

$$\begin{aligned}
\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} &= \begin{pmatrix} \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi \\ \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi \\ \cos \theta & -r \sin \theta & 0 \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix} \\
&= A \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix} &= \begin{pmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \frac{1}{r} \cos \theta \cos \phi & \frac{1}{r} \cos \theta \sin \phi & -\frac{1}{r} \sin \theta \\ -\frac{1}{r} \frac{\sin \phi}{\sin \theta} & \frac{1}{r} \frac{\cos \phi}{\sin \theta} & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \\
&= A^{-1} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} \partial x \\ \partial y \\ \partial z \end{pmatrix} &= \begin{pmatrix} \sin \theta \cos \phi & \frac{1}{r} \cos \theta \cos \phi & -\frac{1}{r} \frac{\sin \phi}{\sin \theta} \\ \sin \theta \sin \phi & \frac{1}{r} \cos \theta \sin \phi & \frac{1}{r} \frac{\cos \phi}{\sin \theta} \\ \cos \theta & -\frac{1}{r} \sin \theta & 0 \end{pmatrix} \begin{pmatrix} \partial r \\ \partial \theta \\ \partial \phi \end{pmatrix} \\
&= (A^{-1})^T \begin{pmatrix} \partial r \\ \partial \theta \\ \partial \phi \end{pmatrix}
\end{aligned}$$