A000172(014)

B. Tech. (Hon's) (First Semester) Examination

Nov.-Dec. 2023

(AICTE Scheme)

ENGINEERING MATHEMATICS-I

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : *Each question contains four parts. Part (a) of each question is compulsory. Attempt any* ***two*** *parts from (b), (c) and (d) of each question. The figure in the right-hand margin indicates marks.*

Unit-I

1. (a) Evaluate $\lim_{n \to \infty} \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right\}^{1/n}$. 4

(b) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 8

(c) If $y^{1/m} + y^{-1/m} = 2x$, prove that

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0 \quad 8$$

(d) State Lagrange's mean value theorem and verify for the function

$$f(x) = \sqrt{x^2-4} \text{ in the interval } [2, 4]. \quad 8$$

Unit-II

2. (a) Verify Euler's theorem for the function

$$z = \frac{x^{1/3} - y^{1/3}}{x^{1/2} + y^{1/2}} \quad 4$$

(b) (i) Evaluate the integral $\int_0^\infty \int_0^x x \exp\left(-\frac{x^2}{y}\right) dx\, dy$ by changing the order of 8

(ii) Change the order of integration $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} v\, dx\, dy$.

(c) Find the extreme value of the function

$$f(x) = \sin x \sin y \sin(x+y). \quad 8$$

(d) Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $x+y+z=a$. 8

Unit-III

3. (a) State Green's Theorem for a plane and hence evaluate : 4

$\oint_C (x^5+3y)\,dx + \left(2x - e^{y^3}\right)dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

6×8 = 48

6×8

48+8

56

(b) State Gauss's divergence theorem and hence evaluate

$\iint_S \left(y^2z^2\hat{i} + z^2x^2\hat{j} + z^2y^2\hat{k}\right).\hat{n}\,dS$, where S is the part of the sphere $x^2+y^2+z^2=1$ above the xy plane and bounded by xy-plane. 8

(c) Verify Stoke's theorem for a vector field defined by $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ in the rectangular in the xy-plane bounded by line $x = \pm a, y = 0, y = b$. 8

(d) Evaluate $\operatorname{div}\left(r^n\vec{r}\right)$ and find the value of n, if $r^n\vec{r}$ is solenoidal. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at (1, −1, 2) in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$. 8

Unit-IV

4. (a) Show that the function $u = \cos x \cosh y$ is harmonic and find its harmonic conjugate. 4

(b) Prove that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} (z \neq 0)$$

$f(0) = 0$ is continuous and that Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist. 8

(c) If $u + v = \dfrac{2\sin(2x)}{e^{2y} + e^{-2y} - 2\cos(2x)}$ and

$f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in the terms of z. 8

(d) Sate and prove necessary condition for a function to be analytic. 8

Unit-V

5. (a) Using Fourier Series, Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos(nx)}{n^2}; -\pi < x < \pi.$$

Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}$. 4

(b) Find Fourier series of $f(x)$ in range $-\pi < x < \pi$, where

$$f(x) = \frac{\pi}{2\sinh \pi} e^x$$

And prove that sum of series $= \frac{\pi}{2} \coth \pi$. 8

(c) An alternating current after passing through a rectifier has the form

$$I(x)=\begin{cases} I_0 \sin x & , \quad 0 \le x \le \pi \\ 0 & , \quad \pi \le x \le 2\pi \end{cases}$$

Where I_0 is the maximum current and the period is 2π, express $I(x)$ as a Fourier series and evaluate :

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots\infty=?$$ 8

(d) Find sine series of $f(x)$ in half range, where

$$f(x)=\begin{cases} x & , \quad 0 \le x \le \frac{\pi}{2} \\ \pi - x & , \quad \frac{\pi}{2} \le x \le \pi \end{cases}$$

And prove that $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots$ 8