Chapter 6: Solutions to Math and Logic Puzzles

Ruoying Jia (ruoyingj)

6.1 Solution:

Order the bottles with labels from 1 to n. The i pills from bottle i. Scale the whole weight of all the pills as W. Then the number of bottle is (W – 210) / 0.1.

6.2 Solution:

P(game 1) = p

P(game 2) = $C_3^2 p^2(1-p) + p^3$ = $3p^2$-$2p^3$

If we want to win in game 1, then p(game 1) > p(game 2)

p > $3p^2$-$2p^3$ $\Longrightarrow$ 0 < p < 0.5

Thus, we will win in game 1 if 0 < p < 0.5. We will win in game 2 if 0.5 < p < 1.

6.3 Solution:

It is impossible to use 3 dominos to cover the entire board. Let' consider the first row, it will have 6 dominos and one dominos one square in the first row and another square is in the second row. The Same situation will happen in the second row. There are 3 dominos and one which is across two rows. Thus, no matter how many rows we have, it always need a dominos which takes up two squares from the different rows. It is impossible for us to fit all the chessboard with 31 dominos.

6.4 Solution:

The situation that ants will not collide since they will walk in the same direction. There are two directions that are clockwise and counter clockwise.

P(not collision) = 2* $(\frac{1}{2})^3$ = $(\frac{1}{2})^2$

P(collision) = 1 - $(\frac{1}{2})^2$ = $\frac{3}{4}$

If n ants are on an n-vertex polygon:

P(collision) = 1 - $(\frac{1}{2})^{n-1}$

6.5 Solution:

First we fill up 5 quart jug.

Second we fill 3 quart jug with 5 quart. Then 5 quart remains 2 quart.

Make the 3 quart jug empty and fill 5 quart jug with 3 quart jug. There are 2 quart in 3 quart jug.

Fill 5 quart jug and fill 3 quart jug with 5 quart jug. Then 3 quart jug contains 3 quart jug and 5 quart jug contains 4 quart.

We can obtain 4 quart from 5 quart jug.

6.6 Solution:

When there is only one man with blue eyes, then he will find there is no people with blue eyes and he will leave at the first evening.

When there are two men with blue eyes, since they could only observe there are only one man with blue eyes. They do not know whether there are two men with blue eyes or only one man. So if there is no man leaving in the first evening. It will be two men with blue eyes and they will leave in the second evening.

When there are more than two men with blue eyes. Suppose there are three men, they they will observe whether there are two man or three man. If there are no two men leave in the second evening. Both of three will leave in the third evening.

Thus, we can conclude that if there are n men with blue eyes, then there will be n men leave in the nth evening.

6.7 Solution:

Since each family must have one girl, there will be n boys and one girl in each family. Let p represent the probability of having boys.

First, when there is only one girl, p = 0.

Second, when there are one boy and one girl, p = $\frac{1}{4}$.

Third, when there are two boys and one girl, p = $\frac{2}{4}$.

Fourth, when there are three boys and one girl, p = $\frac{3}{16}$.

Let p'represent the total probability of having boys,

p' = $\frac{1}{4} + \frac{2}{4} + \frac{3}{16} + \frac{4}{32} + \frac{5}{64} + \cdots$. It approaches to 1. Thus the probability of having boys is near to 1. They have the same ratio to have a girl and have a boy.

6.8 Solution:

Since we should minimize the number of drops for the worst case, if the number of egg 1 drops increase, we should decrease the number of egg 2 drops.

Suppose the interval of egg 1 drops decrease by 1 each time.

Suppose the start of floor is x. Then x + (x - 1) + (x - 2) + (x - 3) + ⋯ + 1 = 100, we will calculate x = 13 or 14. If x = 13, it will end up at 91 floor. The total step is 21.

If x = 14, it will end up at 99 floor. The total step is 14.

Thus we should pick 14 as the start point for egg 1.

6.9. Solution:

Since the locker is toggled when the time is the factor of its index. The initial situation is open. Then after pass 100 times, only if the toggled times is odd which can make it open. The only way to make toggled time odd is that the index has the number of factors is odd. So the square numbers have odd number of factors. For example 1, 4, 9, 16 , 25, 36, 49, 64, 81, 100.

6.10 Solution:

The minimal day to know the result is seven.

We can use the binary sequence to represent the number of the bottles. Since there are ten strips, it can represent 2^{10} > 1000 bottles. If one bit of a bottle’s binary representation is 1, we drop on the ith strip. After 7 days, we will find which strips have positive results. And we gather index of strips together and can find out which bottle is poisoned.