$$
\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}
$$
$$
= A \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}
$$

$$
\begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \frac{1}{r}\cos\theta\cos\phi & \frac{1}{r}\cos\theta\sin\phi & -\frac{1}{r}\sin\theta \\ -\frac{1}{r}\frac{\sin\phi}{\sin\theta} & \frac{1}{r}\frac{\cos\phi}{\sin\theta} & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}
$$
$$
= A^{-1} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}
$$

$$
\begin{pmatrix} \partial x \\ \partial y \\ \partial z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \frac{1}{r}\cos\theta\cos\phi & -\frac{1}{r}\frac{\sin\phi}{\sin\theta} \\ \sin\theta\sin\phi & \frac{1}{r}\cos\theta\sin\phi & \frac{1}{r}\frac{\cos\phi}{\sin\theta} \\ \cos\theta & -\frac{1}{r}\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \partial r \\ \partial\theta \\ \partial\phi \end{pmatrix}
$$
$$
= (A^{-1})^{\mathrm{T}} \begin{pmatrix} \partial r \\ \partial\theta \\ \partial\phi \end{pmatrix}
$$