JUST A FEW SEEDS MORE: VALUE OF NETWORK INFORMATION FOR DIFFUSION

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INTRODUCTION

- **Diffusion Process:An agent is either informed or uninformed. Once an agent becomes informed, it remains informed forever after. The diffusion process considered here is one in which communication is undirected.**

- **Omniscient:** $\mathbf{H}(\mathrm{OMN}, s) \geq \mathbf{H}(\mathrm{OPT}, s) \geq \mathbf{H}(\mathrm{RAND}, s)$**, ' at least as well as the optimum'**

- **Fixing the number of seeds available to the omniscient seeding strategy, how many additional seeds are required in order for random seeding to perform as well as the omniscient?**

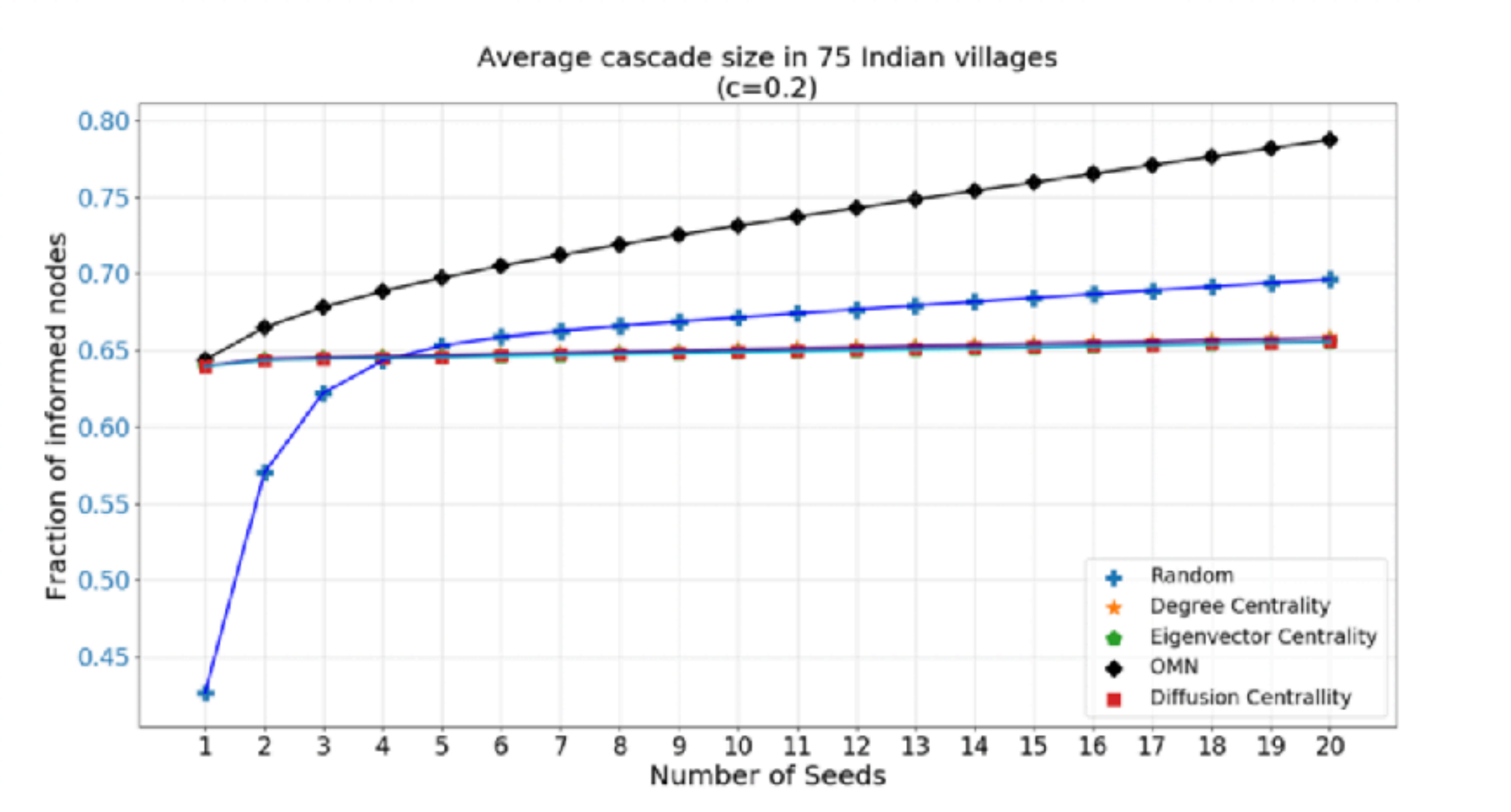
MODEL

- **GOAL:**
  - **max** $H(f,s) = \frac{1}{n}\mathbb{E}_{G\sim\mathbb{P}_n}[\mathbf{h}(G,s,f)]$**, where** $\mathbf{h}(G,s,f) = \mathbb{E}\left[\left|A_T(G,s,f)\right|\right]$**, G: network, s: number of seeds, f: the seeding strategy, n: size of the network.**

- **Inhomogeneous Random Networks (IRN): there is a set of potential "types" and each agent has a specific type :** $\mathrm{T}_\kappa = \left[\kappa_{ij}\right]_{i,j\in[n]}$**,** $\left\|\ \mathrm{T}_\kappa\ \right\|$ **is the type matrix.**
  - **simple Erdős-Rényi graphs : any pair of nodes is connected with the same probability**
  - **networks with homophily: nodes are more intensely connected to nodes with "similar" types**
  - **networks with power-law degree distribution : some individuals are connected to a large fraction of the population**

MAIN THEOREM

- **Idea: Under a set of conditions, the difference in expected fraction of informed individuals between the random seeding strategy with s + x seeds and the omniscient strategy with s seeds vanishes exponentially in x.**
  - **Theorem 1. Consider a sequence of** $\mathrm{IRN}_n(\boldsymbol{p}(\kappa))$ **. Let** s **be the number of seeds,** $\alpha = \lim_{n\to\infty} \mathbf{H}(\mathrm{OMN}, 1)$**. Then,**
  - **if** $\left\| \mathbf{T}_\kappa \right\| > 1/c$**, random seeding catches up to the omniscient seeding at an exponential rate in the number of extra seeds, i.e., α > 0 and for any x,** $\lim_{n\to\infty} \frac{\mathbf{H}(\mathrm{RAND}, s+x)}{\mathbf{H}(\mathrm{OMN}, s)} = 1 - (1-\alpha)^{s+x}$**,**
  - **if** $\left\| \mathbf{T}_\kappa \right\| < 1/c$**, then any seeding strategy diffuses to only a vanishing fraction of the population:** $\lim_{n\to\infty} \mathbf{H}(\mathrm{OMN}, s) = 0$
- **Result:**



ROBUSTNESS AND LIMITATIONS

- **Variance of Random Seeding**
  - **Variance of Random Seeding:** $\lim_{n\to\infty} \mathrm{Var}(\mathbf{H}(\mathrm{RAND}, s)) \leq \alpha^2(1-\alpha)^s\left(1-(1-\alpha)^s\right)$ **for a sequence of** $\mathrm{IRN}_n(\boldsymbol{p}(\kappa))$ **models.**
  - **Speed of Diffusion: depends on the network structure, in general, random seeding is slower.**
- **Other Diffusion Models**
  - **Directed Communication** ✅
  - **Models with threshold** ❌