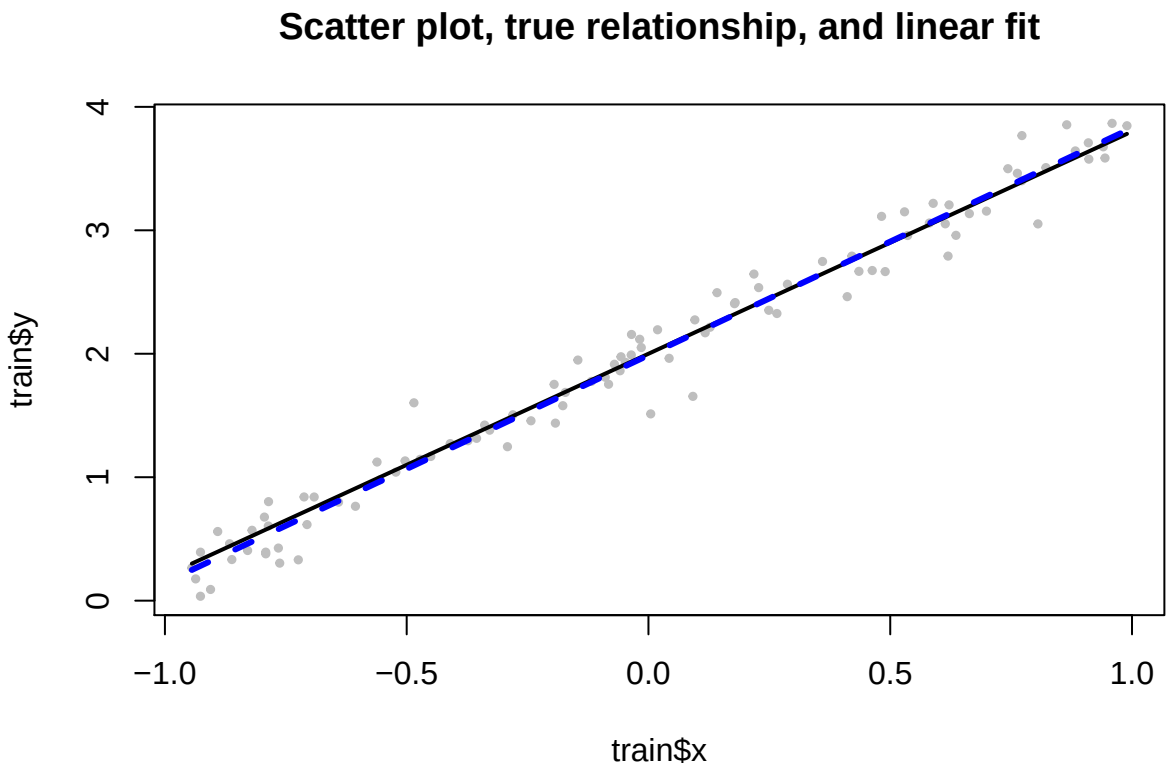
```
func_linear <- function(x){1.8*x+2}
```

Simulate data from the linear function

```
sim_linear <- Simulate(func_linear, N_train, N_test)
```

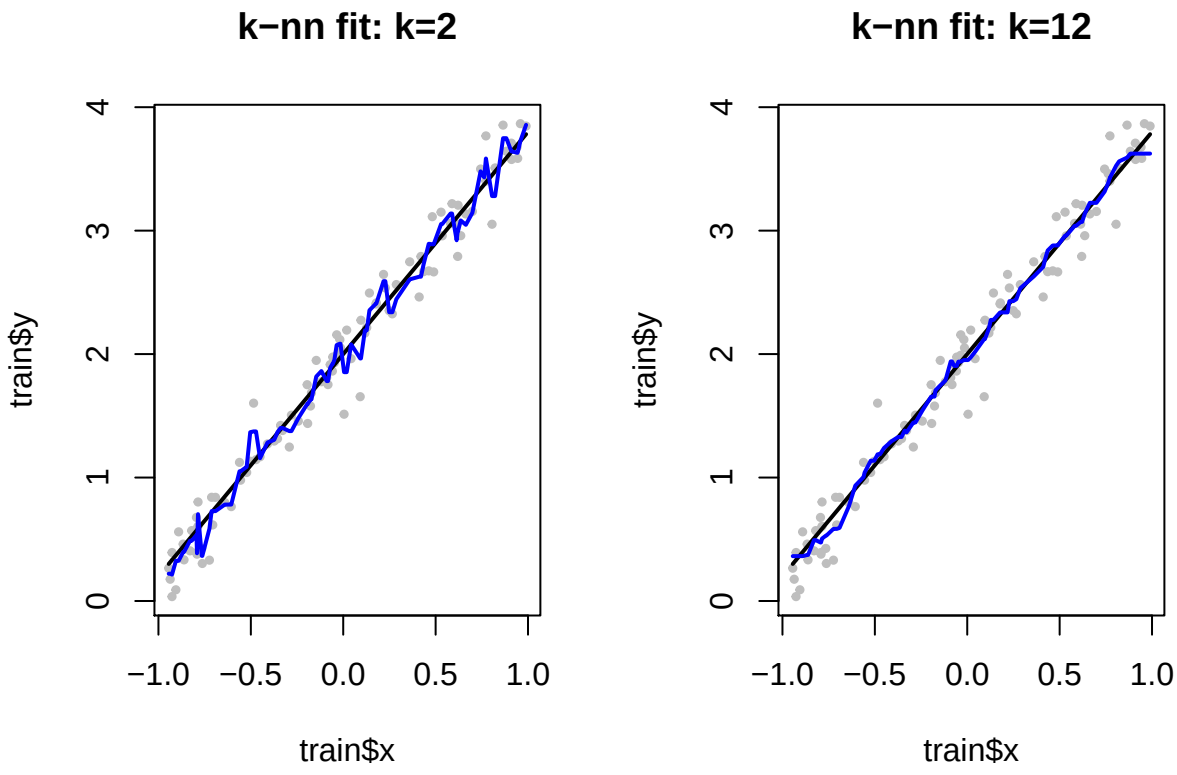
Plot the training dataset, true relationship and best linear fit

```
plot_ls<- Plot(sim_linear, method = "linear regression")
```



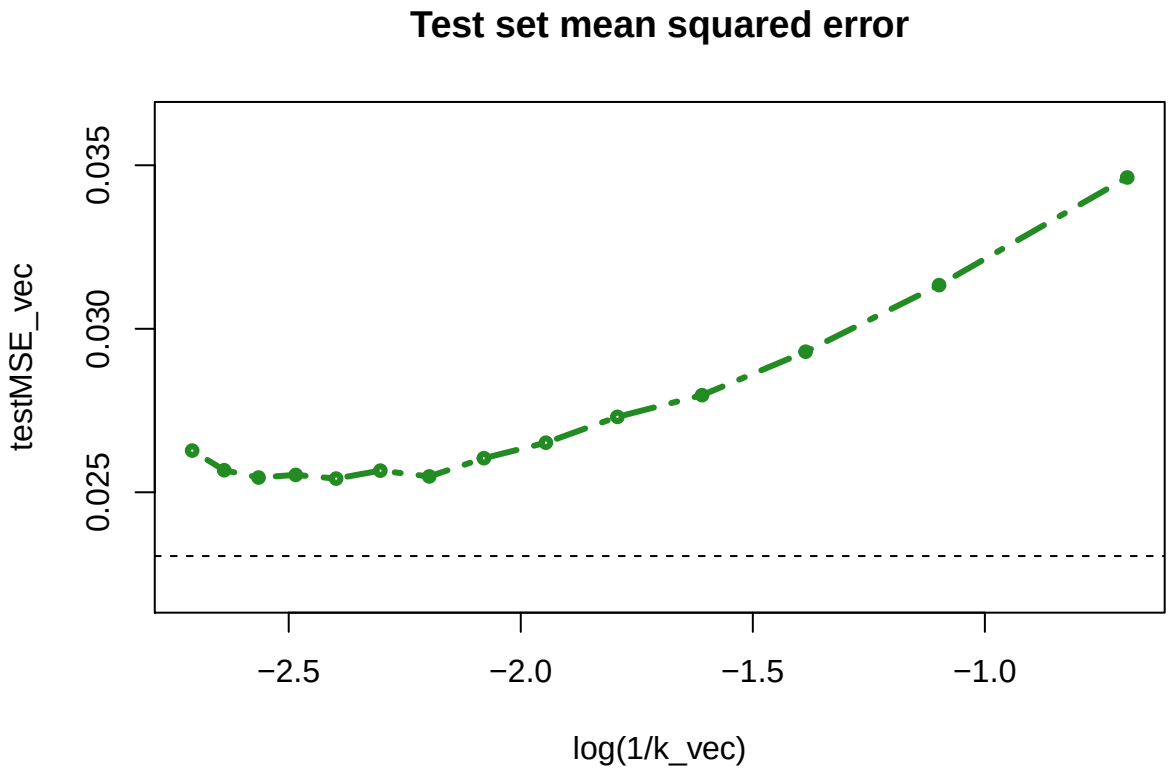
Plot k-NN fit with k=2 and k=12 respectively

```
plot_knn <- Plot(sim_linear, method="k-nn")
```



Plot test set MSE

```
plotMSE <- PlotTestError(sim_linear, plot_ls, plot_knn)
```



- Observation

Linear regression model outperforms k-NN algorithm for any value of k. In fact, when the true relationship is linear, it is hard for a non-parametric approach to compete with linear regression. Also notice that when the value of k is large, i.e. the model is simple, k-NN performs only a little worse than least square regression. It does far worse when k is small.

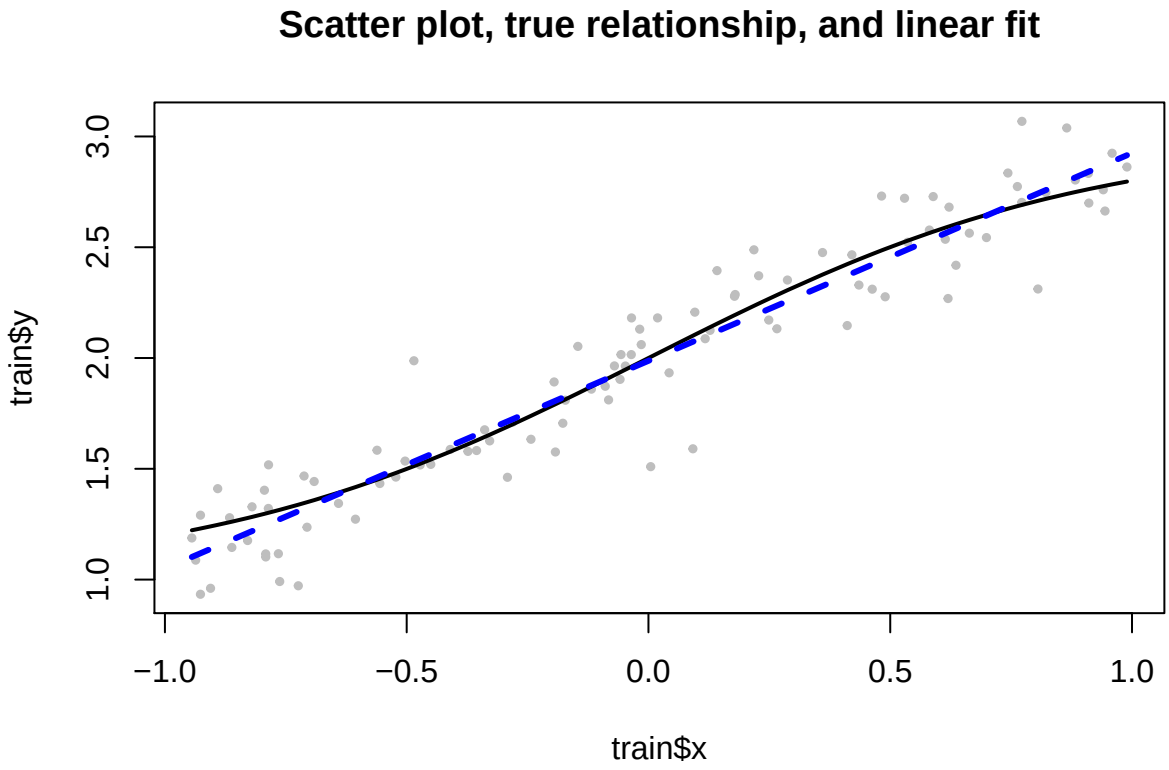
Almost linear model (part 1.6)

Define function

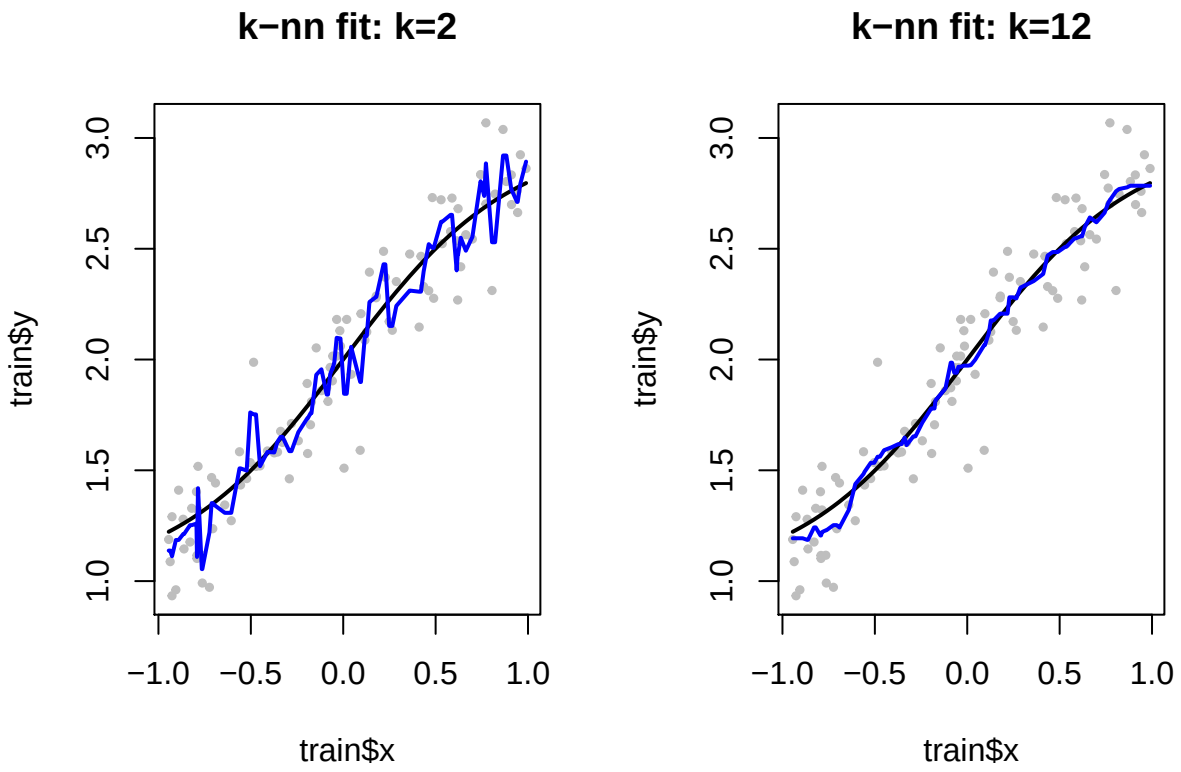
```
func_almost_linear <- function(x){tanh(1.1*x)+2}
```

Simulate data and plot

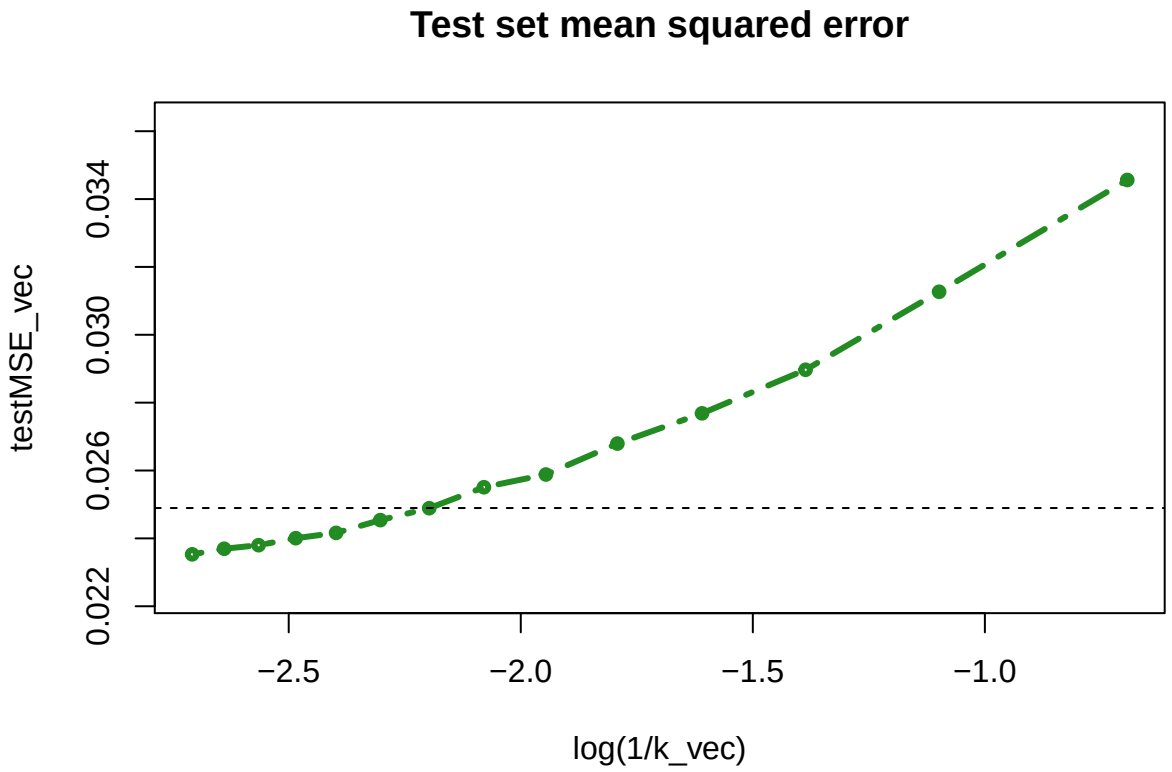
```
sim_aslinear <- Simulate(func_almost_linear, N_train, N_test)
plot_ls_aslinear <- Plot(sim_aslinear, method='linear regression')
```




```
plot_knn_aslinear <- Plot(sim_aslinear, method='k-nn')
```




```
plotMSE_aslinear <- PlotTestError(sim_aslinear, plot_ls_aslinear, plot_knn_aslinear)
```



- Observation

The test MSE for linear regression is superior to that of k-NN for low values of k. However, for larger k, k-NN outperforms linear regression.

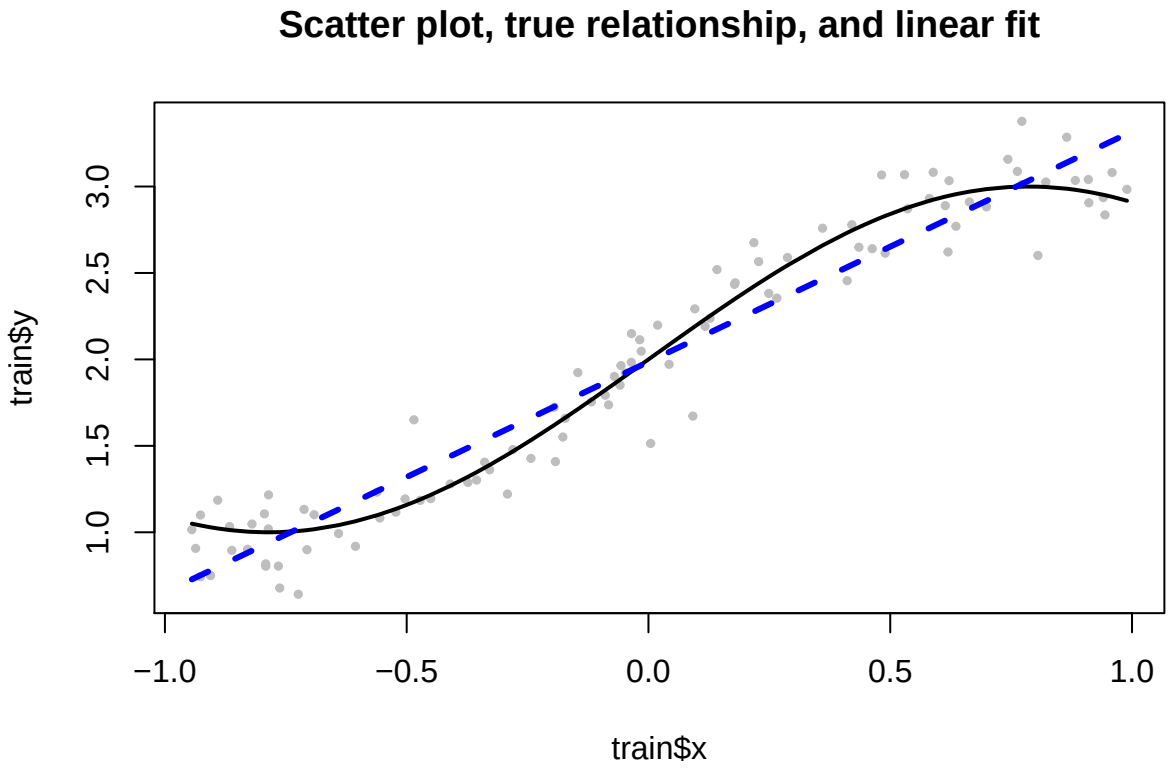
Strongly nonlinear model (part 1.7)

Define function

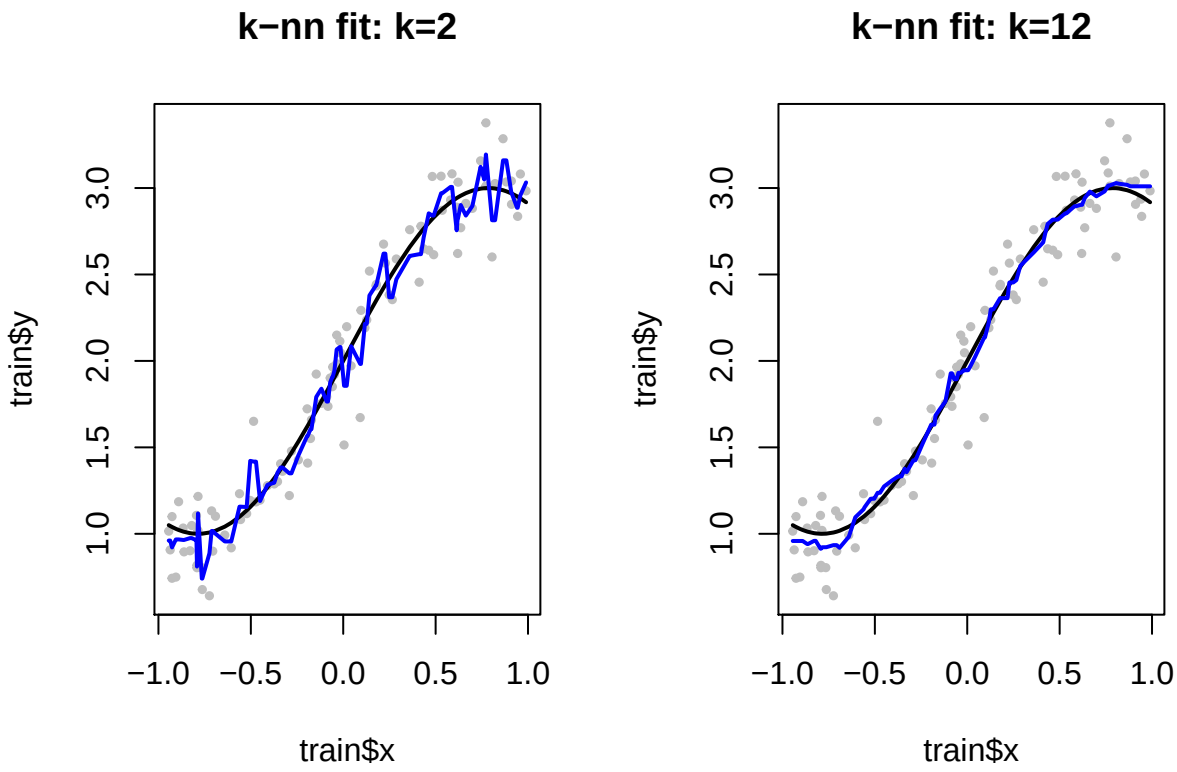
```
func_non_linear <- function(x){sin(2*x)+2}
```

Simulate data and plot

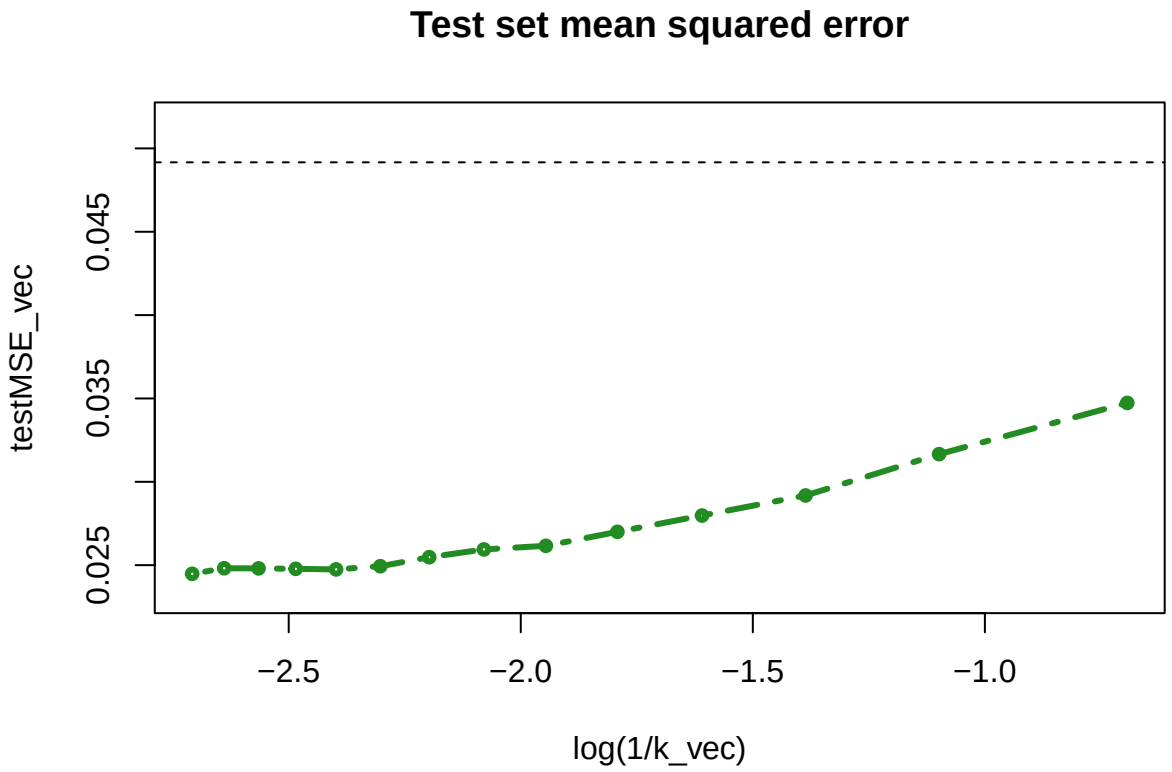
```
sim_nlinear <- Simulate(func_non_linear, N_train, N_test)
plot_ls_nlinear <- Plot(sim_nlinear, method='linear regression')
```




```
plot_knn_nlinear <- Plot(sim_nlinear, method='k-nn')
```




```
plotMSE_nlinear <- PlotTestError(sim_nlinear, plot_ls_nlinear, plot_knn_nlinear)
```



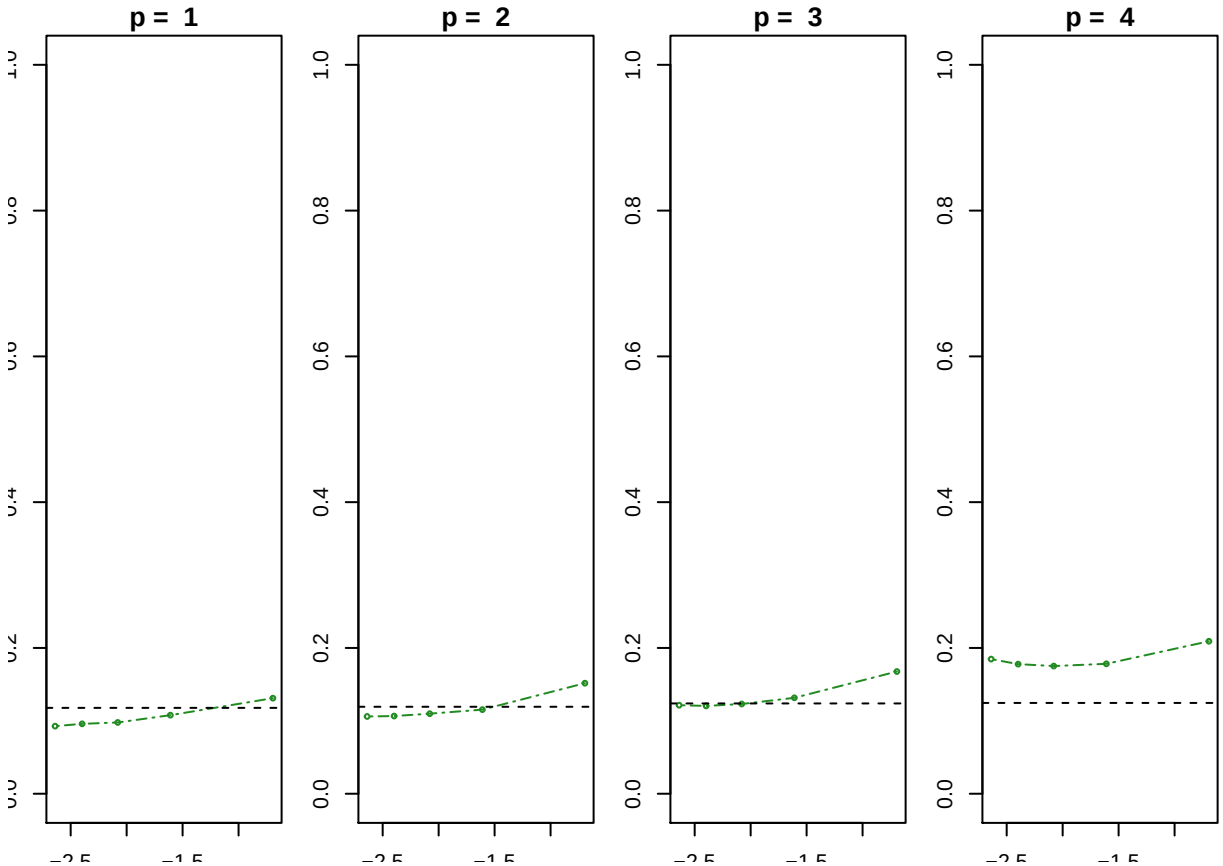
- Observation

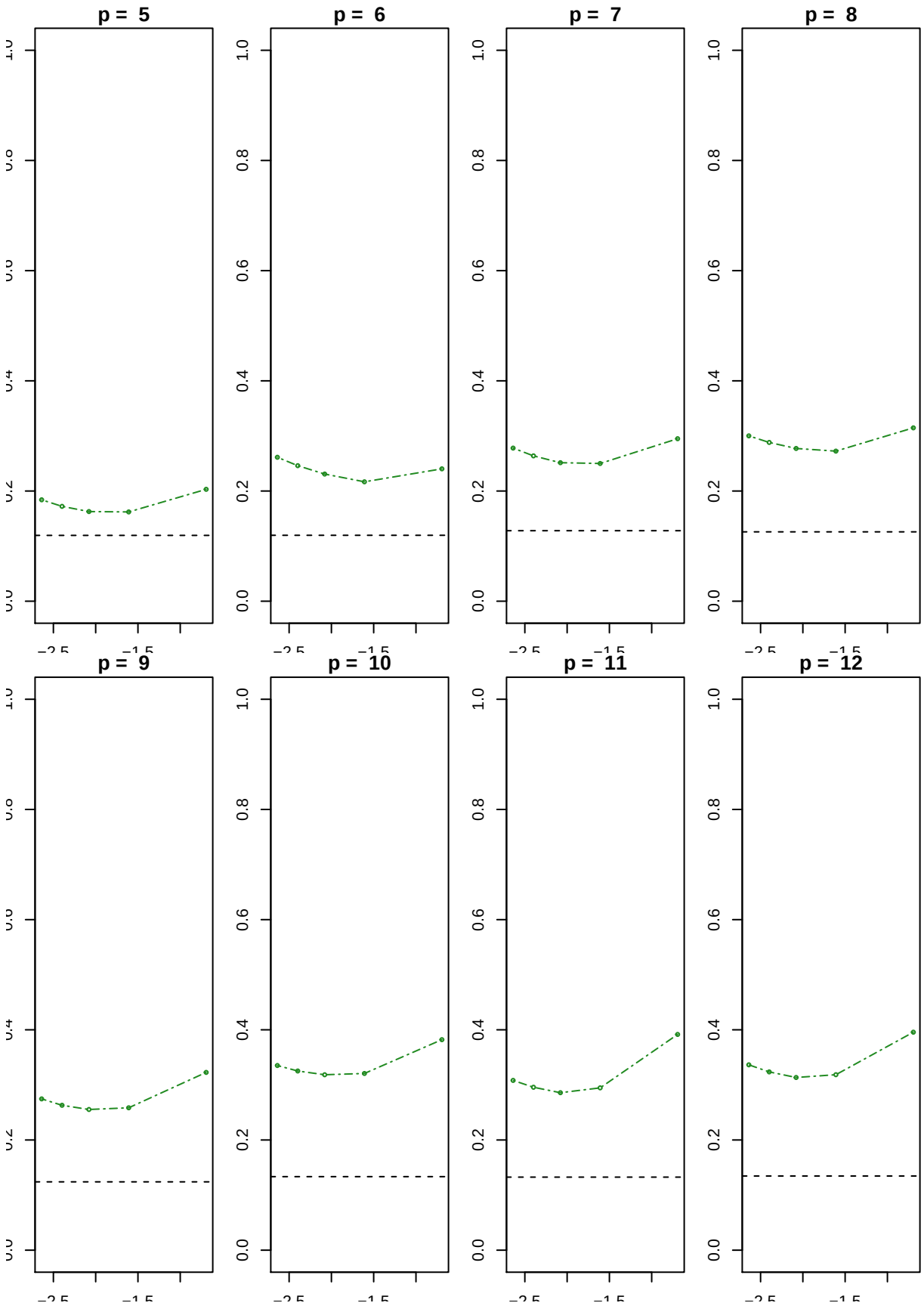
When the true relationship is highly non-linear, k-NN gives much better result than linear regression, for all values of k. The test MSE of linear regression increases dramatically with the extent of non-linearity while that of k-NN changes little.

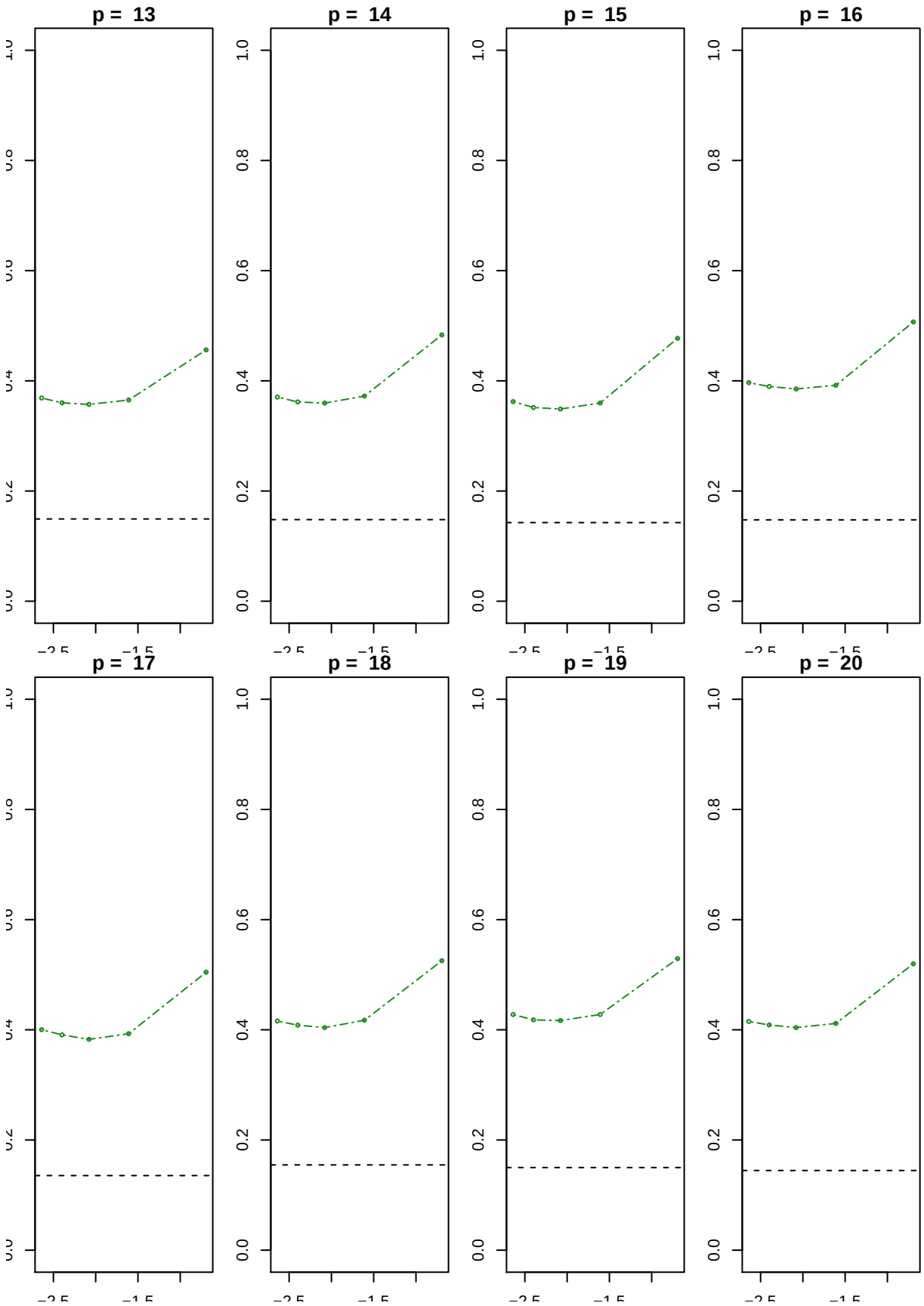
Generate data

```
Xtrain=matrix(runif(N_train*20, -1, 1), nrow=N_train)
Xtest=matrix(runif(N_test*20, -1, 1), nrow=N_test)
Ytrain=sin(2*Xtrain[,1])+2+rnorm(N_train)
Ytest=sin(2*Xtest[,1])+2+rnorm(N_test)

for (pp in 1:5) {
  par(mfrow = c(1, 4), mai=c(0.2,0.2,0.2,0.2))
  for (p in seq((pp-1)*4+1, (pp)*4)) {

    Xtrain=matrix(runif(N_train*p, -1, 1), nrow=N_train)
    Xtest=matrix(runif(N_test*p, -1, 1), nrow=N_test)
    Ytrain=sin(2*Xtrain[,1])+2+0.3*rnorm(N_train)
    Ytest=sin(2*Xtest[,1])+2+0.3*rnorm(N_test)

    train=data.table(y=Ytrain, Xtrain[,1:p])
    colnames(train)=c("y", paste0("x", 1:p))
    test=data.table(y=Ytest, Xtest[,1:p])
    colnames(test)=c("y", paste0("x", 1:p))
    linear=lm(y~., data=train)

    lsMSE=mean((predict(linear, test)-test$y)^2)
```

```
    k_vec=seq(2, 15, 3)
    testMSE_vec=numeric(length=length(k_vec))
    for (i in 1:length(k_vec)){
      near=kknn(y~., train=train, test=test, k=k_vec[i], kernel='rectangular')
      testMSE_vec[i]=mean((near$fitted-test$y)^2)
    }
    plot(log(1/k_vec), testMSE_vec,
         ylim = c(0.,1.),
         type="n")
    title(main=paste("p = ",p))
    lines(log(1/k_vec), testMSE_vec, lty=6, lwd=1, col="forestgreen", type="o", cex=0.5 )
    abline(h=lsMSE, col="black", lty=2)
  }
}
```




p = 5
p = 6
p = 7
p = 8
p = 9
p = 10
p = 11
p = 12
0.0
0.2
0.4
0.6
0.8
1.0
−2.5
−1.5


p = 13
p = 14
p = 15
p = 16
p = 17
p = 18
p = 19
p = 20
0.0
0.2
0.4
0.6
0.8
1.0

- Observation

The increase in dimension only causes a small deterioration in the linear regression test set MSE, but it causes substantial increase in the MSE for k-NN. This is the curse-of-dimensionality problem, which results from the fact that in higher dimensions there is effectively a reduction in sample size. The general rule is linear regression will outperform k-NN method when there is a small number of observations per predictor, though in our simulation exercise this relationship does not hold monotonically.

Optional bonus question

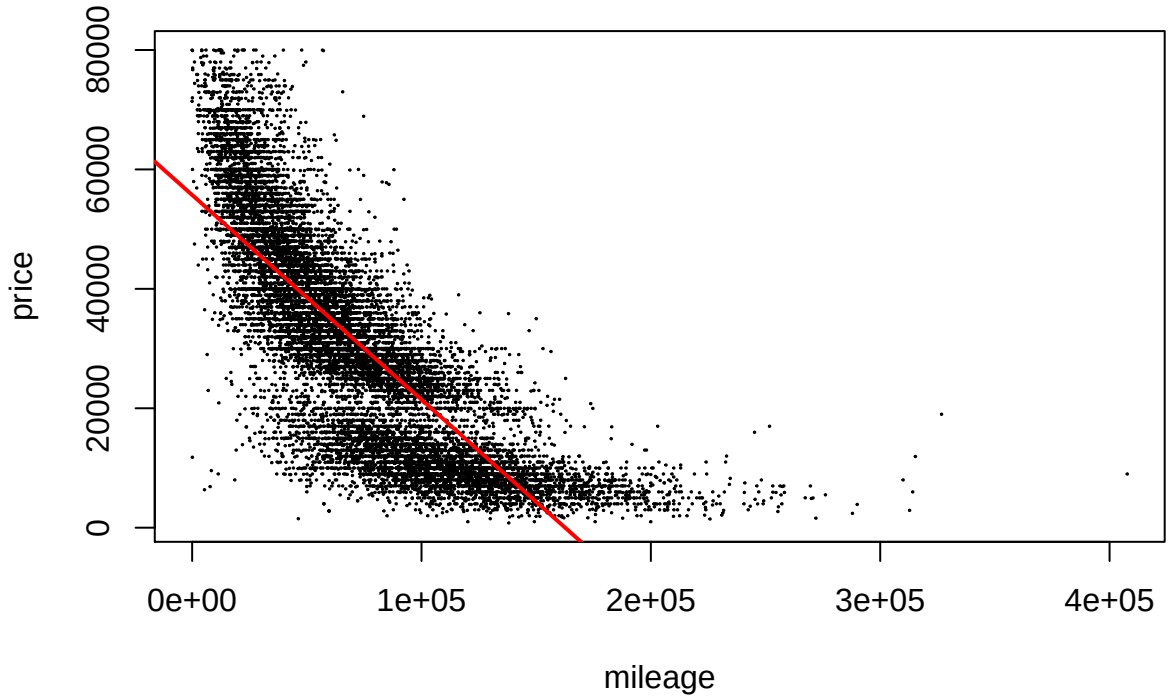
- Our conclusion concerning the relative performance of linear regression and k-NN as the extent of model non-linearity varies will ramain unchanged.
- We will have a wider range of values of k for which k-NN outperforms linear regression. In real world cases where the true relationship is barely linear, for any given p, an increase in the size of training sample boosts the number of sample points within each small neighborhood, which helps to improve the performance of k-NN. In other words, k-NN can now sustain more complex models without the peril of overfitting.
- Generally having a larger training set improves prediction accuracy in terms of test error by reducting the chance of overfitting.

Question 2

```
# set seed
set.seed(100)

# read data
download.file("https://github.com/ChicagoBoothML/MLClassData/raw/master/UsedCars/UsedCars.csv",
              "UsedCars.csv")
UsedCars <- read.csv(file="UsedCars.csv",head=TRUE,sep=",")

# splitting data into training and validation
N = dim(UsedCars)[1]
train_index = sample(N, size = N * 0.75, replace = FALSE)
train = UsedCars[train_index,]
test = UsedCars[-train_index, ]
```

Part 3

```
fit = lm(price ~ mileage, data = train)
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
abline(fit, col = "red", lwd = 2)
```



The linear regression can not capture the non-linear trend of the data.

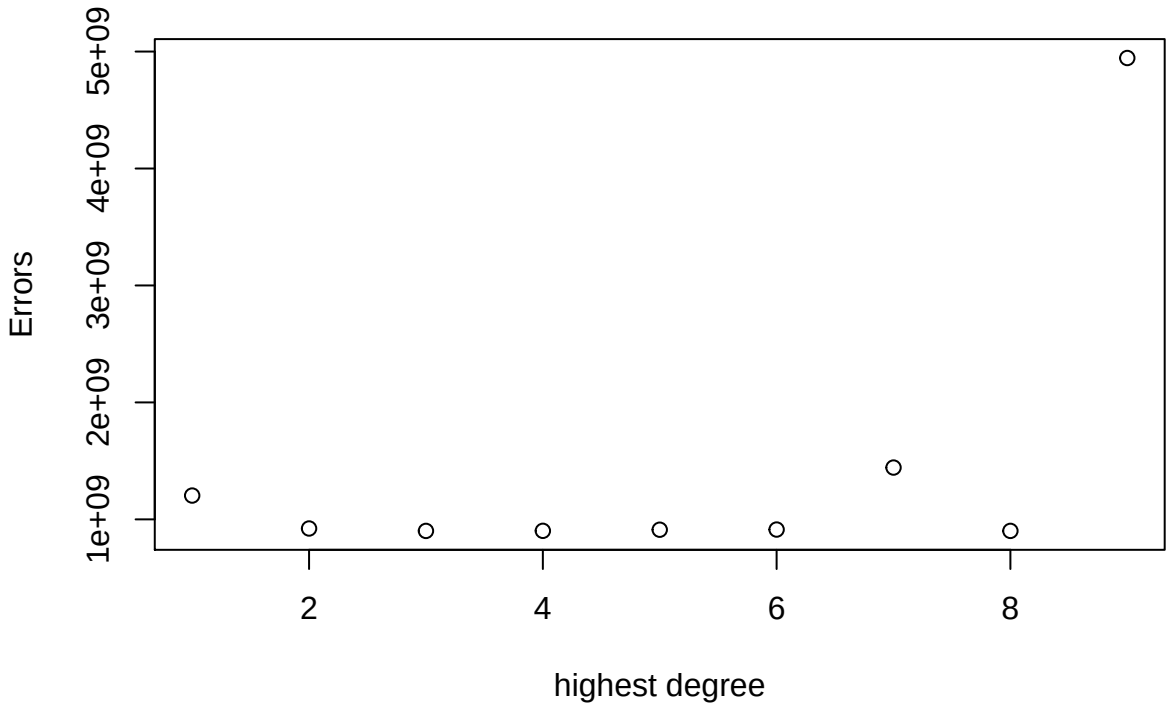
Part 4

```
docv = function(x,y,set,nfold=10,doran=TRUE,verbose=TRUE,...)
{
  #a little error checking
  x = as.matrix(x)
  set = matrix(set, ncol = 1)
  if(!(is.matrix(x) | is.data.frame(x))) {cat('error in docv: x is not a matrix or data frame\n'); retu
  if(!(is.vector(y))) {cat('error in docv: y is not a vector\n'); return(0)}
  if(!(length(y)==nrow(x))) {cat('error in docv: length(y) != nrow(x)\n'); return(0)}

  nset = nrow(set);
  n=length(y) #get dimensions

  if(n==nfold) doran=FALSE #no need to shuffle if you are doing them all.
  cat('in docv: nset,n,nfold: ',nset,n,nfold,'\n')
  lossv = rep(0,nset) #return values
  if(doran) {ii = sample(1:n,n); y=y[ii]; x=x[ii,,drop=FALSE]} #shuffle rows

  fs = round(n/nfold) # fold size
  for(i in 1:nfold) { #fold loop
    bot = (i-1)*fs+1;
    top = ifelse(i==nfold,n,i*fs);
    ii = bot:top
    if(verbose) cat('on fold: ',i,', range: ',bot,':',top,'\n')
    xin = x[-ii,,drop=FALSE];
    yin=y[-ii];
    xout=x[ii,,drop=FALSE];
```

```
    yout=y[ii]
    xin = as.vector(xin)
    xout = as.vector(xout)
    datain = data.frame(x = xin, y = yin)
    dataout = data.frame(x = xout, y = yout)
    for(k in 1:nset)
    { #setting loop
      fit = lm(y ~ poly(x, set[k,]), data = datain)
      yhat = predict(fit,  newdata = dataout)
      lossv[k]=lossv[k]+mean((yout-yhat)^2)
    }
  }
  return(lossv)
}



#### run cv
D = c(1,2,3,4,5,6, 7, 8, 9) # potential degrees
errors = docv(train$mileage, train$price, D, nfold = 10)
```

```
## in docv: nset,n,nfold:  9 15047 10
## on fold:  1 , range:  1 : 1505
## on fold:  2 , range:  1506 : 3010
## on fold:  3 , range:  3011 : 4515
## on fold:  4 , range:  4516 : 6020
## on fold:  5 , range:  6021 : 7525
## on fold:  6 , range:  7526 : 9030
## on fold:  7 , range:  9031 : 10535
## on fold:  8 , range:  10536 : 12040
## on fold:  9 , range:  12041 : 13545
## on fold:  10 , range:  13546 : 15047
```

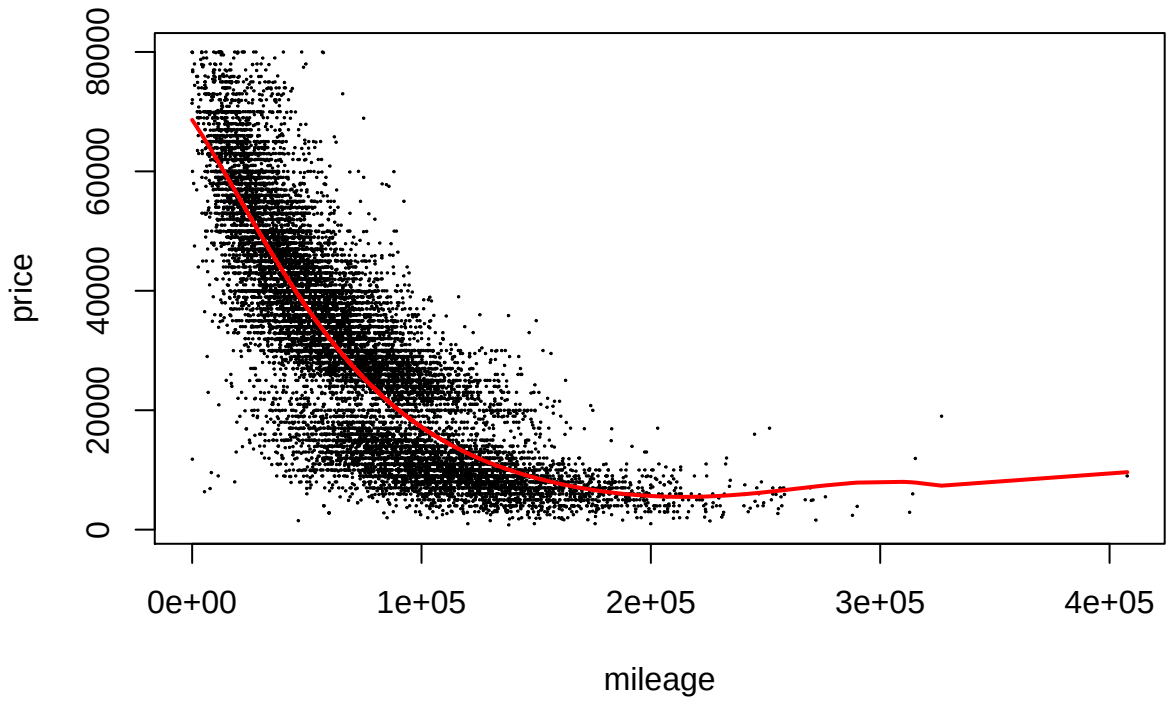
```
plot(D, errors, xlab = "highest degree", ylab = "Errors")
```




```
best.degree = D[which.min(errors)] # find the best degree

best.degree
```

```
## [1] 3
```

```
fit = lm(price ~ poly(mileage, 7), data = train)
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
fitted = predict(fit, newdata = data.frame(mileage = sort(train$mileage)))
lines(sort(train$mileage),fitted,col="red",lwd=2, cex.lab=2)
```



```r
# error
sqrt(mean((test$price -predict(fit, test))^2))
```

```
## [1] 10537.12
```

Due to randomness of the code, the result might be different for each run.

Part 5

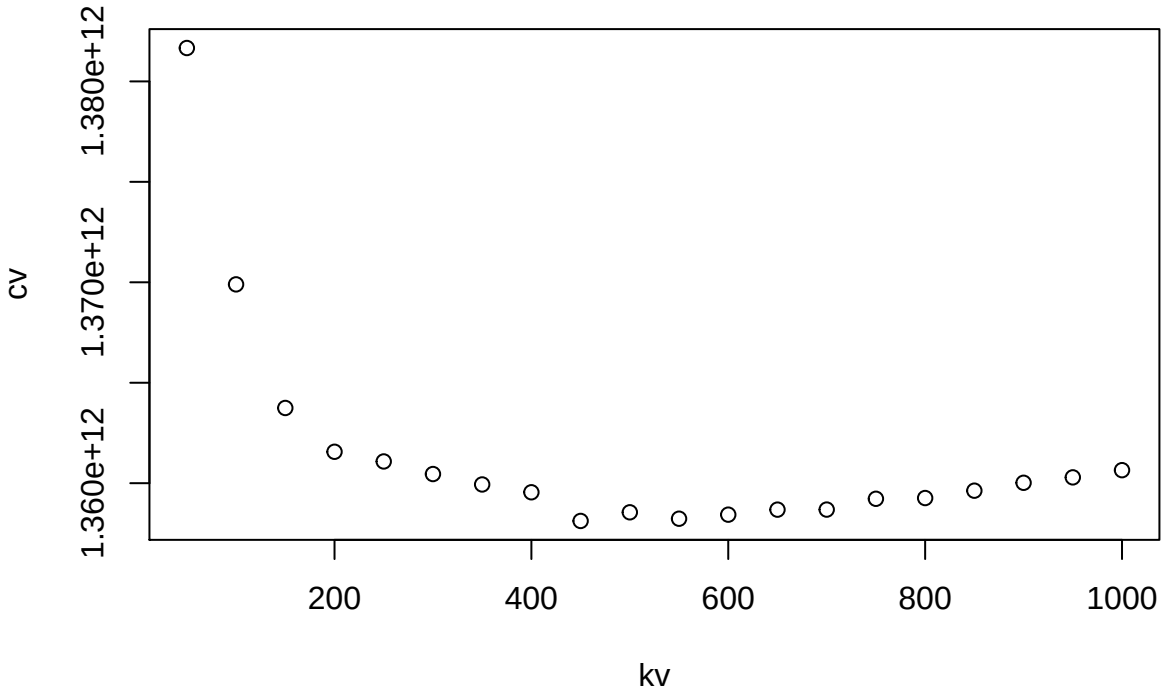
```r
##########
## kNN
## Warning: super slow
##########

library(MASS)
```

```
## Warning: package 'MASS' was built under R version 3.4.3
```

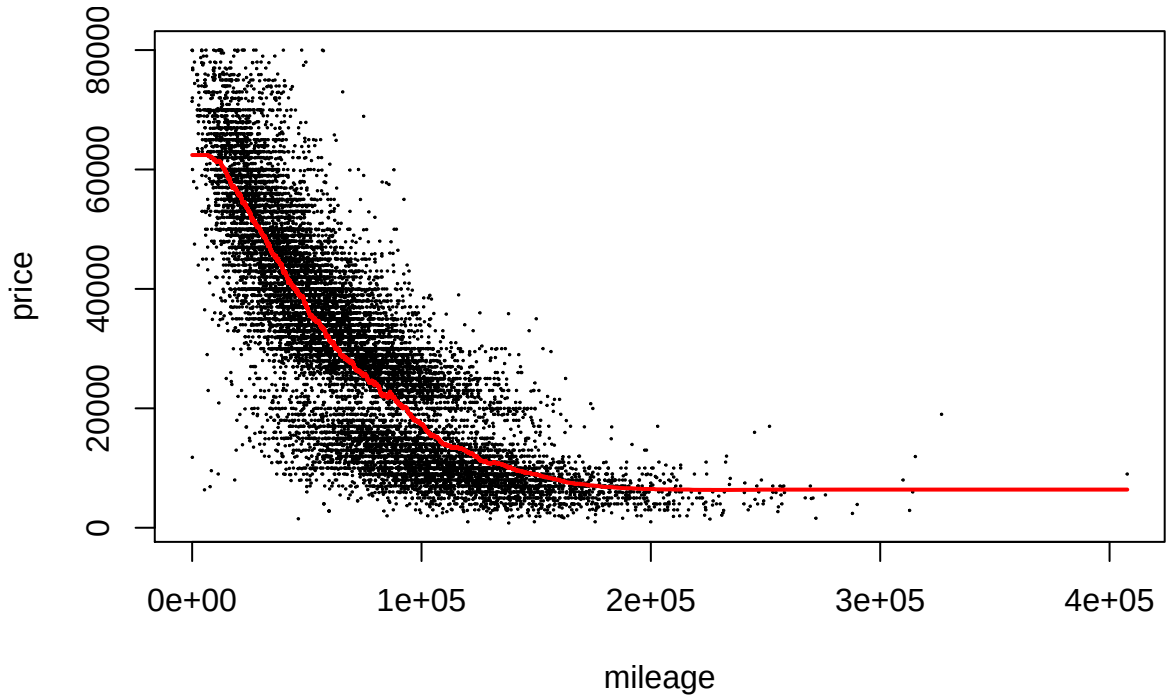
```r
library(kknn)
download.file("https://raw.githubusercontent.com/ChicagoBoothML/HelpR/master/docv.R", "docv.R")
source("docv.R") #this has docvknn used below
set.seed(99) #always set the seed!

kv = 1:20 * 50   #these are the k values (k as in kNN) we will try
# since the possible k might be very large, we pick k every 50, from 50 to 1000

#does cross-validation for training data (x,y).

cv = docvknn(matrix(train$mileage,ncol=1),train$price,kv,nfold=10)
```

```
## in docv: nset,n,nfold:  20 15047 10
## on fold:  1 , range:  1 : 1505
## on fold:  2 , range:  1506 : 3010
## on fold:  3 , range:  3011 : 4515
## on fold:  4 , range:  4516 : 6020
## on fold:  5 , range:  6021 : 7525
## on fold:  6 , range:  7526 : 9030
## on fold:  7 , range:  9031 : 10535
## on fold:  8 , range:  10536 : 12040
## on fold:  9 , range:  12041 : 13545
## on fold:  10 , range:  13546 : 15047
```

```r
plot(kv, cv)
```




```
kbest = kv[which.min(cv)]

kfbest = kknn(price~mileage,train,data.frame(mileage=sort(train$mileage)),k=kbest,kernel = "rectangular
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
lines(sort(train$mileage),kfbest$fitted,col="red",lwd=2, cex.lab=2)
```




```
# error
kfbest = kknn(price~mileage,train,test,k=kbest,kernel = "rectangular")
sqrt(mean((test$price - kfbest$fitted.values)^2))
```

```
## [1] 9471.33
```

```
##########
## Tree
##########

library(rpart)      # package for trees
library(rpart.plot) # package that enhances plotting capabilities for rpart
library(MASS)  # contains boston housing data


big.tree = rpart(price~mileage, data=train,
      control=rpart.control(minsplit=5,cp=0.0001,xval=10))

nbig = length(unique(big.tree$where))
cat('size of big tree: ',nbig,'\n')
```

```
## size of big tree:  158
```

```
(cptable = printcp(big.tree))
```

```
##
## Regression tree:
## rpart(formula = price ~ mileage, data = train, control = rpart.control(minsplit = 5,
##     cp = 1e-04, xval = 10))
##
## Variables actually used in tree construction:
## [1] mileage
##
## Root node error: 5.008e+12/15047 = 332822195
##
## n= 15047
##
##            CP nsplit rel error  xerror      xstd
## 1  0.53908774      0   1.00000 1.00012 0.0091403
## 2  0.07836283      1   0.46091 0.46211 0.0052704
## 3  0.06683151      2   0.38255 0.38425 0.0050372
## 4  0.01045803      3   0.31572 0.31788 0.0043603
## 5  0.00912934      4   0.30526 0.30777 0.0042855
## 6  0.00766216      5   0.29613 0.29937 0.0042399
## 7  0.00738518      6   0.28847 0.29168 0.0041789
## 8  0.00178057      7   0.28108 0.28469 0.0041277
## 9  0.00157085      8   0.27930 0.28273 0.0041088
## 10 0.00134710      9   0.27773 0.28156 0.0041206
## 11 0.00132704     10   0.27638 0.28052 0.0041345
## 12 0.00126318     11   0.27506 0.27973 0.0041277
## 13 0.00110278     12   0.27379 0.27849 0.0041112
## 14 0.00103506     13   0.27269 0.27704 0.0040984
## 15 0.00091311     14   0.27166 0.27612 0.0040947
## 16 0.00043113     15   0.27074 0.27475 0.0040803
## 17 0.00040747     16   0.27031 0.27520 0.0041377
## 18 0.00029303     17   0.26990 0.27489 0.0041368
## 19 0.00028726     18   0.26961 0.27490 0.0041626
## 20 0.00025328     21   0.26875 0.27486 0.0041614
## 21 0.00025269     23   0.26824 0.27508 0.0041753
```

```
## 22 0.00022213     24   0.26799 0.27489 0.0041764
## 23 0.00020287     25   0.26777 0.27495 0.0041802
## 24 0.00018330     26   0.26757 0.27651 0.0042260
## 25 0.00017980     27   0.26738 0.27692 0.0042314
## 26 0.00016158     28   0.26720 0.27726 0.0042383
## 27 0.00015544     29   0.26704 0.27893 0.0042699
## 28 0.00014731     35   0.26611 0.28012 0.0043119
## 29 0.00014230     44   0.26478 0.28121 0.0043374
## 30 0.00014075     46   0.26450 0.28224 0.0043587
## 31 0.00014058     48   0.26422 0.28255 0.0043676
## 32 0.00013823     49   0.26408 0.28261 0.0043695
## 33 0.00013642     57   0.26280 0.28309 0.0043741
## 34 0.00013007     66   0.26158 0.28430 0.0043885
## 35 0.00012671     74   0.26054 0.28505 0.0043973
## 36 0.00012496     75   0.26041 0.28611 0.0044358
## 37 0.00012488     82   0.25950 0.28626 0.0044364
## 38 0.00012329     84   0.25925 0.28628 0.0044364
## 39 0.00012118     85   0.25912 0.28652 0.0044388
## 40 0.00012094     86   0.25900 0.28680 0.0044382
## 41 0.00011797     89   0.25860 0.28718 0.0044439
## 42 0.00011733     90   0.25848 0.28770 0.0044587
## 43 0.00011518    101   0.25715 0.28818 0.0044698
## 44 0.00011452    108   0.25635 0.28859 0.0044759
## 45 0.00011401    109   0.25623 0.28863 0.0044749
## 46 0.00011353    110   0.25612 0.28868 0.0044769
## 47 0.00011290    111   0.25600 0.28873 0.0044682
## 48 0.00011170    113   0.25578 0.28874 0.0044676
## 49 0.00011024    118   0.25520 0.28909 0.0044589
## 50 0.00011006    119   0.25509 0.28933 0.0044742
## 51 0.00010932    120   0.25498 0.28941 0.0044714
## 52 0.00010537    132   0.25358 0.29131 0.0044829
## 53 0.00010398    136   0.25314 0.29348 0.0045150
## 54 0.00010371    137   0.25303 0.29362 0.0045164
## 55 0.00010248    149   0.25171 0.29408 0.0045252
## 56 0.00010151    156   0.25095 0.29446 0.0045291
## 57 0.00010000    157   0.25085 0.29672 0.0045885
```

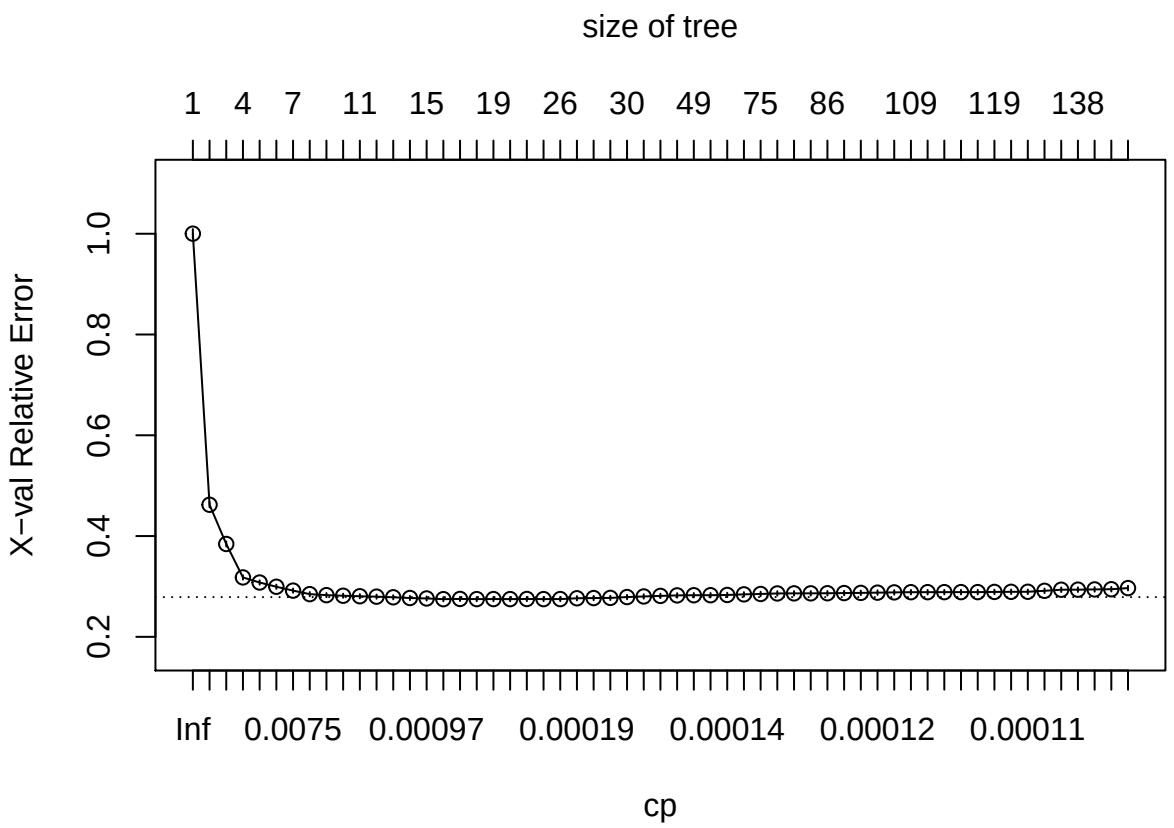
```
##              CP nsplit rel error    xerror        xstd
## 1  0.5390877357      0 1.0000000 1.0001207 0.009140319
## 2  0.0783628292      1 0.4609123 0.4621082 0.005270426
## 3  0.0668315064      2 0.3825494 0.3842476 0.005037162
## 4  0.0104580334      3 0.3157179 0.3178814 0.004360294
## 5  0.0091293448      4 0.3052599 0.3077734 0.004285485
## 6  0.0076621629      5 0.2961306 0.2993714 0.004239868
## 7  0.0073851837      6 0.2884684 0.2916778 0.004178879
## 8  0.0017805692      7 0.2810832 0.2846904 0.004127741
## 9  0.0015708518      8 0.2793026 0.2827267 0.004108769
## 10 0.0013470994      9 0.2777318 0.2815625 0.004120566
## 11 0.0013270404     10 0.2763847 0.2805225 0.004134491
## 12 0.0012631817     11 0.2750576 0.2797316 0.004127658
## 13 0.0011027784     12 0.2737945 0.2784881 0.004111244
## 14 0.0010350642     13 0.2726917 0.2770408 0.004098444
## 15 0.0009131128     14 0.2716566 0.2761213 0.004094681
## 16 0.0004311265     15 0.2707435 0.2747516 0.004080315
```

```
## 17 0.0004074681     16 0.2703124 0.2752013 0.004137656
## 18 0.0002930337     17 0.2699049 0.2748866 0.004136775
## 19 0.0002872586     18 0.2696119 0.2748980 0.004162551
## 20 0.0002532837     21 0.2687501 0.2748595 0.004161396
## 21 0.0002526870     23 0.2682435 0.2750782 0.004175347
## 22 0.0002221260     24 0.2679908 0.2748920 0.004176384
## 23 0.0002028733     25 0.2677687 0.2749462 0.004180235
## 24 0.0001832985     26 0.2675658 0.2765097 0.004225985
## 25 0.0001798018     27 0.2673825 0.2769152 0.004231387
## 26 0.0001615783     28 0.2672027 0.2772587 0.004238251
## 27 0.0001554357     29 0.2670412 0.2789293 0.004269940
## 28 0.0001473055     35 0.2661086 0.2801205 0.004311867
## 29 0.0001422984     44 0.2647824 0.2812088 0.004337355
## 30 0.0001407465     46 0.2644978 0.2822362 0.004358691
## 31 0.0001405768     48 0.2642163 0.2825452 0.004367595
## 32 0.0001382300     49 0.2640758 0.2826120 0.004369508
## 33 0.0001364228     57 0.2628049 0.2830919 0.004374056
## 34 0.0001300727     66 0.2615771 0.2843005 0.004388472
## 35 0.0001267091     74 0.2605365 0.2850488 0.004397319
## 36 0.0001249585     75 0.2604098 0.2861134 0.004435758
## 37 0.0001248838     82 0.2594980 0.2862623 0.004436365
## 38 0.0001232930     84 0.2592482 0.2862825 0.004436424
## 39 0.0001211767     85 0.2591249 0.2865154 0.004438816
## 40 0.0001209394     86 0.2590037 0.2868041 0.004438190
## 41 0.0001179662     89 0.2585956 0.2871786 0.004443877
## 42 0.0001173263     90 0.2584777 0.2876969 0.004458678
## 43 0.0001151809    101 0.2571519 0.2881816 0.004469789
## 44 0.0001145172    108 0.2563457 0.2885877 0.004475851
## 45 0.0001140117    109 0.2562312 0.2886310 0.004474892
## 46 0.0001135314    110 0.2561171 0.2886762 0.004476863
## 47 0.0001129039    111 0.2560036 0.2887261 0.004468185
## 48 0.0001116955    113 0.2557778 0.2887443 0.004467568
## 49 0.0001102378    118 0.2552003 0.2890862 0.004458926
## 50 0.0001100552    119 0.2550900 0.2893328 0.004474158
## 51 0.0001093245    120 0.2549800 0.2894150 0.004471449
## 52 0.0001053704    132 0.2535835 0.2913079 0.004482856
## 53 0.0001039798    136 0.2531352 0.2934839 0.004515004
## 54 0.0001037116    137 0.2530312 0.2936236 0.004516425
## 55 0.0001024819    149 0.2517074 0.2940842 0.004525157
## 56 0.0001015093    156 0.2509533 0.2944600 0.004529065
## 57 0.0001000000    157 0.2508518 0.2967185 0.004588457
```

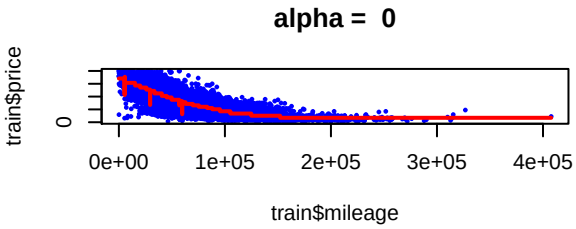
```
(bestcp = cptable[ which.min(cptable[,"xerror"]), "CP" ])   # this is the optimal cp parameter
```

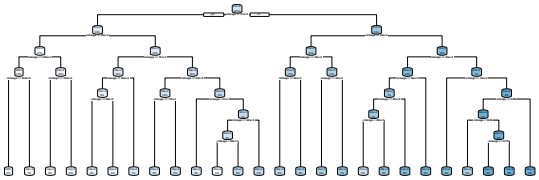
```
## [1] 0.0004311265
```

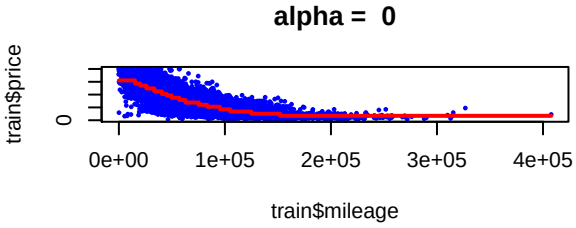
```
plotcp(big.tree) # plot results
```

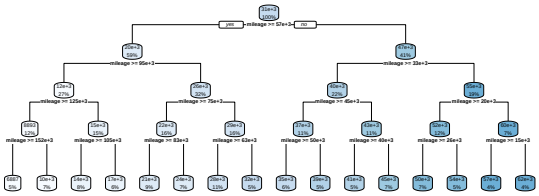


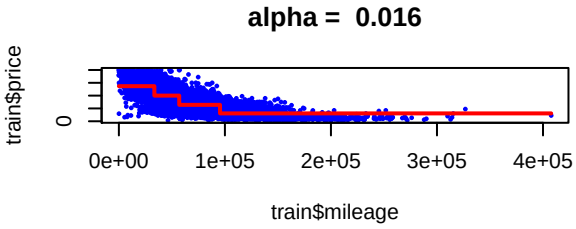

```
# show fit from some trees
oo = order(train$mileage)
cpvec = c(bestcp / 2, bestcp,.0157)

par(mfrow=c(3,2))
for(i in 1:3) {
  plot(train$mileage,train$price,pch=16,col='blue',cex=.5)
  ptree = prune(big.tree,cp=cpvec[i])
  pfit = predict(ptree)
  lines(train$mileage[oo],pfit[oo],col='red',lwd=2)
  title(paste('alpha = ',round(cpvec[i],3)))
  rpart.plot(ptree)
}
```

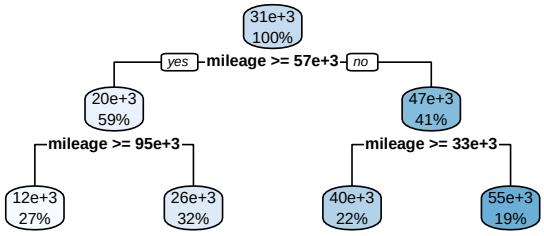




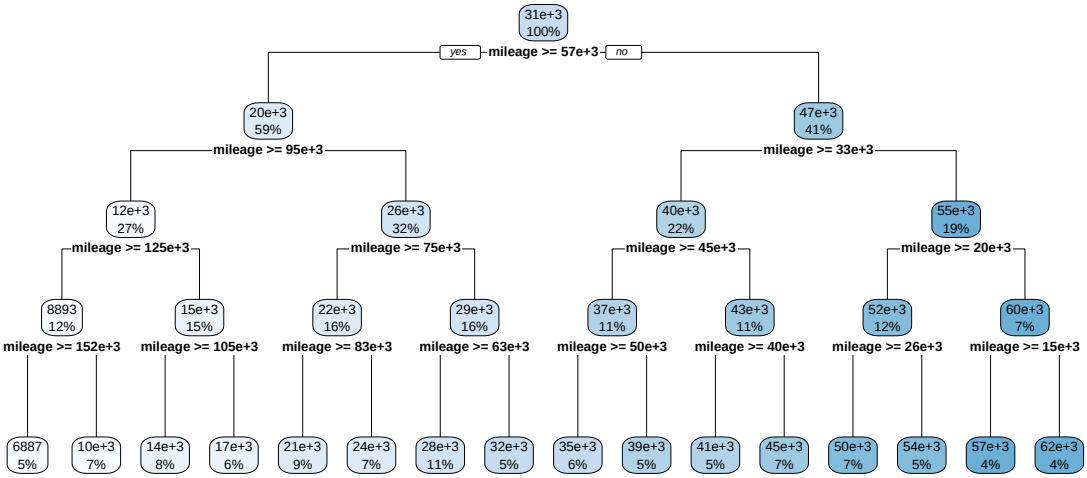




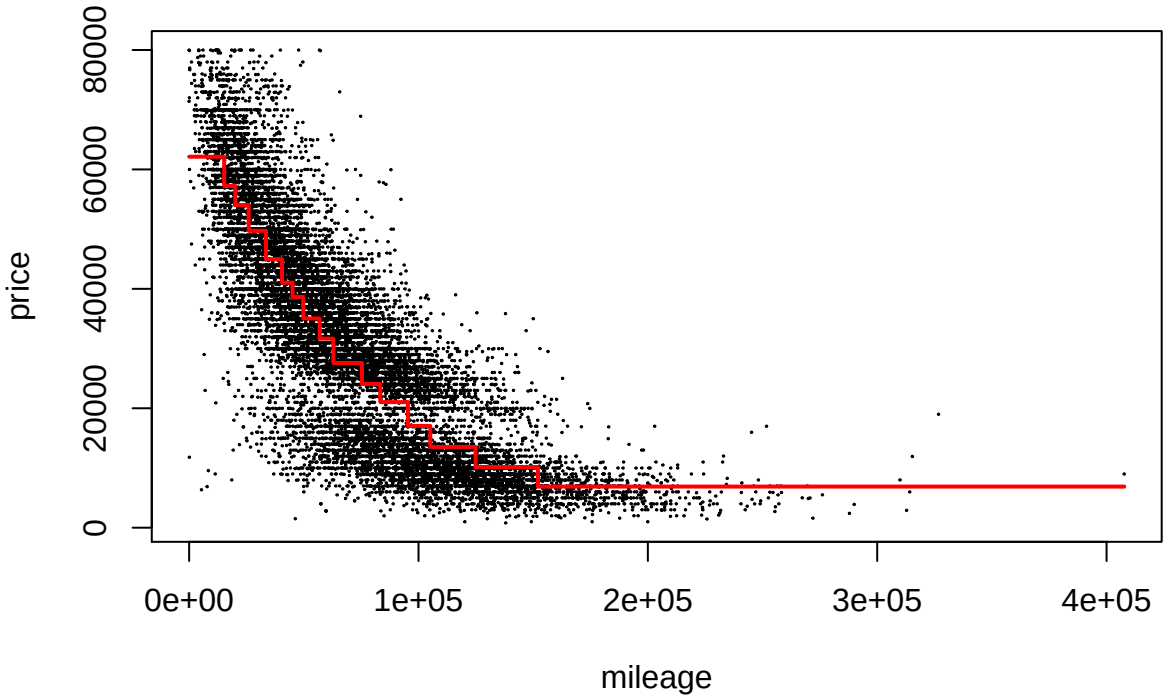





```
par(mfrow=c(1,1))
best.tree = prune(big.tree,cp=bestcp)
rpart.plot(best.tree)
```




```
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
lines(sort(train$mileage),predict(best.tree)[oo],col="red",lwd=2,cex.lab=2)
```




```
# error
sqrt(mean((test$price - predict(best.tree, test))^2))
```

```
## [1] 9545.132
```

From figures above, I would choose kNN because itâ€™s robust and smooth. Tree and high-order polynomials are less smooth and very sensitive to extreme values.

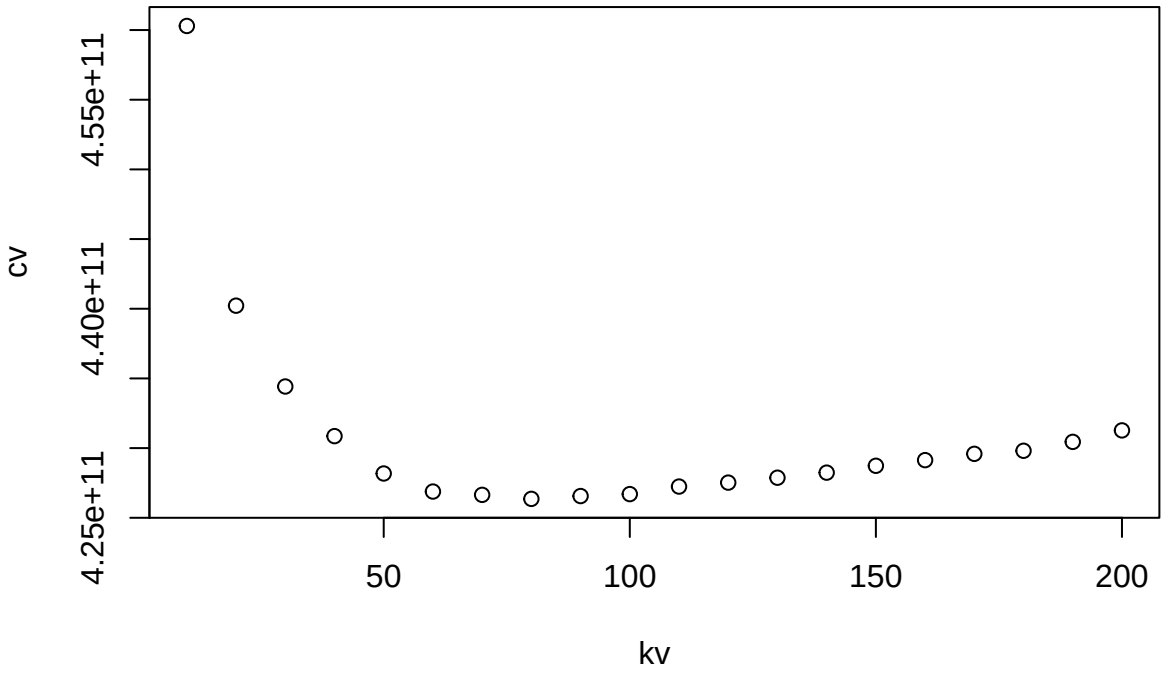
Part 6

```
################################################################################
# 6
# Add one more variable

##########
## kNN
## Warning: super slow
##########



library(MASS)
library(kknn)
set.seed(99) #always set the seed!

kv = 1:20 * 10

cv = docvknn(scale(as.matrix(cbind(train$mileage, train$year))),train$price,kv,nfold=10)
```

```
## in docv: nset,n,nfold:  20 15047 10
## on fold:  1 , range:  1 : 1505
```

```
## on fold:  2 , range:  1506 : 3010
## on fold:  3 , range:  3011 : 4515
## on fold:  4 , range:  4516 : 6020
## on fold:  5 , range:  6021 : 7525
## on fold:  6 , range:  7526 : 9030
## on fold:  7 , range:  9031 : 10535
## on fold:  8 , range:  10536 : 12040
## on fold:  9 , range:  12041 : 13545
## on fold:  10 , range:  13546 : 15047
```

```
plot(kv, cv)
```

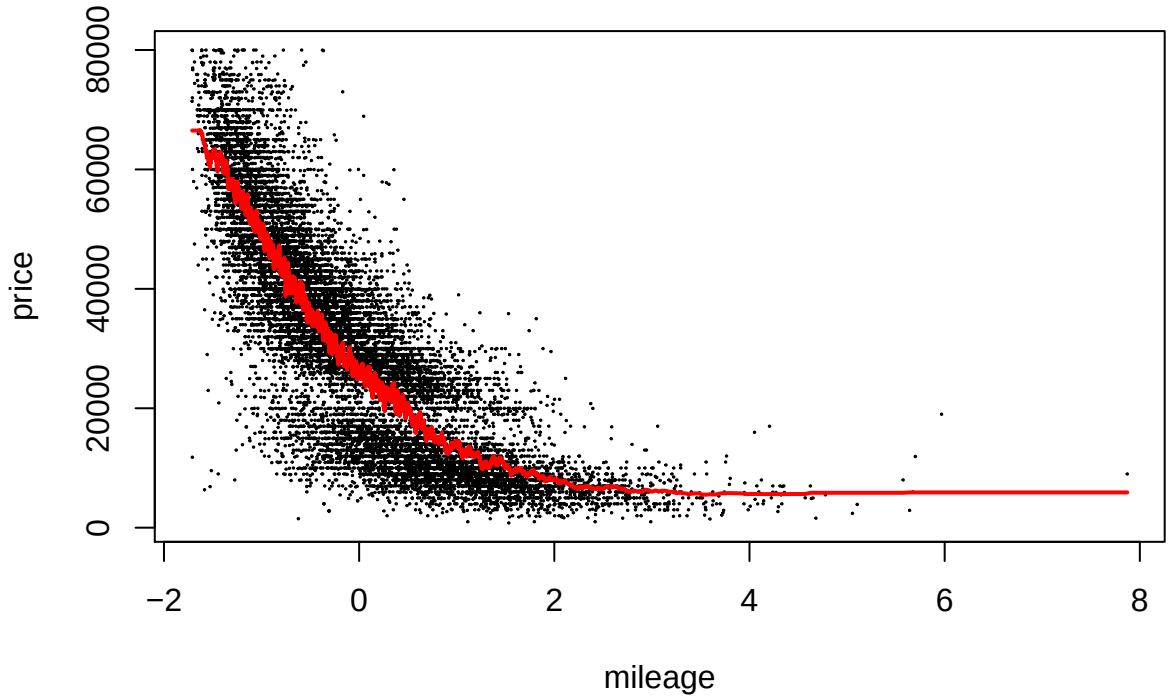



```
kbest = kv[which.min(cv)]

train.scaled = train
train.scaled$year = scale(train.scaled$year)
train.scaled$mileage = scale(train.scaled$mileage)
test.scaled = test
test.scaled$year = scale(test.scaled$year)
test.scaled$mileage = scale(test.scaled$mileage)

kfbest = kknn(price~mileage+price,train.scaled,data.frame(mileage=sort(train.scaled$mileage)),k=kbest,k
```

```
## Warning in model.matrix.default(mt, mf): the response appeared on the
## right-hand side and was dropped
```

```
## Warning in model.matrix.default(mt, mf): problem with term 2 in
## model.matrix: no columns are assigned
```

```
plot(train.scaled$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
lines(sort(train.scaled$mileage),kfbest$fitted,col="red",lwd=2, cex.lab=2)
```




```
# error
kfbest = kknn(price~mileage,train.scaled,test.scaled,k=kbest,kernel = "rectangular")
sqrt(mean((test.scaled$price - kfbest$fitted.values)^2))
```

```
## [1] 9547.901
```

```
##########
## Tree
##########
library(rpart)      # package for trees
library(rpart.plot) # package that enhances plotting capabilities for rpart
library(MASS)  # contains boston housing data


big.tree = rpart(price~mileage+price, data=train,
                 control=rpart.control(minsplit=5,cp=0.0001,xval=10))
```

```
## Warning in model.matrix.default(attr(frame, "terms"), frame): the response
## appeared on the right-hand side and was dropped
```

```
## Warning in model.matrix.default(attr(frame, "terms"), frame): problem with
## term 2 in model.matrix: no columns are assigned
```

```
nbig = length(unique(big.tree$where))
cat('size of big tree: ',nbig,'\n')
```

```
## size of big tree:  158
```

```
(cptable = printcp(big.tree))
```

```
##
## Regression tree:
## rpart(formula = price ~ mileage + price, data = train, control = rpart.control(minsplit = 5,
##     cp = 1e-04, xval = 10))
##
```

```
## Variables actually used in tree construction:
## [1] mileage
##
## Root node error: 5.008e+12/15047 = 332822195
##
## n= 15047
##
##            CP nsplit rel error  xerror      xstd
## 1  0.53908774      0   1.00000 1.00012 0.0091403
## 2  0.07836283      1   0.46091 0.46211 0.0052704
## 3  0.06683151      2   0.38255 0.38425 0.0050372
## 4  0.01045803      3   0.31572 0.31788 0.0043603
## 5  0.00912934      4   0.30526 0.30777 0.0042855
## 6  0.00766216      5   0.29613 0.29937 0.0042399
## 7  0.00738518      6   0.28847 0.29168 0.0041789
## 8  0.00178057      7   0.28108 0.28469 0.0041277
## 9  0.00157085      8   0.27930 0.28273 0.0041088
## 10 0.00134710      9   0.27773 0.28156 0.0041206
## 11 0.00132704     10   0.27638 0.28052 0.0041345
## 12 0.00126318     11   0.27506 0.27973 0.0041277
## 13 0.00110278     12   0.27379 0.27849 0.0041112
## 14 0.00103506     13   0.27269 0.27704 0.0040984
## 15 0.00091311     14   0.27166 0.27612 0.0040947
## 16 0.00043113     15   0.27074 0.27475 0.0040803
## 17 0.00040747     16   0.27031 0.27520 0.0041377
## 18 0.00029303     17   0.26990 0.27489 0.0041368
## 19 0.00028726     18   0.26961 0.27490 0.0041626
## 20 0.00025328     21   0.26875 0.27486 0.0041614
## 21 0.00025269     23   0.26824 0.27508 0.0041753
## 22 0.00022213     24   0.26799 0.27489 0.0041764
## 23 0.00020287     25   0.26777 0.27495 0.0041802
## 24 0.00018330     26   0.26757 0.27651 0.0042260
## 25 0.00017980     27   0.26738 0.27692 0.0042314
## 26 0.00016158     28   0.26720 0.27726 0.0042383
## 27 0.00015544     29   0.26704 0.27893 0.0042699
## 28 0.00014731     35   0.26611 0.28012 0.0043119
## 29 0.00014230     44   0.26478 0.28121 0.0043374
## 30 0.00014075     46   0.26450 0.28224 0.0043587
## 31 0.00014058     48   0.26422 0.28255 0.0043676
## 32 0.00013823     49   0.26408 0.28261 0.0043695
## 33 0.00013642     57   0.26280 0.28309 0.0043741
## 34 0.00013007     66   0.26158 0.28430 0.0043885
## 35 0.00012671     74   0.26054 0.28505 0.0043973
## 36 0.00012496     75   0.26041 0.28611 0.0044358
## 37 0.00012488     82   0.25950 0.28626 0.0044364
## 38 0.00012329     84   0.25925 0.28628 0.0044364
## 39 0.00012118     85   0.25912 0.28652 0.0044388
## 40 0.00012094     86   0.25900 0.28680 0.0044382
## 41 0.00011797     89   0.25860 0.28718 0.0044439
## 42 0.00011733     90   0.25848 0.28770 0.0044587
## 43 0.00011518    101   0.25715 0.28818 0.0044698
## 44 0.00011452    108   0.25635 0.28859 0.0044759
## 45 0.00011401    109   0.25623 0.28863 0.0044749
## 46 0.00011353    110   0.25612 0.28868 0.0044769
```

```
## 47 0.00011290    111   0.25600 0.28873 0.0044682
## 48 0.00011170    113   0.25578 0.28874 0.0044676
## 49 0.00011024    118   0.25520 0.28909 0.0044589
## 50 0.00011006    119   0.25509 0.28933 0.0044742
## 51 0.00010932    120   0.25498 0.28941 0.0044714
## 52 0.00010537    132   0.25358 0.29131 0.0044829
## 53 0.00010398    136   0.25314 0.29348 0.0045150
## 54 0.00010371    137   0.25303 0.29362 0.0045164
## 55 0.00010248    149   0.25171 0.29408 0.0045252
## 56 0.00010151    156   0.25095 0.29446 0.0045291
## 57 0.00010000    157   0.25085 0.29672 0.0045885
```

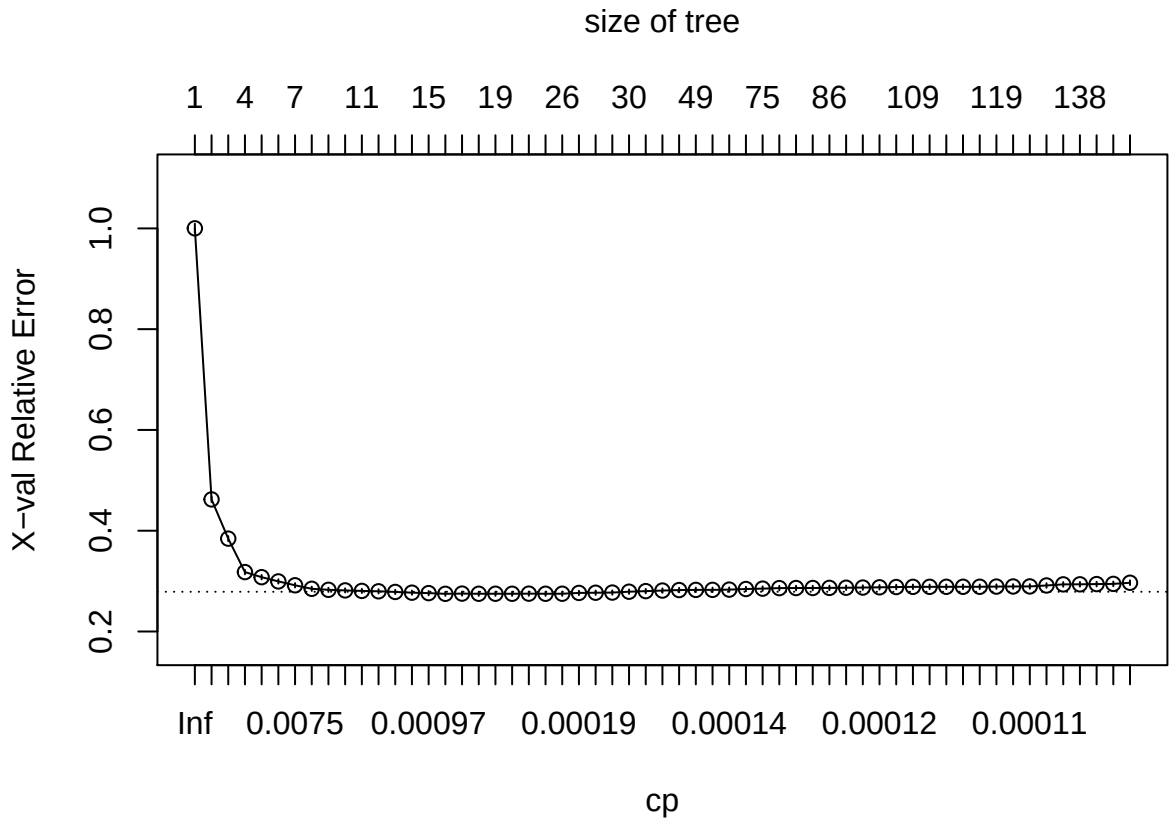
```
##              CP nsplit rel error    xerror        xstd
## 1  0.5390877357      0 1.0000000 1.0001207 0.009140319
## 2  0.0783628292      1 0.4609123 0.4621082 0.005270426
## 3  0.0668315064      2 0.3825494 0.3842476 0.005037162
## 4  0.0104580334      3 0.3157179 0.3178814 0.004360294
## 5  0.0091293448      4 0.3052599 0.3077734 0.004285485
## 6  0.0076621629      5 0.2961306 0.2993714 0.004239868
## 7  0.0073851837      6 0.2884684 0.2916778 0.004178879
## 8  0.0017805692      7 0.2810832 0.2846904 0.004127741
## 9  0.0015708518      8 0.2793026 0.2827267 0.004108769
## 10 0.0013470994      9 0.2777318 0.2815625 0.004120566
## 11 0.0013270404     10 0.2763847 0.2805225 0.004134491
## 12 0.0012631817     11 0.2750576 0.2797316 0.004127658
## 13 0.0011027784     12 0.2737945 0.2784881 0.004111244
## 14 0.0010350642     13 0.2726917 0.2770408 0.004098444
## 15 0.0009131128     14 0.2716566 0.2761213 0.004094681
## 16 0.0004311265     15 0.2707435 0.2747516 0.004080315
## 17 0.0004074681     16 0.2703124 0.2752013 0.004137656
## 18 0.0002930337     17 0.2699049 0.2748866 0.004136775
## 19 0.0002872586     18 0.2696119 0.2748980 0.004162551
## 20 0.0002532837     21 0.2687501 0.2748595 0.004161396
## 21 0.0002526870     23 0.2682435 0.2750782 0.004175347
## 22 0.0002221260     24 0.2679908 0.2748920 0.004176384
## 23 0.0002028733     25 0.2677687 0.2749462 0.004180235
## 24 0.0001832985     26 0.2675658 0.2765097 0.004225985
## 25 0.0001798018     27 0.2673825 0.2769152 0.004231387
## 26 0.0001615783     28 0.2672027 0.2772587 0.004238251
## 27 0.0001554357     29 0.2670412 0.2789293 0.004269940
## 28 0.0001473055     35 0.2661086 0.2801205 0.004311867
## 29 0.0001422984     44 0.2647824 0.2812088 0.004337355
## 30 0.0001407465     46 0.2644978 0.2822362 0.004358691
## 31 0.0001405768     48 0.2642163 0.2825452 0.004367595
## 32 0.0001382300     49 0.2640758 0.2826120 0.004369508
## 33 0.0001364228     57 0.2628049 0.2830919 0.004374056
## 34 0.0001300727     66 0.2615771 0.2843005 0.004388472
## 35 0.0001267091     74 0.2605365 0.2850488 0.004397319
## 36 0.0001249585     75 0.2604098 0.2861134 0.004435758
## 37 0.0001248838     82 0.2594980 0.2862623 0.004436365
## 38 0.0001232930     84 0.2592482 0.2862825 0.004436424
## 39 0.0001211767     85 0.2591249 0.2865154 0.004438816
## 40 0.0001209394     86 0.2590037 0.2868041 0.004438190
## 41 0.0001179662     89 0.2585956 0.2871786 0.004443877
```

```
## 42 0.0001173263     90 0.2584777 0.2876969 0.004458678
## 43 0.0001151809    101 0.2571519 0.2881816 0.004469789
## 44 0.0001145172    108 0.2563457 0.2885877 0.004475851
## 45 0.0001140117    109 0.2562312 0.2886310 0.004474892
## 46 0.0001135314    110 0.2561171 0.2886762 0.004476863
## 47 0.0001129039    111 0.2560036 0.2887261 0.004468185
## 48 0.0001116955    113 0.2557778 0.2887443 0.004467568
## 49 0.0001102378    118 0.2552003 0.2890862 0.004458926
## 50 0.0001100552    119 0.2550900 0.2893328 0.004474158
## 51 0.0001093245    120 0.2549800 0.2894150 0.004471449
## 52 0.0001053704    132 0.2535835 0.2913079 0.004482856
## 53 0.0001039798    136 0.2531352 0.2934839 0.004515004
## 54 0.0001037116    137 0.2530312 0.2936236 0.004516425
## 55 0.0001024819    149 0.2517074 0.2940842 0.004525157
## 56 0.0001015093    156 0.2509533 0.2944600 0.004529065
## 57 0.0001000000    157 0.2508518 0.2967185 0.004588457
```

```
(bestcp = cptable[ which.min(cptable[,"xerror"]), "CP" ])   # this is the optimal cp parameter
```

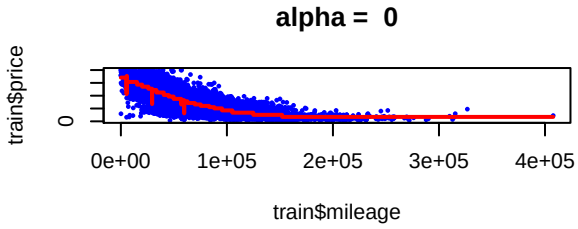
```
## [1] 0.0004311265
```

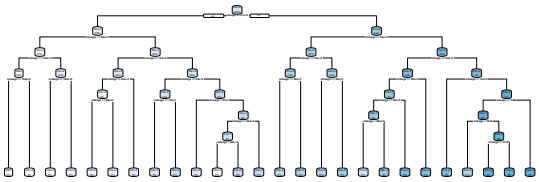
```
plotcp(big.tree) # plot results
```

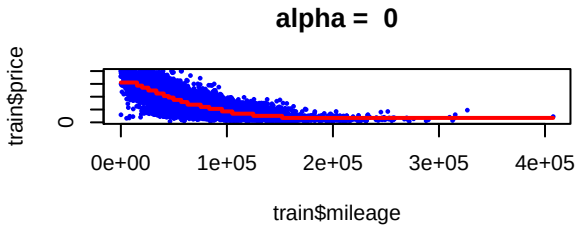


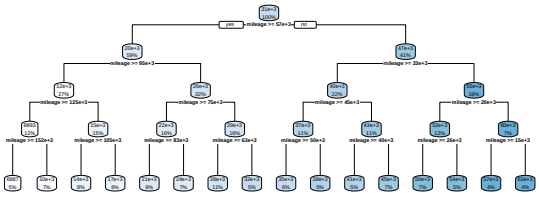

```
# show fit from some trees
oo = order(train$mileage)
cpvec = c(bestcp / 2, bestcp,.0157)

par(mfrow=c(3,2))
for(i in 1:3) {
```

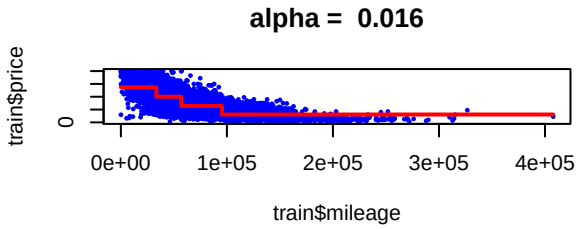
```
  plot(train$mileage,train$price,pch=16,col='blue',cex=.5)
  ptree = prune(big.tree,cp=cpvec[i])
  pfit = predict(ptree)
  lines(train$mileage[oo],pfit[oo],col='red',lwd=2)
  title(paste('alpha = ',round(cpvec[i],3)))
  rpart.plot(ptree)
}
```

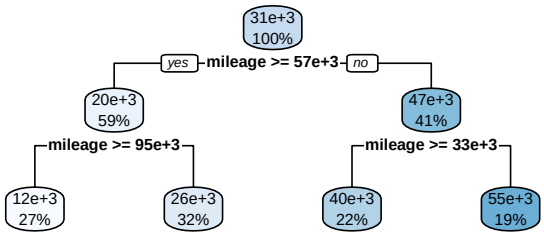




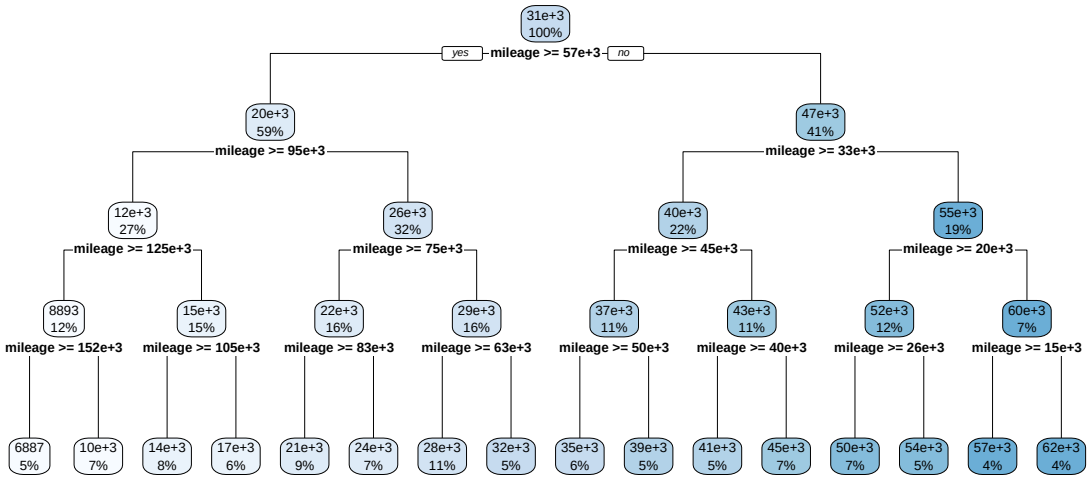




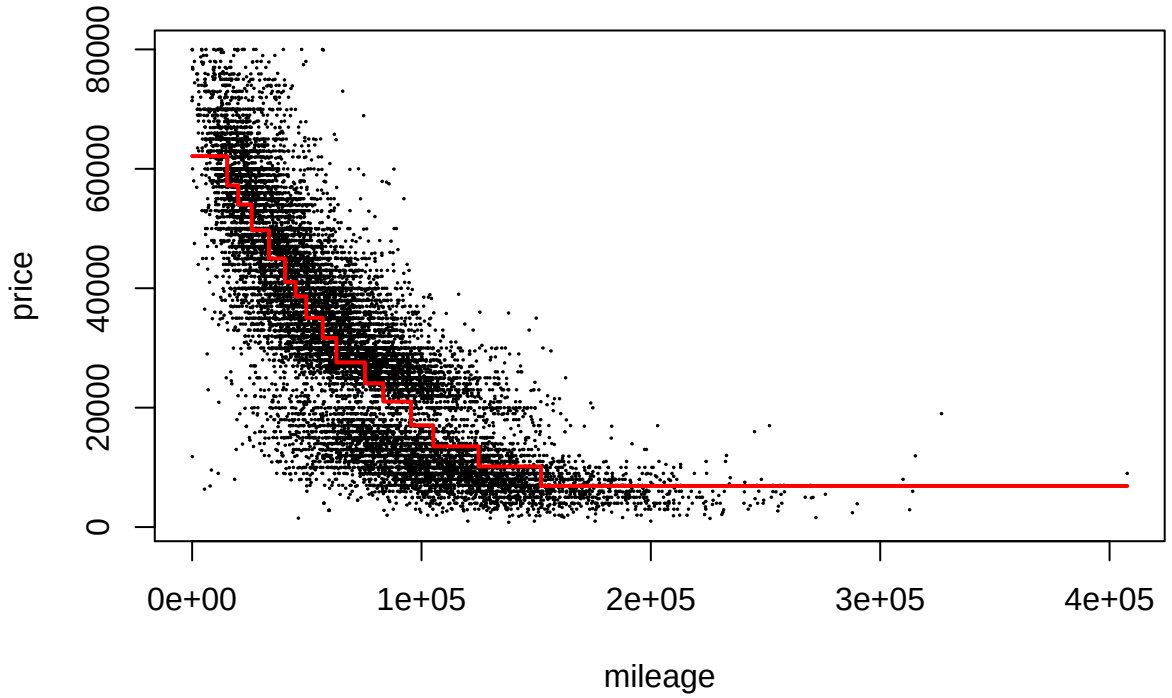





```
par(mfrow=c(1,1))
best.tree = prune(big.tree,cp=bestcp)
rpart.plot(best.tree)
```



```
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
lines(sort(train$mileage),predict(best.tree)[oo],col="red",lwd=2,cex.lab=2)
```




```
# error
sqrt(mean((test$price - predict(best.tree, test))^2))
```

```
## [1] 9545.132
```

Comparing with one variable case, now k of kNN and optimal tree size are smaller. Both method get lower RMSE with one more variable.

Part 7

```
big.tree = rpart(price~., data=train,
                 control=rpart.control(minsplit=5,cp=0.0001,xval=10))

nbig = length(unique(big.tree$where))
cat('size of big tree: ',nbig,'\n')
```

```
## size of big tree:  102
```

```
(cptable = printcp(big.tree))
```

```
##
## Regression tree:
## rpart(formula = price ~ ., data = train, control = rpart.control(minsplit = 5,
##     cp = 1e-04, xval = 10))
##
## Variables actually used in tree construction:
## [1] color        displacement mileage      region       soundSystem
## [6] trim         year
##
```

```
## Root node error: 5.008e+12/15047 = 332822195
##
## n= 15047
##
##            CP nsplit rel error   xerror      xstd
## 1  0.61662322      0  1.000000 1.000109 0.0091413
## 2  0.17316315      1  0.383377 0.386580 0.0043738
## 3  0.05171357      2  0.210214 0.199123 0.0032176
## 4  0.01400228      3  0.158500 0.166033 0.0028922
## 5  0.01333573      4  0.144498 0.145898 0.0026398
## 6  0.00889372      5  0.131162 0.135061 0.0024662
## 7  0.00830282      6  0.122268 0.126808 0.0024102
## 8  0.00698175      7  0.113966 0.118509 0.0024009
## 9  0.00590622      8  0.106984 0.112548 0.0022896
## 10 0.00392076      9  0.101078 0.104663 0.0021261
## 11 0.00379582     10  0.097157 0.100542 0.0020598
## 12 0.00334702     11  0.093361 0.098052 0.0020281
## 13 0.00238000     12  0.090014 0.091071 0.0019290
## 14 0.00165064     13  0.087634 0.088127 0.0018983
## 15 0.00165031     14  0.085983 0.086531 0.0019040
## 16 0.00157980     15  0.084333 0.085890 0.0018872
## 17 0.00156529     16  0.082753 0.084914 0.0018728
## 18 0.00156279     17  0.081188 0.084615 0.0018703
## 19 0.00121171     18  0.079625 0.082570 0.0018387
## 20 0.00120888     19  0.078413 0.080298 0.0017906
## 21 0.00108720     21  0.075996 0.079212 0.0017739
## 22 0.00101711     22  0.074908 0.078106 0.0017524
## 23 0.00082202     23  0.073891 0.076294 0.0017026
## 24 0.00077895     24  0.073069 0.075108 0.0016752
## 25 0.00076646     25  0.072290 0.074394 0.0016570
## 26 0.00064861     26  0.071524 0.073449 0.0016505
## 27 0.00064715     27  0.070875 0.072196 0.0016269
## 28 0.00063162     28  0.070228 0.071891 0.0016226
## 29 0.00063107     30  0.068965 0.071707 0.0016221
## 30 0.00061136     31  0.068334 0.071707 0.0016221
## 31 0.00051406     32  0.067722 0.070558 0.0016065
## 32 0.00050215     33  0.067208 0.070050 0.0016059
## 33 0.00050123     34  0.066706 0.069678 0.0016374
## 34 0.00049495     35  0.066205 0.069600 0.0016367
## 35 0.00043115     36  0.065710 0.068836 0.0016305
## 36 0.00042389     37  0.065279 0.068174 0.0016229
## 37 0.00042116     38  0.064855 0.068043 0.0016223
## 38 0.00041973     39  0.064434 0.067894 0.0016219
## 39 0.00041122     40  0.064014 0.067699 0.0016196
## 40 0.00040802     41  0.063603 0.067316 0.0016169
## 41 0.00037489     42  0.063195 0.066976 0.0016143
## 42 0.00036827     43  0.062820 0.065966 0.0016074
## 43 0.00036290     45  0.062083 0.065665 0.0016019
## 44 0.00034924     46  0.061721 0.065305 0.0015930
## 45 0.00033263     47  0.061371 0.064702 0.0015889
## 46 0.00031679     49  0.060706 0.064416 0.0015811
## 47 0.00026252     50  0.060389 0.064176 0.0016018
## 48 0.00024995     51  0.060127 0.063890 0.0016113
## 49 0.00024629     52  0.059877 0.063896 0.0016114
```

```
## 50 0.00024597     53  0.059630 0.063932 0.0016114
## 51 0.00023915     54  0.059385 0.063800 0.0016108
## 52 0.00023886     55  0.059145 0.063777 0.0016101
## 53 0.00023806     56  0.058907 0.063735 0.0016098
## 54 0.00023748     57  0.058668 0.063735 0.0016098
## 55 0.00021534     58  0.058431 0.063533 0.0016100
## 56 0.00020969     59  0.058216 0.063050 0.0016063
## 57 0.00020357     60  0.058006 0.062801 0.0016054
## 58 0.00020243     61  0.057802 0.062733 0.0016051
## 59 0.00020165     62  0.057600 0.062677 0.0016049
## 60 0.00020060     63  0.057398 0.062651 0.0016044
## 61 0.00019943     64  0.057198 0.062599 0.0016042
## 62 0.00019856     65  0.056998 0.062562 0.0016043
## 63 0.00019494     66  0.056800 0.062444 0.0016040
## 64 0.00018167     67  0.056605 0.062094 0.0016028
## 65 0.00017622     68  0.056423 0.061543 0.0015973
## 66 0.00017249     69  0.056247 0.061328 0.0015964
## 67 0.00017024     70  0.056074 0.061285 0.0015980
## 68 0.00015539     71  0.055904 0.061182 0.0015872
## 69 0.00014705     72  0.055749 0.060708 0.0015814
## 70 0.00014651     73  0.055602 0.060516 0.0015789
## 71 0.00014595     74  0.055455 0.060501 0.0015789
## 72 0.00014485     75  0.055309 0.060477 0.0015788
## 73 0.00013859     76  0.055164 0.060588 0.0015806
## 74 0.00013746     77  0.055026 0.060573 0.0015854
## 75 0.00013527     78  0.054888 0.060506 0.0015842
## 76 0.00013153     79  0.054753 0.060511 0.0015850
## 77 0.00012782     80  0.054622 0.060493 0.0015864
## 78 0.00012754     81  0.054494 0.060450 0.0015862
## 79 0.00012517     82  0.054366 0.060419 0.0015868
## 80 0.00012393     83  0.054241 0.060378 0.0015868
## 81 0.00012384     84  0.054117 0.060315 0.0015860
## 82 0.00011745     85  0.053993 0.060149 0.0015848
## 83 0.00011678     86  0.053876 0.060138 0.0015863
## 84 0.00011466     87  0.053759 0.060123 0.0015869
## 85 0.00011195     88  0.053644 0.060066 0.0015878
## 86 0.00011096     90  0.053420 0.060192 0.0015992
## 87 0.00011011     93  0.053088 0.060208 0.0015995
## 88 0.00010970     94  0.052977 0.060261 0.0016010
## 89 0.00010620     95  0.052868 0.060275 0.0016021
## 90 0.00010612     96  0.052762 0.060308 0.0016021
## 91 0.00010499     97  0.052655 0.060299 0.0016020
## 92 0.00010375     98  0.052550 0.060318 0.0016021
## 93 0.00010284     99  0.052447 0.060302 0.0016020
## 94 0.00010257    100  0.052344 0.060337 0.0016027
## 95 0.00010000    101  0.052241 0.060204 0.0015955
```

```
##              CP nsplit  rel error     xerror        xstd
## 1  0.6166232237      0 1.00000000 1.00010894 0.009141261
## 2  0.1731631458      1 0.38337678 0.38657986 0.004373801
## 3  0.0517135744      2 0.21021363 0.19912337 0.003217621
## 4  0.0140022783      3 0.15850006 0.16603285 0.002892206
## 5  0.0133357270      4 0.14449778 0.14589817 0.002639773
## 6  0.0088937244      5 0.13116205 0.13506067 0.002466198
```

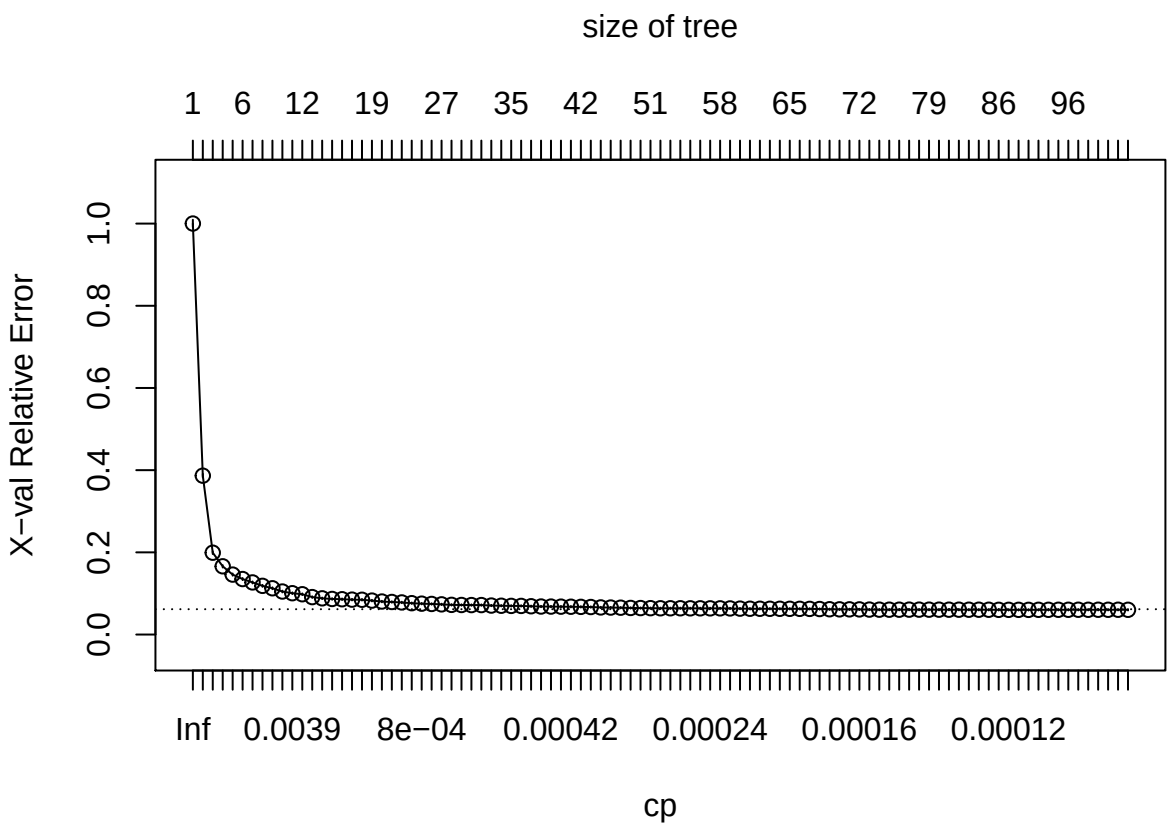
```
## 7  0.0083028208      6 0.12226833 0.12680846 0.002410218
## 8  0.0069817538      7 0.11396551 0.11850926 0.002400890
## 9  0.0059062247      8 0.10698375 0.11254840 0.002289574
## 10 0.0039207634      9 0.10107753 0.10466334 0.002126093
## 11 0.0037958238     10 0.09715676 0.10054233 0.002059784
## 12 0.0033470204     11 0.09336094 0.09805230 0.002028058
## 13 0.0023800029     12 0.09001392 0.09107132 0.001928964
## 14 0.0016506425     13 0.08763392 0.08812712 0.001898294
## 15 0.0016503086     14 0.08598327 0.08653149 0.001903969
## 16 0.0015798041     15 0.08433297 0.08588952 0.001887163
## 17 0.0015652863     16 0.08275316 0.08491373 0.001872834
## 18 0.0015627898     17 0.08118787 0.08461469 0.001870336
## 19 0.0012117063     18 0.07962509 0.08257024 0.001838674
## 20 0.0012088800     19 0.07841338 0.08029771 0.001790634
## 21 0.0010872031     21 0.07599562 0.07921189 0.001773908
## 22 0.0010171066     22 0.07490842 0.07810645 0.001752361
## 23 0.0008220185     23 0.07389131 0.07629371 0.001702584
## 24 0.0007789480     24 0.07306929 0.07510752 0.001675188
## 25 0.0007664638     25 0.07229034 0.07439446 0.001656980
## 26 0.0006486098     26 0.07152388 0.07344922 0.001650486
## 27 0.0006471527     27 0.07087527 0.07219649 0.001626895
## 28 0.0006316216     28 0.07022812 0.07189148 0.001622597
## 29 0.0006310691     30 0.06896487 0.07170724 0.001622119
## 30 0.0006113592     31 0.06833380 0.07170724 0.001622119
## 31 0.0005140579     32 0.06772244 0.07055842 0.001606542
## 32 0.0005021461     33 0.06720839 0.07005013 0.001605866
## 33 0.0005012348     34 0.06670624 0.06967840 0.001637387
## 34 0.0004949475     35 0.06620501 0.06960037 0.001636684
## 35 0.0004311512     36 0.06571006 0.06883566 0.001630478
## 36 0.0004238851     37 0.06527891 0.06817435 0.001622862
## 37 0.0004211600     38 0.06485502 0.06804313 0.001622317
## 38 0.0004197348     39 0.06443386 0.06789383 0.001621858
## 39 0.0004112215     40 0.06401413 0.06769929 0.001619620
## 40 0.0004080176     41 0.06360291 0.06731579 0.001616906
## 41 0.0003748905     42 0.06319489 0.06697575 0.001614312
## 42 0.0003682713     43 0.06282000 0.06596610 0.001607411
## 43 0.0003628976     45 0.06208346 0.06566510 0.001601903
## 44 0.0003492431     46 0.06172056 0.06530490 0.001593040
## 45 0.0003326344     47 0.06137131 0.06470169 0.001588893
## 46 0.0003167887     49 0.06070605 0.06441628 0.001581116
## 47 0.0002625241     50 0.06038926 0.06417602 0.001601761
## 48 0.0002499452     51 0.06012673 0.06388982 0.001611282
## 49 0.0002462906     52 0.05987679 0.06389612 0.001611383
## 50 0.0002459669     53 0.05963050 0.06393228 0.001611444
## 51 0.0002391502     54 0.05938453 0.06380040 0.001610768
## 52 0.0002388580     55 0.05914538 0.06377670 0.001610066
## 53 0.0002380568     56 0.05890652 0.06373504 0.001609767
## 54 0.0002374792     57 0.05866847 0.06373504 0.001609767
## 55 0.0002153422     58 0.05843099 0.06353306 0.001609964
## 56 0.0002096887     59 0.05821564 0.06305050 0.001606263
## 57 0.0002035727     60 0.05800595 0.06280053 0.001605406
## 58 0.0002024271     61 0.05780238 0.06273279 0.001605062
## 59 0.0002016532     62 0.05759996 0.06267657 0.001604932
## 60 0.0002006046     63 0.05739830 0.06265083 0.001604384
```

```
## 61 0.0001994274     64 0.05719770 0.06259863 0.001604172
## 62 0.0001985642     65 0.05699827 0.06256205 0.001604308
## 63 0.0001949440     66 0.05679971 0.06244393 0.001603995
## 64 0.0001816657     67 0.05660476 0.06209374 0.001602798
## 65 0.0001762215     68 0.05642310 0.06154335 0.001597323
## 66 0.0001724918     69 0.05624687 0.06132847 0.001596424
## 67 0.0001702403     70 0.05607438 0.06128473 0.001598030
## 68 0.0001553936     71 0.05590414 0.06118182 0.001587189
## 69 0.0001470496     72 0.05574875 0.06070774 0.001581411
## 70 0.0001465080     73 0.05560170 0.06051559 0.001578906
## 71 0.0001459511     74 0.05545519 0.06050059 0.001578901
## 72 0.0001448544     75 0.05530924 0.06047706 0.001578792
## 73 0.0001385946     76 0.05516439 0.06058809 0.001580629
## 74 0.0001374574     77 0.05502579 0.06057298 0.001585438
## 75 0.0001352742     78 0.05488833 0.06050558 0.001584172
## 76 0.0001315331     79 0.05475306 0.06051140 0.001585038
## 77 0.0001278190     80 0.05462153 0.06049264 0.001586433
## 78 0.0001275400     81 0.05449371 0.06044975 0.001586166
## 79 0.0001251691     82 0.05436617 0.06041898 0.001586791
## 80 0.0001239268     83 0.05424100 0.06037812 0.001586771
## 81 0.0001238351     84 0.05411707 0.06031548 0.001586045
## 82 0.0001174523     85 0.05399324 0.06014890 0.001584807
## 83 0.0001167788     86 0.05387578 0.06013846 0.001586323
## 84 0.0001146643     87 0.05375901 0.06012339 0.001586890
## 85 0.0001119541     88 0.05364434 0.06006562 0.001587803
## 86 0.0001109636     90 0.05342043 0.06019209 0.001599240
## 87 0.0001101065     93 0.05308754 0.06020829 0.001599458
## 88 0.0001097049     94 0.05297744 0.06026117 0.001601028
## 89 0.0001061964     95 0.05286773 0.06027470 0.001602093
## 90 0.0001061154     96 0.05276153 0.06030830 0.001602083
## 91 0.0001049873     97 0.05265542 0.06029884 0.001601989
## 92 0.0001037524     98 0.05255043 0.06031780 0.001602069
## 93 0.0001028405     99 0.05244668 0.06030226 0.001601987
## 94 0.0001025679    100 0.05234384 0.06033734 0.001602654
## 95 0.0001000000    101 0.05224127 0.06020438 0.001595532
```

```
(bestcp = cptable[ which.min(cptable[,"xerror"]), "CP" ])   # this is the optimal cp parameter
```

```
## [1] 0.0001119541
```

```
plotcp(big.tree) # plot results
```




```
# show fit from some trees
oo = order(train$mileage)
cpvec = c(bestcp / 2, bestcp,.0157)

par(mfrow=c(3,2))
for(i in 1:3) {
  plot(train$mileage,train$price,pch=16,col='blue',cex=.5)
  ptree = prune(big.tree,cp=cpvec[i])
  pfit = predict(ptree)
  lines(train$mileage[oo],pfit[oo],col='red',lwd=2)
  title(paste('alpha = ',round(cpvec[i],3)))
  rpart.plot(ptree)
}
```

```
## Warning: labs do not fit even at cex 0.15, there may be some overplotting

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```
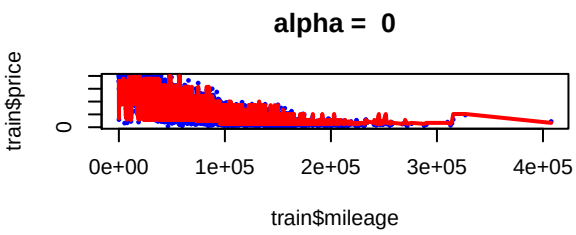
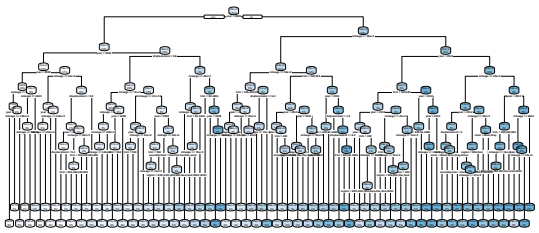
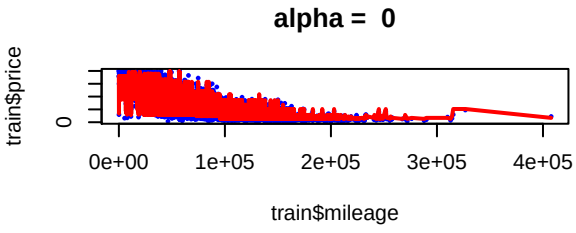
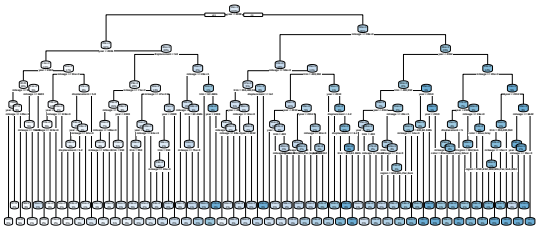
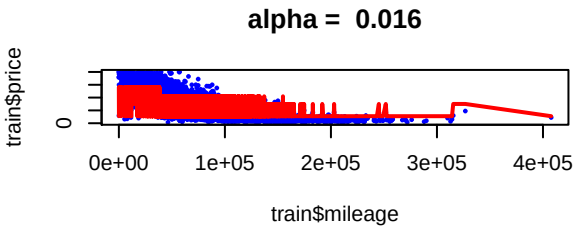
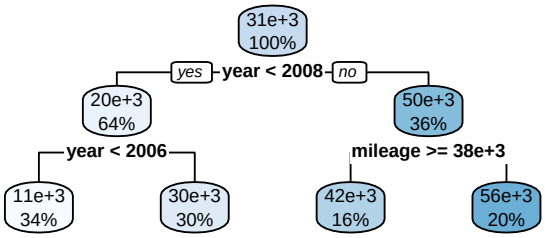







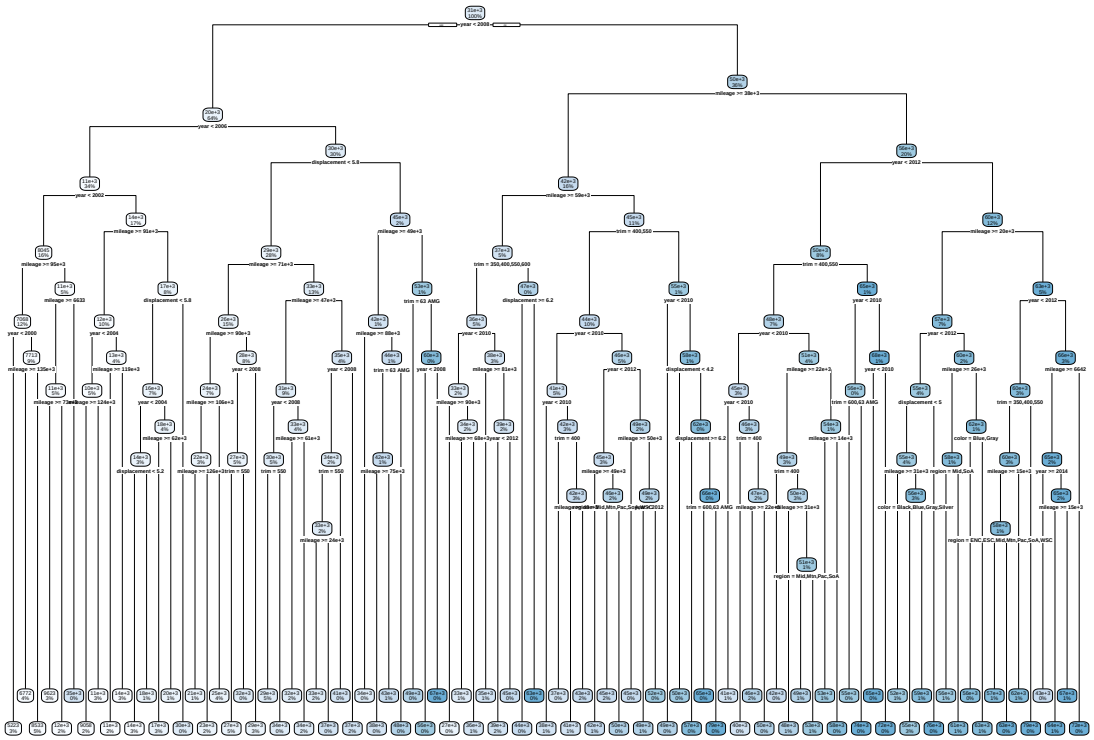




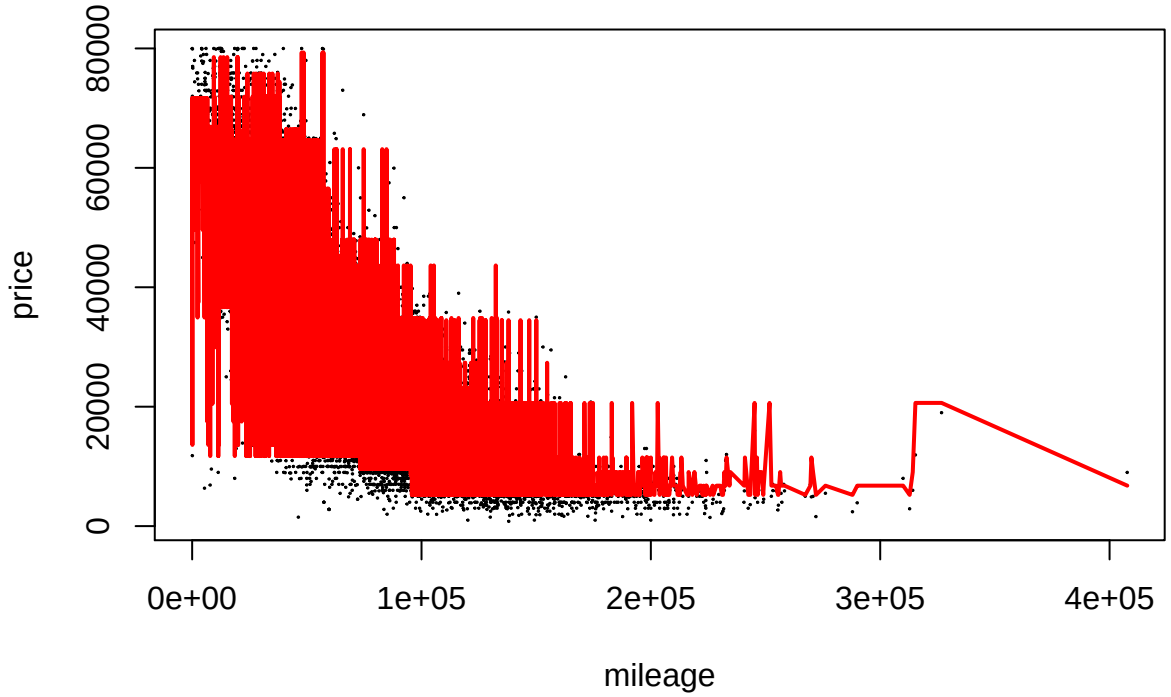



```
par(mfrow=c(1,1))
best.tree = prune(big.tree,cp=bestcp)
rpart.plot(best.tree)
```

```
## Warning: labs do not fit even at cex 0.15, there may be some overplotting
```



```
plot(train$mileage, train$price, xlab = "mileage", ylab = "price", pch = 1, cex = 0.1)
lines(sort(train$mileage),predict(best.tree)[oo],col="red",lwd=2,cex.lab=2)
```




```
# error
sqrt(mean((test$price - predict(best.tree, test))^2))
```

```
## [1] 4490.279
```

Bonus

One possible procedure

- For each fixed k, there are $\binom{p}{k}$ possible k-variable models. Run cross validation and compute RMSE for all possible models. The model with smallest RMSE is the optimal k-variable model.
- For all $k = 1, \ldots, p$. Compare RMSE of all optimal k-variable models and find the one with smallest RMSE.