Stanford

ICME
INSTITUTE for COMPUTATIONAL & MATHEMATICAL ENGINEERING

Understanding Regression-adjusted Control Variate

Sobolev embedding and rare event

Joint work with Jose Blanchet, Haoxuan Chen, Lexing Ying

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https://2prime.github.io/

ML Nowadays

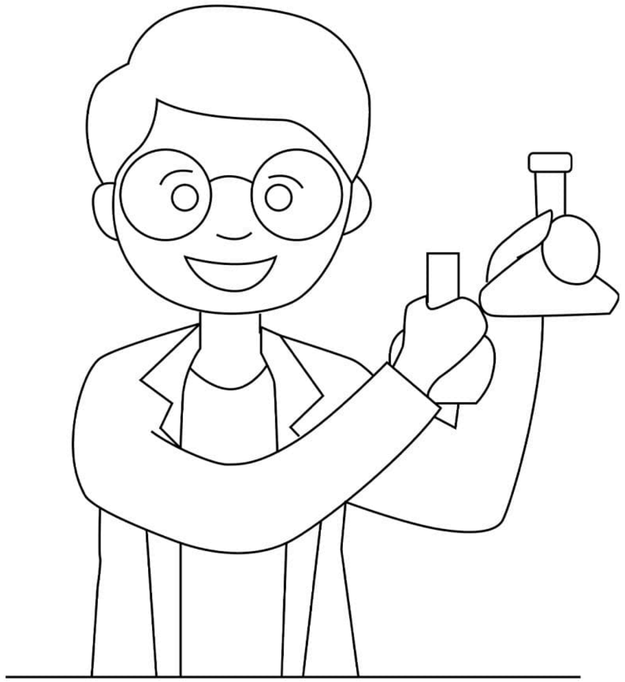
"entertainment, advising"

Midjourney

OpenAI ChatGPT4.0

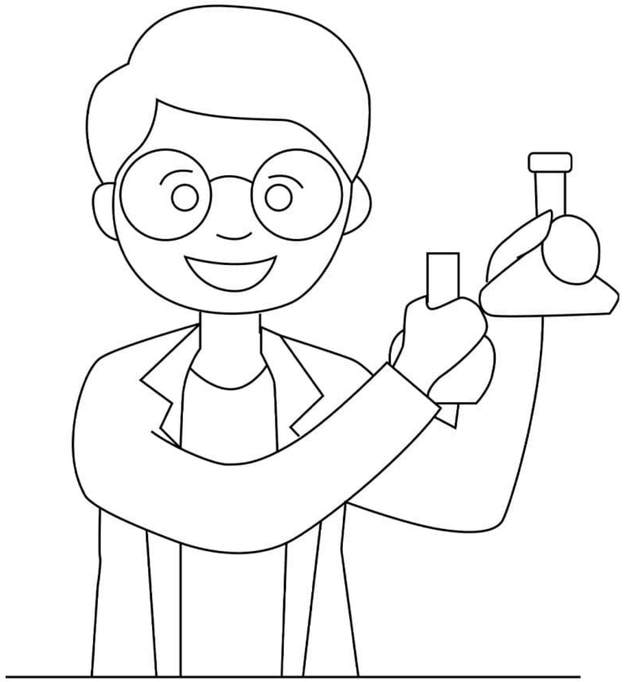
《science》

We want Guarantee!



Theorem If you randomly collect
( ) data, then you can achieve
( ) accuracy with your AI!

We want Guarantee!



Theorem If you randomly collect
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Assumption 1. Xxx
Assumption 2. Xxx
Assumption 3. Xxx
Assumption 4. Xxx

ML for Science nowadays

Debaising ML for Science

You can prove theorem, but I still don't trust you!

Can we debias ML estimator or use it in an unbiased way?

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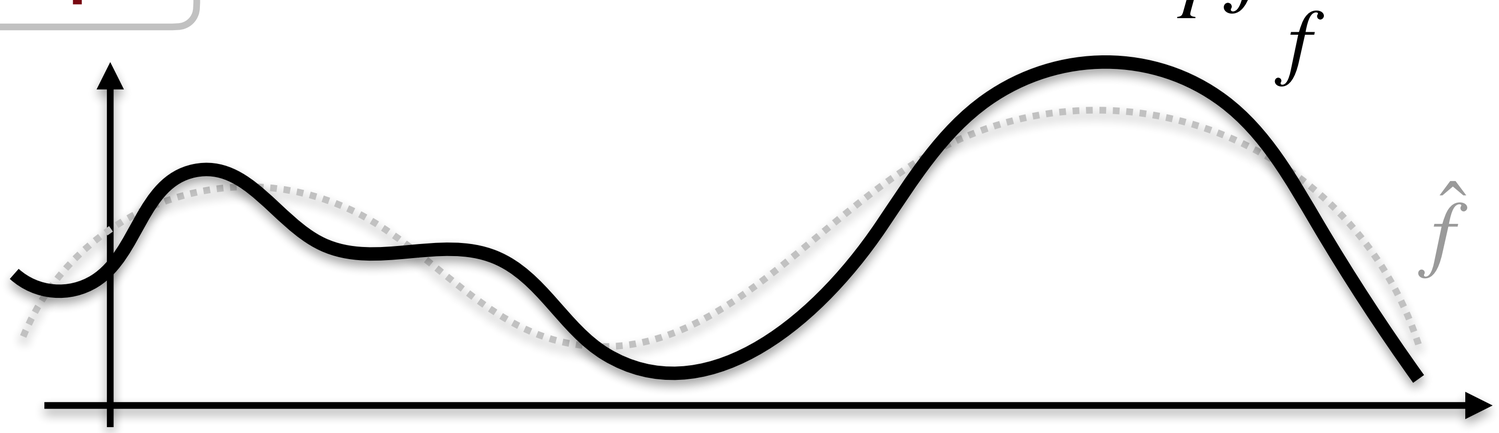
Example Monte Carlo Estimate $\mathbb{E}_P f$



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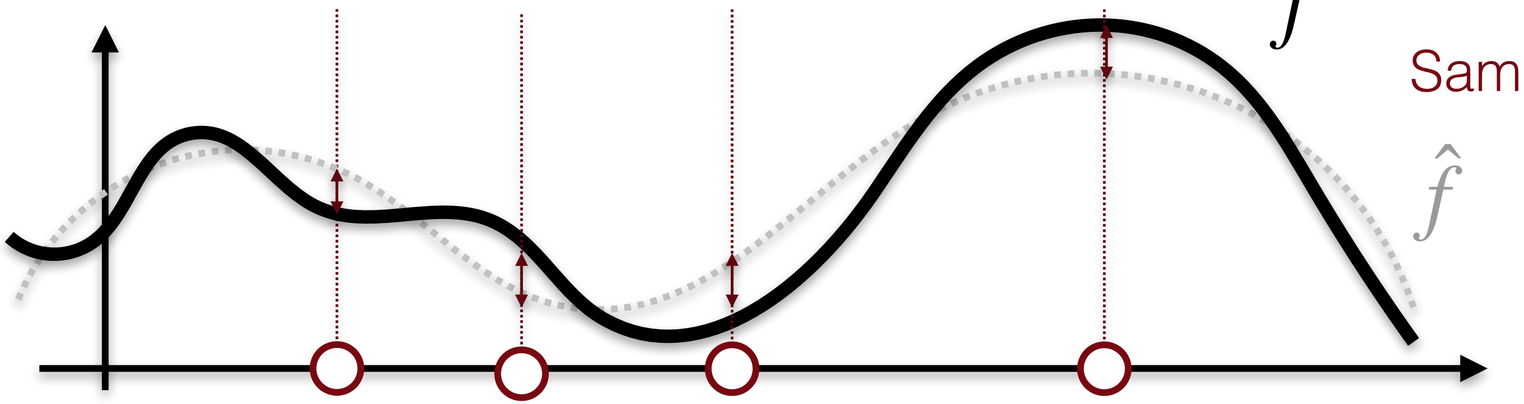

Sample extra data to know $f - \hat{f}$

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"Regression-adjusted control variate"

Can we debias ML estimator or use it in an unbiased way?

Example Monte Carlo Estimate $\mathbb{E}_P f$

Step 1 Using half of the data to estimate $\hat{f}$

Step 2 $\mathbb{E}_P f = \mathbb{E}_P(\hat{f}) + \mathbb{E}_P(f - \hat{f})$

Low order term

“Modern” regression-adjusted cv

Trace estimation:

Hutch++ Lin 17 Numerische Mathematik Mewyer-Musco-Musco-Woodruff 20

Dimension Reduction:

Sobczyk and Luisier Neuips 22

Conformal Prediction:

Conformalized quantile regression Romano-Patterson-Candes Neurips 19

Gradient Estimation

Shi-Zhou-Hwang-Tisias-Mackey Neurips 22 outstanding paper

Causal Inference:

Double Robust estimation ….

Debaising ML for Science

Is this algorithm statistical optimal?

When this improves MC estimator?

Can we debias ML estimator or use it in an unbiased way?

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Understanding this statistically…

Is this algorithm statistical optimal?

Why consider $q-$th moment?

When this improves MC estimator?

Can we debias ML estimator or use it in an unbiased way?

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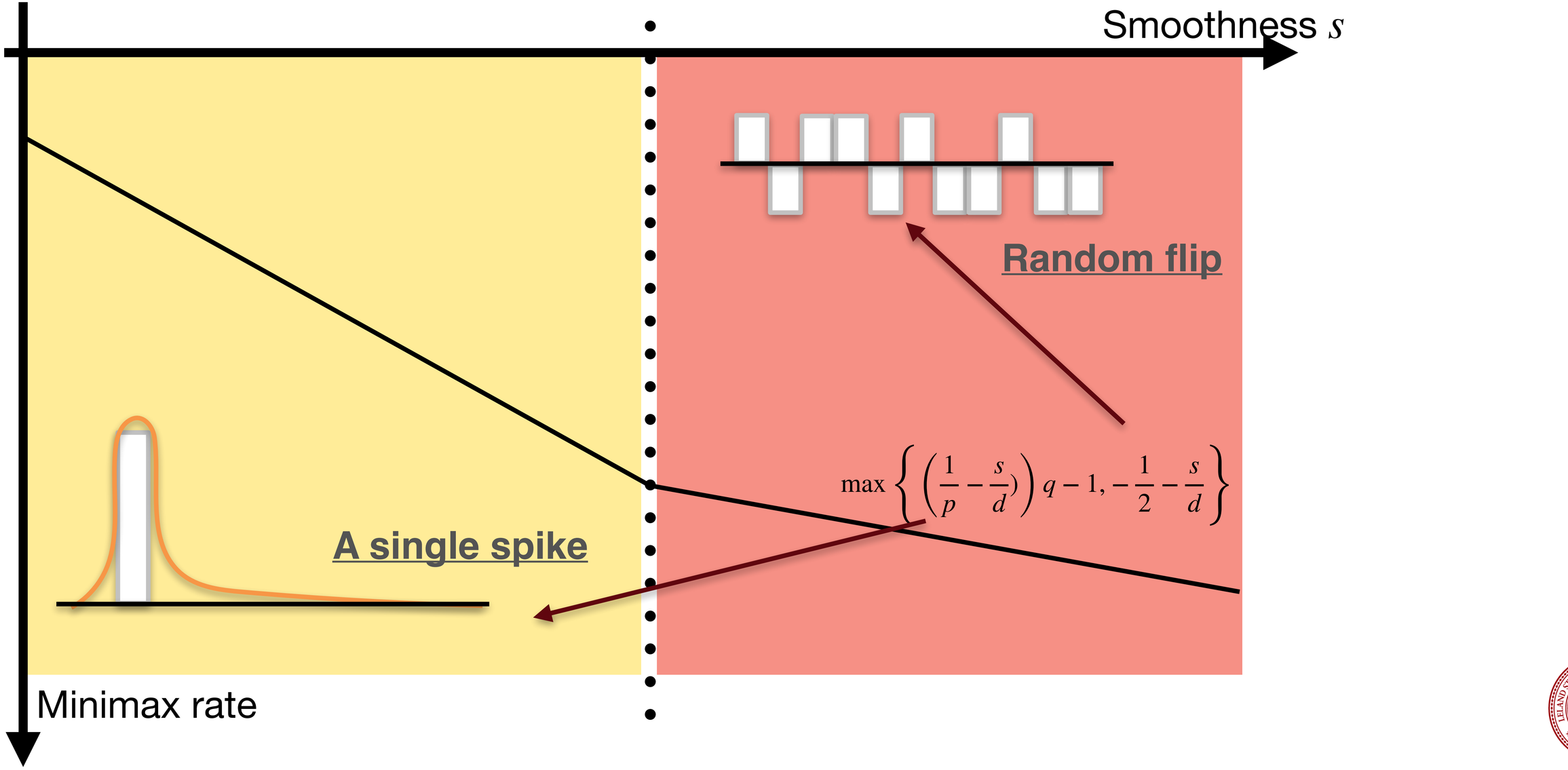
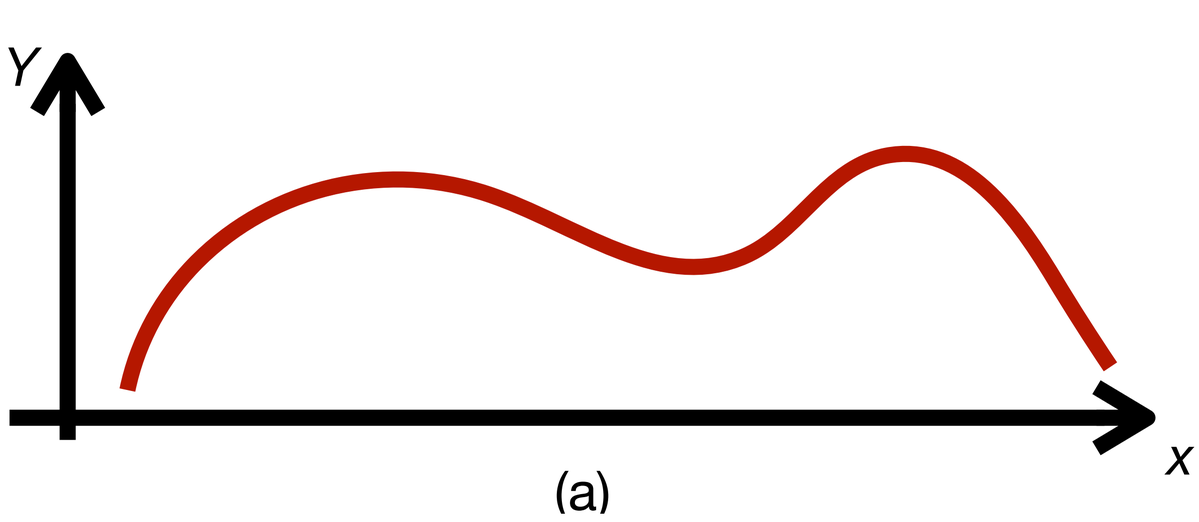
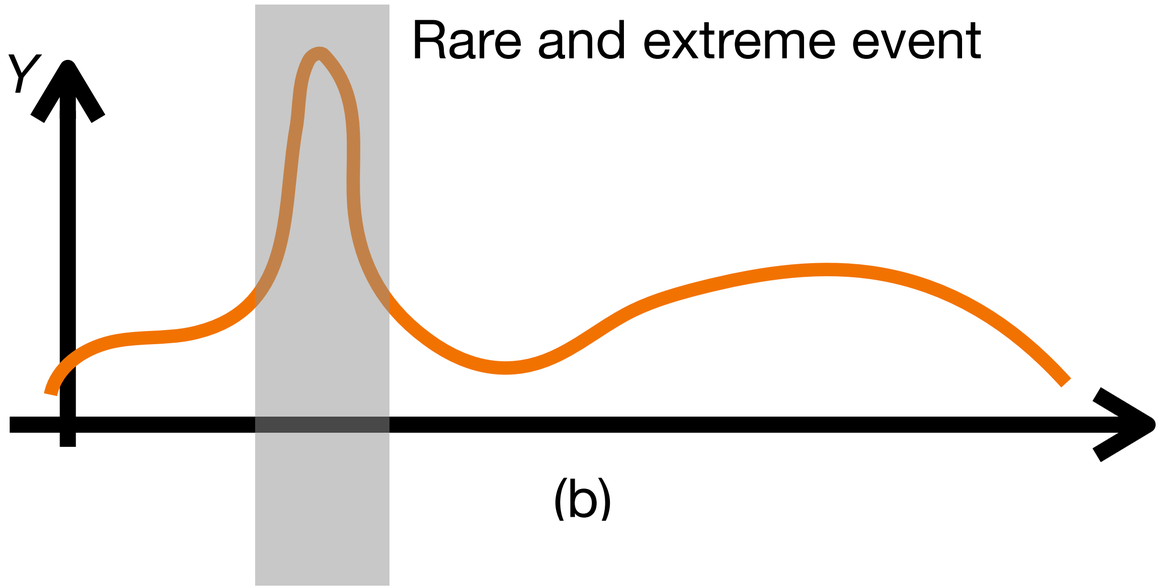
Setting the information theoretical limit

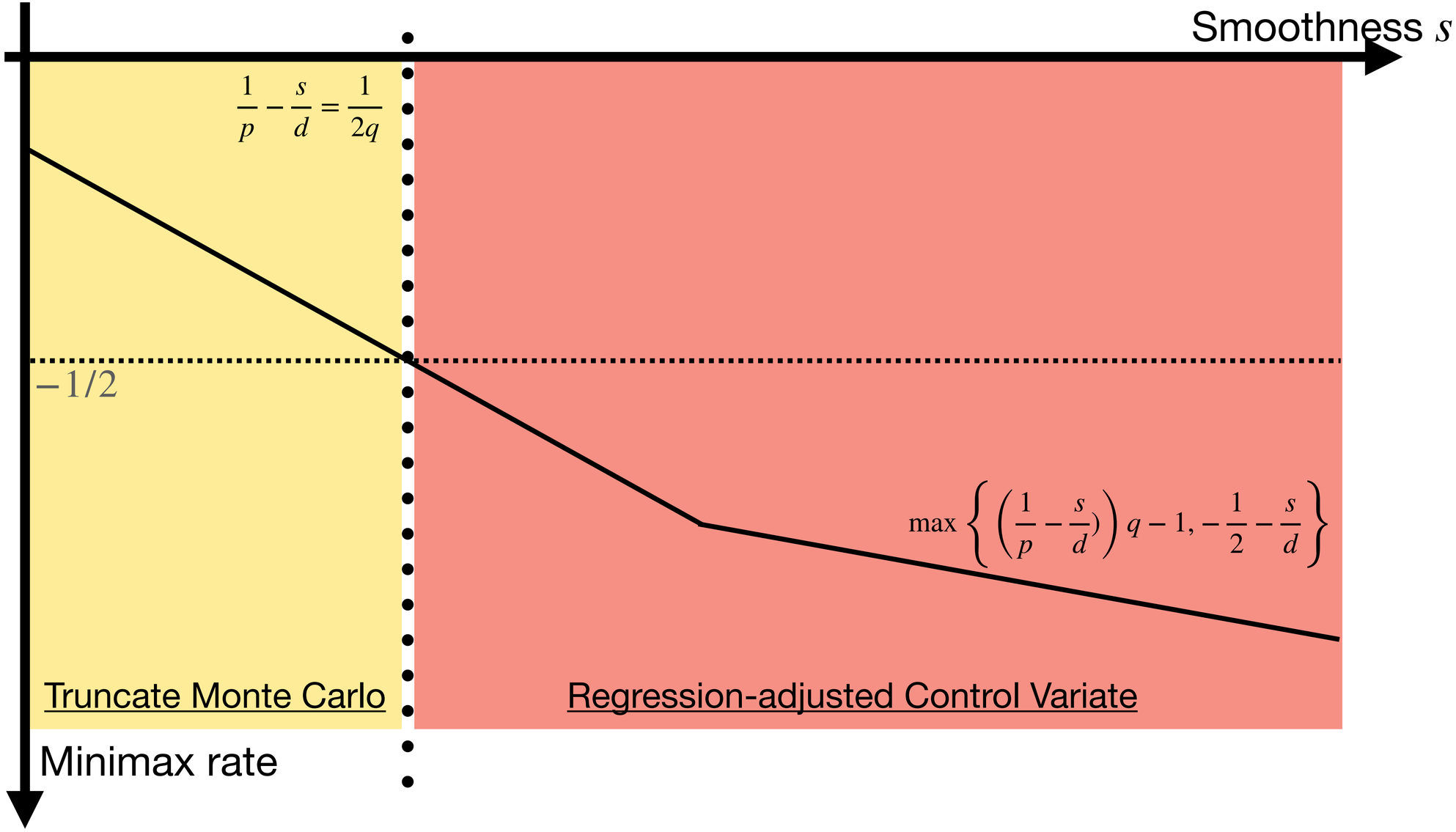


Rare Event and Smoothness…





When the control variate helps



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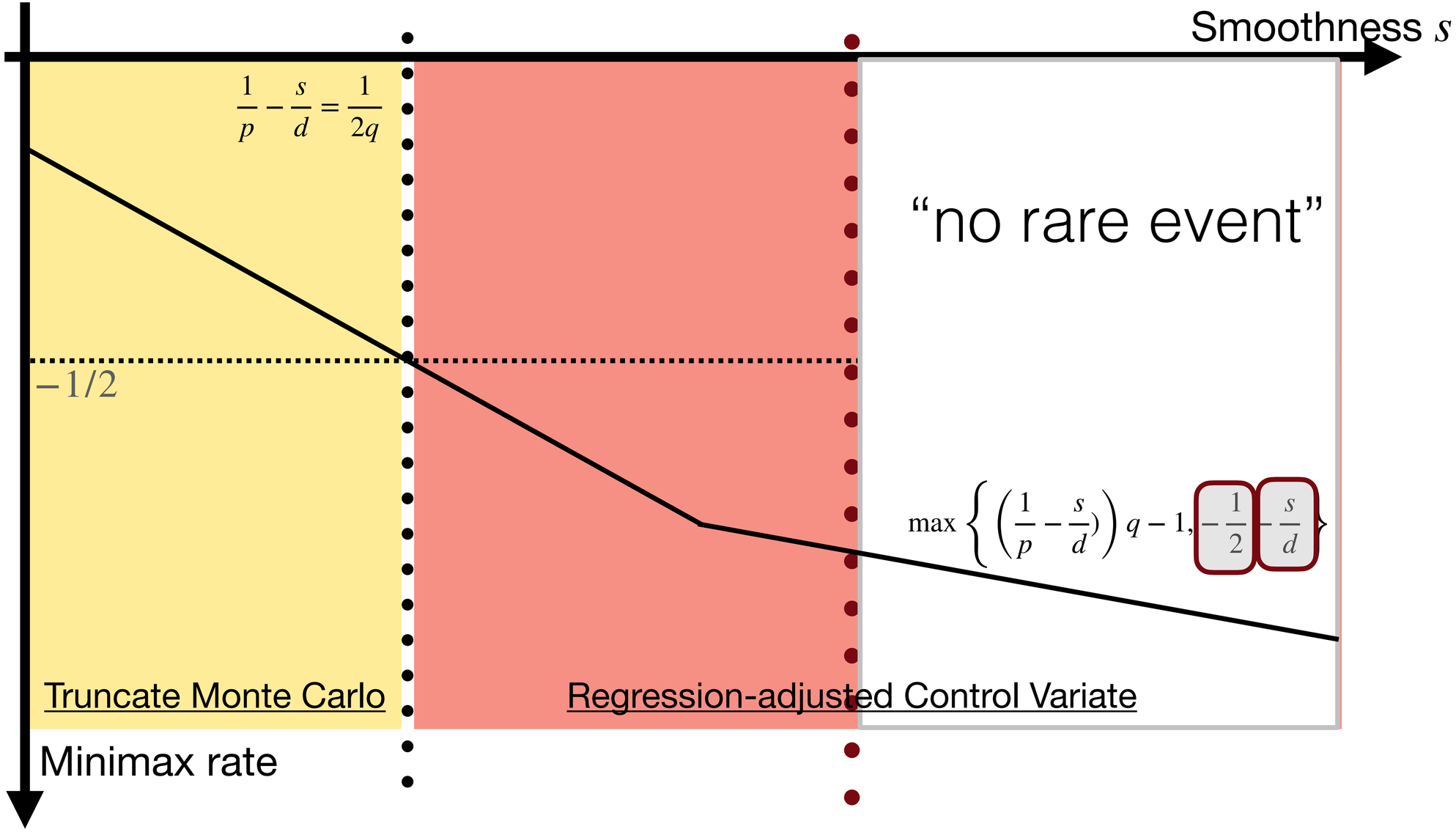

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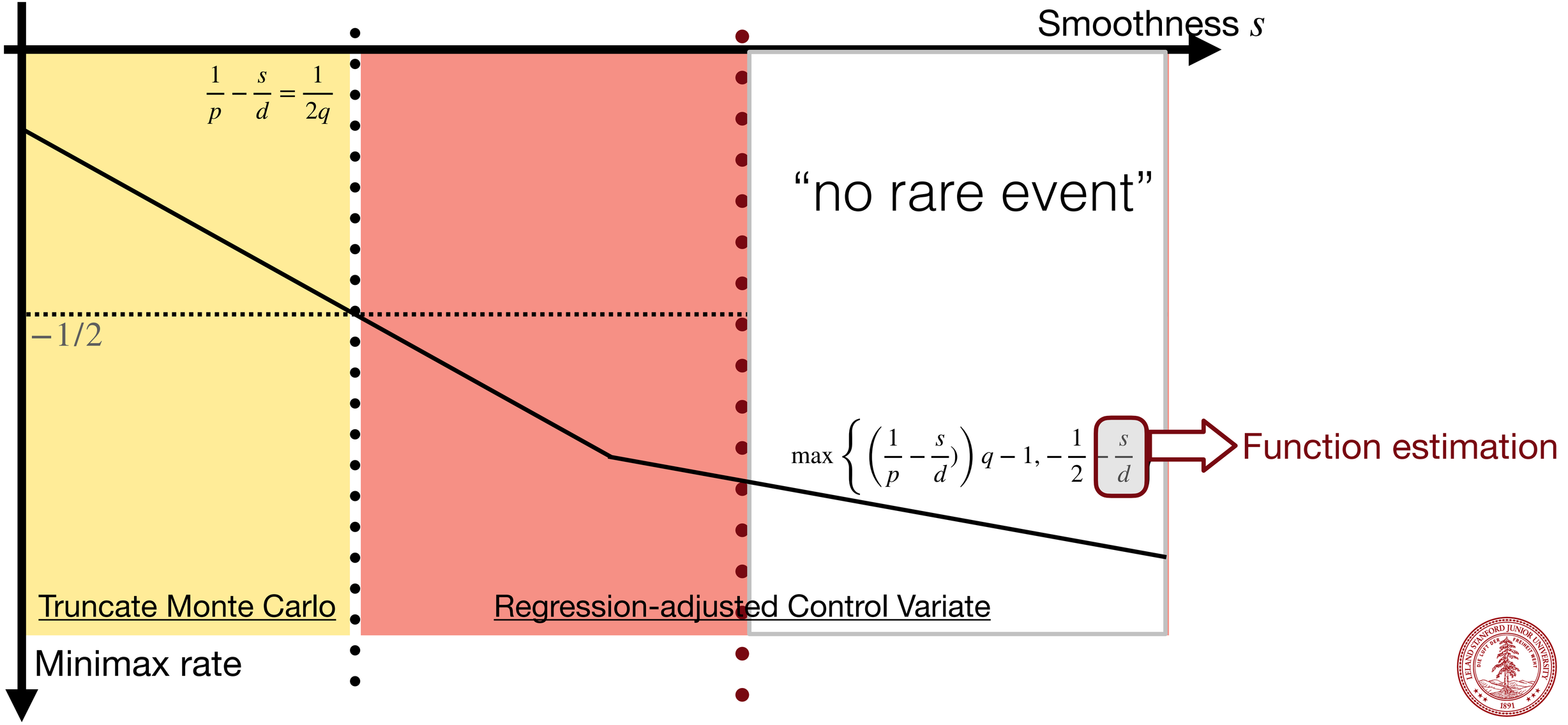

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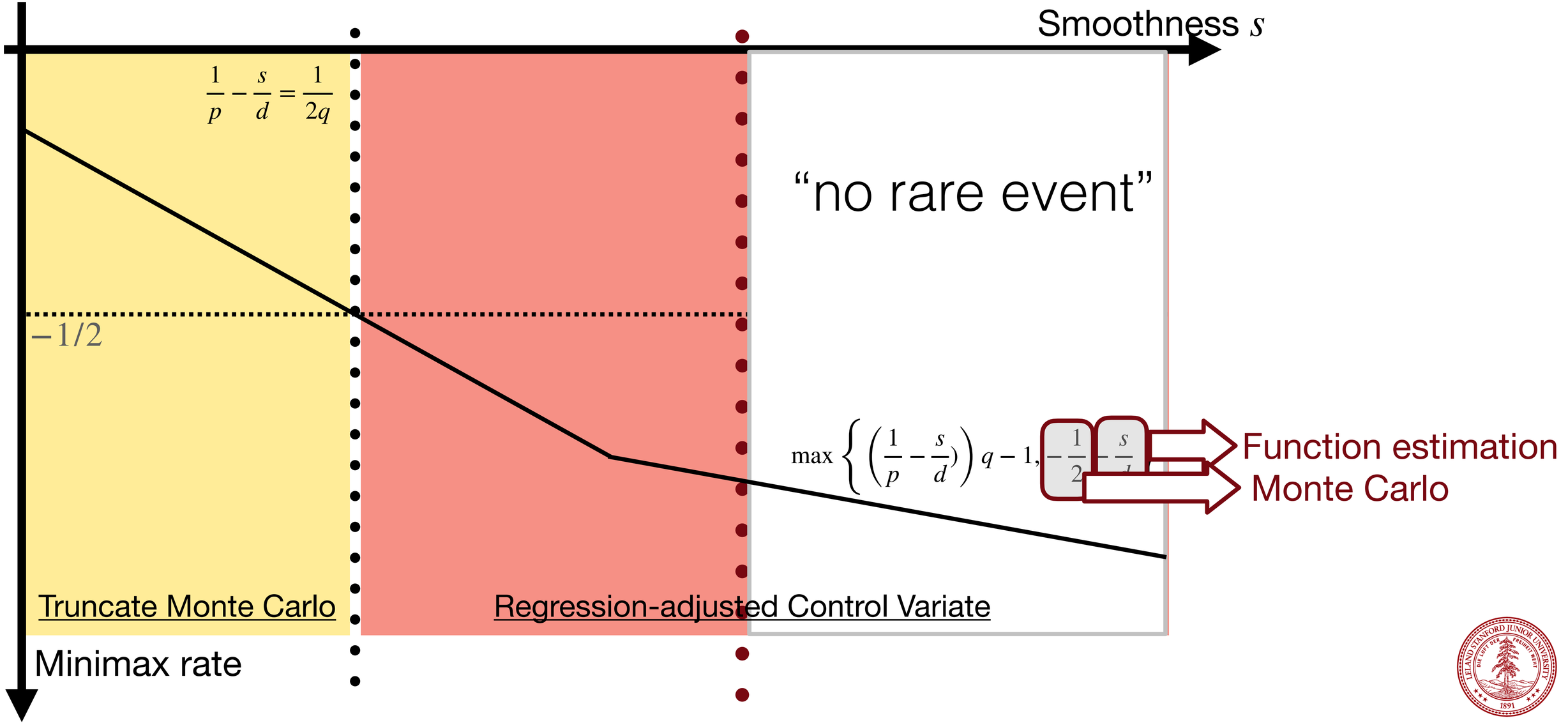

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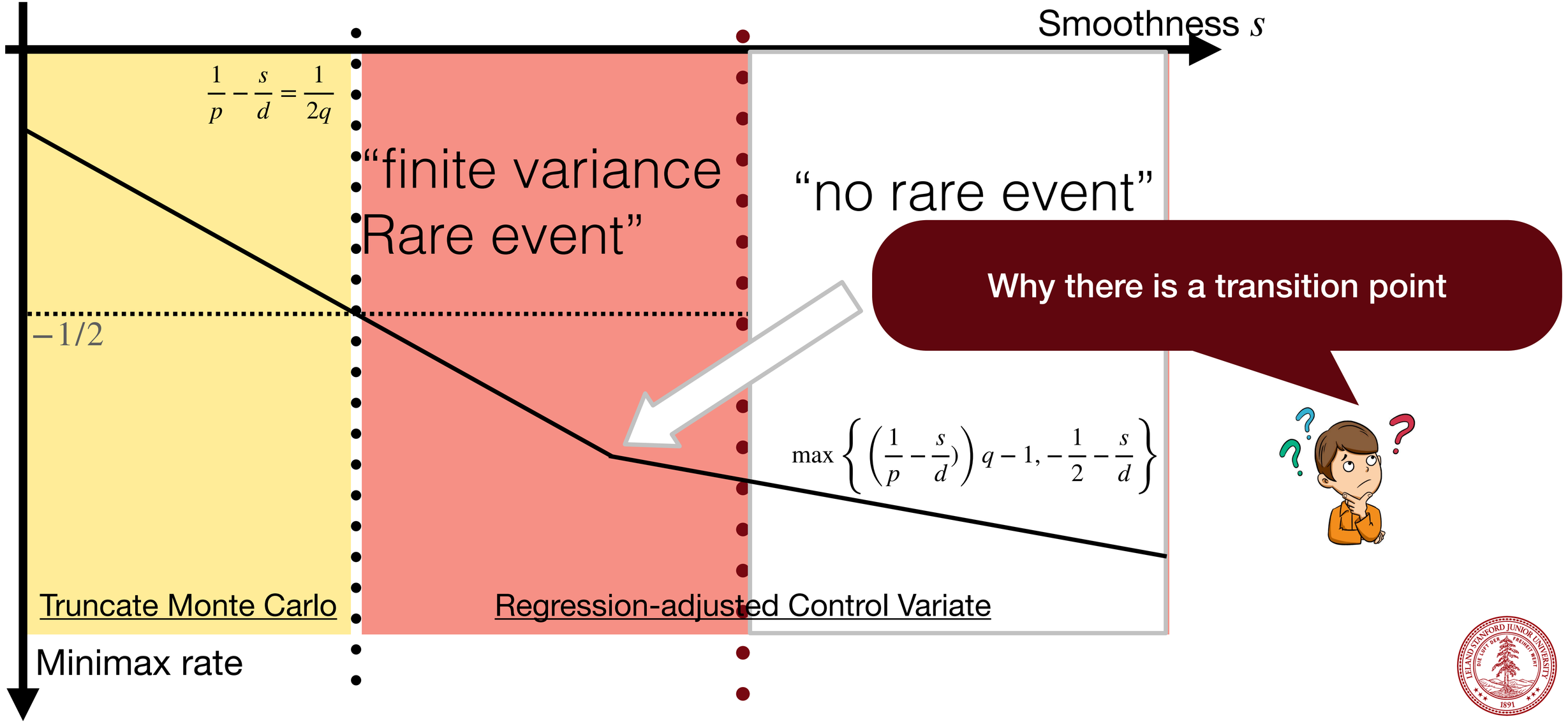

Semi-parametric efficiency…

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"influnce function" (gradient)

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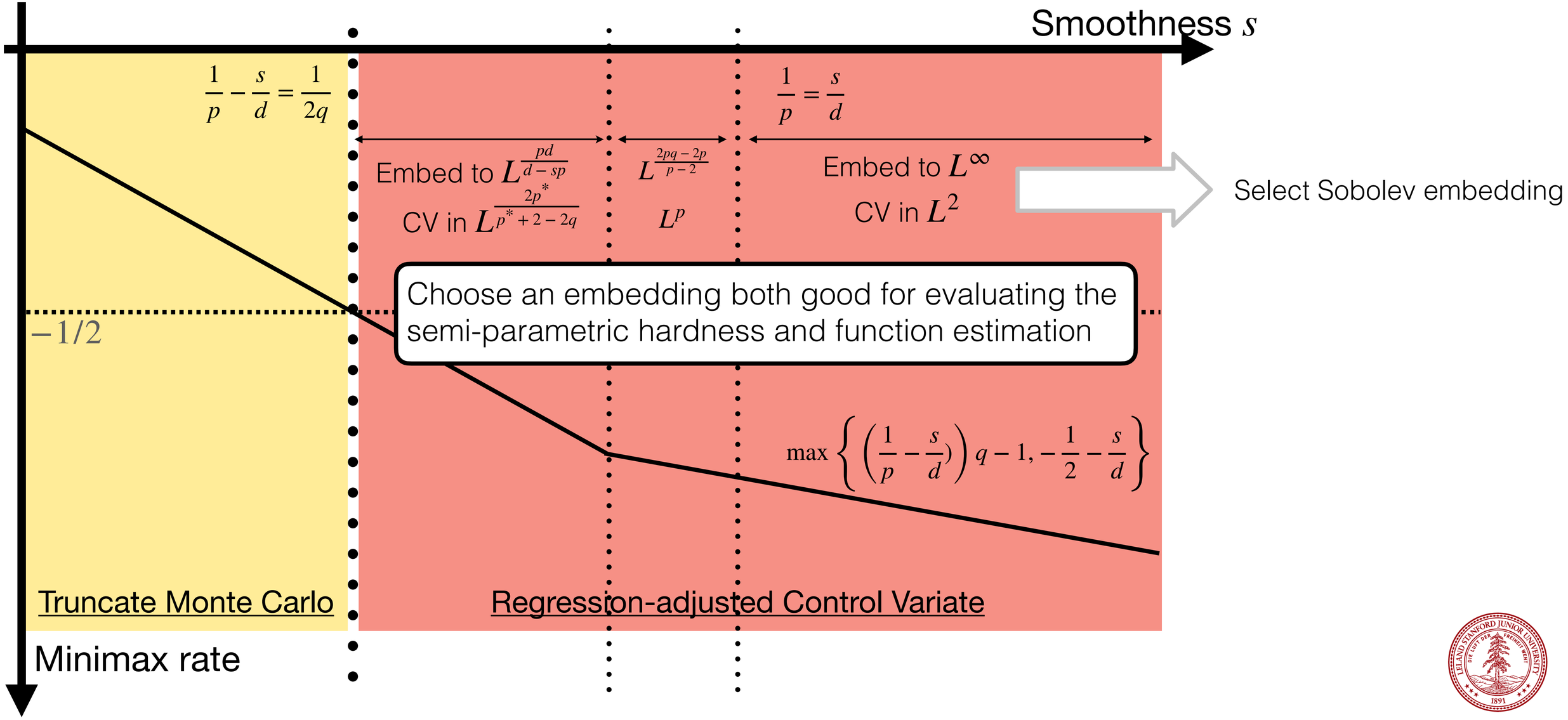
"influnce function" (gradient)

Embed f^{q-1} and $f - \hat{f}$ into "dual" space

How to select the sobolev emebedding

Tricky part of the Proof:select embedding



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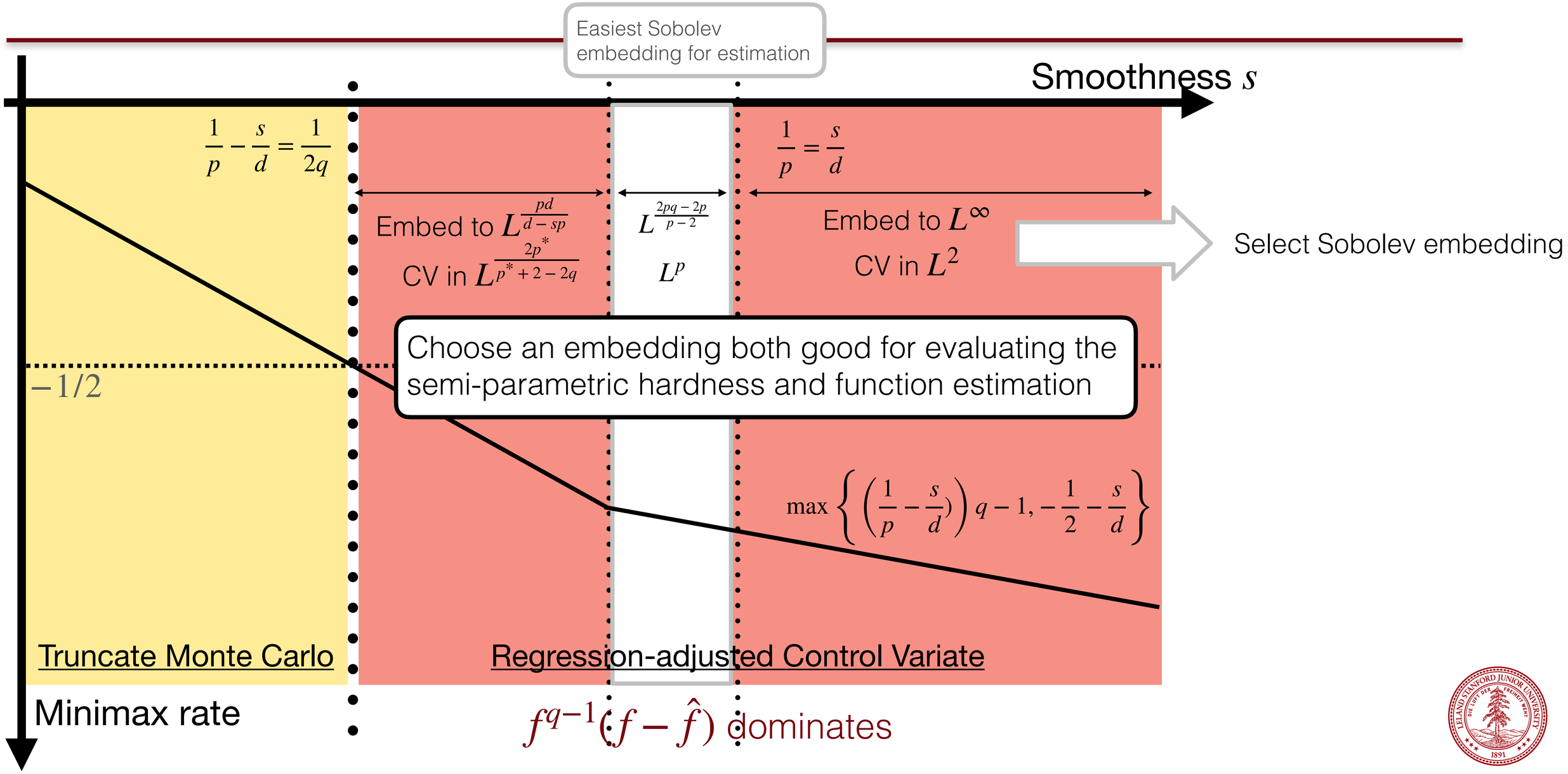

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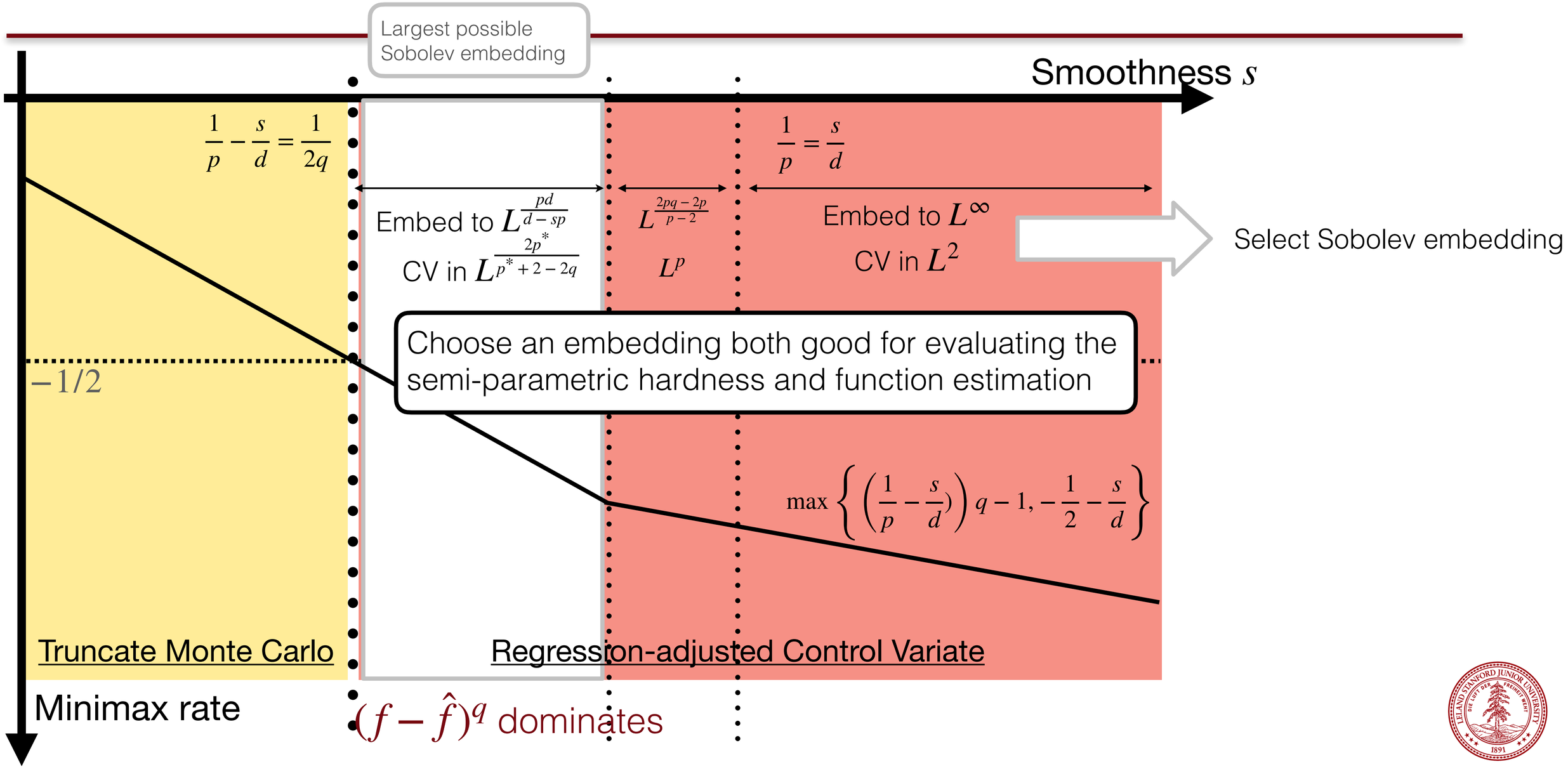
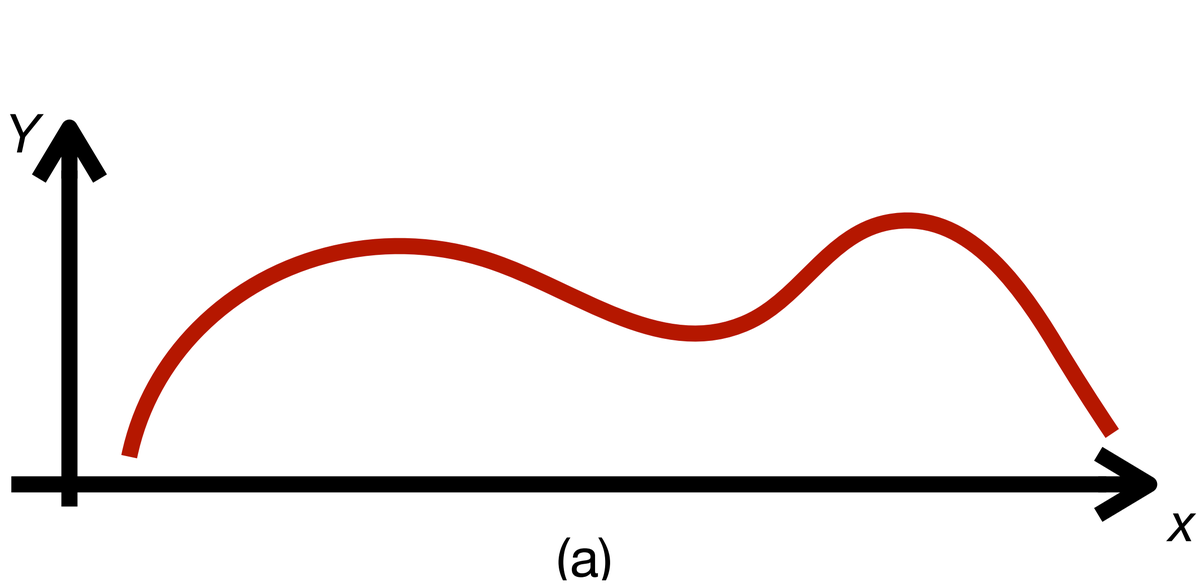
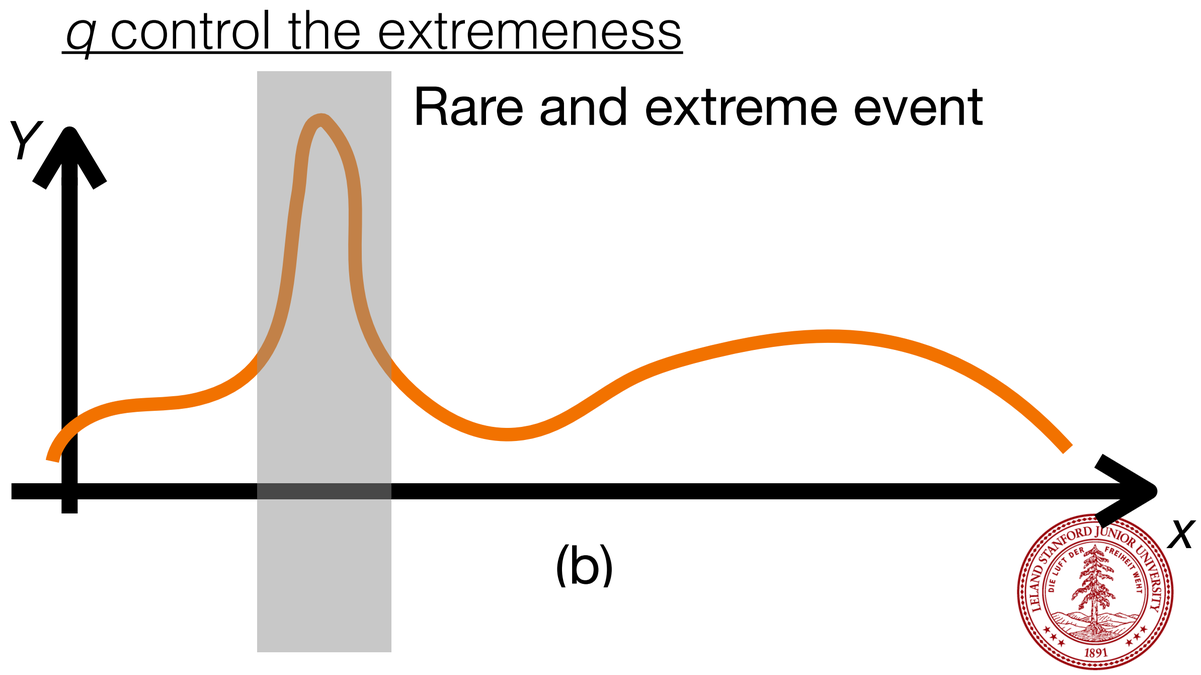


Take home message

a) Statistical optimal regression is the optimal control variate
b) It helps only if there isn't a hard to simulate (infinite variance) Rare and extreme event





Contact: yplu@stanford.edu