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Statement of Purpose
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First and second years of BSMS

When I came to the IISER Mohali campus after completing the first semester of my BSMS online, I had some idea what groups and vector spaces were. The first semester course on mathematics called "symmetry" ended on the definition of symmetry of a subset of $\mathbb{R}^n$ as the group of rotations, reflections and translation that keep it stable. However, I had heard the term *symplectic manifold*, eluding to the subject of geometric mechanics, or more generally topological dynamics on manifolds. Of course the definition of *manifolds*, *Riemannian* or *symplectic manifolds* did not make any sense at the time. It was at this time I opened a book on geometric mechanics in the library at IISER Mohali saying *torque free motion of a rigid body is the geodesic flow (of a left invariant metric) on* $SO(3)$.

The second semester course was on analysis on $\mathbb{R}$, and throughout the course I found myself relearning all the intuitive notions about *functions* I had after learning calculus. The subjects of linear algebra and real analysis seemed to collide in the definition of *total derivative* of a multivariable function. I was also trying to learn about *differential forms* around this time, having learnt it was a "proper" was to integrate on higher dimensional spaces.

Next, we had a course on Gauss's geometry of curves and surfaces. We defined curvature of curves and surfaces in $\mathbb{R}^3$ and I found myself loving to learn and teach differential geometry to my peers! I truly understood the notion of *total derivative* as a *map between tangent spaces that sends velocities to velocities* clearly in the setup of smooth map between surfaces.

After a semester on surfaces, I found the definition of manifolds and smooth maps between them quite simple! Me and and a friend were looking at courses in mathematics we can audit for fun. Given we knew the definition of topological spaces and continuous maps between them, I suggested we look at what happens in the course on *algebraic topology*. After a few classes of looking at homology and homotopy groups and some “ugly” topological spaces, I started to really appreciate the definition of manifolds.

That summer, I luckily became a part of reading group where we proved the de Rham isomorphism using sheaf/Cech cohomology theory from Griffiths and Harris, Bott and Tu, respectively. The worlds of *differential forms* and *algebraic topology* collided in this wonderful isomorphism. Even though I was young in doing mathematics, with practice and some blackboxes I had gained some skill in chasing a diagram of vector spaces and a background on the theory of smooth manifolds.

Third and forth years of BSMS

After this, I began to understand the structure of vector fields and their flows, which are fundamental to the theory of Lie groups, the representation theory of Lie groups, and symplectic geometry. I could now understand what *geodesic flow on* $SO(3)$ meant that I heard about more than a year ago! This is the point where I had my first course on complex analysis. Thus, I could appreciate the topological, homological and Lie theoretic aspects of the first course on complex analysis. Among other things, I discovered the motivation behind sheaf cohomology by the Mitag-Leffler problem on Riemann surfaces (from Griffiths and Harris, parts of which I already read for sheaf theoretic de Rham isomorphism, but somehow missed this section!).

The course ended with the construction of Etale space to define global holomorphic functions. This led me to discover the “Riemann surface of a (germ of a) holomorphic function”. After a back and forth with this definition for a year, I started to understand what the topology on the Etale space is *doing*.

Retrospection on my training

Over the course of my training, I have explored diverse areas of mathematics, including differential geometry, dynamical systems, symplectic geometry, complex geometry, representation theory, Lie groups, ergodic theory and mathematical physics. I have had the opportunity to credit courses on knot theory, Riemannian geometry, elliptic curves, Fourier analysis in my major years at IISER Mohali. Still, I believe the course I benefitted from the most was the one on curves and surfaces in third semester, which led to my interest in the theory of manifolds and algebraic topology quite early.

Engaging in above mentioned subjects led to me inculcate a diverse set of theoretical and problem solving skills, which firstly lets me engage in different conversations quickly and furthermore benefits me from having fruitful conversations with my peers in the mathematics community. I have had fun giving talks for the mathematics club, courses, graduate seminar etc.

Fifth year of BSMS (masters thesis)

After crediting the course on Galois theory and with the prior knowledge of the theory of covering spaces, I started to appreciate the functorial correspondence of topological theory of ramified coverings of compact Riemann surfaces and Galois theory of field extensions of $\mathbb{C}(z)$. I wish to study classification/moduli spaces of such ramified coverings for a fixed ramification profile.

On the other side, after crediting the course on Riemannian geometry (thus looking at the hyperbolic space), I came to know about the Teichmuller and moduli space of these structures on topological surfaces. Further, learning about the history of elliptic and Abelian integrals, period of elliptic curves, leading to the Jacobian of a Riemann surface, and the theorem of Torelli gave me another map from the moduli space of Riemann surfaces to *some* space of lattices in $\mathbb{C}^g$. This is another result I wish to prove someday.

Latter in the summer, I attended a workshop on the *rigidity of discrete groups* held in IISER Mohali. I was fascinated by the theorems on rigidity of hyperbolic 3-manifolds and some complex manifolds, having seen the theory of *deformations* in dimension 2. Eventually, I decided to do my masters thesis on topics related to the strong rigidity of locally symmetric spaces: boundary of (globally) symmetric spaces and application of ergodic theory in this area.

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