Abstract

A heat exchanger is a thermal system that facilitates processes such as heating and cooling, playing a crucial role in various industrial and scientific applications. The pursuit of sharp bounds on heat transfer efficiency remains an important and widely studied topic. In this study, we investigate the lower bound of heat transport in an infinite horizontal strip containing an incompressible fluid, with energy-constrained cooling as the chosen framework. The system is further characterized by mixed boundary conditions.

We begin by defining the steady heat equation within the domain and specifying the corresponding boundary conditions. Subsequently, we introduce the original optimal cooling problem and reformulate it as a minimax problem. To address this reformulation, we employ Fourier expansion and select Chebyshev functions as the basis functions. To cast the problem into a conic programming framework, we impose semidefinite programming (SDP) conditions and incorporate rotated Lorenz cones as constraints. The resulting conic optimization problem is then formulated using the MATLAB toolbox YALMIP and solved with the conic solver MOSEK.

We establish a lower bound and derive an almost matching scaling law of $O(Pe^{-1.46})$ for small Péclet numbers, up to ($Pe = 100$).

Key words: Steady heat equation, minimax problem, conic program