数值分析

复合梯形公式推导

$$\text{梯形公式：} T = \frac{(b-a)}{2} * (f(a) + f(b)) \tag{1}$$

$$\text{求：} In = \int_a^b f(x)dx$$

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设在区间[a,b]上划分n等分，xk=a+kh,h=(b-a)/n,  k=0,1,2,3,4,...n;n在每个区间[xk,xk+1]上使用梯形公式得：
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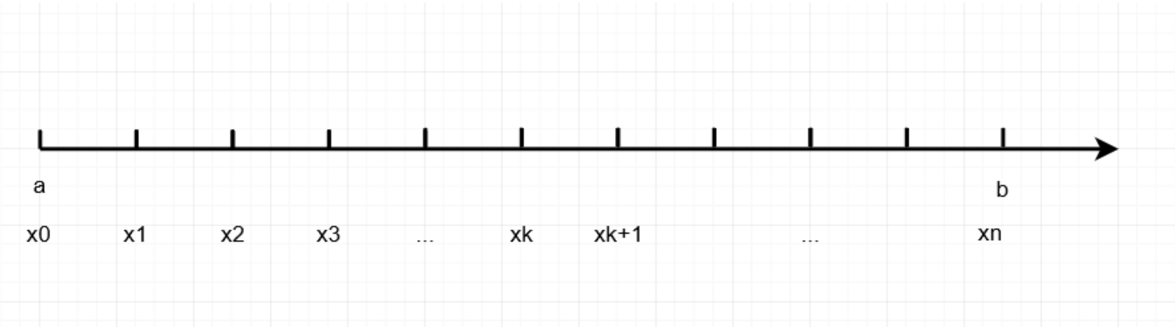


$$In = \int_a^b f(x)dx = \sum_{i=0}^{n-1} \int_{x_k}^{x_k+1} f(x)dx; \tag{2}$$

对子区间上用梯形公式：

$$In = \sum_{i=0}^{n-1} \frac{x_{k+1} - x_k}{2} [f(x_{k+1}) + f(x_k)]$$

因为：

$$h = x_{k+1} - x_k; \text{所以：} In = \frac{h}{2} \sum_{i=0}^{n-1} [f(x_{k+1}) + f(x_k)];$$

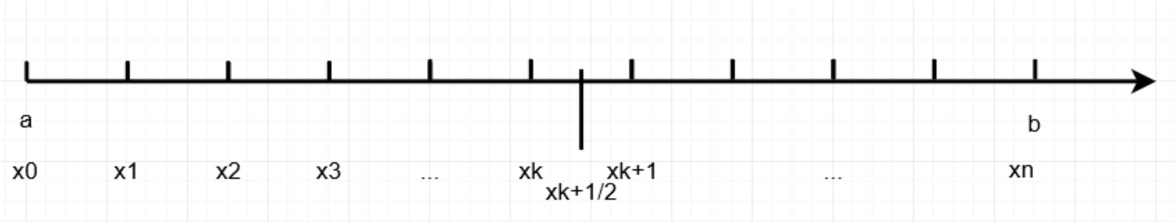
$$In = \frac{h}{2} \{f(x_0) + f(x_1) + f(x_1) + f(x_2) + f(x_2) + f(x_3) + \ldots + f(x_{n-1}) + f(x_n)\}$$

$$x_0 = a, x_n = b; In = \frac{n}{2} [f(a) + f(b) + 2 \sum_{i=1}^{n-1} f(x_k)]$$

$$\text{所以：} Tn = \frac{h}{2} [f(a) + f(b) + 2 \sum_{i=1}^{n-1} f(x_k)] \quad \text{证毕} \tag{3}$$

复合辛普森公式推导

$$\text{辛普森公式：} S = \frac{b-a}{6} [f(a) + 4f(\frac{a+b}{2}) + f(b)] \tag{1}$$



区间划分同梯形公式类似，这里[xk,xk+1]的中点为xk+1/2=xk+(1/2)h;

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\begin{aligned}
S &= \int_a^b f(x)dx = \sum_{i=0}^{n-1}\int_{x_k}^{x_{k+1}} f(x)dx; \qquad (2)\\
&= \sum_{i=0}^{n-1}(\frac{x_{k+1}-x_k}{6})[f(x_k)+4f(\frac{x_{k+1}+x_k}{2})+f(x_{k+1})]\\
&= \sum_{i=0}^{n-1}(\frac{h}{6})[f(x_k)+4f(\frac{x_{k+1}+x_k}{2})+f(x_{k+1})]; \quad \text{因为}: h=(x_{k+1}-x_k)\\
&= \frac{h}{6}[f(x_0)+f(x_1)+f(x_1)+f(x_2)+f(x_2)+f(x_3)+\ldots+f(x_{n-1})+f(x_n)+\sum_{i=0}^{n-1}4f(\frac{x_{k+1}+x_k}{2})]\\
&= \frac{h}{6}[f(a)+f(b)+\sum_{i=0}^{n-1}4f(\frac{x_{k+1}+x_k}{2})+2\sum_{i=1}^{n-1}f(x_k)]\\
&= \frac{h}{6}[f(a)+f(b)+\sum_{i=0}^{n-1}4f(x_{k+\frac{1}{2}})+2\sum_{i=1}^{n-1}f(x_k)]; \quad \text{因为}: x_{k+\frac{1}{2}} = \frac{x_{k+1}+x_k}{2}\text{；证毕}
\end{aligned}
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龙贝格公式推导

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\begin{aligned}
&\text{同梯形公式：将}[x_k,x_{k+1}]\text{中间插入一个二分点}x_{k+\frac{1}{2}} = \frac{1}{2}(x_k+x_{k+1})\\
&\quad\text{则每个子区间用梯形公式求得的积分为：}(\text{步长：}h=\frac{x_k+x_{k+1}}{2})\\
&\qquad\text{所以}:T = \frac{h}{4}[f(x_k)+2f(x_{k+\frac{1}{2}})+f(x_{k+1})]\\
&\qquad T_{2n} = \sum_{i=0}^{n-1}\frac{h}{4}[f(x_k)+2f(x_{k+\frac{1}{2}})+f(x_{k+1})]\\
&\qquad = \frac{h}{4}\sum_{i=0}^{n-1}[f(x_k)+2f(x_{k+\frac{1}{2}})+f(x_{k+1})]\\
&\qquad = \frac{h}{4}\sum_{i=0}^{n-1}[f(x_k)+f(x_{k+1})]+\frac{h}{2}\sum_{i=0}^{n-1}f(x_{k+\frac{1}{2}})\\
&\qquad = \frac{1}{2}T_n+\frac{h}{2}\sum_{i=0}^{n-1}f(x_{k+\frac{1}{2}} \qquad (3)\\
&(\text{注：}T_n\text{表示复合梯形公式})
\end{aligned}
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