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CHAPTER 1: INTRODUCTION

1.1 Introduction

This Chapter provides background and motivation for the study. It is structured as follows: Section 1.2 looks at the background and overview; Section 1.3 shows the motivation of the study; Section 1.4 summarizes research objectives; Section 1.5 indicates the value of the study in the context of South Africa; Section 1.6 outlines the structure of the paper and Section 1.7 lists the definition of terms.

1.2 Background and Overview

Modern capital market analysis was founded by Markowitz (1952), Sharpe (1964), and Lintner (1965), particularly in their analysis of the equilibrium risk-return relationship. According to this risk-return paradigm, investors earn higher returns when they take on more systematic risks. Although this concept is well accepted and employed in almost all empirical asset pricing studies, the proxies of systematic risks are still contentious. In the past decades, numerous factors have been proposed as underlying risk factors, a situation that Cochrane (2011) describes as a "factor zoo". Today, asset pricing faces a fundamental challenge: how to bring discipline to the proliferation of factors.

In the first place is the single-factor Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), and the multi-factor Arbitrage Pricing Theory (APT) of Ross (1976) which have triggered the research upsurge in looking for adaptive risk factors to explain the cross-sectional variance of average asset returns. Other benchmark factors have also been introduced into asset pricing models during the past decades, notably the HML value factor and SMB size factor of the Fama and French (1993) three-factor model (FF3-factor model), the WML momentum factor of Carhart (1997) four-factor model (Carhart4-factor model), the RMW profitability factor and the CMA investment factor of the Fama and French (2015) five-factor model (FF5-factor model).

Research papers describing new risk factors usually provide evidence of their risk premiums. Most of their empirical tests follow the methodology proposed by Fama and MacBeth (1973), in which they assume that the linear relationship between expected return and factor loadings can be determined through the use of a two-pass regression method. In the first pass of Fama-MacBeth (FM) regression, factor loadings (betas) are estimated by using time series regression of individual asset returns on the proposed factors. Furthermore, the second pass regresses the cross-section of asset returns on betas, which were estimated in the previous step, to estimate risk premiums on a time-series scale, and then tests whether the time-series averaged risk premiums are statistically significant. However, the explanatory variables (betas) in the second-pass FM regression are estimated with measurement errors from the first-pass regression, and consequently, introduce the Errors-in-Variable (EIV) problem. Shanken (1992) finds that the FM time-series procedure for computing standard errors fails to reflect measurement error in betas and overstates the precision of the risk premium estimates. However, aside from the biased coefficient of the imprecisely measured variable, there is also a bias in the coefficients of the other variables, which is called contamination bias, and the direction of this bias is in general unknown. Consequently, factor premium estimations will be biased and inconsistent in the cross-sectional FM regression and the direction of bias cannot be determined when there are multiple variables involved in the second pass regressions.

Black et al. (1972) and Fama and MacBeth (1973) note this EIV problem and suggest that this problem can be mitigated by grouping stocks into portfolios rather than using individual stocks as test assets. Since each portfolio has a significant amount of individual stocks, they argued that the beta estimation errors of individual stocks would cancel each other out when they are grouped into portfolios. Since then, it has become a research protocol to use portfolios as test assets when a two-pass regression is applied. However, using the FF3-factor model as an example, people might think how tautological it is that the 25 portfolios double sorted by size and book-to-market ratio are explained by the factors that are also constructed based on the same characteristics. As Lewellen et al. (2010) point out, sorting stocks into portfolios according to specific firm characteristics (like size and value) will impart a strong factor structure into test portfolio returns.

Furthermore, Jegadeesh et al. (2019) suggest that the portfolio grouping procedure according to specific firm attributes is a kind of dimensionality reduction treatment because cross-sectional asset returns would vary with fewer independent variables across portfolios than across individual stocks. On the other hand, using portfolios as test assets will lose many cross-sectional characteristics of individual stocks. Therefore, anomalies that are unrelated to portfolio-grouping procedures may be hidden when the risk factors are orthogonal to the attribute of portfolios. To examine the independent nature of characteristics, it is necessary to sort the portfolios multivariate according to each characteristic to assess their predictive power. However, Cochrane (2011) points out that this approach begins to break down as the number of characteristics increases, allowing only a small number of characteristics to be controlled. A similar argument is made by Ang et al. (2010), who argue that by aggregating stocks into portfolios, the cross-sectional dispersion of betas shrinks, resulting in larger standard errors of risk premium estimates. Nevertheless, the EIV problem is inherent in using individual stocks as test assets under traditional FM procedures.

For mitigating the EIV bias, this study first adopts the instrumental variables (IV) approach in the FM regression, as Jegadeesh et al. (2019) suggest. To be specific, betas are first calculated using a subset of data samples, and they are used in the second-pass regression as the explanatory variables (EVs). Further, betas would be estimated again using another disjoint data sample, and these betas are the IVs in the second pass regression. The point here is that the EVs and IVs are estimated from a disjoint data sample, thus their estimation errors are cross-sectionally uncorrelated. However, this method strongly relies on the assumption that measurement errors are white noise, namely there is no autocorrelation among measurement errors. However, the nature of measurement errors is unobservable, and this problem is elaborated in Section 4.3.2. In light of the potential failure of the FM-IV method, this study adopts another method from Shanken (1992), i.e., Shanken’s asymptotic EIV-corrected estimator. In addition, the original Shanken’s estimator is improved to adopt unbalanced panel data following the instruction of Chordia et al. (2017). By using individual stocks as test assets and mitigating EIV bias, this paper attempts to test whether the factors under the CAPM, the FF3- and the FF5-factor model command risk premiums that are significantly different from zero. The motivation here is that previous studies on the JSE have used portfolios rather than individual stocks as test assets, which could lead to problems with low dimensionality in regression.

Given that beta estimates contain measurement errors, characteristics may be better proxies for “true” betas, and the slope coefficient of characteristics (cross-sectional factors) may reflect the risk premiums of underlying systematic risk factors. For example, in contrast to Fama and French (1993) and Davis et al. (2000), who argue that factor loadings explain expected returns, Daniel and Titman (1997) argue that characteristics explain expected returns. In addition, there are many papers in the literature that present variations of this interpretation, such as Berk et al. (1999), Zhang (2005), and Novy-Marx (2013). In this regard, it is legitimate to ask whether the underlying firm characteristics or the estimated betas do a better job of tracking average returns on the cross-section. For that purpose, this study adopts the way of Fama and French (2019), in which they use the cross-sectional regression approach of FM to construct a series of cross-sectional factors (CS factors) corresponding to the time-series factors of the FF5-factor model and then compare their explanatory powers on average returns, respectively.

Briefly, this paper aims to: 1) eliminate the EIV bias by using the IV approach and revising Shanken's estimator, respectively; 2) compare the explanatory power of TS and CS factor models on average returns by using estimated betas and predetermined characteristics as factor loadings, respectively.

1.3 Motivation of the Study

During the past decades, a wide range of factors have been proposed, a situation that Cochrane (2011) describes as a "factor zoo". Papers proposing new risk factors usually provide evidence of the risk premiums associated with them under the FM regression. However, the EIV problem is inherent in such a two-stage regression. Additionally, the traditional method of using portfolios to mitigate the EIV problem will bring many drawbacks, particularly, it could mask the firm characteristics of individual stocks due to the diversification procedure. Thus, this study attempts to test whether factor risk premiums under the standard CAPM, the FF3- and the FF5-factor models that are still significantly different from zero after correcting for the EIV bias. It is motivated by the fact that most previous studies use portfolio sorts as test assets and fail to account for the EIV bias, which can lead to overstating the precision of risk premium estimates and therefore misidentifying factors that are unrewarded.

This study also aims to solve the puzzle proposed by Jegadeesh et al. (2019), in which they find when using individual stocks as tests assets and correcting for the EIV bias, none of the factors like market, size, value, profitability, and investment is priced on cross-section, while the slope coefficients of their corresponding characteristics are strongly significant. In this context, it is legitimate to ask whether characteristics may serve as better proxies for "true" betas and their corresponding slope coefficients may reflect the risk premiums of underlying factors. To test this assumption empirically, this study employs the methods proposed by Fama and French (2019), in which they construct a series of TS and CS factor models and then compare their explanatory power on the variation of cross-sectional average returns.

1.4 Research Objectives

- ❖ Resolve the EIV problem in the estimation of factor risk premiums by using the IV approach and revising Shanken's estimator, respectively.
- ❖ Compare CS and TS factor models by using prespecified characteristics and estimated betas as factor loadings, respectively.

1.5 Valuation in South Africa

From the literature published in South Africa, there is a variety of evidence pointing to the presence of the value and size premiums, as well as the momentum anomaly. There is also evidence supporting the existence of idiosyncratic risk and low beta. Van Rensburg (2001) and van Rensburg & Robertson (2003), two of the earliest comprehensive assessments of Fama-French APT models on the JSE stock market, demonstrate that value (proxied by price-to-earnings ratio), size and momentum were significantly independent factors on the JSE. Further, Basewicz and Auret (2010) conduct studies similar to Fama and French (1992) and conclude that both size and value effects significantly affect the JSE, while the CAPM model fails to explain these effects. In addition, Gilbert and Kruger (2011) test the JSE cross-sectional returns over a longer period than van Rensburg and Robertson (2003), and they also find significant size and value effect as well as evidence of the low beta anomaly.

More recently, Page and Auret (2017, 2018) conducted an in-depth test of momentum on the JSE, they provide evidence in line with Muller and Ward (2013) that momentum is an independent investment style on the JSE. Moreover, Page and Auret (2019) use panel regression analysis and substantiate the existence of size, value and momentum premiums, as well as a low beta anomaly among shares listed on the JSE. Although the magnitudes and significance levels of certain factors are different in these studies, they all state there is a need for a broader APT-based factor model to model the South African equity markets correctly.

In summary, these papers indicate that the CAPM model has failed to explain the cross-sectional variation of stock returns, while size, value and momentum factors tend to play an important role in the JSE. Further, the FM procedure is applied in most of these papers and double-sorted portfolios have been used as test assets to mitigate the EIV problem as suggested by Fama and French (1992). However, the traditional FM procedure, under the standard assumption, is found to overstate the precision of risk premium estimates and provide biased and inconsistent coefficient estimates. Therefore, Fama and French (1992) and other studies that followed their procedure may have exaggerated their results. It is, therefore, crucial for this study to re-examine these prominent factors as proposed by previous literature more explicitly and accurately. Through the resolution of EIV biases in risk premium estimation and the comparison of CS and TS factor models, this paper aims to help South African investors to understand which style factors are priced and whether CS factors can provide better descriptions of average returns than TS factors.

1.6 Structure of the Study

The paper is organized in the following manner. Chapter 2 provides an introduction to the pertinent literature, together with a review of the work of other researchers. Chapter 3 introduces the sources and collection procedures of the data and the design of the methodology and models that are used in the study. Chapter 4 presents the simulation procedure and the results as corrected for the EIV bias under the FM-OLS, FM-IV, and BJS-Shanken methods, respectively. Chapter 5 presents empirical asset pricing results by using the CAPM, the FF3-factor, the Carhart4-factor, and the FF5-factor models. In Chapter 6, this paper compares the performances of the TS and CS factor models in explaining the variation of cross-sectional average returns by using betas and characteristics as factor loadings respectively. Eventually, Chapter 7 sums up the study and concludes with the findings as well as possible caveats that may have to be considered.

1.7 Definition of Terms

Abbreviation	Full Name
JSE	Johannesburg Stock Exchange
J203T	The All-Share Index on the Johannesburg Stock Exchange
APT	Arbitrage Pricing Theory
CAPM	Capital Asset Pricing Model
FF3-Factor Model	Fama-French Three-Factor Model (1993)
Carhart4-Factor Model	Carhart Four-Factor Model (1997)
FF5-Factor Model	Fama-French Five-Factor Model (2015)
FM	Fama-MacBeth (1973)
BJS	Black, Jensen and Scholes (1972)
MKT	Market Factor
SMB	Small-Minus-Big (Size Factor)
HML	High-Minus-Low (Value Factor)
RMW	Robust-Minus-Weak (Profitability Factor)
CMA	Conservative-Minus-Aggressive (Investment Factor)
Size/MC	Natural Logarithm of Market Capitalization
PE	Price-to-Earnings Ratio
OP	Operating Profitability Ratio
INV	Investment Growth Ratio
EIV	Errors-in-Variables
CS	Cross-Section
TS	Time-Series

LHS	Left-Hand-Side
RHS	Right-Hand-Side
OLS	Ordinary Least Squared
IV	Instrumental Variables
EW	Equal-Weight
VW	Value-Weight

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter reviews the new methods for addressing EIV bias and explores the findings of using predetermined characteristics to replace estimated betas as proxies for true factor loadings. It is a common practice in the previous empirical asset pricing literature to estimate coefficients by using portfolio sorts as test assets. The rationale for using portfolios is that factor loadings of portfolios can potentially be estimated with greater precision than factor loadings of individual stocks. However, when stocks are sorted into portfolios based on certain characteristics, it is possible to mask some important characteristics of the cross-section stock returns. With this being the case, the two-pass FM regression approach may miss some important factors that contribute to risk rewards. In light of this, new methods will be required to allow the use of individual stocks rather than portfolios, and to further account for the EIV problem in a new way.

The remainder of this chapter proceeds as follows: Section 2.2 introduces and compares a selection of the new methods that aim to resolve the EIV bias. Section 2.3 reports the empirical asset pricing findings of using characteristics as loadings and their corresponding slope coefficients as factor returns. Finally, Section 2.4 concludes.

2.2 New Approaches to Resolving the EIV Problem

Shanken (1992) propose that there are two aspects of the EIV problem: 1) a downward bias in the commonly used standard errors for factor risk premium and zero-beta asset return estimates, and consequently overstate the precision of estimation; 2) a classic EIV problem: the small-sample bias in the second stage cross-sectional regression estimates since betas are estimated with measurement errors from the first stage.

Many approaches have been proposed to solve the EIV bias in past decades. For example, Black et al. (1972) and Fama and Macbeth (1973) suggest sorting individual assets into quintile or decile portfolios based on certain firm characteristics and then using them as test assets in a two-pass regression, which is now considered to be a traditional approach. They argue that if the errors in the estimated betas are imperfectly correlated across assets, then the measurement errors would tend to offset each other when the assets are grouped into portfolios. In addition, much other literature also argues that creating portfolios reduces idiosyncratic volatility and allows more precise estimates of factor loadings, and consequently risk premia. As a result, portfolio sorts have been used in almost all asset pricing tests because using individual assets leads to severe estimation bias in estimating risk premiums. However, Ang and Liu (2018) show analytically and confirm empirically that this motivation is wrong, in which smaller standard errors of portfolio beta estimates do not lead to smaller standard errors of cross-sectional coefficient estimates. Especially, they show that portfolio sorts destroy information by shrinking beta dispersion, resulting in larger standard errors. Furthermore. it is also important to note that sorting stocks into portfolios according to certain firm characteristics may hide important cross-sectional characteristics of individual stock returns as suggested by Shanken (1992). Due to this limitation, two-pass regression may fail to identify potential priced factors. In light of this, new methods will be required to allow the use of individual stocks rather than portfolios, and to further account for the EIV problem in a new way.

2.2.1 The Generalized Portfolio Sorts Approach

In a recent paper, Hoechle et al. (2019) demonstrate that portfolio sorts can mislead investors by attributing cross-sectional return predictability to the underlying characteristic that is used in sorting the portfolio. By using portfolio sorts, they assume that the alpha of the top-versus-bottom portfolio is entirely determined by observable firm characteristics underlying the sort. A certain firm characteristic which does not predict the cross-sectional average returns may happen to cluster stocks with a strong performance in the top portfolio and with poor performance in the bottom portfolio. However, the difference between the alpha of the top-versus-bottom portfolios is instead the result of unobservable heterogeneity across firms and, therefore, does not necessarily relate to the underlying firm characteristic. In light of this, an alternative method is proposed by Hoechle et al. (2019) based on the estimation of a panel dataset at the firm level. They regress monthly excess returns over risk-free rate for individual stocks on a series of firm characteristics and market factors using the generalized portfolio sorts (GPS) methodology. In this approach, fixed effects are included in the analysis to address the problem of unobserved heterogeneity across firms. The GPS methodology, however, cannot be guaranteed to be unaffected by the EIV problem because it still uses the traditional OLS estimation in its regression. Hence, the IV approach proposed by Jegadeesh et al. (2019) can be viewed as complementary to the GPS methodology of Hoechle et al. (2019).

2.2.2 The Asymptotic EIV-Corrected Approach

As an early paper that uses individual stocks to test asset pricing models, Litzenberger and Ramaswamy (1979) suggest a modified design matrix and a weighted least squares (WLS) estimator can be used for cross-sectional regression. In that paper, they assume asset returns follow a single-factor model. Further, Shanken (1992) proposes a modification to the two-pass regression method for deriving an asymptotically distributed cross-sectional regression estimator within a multifactor framework. Shaken provides an integrated econometric view of maximum likelihood (ML) methods and two-pass approaches to the evaluation of asset pricing models by expanding the usual framework to incorporate serial correlation in the underlying market factors. The author provides a way to correct for the EIV bias by estimating betas simultaneously with return premiums in a limitation (large N) sense when time T is fixed, i.e., N-consistent. It should be noted that Shanken's bias-adjusted estimator is the only member of a large class of FM-OLS modified estimators that do not require preliminary estimation of the bias adjustment. Additionally, unlike other approaches such as instrumental variable estimation, Shanken's estimator merely requires a panel of asset returns and a sample of factor realizations.

The recent paper from Chordia et al. (2017) further generalized Shanken's framework to a situation involving multiple factors and characteristics under an unbalanced panel dataset. Both papers derive the asymptotic bias associated with the EIV and then undo it analytically. An important contribution of Chordia et al. (2017) is that they correct for the EIV bias which can appear when firm characteristics are time-varying and influenced by past returns, as is the case for size and several other characteristics. This issue can induce cross-sectional correlations between characteristics and measurement errors in betas, which has not previously been considered. Using their EIV-corrected estimator, the authors analyse the finite-sample properties of the new estimator for traded factor models. It is confirmed by simulation results that the EIV bias is largely eliminated, and the usual FM standard errors do not differ much from the simulated standard errors.

2.2.3 The Regression-Calibration Approach

Kim (1995) presents a direct method for correcting cross-sectional regression coefficients, in which the lagged betas are used to obtain the closed-form estimated market risk premium with max likelihood estimation. The correction is particularly useful when individual stocks are used to estimate betas since the coefficients are consistent when a fixed time-series sample size is used and assets, N, are allowed to increase without bound. This property of the estimator is often referred to as N-consistency, following the definition by Shanken (1992). In the recent paper, Kim and Skoulakis (2018) further develop a two-pass cross-sectional regression approach to estimate ex-post premia and construct beta-pricing tests under the same construction as Kim (1995) when time-series length T is fixed, and asset number N tends to infinity. They refer to their method as "the regression-calibration approach". Specifically, the authors correct the betas estimated from the first-pass FM regression by using an EIV-corrected estimator to satisfy the orthogonality condition, thus the estimator of risk premiums provided in the second pass is N-consistent.

2.2.4 The Instrumental Variable Approach

Unlike the above approaches, which estimate ex-post premiums and construct asset pricing tests using large cross-sections, Jegadeesh et al. (2019) propose to mitigate the EIV bias by revising the traditional FM regression with IV estimation. The IV method is a standard econometric solution in dealing with the EIV problem. Specifically, Jegadeesh et al. (2019) focus on ex-ante risk premia estimation of asset pricing models and resort to the original FM approach for computing standard errors and test statistics, as traditionally used when the time-series length T is large. By using individual stocks as test assets, the IV method can still provide consistent estimates of ex-post risk premiums, thus avoiding the limitations associated with portfolio sorts. The key to this IV method is the absence of correlation between the measurement errors of instrumental betas and explanatory betas, which are estimated from disjoint data samples within the same rolling window. This property implies not only the efficient market hypothesis that unexpected returns are serially uncorrelated but also that measurement errors of betas estimated from disjoint sample periods are cross-sectionally uncorrelated. In Section 3.3, this study dives deep into the IV estimation by framing it as a two-stage least square regression.

2.3 Betas versus Characteristics

Many papers in the literature such as Berk et al. (1999), Zhang (2005), and Novy-Marx (2013) suggest that characteristics may serve as better proxies for "true" unobserved factor loadings given that betas are estimated with measurement errors. In particular, Jegadeesh et al. (2019) modify their IV approach to include firm characteristics as explanatory variables and test whether characteristics can be better risk proxies. Unfortunately, their results do not support the "risk-proxy" hypothesis that the significant slope coefficient of characteristics is because characteristics are better proxies for true future betas than past betas. However, as suggested by Chordia et al. (2017) that there need not be a simple relation between betas and characteristics at the individual stock level because the estimated betas can change dramatically from one day to the next. In this regard, it is legitimate to ask whether the underlying firm characteristics or the estimated betas do a better job of tracking average returns on the cross-section, which leads the study to add the "Additional Test Chapter" to compare their explanatory power.

For example, Fama and French (2019) construct a series of CS factors (the slope coefficients of characteristics by using cross-sectional regression) as compared with the TS factors of the Fama-French five-factor model (2015). They find that time-series models that use only CS factors provide better descriptions of average returns than time-series models that use TS factors. This finding is robust when the authors adopt both constant and time-varying factor loadings in both CS and TS factor models. Especially, a combination of CS factors with time-varying factor loadings has the best performance in explaining the cross-sectional asset average returns, i.e., the lowest pricing errors. It should be noted that Fama and French (2019) use a series of 225 double-sorted portfolios as test assets on the LHS of the time-series regression model rather than individual stocks. To avoid the limitation of using portfolio sorts, this study decides to use individual stocks as test assets.

2.4 Conclusion

As all of these standard approaches are based on regressions, which include time-series regression, cross-sectional regression, two-stage FM regression, and stochastic discount factor (SDF) method, their results suffer from very the same problems as all regression-based approaches faced. The EIV problem is among the most prominent regression issues since many papers still use two-step regression as a standard way to evaluate asset pricing models.

In recent empirical asset pricing literature, more refined and alternative approaches have been suggested that address these problems. This chapter reviews new methods that address the EIV problem in factor risk premium estimations. Among these new approaches, the IV approach of Jegadeesh et al. (2019) and the EIV bias-corrected estimator of Shanken (1992) deserve a particular focus because they allow the use of individual stocks rather than portfolio sorts as test assets. In addition, Chordia et al. (2017) and Kim and Skoulakis (2018) have contributed further techniques to remove the bias in factor loading estimation. Due to measurement errors in beta estimates, characteristics may serve as better proxies for "true" betas and their corresponding slope coefficients may reflect the risk premiums of underlying systematic risk factors.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter gives an outline of the research methodology that was followed in the study. It provides information on the data collection and the corresponding processing procedure in Section 3.2. In Section 3.3, the study proposes the revised two-stage BJS/FM regression method. Section 3.4 introduces the OLS, IV, and Shanken's estimators. A bunch of prominent empirical asset pricing models are listed in Section 3.4. Finally, Section 3.5 concludes the chapter.

3.2 Data

Daily data of stock price, market cap, market return, price-to-earnings ratio, operating profitability and change of total asset for listed companies on the JSE are obtained from the Bloomberg terminal at Wits Lab, while the risk-free is collected from the South African Reserve Bank. The 91-day Treasury-bill (T-bill) return rate was obtained by the South African Reserve Bank and used as a proxy for the risk-free rate. Because the 91-day T-bill's return was an annualized return, it is geometrically divided into daily returns. In addition, J203T was obtained from the Bloomberg terminal and used to represent the return of the benchmark market. The reason for using daily data is because it is more reliable and efficient, has higher forecasting power, and fits better with the assumption of market efficiency compared with monthly data. The sample period used in this study spans from Jan 2000 to Dec 2019. Over the entire sample period, the final cross-section of shares is 245 after adjusting for liquidity. A summary of the average number of traded stocks in the universe portfolios and the double-sorted portfolios starting from Jan 2001 to Dec 2019 (one year gap for estimation) is presented in Table 1. The way of sorting stocks based on their firm characteristics and corresponding portfolio returns is explained in detail in the Appendix.

Table 1
The average number of traded stocks in the universe portfolio, double-sorted portfolio

Years	**All**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
Universe	201	120	142	149	160	178	191
Double-Sorted	38	17	30	33	30	32	34
Years	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Universe	193	201	207	211	215	220	228
Double-Sorted	38	34	41	37	44	46	42
Years	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	

Universe	230	233	235	235	238	240
Double-Sorted	45	45	42	46	48	47

Portrayed are the mean number of stocks in the universe portfolios and double-sorted portfolios for all months from Jan 2001 to Dec 2019.

For maintaining the integrity of the test, several filters are applied. Firstly, stocks that miss market cap, operating profitability, investment growth rate, momentum, or positive book-to-market ratio in one or more of the next eleven months are kept for 1 month and removed afterwards. In the month they are kept, they have a -100% return. Secondly, financial companies such as banks, along with pure real estate investment trusts, cash shells, investment trusts and exchange-traded funds were excluded from the sample. Thirdly, to reduce the likelihood of overshooting due to outliers, this study Winsorizes every individual stock return at the 1% and 99% levels during regressions, i.e., stock returns that are more extreme than those levels are replaced with the values at those percentiles.

3.3 BJS/FM Regression Procedure

The majority of empirical tests of asset pricing models, including all of the models mentioned above, employ BJS/FM regression to determine if a linear relationship exists between expected return and betas. Specifically, the following time-series regressions are used in the first pass of BJS/FM regression. Particularly, rolling betas are designed to capture time-variation in betas in FM regression, while BJS regression uses the entire sample period to estimate betas. In this study, a three-year rolling window of past daily stock returns is used to estimate the betas in FM regression. Three years is a reasonable compromise between shorter periods, such as one year, which often lead to greatly increasing estimation errors, and longer periods, such as five years, which typically involve using outdated information and only allow for small variations in the factor loadings over time. In addition, the high volatility of realized stock returns obscures their information about expected returns, thus using a rolling five years or above window period may say little about the cross-section of expected returns.

$$(1)\ r_{i,t} \ = \ a_0 + \sum_{K=1}^{K} \beta_{i,k}\, f_{k,t} + \varepsilon_{i,t}$$

where $f_{k,t}$ represents the pure factor k return in month t, a_0 represents the intercept term, $\beta_{i,k}$ represents the loadings of stock i to factor k, and $\varepsilon_{i,t}$ represents the regression residuals. The second pass of BJS/FM regression is conducted on a time-series scale by regressing the cross-sectional asset returns on betas that were estimated from the first pass, using the following cross-sectional regression:

$$(2)\ r_{i,t} = \gamma_{0,t} + \sum_{K=1}^{K} \hat{\beta}_{i,k}\, \gamma_{k,t} + \xi_{i,t}$$

where $r_{i,t}$ is the excess return of stock i rather than portfolios as many other papers used, $\gamma_{k,t}$ is risk premium of factor k in month t, $\hat{\beta}_{i,k}$ is obtained from the first pass which is estimated with errors.

For revised FM regression with the IV estimator, the betas estimated by using Eq. (1) are divided into the two cross-sectionally uncorrelated parts, namely explanatory and instrumental betas, which are estimated separately using returns on days belonging to odd and even months within the rolling window, respectively. In the second step of this revised FM regression, a two-stage least squares (2SLS) regression is performed at each time t using explanatory betas are explanatory variables and instrumental betas are instrumental variables and then calculating the time-series average of risk premiums for each factor. The IV estimator is described in Section 3.4.2. Specifically, this study fits the regression with daily return data from month t-36 to month t-1 to estimate beta for month t, i.e., the rolling window in this study is 36 months (756 days), and the t-statistics of the mean of zero-beta asset returns and factor premiums are adjusted by Newey-West corrected for autocorrelation and heteroscedasticity.

Another important question is what value of t-statistic one would consider statistically significant. As stated by Harvey et al. (2016) a factor recognized by the academic community could be regarded as significant if its t-statistic value is above 2.0. This threshold takes the multiple testing in asset pricing models into consideration. As a result, the significant level of t-statistics for factor risk premiums in this study is also set to be 2.0 or above.

3.4 Estimators

This section lists three estimators in the BJS/FM regression for this study. The OLS estimator in the second pass of BJS/FM regression is problematic as the explanatory variables (betas) are estimated with measurement errors, i.e., a classic EIV problem. Thus, the study introduces the IV and Shanken's estimator to correct this bias.

3.4.1 OLS Estimator

The second pass of FM regression as Eq. (2) shown can be simply rewritten as:

$$(3)\ r_t = \hat{\gamma}\hat{B} + \xi_t$$

where r_t is a $1 * N$ row vector of excess returns at time t, $\hat{B}$ is the $(K+1) * N$ matrix including the unit vector and K factor betas, and $\hat{\gamma}$ is a $1 * (K+1)$ vector of risk premiums. Thus, the OLS estimator of risk premium can be written as:

$$(4)\ \widehat{\gamma_{OLS,t}}' = \left(\hat{\mathrm{B}}\,\hat{\mathrm{B}}'\right)^{-1}(\hat{\mathrm{B}}\,r_t')$$

$\hat{\mathrm{B}}$ is estimated from the first stage of FM regression with errors, thus the OLS estimations of $\widehat{\gamma_{OLS,t}}$ are biased due to the EIV problem.

3.4.2 IV Estimator

In Jegadeesh et al. (2019), they introduce IV to avoid the EIV problems in Eq. (4) and the IV estimator of risk premium can be written as:

$$(5)\ \widehat{\gamma_{IV,t}}' = \left(\widehat{B_{IV}}\widehat{B_{EV}}'\right)^{-1}(\widehat{B_{IV}}r_t')$$

where B_{IV} and B_{EV} are the matrices of IVs and EVs, respectively. Both betas are calculated simultaneously from two disjoint data in the same rolling window. For example, odd-month betas serve as IVs and even-month betas serve as EVs when month t is even, and vice versa when month t is odd. The EVs and IVs are estimated from a disjoint daily data sample which is divided into odd-month and even-month. Daily data within odd and even months are used to estimate B_{IV} and B_{EV}. Both B_{IV} and B_{EV} are the estimations of true betas with errors, but their errors are not cross-sectionally correlated.

When a large number of stocks are present in the cross-section under mild regularity conditions, which Shanken (1992) describes as N-consistency, EIV can be solved with IV techniques. The following is a formal expression of N-consistency for the IV estimator.

Proposition 1: Assume that stock returns are influenced by an approximate factor structure with K common factors. Applied to a mildly regular sample, the IV estimator given by Eq. (5) is N-consistent if the number of stocks increases without bound.

According to Proposition 1, the IV estimator can be viewed as a cross-sectional regression based on the two-stage least squares (2SLS) method. The first step in the 2SLS process is to regress the explanatory variables against the instrumental variables. The slope coefficients of the first pass regression are as follows:

$$(6)\ \hat{\lambda} = \left(\widehat{B_{IV}}\widehat{B_{EV}}'\right)^{-1}(\widehat{B_{IV}}\widehat{B_{EV}}')$$

An OLS model for this second pass regression is an IV model, as it uses the fitted values from the first pass regression as the explanatory variables. The formula is given below:

$$(7)\ \widehat{\gamma_{IV,t}}' = \hat{\lambda}^{-1}\{\left(\widehat{B_{IV}}\widehat{B_{IV}}'\right)^{-1}(\widehat{B_{IV}}r_t')\}$$

Within braces, the expression in Eq. (7) is an OLS estimate of risk premiums if returns are regressed against IV betas. However, EIV bias can affect these estimates. By multiplying by the inverse of $\hat{\lambda}$, the EIV bias can be corrected, thus yielding consistent estimates.

3.4.3 Shanken's Estimator

Shanken (1992) and Bai and Zhou (2015) among others propose that the FM-OLS second-pass estimator is biased and inconsistent as the explanatory variables (betas) are estimated with measurement errors when T is fixed. On the other hand, if the beta is estimated for each asset over the entire sample period (not rolling window), measurement errors in beta decline with increasing sample size, T. In this case, the cross-sectional OLS estimator is T-consistent. Although the T-consistent property of the cross-sectional OLS estimator is attractive, people are more likely to deal with a "short and wide" panel in practice, (i.e., short period, T, and a large number of cross-sectional units, N).

Black et al. (1972) and Fama and MacBeth (1973) suggest minimizing the measurement error of betas by grouping individual stocks into portfolios. They argue that portfolio betas are free of estimation errors as the stock number in each portfolio increases to infinite and hence the risk premium estimator is N-consistent. However, as Ang et al. (2018) point out, smaller standard errors of portfolio beta estimates do not lead to more precise risk premium estimates. Especially, portfolios destroy information by shrinking the cross-sectional dispersion of betas, leading to larger standard errors of coefficient estimates.

Shanken (1992) examines the estimator from a different analytic perspective; he assumes T is fixed and N is allowed to increase without bound; a situation in which the traditional second-pass estimator is biased and inconsistent. He further shows that this bias can be corrected by a bias-adjusted term as explained in the following equations. One concern here is that the ex-post factor risk premiums cannot be used as a consistent estimator of the ex-ante premiums because the average of factor realizations does not converge in probability to its expectation when T is fixed. However, Bai and Zhou (2015) show that this impact is small in practice.

Assumption 1: The residual vector in Eq. (1) is independently and identically distributed over time, conditional on factor realizations.

Assumption 2: The factor realizations vector in Eq. (1) is generated by a stationary process such that the mean and variance converge in probability to the true moments, as T increases to infinite. Also, the average of factor realizations is asymptotically normally distributed.

Under Assumption 1 and 2, the bias-adjusted estimator and covariance matrix of Shanken (1992) is given below. Denote the trace operator by $tr(\cdot)$ and a K-dimensional vector of zeros by 0_k.

$$(8)\ \lambda_{\widehat{Shanken}} = \left(\Sigma\hat{\beta}\ - \widehat{\Lambda}\right)^{-1} \cdot \frac{\hat{\beta}'\bar{R}}{N}$$

$$where\ \Sigma\hat{\beta} = \frac{\hat{\beta}'\hat{\beta}}{N}, \widehat{\Lambda} = \begin{bmatrix} 0 & 0'_k \\ 0_k & \sigma^2(F'F)^{-1} \end{bmatrix}\ and\ \sigma^2 = \frac{1}{N(T-K-1)} tr(\Sigma)$$

The classic OLS estimator is below, as Shanken point out, $\widehat{\lambda_{OLS}}$ cannot be used as a consistent estimator of the ex-ante risk premiums for a fixed T. Thus, Shanken's bias-adjusted estimator shows a multiplicative adjustment through the term $\left(\Sigma\hat{\lambda}\ - \widehat{\Lambda}\right)^{-1}$.

$$(9)\ \widehat{\lambda_{OLS}} = \left(\hat{\beta}'\hat{\beta}\right)^{-1}\hat{\beta}'\bar{R}$$

Furthermore, Shanken's estimator in Eq. (8) is extended to deal with the more realistic case of unbalanced panels following the proposition of Raponi et al. (2020). The proposed modified estimator of the ex-post risk premia is below

$$(10)\ \lambda_{\widehat{Shanken,u}} = \left(\Sigma\widehat{\beta_u} \ - \widehat{\Lambda_u}\right)^{-1} \cdot \frac{\widehat{\beta_u}'\overline{R_u}}{N}$$

$$where\ \Sigma\widehat{\beta_u} = \frac{\widehat{\beta_u}'\widehat{\beta_u}}{N}, \widehat{\Lambda_u} = \begin{bmatrix} 0 & 0'_k \\ 0_k & {\sigma_u}^2({F_u}'F_u)^{-1} \end{bmatrix}$$

$$and\ \ {\sigma_u}^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{(T_i - K - 1)} tr(\Sigma_u)\right)$$

Since the panel is unbalanced, there is now a sequence of ex-post risk premiums, one for each asset i. In the other words, the sequences of these risk premiums have different time lengths T_i for each asset i. In addition, the elements like $\widehat{\beta_u}, \widehat{\Lambda_u}\ and\ \overline{R_u}$ in Eq. (10) with subscript u mean that they are truncated to have the same time length T_i for each asset i. Raponi et al. (2020) further prove the consistency and asymptotic normality of this modified Shanken's estimator $\lambda_{\widehat{Shanken,u}}$.

Given that the FM procedure tends to overstate the precision (underestimate the standard errors) of risk premium estimates when the sample period is fixed, Shanken proposes an adjustment method on the covariance matrix of factor premium estimates to correct for that. There are two theorems in his paper: the first one is for cross-sectional regression, and the second one is for BJS or FM regression with rolling betas.

Theorem 1:

$$(11)\ T \cdot Cov_{adj}(\hat{\lambda}) = \ (1 + c)\left(T \cdot Cov_{bias}(\hat{\lambda}) - \ \textstyle\sum F^*\right) + \ \textstyle\sum F^*$$

$$where\ T \cdot Cov_{bias}(\hat{\lambda}) = \left[\ (\beta'\beta)^{-1}\beta \textstyle\sum \beta(\beta'\beta)^{-1} \ + \displaystyle\sum F^* \right]$$

$$and\ c = \ \lambda' \sum F^{-1}\, \lambda$$

Theorem 2:

$$(12)\ T \cdot Cov_{adj}(\hat{\lambda}) = (1 + c^*)\big(T \cdot \widehat{W} - \textstyle\sum F^*\big) + \sum F^*$$

$$where\ T \cdot \widehat{W} = \sum_{t=1}^{T} \big(\hat{\lambda} - \bar{\lambda}\big)\,(\hat{\lambda} - \bar{\lambda})'$$

$$and\ c^* = [1 - \frac{(y-1)(y+1)}{3yn}] \cdot c$$

where λ is the vector of zero-beta rate and factor risk premium estimates, β is the matrix of stock intercepts and betas obtained from the first stage of FM regression, $\sum$ is the covariance of residuals, $\sum F$ is the covariance matrix of k factors (without intercept), $\sum F^*$ is the covariance matrix of (k+1) factors (with intercept), c stands for the asymptotic adjustment, y is the length of the rolling window in years, n is the length of sample period after excluding y, and T is the length of the whole sample period. Additionally, c is just the squared value of the well-known Sharpe measure of performance if there is only one factor in the model.

There are several advantages to adopting the Shanken's correction: 1) the Shanken's bias-adjusted estimator is the only member of a large class of FM-OLS modified estimators that do not require preliminary estimation of the bias-adjustment; 2) unlike other approaches such as instrumental variable estimation, Shanken's estimator merely requires a panel of asset returns and a sample of factor realizations.

3.5 Empirical Asset Pricing Models

3.5.1 Risk-Return Model

A variety of asset pricing models have been used to describe the expected returns of test assets and their correlation with certain risk factors. By using the IV approach, this study tests empirically whether risk awards in the CAPM, the FF3-factor model and the FF5-factor model are significantly different from zero. In general, a K-factor asset pricing model looks like this:

$$(13)\ E(r_i) = \gamma_0 + \sum_{K=1}^{K} \beta_{i,k}\, \gamma_k$$

where $E(r_i)$ is the expected excess return on stock i, $\beta_{i,k}$ is the loadings of stock i to factor k, and γ_k is the factor k premium, γ_0 is the excess return on the zero-beta asset. Especially, the return of a zero-beta asset is the risk-free rate and its excess return should be zero if riskless borrowing and lending are allowed, i.e. γ_0=0.

3.5.2 The CAPM Model

There are a number of asset pricing models in the literature that assume just a few factors can convey systematic risk and that the expected return on an asset is linearly proportional to its factor betas. Among them, risk factors are determined by theoretical arguments. The single-factor CAPM (Sharpe, 1964; Lintner, 1965) is used in this study, which states that the market portfolio return is a relevant risk factor. Following is the corresponding time-series regression model:

$$(14)\ r_{i,t} - r_{f,t} \ = \ a_i + \ \beta_{i,MKT}\, MKT_t + \ \varepsilon_{i,t}$$

3.5.3 The FF3-Factor Model

Some models specify factors using economic introspection and intuition. Fama and French (1993), for example, construct factors that capture the size and book-to-market effects and test if these accurately capture systematic risks. Following is the corresponding time-series regression model:

$$(15)\ r_{i,t} - r_{f,t} \ = a_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \ \varepsilon_{i,t}$$

3.5.4 The Carhart4-Factor Model

Jegadeesh and Titman (1993) propose the momentum anomaly in which they construct a long-short portfolio by ranking stocks from “winners” to “losers” based on their cumulative return summation from t-11 to t-1 (a total of 11 months, the latest month is intentionally excluded to avoid the potential short-term reversal phenomenon). As motivated by this study, Carhart (1997) include the momentum factor in the previous FF3-Factor Model. Following is the corresponding time-series regression model:

$$(16)\ r_{i,t} - r_{f,t} \ = a_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,WML} WML_t + \ \varepsilon_{i,t}$$

3.5.5 The FF5-Factor Model

A new five-factor model is proposed by Fama and French (2015) by adding another profitability and investment variable to their previous FF3-factor model. This study adopts all of these three linear beta asset pricing models to estimate various risk premiums, including size, B/M ratio, operating profitability, and investment growth rate. Following is the corresponding time-series regression model:

$$(17)\ r_{i,t} - r_{f,t} = a_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t + \beta_{i,CMA}CMA_t + \varepsilon_{i,t}$$

In the single-factor CAPM, the FF3-factor model, Carhart4-factor the FF5-factor model models, a_i is the intercept and also the mispricing error for left-hands-side stock i, and $r_{i,t}$ is the return of individual stock i at day t, and $r_{f,t}$ is the risk-free rate (91-day Treasury-bill return rate at the beginning of day t), and $\beta_{i,MKT}, \beta_{i,SMB}, \beta_{i,HML}, \beta_{i,RMW}, \beta_{i,CMA}, \beta_{i,WML}$ are the betas with respect to the market, size, earnings-to-price, operating profitability, investment and momentum factors, and $MKT_t, SMB_t, HML_t, RMW_t, CMA_t, WML_t$ are the corresponding factor return realizations. The MKT_t factor is the value-weight stock market return for day t and the remaining four factors are differences between returns on double-sorted portfolios of small and big stocks (SMB_t), high and low BM stocks (HML_t), stocks with robust and weak profitability (RMW_t), stocks of low and high investment firms (CMA_t, conservative minus aggressive), and stocks of winner (high historical cumulative returns) and loser (low historical cumulative returns) firms (WML_t). The definition of these factors, the construction procedure and the corresponding summary statistics are presented in detail in the Appendix.

3.6 Conclusion

This chapter describes the data collection procedure and the corresponding treatments. The regression method that this study used is a revised two-stage FM/BJS regression, in which the IV/Shanken's estimator is used in the second-stage cross-sectional regression rather than the traditional OLS estimator to correct for the EIV bias. Several prominent empirical asset pricing models are tested in this study, including the single-factor CAPM, the Carhart four-factor model, and the Fama and French three- and five-factor models. In the next chapter, the simulation based on the OLS, IV and Shanken's estimators is conducted for comparison.

CHAPTER 4: SIMULATION

4.1 Introduction

This chapter evaluates the OLS and IV methods with two small sample simulations under the single-factor CAPM model. It consists of two main sections: Section 4.2 describes the simulation procedure about how to construct simulating stock returns and market returns by using parameters matched to real data; In Section 4.3, the simulation results of using the FM-OLS, FM-IV and BJS-Shanken methods are compared. Finally, Section 4.5 concludes.

4.2 Simulation Procedure

To intuitively evaluate the bias and root-mean-squared error (RMSE) of OLS and IV estimators, a battery of simulations using the parameters matched to the real data are conducted with the cross-sectional size of N = (500, 2000) stocks and time-series length of T = (20, 57) years. Based on 1000 replications, the average difference between the risk premium estimates and the average factor realization is the ex-post bias, which is the focus of the simulation. In addition, the ex-ante bias relative to the true risk premium is also provided as a supplement. People only observe ex-post realizations in practice, though we know the "true" (ex-ante) risk premium in simulations. The difference between the ex-ante and ex-post risk premium will be narrowed as T increases to infinity.

Before conducting the simulation, time-series regressions by using the CAPM model are fitted for each stock to estimate market betas and residual return standard deviations, which are the two key values in simulating the distribution of data. Table 2 reports the simulation parameters.

Table 2

Simulation Parameters		
Panel A: Market Factor		
	Mean	**Std.Dev.**
Market	6.79%	17.13%
Panel B: Betas and Idiosyncratic Volatility for CAPM		
	Mean	**Std.Dev.**
Betas	0.5078	0.2957
IV	31.84%	10.35%

This table presents the parameters that are used in the simulations. This study sets the risk premium of the market factor and its covariance structure in the simulations equal to the *corresponding sample values during the sample period of January 2000 through December 2019. The means and standard deviations of the market factors and idiosyncratic volatility are annualized and reported in percentages.*

The following steps are taken to randomly generate daily returns:

1) For each stock, we randomly generate a beta and a standard deviation of residuals from normal distributions with means and standard deviations equal to the corresponding means and standard deviations of the real data. Betas and standard deviations of residuals are generated at the beginning of each simulation to keep them constant through 1000 repetitions.

2) For each day, realized market factor returns are generated from a normal distribution with mean and standard deviation equal to the mean and standard deviation from the real market excess return.

3) For each stock, the daily residual returns are generated from a normal distribution with a mean equal to zero and a standard deviation equal to the value generated in step (1).

Finally, the excess return for each stock i on day t can be calculated as:

$$(18)\ r_{i,t} = \beta_i r_{MKT,t} + \varepsilon_{i,t}$$

For the first pass of FM regression in the simulation, β_i is estimated by regressing simulated stock excess returns against simulated market excess returns with a traditional rolling estimation window of three years (756 days)[1]. For the IV method, daily stock excess returns are divided into odd-month and even-month as described above to estimate explanatory and instrumental betas.

The second pass of FM regression is fitted on a monthly scale, thus simulated daily stock and market excess returns are compounded to compute corresponding monthly returns. Further, rolling estimation windows of different intervals (including half even-months and half oven-months) are rolled forward by one month and the IV/OLS estimations are repeated over holding month periods on a time-series scale to estimate the intercepts and market premiums. Finally, the time-series average of the market premiums is calculated and then compare with the true market premium to evaluate the ex-ante and ex-post biases of the IV/OLS estimator.

[1] The simulation assumes each month has 21 trading days, thus 252 days corresponding to one year.

4.3 Simulation Results

Under a single-factor CAPM model, the following sections first compare the simulation results of using FM-IV and FM-OLS to estimate market risk premium and then present the superiority of using BJS-Shanken correction as the final resolution to mitigate EIV bias.

4.3.1 FM-OLS and FM-IV Regression

As the other literature on using the IV approach, this study also conducts weak instruments tests, and their null hypotheses are rejected on average at the 0.01 level meaning that the instrumental betas are strong instruments, while the explanatory betas are statistically significantly correlated with the residual returns on average according to the result of (Wu-)Hausman test for endogeneity. Additionally, the average F-statistics in the first stage of IV-2SLS regression is about 1829.41, which is much higher than the rule of thumb that the regression of explanatory variables on the instrumental variables should have an F statistic of at least 10, meaning that the instrumental variables are strongly correlated with the explanatory variables.

Table 3 presents the ex-ante and ex-post biases and RMSEs. When a rolling window of 1 year (252 days) is used to estimate beats, the OLS estimator is biased by -27.14% relative to the ex-ante risk premium and by -26.19% relative to the ex-post risk premium, respectively. On the other hand, the ex-ante and ex-post biases of the IV estimator are -16.51% and -15.73%, respectively, which are a bit lower than the OLS estimator. When the rolling window is increased to 5 years (1260 days), the average ex-ante and ex-post biases of IV and OLS estimators are -2.37%, -1.89%, 2.52% and 1.65%, respectively, which are much smaller than using one year as the rolling window. Despite this, both estimators' biases fail to converge to zero as the rolling window grows. In addition, the biases of IV estimators are only slightly lower than the OLS estimator about 7% on average. These results contrast with Jegadeesh et al. (2019), which find the biases of the IV estimator are fairly close to zero and statistically insignificant regardless of the different settings of the beta estimation window.

Table 3
Small Sample Properties of IV Risk Premium Estimates (CAPM)

Rolling Period (years)	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
1	OLS	-27.1393	-26.1879	0.0641	0.0742
	IV	-16.5125	-15.7312	0.0691	0.0791

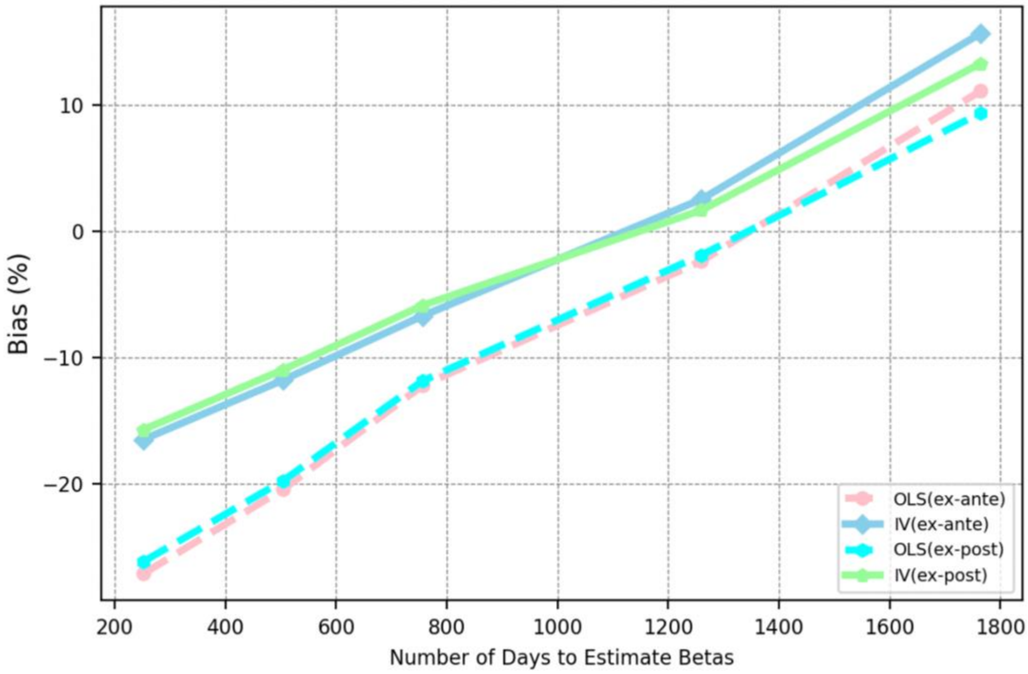
2	OLS	-20.4629	-19.7896	0.0658	0.0778
	IV	-11.8083	-10.9876	0.0685	0.0810
3	OLS	-12.2368	-11.8769	0.0674	0.0803
	IV	-6.7241	-5.8921	0.0692	0.0820
5	OLS	-2.3749	-1.8976	0.0693	0.0801
	IV	2.5237	1.6542	0.0707	0.0812
7	OLS	11.0925	9.3574	0.0704	0.0797
	IV	15.6539	13.2759	0.0714	0.0803

This table presents the biases and root-mean-squared errors (RMSEs) of risk premium estimates when the second-stage regressions are fitted using the OLS and Instrumental Variable (IV) methods under the CAPM model. The simulation uses 500 stocks in the cross-section, and the results are based on 1000 repetitions. The sample period for the simulations is 240 months. For each month, rolling betas are estimated using daily return data over the previous 12/24/36/60/84 months. Ex-ante bias is the difference between the mean risk premium estimate and the corresponding true risk premium. Ex-post bias is the difference between the mean risk premium estimate and the sample mean of the corresponding risk factor realizations in that particular simulation. Ex-ante and ex-post biases are expressed in percentages.

Figure 1 plots the ex-ante and ex-post bias of the IV and OLS estimators as a function of the rolling window length. According to it, the biases of the IV estimator do not converge to zero as the rolling window in the sample grows. Therefore, the IV estimator under the single-factor CAPM model is not unbiased as Jegadeesh et al. (2019) argued when the observation window is finite (T = 5215 days), and the sample size is small (N = 500).

Figure 1

Biases versus Number of Days Used to Estimate Betas (N=500, T=20 years)

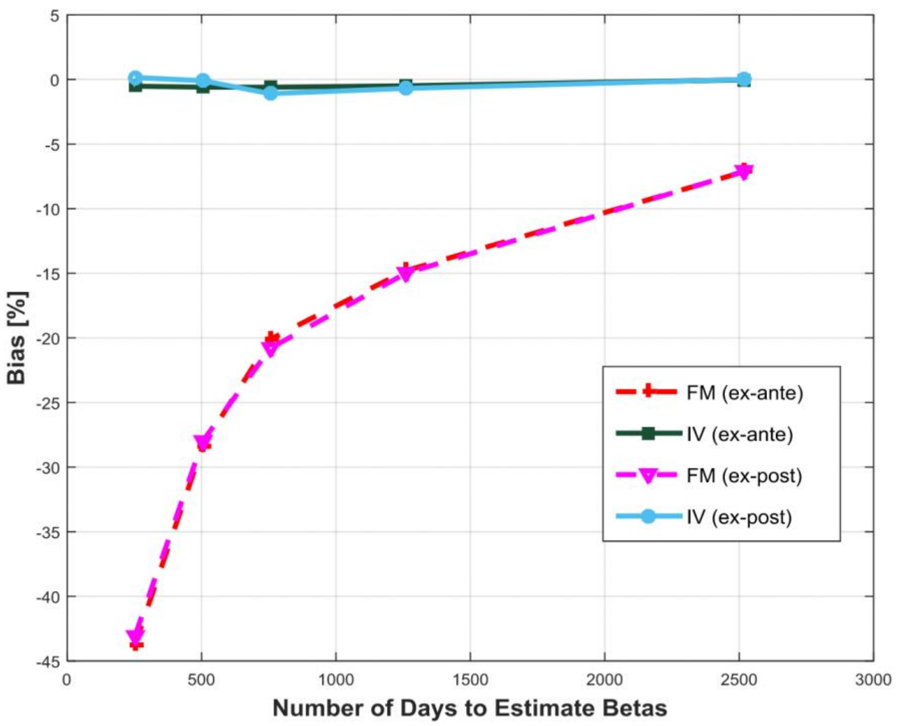


This figure presents the ex-ante and ex-post biases using the ordinary least squares (OLS) and instrumental variables (IV) estimators of the market risk premium under the CAPM, as a function of the number of days in the rolling window to estimate the market betas. The simulations use the market risk premium of 6.79% per annum and 500 individual stocks in the cross-section. The sample period for the simulations is 284 months. The y-axis is biased as a percentage of the true market risk premium, and the x-axis is the number of days in the rolling window to estimate the market betas. These results are based on 1,000 repetitions for each estimation window.

The last two columns in Table 3 present the ex-ante and ex-post RMSEs of both estimators, which measure the combined effect of standard errors and biases. Overall, the ex-ante RMSE for the IV estimator is only slightly higher than that for the OLS estimator, which indicates that the IV estimator is almost the same accurate (or inaccurate) as the OLS estimator in the simulation. For example, the ex-ante and ex-post RMSE for the OLS estimator is 0.06 and 0.07, compared with 0.07 and 0.08 for the IV estimator when the rolling period is one year (252 days), and these differences are statistically insignificant. This result is also in contrast to the findings in Jegadeesh et al. (2019), in which they found that the ex-ante and ex-post RMSEs of the IV estimator are significantly lower than that of the OLS estimator, meaning the IV estimator is more stable or accurate than the OLS estimator.

It should be noted from the above result that the ex-ante and ex-post biases become positive and significantly large as the rolling window increase to 7 years (1750 days), the values of them in Table 3 are 11.09%, 9.35%, 15.65%, 13.27%, respectively. The fact that biases are not converged towards zero as the rolling window increases is in contrast to the proposition of Jegadeesh et al. (2019), in which they suggest that the EIV bias is an attenuation bias (always towards zero), and the IV estimator remains unbiased even with a small sample size as below shown. Figure 2 is from Jegadeesh et al. (2019), which shows that the bias of the IV estimator is always close to zero regardless of the different rolling window settings.

Figure 2
Biases versus Number of Days Used to Estimate Betas (N=2000, T=57 years)



This figure is from Jegadeesh et al. (2019), which presents the ex-ante and ex-post biases using the ordinary least squares (OLS) and instrumental variables (IV) estimators of the market risk premium under the CAPM, as a function of the number of days in the rolling window to estimate the market betas.

The reason for the contrary simulation results found in this study may be attributed to the following reasons: 1) the simulation procedure that this study used is not the same as what Jegadeesh et al. (2019) applied; 2) the original paper exploits the assumed independence of beta measurement errors across time, however, once the measurement errors are auto-correlation, lagged betas are no longer justified to be used as instrumental variables; 3) the IV estimator under FM procedure is not unbiased due to the complexity in a finite sample; 4) Comparatively to 2000 stocks in the original paper, 500 stocks on the cross-section are too small for simulations. In an attempt to dispel such concerns, this study repeats the same size simulation as the original paper, so the length of the observation period will be increased to 57 years and the stock amount will be increased to 2000, and all of the simulation parameters will remain the same as in the original study. Table 4 shows the original simulation parameters.

Table 4
Simulation Parameters

Panel A: MKT Factor		
	Mean	**Std.Dev.**
MKT	5.80%	15.33%

Panel B: Betas and Idiosyncratic Volatility (IV) for CAPM		
	Mean	**Std.Dev.**
Betas	0.95	0.42
IV	58.73%	23.81%

This table presents the parameters that are used in the simulations. This study sets the risk premium of the market factor and its covariance structure in the simulations equal to the corresponding sample values in Jegadeesh et al. (2019). The means and standard deviations of the MKT factors and idiosyncratic volatility are annualized and reported in percentages.

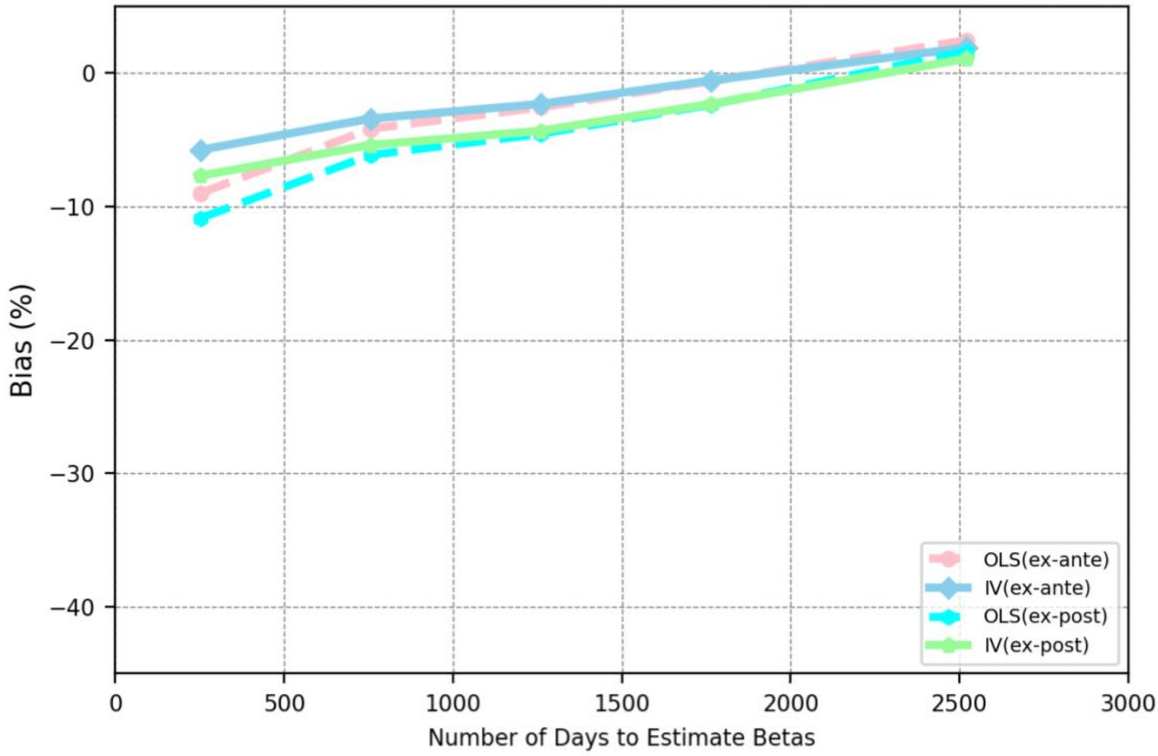
As compared with Table 3, the ex-ante and ex-post biases for both estimators in Table 5 decreased significantly as the number of cross-sectional stocks increased from 500 to 2000. IV estimator biases are negative and significantly different from zero, and they are similar to those of the OLS estimator. In other words, the IV estimator fails to outperform the OLS estimator in terms of correcting for EIV bias. By the time the rolling window reaches 10 years (2520 days), both estimators tend to have significant positive biases, which means that their bias is not attenuated but monotonically increasing. This tendency can also be found in Figure 3. In conclusion, using lagged betas as instrumental variables would not result in unbiased estimates of ex-post risk premiums.

Table 5
Small Sample Properties of IV Risk Premium Estimates (CAPM)

Rolling Period (years)	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
1	OLS	-9.0037	-10.8707		
	IV	-5.7519	-7.6857		
3	OLS	-4.1658	-6.1321		
	IV	-3.4048	-5.3868		
5	OLS	-2.5595	-4.5588		
	IV	-2.3114	-4.3158		
7	OLS	-0.6091	-2.3624		
	IV	-0.5500	-2.3044		
10	OLS	2.4610	1.6327		
	IV	1.9114	1.0795		

This table presents the biases and root-mean-squared errors (RMSEs) of risk premium estimates when the second-stage regressions are fitted using the OLS and Instrumental Variable methods under the CAPM model. The simulation uses 2000 stocks in the cross-section, and the results are based on 1000 repetitions. The sample period for the simulations is 684 months. For each month, rolling betas are estimated using daily return data over the previous 12/36/60/84/120 months. Ex-ante bias is the difference between the mean risk premium estimate and the corresponding true risk premium. Ex-post bias is the difference between the mean risk premium estimate and the sample mean of the corresponding risk factor realizations in that particular simulation. Ex-ante and ex-post biases are expressed as percentages of the true risk premiums.

Figure 3
Biases versus Number of Days Used to Estimate Betas (N=2000, T=57 years)



This figure presents the ex-ante and ex-post biases using the ordinary least squares (OLS) and instrumental variables (IV) estimators of the market risk premium under the CAPM, as a function of the number of days in the rolling window to estimate the market betas. The simulations use the market risk premium of 5.80% per annum and 2000 individual stocks in the cross-section. The sample period for the simulations is 684 months. The y-axis is bias as a percentage of the true market risk premium, and the x-axis is the number of days in the rolling window to estimate the market betas. These results are based on 1,000 repetitions for each estimation window.

4.3.2 The Problem of the FM-IV Method

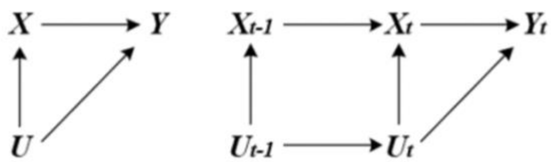
In the recent paper by Pukthuanthong et al. (2019), who are also the authors of Jegadeesh et al. (2019), they did not use the IV approach for attenuating EIV bias due to concerns about "weak instrument" issues given the puzzling findings in their untabulated results. However, this explanation does not seem to be convincible as it is easy to prove that lagged betas would have a strong correlation with current betas. Thus, this study explains the failure of the FM-IV approach from an econometric perspective.

In Jegadeesh et al. (2019), they proposed to use the IV estimation approach to attenuate the errors-in-variables (EIV) bias, which is inherent to a two-stage Fama-MacBeth (FM) regression. The core of their method is that the beta estimation of IVs and EVs is from a disjoint data sample (equal split odd and even months within the rolling window), which allows their measurement errors to be cross-sectionally uncorrelated. In this context, the IVs that they applied are lagged EVs.

For any IVs to be valid, it needs to satisfy two prerequisites, namely relevance and exogeneity. It is obvious that using lagged EV_{t-1} as IVs would have strong correlations with the current EV_t, thus the lagged EVs are strong IVs. In addition, using lagged EVs (EV_{t-1}) as IVs are exogenous to the regression of time t. This is because the regression residuals ($U_t + \varepsilon_t$) at time t are cross-sectionally uncorrelated with lagged EVs (EV_{t-1}). It seems like lagged EVs are an ideal choice for IVs and can be further used in dealing with EIV bias.

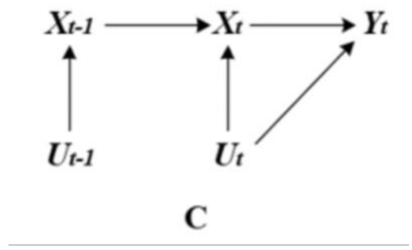
However, the above statement is valid only if the measurement errors do not have serial correlation. Reed (2015) suggests that if unobservable factors (e.g., measurement errors) have autocorrelation then they will cause lagged EVs to be invalid instruments. In the Appendix of Jegadeesh et al. (2019), they prove that the IV estimator is unbiased and N-consistent by simply assuming that measurement errors are white noise, i.e., they do not have any autocorrelation. However, the nature of measurement errors is unobservable, and one cannot test whether it satisfies the hypothesis that there is no serial correlation. Due to the potential autocorrelation, the risk premium estimates in the second pass would neither converge in probability to ex-post nor ex-ante risk premiums, resulting in biased and inconsistent estimates.

The directed acyclic graph A shows a classic endogeneity problem when the measurement errors U can influence Y by $U \rightarrow X \rightarrow Y$ and $U \rightarrow Y$. A common practice in applied econometrics work consists of using lagged EVs as IVs to solve this endogeneity problem caused by the measurement errors in X. However, the lagged measurement errors U_{t-1} can influence Y_t by $U_{t-1} \rightarrow U_t \rightarrow Y_t$ and $U_{t-1} \rightarrow X_{t-1} \rightarrow X_t \rightarrow Y_t$ when the measurement errors have serial correlation as the directed acyclic graph shown in directed acyclic graph B, which will result in the biased and inconsistent estimation of parameter coefficients.



A B

According to Bellemare et al. (2015), when there are unobservable factors in a model, if both of the following two conditions are fulfilled simultaneously: 1) there is no serial correlation in unobservable factors (like measurement errors); 2) EVs are stable autoregressive processes, then lagged EVs (X_{t-1}) can be used as IVs for current EVs (X_t) to solve the endogeneity problem.



In summary, the IV estimator will not eliminate the EIV bias but simply move it back to a point in time when there is autocorrelation in measurement errors, and consequently provide biased and inconsistent estimates.

4.3.3 BJS-Shanken Regression

Traditionally, factor loadings are estimated through time-series regressions of excess asset returns on factor realizations. Further, betas can be estimated by either using the entire period T like BJS (1972), which assumes that betas are constant throughout the entire period T or by using a rolling window like FM (1973), which assumes betas are constant within the specified window. Using rolling betas as explanatory variables, however, does not require the cross-sectional estimator to be T-consistent since expanding the sample period does not eliminate the systematic EIV bias in each cross-sectional regression. While Shanken (1992) describes his adjustment for accommodating "rolling betas", it cannot be considered as formal asymptotic analysis, but rather as a simple heuristic. In light of this, this study decides to employ BJS regression along with Shanken's bias-adjusted estimator.

Table 6 shows the ex-post biases of Shanken's bias-adjusted estimator under the BJS regression of single-factor CAPM. The ex-post bias of Shanken's estimator is only -5.01% of the true simulation risk premiums when using a short sample period of 5 years and 500 stocks on cross-section to estimate betas. The magnitude of this bias is much smaller than that of the FM-OLS and FM-IV in Table 3 and Table 5. For example, when using a sample size (T = 20 years, N = 500) as Table 3 shown, the ex-post bias of OLS and IV estimators are range from -26.19% to 13.28% with an absolute average of 11.66%. In contrast, the ex-post bias of the BJS-Shanken estimator under the same setting is only -1.62%. When using a large sample size (T = 57 years, N = 2000) as Table 5 presented, the ex-post bias of OLS and IV estimators are range from -10.01% to 1.08% with an absolute average of 4.63%, while the bias of Shanken's estimator under the same number of cross-sectional stocks is already fairly close to zero and statistically indistinguishable from zero even with a sample period of 5 years.

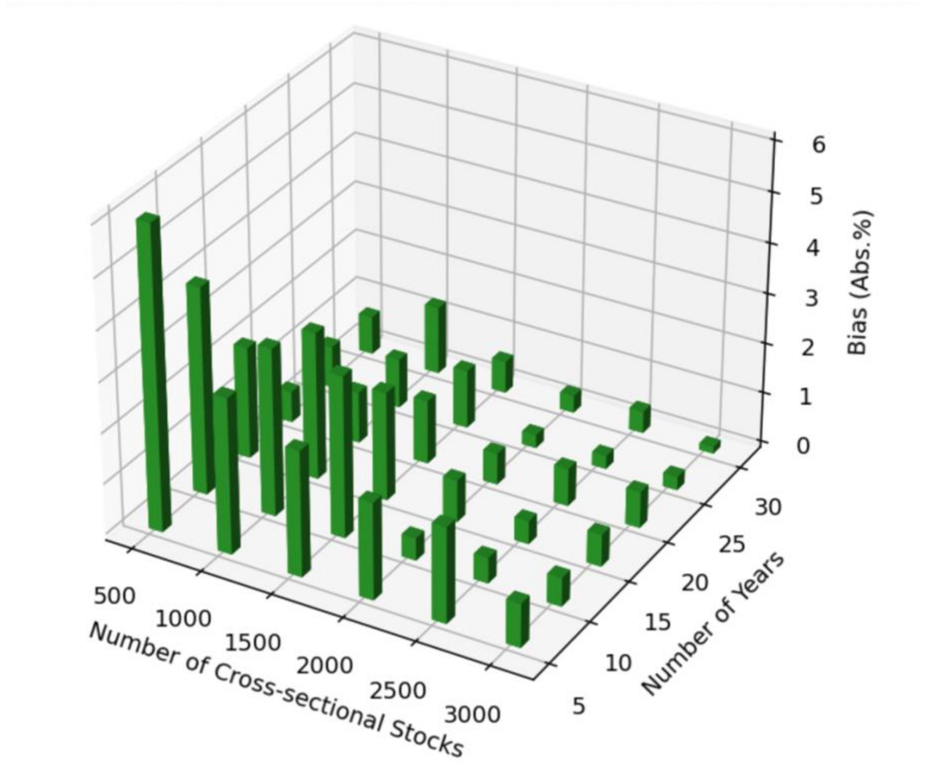
Table 6
The Ex-Post Bias (%) of BJS-Shanken Risk Premium Estimates (CAPM)

T \ N	500	1000	1500	2000	2500	3000
5	-5.0130	-3.0869	-2.4919	-1.6206	-1.2964	-0.8721
10	-4.1233	-3.3234	-3.2082	-0.4455	-0.5088	-0.5721
15	-2.2261	-2.9313	-2.1567	-0.8372	-0.4616	-0.6386
20	-1.6175	-1.0227	-1.2658	-0.6272	-0.7525	-0.7233
25	-1.8410	-0.9819	-1.1634	-0.2663	-0.2680	-0.2697
30	-1.7652	-1.3576	-0.6444	-0.3456	-0.4392	-0.1328

This table presents the ex-post biases of risk premium estimates when the second-stage regressions are fitted using the BJS-Shanken methods under the CAPM model. The simulations use the market risk premium of 6.79% per annum and 500/1000/1500/2000/2500/3000 individual stocks on the cross-section. The sample period for the simulations is 5/10/15/20/25/30 years. The result is based on 100 repetitions. Ex-post bias is the difference between the mean risk premium estimate and the sample mean of the corresponding risk factor realizations in that particular simulation.

Figure 4 plots the ex-post biases using Shanken's estimator, which is expressed as a function of the number of time-series years and cross-sectional stocks to estimate the market factor loadings. The ex-post bias of Shanken's estimator is attenuated and biased towards zero as the cross-sectional stock number grows, which implies the consistent and unbiased property of Shanken's estimator in small sample simulations. This result contrasts with the OLS and IV estimations as shown in Figures 1 and 3, in which they fail to converge to zero and their ex-post biases tend to be monotonically increased as the cross-sectional stock number increases. These results further confirm the superiority of employing BJS-Shanken's adjustment to attenuate the EIV bias rather than using the traditional FM-OLS method or the FM-IV method as proposed by Jegadeesh et al. (2019).

Figure 4
The Ex-post Biases versus Number of Years and Stocks Used to Estimate Betas



This figure presents the ex-post biases using Shanken's estimators of the market risk premium under the single-factor CAPM, as a function of the number of days and cross-sectional stocks in the entire sample period to estimate the market betas. The simulations use the market risk premium of 6.79% per annum and 500/1000/1500/2000/2500/3000 individual stocks on the cross-section. The sample period for the simulations is 5/10/15/20/25/30 years. The z-axis (up) is the ex-post bias as a percentage of the true market risk premium, their signs are all negative but their values are presented as absolute values. The y-axis (right) is the number of years in the sample period to estimate the market betas, and the x-axis (left) is the number of stocks on the cross-section. These results are based on 100 repetitions.

4.4 Conclusion

The IV approach fails to attenuate the EIV bias in FM regression as its ex-post bias is only slightly lower than the OLS method and does not converge to zero as cross-sectional stock number N increases. This result is in contrast to the findings of Jegadeesh et al. (2019), which suggest that the FM-IV estimator is unbiased and N-consistent even with small sample simulations. Furthermore, this study explains that the failure of the IV approach may be due to the serial correlation among unobservable measurement errors as suggested by Bellemare et al. (2015) and Reed (2015) rather than the weak instrument problem. On the other side, the BJS-Shanken simulation results in line with the proposition of Shanken (1992), in which the bias-adjusted estimator is unbiased and consistent even with a small sample simulation. The tables and figures clearly show the superiority of employing Shanken's estimator in dealing with EIV bias.

CHAPTER 5: EMPIRICAL RESULTS

5.1 Introduction

This chapter reports the result of applying BJS-Shanken approaches to estimate the risk premium of factors proposed by prominent asset pricing models when individual stocks are used as test assets. Several empirical asset pricing models are tested in this study, including the single-factor CAPM, the Fama and French three- and five-factor models. The rest of this chapter is organized as follows. Section 5.2 reports the empirical asset pricing result by using BJS regression along with Shanken correction; In Section 5.3, this study provides the results of using panel regression analysis to test the explanatory power of a series of firm characteristics and market beta coefficients; Section 5.4 shows the information coefficient and information ratio results of estimated betas and lagged characteristics in predicting the future stock returns, separately. Finally, Section 5.5 concludes the findings of this chapter.

5.2 BJS regression

This section employs the BJS-Shanken method to estimate the price of risk factors proposed by prominent asset pricing models. Specifically, it tests whether the risk premiums under the CAPM and the FF3- and FF5-factor models are significantly different from zero when using individual stocks as test assets.

5.2.1 The CAPM and The FF3-Factor Model

Table 7 presents risk premium estimates and corresponding t-statistics using Shanken's correction. In addition, the cross-sectional R-squared values are also reported for measuring the goodness of fit. For the single-factor CAPM model in Column (1), the market risk premium estimate is -1.24%, which is statistically significantly different from zero. This result implies that there is strong empirical support for the CAPM with individual stocks. The R-squared value of the regression indicates that the CAPM model seems to explain 14.22% of the total variation in share returns over the sample period.

For the FF3-factor model, Columns (2) shows that the market risk premium estimate is -0.39%, which is not statistically significant, and risk premiums for the SMB and HML factor are 1.00% and 2.47%, respectively, and both are statistically significant meaning that factor risks under the FF3-factor model are priced in the cross-section of individual stock returns except for the market factor. Given the R-squared value of the FF3-factor model is 21.37%, the size and value factor together seem to contribute to a 50.28% improvement in explanatory power over the CAPM model.

The next two columns (3) and (4) includes SIZE (natural logarithm of market capitalization) and BM (book-to-market ratio) as control variables in the second stage of BJS regression. When these two firm characteristics are added to the CAPM in Column (3), the market risk premium estimate is 2.20% and still significant. In addition, the slope coefficients on SIZE and BM are -1.65% and -1.15%, respectively, and both are statistically significant. For the FF3-factor model with control variables in Column (4), the market, SMB and HML factor risk premiums are 2.04%, -1.17% and 3.59%, respectively, and still, all of them are statistically significant. The slope coefficients of SIZE and BM are -1.40% and -1.47%, respectively, and both are statistically significant.

Table 7
Risk Premium Estimates with Individual Stocks:
CAPM and Fama-French Three-Factor Model

	1	**2**	**3**	**4**
Const	**1.4164**	**1.2425**	**1.2238**	**1.5302**
	7.05	**6.18**	**4.62**	**6.49**
MKT	**-1.2364**	-0.3919	**2.1972**	**2.0417**
	-2.80	-0.95	**4.11**	**3.75**
SMB		**0.9977**		**-1.1714**
		2.36		**-2.29**
HML		**2.4715**		**3.5853**
		3.49		**4.54**
SIZE			**-1.6503**	**-1.4004**
			-8.03	**-6.46**
BM			**-1.1453**	**-1.4770**
			-6.32	**-8.69**
R^2	14.22%	21.37%	22.29%	29.06%

This table reports risk premium estimates, in percentage per month, by using individual stocks as test assets. It reports the test results using the BJS-Shanken method in Columns (1) to (4). Rows labelled MKT, SMB, and HML are the risk premiums for the market, size and value factors, respectively, and the corresponding t-statistics are in parentheses (bold if t-statistics are higher than 2). SIZE is the natural logarithm of market capitalization. BM is the book-to-market ratio at the end of the previous month. Betas for each month are estimated using daily returns over the entire sample period. The sample period is from Jan 2000 through Dec 2019.

5.2.2 The Carhart4-Factor Model

Table 8 presents the estimates for the Carhart4-factor model, which includes the momentum factor (WML, winners-minus-losers) in the FF3-factor model. As shown in column (1), using the momentum factor alone can explain 13.81% of the variation of individual stock expected stocks. Further, the coefficient momentum factor risk in Column (2) under the Carhart4-factor model indicates that a one-unit positive change in the momentum factor loadings results in a positive 6.39% change in the expected average return. The R-squared value of the Carhart4-factor model is 24.02%, which is only slightly higher than that of the FF3-factor model (21.37%). In other words, the Carhart4-factor after adding the momentum factor model is not much better than the FF3-factor model in terms of its explanatory power on the variation of cross-sectional stock returns.

From Column (1) to Column (4), the risk premium estimates of momentum factor are all positive and statistically significant and higher than the other risk premiums on average. This suggests that the momentum factor risks are robustly priced in the cross-section of individual stock returns even when adding SIZE and BM to control for omitted variable bias. Moreover, Column (2) shows that the value factor becomes insignificant and small after including the momentum factor in the FF3-factor model. This result implies the momentum factor can be viewed as a competitor to challenge the pricing power value factor.

Table 8
Risk Premium Estimates with Individual Stocks:
Carhart Four-Factor Model

	1	**2**	**3**	**4**
Const	**0.4302**	**0.2230**	**1.7811**	**1.0079**
	2.12	**2.12**	**7.28**	**4.08**
MKT		**2.1527**		**3.0429**
		2.69		**5.19**
SMB		**2.8779**		0.4507
		4.77		0.79
HML		-0.5139		**2.0518**
		0.11		**2.53**
WML	**1.9722**	**6.3882**	**2.6854**	**4.0853**
	2.92	**5.41**	**3.68**	**4.53**
SIZE			**-1.3352**	**-1.1139**
			-6.72	**-5.16**
BM			**-1.1776**	**-1.3963**
			-7.52	**-8.17**
R^2	13.81%	24.02%	23.69%	32.11%

This table reports risk premium estimates, in percentage per month, by using individual stocks as test assets. It reports the test results using the BJS-Shanken method in Columns (1) to (4). Rows labelled MKT, SMB, HML and WML are the risk premiums for the market, size, value and momentum factors, respectively, and the corresponding t-statistics are in parentheses (bold if t-statistics are higher than 2). SIZE is the natural logarithm of market capitalization. BM is the book-to-market ratio at the end of the previous month. Betas for each month are estimated using daily returns over the entire sample period. The sample period is from Jan 2000 through Dec 2019.

5.2.3 The FF5-Factor Model

The purpose of this section is to determine whether profitability (RMW) and investment (CMA) factor risk premiums under the FF5-factor model are significantly different from zero, with and without controlling for their respective characteristics. Table 9 presents the corresponding empirical asset pricing results. From Column (1) and Column (2), both profitability and investment factor premiums are not significantly different from zero, meaning both factor risks may be not priced in the cross-section of stock returns. Column (3) reports the estimates for the FF5-factor model, only the value and investment factor premiums are significant, and this result is robust even when including their corresponding characteristics, i.e., SIZE, BM, operating profitability (OP), and investment (INV) as control variables, for omitted variable bias. On the other side, the slope coefficient of SIZE, PE, OP and INV are -1.51%, -1.46%, -0.16% and -1.36%, respectively, and they are all statistically significant at the 1% level except for INV.

Table 9
Risk Premium Estimates with Individual Stocks:
Fama-French Five-Factor Model

	1	2	3	4	5	6
Const	**0.9011**	**0.8523**	**1.1884**	**0.9253**	**0.7929**	**1.3405**
	4.27	**4.04**	**5.88**	**4.38**	**3.32**	**4.94**
MKT			-0.6737			**1.4508**
			-1.36			**2.57**
SMB			0.6007			**-1.9889**
			1.54			**-3.50**
HML			**3.8691**			**4.0333**
			3.89			**4.71**
RMW	0.6842		0.8320	-0.4082		-1.5831
	1.13		0.64	-0.63		-1.96
CMA		-0.2349	**4.9181**		**2.1875**	**5.6471**
		-0.26	**2.75**		**2.17**	**3.80**
SIZE						**-1.5073**
						-6.45
BM						**-1.4554**

						-7.72
OP				0.2388		-0.1564
				0.82		-0.48
INV					-1.7773	-1.3617
					-1.70	-1.19
R^2	15.19%	15.14%	26.44%	19.17%	19.34%	39.34%

This table reports risk premium estimates from the BJS-Shanken method, in percent per month, their tests using individual stocks as test assets, and corresponding t-statistics in parentheses (bold if t-statistics are higher than 2). Rows labelled MKT, SMB, HML, RMW and CMA are the risk premiums for the market, size, value, profitability, and investment factors, respectively. SIZE is the natural logarithm of market capitalization. BM is the book-to-market ratio at the end of the previous month. OP and INV are the operating profitability and the growth of investment ratio, respectively. Betas for each month are estimated using daily returns data over the entire sample period. The sample period is from Jan 2000 through Dec 2019.

In this section, the BJS-Shanken method is used to test whether the premiums for risk factors proposed by several prominent asset pricing models are reliably different from zero. The asset pricing models tested in this study are the singular-factor CAPM, the FF3-Factor and FF5-Factor models. Previous empirical research use portfolio sorts as test assets and finds significant risk premiums of size and value factors on the JSE, which include the works of Basewicz and Auret (2010) and Van Rensburg and Robertson (2003). After using individual stocks as test assets to avoid the limitation of using portfolios, this study again finds the size, value and momentum factors under these asset pricing models are priced on the cross-section of individual stock returns no matter whether corresponding firm characteristics are included as control variables or not.

On the other hand, the slope coefficients on SIZE, PE, and OP are also significantly different from zero, which makes us wonder whether characteristics may also be alternative proxies for true betas and consequently the slope coefficients on characteristics may reflect the risk premiums of underlying factors. This is the motivation for this study to conduct the additional test in chapter 6 following the instruction from Fama and French (2019) to compare TS factors and CS factors by using time-series regression models.

5.3 Panel Regression

Although the modified Shanken's estimator works well in dealing with the EIV bias, the BJS/FM regression procedure still may be subject to the omitted variables bias. Therefore, it is difficult to rule out the possibility that the significance of firm characteristics arises from the fact that they happen to line up well with the loadings of omitted factors. In this regard, the study employs a firm-level panel regression method, as motivated by Page and Auret (2019), to further test whether lagged firm characteristics along with estimated market beta can robustly explain the cross-sectional variation in average returns on the JSE after including the fixed effects model. The reason for using this approach is that firm-fixed effects can capture the impact of omitted and potentially unobservable firm-specific factors.

This section is conducted in the same vein as Page and Auret (2019), meaning excess geometric individual stock returns are semi-annually calculated at time t, while lagged values of market beta, size, value, momentum, profitability, and investment are estimated six months prior at time t-6. Specifically, these characteristics are intentionally lagged to determine the lead-lag relationship between ex-ante priced factors and ex-post share returns. As a result, the time-series of observations is equal to a maximum of 34, which ranges from July 2003 to Jan 2020, while the cross-section of shares remains the same as described in Section 3.2. Although the panel regression includes a total of six independent variables, the main focus here is the size, value and momentum factors as they are strongly significant in the previous result. Thus, the central issue in this section is whether the significance of these characteristics (size, value and momentum) is due to their correlation with the firm fixed effect. In other words, whether the explanatory power of these characteristics is robust to the inclusion of firm fixed effects.

5.3.1 Pooled OLS

Table 10 presents the results of the pooled OLS regression. The table depicts the coefficient parameters of market beta and firm characteristics applied in regressions one to five, with the regression number presented in the first row. Regression I only considers the market betas and its result is similar to the CAPM model in Table 7 shown. The market beta coefficient is -0.86% and statistically significant, implying that the low beta anomaly is present and significant on the JSE. Regression II and III both consider the FF3-factor, however, Regression II uses the PE ratio as a proxy for value whereas Regression III uses the BM ratio. Both regressions show that the size coefficient is negative and statistically significant. The value coefficient is 0.67% when using the BM ratio as a proxy, i.e. a single unit increase in lagged BM ratio will increase the expected monthly return of 0.67%. The value coefficient, however, is not priced when using the PE ratio as a proxy. This result is in line with Auret and Sinclaire (2006), which found that the BM ratio is the best proxy of value on the JSE. Regression IV also considers the FF3-factor but extends the model to include momentum (historical six minus one-month cumulative return), similar to the Carhart4-factor model. The coefficients of market beta, size and value (BM) are largely unchanged with the same sign, similar size and significance level. In line with what Page and Auret (2017) and Page and Auret (2019) found, the momentum coefficient is positive and both statistically and economically significant. Specifically, its economic size is about five times greater than the size and value coefficients, meaning a one-unit increase in historical momentum will result in an increase of 2.96% in average excess return per month. The final column reports the result of the FF5-factor, the coefficients of profitability and investment are not statistically significant, while the coefficients of the other three factors (market beta, size and value) remain the same sign and similar size. This result is not a surprise as Table 9 in the previous BJS-Shanken regression section also indicated the same finding.

Table 10
Coefficient Estimates under Pooled OLS Regression

	I	II	III	IV	V
Const	**1.2628**	**1.7622**	**1.1529**	**0.9473**	**0.9027**
	3.78	**3.84**	**3.19**	**2.92**	**2.83**
Market beta	**-0.8582**	**-0.7766**	**-0.9297**	**-0.8009**	**-0.7991**
	-2.68	**-2.41**	**-2.24**	**-2.02**	**-2.01**
Size		**-1.1033**	**-0.7091**	**-0.5834**	**-0.6092**
		-3.12	**-2.43**	**-2.59**	**-2.71**
Value (BM)			**0.6703**	**0.5928**	**0.5944**
			17.94	**16.19**	**16.87**
Value (PE)		-0.1086			
		-0.83			
Momentum				**2.9676**	**2.9503**
				4.15	**4.14**
Profitability					0.3125
					1.33
Investment					-0.5543
					-1.62
R^2	5.01%	6.85%	14.50%	17.24%	17.89%

This table reports coefficient estimates from the Pooled OLS regression, in percent per month, their tests using individual stocks as test assets, and corresponding t-statistics in parentheses (bold if t-statistics are higher than 2). Size is the natural logarithm of market capitalization. Value (BM) is the book-to-market ratio and Value (PE) is the price-to-earnings ratio, both are estimated as the median of the previous 12 months. Profitability and Investment are the operating profitability and the growth of investment ratio, respectively, both are estimated as the median of the previous 12 months. Momentum is the historical cumulative returns estimated over the previous six minus one months. Market betas are estimated semi-annually by regressing excess individual stock returns on market returns (J203T) with a rolling window of 3 years (36 months). All of these characteristics and estimated market *betas are lagged for 6 months in the panel regression for determining the lead-lag relationship. The sample period ranges from July 2003 to Jan 2020.*

The final row of Table 10 presents the R-squared value under each regression, which is a measurement of the goodness-of-fit. The final row of Regression II (6.85%) illustrates that the inclusion of the size factor contributed 36% to the increase in the explanatory power of the market beta model (5.01%). The R-squared value of Regression III (FF3-factor) is 14.50%, which means the inclusion of value (BM) factors increases the explanatory power of the market beta and size models by 110%. Moreover, the R-squared value of Regression IV (Carhart4-factor) is 17.24%, implying a 19% increase in explanatory power over Regression III due to the inclusion of historical momentum. On the other side, the contribution of investment and profitability factors are extremely small as they are not priced in Regression V.

Table 11 and Table 12 presents the results of the heteroskedasticity test by using White- and the Breusch-Pagan-Test, and the results of the auto-correlation test by using the Durbin-Watson-Test, respectively. These two tables show that there is heteroskedasticity and autocorrelation among the residuals under all regressions (from Regression I to V). Thus, it seems like Fixed Effect- (FE) or Random Effect- (RE) model will be more suitable.

Table 11
Heteroskedasticity Test Results

White	I	II	III	IV	V
LM Stat	**310.34**	**357.64**	**895.28**	**1038.59**	**1061.18**
LM p-value	0.0000	0.0000	0.0000	0.0000	0.0000
F Stat	**182.03**	**47.73**	**173.43**	**146.68**	**78.92**
F p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Breusch-Pagan	I	II	III	IV	V
LM Stat	**61.76**	**81.53**	**120.07**	**121.98**	**127.94**
LM p-value	0.0000	0.0000	0.0000	0.0000	0.0000
F Stat	**63.58**	**28.22**	**42.38**	**32.31**	**22.64**
F p-value	0.0000	0.0000	0.0000	0.0000	0.0000

This table summarizes the result of using the White- and Breusch-Pagan-Test to identify the heteroscedasticity among regression residuals. LM Stat is Lagrange multiplier statistic, while F Stat is F statistics. In addition, the table also reports the corresponding p-values of these statistics (bold if the p-value is smaller than 1%). The null hypothesis for the White- and Breusch-Pagan-Test is that the variances for the errors (regression residuals in the sample) are equal.

Table 12
Autocorrelation Test Results

Durbin-Watson	I	II	III	IV	V
Result	1.44	1.47	1.79	2.06	2.08

This table summarizes the result of using the Durbin-Watson-Test to identify the autocorrelation among regression residuals. The test will have one output ranges from 0 to 4. The mean equal to 2 indicates that there is no autocorrelation identified, range (0,2) means that there is positive autocorrelation (the nearer to zero the higher the correlation), and range (2,4) means that there is negative autocorrelation (the nearer to four the higher the correlation).

5.3.2 FE Model

The FE models determine firm-fixed (heterogeneous) effects as constant variables over time and account for that in the intercept term. Table 13 summarizes the results of FE regression (estimated assuming firm fixed effects). The coefficients of market beta, value (BM) and momentum across from Regression I to V are strongly significant and consistent in terms of sign and magnitude, and these results are similar to Table 10 (Pooled OLS regression). For example, the market beta coefficient indicates that a single unit increase in the beta will result in a -0.85% decrease in the expected monthly return on average, while the value (BM) and momentum coefficients imply that a one-unit increase in the lagged BM ratio and historical momentum will end up with an increase in expected monthly return of 0.53% and 1.98% on average, respectively.

In line with Page and Auret (2019), the table also presents that the application of the FE model seems to have a major impact on the size coefficient. In Table 13, the size coefficients remain significant and negative, but its economic size is now 5.05% on average, which is about 6.7 times larger than that in Table 10. This result implies that a single unit increase in the lagged size (natural log of market capitalization) will now result in a decrease in the expected average return of -5.05% per month. In addition, the profitability coefficient is now significant and positive, meaning a one-unit increase in lagged operating profitability ratio results in a 0.82% increase in expected average monthly return.

Table 13
Coefficient Estimates under Fixed Effects Regression

	I	II	III	IV	V
Const	**1.2597**	**3.9311**	**3.0524**	**2.7650**	**2.8057**
	8.54	**16.43**	**12.03**	**10.62**	**10.51**
Market beta	**-0.8565**	**-0.8565**	**-0.9506**	**-0.8686**	**-0.8530**
	-9.87	**-10.3**	**-11.71**	**-10.6**	**-10.37**
Size		**-6.4033**	**-4.8475**	**-4.3248**	**-4.6329**
		-13.79	**-10.76**	**-9.62**	**-9.68**
Value (BM)			**0.5588**	**0.5134**	**0.5117**
			10.82	**9.86**	**9.85**
Value (PE)		-0.1036			

		-0.72			
Momentum				**2.0141**	**1.9583**
				5.35	**5.19**
Profitability					**0.8197**
					2.27
Investment					-0.3141
					-0.77
R^2	4.63%	12.83%	17.59%	18.74%	18.97%

This table reports coefficient estimates from the FE regression, in percent per month, their tests using individual stocks as test assets, and corresponding t-statistics in parentheses (bold if t-statistics are higher than 2). Size is the natural logarithm of market capitalization. Value (BM) is the book-to-market ratio and Value (PE) is the price-to-earnings ratio, both are estimated as the median of the previous 12 months. Profitability and Investment are the operating profitability and the growth of investment ratio, respectively, both are estimated as the median of the previous 12 months. Momentum is the historical cumulative returns estimated over the previous six minus one months. Market betas are estimated semi-annually by regressing excess individual stock returns on market returns (J203T) with a rolling window of 3 years (36 months). All of these characteristics and estimated market betas are lagged for 6 months in the panel regression for determining the lead-lag relationship. The sample period ranges from July 2003 to Jan 2020, with a total of 34 time-series observations.

5.3.3 RE Model

Unlike the FE model, the RE models determine firm-fixed (heterogeneous) effects as random variables over time and can therefore be incorporated within the error term. The question of choosing between the FE and RE model depends on whether the unobserved heterogeneity is a constant or random effect. For that, Table 14 depicts the result of the Hausman-Test, the null hypothesis of it is that the covariance between independent variables (characteristics) and intercept term (firm fixed effects) is zero. From the table, the null hypothesis is rejected at a significant level for all regression models except for Regression I, meaning the FE model seems to be the most suitable for Regression II to V because of the unobserved heterogeneity under these regressions is constant rather than random. On the other side, the RE model is the preferred solution for Regression I.

Table 14
Hausman-Test for Random Effects

Hausman	I	II	III	IV	V
chi-Squared	0.0606	**149.39**	**91.74**	**174.85**	**44.83**
p-Value	0.9701	**0.0000**	**0.0000**	**0.0000**	**0.0000**

This table reports the result of using the Hausman-Test to identify the potential correlation between firm fixed effects and independent variables. The null hypothesis here is that the covariance between independent variables and firm fixed effects (intercept terms) is zero. If this is the case, then the preferred model is random effects.

The result of RE regression is presented in Table 15. Regression I shows that the market coefficient is -0.85% with a t-statistic of -10.28, indicating a significant low beta premium is present on the JSE. Consistent with the result of Pooled OLS and FE regressions, the coefficient estimates of the characteristics underlying the CAPM, FF3- and Carhart4-factor model are all statistically significant and economically large with the correct sign as previous literature suggested. The profitability and investment factor under the FF5-factor model are still not priced. Unlike the FE model, the size coefficient is -0.88% on average, which is close to the estimates under Pooled OLS regression, indicating that a one-unit increase in the natural log of market capitalization results in a −0.88% decrease in the expected return per month. Although the coefficient parameters in Table 15 are similar to Table 10, the RE model still has the benefit of dealing with the serial correlation among intercept terms.

Table 15
Coefficient Estimates under Random Effects Regression

	I	II	III	IV	V
Const	**1.2440**	**1.9052**	**1.2424**	**0.9409**	**0.8947**
	7.32	**9.89**	**6.55**	**5.63**	**5.26**
Market beta	**-0.8495**	**-0.7853**	**-0.9209**	**-0.7922**	**-0.7887**
	-10.28	**-9.55**	**-11.57**	**-10.06**	**-9.95**
Size		**-1.4141**	**-0.9087**	**-0.5852**	**-0.6126**
		-7.2	**-4.92**	**-3.65**	**-3.72**
Value (BM)			**0.6623**	**0.5896**	**0.5912**
			13.37	**11.96**	**11.95**
Value (PE)		-0.0902			
		-0.65			
Momentum				**3.0151**	**2.9964**
				8.44	**8.38**
Profitability					0.3224
					1.08
Investment					-0.6092
					-1.49
R^2	4.81%	7.14%	14.41%	17.28%	17.40%

This table reports coefficient estimates from the RE regression, in percent per month, their tests using individual stocks as test assets, and corresponding t-statistics in parentheses (bold if t-statistics are higher than 2). Size is the natural logarithm of market capitalization. Value (BM) is the book-to-market ratio and Value (PE) is the price-to-earnings ratio, both are estimated as the median of the previous 12 months. Profitability and Investment are the operating profitability and the growth of investment ratio, respectively, both are estimated as the median of the previous 12 months. Momentum is the historical cumulative returns estimated over the previous six minus one months. Market betas are estimated semi-annually by regressing *excess individual stock returns on market returns (J203T) with a rolling window of 3 years (36 months). All of these characteristics and estimated market betas are lagged for 6 months in the panel regression for determining the lead-lag relationship. The sample period ranges from July 2003 to Jan 2020, with a total of 34 time-series observations.*

Table 16 summarizes the results of the panel regression analysis across different models. The last two columns show that the coefficients of market beta, size, value, and momentum are all statistically significant and their signs are in line with local and international literature. Especially, the market beta coefficient is significant and negative, indicating that the low beta premium exists on the JSE. Additionally, the application of the fixed effects model appears to have a significant effect on the size coefficient. Further, the economic magnitude of the momentum coefficient is about five times greater than the size and value coefficients. The results in this section support the findings of Page and Auret (2019) in terms of the signs, significance and economic magnitude of these coefficient estimates as listed in Table 16, while this section extends the study to include value (PE), profitability, and investment factors as FF5-factors, however, none of which are significantly priced.

Table 16
A Summary of Panel Regression

Variable	Pooled OLS	Fixed Effects	Random Effects	Significance proportion	Relationship consistent with the literature
Market Beta	-0.8329%	-0.8770%	-0.8273%	100%	100%
Value (BM)	0.6192%	0.5280%	0.6143%	100%	100%
Size	-0.7513%	-5.0521%	-0.8801%	100%	100%
Momentum	2.9590%	1.9862%	3.0057%	100%	100%

In summary, the significance of size, value (BM) and momentum coefficients are not due to their correlation with omitted and potentially unobserved firm-specific factors. In other words, this section shows that these characteristics along with low market beta are significant and independently priced factors on the JSE, which is in line with the Carhart4-factor model. Accordingly, it is legitimate to ask whether these lagged firm characteristics or estimated betas would do a better job of tracking average returns on the cross-section.

5.4 Rank IC and IR of Betas and Characteristics

In this study, when individual stocks are used as test assets, prominant factors like size, value and momentum are priced, except for profitability and investment, along with the slope coefficients of their corresponding characteristics. This implies that factor loadings and characteristics may both explain variation in cross-sectional average returns. Jegadeesh et al. (2019) propose that characteristics may serve as better proxies for true factor loadings because betas are estimated with measurement errors. In this section, the rank information coefficient (IC) series and information ratio (IR) are used to compare the predictive ability of estimated betas and lagged characteristics on future stock returns. A higher IC value indicates stronger predictive ability or smaller tracking errors, while a higher IR value means more stable predictive ability.

Table 17 presents the annualized average of rank IC series and IR series for betas (rolling and constant) and lagged characteristics. The left-side columns use rolling betas, which are estimated for each month based on daily data from the previous 36 months, while the right-side columns use constant betas, which are estimated based on daily data from the whole sample period. It shows that the rolling betas of size factor (SMB beta) and the natural logarithm of market capitalization (SIZE) tend to have a similar predictive ability as their average IC values are very close, 0.08 and 0.07, respectively. However, the IR of SMB betas are 40% higher than that of the SIZE characteristic, meaning the predictive ability of size factor loadings is more stable than its underlying characteristic.

For value and momentum factors, the underlying characteristics seem to have stronger predictive ability than factor loadings. For example, the average IC values of MOM (historical cumulative momentum estimated over the previous twelve minus one months) are four times larger than that of WML factor loadings. On the other side, the predictive ability of underlying characteristics for value and momentum factors are also much more stable than their betas. Profitability and investment factors are also provided for comparison, but they are not of our interest since they are not priced on the cross-section.

The results on the right-side columns use constant betas rather than rolling betas. The average rank correlation between size factor loadings and future returns became negative, while its predictive ability was still higher than the underlying characteristics and more stable. For the value factor, the result in terms of predictive ability under rolling beta and constant betas is similar, meaning BM ratios tend to do a better job than HML betas in predicting individual stock returns and are more stable.

Table 17
IC and IR of Betas and Characteristics

	Rolling Betas (t = 3 years)		Constant Betas (T = 15 years)	
	IC	IR	IC	IR

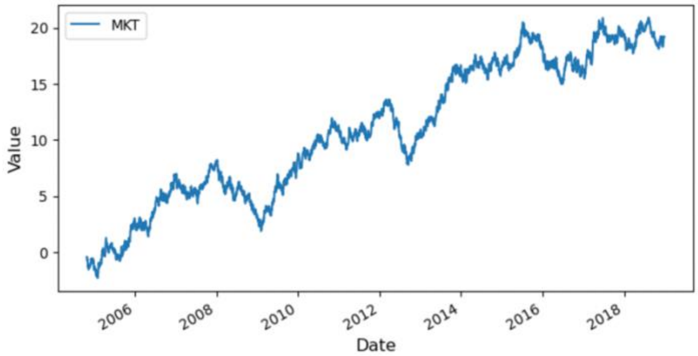
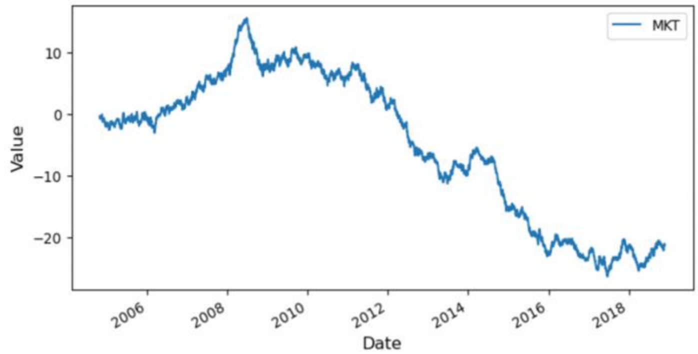
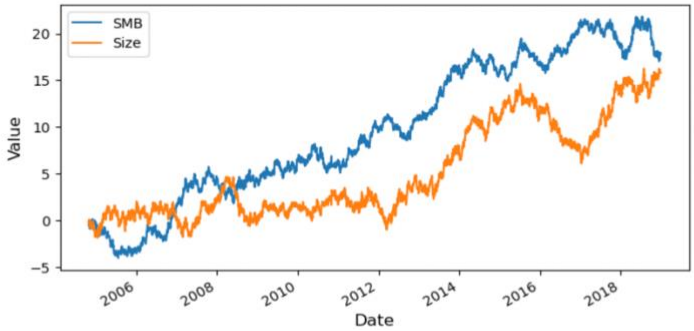
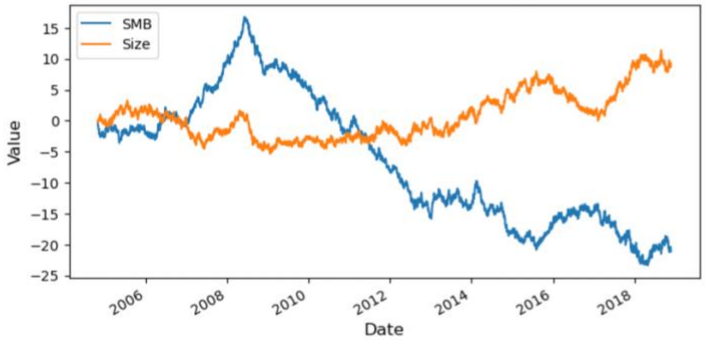
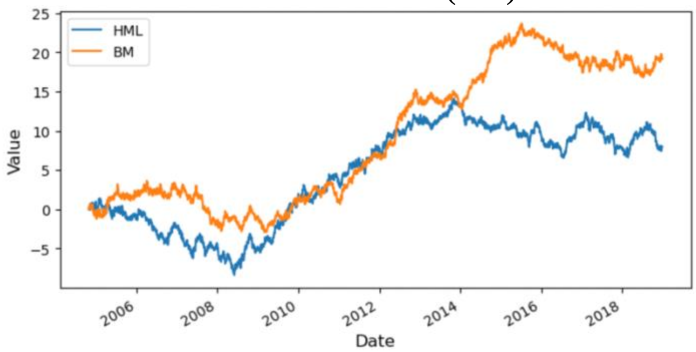
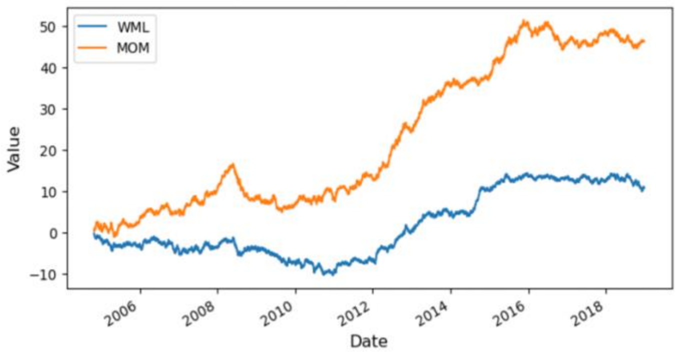
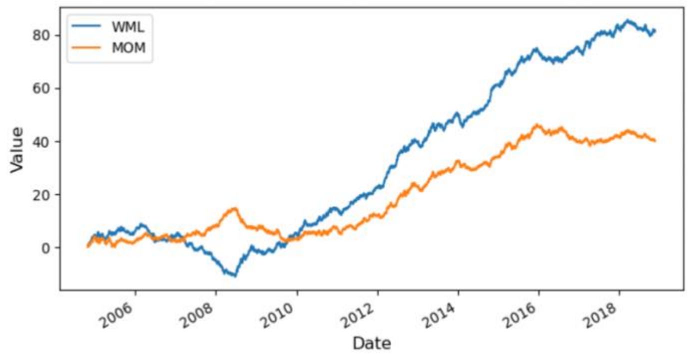
Market Beta	0.0861	0.4633	-0.0957	-0.3901
SMB Beta	0.0797	0.4282	-0.0951	-0.3957
SIZE	0.0708	0.3066	0.0419	0.1758
HML Beta	0.0357	0.1954	0.1165	0.5402
BM	0.0858	0.4865	0.1785	0.9918
WML Beta	0.0488	0.2651	0.3682	1.3424
MOM	0.2078	0.9312	0.1807	0.7871
RMW Beta	0.1666	0.8999	0.2688	0.9952
OP	0.1582	0.7946	0.1537	0.7526
CMA Beta	0.0899	0.4777	-0.2000	-0.8929
INV	0.0575	0.3204	0.0244	0.1324

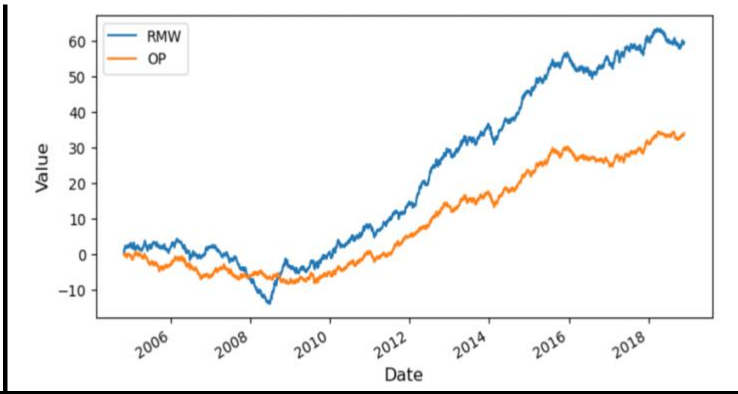
This table summarizes the annualized average of Rank IC and IR series of estimated betas and lagged characteristics in predicting the individual stock returns, which range from Oct 2004 to Dec 2019. Rolling Betas are estimated for each month using daily returns data from the previous 36 months. Constant Betas are estimated for the entire sample period using daily return data. SIZE is the natural logarithm of market capitalization and BM is the book-to-market ratio. OP and INV are the operating profitability and investment/total asset, respectively. MOM is the historical cumulative momentum estimated over the previous twelve minus one months. All of these characteristics lagged for one month. A higher IC value indicates stronger predictive ability or smaller tracking errors, while a higher IR value means more stable predictive ability.

Figure 5 plots the cumulative sum of rank IC series for estimated betas and characteristics, respectively. For size factor, it shows that its estimated rolling betas tend to have higher and more stable predictive ability than its lagged underlying characteristics, and this is robust when using constant betas, while the relationship between estimated betas and future returns becomes negative. On the other side, the lagged underlying characteristics of value factors outperform both rolling and constant betas in terms of predictive ability. For the momentum factor, MOM outperforms rolling factor loadings but is beaten by the constant ones. Furthermore, the lagged underlying characteristics and estimated betas of size, value, and momentum factors are positive indicators of future returns except for the constant betas of the size factor.

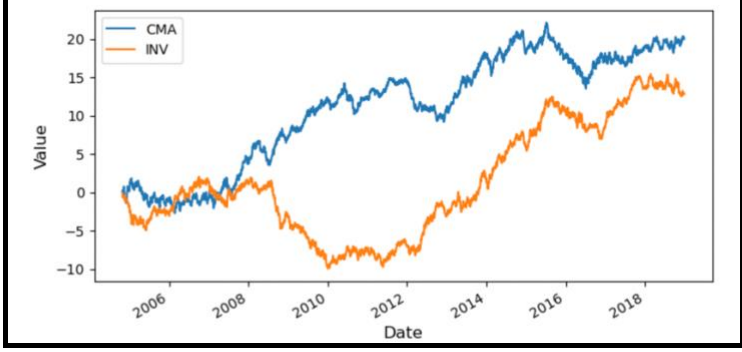
Figure 5
Cumulative Rank IC Series of Betas Versus Characteristics

Rolling Betas (t = 3 years)	Constant Betas (T = 15 years)
Market Beta	Market Beta

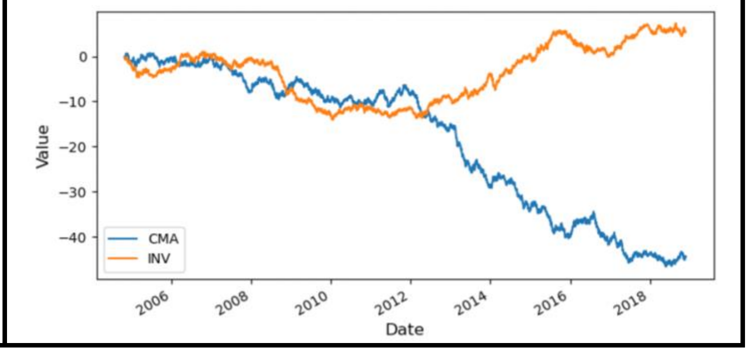
	
SMB Beta vs Size	SMB Beta vs Size
	
HML Beta vs Value (BM)	HML Beta vs Value (BM)
	
WML Beta vs Momentum (t-12 to t-1)	WML Beta vs Momentum (t-12 to t-1)
	
RMW Beta vs OP	RMW Beta vs OP



CMA Beta vs INV



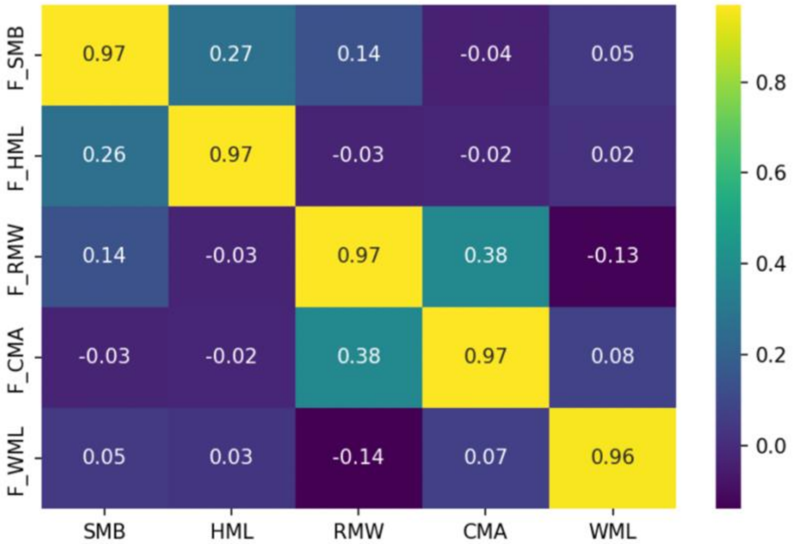
CMA Beta vs INV



This figure summarizes the average of Rank IC and IR series of estimated betas and lagged characteristics in predicting the individual stock returns, which range from Oct 2004 to Dec 2019 (15 years). Rolling Betas are estimated for each month using daily returns data from the previous 36 months. Constant Betas are estimated from the entire sample period using daily return data. SIZE is the natural logarithm of market capitalization and BM is the book-to-market ratio. MOM is the historical cumulative momentum estimated over the previous twelve minus one months. OP and INV are the operating profitability and investment/total asset, respectively. All of these characteristics lagged for one month (21 days). A higher IC value indicates stronger predictive ability or smaller tracking errors, while a higher IR value means more stable predictive ability.

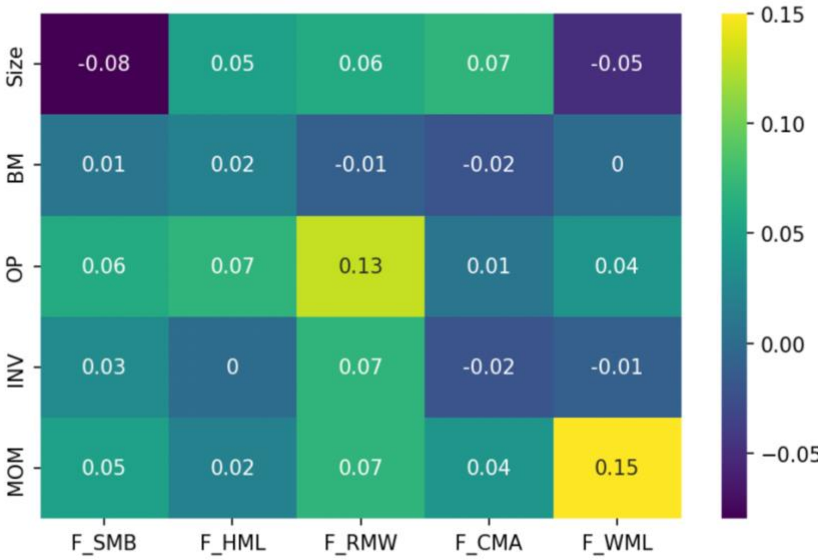
The IC and IR results show that the estimated betas and lagged characteristics are neck and neck in predicting future returns. The “risk-proxy” hypothesis as proposed in Jegadeesh et al (2019), which characteristics may serve as better proxies for “true” future betas than the betas estimated from past data, implies that characteristics should be more highly correlated with the future betas. Therefore, the study further compares the correlation between past betas and future betas, as well as lagged characteristics and future betas, respectively. The diagonal of Figure 6 and Figure 7 present that, under the CAPM, FF3- and FF5-factor models, the average cross-sectional correlation between past betas and future betas are neatly perfectly correlated, while lagged characteristics are poorly correlated with future betas on average. However, as suggested by Chordia et al. (2017) that there need not be a simple relation between betas and characteristics at the individual stock level because the estimated betas can change dramatically from one day to the next. In this regard, it is legitimate to ask whether the underlying firm characteristics or the estimated betas do a better job of tracking average returns on the cross-section, which leads the study to add the “Additional Test Chapter” to compare their explanatory power.

Figure 6
The Average Cross-sectional Correlation Among Past Betas and Future Betas



This table presents the average cross-sectional correlations between past betas (SMB, HML, RMW, CMA, and WML) and future betas (F_SMB, F_HML, F_RMW, F_CMA, and F_WML). Betas are estimated for each month using daily returns data from the previous 36 months, while future betas are one period ahead of past betas.

Figure 7
The Average Cross-sectional Correlation Among Lagged Characteristics and Future Betas



This table presents the average cross-sectional correlations between characteristics (Size, BM, OP, INV, and MOM) and future betas (F_SMB, F_HML, F_RMW, F_CMA, and F_WML). Betas are estimated for each month using daily returns data from the previous 36 months, while future betas are one period ahead of past betas. SIZE is the natural logarithm of market capitalization *and BM is the book-to-market ratio. MOM is the historical cumulative momentum estimated over the previous twelve minus one months. OP and INV are the operating profitability and investment/total asset, respectively. All of these characteristics lagged for one month (21 days).*

5.5 Conclusion

Overall speaking, the factor risks of market, SMB, HML and WML are all significantly priced in the cross-section of individual stock returns on the JSE, while the slope coefficients of their corresponding characteristics (SIZE, BM, MOM) are also statistically significant under the BJS-Shanken regression. In the further panel regression analysis, the study finds that the significance of characteristics is not due to their correlation with omitted and potentially unobserved firm-specific factors. The rank IC and IR comparison for estimated betas and lagged characteristics suggest that they are neck and neck in predicting future returns and the significance of firm characteristics is not because they happen to line up well with future betas. In summary, it is legitimate to ask whether the underlying firm characteristics or the estimated betas do a better job of tracking average returns on the cross-section. Therefore, in the next chapter, the study tests the explanatory power of slope coefficients of characteristics (CS factors) and TS factors on the variation of cross-sectional average returns.

CHAPTER 6: ADDITIONAL TESTS

6.1 Introduction

In light of the fact that beta estimates include measurement errors, it may be more accurate to use firm characteristics as proxies for "true" unobserved betas, and the slope coefficients of these characteristics may actually reflect the risk premiums of underlying systematic factors. For example, in contrast to Fama and French (1993) and Davis et al. (2000), who argue that factor loadings explain expected returns, Daniel and Titman (1997) argue that characteristics explain expected returns. Further, there are many papers in the literature that present variations of this interpretation, such as Berk et al. (1999), Zhang (2005), and Novy-Marx (2013).

In this study, when individual stocks are used as test assets, traditional factors like market, size, value and momentum are priced, except for profitability and investment, and the slope coefficients of their corresponding characteristics are also strongly significant. Therefore, it is legitimate to ask whether the underlying firm characteristics or the factor loadings do a better job of tracking average returns in the cross-section. Therefore, in this chapter, the cross-sectional FM regression approach is used to construct CS factors corresponding to the TS factors of Carhart (1997), and the goal here is to compare the explanatory power of TS and CS factor models on average asset returns, respectively. The reason for using the Carhart4-factor model is because many South African studies have found evidence of the presence of the value, size and momentum risk premiums. For example, van Rensburg and Robertson (2003) and its extension of Basiewicz and Auret (2009) find a significant size and value premium on the JSE, while Auret and Sinclaire (2006) find that the book-to-market ratio was the best proxy for value. More recently, Page and Auret (2018) provide evidence in line with Muller and Ward (2013) that the momentum effect is strong on the JSE.

The rest of this chapter is organized as follows. Section 6.2 describes the methodology of constructing one TS and two CS factor models. Section 6.3 lists a series of evaluation metrics including GRS and Alpha tests. Section 6.4 reports the empirical asset pricing result over individual stock returns by using TS and CS factor models. Finally, Section 6.5 concludes the findings.

6.2 TS and CS Factor Models and Metrics

This section lists three time-series regression models that are later used to describe average returns for a wide range of test assets, and their performances are compared in Section 6.4. The TS and CS factor models differ in that the TS factors are arbitrary, while the CS factors estimated by the FM procedure are optimized through OLS regression daily. Thus, at the centre of this chapter is the question of whether the optimization of the CS factors improves the description of average returns for test assets beyond the ones producing the factors themselves.

6.2.1 TS Factor Model I

A total of three factor models (one for the TS factor model and two for the CS factor models) in Fama and French (2019) are chosen to conduct the comparison in this paper. There are a total of four different models in that paper, but the TS factor model II with interaction terms is not the main purpose of the comparison. The TS factor model I is the model of Carhart (1997) as Eq. (19) shows.

TS Factor Model I:

$$(19)\ R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,WML} WML_t + \varepsilon_{i,t}$$

Where a_i is the intercept and also the mispricing error for LHS individual stock i, and $R_{i,t}$ is the return of individual stock i at day t, and $r_{f,t}$ is the risk-free rate at day t and $\beta_{i,MKT}, \beta_{i,SMB}, \beta_{i,HML}, \beta_{i,WML}$ are the betas with respect to the market, size, value and momentum factors, and $MKT_t, SMB_t, HML_t, WML_t$ are the respective factor return realizations, i.e., double-sorted portfolio returns. Specifically, the MKT_t factor is the value-weight stock market return for day t and the remaining four factors are differences between returns on double-sorted portfolios of small and big stocks (SMB_t), high and low BM stocks (HML_t), stocks of winner (high historical cumulative returns) and loser (low historical cumulative returns) firms (WML_t). The definition of these variables is explained in detail in the Appendix.

6.2.2 CS Factor Model I

The values of firm characteristics are used as explanatory variables in the second stage of FM regression rather than estimated betas which can avoid the EIV problem. By utilizing cross-sectional FM regression, this study CS factors that correspond to the TS factors of Carhart (1997). Fama and French (2006) and Lewellen (2015) are recent examples that treat FM regressions as a way to allow for variation in average returns related to characteristics.

The following Eq. (20) is used in the second stage of FM regression to estimate the time series of slope coefficients for each firm characteristic, as indicated by the notation, and that will be later used as pure factor returns in Eq. (21). In addition, the reason for applying cross-sectional FM regression is because it can deal with the cross-sectional correlation (time-effect) in returns data and acquire consistent standard errors.

$$(20)\ R_{x,t} \ = \ R_{z,t} + R_{MC,t}MC_{x,t-1} + R_{BM,t}BM_{x,t-1} + R_{MOM,t}MOM_{x,t-1} + \varepsilon_{x,t}$$

Where, the factor loadings are the previously observed values of $MC_{x,t-1}$ (the natural logarithm of market capitalization), $BM_{x,t-1}$ (the book-to-market ratio) and $MOM_{x,t-1}$ (the cumulative return for day t-252 to t-21) for standard portfolios x at day $t-1$. Specifically, the value of $MC_{x,t-1}$, $\mathrm{BM}_{x,t-1}, and\ \mathrm{MOM}_{x,t-1}$ are all cross-sectional standardized by using z-scores. As a result, the average of the 12 standard portfolios each time t is zero and the standard deviation is one. $R_{x,t}$ is the standard portfolio return x at time t, where x is a vector of 12 elements, and the intercept $R_{z,t}$ is the time t return on a standard portfolio of the LHS assets ($R_{x,t}$), both will be explained in detail in the following. Further, $R_{MC,t}$, $R_{BM,t}$, $R_{MOM,t}$ are pure factor returns, and $\varepsilon_{i,t}$ is the residual returns.

$R_{z,t}$ are used in the following CS factor model I as the intercept, meaning that the CS factor model I is not used to explain the excess return of $R_{i,t}$ relative to $R_{f,t}$ as the TS factor model I or CS factor model II shown, but the excess return of $R_{i,t}$ relative to $R_{z,t}$. This is somewhat contrary to our intuition since our purpose is to compare TS and CS factor models and it seems like they cannot be compared given that the benchmarks are different. Fama and French (2019), explain this by proposing that the TS and CS factors must be derived from the same portfolios (using the classic 2*3 double sorts, a total of 12 portfolios). In other words, CS factors are constructed by using the same portfolios that produced TS factors. As a result, their corresponding models can be compared efficiently.

Fama and French (2019) claim that factor loadings in Eq. (20) are prespecified and can be used in cross-sectional FM regression without causing the EIV problem, which may appear to minimize the sum of squared residuals. Furthermore, each CS factor is a pure factor which is optimized by cross-sectional regression, meaning that the CS factor is only exposed to the target factor and has no exposure to the other non-target factors. As a result, the Eq. (13) that generates CS factors is optimized to describe the performance of each of these portfolios. The TS factors, on the other hand, are defined arbitrarily for these portfolios. In other words, the factors obtained from 2 × 3 sorts are not optimized, and each factor may be exposed to other factors randomly.

Taking into account that the same portfolios produce the TS and CS factors, one central question in the asset pricing tests concerns whether the optimization of the CS factors enhances the descriptions of average returns for test assets that are not part of the 2*3 double sorts. This highlights the importance of using the same portfolios (from the 2*3 double sorts) to produce TS and CS factors. Therefore, the assets on the LHS of Eq. (20) are not the individual stock returns ($R_{i,t}$) but standard portfolio returns ($R_{x,t}$), which are closely related to the Carhart4-factor. In the Carhart4-factor model, excluding the market factor (MKT), the other four factors (SMB, HML, and WML) are constructed by grouping stocks into portfolios based on size and the other firm characteristics. At time t, stocks are first sorted into 2 sub-portfolios based on their market capitalizations. Now, within these 2 sub-portfolios, each will be further sorted into 3 sub-portfolios based on the book-to-market ratio and historical cumulative momentum, respectively, from high to medium to low. Thus, each intersection will have 6 (2*3) portfolios. Since there are two factors outside the SMB factor, there are a total of 12 (6*2) standard portfolios in $R_{x,t}$. Fama and French (2019) use these 12 standard portfolios as the LHS asset in the second stage of FM regression to estimate the CS factor returns as Eq. (20) shows. Especially, in the estimates of the Eq. (20), the characteristics of these 12 standard portfolios are also z-scores: each characteristic is rescaled so that each day the cross-sectional mean and standard deviation of these 12 portfolios will be 0 and 1, respectively.

Now, the calculation of $R_{z,t}$ can be explained by assuming that R_x in Eq. (21) is the equal-weight average return of the 12 standard portfolios. In other words, R_x has zero factor loadings on each of these three CS factors. As Eq. (21) shown, the intercept term $R_{z,t}$ in the FM cross-section regression can be explained as the day t return on a standard portfolio of the LHS assets (R_x) with weights that sum to one and zero out each explanatory variables, which is common to all assets and not captured by the regression explanatory variables. It should be noted that each FM slope portfolio does not require any net investment, this is accomplished by financing long positions in LHS assets with short positions in other LHS assets. Further, each CS factor for day t is equal to the return on a zero-investment portfolio, in which the portfolio's standardized characteristic is equal to one, and the values of other standardized characteristics are equal to zero. Equivalently, each CS factor sets the value of its unstandardized characteristic one standard deviation above its cross-section mean for the month and sets other unstandardized characteristics to their cross-section means.

$$(21)\ R_{x,t} = R_{z,t} + 0 * R_{MC,t} + 0 * R_{BM,t} + 0 * R_{WML,t};\ \text{thus}\ R_{x,t} = R_{z,t}$$

However, Eq. (20) does not look like an asset pricing model. It is a cross-sectional FM regression model which would be conducted on a time series scale to estimate the pure factor returns of CS factors. In Fama and French (2019), firstly move the intercept $R_{z,t}$ to the left side of the Eq. (20) so that LHS returns are in excess of $R_{z,t}$, and then interchange the position of firm characteristics and CS factors to derive the Eq. (22) (or CS factor model I). Their insight here is that when the cross-sectional regression Eq. (22) is stacked across t, it becomes an asset pricing model that can be used in time-series applications. As a result, the FM cross-section regression becomes a time-series factor model with prespecified time-varying factor loadings.

CS Factor Model I:

$$(22)\ R_{i,t} - R_{z,t} = MC_{i,t-1}R_{MC,t} + BM_{i,t-1}R_{BM,t} + MOM_{i,t-1}R_{MOM,t} + \varepsilon_{i,t}$$

Equation (22) is a three-factor model in which the factors used to explain asset returns in excess of $R_{z,t}$ are $R_{MC,t}$, $R_{BM,t}$, $R_{MOM,t}$ and the factor loadings are the time-varying previous observed values of market capitalization ($MC_{i,t-1}$), the book-to-market ratio ($BM_{i,t-1}$) and the historical cumulative momentum ($MOM_{i,t-1}$) for each asset i at day t. These prespecified characteristics are standardized using the time-series mean and standard deviation of the characteristics for the 12 portfolios in Eq. (20). It should be noted that the result of pricing errors and predictions is not affected by standardization. Daily CS factors are rescaled via standard characteristics in Eq. (20), but the rescaling is reversed in Eq. (22) where the standardized characteristics are also used as loadings. Therefore, the result will be the same regardless of whether the characteristics are raw or standardized. The test asset on the LHS are individual stock returns ($R_{i,t}$). The time-series average of the residual $\varepsilon_{i,t}$ is the pricing error for stock i.

Note that the CS factor model I is used as a time-series model to describe average returns for a wide range of LHS assets, not for regression. In short, the procedure of calculating $R_{z,t}$ and using CS factor model I as an asset pricing model can be summarized as the following three steps:

1) At each time t, the intercept $R_{z,t}$ and the CS pure factor returns are obtained by conducting the Eq. (20) on a cross-sectional scale.

2) For a given asset i (Note that the assets here are not the LHS of the Eq. (20), which are the 12 standard portfolios, but individual stocks), calculate its three standardized (z-score) firm

characteristics and then use the three pure factor returns and $R_{z,t}$ obtained by the first step to form its expected return rate.

3) At each time t, calculate the difference between the expected return rate and the real return rate of asset i, and that is the pricing error of CS factor model I for the asset i at time t. Finally, calculate the time-series average of this pricing error $\varepsilon_{i,t}$.

In the TS Factor Model I, the factors are prespecified, which are double sorted based on firm characteristics without any attempt to optimize. Instead, the time-series regression optimizes an asset's betas on the prespecified factors, subject to the constraint that the betas are constant within the estimation window and assuming the regression residuals to be independent and identically distributed (iid). In short, the TS factor model I optimizes loadings on factors that are not themselves optimized.

In contrast, the factor loadings (firm characteristics) in CS factor model I are prespecified and can be directly used in the cross-sectional FM regression without causing the EIV problem like estimated betas, this can also be viewed as an attempt to minimize the sum of squared residuals. As a result, the CS factors are optimized to the prespecified factor loadings and LHS returns daily but under the restricted assumption that the disturbances in Eq. (20) are cross-sectionally iid.

6.2.3 CS Factor Model II

The CS factor model II can be viewed as a substitute for the TS factor model I, however, the performance of size, value and momentum factors in CS factor model II is derived from FM cross-sectional regression. In other words, CS factors are just a different way of constructing the Carhart4-factor in TS factor model I, except for the market factor. After obtaining the CS factors ($R_{MC,t}$, $R_{PE,t}$, $R_{OP,t}$ and $R_{INV,t}$) from Eq. (20), it is natural to interchange the position of firm characteristics and CS factors to derive the Eq. (23) (CS factor model II). In addition, the market factor is also added in this second CS factor model. As a result, CS factor model I is a combination of CS factors and time-series regression.

CS Factor Model II:

$$(23)\ R_{i,t} - R_{f,t} = a_i + \beta_{i,1} R_{MKT,t} + \beta_{i,2} R_{MC,t} + \beta_{i,3} R_{BM,t} + \beta_{i,4} R_{MOM,t} + \varepsilon_{i,t}$$

Where, $R_{MC,t}$, $R_{BM,t}$ and $R_{MOM,t}$ are all pure CS factor returns obtained from running Eq. (20) to replace the TS factor realizations in Eq. (19). Thus, CS factor model II is a direct competitor for TS factor model I, in which the Eq. (23) can be viewed as a time-series regression in the same family as Eq. (19) in that LHS returns are in excess of the risk-free rate. Again, the time-series average of the residual $\varepsilon_{i,t}$ is the mispricing error for stock i. CS factor model II is different to CS factor model I, the latter is a rearrangement of the cross-sectional FM regression in Eq. (20) with LHS asset returns in excess of $R_{z,t}$ and only CS factors on the RHS without the market factor.

6.3 GRS and Alpha Tests (Equal-Weight Metrics)

When evaluating a multi-factor asset pricing model, one approach is to jointly test whether the pricing error of testing assets is zero and an alternative way is to test the pricing error of each asset and compare it with zero separately. GRS test proposed by Gibbons et al. (1989) or mean-variance spanning test can be adopted if the target is to jointly test for pricing error. In addition, if the goal is to treat pricing errors independently, then one can use the Alpha test. Thus, GRS and Alpha tests, as well as a bunch of other equal-weight metrics, are used to evaluate the performance of CS and TS factor models.

GRS test statistic:

$$(24)\frac{T-N-K}{N}\ (1+E[\lambda_t]'\widehat{\Sigma_\lambda}^{-1}E[\lambda_t])^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\ \sim F_{N,T-N-K}$$

$$where\ \widehat{\Sigma_\lambda} = \frac{1}{T}\sum_{t=1}^{T}[\lambda_t - E[\lambda_t]][\lambda_t - E[\lambda_t]]'$$

$$and\ \hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_t{}'$$

Where $\hat{\alpha}$ is the vector of pricing errors for N assets, $\hat{\varepsilon}_t$ is the vector of regression residuals for N assets at time t, λ_t is the vector of K factor returns at time t.

For conducting the Alpha test, the pricing error (α) of each asset under the asset pricing models needs to be estimated and then obtains the absolute value of these errors (α) and calculate the corresponding standard errors. The reason for using absolute value is because the focus here is the magnitude of pricing error rather than its direction. Using A and V to indicate a cross-sectional average and variance, respectively. Therefore, the mean of intercepts $(A|\alpha|)$ and the mean of its corresponding t-statistics $(A|t(\alpha)|)$ are the two main evaluation indicators in the Alpha test. Further, there are some variants based on these equal-weight (EW) metrics. The proportion of the cross-section dispersion in average returns missed by a model can be measured in two means. Using the first way, the average of squared intercepts is divided by the cross-sectional variance of the LHS average returns ($A\alpha^2/V\bar{r}$). The second way is to subtract the squared standard error of each intercept, $S^2(\alpha)$, which reduces the noise in the estimates, thus the adjusted estimate of the proportion of cross-sectional dispersion in average returns missed by a model is $A(\alpha^2 - S^2(\alpha))/V\bar{r}$ (or $A(\lambda^2)/V\bar{r}$). In short, the alpha test includes four parts, which are $A|\alpha_i|$, $A|t(\alpha)|$, $A\alpha^2/V\bar{r}$ and $A(\lambda^2)/V\bar{r}$, and they called EW metrics because each regression contributes the same weight in the calculation. The low value of $A\alpha^2/V\bar{r}$ or $A(\lambda^2)/V\bar{r}$ are good news for a model, meaning that the dispersion of its pricing errors is small relative to the dispersion of test asset average returns.

In addition, the three measures of regression fit are added for comparison following Fama and French (2019): the cross-sectional average of the standard errors of the intercepts, $As(\alpha)$, the average of regression R-squared (goodness of fit), AR^2, and the average of the standard deviations of the regression residuals, $As(e)$. Additionally, this study adds another goodness of fit measurement to intuitively compare the explanatory power of TS and CS models on the variation of cross-sectional average returns, the cross-sectional R-squared as Eq. (20) shown. The higher value of R_C^2 means the stronger explanatory power of an asset pricing model.

$$(25)\ R_C^2 = 1 - \frac{Var(\hat{\alpha})}{Var(\bar{R})}$$

where $Var(\hat{\alpha})$ is the variance of pricing errors on cross-section, $Var(\bar{R})$ is the variance of the average of excess stock returns on cross-section.

6.4 Empirical Asset Pricing Results

Table 18 and Table 19 examine how well TS factor model I and CS factor model I&II explain the average returns on the individual stock returns. The performance evaluation metrics include the GRS test and a series of alpha tests (equal-weight metrics), and four measures of regression fit. Panel A in Table 18 provides evidence on TS model I and CS model II, both estimate constant factor loadings by using OLS regression slopes and explain the LHS returns over the level return $R_{f,t}$. The specific issue here is whether the time-series SMB, HML, and WML factors of TS model I generate smaller pricing error metrics than the corresponding cross-sectional SIZE, BM, and MOM factors of CS factor model II. According to the result, TS factor model I and CS factor model II have very similar monthly pricing errors, 0.74% and 0.82%, respectively. Additionally, the dispersion of their pricing errors relative to the dispersion of test asset average returns are also close, which are 1.01 and 1.16 without noise adjustment, then 0.78 and 0.92 after it is adjusted, respectively. In short, the TS factors only win the CS factors by a small margin of 10.81%. Therefore, there is no compelling evidence to replace TS factors with CS factors in the time-series regression model. In other words, simply substituting CS factors for TS factors does not improve much on the precision of the Carhart4-factor model.

Panel B of Table 18 presents the performance of CS factor model I in which LHS returns are over the level return $R_{z,t}$ rather than $R_{f,t}$. For these three versions of CS factor model I, the biggest difference is that the factor loadings of Panel B1 are OLS estimates, while the factor loadings of Panels B2 and B3 are prespecified characteristics. Panel B1 shows the result of the model when using OLS slopes as constant factor loadings, which has the highest pricing error of 0.61% and the highest dispersion degree (0.25 without noise adjustment and 0.17 after it is adjusted) compared to the other two versions of the model. However, it is not natural to use OLS slopes as the constant-loading version for CS factor model I. Thus, Panel B2 presents the result of using the time-series average of characteristics as the constant-loading version for the model, in which the loadings for cross-sectional SIZE, BM and MOM factors ($R_{MC,t}$, $R_{BM,t}$ and $R_{MOM,t}$) are the average of prespecifies characteristics ($MC_{i,t-1}$, $BM_{i,t-1}$ and $MOM_{i,t-1}$). After using average characteristics as factor loadings, the pricing error of the model is reduced to 0.59% and the dispersion degree is also decreased to 0.23 (without noise adjustment) and 0.15 (with noise adjustment).

Further, Panel B3 shows that using time-varying characteristics as factor loadings for the CS factor model I perform better on all metrics than the other versions of Panels B1 and B2, which has the lowest pricing errors (0.57%) and dispersion degree (0.14 without adjustment and 0.11 after adjustment). This result is also suggested by Fama and French (2019), which also find that the most ideal factor loadings for CS factors are their predetermined time-varying characteristics. However, it should be noted that using time-varying characteristics rather than constant characteristics as factor loadings does not lead to significant improvement given that the average pricing error in Panel B2 is 0.59% and in Panel B3 is 0.57%. This implies that the success of CS factor model I is due more to its CS factors rather than using their corresponding time-varying characteristics.

The last two columns of Table 18 show that the TS and CS factor models are all strongly rejected on GRS and Hotelling's test, meaning that none of them provides complete descriptions of average returns on the LHS assets. The study replaces GRS with the F-statistic of Hotelling's T-squared which tests whether the expected values of the pricing errors for LHS returns are jointly zero. This is because the factor loadings of Panel B2 and B3 are not estimates but prespecified characteristics and the tests assets are not the same that produce the CS factors.

Table 18

Summary of Average Errors from Regressing with Individual Stocks

Panel A: TS Factor Model I & CS Factor Model II (Returns in Excess of Rf) Factor Loadings are Constant Regression Slopes						
Model	$A\|\alpha\|$	$A\|t(\alpha)\|$	$A\alpha^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	GRS	p(GRS)
Rm-Rf, SMB, HML, WML	0.74%	0.98	1.01	0.78	2.09	0.0000
Rm-Rf, RMC, RBM, RMOM	0.82%	1.14	1.16	0.92	2.15	0.0000
Panel B: CS Factor Model I (Returns in Excess of Rz)						
Panel B1: Factor Loadings are Constant Regression Slopes						
Model	$A\|\alpha\|$	$A\|t(\alpha)\|$	$A\alpha^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	GRS	p(GRS)
RMC, RBM, RMOM	0.61%	1.12	0.25	0.17	2.24	0.0000
Panel B2: Factor Loadings are Average Characteristics						
Model	$A\|\alpha\|$	$A\|t(\alpha)\|$	$A\alpha^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	T^2	P(T^2)
RMC, RBM, RMOM	0.59%	1.00	0.23	0.15	2.07	0.0000
Panel B3: Factor Loadings are Time-Varying Characteristics						
Model	$A\|\alpha\|$	$A\|t(\alpha)\|$	$A\alpha^2/V\bar{r}$	$A\lambda^2/V\bar{r}$	T^2	P(T^2)
RMC, RBM, RMOM	0.57%	0.94	0.14	0.11	2.03	0.0000

The tests use individual stocks as test assets rather than portfolio sorts. The regressions in Panel A explain returns in excess of the risk-free rate and include the excess market return Rm-Rf. They differ on whether they also use TS factors or CS factors. Panels A summarize results from the time-series regressions of TS factor model I and CS factor model II, respectively. The table shows $A|\alpha|$ and $A|t(\alpha)|$, the average absolute intercept and average absolute t-statistic for the intercepts; $A\alpha^2/V\bar{r}$, the average squared intercept over the cross-sectional variance of $\bar{r}$, the average returns on the LHS portfolios; $A(\lambda^2)/V\bar{r}$, the average difference between each squared intercept and its squared standard error, $S^2(\alpha)$, divided by the variance of $\bar{r}$. GRS and p(GRS) are the GRS statistic of Gibbons et al. (1989); and the p-value of GRS. Panel B summarizes results for different versions of CS factor model I in which LHS returns are overR_z. Panels B1 add an intercept to CS factor model I and use OLS regression slopes as factor loadings. Panels B2 substitute time-series average values for the time-varying characteristics of CS factor model I. The time-varying factor loadings in Panel B3 are prespecified MC, BM and MOM characteristics. Especially, the absolute average of intercepts $A|\alpha|$, the average of the standard errors of the intercepts and the average of the standard deviations of the regression residuals are reported as monthly values.

Table 19 summarises the results of regression fit measurement, the values of AR^2 in Panels B2 and B3 are about two times more than those of Panel A. However, this result could be misleading as the

LHS returns in Panel A are over $R_{f,t}$, while those in Panel B are over the level return $R_{z,t}$. For example, in Panel B, AR^2 measures the average fraction of leftover variance that is explained by other factors after considering the explanatory power of $R_{z,t}$. As suggested by Fama and French (2019), $As(e)$, which is the average of the time-series standard deviations of unexplained LHS returns (residuals), is in any case key to comparing the models of Panels A and B. On the other hand, $As(\alpha)$ is the measure of determining the average precision of pricing errors. From Panels A to Panel B, the values of $As(e)$ and $As(\alpha)$ under TS and CS factor models are all very similar, meaning that the average precision of their pricing error measurements is also very close. Furthermore, the similar $As(e)$ values in Panels B2 and B3 indicate that using prespecified characteristics as factor loadings for CS factors cannot increase the explanatory power of a model. R_c^2 in the last column of Table 19 is used as a supplementary measure to compare the explanatory power for a model on the variation of LHS average returns. In Panel A, the R_c^2 of CS factor model II is 0.09 versus 0.07 for TS factor model I, this implies that simply substituting CS factors for TS factors in the Carhart4-factor model does not lead to significant improvement in the description of average returns. When using constant regression slopes as factor loadings, the R_c^2 in Panel B1 is 0.21 versus 0.07 and 0.09 in Panel A, meaning that CS factor model I has significantly better performance in explaining the variation of cross-sectional LHS average returns than TS factor model I and CS factor model II. Furthermore, Panel B3 shows that CS factor model I has the highest R_c^2 (0.27) when using its natural loadings (time-varying characteristics). However, this improvement of using time-varying characteristics is relatively small given that the R_c^2 of using constant (average) characteristics is 0.25, meaning the dominance of CS factor model I is due more to CS factors than their corresponding time-varying characteristics.

Table 19

Summary of Regression Fit from Regressing with Individual Stocks

Panel A: TS Factor Model I & CS Factor Model II (Returns in Excess of Rf) Factor Loadings are Constant Regression Slopes				
Model	AR^2	$As(\alpha)$	$As(e)$	R_c^2
Rm-Rf, SMB, HML, WML	0.11	0.0014	0.09	0.07
Rm-Rf, RMC, RBM, RMOM	0.12	0.0013	0.09	0.09
Panel B: CS Factor Model I (Returns in Excess of Rz)				
Panel B1: Factor Loadings are Constant Regression Slopes				
Model	AR^2	$As(\alpha)$	$As(e)$	R_c^2
RMC, RBM, RMOM	0.13	0.0013	0.10	0.21
Panel B2: Factor Loadings are Average Characteristics				

Model	AR^2	$As(\alpha)$	$As(e)$	R_c^2
RMC, RBM, RMOM	0.17	0.0014	0.11	0.25
Panel B3: Factor Loadings are Time-Varying Characteristics				
Model	AR^2	$As(\alpha)$	$As(e)$	R_c^2
RMC, RBM, RMOM	0.19	0.0013	0.09	0.27

The tests use individual stocks as test assets rather than portfolio sorts. The regressions in Panel A explain returns in excess of the risk-free rate and include the excess market return Rm-Rf. They differ on whether they also use TS factors or CS factors. Panels A summarize results from the time-series regressions of TS factor model I and CS factor model II, respectively. The table shows AR^2, the average regression R^2; $As(\alpha)$, the average standard error of the intercepts; $As(e)$, the average residual standard deviation; R_c^2, the cross-sectional R-squared. Panel B summarizes results for different versions of CS factor model I in which LHS returns are overR_z. Panels B1 add an intercept to CS factor model I and use OLS regression slopes as factor loadings. Panels B2 substitute time-series average values for the time-varying characteristics of CS factor model I. The time-varying factor loadings in Panel B3 are prespecified MC, BM, and MOM characteristics.

6.5 Conclusion

In conclusion, using asset returns over a level return R_z and time-varying characteristics for CS factors provide the best descriptions of cross-sectional average returns than TS factor model I and CS factor model II in Panel A. This result is the same as Fama and French (2019). In addition, simply substituting TS factors for CS factors in the Carhart4-factor model does not lead to significant improvement since the pricing errors and goodness-of-fit of CS factor model II and TS factor model I are similar. Further, the dominance of CS factor model I is due more to its CS factors than their matching time-varying characteristics since using constant (average) characteristics as factor loadings has similar pricing errors and goodness-of-fit as using time-varying characteristics. The benefit of using CS factors is that they are estimated daily by using the cross-sectional FM OLS regression, which aims to optimize the daily description of returns on the 12 double-sorted portfolios. In contrast, the TS factors are arbitrarily constructed by the same 12 double-sorted portfolios without any optimizations.

CHAPTER 7: CONCLUSIONS

7.1 Conclusions

Previous empirical research use portfolios as tests for assets for two reasons: 1) creating portfolios reduce idiosyncratic volatility and allows more precise estimates of factor loadings, and consequently risk premiums; 2) if the errors in the estimated betas are imperfectly correlated across assets, then the measurement errors would tend to offset each other when the assets are grouped into portfolios. However, sorting stocks into portfolios according to specific characteristics may mask the features of stocks and impart a strong factor structure to test portfolio returns. Due to this limitation, two-pass regression may be unable to identify potential priced factors or misidentify factors that are actually unrewarded, resulting in a "factor zoo". In light of this, new methods will be required to allow the use of individual stocks rather than portfolios, and to further account for the EIV problem in a new way.

Among a bunch of new methods that aim to address the EIV problem, the IV approach in Jegadeesh et al. (2019) deserves a particular focus because it allows the use of individual stocks rather than portfolios as test assets and their result remains robust when using rolling betas in finite-sample. However, the profound advantage of the IV approach in risk premium estimation as Jegadeesh et al. (2019) argued failed to be found in this study as the IV estimator is biased and inconsistent in the small-sample simulation with the standard CAPM model. Further analysis reveals that this contrasting result is not caused by the "classic" weak-instruments problem but rather the time-series correlation of the measurement errors. The foundation stone of the IV approach proposed by Jegadeesh et al. (2019) is that the measurement errors of the explanatory and instrumental betas are cross-sectionally uncorrelated because they are estimated from the disjoint sample (odd and even) within a rolling window. Although this statement is uncontroversial, the measurement errors may still have serial correlation. In this context, the IV estimator will not eliminate the EIV bias but simply move it back to a point in time, resulting in inconsistent estimates. For example, Bellemare et al. (2015) and Reed (2015) suggest that when measurement errors are serial autocorrelation, using lagged explanatory variables as instrumental variables to address the EIV bias will be invalid.

Given that the IV estimator does not save the traditional FM procedure from the EIV problem, this study adopts Shanken's (1992) adjustment on the estimates of coefficients and standard errors. Especially, BJS regression is used in this study namely the betas are estimated using the entire sample period rather than using rolling windows as FM suggested. In the Appendix of Shanken (1992), he clarifies that his N-consistent bias-adjusted estimator should not be viewed as a formal asymptotic analysis for FM procedure with rolling betas but rather as a simple heuristic. According to the simulation result, the ex-post risk premium estimates under the BJS regression with Shanken's estimator are consistent and nearly unbiased in small samples.

The BJS-Shanken approach is further used to test whether the premiums of risk factors as suggested by a number of popular asset pricing models are indeed significantly different from zero. Specifically, the standard CAPM, the FF5-Factor, and the FF3-Factor models are used in this study. In addition, the sample period spans from Jan 2000 to Dec 2019. A total of 245 individual stocks (after adjusting for size and turnover ratio) enter the sample at different points in time. Previous empirical research on the JSE finds the significant premiums for size, value and momentum factors when using double-sorted portfolios as test assets. This includes the works of Van Rensburg and Robertson (2003) and Basewicz and Auret (2010), in which they conduct studies similar to Fama and French (1992) and conclude that both size and value effects significantly affect the JSE. More recently, Page and Auret (2018) provide evidence in line with Muller and Ward (2013) that the momentum effect is strong on the JSE. Their results are further confirmed in this study as these factors are significantly priced on the cross-section when using individual stocks as test assets, regardless of whether corresponding firm characteristics are controlled or not. On the other side, the slope coefficients of characteristics like SIZE, BM and MOM are also strongly significant. In the further panel regression analysis, the study finds that the significance of these characteristics is not due to their correlation with omitted and potentially unobserved firm-specific factors. In addition, the rank IC and IR results show that prespecified characteristics and estimated betas are both reliable and stable predictors of future returns and the significance of firm characteristics is not because they happen to line up well with future betas ("risk-proxy" hypothesis). These findings imply that firm characteristics could also be a choice for proxying the unobservable factor loadings since beta estimates contain measurement errors, and their corresponding slope coefficients may reflect the risk premiums of underlying factors. In Jegadeesh et al. (2019), the authors fail to provide empirical evidence to support this assumption, and as such, they refer to this as a puzzle.

To unravel this puzzle, this study adopts the approaches of Fama and French (2019), in which they construct a series of asset pricing models by using estimated betas and prespecified characteristics as factor loadings for TS and CS factors, respectively, and then compare their explanatory power on cross-sectional average returns. An attractive feature of this study is that the results are robust since individual stocks are used as test assets to avoid the low dimensionality problem of using portfolios and a bunch of evaluation metrics are jointly applied in the comparison. The results indicate that slope coefficients (CS factors) with their matching characteristics provide better descriptions for average returns on LHS assets (listed stocks on the JSE) than factor premiums (TS factors) with estimated betas. This result is consistent regardless of whether the factor loadings are time-varying or constant. In other words, the dominance of the CS factor model is due more to CS factors than the versions of their matching loadings. It should be noted that simply substituting CS factors for TS factors in regression models like the Carhart4-factor model does not lead to significant improvement in the description of average returns. The competitive edge of CS factors is that they are estimated through running the cross-sectional FM-OLS regression, which aims to optimize the daily description of returns on the double-sorted portfolios that produce TS factors. In contrast, the TS factors are arbitrarily constructed by the same portfolios without any optimizations. This is the key to answering the "betas versus characteristics" puzzle.

In conclusion, this study first provides the simulation evidence that the FM-IV method as provided by Jegadeesh et al. (2019) is problematic because the unobserved factors (measurement errors) may have serial correlation. Further, the study shows that the BJS-Shanken bias-adjusted estimator is N-consistent and nearly unbiased in small sample simulation. Further, the empirical asset pricing results show that the market, size, value and momentum factors under the CAPM, FF3-factor and Carhart4-factor models are significant along with the slope coefficients of their corresponding characteristics. In Chapter 6, the study provides empirical evidence that characteristics are more suitable proxies for true factor loadings since betas are estimated with measurement errors. Furthermore, their corresponding slope coefficients estimated by cross-sectional FM-OLS regression, which aims to optimize the description of average returns, tend to have a stronger explanatory power than TS factors that are arbitrarily constructed from the same portfolio sorts. However, from an academic point of view, the unobservable nature of true factor loadings and the endogeneity issue that arises in the application of regression-based approaches to financial data still contaminate the identification of real factors, and as such, we may still have a long way to go.

7.2 Limitations

Are Characteristics the True Factors?

The majority of academics and finance professionals agree that pervasive factors affect observed returns. However, there is considerable disagreement regarding which factors are responsible for observed returns, and whether they represent risks, anomalies, or something else. In Pukthuanthong et al. (2019), they try to bring a new protocol for factor identification, and they list three fundamental attributes for a true factor: 1) It should be unpredictable; 2) The variations in its value affect asset prices; 3) There is a risk premium associated with it. They further point out that characteristics should not be viewed as true factors since they can be observed in advance, which contrasts with the unknown nature of real factors. However, characteristics can be related to average returns either because they happen to line up with true factor loadings, i.e., they are good proxies, or because they can capture mispricing. As a result, characteristics that are not related to risk but can offer potential profit opportunities as this study proved deserve a special focus.

Are Characteristics Immune to the EIV Bias?

In this study, characteristics are assumed to be superior proxies of true factor loadings since the beta estimates contain measurement errors. However, it is not clear if firm characteristics are completely immune to the risk of EIV problems, even though they show better predictability than time-series regression betas as this study proved. For example, time-varying characteristics may be influenced by past returns, which induces cross-sectional correlations between characteristics and the measurement errors in betas as Chordia et al. (2017) suggested. Therefore, even if firm characteristics may serve as better proxies for factor loadings, the estimation of factor premium is also affected by the EIV problem. This point should be noted in future research. However, this study and Fama and French (2019) proved that the superior CS factor model is due more to its CS factors than the time-varying characteristics.

The Deficiency of the IV Approach

Since most of the standard approaches to empirical asset pricing rely on regressions, their results are subject to similar problems as most regression-based approaches. Two of the most significant regression issues are the EIV problem and the omitted-variable bias (OVB), which refers to the bias in the coefficient estimates obtained from regressions when some important explanatory variables are omitted from the model. The EIV bias is mostly limited to the estimation of factor premiums through a two-pass regression, while the OVB is present in almost all standard statistical methods of empirical asset pricing. Although the IV approach proposed by Jegadeesh et al. (2019) may help to solve the EIV problem, it may still be subject to the OVB. This is because the FM regression does not take into account the time-series dependence among asset returns (firm-specific fixed effects), it was designed to address cross-sectional correlations (time effects). Combining the GPS framework— proposed by

Hoechle et al. (2019) as outlined in Chapter 2—with the IV approach, one may expect to solve both OVB and EIV problems at once.

In conclusion, it is imperative and urgent for us to examine more rigorous methods in identifying factors involved in empirical asset pricing, as factor-based investing has grown explosively in recent years and there has been a "factor zoo". Nevertheless, most of the new approaches presented are limited to addressing a single aspect of the multiple flaws in traditional regression-based approaches. Therefore, it will be crucial in the future to find ways to synthesize the most promising approaches into one widely accepted procedure that can be applied to empirical asset pricing studies as a new protocol.

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APPENDIX

A.1 The Definition of TS and CS Factors

The market factor (MKT) of the CAPM model, the FF3-factor model, and the FF5-factor model is defined as the excess return on the market, which is calculated as the difference between the market return (J203T return rate is used as the proxy) and risk-free return (91-day T-bill return rate is used as the proxy). The size (SMB), value (HML), profitability (RMW), and investment (CMA) factors of the FF3-factor model and the FF-5 factor model are formed at the end of each year, which start from Dec 2002.

A.1.1 FF3-Factor

For the FF3-factor model, stocks are first sorted into two size groups (small and big), by using the cross-sectional median of their market capitalization as breakpoints. Further, stocks are sorted independently into three groups (high, neutral, and low) on the reciprocal of their price-to-earnings ratio (PE) using the top 30 and bottom 70 percentiles as the breakpoints. In other words, high PE ratio stocks (70^{th} PE percentile) are viewed as growth stocks and low PE ratio stocks (30^{th} PE percentile) stand for value stocks. The intersection of the 2*3 size and PE sorts produces a total of six equal-weight (EW) portfolios as Table A1 shows.

Table A1
Double-Sorted Portfolios (2*3 sorts)

Size / PE	Small	Big
Value	Small Value	Big Value
Neutral	Small Neutral	Big Neutral
Growth	Small Growth	Big Growth

As a result, SMB (small-minus-big) is the average return on the three small portfolios minus the average return on the three big portfolios and HML (high-minus-low) is the average return on the two value portfolios minus the average return on the two growth portfolios as below equation (A1) and (A2) shown.

$$(A1)\ SMB = \frac{1}{3}\ (Small\ Value + Small\ Neutral + Small\ Growth) \\ - \frac{1}{3}(Big\ Value + Big\ Neutral + Big\ Growth)$$

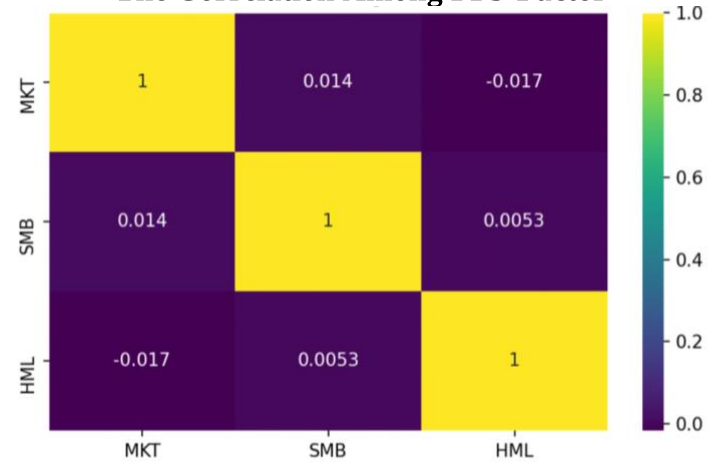
$$(A2)\ HML = \frac{1}{2}(Small\ Value + Big\ Value) + \frac{1}{2}(Small\ Growth + Big\ Growth)$$

Figure A1 shows the cumulative return of the FF3-Factor (MKT, SMB, and HML factors), by initially investing 1 Rand in these three-factor portfolios starting from Jan 2001, respectively.

Figure A1
Cumulative Log-Performance of FF3-Factor (Equal-Weight)

Figure A2 presents the serial correlation among MKT, SMB, and HML factors (double-sorted mimicking portfolios). As we can see from the figure, the correlation among them is fairly small, and consequently, the possibility of multicollinearity bias in regression is low.

Figure A2
The Correlation Among FF3-Factor



A.1.2 FF5-Factor

The operating profitability (OP) ratio of a firm is calculated by dividing its earnings before interest and taxes (EBIT) by the average of its total assets. In addition, the investment (INV) ratio of a firm is determined by calculating the rate of growth of its total assets, i.e., ln (A_{T-1}/A_{T-2}). For the FF5-factor model, stocks are also first sorted into two size groups and then they are independently sorted into value, profitability, and investment group, which are based on their PE ratios, the rates of operating profitability and the rates of growth of total assets, respectively as Table A2 shown. The breakpoints of each second sort are on the 30th and 70th percentiles, respectively.

Table A2
Double-Sorted Portfolios (2*3*3)

Size / PE	Small	Big
Value	Small Value	Big Value
Neutral	Small Neutral	Big Neutral
Growth	Small Growth	Big Growth

Size / OP	Small	Big
Robust	Small Robust	Big Robust
Neutral	Small Neutral	Big Neutral
Weak	Small Weak	Big Weak

Size / INV	Small	Big
Conservative	Small Conservative	Big Conservative
Neutral	Small Neutral	Big Neutral
Aggressive	Small Aggressive	Big Aggressive

Thus, the SMB factor for the FF5-factor model can be calculated as the sum of the returns on the nine small stock equal-weight portfolios of the three 2*3 sorts less the sum of the returns on the nine big stock equal-weight portfolios. Further, the HML factor is the average return on the two value portfolios minus the average return on the two growth portfolios, and RMW (robust-minus-weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios, and CMA (conservative-minus-aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios, as below equation (A3), (A4), (A5) and (A6) shown.

$$(A3)\ SMB = \frac{1}{3}\Big(\frac{1}{3}(Small\ Value + Small\ Neutral\ PE\ + Small\ Growth)$$
$$+\frac{1}{3}(Small\ Robust + Small\ Neutral\ OP\ + Small\ Weak)$$
$$+\frac{1}{3}(Small\ Conservative + Small\ Neutral\ INV + Small\ Aggressive)$$
$$-\frac{1}{3}(Big\ Value + Big\ Neutral\ PE\ + Big\ Growth)$$
$$-\frac{1}{3}(Big\ Robust + Big\ Neutral\ OP\ + Big\ Weak)$$
$$-\frac{1}{3}(Big\ Conservative + Big\ Neutral\ INV + Big\ Aggressive)\Big)$$

$$(A4)\ HML = \frac{1}{2}(Small\ Value + Big\ Value) + \frac{1}{2}(Small\ Growth + Big\ Growth)$$

$$(A5)\ RMW = \frac{1}{2}(Small\ Robust + Big\ Robust) + \frac{1}{2}(Small\ Weak + Big\ Weak)$$

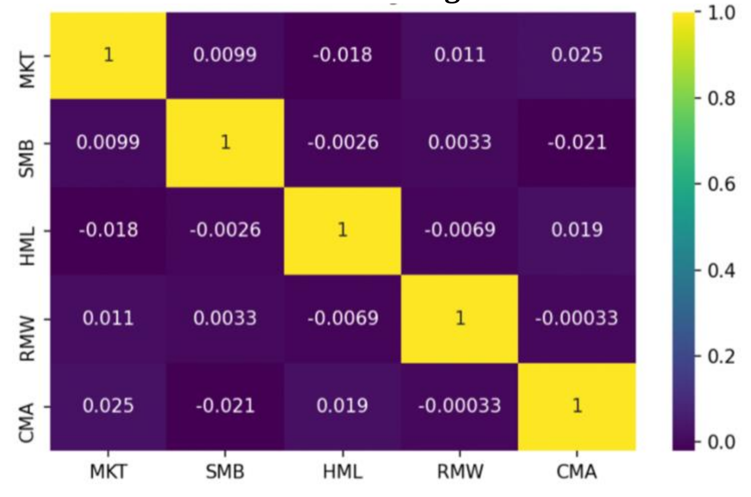
$$(A6)\ CMA = \frac{1}{2}(Small\ Conservative + Big\ Conservative) + \frac{1}{2}(Small\ Aggressive + Big\ Aggressive)$$

Figure A3 shows the cumulative return of the FF5-factor (MKT, SMB, HML, OP, INV factors), by initially investing 1 Rand in the five-factor portfolios starting from Jan 2001, respectively.

Figure A3
Cumulative Log-Performance of FF5-Factor (Equal-Weight)

Figure A4 presents the serial correlation among MKT, SMB, HML, RMW and CMA factors (double-sorted mimicking portfolios). As we can see from the figure, the correlation among them is fairly small, and consequently, the possibility of multicollinearity bias in regression is low.

Figure A4
The Correlation Among FF5-Factor



A.1.3 Carhart4-Factor (TS Factors)

The intersection of the 2*3 size and momentum (WML) sorts produces a total of six equal-weight (EW) portfolios as Table A3 shows.

Table A3
Double-Sorted Portfolios (2*3*3)

Size / PE	Small	Big
Winner	Small Momentum	Big Momentum
Neutral	Small Neutral	Big Neutral
Loser	Small Momentum	Big Momentum

Figure A5 shows the cumulative return of the momentum factor, by initially investing 1 Rand in the momentum portfolio starting from Jan 2001.

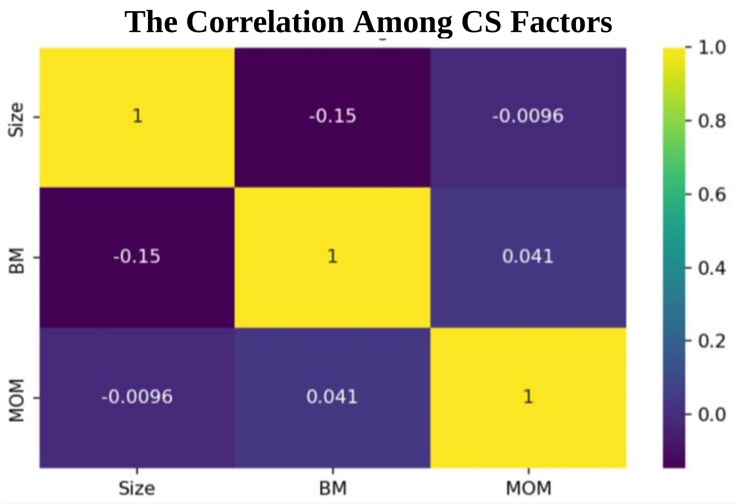
Figure A5
Cumulative Log-Performance of Carhart4-Factor (Equal-Weight)

A.1.4 CS Factors

In this study, the CS factors are formed by regressing Eq. (20) rather than double-sorted portfolios as the above sub-sections show. Specifically, the same portfolios (those from the 2*3 sorts, a total of 12 EW portfolios) are used as the LHS asset in Eq. (20) to produce CS factors and intercepts ($R_{z,t}$). This is important because the additional test aims to compare the explanatory power of TS and CS factors on the average stock returns.

Fama and French (2019) argue that the factor loadings in Eq. (20) are prespecified and can be used in the FM cross-sectional regression without causing an EIV problem, which may appear to be an attempt to minimize the sum of squared residuals. Especially, each factor in the CS factor models is a pure factor which is optimized by cross-sectional regression, meaning that the CS factor is only exposed to the target factor and has no exposure to the other non-target factors. As a result, the Eq. (20) that generates CS factors is optimized to describe the performance of each of these portfolios daily. The TS factors, on the other hand, are defined arbitrarily from these portfolios. In the TS factor model, the factors obtained by 2×3 double sorts do not have such properties, and each factor may be exposed to other factors randomly. Given that the same portfolios generate both the TS and CS factors, a central question concerns whether optimizing the CS factors enhances the description of the average return in the context of test assets beyond the portfolios that generate the factors. Figure A6 presents the correlation among CS factors (pure factor portfolios). As we can see from the figure, the correlation among them is fairly small.

Figure A6



A.2 Summary Statistics

Table A4 shows summary statistics for both TS and CS factors, which includes the time-series averages and the corresponding standard deviations and t-statistics.

Table A4
Annual TS Factors and CS Factors Summary Statistics

Panel A: Summary statistics for TS factor returns				
TS Factor Model: $R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,WML}WML_t + \varepsilon_{i,t}$				
	MKT	**SMB**	**HML**	**WML**
Mean	8.39%	-4.96%	7.32%	14.71%
Std Dev	18.44%	11.03%	11.84%	13.68%
t-statistics	**1.98**	**-2.26**	**2.89**	**4.18**
Panel B: Summary statistics for CS factor returns				
CS Factor Model I: $R_{i,t} - R_{z,t} = MC_{i,t-1}R_{MC,t} + BM_{i,t-1}R_{BM,t} + MOM_{i,t-1}R_{MOM,t} + \varepsilon_{i,t}$				
	$\boldsymbol{R_{z,t}}$	$\boldsymbol{R_{MC,t}}$	$\boldsymbol{R_{BM,t}}$	$\boldsymbol{R_{MOM,t}}$
Mean	3.24%	-0.07%	-1.62%	5.13%
Std Dev	7.19%	4.62%	3.02%	3.47%
t-statistics	**1.92**	-0.06	**-2.35**	**6.27**

This table reports annualized summary statistics of TS and CS factors, and corresponding standard deviation and t-statistics in parentheses (bold if t-statistics are higher than 2). Columns labelled MKT, SMB, HML, and WML are the factor realizations (double-sorted portfolio returns) of the market, size, value, and momentum factors, respectively. MC is the natural logarithm of market capitalization. BM is the book-to-market ratio estimated from the previous 12 months. Momentum is the historical cumulative returns estimated over the previous twelve minus one months. Further, $R_{z,t}$, $R_{MC,t}$, $R_{BM,t}$, and $R_{MOM,t}$ *are the CS factor returns, which are estimated by regressing excess stock returns on the standardized characteristics of the 12 portfolios that also produce the TS factors as Section 6.2.2 described. The sample period is from Jan 2001 through Mar 2019.*

Table A5 depicts the annual summary statistics for a series of factor returns. These factor returns are constructed by double sorting individual stocks into long-short portfolios based on specific underlying firm characteristics as Section A.1 described. The sample period ranges from Jan 2001 to Mar 2019.

Table A5
Annual Equal-Weight Long-Short Factor Summary Statistics

Rf = 5.25%	**Market**	**Size**	**Value**	**Profitability**	**Investment**	**Momentum**
Exp. Return (CARG)	8.39%	-4.96%	7.32%	6.69%	-2.21%	14.71%
Volatility	18.44%	11.03%	11.84%	12.04%	12.53%	13.68%
Return Range	35.93%	29.69%	46.44%	46.62%	54.71%	74.26%
Min Return	-18.67%	-16.38%	-32.38%	-23.36%	-33.92%	-20.08%
Max Return	17.26%	13.31%	14.06%	23.26%	20.79%	54.18%
Sharpe Ratio	0.45	-0.93	0.17	0.12	-0.60	0.69
Max Drawdown	-50.23%	-29.73%	-23.38%	-25.42%	-29.39%	-27.37%
t-statistics	**1.98**	**-2.26**	**2.89**	**2.41**	-0.83	**4.18**