The Inattentive Consumer: Sentiment and Expectations

Rupal Kamdar*

U.C. Berkeley
Department of Economics

Preliminary
Click here for most recent version

September 22, 2018

Abstract

In macroeconomic models, expectations play a crucial role and are commonly assumed to be full-information rational. The reality, however, is information is costly to obtain. I show consumer survey-based beliefs about economic variables are driven by a single component: sentiment. "Optimistic" consumers expect typical expansionary outcomes but inflation to *decline*. I develop a model of a rationally inattentive consumer who faces fundamental uncertainty and information costs. Rather than learning about the fundamentals independently, the consumer optimally chooses signals that are linear combinations of them. The consumer decides to be best informed along the dimension most costly to misunderstand. The signal in this dimension can be interpreted as rationally obtained sentiment. Moreover, the covariances of the posterior beliefs differ from the covariances of the underlying data-generating process. In particular, I show that optimal signal choice leads to countercyclical price beliefs.

Keywords: Expectations, Rational Inattention, Surveys
JEL Codes: E31, E32, E7

*Email: rupal.kamdar@econ.berkeley.edu. I am grateful to Yuriy Gorodnichenko for invaluable advising and continual support on this project. I thank Pierre-Olivier Gourinchas, Raymond Hawkins, Amir Kermani, Filip Matějka, Byoungchan Lee, Walker Ray, and Mauricio Ulate for excellent comments. All errors are my own.

1 Introduction

All economic decisions are based on agents' perceptions about the current economy and/or expectations about future economic outcomes. The workhorse approach to modeling perceptions and expectations has been full-information rational expectations (FIRE). However, this assumption is heroic in that it needs the agent to understand the model and know all available information, past and present. A growing literature has shown that survey-based measures of perceptions and expectations deviate from FIRE in systematic ways.[1] So how do agents form their economic beliefs? The answer is crucial to understanding macroeconomic dynamics and policy-making.

In fact, policymakers often discuss the need to know more about how expectations are formed. Inflation expectations are of particular importance, as they play a key role in determining current inflation according to the New Keynesian Phillips curve. Former Fed Chair Janet Yellen stated, "most importantly, we need to know more about the manner in which inflation expectations are formed and how monetary policy influences them."[2] Similarly Former Fed Chair Greenspan said, "I am not saying what [inflation expectations] is a function of. We know it's a very difficult issue, but that is the key variable. It's important, but just because we can't make a judgment as to what these driving forces are in an econometric sense doesn't mean that it's not real."[3]

In this paper, I use consumer survey data on perceptions (beliefs about the past and present) and expectations (beliefs about the future) to study belief formation. I establish a novel dichotomy; consumers tend to have optimistic or pessimistic expectations about all macroeconomic variables. This results in correlations across variables that are inconsistent with macroeconomic models and experience. For example, 'pessimistic' consumers tend to expect unemployment and inflation to rise along with a deterioration of both general business conditions and their personal financial situation. Notice that these beliefs are not consistent with a typical recession, in particular, the expectation of higher inflation.

Why do otherwise pessimistic individuals expect inflation will rise, an outcome generally associated with expansions? There is widespread consumer contempt for inflation. Shiller (1996) documents that consumers worry inflation will lower their standard of living, by increasing costs without a commensurate increase in income.

[1] See Coibion et al. (2017) for a history of how survey-based measures have been used and the progression of the field from viewing them with skepticism to increased acceptance.

[2] "The Elusive 'Great' Recovery: Causes and Implications for Future Business Cycle Dynamics" 60th annual economic conference sponsored by the Federal Reserve Bank of Boston on 10/14/2016.

[3] Federal Open Market Committee meeting transcript from 7/5-6/1994.

Therefore pessimistic consumers who believe everything will go poorly, expecting typical recession-period outcomes, also expect inflation will be high. The positive correlation between unemployment and inflation expectations is at odds with the last four decades of U.S. experience, but the correlation is robust across consumer surveys, income quartiles, education levels and withstands inclusion of time fixed effects and consumer fixed effects. In contrast to consumers, professional forecaster's expectations demonstrate a correct understanding of the negative co-movement of inflation and unemployment.

Next, I conduct a component analysis on consumer beliefs. The goal of this analysis is to understand (i) the number and (ii) the characteristics of the component(s) that drive consumer beliefs. The first component explains a large portion of the variation in consumer beliefs, with estimates ranging from 30% to 76%.[4] I argue that the first component, which is driving beliefs, is sentiment because of the signs of the loadings and similarity in the aggregate to popular confidence indices. Inflation has a negative loading in the first component.

The finding that consumers incorrectly believe inflation is countercylical is robust. So it must be something fundamental about how people form beliefs, not one off mistakes or errors that get corrected over time. To match the empirical facts, I turn to models of rational inattention. This approach is appealing in that it is not ad-hoc. Agents behave optimally in the face of information constraints.

To develop intuition, I begin with a stylized, static, partial-equilibrium model of a rationally inattentive consumer. The model's consumer has one choice variable, labor, and two unknown state parameters, idiosyncratic wage and idiosyncratic price index. Consumption is the residual so that the budget constraint holds not in expectations, but in realizations of wage and price. The consumer is allowed to obtain noisy signal(s) of any combination of the state parameters, but at a cost dictated by the Shannon mutual information. They optimally choose to receive one noisy signal that is a linear combination of state parameters, rather than receiving noisy independent signals on each parameter. After receiving their signal, the agent updates their beliefs about the state parameters and decides on the value of the choice variable.

In line with the empirical stylized facts, the covariance of the state parameter beliefs can have a sign that is inconsistent with the underlying data-generating process. Furthermore, the consumer chooses one signal and forms beliefs about two state

[4]The lower estimates are for principal component analyses on continuous questions (e.g. what will inflation be over the next 12 months?), while the higher estimates are for a multiple correspondence analyses on categorical questions (e.g. will inflation increase, decrease, or stay the same over the next 12 months?). See Section 2.3 for further details.

variables. The one signal could be viewed as the consumers' 'sentiment'. From this perspective, 'sentiment' is driven by optimal signal choice rather than amorphous animal-spirits.

In an extension of the baseline static model, I add dynamics by assuming wage and price follow independent stochastic processes. The consumer, again, chooses a one dimensional signal that is a linear combination of state parameters. The intuition behind the solution is the same as the static model; however, the dynamic model allows for investigations into how one-time shocks propagate in beliefs. For example, suppose the price level experiences a one time positive shock (a surprise expansion of the money supply), the labor choice response is delayed and muted in comparison to the response under full-information. The sluggish response to shocks is consistent with existing models of rational inattention. Furthermore and novelly, I find that beliefs about price and wage move in opposite directions. In response to a positive price shock, the consumer's beliefs about price increase on impact, but their beliefs about wage drop. This is due to the consumer optimally selecting a signal format that best informs them about their one choice variable, labor, rather than learning about state parameters individually.

This paper contributes to two literatures: (i) empirical investigations into how agents form expectations and (ii) models of rationally inattentive agents. First, I add to the large and growing literature that uses survey-based expectations to study how agents form expectations. Recent papers have proposed lived experiences affect expectations (Malmendier and Nagel (2016) and Kuchler and Zafar (2015)) or that agents have time-varying concerns for model misspecification (Bhandari et al. (2016)). Related research found that consumers do not understand basic macroeconomic relationships such as the income Fisher equation, the Taylor rule, or the Phillips curve (Dräger et al. (2016) and Carvalho and Nechio (2014)). I contribute to this literature by proposing that consumers have binary perceptions and expectations. Consumers are either optimistic or pessimistic about all their beliefs. I demonstrate this behavior exists in consumer survey-data using a component analysis. The first dimension explains the overwhelming majority of the variation in perceptions and expectations and can plausibly be interpreted as a measure of sentiment.

Second, I contribute to the rational inattention literature by developing both static and dynamic partial-equilibrium consumer models that allow for multi-dimensional signals.[5] I focus on the sign of the covariances of posterior mean, which to my knowledge, is unexplored in the literature. The models in this paper build upon the

[5]The existing rational inattention literature is summarized in Sims (2010) and Veldkamp (2011).

work of Sims (2003) which began literature on rational inattention, Maćkowiak and Wiederholt (2009) which formulates a partial equilibrium firm problem, Kőszegi and Matějka (2018) that allow for multi-dimensional signals, and Maćkowiak et al. (2018) that develop analytical results for dynamic rational inattention problems.

This paper proceeds as follows. I present the empirical results in the next section. Section 3 develops a static, partial-equilibrium model and discusses the intuition behind the consumer's optimal signals. Section 4 extends the model to a dynamic framework, calibrates it, and investigates the impulse response functions of beliefs to money supply shocks. I conclude in Section 6 with a summary and avenues for further work.

2 Empirics

In this section, I discuss the two consumer surveys utilized, document a positive correlation between inflation and unemployment expectations, present evidence that consumers form expectations based primarily on one principal component, and argue that the one principal component is a measure of sentiment. The findings suggest consumers have dichotomous expectations; consumers form a large portion of their expectations based on if they are optimistic or pessimistic. An optimistic consumer expects expects typical expansion-period outcomes such as unemployment declines and improved business conditions, while also (surprisingly) expecting lower inflation. Shiller (1996) documented that consumers dislike inflation; therefore, optimistic consumers may believe inflation will fall.

The importance of consumer beliefs is clear in macroeconomic-models. A consumer's beliefs informs their actions. For example, in the basic New Keynesian model the consumer's consumption choice depends on their FIRE expectation of next period consumption and inflation. However, one may wonder if survey-based expectations inform real-world choices of the respondents. There are two strands of literature that suggest expectations solicited through surveys are informative of actions. First, survey-based confidence indices contain information about the future aggregate consumer expenditure (Carroll et al. (1994), Bram and Ludvigson (1998), and Ludvigson (2004)). Second, self-reported expectations influence savings decisions (Arnold et al. (2014)) and choices in a financially-incentivized experiments (Armantier et al. (2015)). Therefore the sentiment-based dichotomy in consumer expectations that I document is likely to affect real-world actions. Appendix A.2 presents additional evidence that survey-based expectations are correlated with choices.

I conclude this section with an analysis of a professional forecasters' expectations. In contrast to the consumer survey results, I show that professional forecasters correctly understand the correlation of unemployment and inflation expectations to be negative. Their first principal component, like with consumers, appears to be a measure of sentiment; however, professionals get the sign on inflation correct. That is, optimistic professionals expect higher inflation along with other expansion-period outcomes.

2.1 Consumer Survey Data

I use two surveys of consumer expectations, the Michigan Survey of Consumers (MSC) and the Federal Reserve Bank of New York's Survey of Consumer Expectations (SCE). Both are monthly surveys where some participants get resampled. They differ in their sample size, with the MSC surveying approximately 500 consumers and the SCE surveying approximately 1,300. The MSC has a long time series having begun in 1978, whereas the SCE only began in 2013.

The MSC and SCE ask comparable, but not identical questions. Their questions differ in phrasing and/or the types of responses allowed (categorical versus continuous). The MSC tends to ask questions that allow categorical responses, while the SCE tends to ask questions that allow continuous responses. Given the differences in questions and answer types, the MSC and SCE empirical analysis approaches need to be slightly different. I note the differences as they arise.

To get a sense of the question format, I discuss the two most relevant survey questions. First, the inflation questions in the MSC and SCE differ only in the phrasing used. The MSC asks, "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?" The SCE asks, "What do you expect the rate of (inflation/deflation) to be over the next 12 months?" Both questions solicit the consumers' expected inflation rate, in percent, over the next year. Second, the unemployment rate questions differ in the phrasing used and the type of response requested. The MSC asks a categorical question on the expected change in the unemployment rate, "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?" The SCE solicits a numerical answer on the probability of unemployment rising with, "What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?"

2.2 Inflation and Unemployment

The New Keynesian Phillips curve is the benchmark structural relationship between inflation and output gap (or more generally, a measure of economic slack). Equation (1) is the New Keynesian Phillips curve where π_t is inflation, $\mathbb{E}_t[\pi_{t+1}]$ is the time t FIRE expectation of $t+1$ inflation, X_t is the output gap, β is the discount rate, and κ is related to the parameters of the model. All reasonable parameterizations will result in $\kappa > 0$, such that a higher output gap is associated with higher inflation.[6]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa X_t \tag{1}$$

Additionally, higher expectations of the next period output gap are positively correlated with higher expectations of next period inflation. See this by shifting the New Keynesian Phillips curve one period forward and taking the period t expectation of both sides.

$$\begin{aligned}\mathbb{E}_t \pi_{t+1} &= \beta \mathbb{E}_t \mathbb{E}_{t+1} \pi_{t+2} + \kappa \mathbb{E}_t X_{t+1} \\ &= \beta \mathbb{E}_t \pi_{t+2} + \kappa \mathbb{E}_t X_{t+1}\end{aligned}$$

The New Keynesian Phillips curve contains the output gap; however, more generally it can be estimated using measures of economic slack. The less slack in the economy (e.g., high output gap, low unemployment), the New Keynesian model predicts higher inflation.

Historical experience generally confirms model predictions of a negative correlation between economic slack and inflation. In fact, the original 'Phillips curve' was an empirical negative correlation between inflation and unemployment in the United Kingdom (Phillips (1958)). Recent experience in the U.S. also suggests inflation and unemployment have a negative correlation. Figure 1 plots the time series of the inflation and unemployment rate in the U.S. Visually, the series appear to negatively co-move, with the exception of stagflation in the 1970's when inflation and unemployment were increasing together. In Figure 2, I plot the the slope coefficient for $\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$ for ten year rolling window regression. The primary outlier in the correlation is the stagflation period.

In contrast, consumer survey-based expectations of inflation and unemployment

[6] As in Galí (2008), $\kappa \equiv \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\left(\frac{1-\alpha}{1-\alpha+\alpha\epsilon}\right)\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ is positive if $\alpha \in (0,1)$. This implies firms have decreasing returns to scale in labor. If firms had increasing returns to scale, it would lead to indeterminacy.

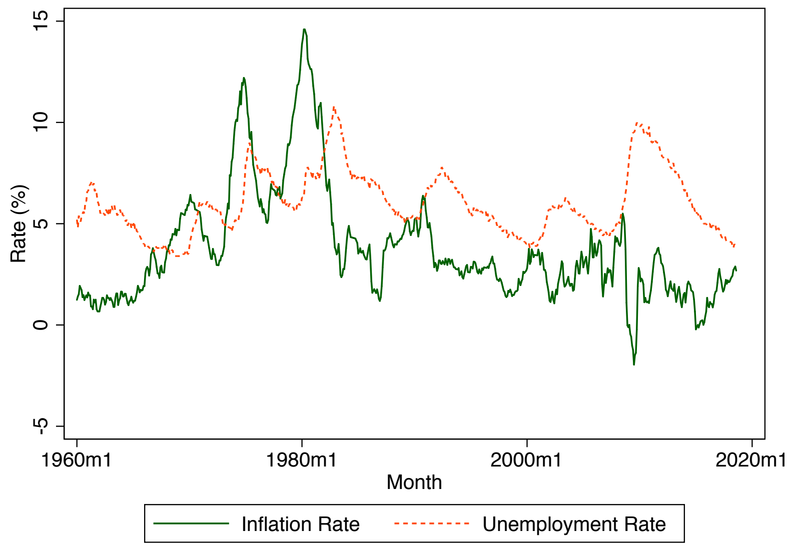

Figure 1: Inflation and Unemployment Rates

Notes: Data are from FRED. The inflation rate is the year-over-year percent change in the consumer price index for all urban consumers.

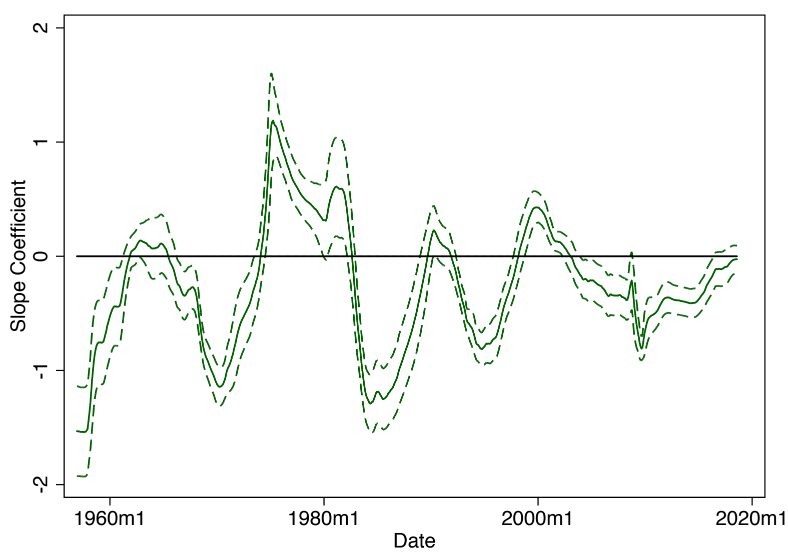

Figure 2: Correlation of Inflation and Unemployment Rates

Notes: Data are from FRED. The inflation rate is the year-over-year percent change in the consumer price index for all urban consumers. Dotted lines represent the 95% confidence interval. Ten year rolling window slope regression coefficient of $\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$ is plotted on the y-axis. The end date of the rolling regression sample is on the x-axis.

are positively correlated. Figure 3 uses MSC data and plots, for each year, the difference in inflation expectations relative to consumers that believe unemployment will stay the same. Consumers that expect unemployment to rise have higher inflation expectations, on average, compared to those that say unemployment will stay the same or decrease, for all periods. Conversely, consumers that expect unemployment will fall have lower inflation expectations, on average.

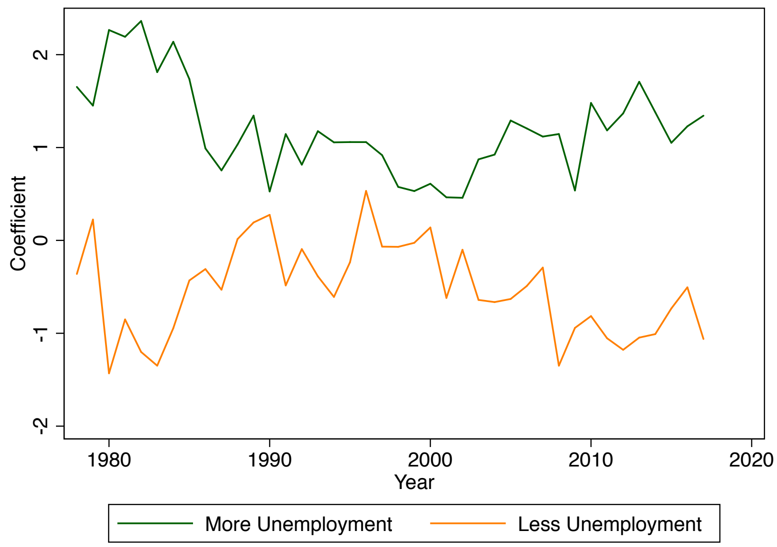

Figure 3: Unemployment and Inflation Expectations (MSC)

Notes: Data are from the MSC. The regression coefficients of $E_{j,t}\pi_{t+1} = \alpha_t + \beta_t^{more} D_{j,t+1}^{more} + \beta_t^{less} D_{j,t+1}^{less} + \epsilon_{j,t}$ are plotted across t. Subscripts j and t denote consumer and year respectively. $D_{j,t+1}^{less}$ is a dummy for if consumer j stated there would be less unemployment in 1 year. $D_{j,t+1}^{more}$ is a dummy for if consumer j stated there would be more unemployment in 1 year. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall).

A regression of expected inflation on the indicator of expected change in unemployment is shown in Table 1. In comparison to consumers that expect unemployment will stay roughly the same over the next year, consumers who expect unemployment will rise expect higher inflation and consumers who expect unemployment will fall expect inflation will be lower. Using the panel structure of the survey, I can absorb monthly fixed effects and/or consumer fixed effects. Column (2) adds monthly fixed effects, and the qualitative results remain.

Before adding consumer fixed effects, column (3) runs the regression from the previous column on the sample of consumers that were surveyed more than once (this is the sample that will not get absorbed into household fixed effects). The sample restriction does not qualitatively change the regression coefficients or significance. Column (4) includes monthly fixed effects and consumer fixed effects. The coefficients decrease in magnitude but remain significant. Why did the inclusion of household fixed effects attenuate the coefficients? Note that in the MSC, respondents that are re-sampled are surveyed a total of twice. The initial survey and another survey six months later. Because of the tight re-sampling window, the addition of household fixed effects removes experience-based explanations. Suppose living through stagflation permanently makes a consumer expect higher inflation when unemployment is rising. This effect would get absorbed into the consumer fixed effects. Accounting for personal experiences is plausibly what attenuates the coefficients, but what remains cannot be explained by experience-based stories.

Furthermore, Appendix A.1 demonstrates that across education and income distributions, consumers believe inflation is countercylical. The magnitudes of the coefficients are attenuated at higher education and income levels, but always significant at the 1% level.

A similar result is found in the SCE as shown in Table 2. Here the unemployment question is not categorical (as in the MSC), but rather a continuous measure of the consumer's percent chance unemployment will be higher in one year. Consumers that assign a higher probability to unemployment rising have higher inflation expectations. The inclusion of monthly fixed effects in column (2) does not change the qualitative findings. Column (3) restricts to the sample of consumers that were surveyed more than once and Column (4) has both monthly and consumer fixed effects. The significant positive coefficient remains, although the coefficient is attenuated with the addition of consumer fixed effects.

Despite macroeconomic theory and recent U.S. experience suggesting inflation is procyclical, consumers believe inflation will be higher when unemployment rises. Section 2.5 conducts a similar exercise for professional forecasters and finds that forecasters have expectation correlations consistent with theory and recent U.S. experience.

2.3 Component Analysis

What is driving the surprising correlation between inflation expectations and unemployment expectations in consumer surveys? The surveys contain a number of other questions and utilizing them in a component analysis sheds light on what is occurring. Both consumer surveys' have a first component that explains a large portion of the variation in responses and resembles a measure of sentiment. I discuss the results for

	(1)	(2)	(3)	(4)
More unemployment	1.590***	1.268***	1.183***	0.408***
	(0.031)	(0.029)	(0.032)	(0.044)
Less unemployment	-0.677***	-0.618***	-0.651***	-0.277***
	(0.033)	(0.032)	(0.034)	(0.048)
Monthly FE	N	Y	Y	Y
Household FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.116	0.057	0.343
N	240356	240356	165900	165900

Table 1: Positive Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Regression results from $E_{j,t}\pi_{t+12} = \alpha + \beta_t^{more} D_{j,t+12}^{more} + \beta_t^{less} D_{j,t+12}^{less} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D_{j,t+12}^{less}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D_{j,t+12}^{more}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. Columns (3) and (4) restrict the sample to households surveyed more than once. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

	(1)	(2)	(3)	(4)
$E_{i,t}(Prob(\Delta Unemp_{t+12} > 0))$	0.070***	0.069***	0.066***	0.034***
	(0.003)	(0.003)	(0.003)	(0.003)
Monthly FE	N	Y	Y	Y
Household FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.022	0.021	0.396
N	50660	50660	49172	49172

Table 2: Positive Correlation of Inflation and the Probability of Unemployment Rising (SCE)

Notes: Data are from the SCE. Regression results from $E_{j,t}\pi_{t+12} = \alpha E_{j,t}(Prob(\Delta Unemp_{t+12} > 0)) + \mu_t + \mu_i + \epsilon_{i,t}$ are reported. Subscripts j and t denote consumer and month respectively. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

each survey in turn, because the question types (categorical vs. continuous) requires differential treatment.

First, let us consider the MSC. It tends to ask categorical questions. The responses are coded as numeric values; however, the value nominal in nature, and the distance between the values does not hold any meaning. Accordingly a multiple correspondence analysis (MCA), the categorical analog of principal component analysis (PCA), is appropriate. MCA addresses the nominal nature of the survey data, by transforming the data into an indicator matrix. The rows represent an individual's responses and the columns are indicators for each category of variables.

I include forward-looking variables (over the next year personal financial conditions, personal real income, expected rates, expected business conditions, expected unemployment, expected inflation) and backward-looking variables (last year's change in personal financial conditions, last year's change in business conditions, current government policy) in the MCA. All answers are originally categorical, with the exception of expected inflation, which I bin into three categories (deflation, inflation between 0% and 4%, inflation above 4%). The first component alone accounts for an extraordinary 76% of the variation in consumer expectations and perceptions.

What is this important first component? Although a component analysis cannot tell us the meaning of the component, the first component loadings are consistent with a measure of sentiment. The ordering of the first component loadings for all variables is such that a pessimistic expectation has a negative loading whereas an optimistic expectation has a positive loading. Table 3 presents the MCA results.

Let us consider the question on unemployment which asks, "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?" In the MCA the first component loadings are -1.54 (more unemployment), .485 (same unemployment), and 1.62 (less unemployment). Consistent with the sentiment ordering, the pessimistic opinion that unemployment will rise has the smallest loading; whereas the optimistic opinion that unemployment will fall has the largest loading.

Alternatively, we can consider the inflation question, "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?"[7]. What response is pessimistic or optimistic is not immediately obvious. Shiller (1996) provides insight into what inflation outcomes consumers prefer. He documents through a series of surveys that consumers dislike inflation because they believe inflation lowers their standard of living. When the surveyor pointed out that nominal incomes would rise to match inflation, respondents often stated their concern about when and if their nominal incomes would adjust sufficiently to match inflation. This suggests

[7] Recall for the MCA I binned the responses into deflation, inflation between 0% and 4%, inflation above 4%

that the consumers that are pessimistic will report high inflation while those that are optimistic will report low inflation or deflation. The MCA first component loadings are -0.80 (inflation above 4%), 0.43 (inflation between 0% and 4%), and 0.80 (less than 0%). Consistent with the sentiment ordering, the pessimistic opinion of high inflation has the smallest loading; whereas, the optimistic opinion of deflation has the largest loading.

	(1) "optimistic"	(2) "same"	(3) "pessimistic"
Unemployment will:			
	decrease	same	increase
	1.62	0.485	-1.54
Inflation will be:			
	$\leq 0\%$	$> 0\%$ and $\leq 4\%$	$\geq 4\%$
	0.80	0.43	-0.80
Personal financial conditions will:			
	improve	same	decline
	1.04	-0.15	-2.40
Real income will:			
	increase	same	decrease
	1.44	0.46	-1.27
Rates will:			
	decrease	same	increase
	0.13	0.31	-0.23
Business conditions will:			
	improve	same	decline
	1.38	0.05	-2.15
Personal financial conditions have:			
	improved	same	declined
	.95	-.10	-1.22
Business conditions have:			
	improved	same	declined
	1.22	0.11	-1.20
Economic policy is:			
	good	fair	poor
	1.60	0.25	-1.56

Table 3: 1st Dimension Loadings for an MCA on MSC

Notes: Data are from the MSC. Multiple correspondence analysis' first component loadings are reported. Forward looking questions compare the 12 month expectation to the present. Backward looking questions compare the present to 12 months ago. The inflation response is a continuous measure; however, for the MCA I bucket the values.

Second, I conduct a component analysis for the SCE as a robustness check to the MSC component analysis results. The SCE's questions most commonly solicit numeric responses. Accordingly I can use PCA, rather than MCA, to find the first component. I include forward-looking questions (expected inflation, chance unemployment rises, chance savings rate rises, chance stock market rises) and one backward-looking question (last year's change in personal financial conditions) in the PCA.

The resulting first component loadings are in Table 4 column (1). The signs of the loadings for all questions is such that a pessimistic expectation has a negative loading whereas an optimistic expectation has a positive loading. This is consistent with the MSC findings, and suggests the first component in consumer expectations is sentiment. The first component explains approximately 30% of the variance in the responses. This is lower than the MSC findings because the SCE solicits continuous responses resulting in more variation in the data.

To further test if the first component is a measure of sentiment, I compare home-owners and non-home-owner expectations about average home price appreciation. One would expect home-owners to enjoy home price appreciation as their asset gains value, and rent-paying non-home-owners to dislike home price appreciation. In Table 4 columns (2) and (3), the PCA sample is restricted to home-owners only and non home-owners, respectively. The expectation of average home price appreciation has a positive first component loading for home-owners and a negative first component loading for non-home-owners.

2.4 Sentiment Index

How does the first component compare to commonly used measures of sentiment such as the Conference Board Consumer Confidence Index and the Michigan Survey of Consumer's Sentiment Index? I have argued that the signs of the loadings in the component analyses suggest that the first component is a measure of sentiment; however, a direct comparison to confidence indices is another way to assess the claim.

To begin, I create an aggregate time series for the first component of both (i) the Michigan Survey of Consumers' MCA and (ii) the Survey of Consumer Expectations' PCA. The first component is found for each respondent and averaged across respondents for a given response month within a survey. Recall that the MSC starts in 1978 and the SCE began in 2013. The SCE is so recent that it has not even experienced a whole business cycle, and so comparisons to the sentiment indexes will focus of the MSC. For completeness sake, Figure 4 plots the MSC and SCE

	(1)	(2)	(3)
Inflation rate will be:			
	-0.2228	-0.2320	-0.2463
% chance unemployment will rise:			
	-0.1094	-0.1496	-0.0210
% chance savings rate will rise:			
	0.4132	0.3840	0.4617
% chance stock market will rise:			
	0.4298	0.4172	0.4411
Will you be financially better off:			
	0.5394	0.5501	0.4999
Have you become financially better off:			
	0.5404	0.5465	0.5079
% change in average home price :			
		0.0336	-0.1523
N:	49977	36625	13307
Restrictions:	n/a	home-own	non-home-own
Variance explained:	0.2971	0.2608	0.2470

Table 4: 1st Dimension Loadings for a PCA on SCE

Notes: Data are from the SCE. Principal component analysis' first component loadings are reported. Forward looking questions have a 12 month horizon. The one backward looking question compares the present to 12 months ago.

first component for the time period both are available. Notice they track each other closely, despite being based on different surveys.

Next, I compare the first component time series to popular sentiment indices, the Conference Board Consumer Confidence Index and the Michigan Survey of Consumer's Sentiment Index. The Conference Board index relies on their own internal survey of consumers. The Michigan Sentiment Index does rely on the same underlying survey as the calculated first component; however the questions relied upon and methodology to construct the indices is different.[8] The first component is compared to the Conference Board index and the Michigan index in Figures 5 and 6, respectively. Indeed, the first component series looks very similar to both of the commonly used measures of sentiment, supporting the hypothesis that the MCA's first component is a measure of sentiment.

[8]To construct the Consumer Sentiment Index, the MSC considers five categorical questions. For each question, the 'relative score' is calculated as the percent of consumers giving favorable responses minus the percent giving unfavorable responses, plus 100 and rounded to the nearest whole number. The five relative scores are added together, divided by the 6.7558 for the base year of 1966, and a constant is added to correct for sample design changes.

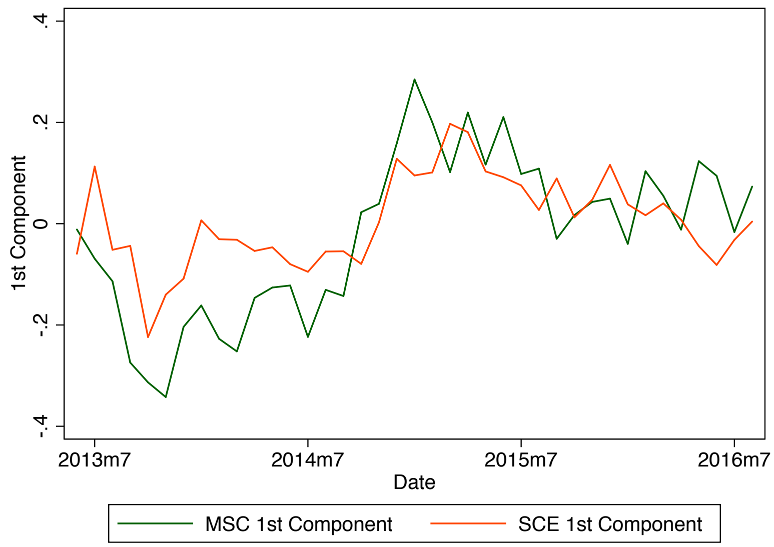

Figure 4: Comparison of 1st Components of MSC and SCE

Notes: Data are from the MSC and SCE. The MSC 1st component is based on a multiple correspondence analysis. The SCE 1st component is based on a principal component analysis. The aggregate time series are calculated as the average of 1st component values for a given month.

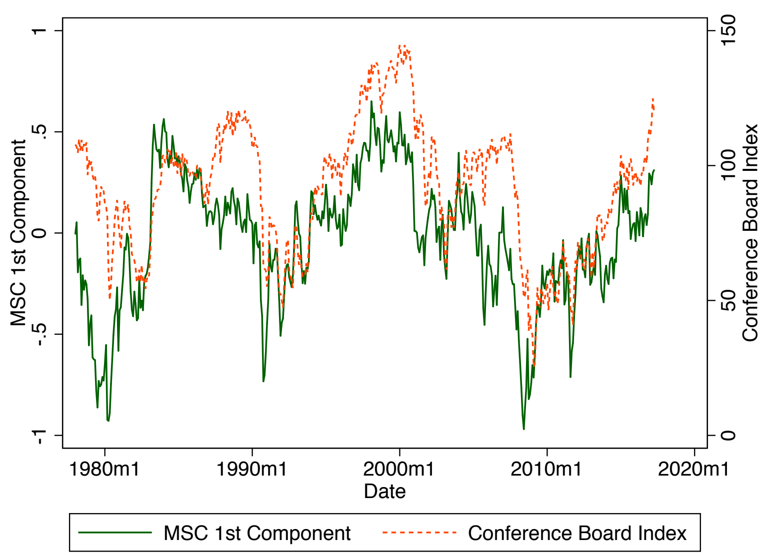

Figure 5: Comparison of MSC's 1st Component and Conference Board Confidence Index

Notes: Data are from the MSC and the Conference Board. The MSC 1st component is based on a multiple correspondence analysis. The aggregate time series is calculated as the average of 1st component values for a given month.

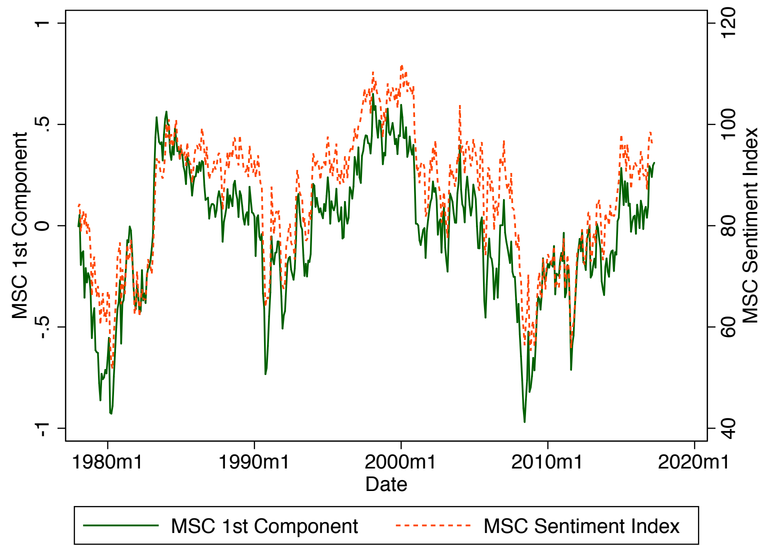

Figure 6: Comparison of MSC's 1st Component and MSC Sentiment Index

Notes: Data are from the MSC. The MSC 1st component is based on a multiple correspondence analysis. The aggregate time series is calculated as the average of 1st component values for a given month.

2.5 Professional Forecasters

The empirics so far have focused on consumers expectations. However, it is interesting to assess how and if the stylized facts documented for consumers differ for professional forecasters. Professional forecasters may have a lower cost of acquiring information or a more precise prior. If so, forecasters may get the sign of the variance of the posterior beliefs of inflation and unemployment correct. I use the Survey of Professional Forecasters (SPF) to test this hypothesis.

The SPF began running quarterly surveys in 1968. The first year with both inflation and unemployment questions was 1981. The number of responses vary, but recent surveys have approximately 40 responses. Some respondents are repeatedly sampled resulting in a panel structure. The respondents' forecasts are often based on a combination of models, experience, and intuition (Stark (2013)).

Table 5 contains the results of regressing inflation expectations on unemployment expectations for the SPF. The coefficient is positive, but quarter fixed effects are added in column (2) and the sign becomes negative.[9] The coefficient remains negative

[9]The positive coefficient in Table 5 column (1) is due to the stagflation expectations in the early 80's. If the sample is restricted to 1985 onwards (rather than 1981 onwards) the regression coefficient is negative without any fixed effects.

with the addition of respondent fixed effects. In line with standard macro-models, higher unemployment expectations is associated with lower inflation expectations for professional forecasters.

	(1)	(2)	(3)
$E_{i,t}Unemp_{t+1}$	0.153***	-0.443***	-0.327***
	(0.015)	(0.056)	(0.052)
Quarter FE	N	Y	Y
Respondent FE	N	N	Y
R-squared	0.033	0.732	0.796
N	4853	4853	4830

Table 5: Negative Correlation of Inflation and Unemployment Expectations (SPF)

Notes: Data are from the SPF. Regression results from $E_{j,t}\pi_{t+4} = \alpha E_{j,t}Unemp_{t+4} + \mu_t + \mu_j + \epsilon_{j,t}$. Subscripts j and t denote forecaster and quarter respectively. The dependent variable is the average of the annualized forecast for CPI inflation for the next four quarters. The independent variable, $E_{i,t}Unemp_{t+4}$, is the average of the forecast for unemployment for the next four quarters. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

	(1)	(2)	(3)
$E_{i,t}[\Delta Unemp_{t+1}]$	-0.416***	-0.496***	-0.377***
	(0.071)	(0.069)	(0.066)
Quarter FE	N	Y	Y
Respondent FE	N	N	Y
R-squared	0.010	0.730	0.796
N	4852	4852	4829

Table 6: Negative Correlation of Inflation and Change in Unemployment Expectations (SPF)

Notes: Data are from the SPF. Regression results from $E_{j,t}\pi_{t+4} = \alpha E_{j,t}[\Delta Unemp_{t+4}] + \mu_t + \mu_j + \epsilon_{j,t}$. Subscripts j and t denote forecaster and quarter respectively. The dependent variable is the average of the annualized forecast for CPI inflation for the next four quarters. The independent variable, $E_{i,t}\Delta Unemp_{t+4} = E_{i,t}Unemp_{t+4} - E_{i,t}Unemp_t$, is the average of the forecast for unemployment for the next four quarters minus the current quarter belief about unemployment. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Recall that the consumer surveys regressed inflation expectations on beliefs about the change in unemployment. To compare the SPF results directly to the consumer survey results, I construct a measure of the change in unemployment expectations. This independent variable is the respondents' average of the next four quarters unemployment rate minus the current quarter belief for the unemployment rate. The coefficient is negative both with and without quarter and respondent fixed effects. Professional forecasters who believe unemployment is going to increase have a lower inflation rate expectation on average. This stands in contrast to consumer surveys but is in line with U.S. experience and standard macro-models. From the perspective of the rational inattention models developed later, this may be because professionals have a lower cost of information or a more precise prior than consumers.

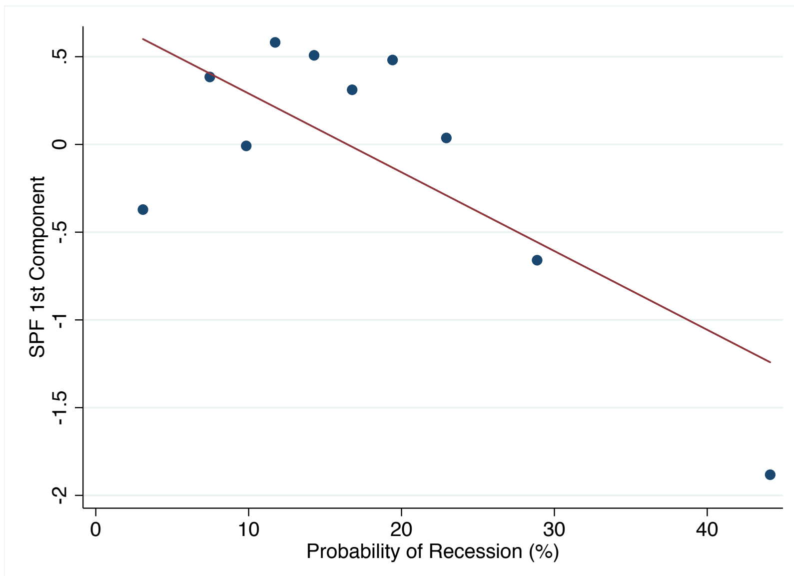

Figure 7: Binscatter of Professional's 1st Components and Their Probabilities of Recession

Notes: Data are from SPF. The first component is from a principal component analysis that contains annual expectations and current-quarter perceptions of consumer price index, core consumer price index, personal consumption expenditures, real gross domestic product, unemployment rate, housing price inflation, and nominal gross domestic product. The probability of recession is the average of the probabilities of recession for the next four quarters.

A principal component analysis on SPF data on annual expectations and current-quarter perceptions of consumer price index, core consumer price index, personal consumption expenditures, real gross domestic product, unemployment rate, housing price inflation, and nominal gross domestic product was conducted.[10] It has a negative loading on unemployment and positive loadings on inflation and the other variables.

[10]All variables are only available for 2007 and after. The first component explains a large 41% of the variation in beliefs

The signs of the loadings suggest the first component is sentiment; however, unlike with consumers, professionals associate other expansionary outcomes with inflation.

The first component, like in the consumer surveys, appears to be a measure of sentiment or a business-cycle measure. In fact, the SPF has a question that solicits respondents to provide their subjective probability of recession in future quarters. The first component of the PCA is negatively correlated with the probability of recession, as shown in Figure 7.

3 Static Rational Inattention Model

What modeling approach can capture the stylized facts: (i) inflation and unemployment expectations have positive covariance and (ii) consumers have one principal component (or signal) that drives the majority of their perceptions and expectations? Common approaches to modeling beliefs such as FIRE, sticky information, or learning will not suffice. FIRE assumes that consumers understand the 'model'; however, consumers consistently do not understand the role inflation. In sticky information models, when an agent updates their information set they achieve FIRE. There are no implications about how information is gathered or the relative accuracy of beliefs across variables. Furthermore, the empirical results are stable across time suggesting learning is also not the correct theoretic underpinning.[11]

A multi-dimensional rational inattention model is capable of matching the key stylized facts. This section develops a static, partial equilibrium model for a consumer under multi-dimension rational inattention. The model is stylistic so as to clearly develop the intuition for how information is optimally gathered. The consumer has one choice variable, but faces two unknown state variables. The consumer is allowed to obtain costly, noisy signal(s) that are any linear combination of the state variables. It turns out the consumer optimizes by choosing one signal based that is a linear combination of state variables. The consumer problem setup is similar to that of Maćkowiak and Wiederholt (2009)'s for the firm and the multi-dimensional rational inattention solution is from Kőszegi and Matějka (2018).

[11]If learning were important, after stagflation one would expect consumers to have a strong positive correlation between inflation and unemployment expectations. Over time, we would then expect that relationship to weaken and eventually flip negative. In the consumer surveys, the positive correlation is strong and stable throughout and after the stagflation period.

3.1 Consumer Problem

Each consumer, denoted by j, consumes and supplies labor. The consumers do not know their idiosyncratic wage W_j or their idiosyncratic price index P_j; however, they may obtain optimal signal(s) about them. The static problem is broken into three sequential steps: (i) obtain noisy signal(s) (ii) commit to amount of labor supplied L_j and (iii) consume so that the budget constraint binds. The timing forces the budget constraint to hold in realization and not in the consumer's expectation. There are no intertemporal dynamics so the problem is static.

Any utility function where the agent chooses consumption and labor and the budget constraint strictly binds, can be re-written as a function of either consumption or labor by substituting in the budget constraint. For the consumer's per-period utility, I could use a direct utility function $U(L_j, C_j)$ with a budget constraint $C_j P_j = L_j W_j$. Or use an indirect utility function, $U(L_j, W_j, P_j)$ with no budget constraint. I choose to do the latter for mathematical ease. Furthermore, the consumer's labor choice will be a function of its beliefs about their wage and price index. Let $\mathbb{E}_j$ be the expectation operator conditioned on the information the consumer obtains from their signal(s). The consumer problem is as follows.

$$\max_{L_j} \; U\Bigg(L_j\Big(\mathbb{E}_j[W_j], \mathbb{E}_j[P_t]\Big), W_j, P_j\Bigg)$$

I remain agnostic about the exact specification of the utility function for now, but Section 3.6 will put structure on it.

3.2 Information Cost

The cost of information is the friction that prevents rationally inattentive agents from achieving FIRE. Cost of information is commonly measured using Shannon mutual information. Shannon mutual information is the expected reduction of entropy (a measure of uncertainty) from the prior to the posterior. Intuitively, the more precise the posterior, the higher the Shannon mutual information.

For flexibility, the cost of information I use is Shannon mutual information times a scaling parameter, $\lambda \in \mathbb{R}_+$. If $\lambda = 0$, there is zero cost of information, and the agent will obtain FIRE. If λ is extremely high, the agent will choose not to obtain a signal (or obtain a signal with infinite variance-covariance), and will thus not update from their prior beliefs about the state. The interesting cases will arise for intermediate values of λ.

3.3 Non-Stochastic Steady State

With no uncertainty, $W_j = \bar{W}$ and $P_j = \bar{P}$. The labor supplied by the consumer will solve the first order condition below. All consumers will pick the same labor, $\bar{L}$, for a symmetric equilibrium.

$$U_1(L_j, \bar{W}, \bar{P}) = 0$$

Subscripts on the utility function denote derivates with respect to the input order variable. The "1" subscript above denotes the derivative with respect to the first input (labor).

3.4 Second-Order Approximation

Next, I find the log-quadratic approximation of the utility function around the non-stochastic solution. Denote log-deviations with lower case variables (e.g., $p_j = lnP_j - ln\bar{P}$). Let $\hat{u}$ be the utility function expressed in terms of log-deviations $\hat{u}(l_j, w_j, p_j) = U(\bar{L}e^{l_j}, \bar{W}e^{w_j}, \bar{P}e^{p_j}) = U(L_j, W_j, P_j)$. Let $\tilde{u}$ be the second-order Taylor approximation of the function $\hat{u}$ at the steady state.

$$\tilde{u}(l_j, w_j, p_j) = \hat{u}_1 l_j + \frac{1}{2}\hat{u}_{11} l_j^2 + \hat{u}_{12} l_j w_j + \hat{u}_{13} l_j p_j + \text{terms independent of labor}$$

Subscripts on $\hat{u}$ denote derivatives with respect to the input order variable, evaluated at the non-stochastic steady state. For example, $\hat{u}_1$ is the derivate of $\hat{u}$ with respect to labor, evaluated at the non-stochastic steady state. Since labor is the choice variable, it must be that $\hat{u}_1 = 0$. Additionally, assume standard convexity such that $\hat{u}_{11} < 0$.

$$\tilde{u}(l_j, w_j, p_j) = \frac{1}{2}\hat{u}_{11} l_j^2 + \hat{u}_{12} l_j w_j + \hat{u}_{13} l_j p_j$$

Let $l_j^\diamond$ be the utility maximizing labor under perfect information. However, the consumer does not have perfect information, so they cannot compute $l_j^\diamond$, they must calculate the expectation of it given the signal(s) received.

$$l_j^* = \mathbb{E}[l_j^\diamond | S_j]$$

The first order condition with respect to l_j is as follows.

$$0 = \hat{u}_{11} l_j^\diamond + \hat{u}_{12} w_j + \hat{u}_{13} p_j$$

$$l_j^{\diamond} = \frac{1}{|\hat{u}_{11}|}(\hat{u}_{12}w_j + \hat{u}_{13}p_j)$$

3.5 Multi-Dimensional Rational Inattention Solution

In the consumer problem, we have one choice variable (labor) and two unknown state variables (idiosyncratic wage and idiosyncratic price index). Let y be the choice variable and x be the vector of unknown state variables.

$$y = l_j$$

$$x = \begin{bmatrix} w_j \\ p_j \end{bmatrix}$$

To utilize the solution methodology of Kőszegi and Matějka (2018), one must be maximizing a quadratic function of the form $-y'Dy + x'By$ where D is symmetric and positive-semidefinite. The log-quadratic approximation of the utility function ($\tilde{u}(l_j, w_j, p) = \frac{1}{2}\hat{u}_{11}l_j^2 + \hat{u}_{12}l_jw_j + \hat{u}_{13}l_jp_j +$ terms independent of l) is quadratic and around the optimum taking a log-quadratic approximation is reasonable.

$$D = \frac{|\hat{u}_{11}|}{2}$$

$$B = \begin{bmatrix} \hat{u}_{12} \\ \hat{u}_{13} \end{bmatrix}$$

One could solve the maximization of the utility function subject to information cost constraints directly; however, it is more tractable to solve a transformed problem that is a function of (i) misperceptions about the state and (ii) the cost of information. This takes three steps. First, find the action y the agent would choose given some posterior mean of the state, $\tilde{x}$. Re-arranging the utility function, as below, it is clear that for any posterior mean $\tilde{x}$, the agent would choose action $y = \frac{D^{-1}B'}{2}\tilde{x}$.

$$\begin{aligned} U(y, x) &= -y'Dy + x'By \\ &= -\left(y - \frac{D^{-1}B'}{2}x\right)' D \left(y - \frac{D^{-1}B'}{2}x\right) + \frac{x'BD^{-1}B'x}{4} \end{aligned}$$

Second, express the utility function as function of the posterior mean of the state, $\tilde{x}$, rather than action y. Substitute $y = \frac{D^{-1}B'}{2}\tilde{x}$ into the utility function, as below. Note the term that only contains the true state and parameters cannot be affected by the consumer's choice. It is a level shift in the utility function and therefore can be dropped for the optimization problem. The new expression for the utility is a function of the agent's misperceptions about the state, $\tilde{x} - x$, and Ω. How severe the utility loss is for misperceiving the state is governed by Ω, and so Ω can be viewed as a loss matrix.

$$
\begin{aligned}
\tilde{U}(\tilde{x}, x) &= -\left(\frac{D^{-1}B'}{2}\tilde{x} - \frac{D^{-1}B'}{2}x\right)' D \left(\frac{D^{-1}B'}{2}\tilde{x} - \frac{D^{-1}B'}{2}x\right) \\
&= -\left(\tilde{x} - x\right)' \Omega \left(\tilde{x} - x\right)
\end{aligned}
$$

Where:

$$
\Omega \equiv \frac{BD^{-1}B'}{4} = \frac{1}{2|\hat{u}_{11}|}\begin{bmatrix} \hat{u}_{12}^2 & \hat{u}_{12}\hat{u}_{13} \\ \hat{u}_{12}\hat{u}_{13} & \hat{u}_{13}^2 \end{bmatrix}
$$

Third, I quantify the cost of information. The posterior will be Gaussian. A general n-dimensional vector that has the multivariate normal distribution of $N(mean, var)$ has entropy $\frac{n}{2} + \frac{n}{2}log(2\pi) + \frac{1}{2}log|var|$. The Shannon mutual information is the change in entropy from the prior to the posterior. The only term in the Shannon mutual information that can be affected by the consumer's decisions is $\frac{1}{2}log|\Sigma|$, where Σ is the posterior variance-covariance. Therefore it is the only cost of information term one needs to consider in the consumer problem. For flexibility, I multiply the Shannon mutual information by a scaling factor, λ, to get the cost of information.

The agent's maximization problem can now be re-written as the sum of (i) the expected utility (a function of misperceptions about the state) and (ii) the cost of information. See equation (2). The choice variable is the posterior variance-covariance matrix, Σ. Such that the agent is picking the precision of their posterior. Equation (3) simplifies equation (2).

$$
\max_{\Gamma \geq \Sigma} -\mathbb{E}_j\left[(\tilde{x} - x)'\Omega(\tilde{x} - x)\right] + \frac{\lambda}{2}log|\Sigma| \tag{2}
$$

$$
\max_{\Gamma \geq \Sigma} -Tr(\Omega\Sigma) + \frac{\lambda}{2}log|\Sigma| \tag{3}
$$

Let $\Gamma = \sigma_0^2 I$ be the prior variance-covariance. The restriction of $\Gamma \geq \Sigma$ in the optimization implies $\Gamma - \Sigma$ must be positive semidefinite. This restriction forces the prior variance-covariance (Γ) to be no more precise in any dimension than the posterior (Σ). Intuitively, an agent is not allowed to forget information contained in their prior in exchange for more information in a dimension the agent cares more about.

Let $v^1, ..., v^k$ be an orthonormal basis of eigenvectors of the loss matrix Ω (which is symmetric). Let the matrix consisting of columns $v^1, ..., v^k$ be called V. The eigenvalue corresponding to v^i is Λ_i. Let Λ be the matrix with Λ_i elements on the diagonal and 0 entries elsewhere. Decomposing the loss matrix, Ω, into its eigenvalues and eigenvectors results in $\Omega = V\Lambda V'$. Note that because Ω is symmetric, the eigenvectors will be orthogonal. The consumer problems' eigenvalues of Ω are $\Lambda_1 = 0$ and $\Lambda_2 = \frac{1}{2|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]$. So the matrix of eigenvalues is:

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2] \end{bmatrix}$$

The corresponding eigenvectors and the resulting matrix of the orthonormal basis of eigenvectors are:

$$v^1 = \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}} \\ 1 \end{bmatrix} \text{ and } v^2 = \begin{bmatrix} \frac{\hat{u}_{12}}{\hat{u}_{13}} \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}} \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{\hat{u}_{12}}{\hat{u}_{13}} \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \\ \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \end{bmatrix}$$

The agent will not update along the first eigenvector since it has an eigenvalue of zero. However, the agent will choose to get a signal along the second eigenvector. Intuitively, the agent is transforming the problem into ' directions' and only choosing a signal in a 'direction' that is most important to him. Notice that the second eigenvector multiplied by x, is the 'direction' of optimal labor under perfect information.

Let $J = V^{-1}\Sigma V = V'\Sigma V$ be the variance-covariance of the posterior in the basis of the eigenvectors of Ω. Note since V is orthogonal its inverse is the same as its transpose. If we find J, we can pin down Σ. See Kőszegi and Matějka (2018) for the proof of the following general solution.

$$J_{ij} = 0 \text{ for all } i \neq j$$

$$J_{ii} = \min\left(\sigma_0^2, \frac{\lambda}{2\Lambda_i}\right)$$

For the consumer problem specified here, the solution for S is:

$$J_{12} = J_{21} = 0$$

$$J_{11} = \sigma_0^2$$

$$J_{22} = \min\left(\sigma_0^2, \frac{\lambda}{\frac{1}{|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]}\right)$$

Assuming the cost parameter λ is such that J_{22} is not the prior variance and the agent gets a signal in one direction, the posterior variance-covariance is:

$$\Sigma = VJV'$$

$$\Sigma = \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}}\frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{\hat{u}_{12}}{\hat{u}_{13}}\frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \\ \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \end{bmatrix} \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \frac{\lambda}{\frac{1}{|\hat{u}_{11}|}[\hat{u}_{12}^2+\hat{u}_{13}^2]} \end{bmatrix} \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}}\frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} \\ \frac{\hat{u}_{12}}{\hat{u}_{13}}\frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} & \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \end{bmatrix}$$

Let $\bar{x} = 0$ be the prior mean, S be the realized signal, ϵ be the signal noise, and Σ_ϵ be the variance-covariance of the signal error. Assuming one N-dimension Gaussian signal (rather than repeated signals), the posterior mean is:

$$\begin{aligned} \tilde{x} &= \Gamma(\Gamma + \Sigma_\epsilon)^{-1}S + \Sigma_\epsilon(\Gamma + \Sigma_\epsilon)^{-1}\bar{x} \\ &= \Gamma(\Gamma + \Sigma_\epsilon)^{-1}S \end{aligned}$$

In the data, I calculated the covariances of expectations across people. The analytical counterpart to that is the covariance of the posterior means of wage and price. The variance-covariance matrix of the posterior means is:

$$\begin{aligned} var(\tilde{x}) &= var(\Gamma(\Gamma + \Sigma_\epsilon)^{-1}S) \\ &= \Gamma(\Gamma + (\Sigma^{-1} - \Gamma^{-1})^{-1})^{-1}\Gamma' \end{aligned}$$

Going back to the consumer problem, I calculate $var(\tilde{x})$. Expressed in equation (4) is the element of interest, the covariance term of the posterior wage and price.

$$cov(\tilde{w}, \tilde{p}) = \frac{\hat{u}_{12}\hat{u}_{13}\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{\hat{u}_{12}^2+\hat{u}_{13}^2}\right)}{\hat{u}_{12}^2 + \hat{u}_{13}^2} \tag{4}$$

What is the sign of the posterior means' covariance term? Everything is known to be positive with the exception of $\hat{u}_{12}\hat{u}_{13}$. So the sign of the covariance term, will be the sign of $\hat{u}_{12}\hat{u}_{13}$.

In the model we have idiosyncratic wages and idiosyncratic price uncertainty. Although these are not direct counterparts to unemployment and inflation expectations observed in the data, they are similar. High wage posteriors are similar to low unemployment expectations. So for the model to match the empirical results, the covariance of the posterior means of wage and price should be negative. Those who expect low wages (high unemployment) also expect high prices and vice versa.

3.6 Utility Function

So far I have not taken a stand on the form of the utility function, but I now assume the canonical utility function in equation (5). Does it have a negative variance-covariance of the posterior means of wages and the price index? If so, the rational inattention approach with a typical utility function can rationalize the positive covariance of inflation and unemployment expectations across people.

$$U(C_j, L_j) = \frac{C_j^{1-\varphi}}{1-\varphi} - \frac{L_j^{1+1/\eta}}{1+1/\eta} \tag{5}$$

Parameter φ is the inverse of the intertemporal elasticity of substitution and η is the Frisch labor supply elasticity. Substituting the budget constraint $C_j = \frac{W_j L_j}{P}$ into the utility function, I get rid of consumption.

$$U(L_j, W_j, P_j) = \frac{\left(\frac{W_j L_j}{P_j}\right)^{1-\varphi}}{1-\varphi} - \frac{L_j^{1+1/\eta}}{1+1/\eta}$$

The utility function written in log-deviations can be written as:

$$\hat{u}(l_j, w_j, p_j) = U(\bar{L}e^{l_j}, \bar{W}e^{w_j}, \bar{P}e^{p_j}) = \frac{\left(\frac{\bar{W}e^{w_j}\bar{L}e^{l_j}}{\bar{P}e^{p_j}}\right)^{1-\varphi}}{1-\varphi} - \frac{(\bar{L}e^{l_j})^{1+1/\eta}}{1+1/\eta}$$

The sign of the posterior's mean's covariance between price and wage is equal to the sign of $\hat{u}_{12}\hat{u}_{13}$. For any parametrization where $\varphi \neq 1$, indeed $\hat{u}_{12}\hat{u}_{13}$ is negative. In fact, as shown in Appendix B.1, $\hat{u}_{12} = -\hat{u}_{13}$. The covariance of the posterior wage and price beliefs, when the agent chooses to get one signal, expressed in equation (4) can be simplified to:

$$cov(\tilde{w}, \tilde{p}) = -\frac{1}{2}\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{2\hat{u}_{12}^2}\right) < 0$$

Figure 8 plots the covariance of the posterior wage and price for varying information costs. Recall that the true underlying data-generating process has zero covariance between wage and price. At high information costs, the agent receives no signals and the posterior covariance between wage and price is the same as the prior covariance

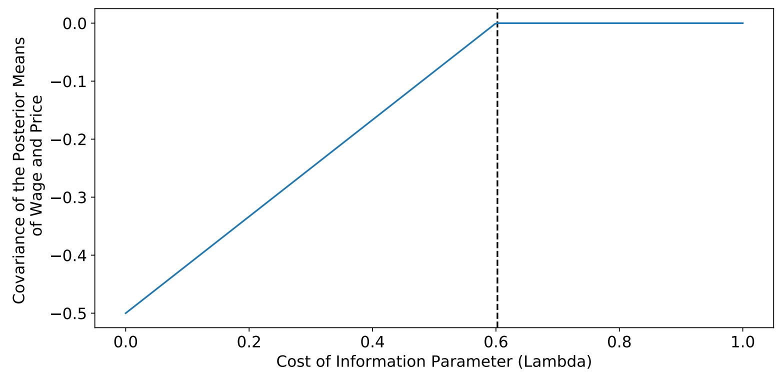

Figure 8: Covariance of Posterior Means, Static Model

Notes: The covariance of the posterior means of wage and price are plotted for varying information costs. For high information costs (λ large), the consumer gets no signals and the covariance is zero. For low information costs (λ small), the agent gets one signal and the covariance is negative. The prior variance-covariance is assumed to be $\sigma_0^2 I$. Parameterization values for the plot are: $\eta = 3$, $\varphi = .5$, $\sigma_0^2 = 1$.

(assumed to be zero). However when information costs are sufficiently low, the agent decides to collect one signal and the posterior covariance between wage and price becomes negative. Optimal signal collection results in price beliefs that are countercylical.

Notice that as information costs fall zero the covariance approaches -.5, rather than the zero covariance of the underlying data-generating process.[12]. This is driven by the fact the agent has one choice variables and faces two unknown state variables. At zero information costs, the consumer will know the optimal labor choice (their one and only choice) but will not know the precise values of wage and price. The consumer has no incentive to gather further information and pay higher costs, as they have already determined the optimal labor choice.

Later, I develop a two-period model where the consumer has two choice variables and two unknown states. In that extension, at sufficiently low information costs, the agent will obtain two orthogonal signals. The consumer's posterior covariance of wage and price will approach zero as information becomes free.

[12]When the agent obtains one signal, the posterior covariance of wage and price beliefs is $-1/2\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{2\hat{u}_{12}^2}\right) = -.5$ when $\sigma_0^2 = 1$ and $\lambda = 0$.

4 Dynamic Rational Inattention Model

The previous section's static model demonstrated how and why agents choose optimal signals that are linear combinations of the state, possibly resulting in incorrect covariances in posterior beliefs. This section extends the static consumer problem to a dynamic problem. I solve the model numerically, using the approach and findings of Maćkowiak et al. (2018). Then, I calibrate the model and use it to investigate the impulse response functions of beliefs to a shock to the price level.

4.1 Set-Up

Let time be discrete and denote it with t. As before, the agent consumes and supplies labor. They do not know their wage or the price index they face, but may obtain optimal signal(s) about them. Every period is broken into three sequential steps: (i) obtain noisy signal(s) (ii) commit to amount of labor supplied L_{jt} and (iii) consume so that the budget constraint binds.The timing forces the budget constraint to hold in realization and not in the consumer's expectations. Notice the consumer is not allowed to hold savings. The consumer can update their labor choice each period.

Unlike in the static approach, a dynamic approach requires a specification of how the fundamentals evolve. Let the log-deviations from steady state of the wage and price index follow the AR(1) processes in equations (6) and (7). The errors, $\epsilon^w_{j,t}$ and $\epsilon^p_{j,t}$, are independent and drawn from a standard normal distribution.

$$w_{j,t} = \phi_w w_{j,t-1} + \theta_w \epsilon^w_{j,t} \tag{6}$$

$$p_{j,t} = \phi_p p_{j,t-1} + \theta_p \epsilon^p_{j,t} \tag{7}$$

Signals can be any linear combination of the log-deviations of current or past period wages ($\{w_{j,t}, ..., w_{j,t-M}\}$), prices ($\{p_{j,t}, ..., p_{j,t-M}\}$), wage errors ($\{\epsilon^w_{j,t}, ..., \epsilon^w_{j,t-M}\}$), and price errors ($\{\epsilon^p_{j,t}, ..., \epsilon^p_{j,t-M}\}$).[13] The consumer chooses the weights to put on each and the standard deviation of the signal error, to optimally learn about their 'best' labor choice subject to information costs. The precise objective function is to minimize the present value of the expected mean-squared error between the optimal labor choice and their belief about the optimal labor choice plus the information cost. Subsection 4.3 discusses the objective in further detail.

Searching over the large set of possible weightings and signal error variance is time-intensive. Fortunately, the results of Maćkowiak et al. (2018) show that (in

[13] M is arbitrarily large.

the set-up used here) the consumer will optimally choose to get one signal and it will be a linear combination of current wage and price. This restriction significantly reduces the computational time needed to solve the model. Optimal signals will be of the form in equation (8) where $S_t \in \mathbb{R}$ is the signal, $h_1 \in \mathbb{R}$ and $h_2 \in \mathbb{R}$ are signal weights, $\epsilon_t \in \mathbb{R}$ is Gaussian white noise, and $\sigma_\epsilon^2 \in \mathbb{R}_+$ is the variance of the signal error. The consumer will pick their optimal signal weights (h_1 and h_2) and variance of the signal error (σ_ϵ^2).

$$S_t = h_1 w_{j,t} + h_2 p_{j,t} + \epsilon_t \tag{8}$$

4.2 Information Set and Costs

In the static set-up, a signal informs the agent about the current state. However in a dynamic model, the current signal serves two purposes. It informs the agent about the current state and stays forever in the agent's information set possibly informing the agent about future states. The information set at time t contains the current signal (S_t), all previous signals ($\{S_1, ..., S_{t-1}\}$), and the initial information set ($\mathcal{I}_0$). The information set at time 0 is an infinite set of signals so that the agent's conditional variance-covariance of the true state is not time-dependent.

$$\mathcal{I}_t = \mathcal{I}_0 \cup \{S_1, ..., S_t\} \tag{9}$$

In line with the static model, the dynamic information cost is the Shannon mutual information scaled by λ. The information cost, $\frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right)$, is derived in Lemma 2 of Maćkowiak et al. (2018).[14]

4.3 Consumer Problem and Solution

If the consumer had full information about wage and price, they would chose optimal labor ($l^\diamond_{j,t} = \frac{\hat{u}_{12}}{|\hat{u}_{11}|} w_{j,t} + \frac{\hat{u}_{13}}{|\hat{u}_{11}|} p_{j,t}$) every period. Without full information, the consumer seeks to minimize the present value of the expected mean-squared error between the optimal labor choice and their belief about optimal labor, $\mathbb{E}_j[\sum_{t=1}^{\infty} \beta^t (l^\diamond_{j,t} - \mathbb{E}_j(l^\diamond_{j,t}|\mathcal{I}_t))^2]$, plus the present value of the information cost, $\mathbb{E}_j[\sum_{t=1}^{\infty} \beta^t \frac{\lambda}{2}\log_2(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2})]$.

The discount factor, $\beta \in (0, 1)$, is assumed to be known by the consumer. Furthermore, as discussed above, the agent's expected mean-squared error between the optimal labor choice and their belief about optimal labor, is not time-independent.

[14] Σ_1 is the limit as t approaches infinity of the variance-covariance of $\begin{bmatrix} w_{j,t} \\ p_{j,t} \end{bmatrix}$ given the information set at $t-1$.

The form of the signal is also the same across time, so the information cost is constant across periods. Together this implies that the loss function is proportional to $\mathbb{E}_j[(l^{\diamond}_{j,t} - \mathbb{E}_j(l^{\diamond}_{j,t}|\mathcal{I}_t))^2 + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma^2_\epsilon}\right)]$.

Putting everything together, the consumer minimizes the per-period expected mean-squared error of optimal labor plus the information cost (equation 10) subject to the wage and price AR(1) processes (equations 6 and 7), the signal form (equation 8), and the information set (equation 9).

$$\min_{h,\sigma_\epsilon} \; \mathbb{E}_j\left[\left(l^{\diamond}_{j,t} - \mathbb{E}_j(l^{\diamond}_{j,t}|\mathcal{I}_t)\right)^2\right] + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma^2_\epsilon}\right) \tag{10}$$

I numerically solve the consumer problem using the algorithm discussed in detail in Appendix B.2. First, I find the consumer's optimal signal (signal weights and variance of the signal error) that minimizes the mean-squared error of the labor choice plus the information cost. Second, I use standard recursive Kalman filtering updating (e.g., Hamilton (1994), Bougerol (1993)) to solve for how the consumer will update their beliefs of labor, price, and wage in response to signals.

4.4 Impulse Response Functions

The model contains seven parameters to calibrate. Four parameters are associated with the AR(1) processes for log-deviations in wage and price: ϕ_w, θ_w, ϕ_p, and θ_p. Two parameters are associated with the utility function, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$. The cost of information scaling factor, λ, adds one last parameter.

I estimate the AR(1) coefficients for log-deviations in price and wage using quarterly, seasonally-adjusted data on the consumer price index and average weekly real earnings for full-time employees from 1980 onwards.[15] AR(1) processes are estimated on the cyclical component of each series obtained using a Hendrick-Prescott filter with a smoothing parameter of 1600. The autoregressive coefficients are $\phi_w = .600$ and $\phi_p = .813$. I set $\theta_w = 1$ and $\theta_p = 1$.

The weights on wage and price log-deviations in optimal labor log-deviations are $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$, respectively. Assume the utility function in equation (5) and $\varphi \neq 1$. Then, as shown in Appendix B.1, the weights must be equal in magnitude but of opposite signs. The precise values will depend on the steady state values of wage, price, labor, and the values of φ and η in the utility function. Assume the steady state wage, price, and labor are 1, $\varphi = .5$, and $\eta = 3$. Then, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$.[16]

[15] Data from FRED.

[16] At the steady state, $\hat{u}_1 = 0$.

I have set $\lambda = 1$ for the baseline results, but will vary it to assess the impact of scaling the cost of information.

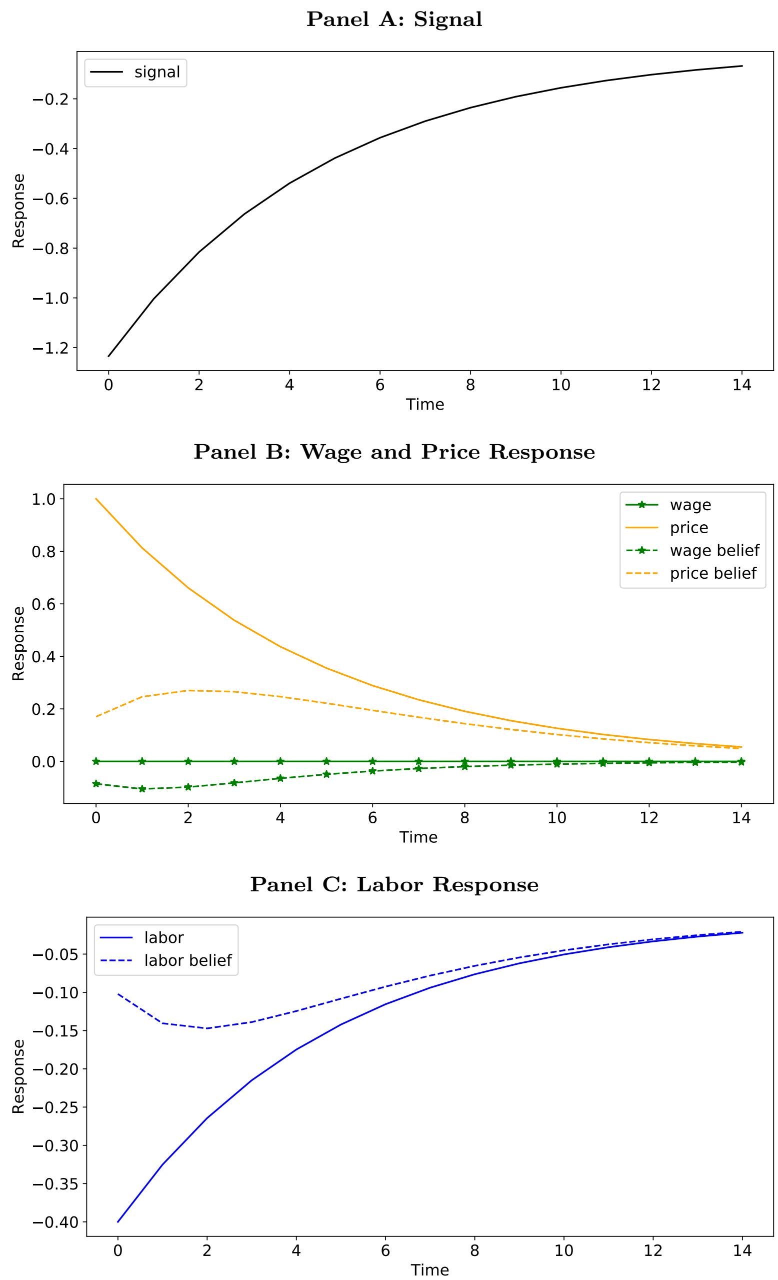

Figure 9: Impulse Responses After a Standard Deviation Shock to Price

Notes: Signal and response of log-deviations in wage, price, and optimal labor (actual and beliefs) to one standard deviation shock to price. Parameter values are: $\phi_w = .600$, $\theta_w = 1$, $\phi_p = .813$, $\theta_p = 1$, $\lambda = 1$, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$. For this calibration, the optimal signal weights are $h_1 = 1$ and $h_2 = -1.23$.

With the parameters set, consider the effects of an exogenous, positive, one standard-deviation shock to the log-deviation in price. This shock can be interpreted as a positive money supply shock. The impulse response functions of the signal, the true log-deviations of wage, price and optimal labor, as well as the beliefs about the log-deviations of wage, price, and optimal labor are plotted in Figure 9. The true value of the log-deviation of price jumps up on impact and reverts back to steady-state following its AR(1) process. Log-deviations of wage are not affected by the price shock so the log-deviation of wage stays at zero. Optimal labor's log-deviation falls by .4 on impact because wage was unaffected, price rose by 1 log-deviation, and the coefficient on price on optimal labor is -.4.

Panel A shows the evolution of the signal in response to a one standard deviation shock to price. The optimal signal weights for this calibration are 1 and -1.23 for wage and price log-deviations, respectively. Therefore the shock to price results in a simultaneous signal of -1.23. The signal then reverts back to zero. Reversion speed is dependent on the signal weight on price log-deviations and the AR(1) process that governs the return of price to steady state.

What happens to labor beliefs (and thus the consumer's labor choice) in response to this money supply shock? The consumer does not fully 'believe' the signal because they understand the signal is noisy. Accordingly, they use recursive Kalman filtering updating to form their labor belief and under-react to shocks on impact. The consumer's labor choice falls less than the optimal labor choice, in response to an expansion of the money supply. This under-reaction to shocks is typical of rational inattention models.

The consumer optimized their signal so as to minimize the mean-squared error of their labor choice. They did not care about the wage or price independently, and only cared about wage and price to the extent they entered the optimal labor choice. Upon getting a negative signal due to a money supply shock, they are not sure if it came from a negative wage shock or a positive price shock (or even noise for that matter). The consumer, if asked to provide their best estimates of wage and price, would use recursive Kalman filtering updating to form their beliefs about wage and price. Their price belief jumps up (less than the price shock) and their wage belief jumps down (despite wage still being at steady state) on impact. As time passes and they recursively update, wage and price beliefs approach true wage and true price, respectively. Note that if the agent had obtained independent signals on wage and price, wage beliefs would not have reacted to the price shock.

How does varying the information cost affect the impulse response functions?

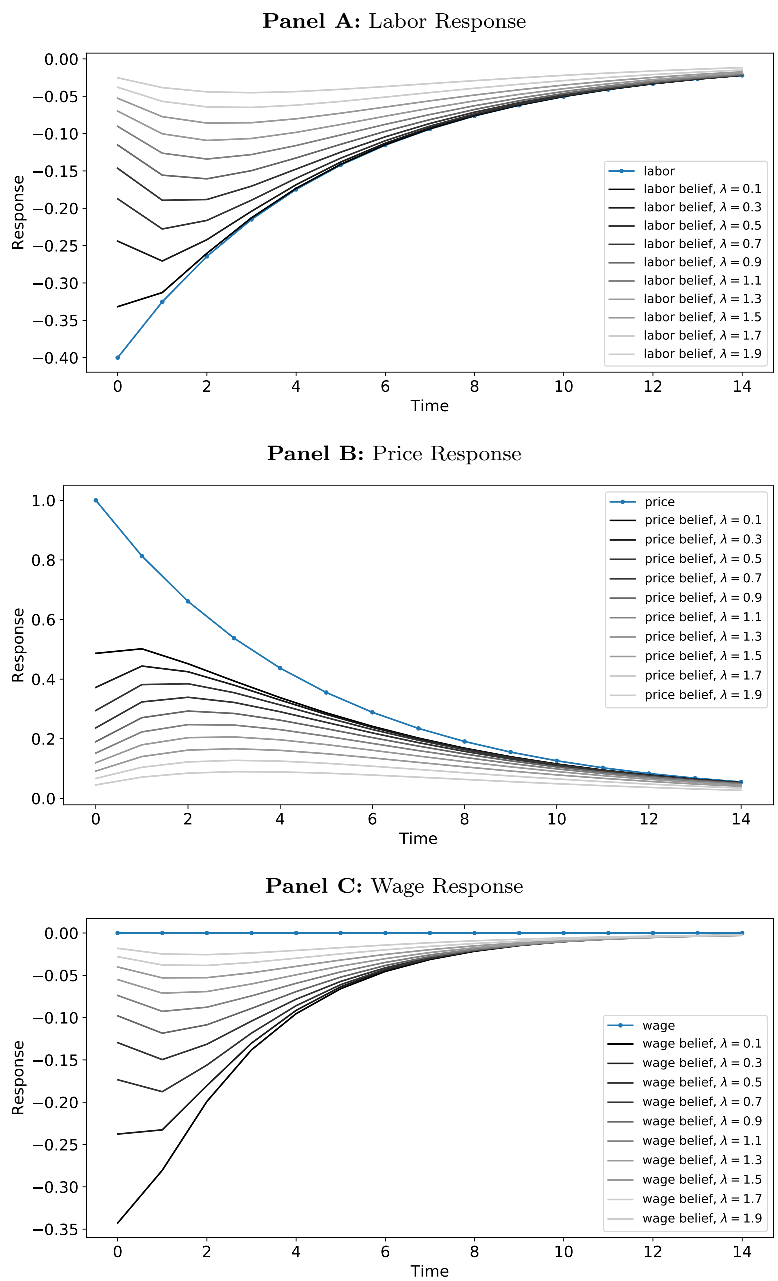

Figure 10: Varying the Cost of Information, Impulse Responses After a Standard Deviation Shock to Price

Notes: Response of labor beliefs for different information costs to one standard deviation shock to price. Parameter values are: $\phi_w = .600$, $\theta_w = 1$, $\phi_p = .813$, $\theta_p = 1$, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$

The cost of information influences the consumer's choice of optimal signal and thus their response to shocks. Figure 10 plots the impulse response functions for labor, wage, and price in response to a price shock, for varying costs of information. As the cost of information decreases (λ declines), the consumer chooses to have less noise in their signal. In the limit, the consumer learns the exact optimal labor choice. The consumer's price and wage beliefs, in contrast to their labor beliefs, do not reach the true values as information costs go to zero. This is because the consumer does not care about price or wage independently; they only seek to know their optimal labor choice.[17]

Notice that as information costs fall, the consumer's price beliefs rise (closer to the true price) and wage beliefs decrease (further from the true wage). Why is the agent's wage belief getting further from the truth? As the information cost goes to zero, the consumer optimally chooses decreasing noise in their signal, and increasingly 'believes' the signal. Due to the signal being one dimensional, the consumer does not know if a low (high) signal should be attributed to a negative wage shock or a positive price shock (positive wage shock or negative price shock). They therefore update their beliefs about both state parameters, despite the fact, that in reality there was only a shock to one state parameter.

5 Two Period Models

5.1 Two Choices and Two States

In the static and dynamic models already presented, the household has one choice variable and faced two unknown state variables each period. As the information cost goes to zero, the agent perfectly learns about their optimal choice by receiving one signal that is the appropriate linear combination of the state variables. Regardless of how low the information cost gets, the agent never chooses to get a second signal (the eigenvalue is zero). Therefore the covariance of the posterior means of wage and price will be negative whenever the agent gets a signal.

This may be unsatisfying in that, as information costs go to zero, we may want the agent to (i) obtain full information about all variables and (ii) have the covariance of the posterior means of wage and price approach the underlying data-generating covariance of zero. In this section, I develop a two-period model with two actions and two state variables. At high levels of information costs, the agent gets no signal (as before). At intermediate values of information costs, the agent obtains one signal along the eigenvector that is most important to the agent. With low information costs, the

[17]Suppose, contrary to the model set-up, the consumer did care about price and wage independently. Then as the cost of information went to zero, the agent would learn about both state parameters fully.

agent will choose to obtain two signals, one along each of the orthogonal eigenvectors. The agent will learn about both state parameters perfectly when information costs are sufficiently low.

There are two periods. The consumer is denoted with subscript j. In the first period, the consumer chooses how much labor to supply (L_j) and how much to save (S_j). The agent does not know their wage (W_j) or the price index (P_j), but may obtain costly signals about them. First period consumption ($C_{j,1}$) is the value that makes the budget constraint bind ($P_{j,1}C_{j,1} = W_jL_j - S_j$). In the second period, the consumer consumes all of their savings, which have grown at rate R. Assume that the price index in both periods are the same ($P_{j,1} = P_{j,1} = P_j$). Assume the consumer knows their discount rate (β) and the savings interest rate (R). The consumer has two choice variables (labor and savings) and two unknown state variables (price index and wage). Equation (11) is the present value of the consumer's utility. Assume utility from consumption and disutility from labor take the forms $u(C) = \frac{C^{1-\varphi}}{1-\varphi}$ and $v(L) = \frac{L^{1+1/\eta}}{1+1/\eta}$, respectively.

$$U(L, C_1, C_2) = u(C_1) - v(L) + \beta u(C_2) \tag{11}$$

Each period's budget constraint must bind: $PC_1 = WL - S$ and $PC_2 = (1 + r)S$. Substituting in the constraints, we obtain utility as a function of the two choice variables and the two unknown state variables.

$$U(L, S, W, P) = u\left(\frac{WL - S}{P}\right) - v(L) + \beta u\left(\frac{(1 + r)S}{P}\right)$$

As in the static model, let lower case variables denote log-deviations from steady state, $\hat{u}$ be the utility function expressed in terms of log-deviations from steady state, and $\tilde{u}$ be the second order approximation at the steady state ($t.i.c$ are terms independent of choice variables labor and savings). Subscripts on $\hat{u}$ indicate derivates with respect to the input variable, evaluated at the steady state.

$$\begin{aligned}\tilde{u}(l_j, s_j, w_j, p_j) =& \hat{u}_1 l_j + \hat{u}_2 s_j + \frac{1}{2}\hat{u}_{11} l_j^2 + \frac{1}{2}\hat{u}_{22} s_j^2 + \hat{u}_{12} l_j s_j + \\ & \hat{u}_{13} l_j w_j + \hat{u}_{14} l_j p_j + \hat{u}_{23} s_j w_j + \hat{u}_{24} s_j p_j + t.i.c.\end{aligned}$$

Optimality of the labor and savings choices implies that $\hat{u}_1 = 0$ and $\hat{u}_2 = 0$. The steady state is found by normalizing the wage and labor steady states to one ($\bar{L} = \bar{W} = 1$) and solving for the steady state of savings ($\bar{S}$) and labor ($\bar{L}$) so that

$\hat{u}_1 = 0$ and $\hat{u}_2 = 0$.

The log-quadratic utility can be expressed as $-y'Dy + x'By$ where

$$y = \begin{bmatrix} l_j \\ s_j \end{bmatrix}, \; x = \begin{bmatrix} w_j \\ p_j \end{bmatrix}, \; D = -\frac{1}{2}\begin{bmatrix} \hat{u}_{11} & \hat{u}_{12} \\ \hat{u}_{12} & \hat{u}_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} \hat{u}_{13} & \hat{u}_{23} \\ \hat{u}_{14} & \hat{u}_{24} \end{bmatrix}$$

The consumer problem can be solved using the methodology of Kőszegi and Matějka (2018). In the static model, the loss matrix due to misperceptions had one zero and one nonzero eigenvalue. Now, the loss matrix due to misperceptions has two nonzero eigenvalues. The agent will, depending on the information cost, either obtain (i) no signal and stay with their prior, (ii) one signal along the eigenvector direction with the higher eigenvalue, or (iii) two signals, one along each eigenvector direction. As information costs go to zero, the consumer will correctly choose optimal labor and optimal savings, as well as correctly know the values of wage and price.

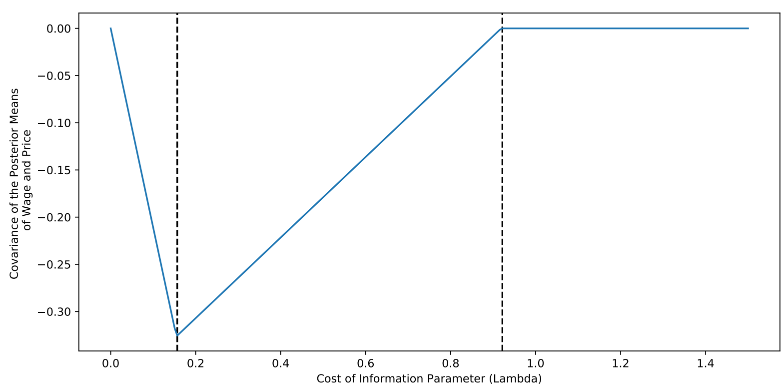

Figure 11: Covariance of Posterior Means, Two-Period Model

Notes: The covariance of the posterior means of wage and price are plotted for varying information costs. For high information costs (λ large), the consumer gets no signals and the covariance is zero. For intermediate information costs, the consumer gets one signal and the covariance of the posterior wage and price beliefs is negative. For low information costs (λ small), the agent gets two signals and the covariance is negative; however, in the limit it approaches zero. The prior variance-covariance is assumed to be $\sigma_0^2 I$. Parameterization values for the plot are: $\eta = 3$, $\varphi = .5$, $\beta = .95$, $R = .05$, $\sigma_0^2 = 1$.

The covariance of the posterior mean of wage and price, the model analog to taking the covariance of beliefs in survey data, will vary across information costs. For high information costs (λ large), the consumer gets no signals and the covariance is their prior (assumed to be zero). For intermediate information costs, the consumer gets one signal and the covariance of the posterior wage and price beliefs is negative. For low information costs (λ small), the agent gets two signals and the covariance is negative; however, in the limit it approaches zero. Therefore in the limit of costless information, the covariance of the posterior wage and price beliefs matches the zero covariance in the true underlying data-generating process. Figure 11 plots the covariance of the posterior means of wage and price across information costs for one parameterization.

6 Conclusion

This paper documents how consumers tend to have optimistic or pessimistic expectations about all macroeconomic variables, resulting in correlations across variables that have the opposite sign of historic macroeconomic data. Most notably, a positive covariance of expected inflation and unemployment. Furthermore the first principal component, plausibly a measure of sentiment, explains the large majority of the variance in expectation measures in survey-data. Motivated by the empirical results, I develop static and dynamic models of a rationally inattentive consumer that can generate (i) the observed counter-intuitive covariance of expectations and (ii) consumers only obtaining information on one 'dimension'.

There are many avenues for important and related future work. Further research should be conducted into better understanding the expectation formation process as revealed in survey data. Are there additional factors that affect consumer expectations that FIRE does not capture? If so, we should incorporate them into general equilibrium models, determine the ramifications to monetary policy propagation, and assess optimal monetary policy.

References

Armantier, O., de Bruin, W. B., Topa, G., Klaauw, W., and Zafar, B. (2015). Inflation Expectations And Behavior: Do Survey Respondents Act On Their Beliefs? *International Economic Review*, 56:505–536.

Arnold, E., Dräeger, L., and Fritsche, U. (2014). Evaluating the Link between Consumers' Savings Portfolio Decisions, their Inflation Expectations and Economic News. Macroeconomics and Finance Series 201402, University of Hamburg, Department of Socioeconomics.

Bhandari, A., Borovička, J., and Ho, P. (2016). Identifying Ambiguity Shocks in Business Cycle Models Using Survey Data. NBER Working Papers 22225.

Bougerol, P. (1993). Kalman Filtering with Random Coefficients and Contractions. *SIAM Journal on Control and Optimization*, 31(4):942–959.

Bram, J. and Ludvigson, S. (1998). Does consumer confidence forecast household expenditure? a sentiment index horse race. *Economic Policy Review*, (Jun):59–78.

Carroll, C. D., Fuhrer, J. C., and Wilcox, D. W. (1994). Does Consumer Sentiment Forecast Household Spending? If So, Why? *American Economic Review*, 84(5):1397–1408.

Carvalho, C. and Nechio, F. (2014). Do People Understand Monetary Policy? *Journal of Monetary Economics*, 66(C):108–123.

Coibion, O., Gorodnichenko, Y., and Kamdar, R. (2017). The Formation of Expectations, Inflation and the Phillips Curve. NBER Working Paper 23304.

Dräger, L., Lamla, M. J., and Pfajfar, D. (2016). Are Survey Expectations Theory-Consistent? The Role of Central Bank Communication and News. *European Economic Review*, 85(C):84–111.

Galí, J. (2008). Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. In *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Introductory Chapters. Princeton University Press.

Hamilton, J. D. (1994). Time Series Analysis. Introductory Chapters. Princeton University Press.

Kőszegi, B. and Matějka, F. (2018). An Attention-Based Theory of Mental Accounting. Working paper.

Kuchler, T. and Zafar, B. (2015). Personal Experiences and Expectations About Aggregate Outcomes. Staff Reports 748, Federal Reserve Bank of New York.

Ludvigson, S. C. (2004). Consumer Confidence and Consumer Spending. *Journal of Economic Perspectives*, 18(2):29–50.

Maćkowiak, B., Matějka, F., and Wiederholt, M. (2018). Dynamic Rational Inattention: Analytical Results. *Journal of Economic Theory*, 176(C):650–692.

Maćkowiak, B. and Wiederholt, M. (2009). Optimal Sticky Prices under Rational Inattention. *American Economic Review*, 99(3):769–803.

Malmendier, U. and Nagel, S. (2016). Learning from Inflation Experiences. *The Quarterly Journal of Economics*, 131(1):53–87.

Phillips, A. W. (1958). The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957. *Economica*, 25(100):283–299.

Shiller, R. J. (1996). Why Do People Dislike Inflation? Cowles Foundation Discussion Papers 1115, Cowles Foundation for Research in Economics, Yale University.

Sims, C. A. (2003). Implications of Rational Inattention. *Journal of Monetary Economics*, 50(3):665–690.

Sims, C. A. (2010). Rational Inattention and Monetary Economics. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 4, pages 155–181. Elsevier.

Stark, T. (2013). SPF Panelists' Forecasting Methods: A Note on the Aggregate Results of a November 2009 Special Survey. Research Department, Federal Reserve Bank of Philadelphia.

Veldkamp, L. L. (2011). *Information Choice in Macroeconomics and Finance*. Number 9621 in Economics Books. Princeton University Press.

Appendix A Empiric Robustness

A.1 Inflation and Unemployment Expectations

Consumers believe inflation is countercylical across income and unemployment groups. Table 7 demonstrates that consumers that expect unemployment will rise have higher inflation expectations on average, across highest educational degree achieved. Column (1) uses consumers without a degree, column (2) uses consumers whose highest degree is high-school and column (3) uses consumers who hold a college degree. The coefficients' magnitudes decline as education levels increase, however they remain significant.

Table 8 shows that regardless of income quartile, consumers believe inflation is countercylical. A consumer's income quartile is based on their income relative to the distribution of incomes reported for that month in the MSC. The number of observations is not constant due to bunching at the cutoffs between quartiles. The coefficients are slightly attenuated for higher incomes, but the qualitative takeaways are the same across quartiles.

	(1)	(2)	(3)
More unemployment	0.634**	0.467***	0.282***
	(0.254)	(0.062)	(0.055)
Less unemployment	-0.811***	-0.267***	-0.191***
	(0.309)	(0.071)	(0.059)
Monthly FE	Y	Y	Y
Household FE	Y	Y	Y
Highest Degree	none	high-school	college
R-squared	0.292	0.345	0.349
N	11979	85322	61502

Table 7: By Education Level, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Regression results, by highest degree obtained groups, from $E_{j,t}\pi_{t+12} = \alpha + \beta_t^{more} D_{j,t+12}^{more} + \beta_t^{less} D_{j,t+12}^{less} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D_{j,t+12}^{less}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D_{j,t+12}^{more}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

	(1)	(2)	(3)	(4)
More unemployment	0.571***	0.604***	0.272***	0.320***
	(0.138)	(0.105)	(0.095)	(0.074)
Less unemployment	-0.512***	-0.431***	-0.190*	-0.223***
	(0.159)	(0.125)	(0.103)	(0.080)
Monthly FE	Y	Y	Y	Y
Household FE	Y	Y	Y	Y
Income Quartile	1 (poor)	2	3	4 (rich)
R-squared	0.301	0.353	0.344	0.380
N	27613	26359	25686	32156

Table 8: By Income Quartile, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Income quartiles are based on the consumer's reported income relative to the distribution of incomes reported that month. Regression results, by income quartile, from $E_{j,t}\pi_{t+12} = \alpha + \beta_t^{more} D_{j,t+12}^{more} + \beta_t^{less} D_{j,t+12}^{less} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D_{j,t+12}^{less}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D_{j,t+12}^{more}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

A.2 Expectations to Actions

As discussed in Section 2, consumer beliefs affect their actions in macroeconomic-models. The empirical literature also suggests survey-based consumer expectations predict outcomes such as savings decisions and contain information on future aggregate outcomes. This paper does not focus on investigating the empirical relationship between survey-based expectations and actions since the relationship is already well-established, and the surveys used in this paper (MSC, SCE, SPF) do not contain direct data on the respondent's actions or choices.

There is however data that may be correlated related to choices. That is, the MSC asks three questions (listed below) on if it is a good or bad time to buy a home, durable household goods, and vehicles. It is plausible to expect that people that say it is a good time to buy an item are more likely to buy that item.

- "Generally speaking, do you think now is a good time or a bad time to buy a house?"

- "About the big things people buy for their homes – such as furniture, a refrigerator, stove, television, and things like that. Generally speaking, do you think now is a good or a bad time for people to buy major household items?"

- "Speaking now of the automobile market – do you think the next 12 months or so will be a good time or a bad time to buy a vehicle, such as a car, pickup, van or sport utility vehicle?"

Figure 12: Average 1st Components by Response to Choice-Related Questions

Notes: Data are from MSC. The y-axis is the average first component of the MCA containing questions about expectations and perceptions, across consumers that responded to the choice-related questions

The baseline MCA empirical results did not utilize these questions. This Appendix incorporates the questions in three ways. First, the consumers who state it is a good time to buy items have, on average, a higher first dimension component in the baseline MCA. Second, the addition of the 'choice-related' questions to the baseline MCA does not alter the qualitative takeaways. Third, the first component of an MCA with only the 'choice-related' questions is highly correlated with the first component of the baseline MCA. Taken together, these findings suggest that consumers' choices are related to their expectations, and 'optimistic' consumers are more likely to purchase a home, vehicle, or household durable because they think it is a good time to do so.

To begin, I plot the average 1st dimension component from the baseline MCA, calculated across responses to the 'choice-related' questions in Figure 12. Consumers who respond it is a good time to buy household durables, vehicles, and cars have a higher average 1st component (i.e. are relatively optimistic). Whereas consumers who respond it is a bad time to buy these items have a lower average 1st component (i.e. are relatively pessimistic).

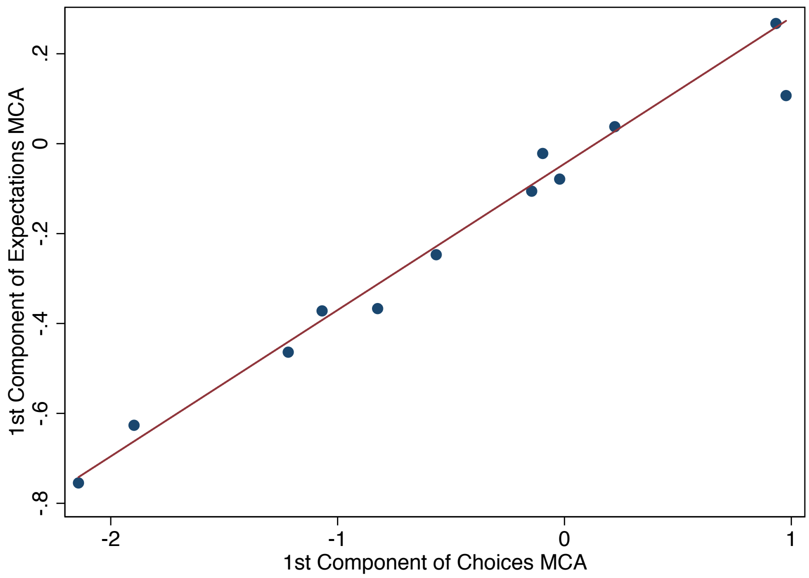

Figure 13: Binscatter of 1st Component of Expectation Questions and 1st Component of Choice-Related Questions (MSC)

Notes: Data are from MSC. The x-axis is the first component of an MCA containing three questions: is it a good or bad time to buy a vehicle, home, household durable. The y-axis is the first component of the MCA containing questions about expectations and perceptions.

Table 9 presents the first component loadings for an MCA with all baseline questions and the three 'choice-related' questions. As with the baseline results, the signs of the loadings are such that a 'pessimistic' belief has a negative loading whereas a 'optimistic' belief has a positive loading.[18] For example, the 'optimistic' belief that it is a good time to purchase a home, vehicle, or household durable all have positive loadings in the first component.

Next, I conduct an MCA on only the 'choice-related' questions. The first component of the choice-related questions and the first component of the baseline MCA are plotted in Figure 13's binscatter. The first component of choice-related-questions is strongly correlated with the first component of expectation-questions.

[18]The one question that deviates from the pattern is the respondent's expectation on how rates will change. Responding decrease or increase has a small negative loading and responding stay the same has a positive loading. This could be because whether rate changes are 'good' or 'bad' for a respondent depends on their savings and debt.

(1) “optimistic”	(2) “same”	(3) “pessimistic”
Unemployment will:		
decrease	same	increase
1.60	0.55	-1.63
Inflation will be:		
$\leq 0\%$	> 0 and $\leq 4\%$	≥ 4
0.68	0.57	-0.88
Personal financial conditions will:		
improve	same	decline
1.03	-0.14	-2.44
Real income will:		
increase	same	decrease
1.46	0.45	-1.28
Rates will:		
decrease	same	increase
-.17	0.33	-0.08
Business conditions will:		
improve	same	decline
1.36	0.08	-2.22
Personal financial conditions have:		
improved	same	declined
.98	-0.07	-1.31
Business conditions have:		
improved	same	declined
1.33	0.18	-1.31
Economic policy is:		
good	fair	poor
1.65	0.26	-1.61
Good/bad time to buy a house:		
good time	pro-con	bad-time
0.55	-1.10	-1.31
Good/bad time to buy household durables:		
good time	pro-con	bad-time
0.53	-0.75	-1.57
Good/bad time to buy a vehicle:		
good time	pro-con	bad-time
0.64	-0.77	-1.30

Table 9: 1st Dimension Loadings for an MCA on MSC, Includes Choice-Related Questions

Notes: Data are from the MSC. Multiple correspondence analysis' first component loadings are reported. Forward looking questions compare the 12 month expectation to the present. Backward looking questions compare the present to 12 months ago. The inflation response is a continuous measure; however, for the MCA I bucket the values. The rates expectation question is colored in gray as it is the only question whose coefficients are not ordered from largest to smallest going from answers that are optimistic to those that are pessimistic.

In summary, responses to choice-related questions are highly correlated to expectation-related questions, with 'optimistic' consumers saying it is a good time to make purchases. Assuming a respondent who says it is a good time to buy a home, vehicle, or household durable is more likely to do so, then expectations will be correlated to real actions.

Appendix B Proofs

B.1 Utility Function: Second Derivatives

This appendix demonstrates that whenever $\varphi \neq 1$ (i) the sign of $\frac{\hat{u}_{13}}{\hat{u}_{12}} = -1$ and (ii) the weights on wage and price log-deviations in optimal labor are equal, but opposite signs. First, recall the utility function:

$$U(C_j, L_j) = \frac{C_j^{1-\varphi}}{1-\varphi} - \frac{L_j^{1+1/\eta}}{1+1/\eta}$$

Substituting the budget constraint $C_j = \frac{W_j L_j}{P}$ into the utility function results in:

$$U(L_j, W_j, P_j) = \frac{\left(\frac{W_j L_j}{P_j}\right)^{1-\varphi}}{1-\varphi} - \frac{L_j^{1+1/\eta}}{1+1/\eta}$$

The utility function written in log-deviations is:

$$\hat{u}(l_j, w_j, p_j) = \frac{\left(\frac{\bar{W}e^{w_j}\bar{L}e^{l_j}}{\bar{P}e^{p_j}}\right)^{1-\varphi}}{1-\varphi} - \frac{(\bar{L}e^{l_j})^{1+1/\eta}}{1+1/\eta}$$

For $\varphi \neq 1$, the second order partial derivative of $\hat{u}$ with respect to labor and wage evaluated at the steady state is:

$$\hat{u}_{12} = (1-\varphi)\left(\frac{\bar{W}\bar{L}}{\bar{P}}\right)^{-\varphi+1}$$

Similarly, for $\varphi \neq 1$, the second order partial derivative of $\hat{u}$ with respect to labor and price evaluated at the steady state is:

$$\hat{u}_{13} = (\varphi-1)\left(\frac{\bar{W}\bar{L}}{\bar{P}}\right)^{-\varphi+1}$$

The ratio of the second order partial derivatives is negative one when $\varphi \neq 1$:

$$\frac{\hat{u}_{12}}{\hat{u}_{13}} = -1$$

Recall that the weights on wage and price log-deviations on optimal labor log-deviations were, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$ and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$, respectively. If $\varphi \neq 1$, then the weights will be of equal magnitude but have opposite signs.

$$\hat{u}_{12} = -\hat{u}_{13} \Rightarrow \frac{\hat{u}_{12}}{|\hat{u}_{11}|} = -\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$$

B.2 Dynamic Model Solution Algorithm

This appendix explains the numerical solution of the dynamic rational inattention model. I begin with notation and the Kalman filter equations and then proceed to describe how to obtain the format of the optimal signal and the variance of the signal error.

Equation (12) is the state-space representation of the AR(1) processes that govern wage and price. Equation (13) is the period t signal. The signal error, ϵ_t, is normally distributed with mean zero and standard deviation σ_ϵ.

$$\xi_{t+1} = F\xi_t + \epsilon^{\xi}_{t+1} \tag{12}$$

$$S_t = h'\xi_t + \epsilon_t \tag{13}$$

$$\text{where } \xi_t \equiv \begin{bmatrix} w_{j,t} \\ p_{j,t} \end{bmatrix}, \; F \equiv \begin{bmatrix} \phi_w & 0 \\ 0 & \phi_p \end{bmatrix}, \; \epsilon^{\xi}_{t+1} \equiv \begin{bmatrix} \theta_w \epsilon^w_{j,t} \\ \theta_p \epsilon^p_{j,t} \end{bmatrix}, \; h \equiv \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

Let $\Sigma_{t|t-1}$ and $\Sigma_{t|t}$ be the variance-covariance matrices of ξ_t conditional on $\mathcal{I}_{t-1}$ and $\mathcal{I}_t$, respectively. Define $\Sigma_1 \equiv \lim_{t\to\infty} \Sigma_{t|t-1}$ and $\Sigma_0 \equiv \lim_{t\to\infty} \Sigma_{t|t}$. Let Q be the variance-covariance matrix of ϵ^{ξ}_{t+1}. From the Kalman filter equations (Hamilton (1994)), we have equation (14) and (15) that govern how the conditional variance-covariance matrices update.

$$\Sigma_{t+1|t} = F\Sigma_{t|t}F' + Q \tag{14}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}h \left(h'\Sigma_{t|t-1}h + \sigma^2_\epsilon\right)^{-1} h'\Sigma_{t|t-1} \tag{15}$$

Taking limits, Σ_1 and Σ_0 are:

$$\Sigma_1 = F\Sigma_0 F' + Q$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 h \left(h'\Sigma_1 h + \sigma_\epsilon^2\right)^{-1} h'\Sigma_1$$

Recall from the paper that the agent is seeking to minimize equation (16).

$$\min_{h,\sigma_\epsilon} \ \mathbb{E}_j \left[\left(l_{j,t}^\diamond - \mathbb{E}_j(l_{j,t}^\diamond | \mathcal{I}_t) \right)^2 \right] + \frac{\lambda}{2} \log_2 \left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2} \right) \tag{16}$$

The following two points allow the minimization problem in equation (16) to be re-written as equation (17). First, for a given prior variance-covariance ($\Sigma_{t|t-1}$), the posterior variance-covariance ($\Sigma_{t|t}$) evolves according to the Kalman filter dynamic equations above, and converges to limiting conditional variance-covariance Σ_0. Since the agent at time zero has received an infinite set of signals, their posterior variance-covariance after time zero does not change and remains at Σ_0. Second, recall that optimal labor is a linear combination of wage and price ($l_j^\diamond = \frac{\hat{u}_{12}}{|\hat{u}_{11}|} w_j + \frac{\hat{u}_{13}}{|\hat{u}_{11}|} p_j$). So the conditional variance-covariance of $l_j^\diamond$, is the conditional variance of $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} w_j + \frac{\hat{u}_{13}}{|\hat{u}_{11}|} p_j$.

$$\min_{h,\sigma_\epsilon} \ \mathbb{E}_j [w'\Sigma_0 w] + \frac{\lambda}{2} \log_2 \left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2} \right), \text{ where } w = \begin{bmatrix} \frac{\hat{u}_{12}}{|\hat{u}_{11}|} \\ \frac{\hat{u}_{13}}{|\hat{u}_{11}|} \end{bmatrix} \tag{17}$$

Now, finding the signal weights and the variance of the signal error that optimize the objective function is straightforward. It amounts to searching over signal weights h and signal variance σ_ϵ to minimize equation (17). For any combination of h and σ_ϵ, Σ_1 can be solved by iterating equation (18) to a fixed point. Once Σ_1 is found, equation (19) solves for Σ_0.

$$\Sigma_1 = F \left(\Sigma_1 - \Sigma_1 h (h'\Sigma_1 h + \sigma_\epsilon^2)^{-1} h'\Sigma_1 \right) F' + Q \tag{18}$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 h (h'\Sigma_1 h + \sigma_\epsilon^2)^{-1} h'\Sigma_1 \tag{19}$$