lab2

2025-09-15

```
install.packages("tinytex")
tinytex::install_tinytex()
```

##Q1

(a) $\mu = E[X]$

Answer:

$$\mu = E(X) = \int_a^b \frac{x}{b-a}\,dx = \frac{x^2}{2(b-a)}\Big|_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

(b) $E[X^2]$

Answer:

$$E[X^2] = \int_a^b \frac{x^2}{b-a}\,dx = \frac{x^3}{3(b-a)}\Big|_a^b = \frac{b^3-a^3}{3(b-a)} = \frac{a^2+ab+b^2}{3}$$

(c) $E[X^3]$

Answer:

$$E[X^3] = \int_a^b \frac{x^3}{b-a}\,dx = \frac{x^4}{4(b-a)}\Big|_a^b = \frac{b^4-a^4}{4(b-a)} = \frac{a^3+a^2b+ab^2+b^3}{4}$$

(d) $E[(X-\mu)^3]$

Answer:

$$E[(X-\mu)^3] = E[X^3 - 3\mu X^2 + 3\mu^2 X - \mu^3] = E[X^3] - 3\mu E[X^2] + 3\mu^2 E[X] - \mu^3$$

Since

$$\mu = \frac{a+b}{2},$$

we have

$$E[(X-\mu)^3] = \frac{a^3+a^2b+ab^2+b^3}{4} - 3\cdot\frac{a+b}{2}\cdot\frac{a^2+ab+b^2}{3} + 3\cdot\frac{(a+b)^2}{4}\cdot\frac{a+b}{2} - \frac{(a+b)^3}{8} = 0$$

(e) $E[(X-\mu)^4]$

Answer:

$$E[(X-\mu)^4] = E[X^4] - 4\mu E[X^3] + 6\mu^2 E[X^2] - 4\mu^3 E[X] + \mu^4$$

Apply

$$\mu = \frac{a+b}{2}$$

to the equation, we can get:

$$E[(X-\mu)^4] = \frac{(b-a)^4}{80}$$

##Q2

```
set.seed(123)
X <- runif(100000, 0, 1)

sample_mean <- mean(X)
theoretical_mean <- 0.5

sample_mean_X3 <- mean(X^3)
theoretical_mean_X3 <- 1/4

sample_central3 <- mean((X - mean(X))^3)
theoretical_central3 <- 0

cat("(a) mean(X):", sample_mean, " (theoretical =", theoretical_mean, ")\n")
cat("(b) mean(X^3):", sample_mean_X3, " (theoretical =", theoretical_mean_X3, ")\n")
cat("(c) mean((X-mean(X))^3):", sample_central3, " (theoretical =", theoretical_central3, ")\n")
```

I got the mean of X as 0.4993, which is close to the theoretical value of 0.5. I got the mean of X3 as 0.2492, which is close to the theoretical value of 0.25. I got the mean of (X − mean(X)) 3 as 0.00014, which is close to the true mean of 0 (According to Question 1 (d), we know the theoretical value is 0.

##Q3

```
set.seed(123)

binsim <- rbinom(n = 10000, size = 20, prob = 0.3)
```

```
p_le_5 <- sum(binsim <= 5) length(binsim)
print(paste("Estimated P(X <= 5):", p_le_5))
```

[a] "Estimated P(X <= 5): 0.4159"

```
p_eq_5 <- sum(binsim == 5)  length(binsim)
print(paste("Estimated P(X = 5):", p_eq_5))
```

[b] "Estimated P(X = 5): 0.1802"

```
estimated_mean <- mean(binsim)
print(paste("Estimated E[X]:", estimated_mean))
```

[c] “Estimated E[X]: 5.9854”

```
estimated_variance <- var(binsim)
print(paste("Estimated Var(X):", estimated_variance))
```

[d] “Estimated Var(X): 4.10859769976998”

##Q4

```
set.seed(42)

poisson_data <- rpois(n = 10000, lambda = 7.2)
estimated_mean <- mean(poisson_data)

estimated_variance <- var(poisson_data)

print(paste("Estimated Mean:", estimated_mean))
print(paste("Estimated Variance:", estimated_variance))

theoretical_mean <- 7.2
theoretical_variance <- 7.2

print(paste("Theoretical Mean:", theoretical_mean))
print(paste("Theoretical Variance:", theoretical_variance))
```

The mean and variance of the simulated data are 7.1893 and 7.31579708970897. They are close to the theoretical values (both mean and variance are 7.2).

##Q5

```
set.seed(123)
P1 <- rpois(10000, 5)
P2 <- rpois(10000, 25)
P3 <- rpois(10000, 125)
P4 <- rpois(10000, 625)
est_fuc <- function(x){
mean_x <- mean(x)
mean_sqrt_x <- mean(sqrt(x))
var_sqrt_x <- var(sqrt(x))
var_x <- var(x)
return(list(mean_x, mean_sqrt_x, var_sqrt_x, var_x))
}
print(est_fuc(P1))
print(est_fuc(P2))
print(est_fuc(P3))
print(est_fuc(P4))
```

λ	E[X]	E[√X]	Var(√X)	Var(X)
5	4.9746	2.166261	0.2819427	4.896444
25	24.9764	4.972036	0.2552792	25.09295

λ	E[X]	E[√X]	Var(√X)	Var(X)
125	125.0207	11.17022	0.2469648	123.2074
625	625.0354	24.99567	0.2518657	629.3937

b.

A properly tuned multiplicative congruential generator cannot produce a sequence of truly random numbers.The generator is considered a pseudorandom number generator because it operates on a deterministic, predictable mathematical formula. This means that given the same initial seed value, the generator will always produce the exact same, repeatable sequence of numbers. As the generation process is fully deterministic, the numbers cannot be truly random.