Rationally Inattentive Monetary Policy

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Abstract

This paper studies optimal monetary policy under rational inattention: the policy maker optimally chooses her information subject to a processing constraint. Our analytical results emphasize how the policy maker's information choices shape her expectations and the dynamics of the macroeconomy. Paying attention to demand shocks lowers output volatility and causes untracked supply shocks to drive inflation. Because persistent supply shocks have a minor impact on interest rates under full information in the New Keynesian model, the policy maker should focus her limited attention on demand shocks. Improvements in information can explain a declining slope of the empirical Phillips curve.

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JEL Classification: D8, E3, E5

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1 Introduction

Information is a key component of monetary policy decision-making. Policy makers must choose what information to pay attention to and what to ignore. As emphasized by recent chairmen of the Federal Reserve, limited information leads to considerable uncertainty about the current state of the economy, the trajectory of the economy, and whether this trajectory is predominantly driven by demand or supply-side forces. For example, Ben Bernanke said: "Uncertainty about the current state of the economy is a chronic problem for policymakers. At best, official data represent incomplete snapshots of various aspects of the economy, and even then they may be released with a substantial lag and be revised later."[1] Although important in practice, this informational choice is absent from existing analyses of optimal monetary policy which assume policy makers have access to all available information about the economy or an exogenously restricted subset. In this paper, we relax this assumption. We study the determination of policy makers' information choices, the optimal monetary policy, and the implied equilibrium dynamics. Our main contribution uses closed-form analytics to show how the policy maker's information choices shape her expectations and the equilibrium dynamics of the economy.

To accomplish our goal, we add rational inattention (Sims, 2003) to the optimal monetary policy problem of a canonical New Keynesian model driven by exogenous demand and supply shocks (e.g. Galí, 2015). Rational inattention constrains the amount of information that the policy maker can obtain about the state of the economy and use in monetary policy design. This forces her to trade-off her attention between demand factors and supply factors: paying more attention to one necessitates paying less attention to the other. To build intuition for the effects of this trade-off and to derive analytical results, we first consider the case in which demand and supply shocks are independent and identically distributed (i.i.d.). We obtain expressions for the policy maker's expectations of these shocks, the paths of nominal interest rates, output, and inflation, and the optimal attention allocation over the shocks.[2]

When information processing capacity is limited, the policy maker must divide her attention between knowing about prevailing supply-side and demand-side conditions. This trade-off affects expectation formation through two channels. First, limited information

[1] Similarly, Jerome Powell at the Jackson Hole Symposium (2018) discussed the of difficulty implementing monetary policy because 'star' variables like the natural rate of unemployment (u^*) or the neutral real rate (r^*) are unknown and moving. "Guiding policy by the stars... has been quite challenging of late because our best assessments of the location of the stars have been changing significantly." The quotation from Ben Bernanke is from the Economic Policy Conference (2007).

[2] In reality, policy makers do not literally track such shocks. Instead, the shocks capture the unanticipated variations in demand and supply conditions that the policy maker would like to pay attention to. For example, the Federal Reserve's Beigebooks frequently discuss the results of consumer surveys, firm surveys, and supply chain information, indicating the policy makers' interests in the prevailing demand and supply conditions.

about current shocks causes the policy maker to place positive weight on her prior beliefs. In the i.i.d. case, this attenuates her posterior expectations of the shocks towards zero. The informational trade-off implies that dampening the expectation attenuation of one shock comes at the cost of strengthening the attenuation of the other shock. Second, the noisy information acquired under rational inattention creates endogenous and stochastic variation in the policy maker's optimal expectations over time. This noise is subject to the same informational trade-off: forming more precise expectations of one shock necessitates forming noisier expectations of the other.

Output dynamics are determined by the informational focus of the policy maker as well as the usual intertemporal substitution channel. In equilibrium, the policy maker adjusts interest rates as a function of her expectations of demand and supply shocks: she lowers rates to accommodate increases in supply, and hikes rates to offset inefficient increases in demand. Optimality implies that monetary policy responds more strongly and more precisely to the shock that the policy maker pays more attention to. As a result, output fluctuations are larger when the policy maker focuses more on supply shocks because interest rate policy is more accommodating of changes in supply, and does not fully offset changes in demand due to a lack of information. Conversely, an increased focus on demand shocks dampens output fluctuations as policy aggressively offsets changes in demand but does not accommodate changes in supply as much.

Deviations in output from its efficient path create output gaps which cause inflation responses that are absent in the complete information economy. Inflation responds positively to demand shocks and negatively to supply shocks. The sizes of these responses depend on the informational trade-off. In particular, inflation responds more to the shock that the policy maker devotes less attention to. Intuitively, if the policy maker focuses mainly on supply shocks, then output gaps and hence inflation are mainly driven by changes in demand that are not offset by monetary policy. The informational focus of policy makers also determines the sign and strength of the co-movement between inflation and real activity. A stronger focus on supply shocks generates more volatile output and a strong positive co-movement between inflation and output driven by demand shocks. In contrast, if the policy maker more carefully tracks changes in demand, then output will be less volatile and will exhibit a weaker and negative co-movement with inflation driven by supply shocks. Thus, the disappearance of the empirical Phillips curve in recent data (Hall, 2013) may be partially attributable to increased attention on demand factors by policy makers.[3]

Noisy expectations create a stochastic component of monetary policy that is orthogonal

[3]In contrast, McLeay and Tenreyro (2020) argue that the disappearance of the empirical Phillips curve is attributable to optimal monetary policy responses to cost-push shocks under complete information.

to the exogenous demand and supply shocks. In equilibrium, these shocks cause a positive co-movement between output and inflation, and so operate like traditional monetary policy shocks (Christiano et al., 2005). However, rather than being exogenous, our monetary policy shocks are an optimal and endogenous response to the policy maker's information constraint, and have a natural origin: limited information leads to noisy monetary policy.

We show that the optimal monetary policy can be implemented using a feedback rule that depends on noisy observations of output and inflation. Unlike the existing literature, we do not assume that these observations are available to the policy maker ex-ante (e.g. Aoki, 2003). Instead, the policy maker endogenously chooses to include these observations in her information set to best inform her beliefs about the economy, thus providing a justification for why policy makers should focus on the dynamics of output and inflation when designing monetary policy. Furthermore, our implementation shows that a focus on demand shocks by the policy maker is associated with the precise measurement and stabilization of output, while a focus on supply shocks is implemented via the more precise measurement and stabilization of inflation. Hence, the modern "price stabilization" goal of central banks around the world can be interpreted as an implicit focus on tracking supply shocks more than demand shocks.[4] Intuitively, stabilizing output reduces both inefficient demand-driven output volatility and also efficient supply-driven volatility, which is optimal for a demand-focused policy maker. In contrast, traditional inflation stabilization mostly offsets demand-driven output gaps, and so is consistent with a policy maker focused on accommodating supply changes. Finally, under such a rule, we show that a sufficiently limited information capacity causes equilibrium indeterminacy because the the policy maker's optimal responses to shocks become too weak to rule out the existence of sunspot equilibria.

The optimal informational focus of the policy maker depends crucially on the persistence of exogenous shocks. In the i.i.d. case, we show analytically that the information processing capacity allocated to a shock is intuitively increasing in that's shock's variance. However, when we calibrate the shock persistences to plausible values, the optimal information allocation is heavily biased towards demand shocks. This asymmetric allocation of processing capacity reflects how each shock affects the efficient real interest rate that the policy maker would like to imitate through monetary policy. Intuitively, very persistent supply shocks do not have large effects on expected growth rates of output and consumption, which are the main determinants of interest rates in equilibrium. Hence, monetary policy is best served by focusing on demand-side factors which have a stronger impact on efficient interest rates that policy makers wish to replicate.

[4] Relatedly, the "dual mandate" present in the Federal Reserve's focus on both prices and employment may be interpreted as balancing both demand and supply considerations.

Finally, we explore how increasing the policy maker's information capacity affects model outcomes, and compare them to macroeconomic trends in post-WW2 data. As the capacity increases, equilibrium dynamics converge to their efficient paths, volatility declines, and the co-movement of inflation with output growth falls. These patterns are consistent with the increasing accuracy of Federal Reserve beliefs as stated in Greenbooks (Tulip, 2009), the decline in empirical macroeconomic volatility (the "Great Moderation"), and offers a new interpretation of the disappearance of the Phillips curve linking inflation and real activity. Through the lens of our model, higher information capacity leads to less price volatility and a weaker correlation between inflation and output growth, even though the structural Phillips curve is stable over time.

Related Literature To the best of our knowledge, we are the first to study optimal monetary policy with endogenous information acquisition. Aoki (2003) studies optimal monetary policy when the policy maker has complete knowledge of the economy's past, but only observes exogenously noisy measures of current output and inflation. We instead use rational inattention to fully endogenize the policy maker's information set. Our approach yields new insights that exogenous information settings obscure. First, we can analyze the important interactions between the policy maker's information choices, her expectations, and the equilibrium dynamics. Second, we find that monetary policy shocks arise endogenously and are an optimal response to limited information rather than a purely exogenous disturbance. Third, we show how the optimal policy can be implemented using noisy measures of output and inflation, where the coefficients on output and inflation, and the noise in the observations themselves are endogenous functions of the optimal information allocation. This dependence allows us to derive the conditions under which the policy maker should track inflation more accurately than output, and when the policy maker should respond more strongly to inflation than output. This analysis is impossible in an exogenous information setting. The advantages of our approach also apply to Boehm and House (2019) who study optimal monetary policy with exogenously restricted information and the additional assumption that policy must be conducted using a Taylor rule.

Although our contribution is applied in nature, we also connect our results to the theoretical results of Svensson and Woodford (2003, 2004), who study linear-quadratic optimal monetary policy problems when policy makers only observe exogenously noisy indicators of economic variables. They show that uncertainty does not affect the conduct of optimal policy when it uses optimal estimates of the variables ("certainty equivalence"), and that the solution to the expectation formation problem cannot be separated from the solution to the equilibrium dynamics problem when policy makers have less information than the private sector. We confirm that both of these properties continue to hold with endogenous information acquisition. In addition, we show how information acquisition choices interact with both the expectation formation process and the determination of equilibrium dynamics. These feedback mechanisms are completely absent in settings with exogenous information.

Our analysis also relates to the recent literature that merges the New Keynesian framework with models of incomplete information. For instance, Woodford (2010) and Adam and Woodford (2012) characterize robust, in the sense of Hansen and Sargent (2005), monetary policy under uncertainty about private sector beliefs. Alternatively, Paciello and Wiederholt (2014) and Angeletos and La'O (2020) study optimal monetary policy when firms have limited information, while Maćkowiak and Wiederholt (2009), and Afrouzi and Yang (2020) study firm pricing behavior under rational inattention. Finally, Maćkowiak and Wiederholt (2015) study business cycle dynamics when both firms and households face inattention constraints. We instead focus on the effects of limited information for the policy maker, and stress how her informational choices shape and are shaped by her expectations and the equilibrium dynamics.

Finally, our results contribute to the discussion around the flattening of the empirical Phillips curve. Closest to us, McLeay and Tenreyro (2020) show how optimal monetary policy under full information can impart a negative correlation between inflation and output in the presence of cost-push shocks. In contrast, we show how limited information can affect the empirical correlation by affecting whether demand or supply shocks drive business cycle fluctuations. At low information, demand shocks dominate and induce a positive correlation between inflation and real activity. As information improves, supply shocks play a larger role and weaken the correlation towards zero. Our approach complements others who have emphasized the role of private sector inflation expectations in estimations of the structural Phillips curve. Hazell et al. (2020) and Jørgensen and Lansing (2021) highlight the importance of anchored inflation expectations over the past two decades, while Coibion et al. (2018) demonstrate the stability of the Phillips curve when estimated using survey-based inflation expectations of households.

The paper proceeds as follows. Section 2 describes the economic environment, information frictions, and sets up the optimal monetary policy problem. We present our main analytical results in Section 3, and the extension to persistent shocks in Section 4. We discuss the link between information amd macroeconomic trends in Section 5. Section 6 concludes.

2 Setup

We conduct our analysis in a standard sticky-price New Keynesian model augmented to allow for endogenous information choices by the policy maker. Given its familiarity, we leave a full description of the model to Appendix A. Here, we briefly describe the key equations before introducing the policy maker's information frictions. Throughout, "hatted" variables denote log deviations from the deterministic steady state.

Aggregate Shocks The economy is driven by two exogenous stochastic processes that drive the household discount rate $\hat{\rho}_t$ and total factor productivity (TFP) $\hat{a}_t$,

$$\hat{\rho}_t = \delta_\rho \hat{\rho}_{t-1} + e_{\rho,t} \tag{1}$$

$$\hat{a}_t = \delta_a \hat{a}_{t-1} + e_{a,t} \tag{2}$$

where $\delta_\rho, \delta_a \in [0,1)$, $e_{\rho,t} \sim N\left(0, \sigma_\rho^2\right)$, and $e_{a,t} \sim N\left(0, \sigma_a^2\right)$.[5] We interpret and refer to these shocks as sources of exogenous variation in demand and supply respectively. Intuitively, a positive discount rate shock increases the marginal utility of current consumption and hence raises contemporaneous aggregate demand, while a positive TFP shock increases the economy's production capacity holding inputs fixed.[6]

The Efficient Flexible Price Benchmark When prices are flexible and monetary policy is neutral, outcomes are Pareto efficient, and are described by "starred" variables for output and the real interest rate that satisfy the log-linear equations

$$\hat{y}_t^* = \tfrac{1+\varphi}{1/\gamma+\varphi} \hat{a}_t \tag{3}$$

$$r_t^* = \rho + \hat{\rho}_t - \tfrac{1+\varphi}{1+\gamma\varphi} \left(1 - \delta_a\right) \hat{a}_t \tag{4}$$

where $\gamma > 0$ is the elasticity of intertemporal substitution, and $1/\varphi > 0$ is the Frisch elasticity of labor supply.

Equilibrium Dynamics under Sticky Prices As is standard, we analyze outcomes under sticky prices in terms of their deviations from the efficient benchmark. Outcomes are

[5] We assume aggregate shocks are independent to clearly deliver the main insights for optimal monetary policy. Correlated shocks would result in the monetary authority's limited information about one shock to effect her beliefs about both shocks.

[6] An alternative supply shock common in the New Keynesian literature is a shock to firms' mark ups. We show that our analysis naturally extends to these shocks in Appendix D.

described by two log-linear equations,

$$\mathbb{E}_t \tilde{y}_{t+1} - \tilde{y}_t = \gamma \left(\iota_t - \mathbb{E}_t \pi_{t+1} - r_t^*\right) \tag{5}$$

$$\pi_t = \varphi_y \tilde{y}_t + \frac{1}{1+\rho} \mathbb{E}_t \pi_{t+1} \tag{6}$$

The Euler equation (5) describes the dynamics of the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^*$, defined as the log deviation of equilibrium output $\hat{y}_t$ from its efficient level. $\iota_t - \mathbb{E}_t \pi_{t+1}$ is the equilibrium real interest rate where ι_t is the nominal interest rate and π_t is inflation. γ governs how strongly output gaps respond to the gap between the equilibrium real interest rate and its efficient counterpart r_t^*. The New Keynesian Phillips Curve (6) describes the dynamics of inflation. $\varphi_y > 0$ governs how strongly inflation responds to output gaps. Note that the expectation operator $\mathbb{E}_t$ is the standard full-information rational expectations operator. We do not incorporate any information constraints on households or firms to cleanly identify the effects of policy maker inattention on optimal monetary policy.[7,8]

2.1 Limited Information under Rational Inattention

To model endogenous information with limited processing capacity, we follow the literature on rational inattention (Sims, 2003). We use the concept of mutual information to quantify how much information the policy maker processes and ultimately uses to implement the path of nominal interest rate rates $\{\iota_t\}$. Formally, mutual information uses entropy to measure uncertainty and quantifies information flow as reduction in uncertainty.[9] For example, if random vectors $\mathbf{Y} = (Y_t, Y_{t-1}, ..., Y_{t-T})$ and $\mathbf{X} = (X_t, X_{t-1}, ..., X_{t-T})$ are jointly normal, their mutual information is

$$I(\mathbf{X}; \mathbf{Y}) = \frac{1}{2} \log_2[(2\pi e)^{T+1} \det \Omega_{\mathbf{X}}] - \frac{1}{2} \log_2[(2\pi e)^{T+1} \det \Omega_{\mathbf{X}|\mathbf{Y}}] \tag{7}$$

where $\Omega_{\mathbf{X}}$ is the unconditional covariance matrix of $\mathbf{X}$, and $\Omega_{\mathbf{X}|\mathbf{Y}}$ is the covariance matrix conditional on $\mathbf{Y}$. The first term computes the entropy of $\mathbf{X}$, while the second computes

[7] In contrast, Maćkowiak and Wiederholt (2015) study a general equilibrium environment where households and firms are rationally inattentive but the monetary authority is not.

[8] We acknowledge that in reality the private sector may also be imperfectly informed, and this may have implications for optimal monetary policy. For instance, the monetary authority's actions could have an informational effect on the private sector (e.g., Nakamura and Steinsson (2018) and Kohlhas (2020)). The question of how private sector and central bank information frictions interact are outside the scope of this paper and left for future work.

[9] Although common in the literature, mutual information is not the only way of modeling rational inattention. In Appendix E, we show that our analysis is robust to modeling rational inattention using the information cost proposed by Hébert and Woodford (2020).

the entropy of $\mathbf{X}$ conditional on $\mathbf{Y}$. Mutual information measures the information about $\mathbf{X}$ contained in $\mathbf{Y}$ by computing the reduction in uncertainty about $\mathbf{X}$ attained by conditioning on $\mathbf{Y}$. As $T \to \infty$, we can compute the average per-period mutual information between the stochastic processes: $\mathcal{I}(\{X\};\{Y\}) = \lim_{T\to\infty} \frac{1}{T} I(\mathbf{X};\mathbf{Y})$ (Maćkowiak and Wiederholt, 2009).

In our setting, we will set $X_t = (\hat{a}_t, \hat{\rho}_t)'$, and we will derive the optimality condition $Y_t = \mathbb{E}_{M,t} X_t$ where $\mathbb{E}_{M,t}$ is the policy maker's endogenous expectation operator. We then adopt a mutual information constraint $\mathcal{I}(\{X\};\{Y\}) = \kappa_M$, where $\kappa_M \geq 0$ is the exogenous per-period information capacity of the policy maker. $\mathcal{I}$ measures the per-period mutual information between the stochastic histories of X and the policy maker's expectations of them. When $\kappa_M = 0$, the policy maker cannot process any information. Therefore Y_t does not contain any information about X_t. In the limit as $\kappa_M \to \infty$, the policy maker can process sufficient information to know X_t with certainty, and can set $Y_t = X_t$.[10]

Finally, to maintain tractability, we follow the rational inattention literature and assume that the expectation formation mapping from X_t to Y_t is stationary for all $t \geq 0$ (Maćkowiak and Wiederholt, 2009). This can be achieved by the policy maker receiving a long history $\mathbf{Y}_\infty = (Y_{-1}, Y_{-2}, ..., Y_{-\infty})$ at the start of period $t = 0$, that summarizes the information collected by the policy maker's predecessors.

2.2 The Optimal Monetary Policy Problem

The optimal monetary policy problem can be solved in two stages. First, in period $t = -1$ the policy maker chooses her information structure, that is how much attention to allocate to supply shocks and how much attention to allocate to demand shocks. This division of attention characterizes how her expectations, $\mathbb{E}_M$, are formed before any information is received. Then in each period $t \geq 0$, the policy maker chooses ι_t given her information set. We now discuss key features of the optimal policy problem with detailed derivations in Appendix B. We provide a complete closed form characterization of the optimal monetary policy when aggregate shocks are i.i.d. in Section 3.

Optimal Interest Rate Choice To determine the optimal choice of ι_t, we assume that the policy maker wishes to minimize the deviations of the equilibrium from the efficient benchmark. To second order, these deviations are summarized by the per-period household

[10] We have stated the rational inattention constraint directly in terms of beliefs. An alternative would be to state the problem in terms of optimal signals which would be of the form X_t plus noise, and would inform the expectations. As shown by Maćkowiak and Wiederholt (2009), these approaches are equivalent.

utility loss

$$\ell_t = \frac{1}{2}\left(\tilde{y}_t^2 + \frac{\xi}{1/\gamma + \varphi}\pi_t^2\right) \tag{8}$$

where $\xi > 0$ governs the size of the losses due to price adjustments.

We solve for the optimal monetary policy under discretion. This means that the policy maker chooses the path of nominal interest rates sequentially. In addition to its tractability, we believe that this approach is reasonable given the informational frictions faced by the policy maker that would make policy commitments difficult to enforce. It also makes our results easily comparable to the case of exogenous information in Aoki (2003).

In each period $t \geq 0$, the policy maker solves

$$\min_{\iota_t} \mathbb{E}_{M,t}\left[\frac{1}{2}\left(\tilde{y}_t^2 + \frac{\xi}{1/\gamma + \varphi}\pi_t^2\right)\right] \tag{9}$$

subject to (5) and (6). This minimization problem has an intuitive solution.

Proposition 1. *The optimal discretionary monetary policy satisfies $\iota_t = \mathbb{E}_{M,t}\left[r_t^*\right]$, so that using* (4)*, we obtain*

$$\iota_t = \rho + \mathbb{E}_{M,t}\left[\hat{\rho}_t\right] - \frac{1+\varphi}{1+\gamma\varphi}\mathbb{E}_{M,t}\left[\hat{a}_t\right]. \tag{10}$$

Proposition 1 echoes the certainty equivalence result in Aoki (2003). The linear-quadratic structure of the policy problem implies that the optimal monetary policy is simply the subjective expectation of the optimal policy under complete information. The policy equation (10) also shows that the policy maker's expectations of demand and supply shocks play a key role in the conduct of optimal monetary policy, and justifies setting $X_t = (\hat{a}_t, \hat{\rho}_t)'$ and $Y_t = \mathbb{E}_{M,t}X_t$ in our application of rational inattention theory.

Our formulation of the information constraint $\mathcal{I}(\{X\};\{Y\}) = \kappa_M$ implies that the policy maker must first collect information about demand and supply conditions, and then use that information to form a belief about the efficient real interest rate. We believe this approach is most consistent with how policy makers gather and use information in reality when setting monetary policy. For example, the Federal Reserve's Beigebooks frequently discuss the results of consumer surveys, firm surveys, and supply chain information, indicating the policy makers' interests in learning about the prevailing demand and supply conditions. We do not allow the policy maker to learn about r_t^* directly since it seems unrealistic, and would abstract from the key informational trade-off in which we are interested.[11]

Optimal Information Structure Choice To write down the first subproblem, we first simplify the information constraint $\mathcal{I}\left(\{(\hat{\rho}_t, \hat{a}_t)'\}; \{(\mathbb{E}_{M,t}[\hat{\rho}_t], \mathbb{E}_{M,t}[\hat{a}_t])'\}\right) \leq \kappa_M$. Following Maćkowiak and Wiederholt (2009), we can exploit the independence of the exogenous shocks and write the constraint as

$$\underbrace{\mathcal{I}\left(\{\hat{\rho}_t\}; \{\mathbb{E}_{M,t}[\hat{\rho}_t]\}\right)}_{=\kappa_\rho} + \underbrace{\mathcal{I}\left(\{\hat{a}_t\}; \{\mathbb{E}_{M,t}[\hat{a}_t]\}\right)}_{=\kappa_a} \leq \kappa_M \tag{11}$$

where $\kappa_\rho \geq 0$ and $\kappa_a \geq 0$ denote the mutual information concerning demand shocks and supply shocks respectively. (11) states that the information processing capacity devoted to each shock cannot exceed the total capacity available: $\kappa_\rho + \kappa_a \leq \kappa_M$.

Given this simplification, we show in Appendix B that the stochastic processes for the policy maker's expectations, ι_t, $\tilde{y}_t$, and π_t are all functions of κ_ρ and κ_a. Therefore, let $\ell(\kappa_\rho, \kappa_a)$ denote the optimal value of (9), taking information as given. The first subproblem is stated as $\min_{\kappa_\rho,\kappa_a} \mathbb{E}\ell(\kappa_\rho, \kappa_a)$ subject to $\kappa_\rho + \kappa_a \leq \kappa_M$, where $\mathbb{E}$ is the ex-ante expectation operator defined before the policy maker receives any information.

3 A Closed Form Solution

To solve the optimal policy problem in closed form, we temporarily assume that aggregate shocks are i.i.d. over time.[12]

Assumption 1. $\delta_\rho = \delta_a = 0$.

To build intuition for the solution to the Ramsey problem, we proceed in stages. We first characterize the the policy maker's optimal expectations. We then derive the equilibrium paths for macroeconomic variables and show how to implement the optimal policy using a feedback rule that specifies the nominal interest rate as a function of observables. Throughout, we stress the effects of informational choices. Finally, we discuss the optimal information allocation that underlies the optimal expectation formation and equilibrium dynamics.

[11]If we were to take this alternative approach, the monetary authority would form an optimal expectation of the efficient real rate directly, rather than of the primitive shocks. However, as argued in Maćkowiak and Wiederholt (2009), this approach is unappealing in contexts when the decision maker cannot realistically attend to the optimal action directly.

[12]We provide a partial analytical characterization of the persistent shock case in Section 4, which also includes a numerical analysis.

3.1 Optimal Expectations

Recall that optimal monetary policy satisfies $\iota_t = \rho + \mathbb{E}_{M,t}[\hat{\rho}_t] - \frac{1+\varphi}{1+\gamma\varphi}\mathbb{E}_{M,t}[\hat{a}_t]$. In the i.i.d. case, the expectations take on a simple form.

Proposition 2. *Under Assumption 1 the policy maker's expectations satisfy*

$$\mathbb{E}_{M,t}[\hat{a}_t] = (1 - 1/2^{2\kappa_a})\hat{a}_t + (\sqrt{2^{2\kappa_a} - 1}/2^{2\kappa_a})\sigma_a v_t \tag{12}$$

$$\mathbb{E}_{M,t}[\hat{\rho}_t] = (1 - 1/2^{2\kappa_\rho})\hat{\rho}_t + (\sqrt{2^{2\kappa_\rho} - 1}/2^{2\kappa_\rho})\sigma_\rho u_t \tag{13}$$

where v_t and u_t are i.i.d. standard normal random variables.

To understand Proposition 2, we first consider the equations separately, and then discuss their joint relationship. In isolation, it is sufficient to discuss (12); the interpretation of (13) is analogous. The expression for $\mathbb{E}_{M,t}[\hat{a}_t]$ is composed of two pieces. The first shows that the policy maker attenuates her expectation of the supply shock towards zero. Intuitively, when the policy maker updates her posterior mean using her limited information, she places positive weight on her prior mean, which is zero under Assumption 1. Furthermore, the extent of attenuation is decreasing in the endogenously chosen information capacity variable κ_a. As the policy maker devotes more attention to supply shocks so that κ_a increases, she lowers the weight on her prior when forming her expectation of the current shock.

The second piece is driven by the random variable v_t, and adds pure noise to the policy maker's expectation. This noise reflects the uncertainty that the policy maker faces about the true supply shock, and causes her expectation to stochastically deviate from her mean posterior expectation over time. We stress that this noise is an endogenous outcome, and is generated by the information constraint. Furthermore, the variance of the noise also depends on the information capacity choice, but now in a non-monotonic manner. When κ_a is low, the policy maker pays little attention to tracking supply shocks and so does not introduce much noise into her expectation, which is closely tied to the long run mean of zero. As κ_a increases, more attention initially introduces more noise to the expectation, until eventually the policy maker obtains sufficient information to know the shock with certainty, reducing the noise component to zero.

In equilibrium, these expectations are jointly determined, and the information capacity choices must satisfy the binding constraint $\kappa_a + \kappa_\rho = \kappa_M$. Therefore, if the policy maker wishes to obtain more accurate expectations of one shock (less attenuation and noise), she must accept less accurate expectations of the other shock. The existence of this informational trade-off contrasts sharply with the previous literature that takes information as given (Aoki, 2003; Svensson and Woodford, 2004), and highlights how endogenous information choices determine the optimal expectations. As we show below, these choices also feed into the equilibrium dynamics of output and inflation, which in turn feedback into the policy maker's optimal choices of κ_a and κ_ρ.

Having evaluated the expectations, we can now write the optimal path of nominal (equal to real) rates in explicit form,

$$\begin{aligned} \iota_t = \rho &+ (1 - 1/2^{2\kappa_\rho})\hat{\rho}_t + (\sqrt{2^{2\kappa_\rho} - 1}/2^{2\kappa_\rho})\sigma_\rho u_t \\ &- \tfrac{1+\varphi}{1+\gamma\varphi}((1 - 1/2^{2\kappa_a})\hat{a}_t + (\sqrt{2^{2\kappa_a} - 1}/2^{2\kappa_a})\sigma_a v_t). \end{aligned} \tag{14}$$

Comparing (14) to the efficient path of real interest rates (4) shows that the expectation attenuation channel mutes the equilibrium real rate responses to exogenous demand and supply shocks. In addition, the noise channel introduces endogenous and stochastic shocks to optimal monetary policy that a policy maker with full information could avoid. We also note the effect of the informational trade-off: monetary policy responds more strongly and more precisely to the shock that the policy maker pays more attention to.

3.2 Equilibrium Dynamics

Combining (14) with the Euler equation (5) and the New Keynesian Phillips curve (6) yields a three equation system, that we solve for the full equilibrium dynamics of the economy.

Proposition 3. *Under Assumption 1, the optimal paths for output and inflation are*

$$\begin{aligned} \hat{y}_t = \gamma(\hat{\rho}_t/2^{2\kappa_\rho} &- (\sqrt{2^{2\kappa_\rho} - 1}/2^{2\kappa_\rho})\sigma_\rho u_t) \\ &+ \tfrac{1+\varphi}{1/\gamma+\varphi}((1 - 1/2^{2\kappa_a})\hat{a}_t + (\sqrt{2^{2\kappa_a} - 1}/2^{2\kappa_a})\sigma_a v_t), \end{aligned} \tag{15}$$

$$\begin{aligned} \pi_t = \varphi_y\gamma(\hat{\rho}_t/2^{2\kappa_\rho} &- (\sqrt{2^{2\kappa_\rho} - 1}/2^{2\kappa_\rho})\sigma_\rho u_t) \\ &+ \varphi_y\tfrac{1+\varphi}{1/\gamma+\varphi}(-\hat{a}_t/2^{2\kappa_a} + (\sqrt{2^{2\kappa_a} - 1}/2^{2\kappa_a})\sigma_a v_t). \end{aligned} \tag{16}$$

Consider first the path of output, which is determined by the textbook intertemporal substitution channel (Galí, 2015). Comparing (15) to the efficient path (3), $\hat{y}_t^* = \frac{1+\varphi}{1/\gamma+\varphi}\hat{a}_t$, shows that the output response to supply shocks is muted, while the response to demand shocks is amplified. These differences stem from the muted responses of the optimal real interest rate to exogenous shocks. When the real interest rate response to supply shocks is muted, households do not substitute consumption across periods as much. Hence the output response to supply shocks is also muted. In contrast, the muted real rate response to demand shocks creates larger deviations between interest rates and the household's discount rate, which result in more intertemporal substitution of consumption and larger output responses to demand shocks than in the efficient economy.

Crucially, the strength of these effects depends on the information allocation chosen by the policy maker. For example, when the policy maker shifts her attention towards supply shocks so that κ_a rises but κ_ρ falls, output becomes more sensitive to both supply and demand shocks as interest rate policy provides more accommodation of changes in supply, but does not offset changes in demand as strongly. In contrast, shifting attention towards demand shocks reduces the sensitivity of output to both shocks as interest rates offset changes in demand more aggressively but do not accommodate changes in supply as much. In short, output volatility is a function of the information that the policy maker chooses to attend to.

Substituting the dynamics for output into the New Keynesian Phillips Curve (6) yields the dynamics of inflation. Proposition 3 shows that inflation responds to both demand and supply shocks, in sharp contrast with its stability under the full information policy. Inflation responds positively to demand shocks, but negatively to supply shocks. In line with the textbook intuition, a positive demand shock raises demand holding long run productive capacity fixed, and so causes firms to increase their prices in the short run. On the other hand, a positive supply shock lowers firms' marginal cost of production and results in higher output at lower prices.

The strengths of the inflation responses to each shock also depend on the information allocation. Expression (16) shows that inflation responds more to the shock that the policy maker devotes less attention to. For example, if the policy maker devotes more attention to supply shocks so that κ_a rises but κ_ρ falls, inflation will respond more to demand shocks. This follows from the implied dynamics of the output gap. When the policy maker focuses more on supply shocks, deviations in output from its efficient level are due mainly to demand shocks that the policy maker does not track as much. Since inflation depends on the path of output gaps, its dynamics become mainly driven by demand shocks.

Finally, combining (15) and (16) shows how the information allocation determines both the sign and the size of the equilibrium co-movement between output and inflation. If the policy maker shifts her attention to supply shocks, then output is more volatile, and inflation is mainly driven by demand shocks. As a result, output and inflation exhibit a strong positive co-movement. In contrast, if the policy maker shifts her attention towards demand shocks, then output volatility declines, and inflation becomes mainly driven by supply shocks. In this case, output and inflation will exhibit a weaker and negative co-movement. This finding offers a tentative link between the strength of the empirical Phillips curve (the co-movement between inflation and real activity) and the informational focus of monetary policy, and suggests that the Phillips curve is strongest when policy is focused on supply side factors.

Endogenous Monetary Policy Shocks In addition to exogenous demand and supply shocks, Proposition 3 also shows how the macroeconomy is driven by the endogenous shocks, v_t and u_t that arise due to the policy maker's limited information. Therefore, the economy is subject to endogenous and stochastic fluctuations under the optimal monetary policy. These fluctuations are caused by the information constraint, which creates stochastic deviations between the nominal interest rate and the efficient real interest rate. In equilibrium, these deviations affect the paths for output and inflation.

The endogenous shocks v_t and u_t cause a positive co-movement of output and inflation, but a negative co-movement of output and the nominal interest rate. Therefore, v_t and u_t have the characteristics of monetary policy shocks, as defined in the vast literature on monetary policy transmission (e.g. Christiano et al., 2005). The information constraint offers an explanation for the origin of such shocks. In equilibrium, there are unanticipated shocks to the nominal interest rate that stem from the limited information that the policy maker has about the efficient path of the economy she would like to target. In contrast to the literature with exogenous information, the variances of these shocks are determined endogenously as part of the optimal policy, and reflect both the total information processing capacity of the policy maker, and her optimal division of this capacity between competing information sources.

3.3 Implementation

Having characterized the equilibrium dynamics, we now derive a feedback rule that specifies the optimal nominal interest rate as a function of noisy measures of output and inflation. Crucially, the noise respects the informational constraint, so that the measures of output and inflation can be interpreted as noisy observational data that the policy maker could collect.

Proposition 4. *Under Assumption 1, the optimal monetary policy can be implemented using the rule*

$$\iota_t = \rho + \tfrac{2^{2\kappa_a}-1}{2^{2\kappa_a}} \tfrac{2^{2\kappa_\rho}}{\gamma\varphi_y} \pi_t^o + \tfrac{2^{2\kappa_\rho}-2^{2\kappa_a}}{2^{2\kappa_a}} \tfrac{1}{\gamma} \hat{y}_t^o \tag{17}$$

where π_t^o and $\hat{y}_t^o$ are noisy observations of output and inflation that satisfy

$$\pi_t^o = \pi_t - \gamma\varphi_y \tfrac{1+\varphi}{1+\gamma\varphi} \sqrt{1/(2^{2\kappa_a}-1)}\sigma_a v_t, \tag{18}$$

$$\hat{y}_t^o = \hat{y}_t + \gamma \tfrac{2^{2\kappa_a}}{2^{2\kappa_\rho}-2^{2\kappa_a}} \sqrt{2^{2\kappa_\rho}-1}\sigma_\rho u_t. \tag{19}$$

While this implementation bears some similarities to existing work with exogenously restricted information (e.g. Aoki, 2003), there are two key differences. First, the policy maker's information set is endogenously determined in our setting, and does not necessarily have to contain y_t^o and π_t^o. Therefore, our model shows that it is indeed optimal for a policy maker to use noisy indicators of output and inflation to set monetary policy, rather than exogenously assuming that she does. Second, both the coefficients in the rule and the measurement errors that contaminate the policy maker's observations of output and inflation are endogenously determined and linked to the optimal information allocation over demand and supply shocks. This sheds light on how the informational trade-off affects monetary policy implementation.

Proposition D shows that focusing more attention on demand shocks is associated with the precise measurement and stabilization of output, while focusing more on supply shocks is best implemented using the precise measurement and stabilization of inflation. To see this, define $m_t^y = \hat{y}_t^o - \hat{y}_t$ and $m_t^\pi = \pi_t^o - \pi_t$ as the measurement errors of output and inflation. As the policy maker shifts her attention towards demand shocks, κ_ρ rises and κ_a falls, and the variance of m_t^y decreases while the variance of m_t^π increases. At the same time, the coefficient on output in (17) increases while the coefficient on inflation changes ambiguously.[13] Hence, an increased focus on demand shocks is associated with more precise measurements of output relative to inflation, and a relatively stronger response of interest rates to changes in output. Intuitively, we know from Proposition 3 that when the policy maker pays more attention to demand shocks, she sacrifices some efficient supply-driven output volatility to further reduce inefficient demand-driven volatility. These output dynamics can be implemented by measuring output more precisely, and by raising interest rates more aggressively in response to increases in output. In contrast, paying more attention to supply shocks is associated with more precise measurements of inflation relative to output, and a weaker (possibly even negative) response of interest rates to output. In this case, the policy maker allows for more inefficient demand-driven output volatility to accommodate more efficient supply-driven volatility. These dynamics can be implemented by increasing interest rates in response to increases in more precisely measured inflation, which allows the policy maker to offset demand shocks to some extent. This result offers a tentative interpretation of the "price stability" objective of central banks around the world as an implicit focus on supply factors over demand factors. Relatedly, the "dual mandate" present in the Federal Reserve's focus on both prices and employment may be interpreted as balancing both demand and supply considerations.

Finally, we note that in the knife-edge case in which the policy maker devotes equal

[13]The coefficient on inflation falls for a large enough increase (decrease) in κ_ρ (κ_a).

attention to demand shocks and supply shocks, $\kappa_a = \kappa_\rho$, and the policy maker needs only to respond to inflation in order to implement the optimal policy. In this special case, (16) shows that inflation depends linearly on the path of efficient real rate deviations $\hat{\rho}_t - \frac{1+\varphi}{1+\gamma\varphi}\hat{a}_t$, and so is a sufficient statistic for the necessary adjustment in monetary policy. Therefore, linear dependence of the nominal rate on inflation is sufficient to implement the optimal path of nominal interest rates. This result also offers an information-based interpretation of the well known result that a Taylor rule with an infinitely large coefficient on inflation can implement the efficient equilibrium with arbitrary accuracy. If we let $\kappa_M \to \infty$ so that $\kappa_a, \kappa_\rho \to \infty$ also, then $\kappa_a/\kappa_\rho \to 1$ and the logic above applies: the coefficient on output tends to zero, while the coefficient on inflation goes to infinity.[14] In this sense, our implementation result converges to the standard model as $\kappa_M \to \infty$.

3.4 Optimal Information Allocation

Having characterized the optimal monetary policy and the equilibrium dynamics, we can now solve for the optimal information allocation (the first subproblem). To do this, it is useful to note that the ex-ante expected utility loss is given by

$$\mathbb{E}\ell(\kappa_\rho, \kappa_a) = \frac{1}{2}\left(\frac{\sigma_\rho^2}{2^{2\kappa_\rho}} + \left(\frac{1+\varphi}{1+\gamma\varphi}\right)^2 \frac{\sigma_a^2}{2^{2\kappa_a}}\right) \tag{20}$$

Minimizing (20) subject to $\kappa_\rho + \kappa_a \leq \kappa_M$ yields the following solution.

Proposition 5. *Under Assumption 1, the optimal information allocation satisfies*

$$\kappa_a = \begin{cases} 0 & \textit{if} \quad \log_2(\frac{1+\varphi}{1+\gamma\varphi}\sigma_a/\sigma_\rho) \leq -\kappa_M \\ \frac{1}{2}\kappa_M + \frac{1}{2}\log_2(\frac{1+\varphi}{1+\gamma\varphi}\sigma_a/\sigma_\rho) & \textit{if} \quad \log_2(\frac{1+\varphi}{1+\gamma\varphi}\sigma_a/\sigma_\rho) \in (-\kappa_M, \kappa_M) \\ \kappa_M & \textit{if} \quad \log_2(\frac{1+\varphi}{1+\gamma\varphi}\sigma_a/\sigma_\rho) \geq \kappa_M \end{cases} \tag{21}$$

and $\kappa_\rho = \kappa_M - \kappa_a$.

In the model, when choosing her optimal information allocation, the policy maker takes into account how her choices affect the equilibrium dynamics of output and inflation and thus impact the household's welfare. Since welfare is higher when nominal rates more closely track efficient real rates this decision depends on whether efficient real interest rates are mainly driven by changes in demand or supply. Intuitively, if the efficient real rate is mainly driven by supply shocks such that $\frac{1+\varphi}{1+\gamma\varphi}\sigma_a > \sigma_\rho$, then it is optimal for the policy maker to devote

[14] That $\kappa_a, \kappa_\rho \to \infty$ when $\kappa_M \to \infty$ follows from Proposition 5.

more of her information processing capacity to these shocks. If the relative variance of supply shocks is sufficiently high or low, then we obtain a corner solution $\kappa_a = \kappa_M$ or $\kappa_a = 0$.

Equilibrium Determinacy Combining the optimal policy rule (17) with the Euler equation (5) and New Keynesian Phillips curve (6) yields a three equation system. The following result uses the determinacy of the system together with the optimal information allocation (79) to establish when the optimal policy equilibrium is locally unique.

Proposition 6. *The optimal equilibrium can be implemented uniquely when*

$$\kappa_M > \log_2 \left(\frac{1}{1+\rho} \frac{1+\gamma\varphi}{1+\varphi} \frac{\sigma_\rho}{\sigma_a} + \left(\frac{\rho}{1+\rho} + \gamma\varphi_y \right) \frac{1+\varphi}{1+\gamma\varphi} \frac{\sigma_a}{\sigma_\rho} \right). \tag{22}$$

Similar to the textbook New Keynesian model, determinacy holds when the coefficients in the policy rule are large enough, thus ruling out self-fulfilling sunspot equilibria. In our setting, this condition holds when the policy maker's information processing capacity is large enough to guarantee that the policy maker can track changes in inflation and output with sufficient accuracy. This ensures that the nominal rate responds enough to rule out sunspot equilibria. For example, a large enough κ_M guarantees that the real interest rate increases in response to inflation, which rules out equilibria in which an increase in inflation driven by a sunspot is supported by a fall in the real interest rate and an increase in output.[15]

4 The Case of Persistent Shocks

We now relax Assumption 1 and return to the case of persistent shocks. We highlight the key analytical differences when aggregate shocks are persistent, and then solve for the optimal information allocation numerically.

4.1 Equilibrium Dynamics and Existence

When aggregate shocks are persistent, equilibrium dynamics can change qualitatively from the i.i.d. case. Furthermore, the equilibrium may cease to exist. To shed light on these issues, it is sufficient to examine the responses of output and inflation to persistent supply shocks

[15] This stands in contrast to the generic indeterminacy result in Lubik et al. (2019). In their setting, the imperfectly informed central bank obtains noisy signals about inflation, and there is a complementarity between the Taylor rule's response to inflation and the central bank's Kalman gain. Intuitively, if the central bank stabilizes inflation successfully through a strong response to perceived inflation, the smaller the signal-to-noise ratio, and the central bank's inflation beliefs respond less to signals.

under the optimal monetary policy (an analogous argument applies to demand shocks and endogenous monetary policy shocks). In Appendix B, we show that the impulse responses of the output gap and inflation to a unit supply shock that hits in period t are given by

$$\tilde{y}_{t+s} = -\frac{\gamma}{2^{2\kappa_a}} \frac{\frac{1+\varphi}{1+\gamma\varphi}\left(1-\delta_a\right)\left(1-\tilde{\delta}_a/\left(1+\rho\right)\right)}{(1-\tilde{\delta}_a/(1+\rho))(1-\tilde{\delta}_a)-\gamma\varphi_y\tilde{\delta}_a}\tilde{\delta}_a^s \tag{23}$$

$$\pi_{t+s} = -\frac{\gamma\varphi_y}{2^{2\kappa_a}} \frac{\frac{1+\varphi}{1+\gamma\varphi}\left(1-\delta_a\right)}{(1-\tilde{\delta}_a/(1+\rho))(1-\tilde{\delta}_a)-\gamma\varphi_y\tilde{\delta}_a}\tilde{\delta}_a^s \tag{24}$$

for $s \geq 0$, where $\tilde{\delta}_a = \delta_a/2^{2\kappa_a} \in [0,1)$ governs the persistence of the response, and is decreasing in the information allocation choice variable κ_a. Intuitively, as the policy maker allocates more information capacity to supply shocks, the persistence of supply-driven output gaps and inflation declines. When $\delta_a = 0$, the responses simplify to Proposition 3.

The crucial term in these expressions is the denominator function

$$\mathcal{P}(\tilde{\delta}_a) = (1-\tilde{\delta}_a/(1+\rho))(1-\tilde{\delta}_a)-\gamma\varphi_y\tilde{\delta}_a \tag{25}$$

$\mathcal{P}$ is a decreasing function of $\tilde{\delta}_a$ over its domain $\tilde{\delta}_a \in [0,1)$. Note that in the i.i.d. case $\mathcal{P}(0) = 1$. Furthermore, there exists a $\tilde{\delta}_0 \in (0,1)$ such that $\mathcal{P}(\tilde{\delta}_0) = 0$ given by

$$\tilde{\delta}_0 = \frac{1+1/(1+\rho)+\gamma\varphi_y - \sqrt{\left(1+1/(1+\rho)+\gamma\varphi_y\right)^2 - 4/(1+\rho)}}{2/(1+\rho)}$$

There are then three cases to consider. First, suppose $\tilde{\delta}_a < \tilde{\delta}_0$ or equivalently $\mathcal{P}(\tilde{\delta}_a) > 0$. Then, equilibrium dynamics resemble a persistent version of the i.i.d. case in Proposition 3. Notice that the dynamics become more volatile as $\tilde{\delta}_a$ increases and $\mathcal{P}(\tilde{\delta}_a)$ approaches zero.

Second, suppose $\tilde{\delta}_a > \tilde{\delta}_0$ so that $\mathcal{P}(\tilde{\delta}_a) < 0$. In this case, the equilibrium dynamics switch sign so that output gaps and inflation respond positively to increases in supply. Therefore, if $\tilde{\delta}_a$ and $\tilde{\delta}_0$ are close, a small change to the information choice κ_a can drastically alter the way in which the economy responds to persistent aggregate shocks (switching between the first and second case).

Finally, when $\tilde{\delta}_a = \tilde{\delta}_0$, $\mathcal{P}(\tilde{\delta}_a) = 0$ and $1/\mathcal{P}(\tilde{\delta}_a)$ is undefined. As such the responses (23) and (24) do not exist. Since $\tilde{\delta}_a$ depends on the information choice variable κ_a, the policy maker must take this possibility of non-existence into account when choosing the optimal information allocation. Note that $\tilde{\delta}_0 \in (0,1)$ for all $\rho, \gamma, \varphi_y > 0$ so that non-existence is a general feature of the economy.

To understand the source of these effects, use (23) and (24) to write the Euler equation

(5) in the form

$$\tilde{\delta}_a \gamma \pi_t - (1 - \tilde{\delta}_a)\tilde{y}_t = \gamma\left(\iota_t - r_t^*\right) \tag{26}$$

which shows how the size of $\tilde{\delta}_a$ determines whether changes in the interest rate gap $\iota_t - r_t^*$ driven by supply shocks transmit mainly to inflation or output gaps. When κ_a is large, $\tilde{\delta}_a$ is close to zero. In this case, (26) approximates to $\tilde{y}_t \approx -\gamma\left(\iota_t - r_t^*\right)$, so that the output gap (and current inflation by the New Keynesian Phillips curve (6)) decreases in response to the rise in the interest rate gap $\iota_t - r_t^*$ driven by a positive supply shock.[16] As $\tilde{\delta}_a$ increases, $\iota_t - r_t^*$ rises more in response to the shock, the weight on the output gap term in (26) falls, while the weight on the inflation term increases. In equilibrium, the size of the negative output gap response must therefore increase, resulting in more volatile dynamics. However, because more negative output gaps induce higher deflation, as $\tilde{\delta}_a$ approaches $\tilde{\delta}_0$ from below, the negative inflation term eventually dominates and the equilibrium ceases to exist.

When κ_a is small, $\tilde{\delta}_a$ is close to unity. In this case, (26) approximates to $\pi_t \approx \frac{\iota_t - r_t^*}{\tilde{\delta}_a}$, so that there is inflation, and hence positive output gaps in response to a positive supply shock. As $\tilde{\delta}_a$ decreases, $\iota_t - r_t^*$ rises less in response to the shock, the weight on the negative output gap term rises, while the weight on the positive inflation term decreases. In equilibrium, the size of the inflation response must therefore increase, resulting in more volatile dynamics. However, because more inflation comes with larger positive output gaps, as $\tilde{\delta}_a$ approaches $\tilde{\delta}_0$ from above, the negative output gap term eventually dominates and an equilibrium no longer exists.

4.2 Optimal Information Allocation

To compute the optimal information allocation when aggregate shocks are persistent, we proceed numerically.

Parameter Calibration Table 1 summarizes the calibration of the model at a quarterly frequency. We set the parameters governing preferences, technology, and exogenous shocks in line with the vast literature that studies quantitative New Keynesian models (e.g. Smets and Wouters, 2007). For robustness, we solve the model for 10 values of κ_M over the interval $[0.3, 1.1]$. Over most of this range, endogenous monetary shocks account for less than 25% of output volatility (Figure 5), which is in line with Christiano et al. (2005).

[16] In response to a positive supply shock, the monetary authority's expectation of the supply shock, $\mathbb{E}_{M,t}\left[\hat{a}_t\right]$, increases. However, under rational inattention, the rise in her beliefs is less than the true rise of $\hat{a}_t$. Therefore the change in the interest gap, $\Delta(\iota_t - r_t^*) = \Delta\left(-\frac{1+\varphi}{1+\gamma\varphi}\left[\mathbb{E}_{M,t}\left[\hat{a}_t\right] - \hat{a}_t\right]\right)$, is positive.

Parameter	Value	Description
γ	1	Elasticity of Intertemporal Substitution
φ	1	Frisch Elasticity of Labor Supply
ρ	0.01	Discount Rate
Φ	6	Elasticity of Substitution
ξ	100	Price Adjustment Cost
δ_a	0.95	Persistence of TFP shocks
σ_a	0.01	Standard Deviation of TFP innovations
δ_ρ	0.95	Persistence of Discount Rate shocks
σ_ρ	0.01	Standard Deviation of Discount Rate innovations
κ_M	$[0.3, 1.1]$	Range of Information Processing Capacities

Table 1: Model calibration. Standard parameters are set in line with the New Keynesian literature (e.g. Smets and Wouters, 2007). We solve the model for 10 values of κ_M over the interval $[0.3, 1.1]$.

Table 2 reports the optimal information allocation (κ_a, κ_ρ) expressed as percentages of κ_M. For all values of κ_M, the processing capacity devoted to supply shocks never exceeds 3% of the total capacity. This finding is far from the optimal information allocation when shocks are i.i.d., which Proposition 5 implies would be a 50-50 split given our calibration. To understand this extreme finding, recall from (4) that the efficient real rate depends on supply shocks with a coefficient $1 - \delta_a$. Under our textbook calibration, this coefficient is close to zero, and the efficient real rate is predominantly driven by demand shocks instead of supply shocks, irrespective of demand shock persistence. Therefore, it is never optimal for the policy maker to adjust monetary policy a lot in response to supply shocks. Hence, the policy maker need not devote much attention to them.

κ_M	0.3	0.39	0.48	0.57	0.66	0.74	0.83	0.92	1.01	1.1
κ_a	0	0	0	0	0	0	1	2	2	3
κ_ρ	100	100	100	100	100	100	99	98	98	97

Table 2: Optimal information allocation, expressed as percentages of κ_M.

4.3 Impulse Response Functions

To see the consequences of this information allocation for equilibrium dynamics, Figure 1 plots the impulse responses of output and inflation to one standard deviation demand, supply, and endogenous monetary policy shocks. Solid curves report responses when information processing capacity is low ($\kappa_M = 0.39$), while dashed curves report the responses when the capacity is high ($\kappa_M = 0.92$). We also plot the efficient response of output in the flexible price economy using a dashed-dotted curve. All responses are expressed in percentage points and are plotted for the first 8 quarters. For context, one may think of the low capacity regime as capturing macroeconomic policy prior to the Great Moderation, and more recent years being captured by the high capacity regime. We discuss this interpretation further in Section 5.

Consider first the low capacity case. In line with the analytical results, the output response to demand shocks is significantly amplified relative to the efficient economy (recall that efficient output does not respond to demand shocks at all). However, because the policy maker obtains little information about the persistent supply shocks, the output and inflation responses are actually more volatile than the efficient economy. However, the high persistence of supply shocks renders the gap between the equilibrium and efficient responses very small. Intuitively, (4) shows that the efficient real rate does not respond strongly to persistent supply shocks, so that an inertial monetary policy response is close to efficient. Finally, endogenous monetary policy shocks have large and persistent effects on output and inflation driven by the policy maker's persistent errors in tracking demand shocks. Formally, in Appendix B we follow Maćkowiak and Wiederholt (2009) and derive the following expression for the policy maker's expectation of persistent demand shocks,

$$\mathbb{E}_{M,t}\hat{\rho}_t = \sum_{s=0}^{\infty}\left(\delta_\rho^s - \frac{1}{2^{2\kappa_\rho}}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s\right)e_{\rho,t-s} + \sum_{s=0}^{\infty}\sqrt{\frac{1}{2^{2\kappa_\rho}}\frac{2^{2\kappa_\rho}-1}{2^{2\kappa_\rho}-\delta_\rho^2}}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s\sigma_\rho u_{t-s}$$

which shows that the persistence of the policy maker's mistakes $\delta_\rho/2^{2\kappa_\rho}$ is decreasing in κ_ρ. Therefore, errors are more persistent when information capacity is low.

Now consider how the responses change when the policy maker has higher information processing capacity. The responses to demand shocks converge to the efficient path within one year, while the responses to supply shocks remain unchanged. In line with these results, the response to monetary shocks are muted and also die away more quickly. Intuitively, high information capacity implies smaller and less persistent mistakes in expectation formation.

Interest Rate Responses Figure 2 plots the responses of real and nominal interest rates to the three shocks, in low and high information capacity environments. When information capacity is low, the nominal interest rate responds strongly to endogenous monetary policy shocks, but exhibits a muted hump-shaped response to demand shocks. In combination with positive response of expected inflation, the real interest rate response to demand shocks is very muted on impact before converging to its efficient response dictated by (4). These patterns are dampened when the information capacity is increased. In both cases, the responses to supply shocks are very small, consistent with their low information allocation and minor effect on efficient real interest rates.

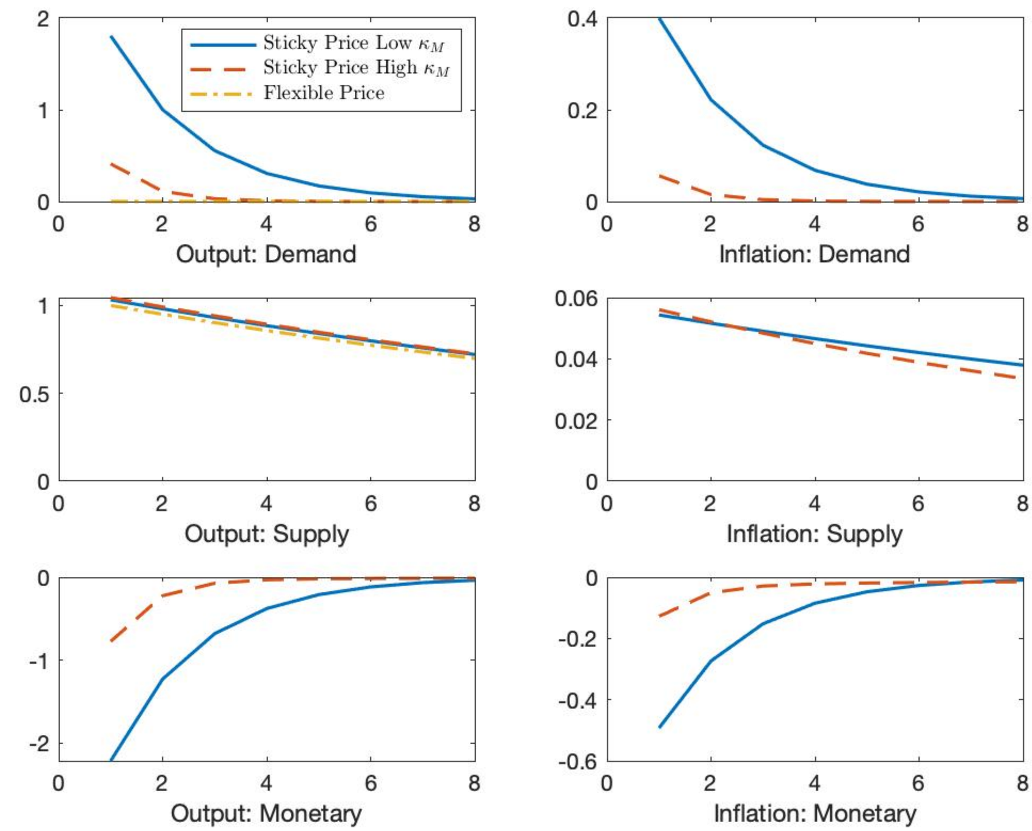


Figure 1: Impulse Response Functions of output and inflation to one standard deviation demand, supply, and endogenous monetary policy shocks. Responses are in percent deviations from steady state, and are plotted at a quarterly frequency. Low $\kappa_M = 0.39$. High $\kappa_M = 0.92$.

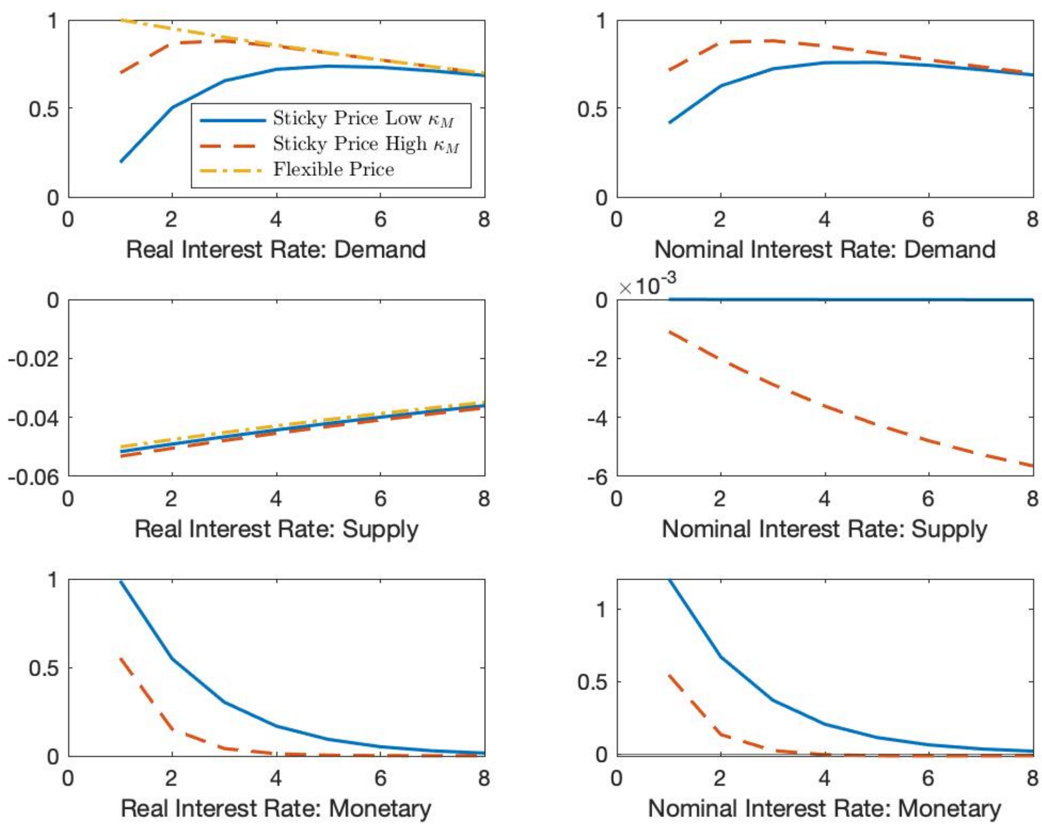


Figure 2: Impulse Response Functions of real and nominal interest rates to one standard deviation demand, supply, and endogenous monetary shocks. Responses are in percent deviations from steady state, and are plotted at a quarterly frequency. Low $\kappa_M = 0.39$. High $\kappa_M = 0.92$.

5 The Macroeconomic Effects of Information Frictions

Next, we study how improvements in policy makers' information processing capabilities have played a role in determining notable macroeconomic trends over the period 1960Q1-2006Q4. The Federal Reserve's processing of information has improved over this time as evidenced by increased accuracy of the Federal Reserve Greenbooks' nowcasts and forecasts of inflation and output (Tulip, 2009). To examine the connection between this improved information processing and macroeconomic outcomes, we describe the empirical patterns, and then compare them to model outcomes for an increasing sequence of κ_M values, which captures improvements in policy makers' information processing in a simple manner. This experiment complements Maćkowiak and Wiederholt (2015), who study how changes in the monetary policy rule affect macroeconomic volatility when the private sector faces informational constraints.[17]

5.1 Empirical Trends

Figures 3a, 3b, and 3c plot the standard deviations of output growth, inflation, and the federal funds rate for a rolling window of 20 years. The standard deviation is plotted at the date corresponding to the end of the 20 year sample. There has been there has been a consistent downward trend in the standard deviation of all three variables.[18] This decline in volatility is often referred to as the Great Moderation, and a variety of reasons have been put forward to explain it such as structural changes (e.g. business practices, technology), improved monetary policies, and good luck. Through the lens of our model, we argue that the decline in volatility of these macroeconomic variables is the result of increased information processing capacity of the Federal Reserve.

Over the same period, the correlation between inflation and real activity has weakened, which is associated with a flattening of the Phillips curve. Figure 3d plots the slope coefficient from regressing inflation on output growth over a rolling window of 20 years.[19] After the early 1980's the relationship between inflation and output growth has diminished. This is, in

[17]To compare outcomes across models with different processing capacities, we assume that the policy maker's history is revised to reflect improvements in information processing when κ_M increases. This captures the data revision process that occurs in reality at modern central banks, and simplifies the analysis by obviating the need for transitional learning dynamics. Our approach is comparable to that taken in Maćkowiak and Wiederholt (2015).

[18]One exception is that the standard deviation of output growth increased following the Great Recession. However, it did not increase to the level seen in the samples ending from in the late 1900s.

[19]The original Phillips curve was the empirical relationship between inflation and unemployment; however the relationship has been extended to use a variety of measures of real activity. We use output growth as it can be easily tied to the model.

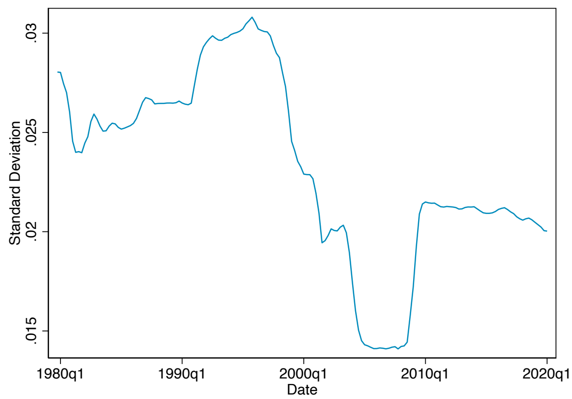
part, why some have declared the Phillips curve dead (Hall, 2013). We argue that this trend can in part be attributed to improvements in information processing by the central bank.

5.2 Model Results

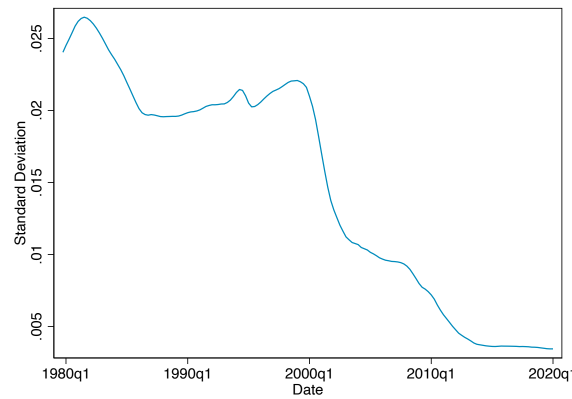
The model-based impulse responses suggest that higher information capacity reduces inefficient macroeconomic volatility. To further isolate this effect, Figure 4 conducts a comparative statics exercise. It details how the four measures of empirical macroeconomic volatility discussed above vary as we increase κ_M. As the policy maker's information processing capacity increases, macroeconomic volatility and the strength of the co-movement between output growth and inflation all decline. In particular, output growth volatility shrinks by a factor of around 5, while the strength of its co-movement with inflation declines by over 60%. It is interesting to note that the absolute levels of volatility and the corresponding declines match the data well. In that sense, an improvement in information processing capacity provides a parsimonious explanation for the overall decline in macroeconomic volatility that we have documented empirically. Intuitively, as κ_M increases, outcomes converge to their flexible price counterparts. Along this path, macroeconomic volatility due to endogenous monetary policy shocks shrinks, leaving only the volatility driven by fundamental demand and supply shocks. In addition, inflation dynamics converge to zero, so that its co-movement with real activity also goes to zero.

Forecast Error Variance Decompositions Another way of visualizing the quantitative impact of increasing κ_M is to consider the forecast error variance decompositions (FEVD) of output, inflation, and nominal rates into demand, supply, and endogenous monetary shocks. We compute these decompositions at a 20 quarter horizon, and report them in Figure 5 as a function of κ_M.

Consider first the FEVDs of output. As κ_M increases, the fraction of variance explained by supply shocks increases, while the fractions attributable to demand and monetary shocks decrease. This pattern is consistent with the fact that output is driven only by supply shocks in the efficient equilibrium. Similarly, the fraction of inflation variance explained by supply shocks increases as κ_M increases, mainly at the expense of demand shocks. Finally, as κ_M increases, the fraction of variance in the optimal nominal rate explained by endogenous monetary shocks naturally declines, while the fraction explained by demand shocks increases dramatically. Supply shocks play a negligible role in nominal rate dynamics. This pattern echoes the optimal information allocation in Figure 2, where supply shocks are essentially ignored by the policy maker for all values of κ_M. Intuitively, very persistent supply shocks

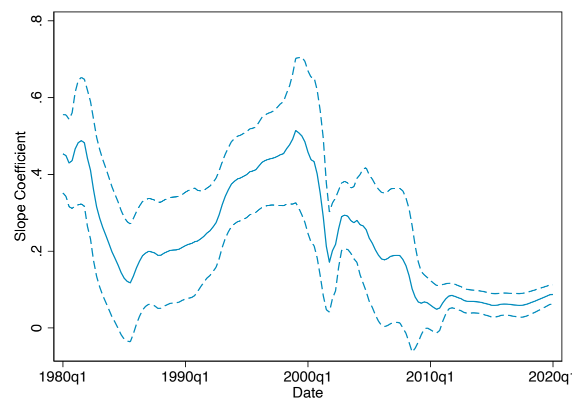


(a) Standard Deviation of Output Growth



(b) Standard Deviation of Inflation

(c) Standard Deviation of the Federal Funds Rate



(d) OLS Slope of Inflation on Output Growth

Figure 3: Empirical moments. A rolling window of 20 years is used and the estimated moment is plotted at the end of the sample window. Output growth in quarter t is $(GDP_t - GDP_{t-4})/(GDP_{t-4})$ where GDP is real GDP. Inflation is $(PCE_t - PCE_{t-4})/(PCE_{t-4})$ where PCE is the Personal Consumption Expenditure Index. The dotted lines represent a 95% confidence interval. Data are from FRED and range from 1960Q1 to 2020Q1.

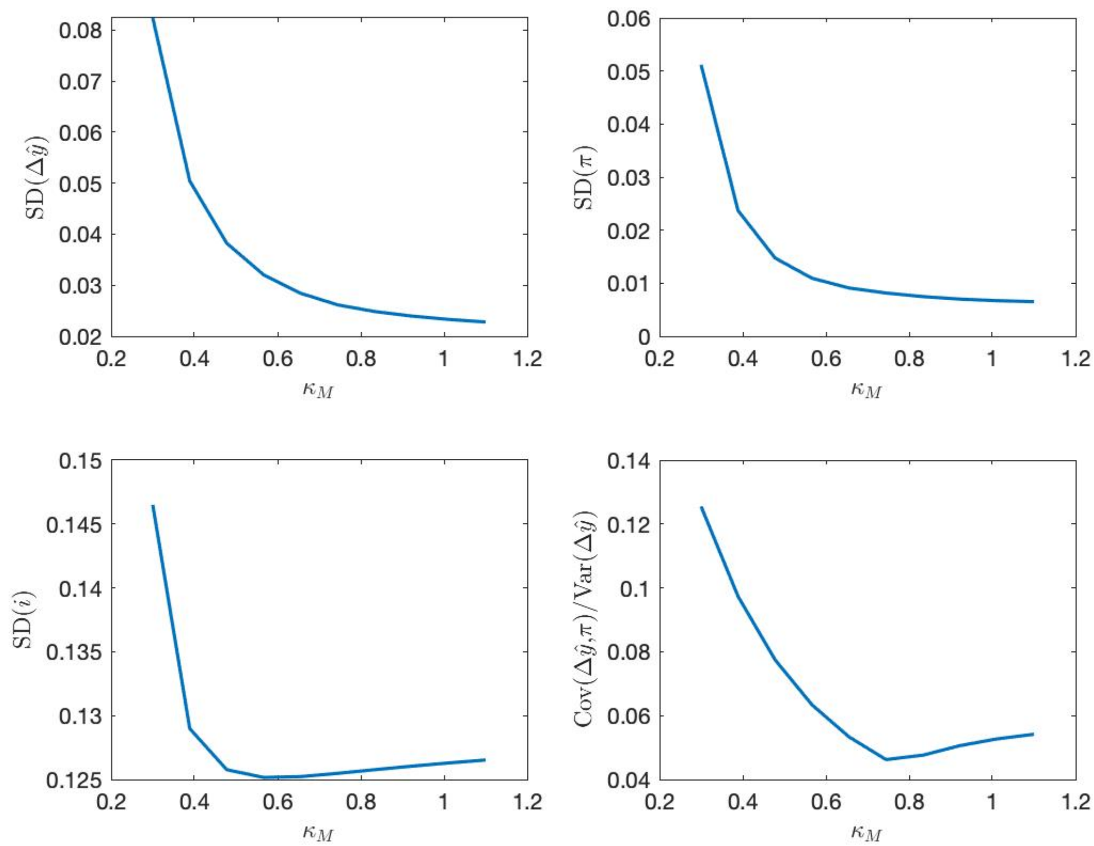


Figure 4: Model moments as a function of κ_M. SD is the standard deviation, Cov is the covariance, and Var is the variance. The top row plots the standard deviations of output growth and inflation. The bottom row plots the standard deviation of the nominal interestrate, and the slope coefficient from a regression of inflation on output growth. Output growth and inflation are computed as annual rates reported at a quarterly frequency, like in the data.

do not affect the efficient real rate, so the policy maker does not need to adjust the nominal rate in response to them.

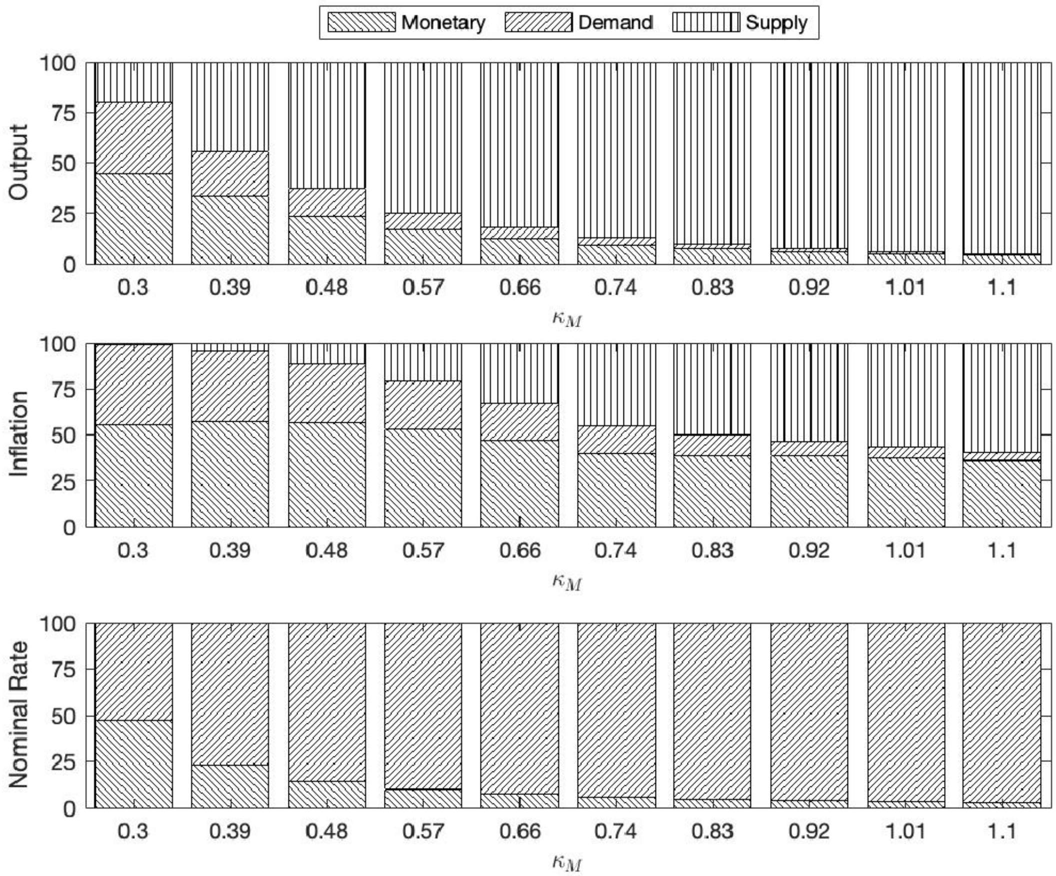


Figure 5: Forecast Error Variance Decompositions at a 20 quarter horizon for different values of κ_M.

6 Conclusion

Monetary authorities face significant uncertainty about the state of the economy due to data lags or cognitive limitations, yet optimal monetary policy is often studied in a full-information setting. We relax this assumption and embed a rationally inattentive policy maker in an otherwise standard New Keynesian model. The policy maker faces an information processing constraint that forces her to tradeoff learning about supply and demand shocks. How the monetary authority allocates her attention not only affects her expectations of supply and demand factors, but importantly affects the dynamics of the macroeconomy. We show that it is optimal for the policy maker to largely focus on demand shocks, since persistent supply shocks have limited effects on the efficient real interest rate. Furthermore, our model shows that improved information processing capabilities by a policy maker will reduce macroeconomic volatility and reduce the empirical Phillips curve slope. This suggests the Great Moderation's reduction in volatility and flattening of the estimated Phillips curve, may be partially attributed to the policy maker's information improving, allowing her to more closely track optimal monetary policy under full-information.

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A Model Details

We conduct our analysis in a New Keynesian model augmented to allow for endogenous information choices by the policy maker. To cleanly isolate the effects of these choices relative to the complete information benchmark, we follow Svensson and Woodford (2004) and maintain full information in the private sector. This results in standard private sector equilibrium conditions and implies that all departures from the standard model are due to the policy maker's information choices.

Households A representative household consumes a unit mass of final goods and supplies labor in each period. She has preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \zeta_t \left(\frac{C_t^{1-1/\gamma}}{1-1/\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{27}$$

where $C_t = (\int_0^1 c_{j,t}^{(\Phi-1)/\Phi} dj)^{\Phi/(\Phi-1)}$ is a consumption index over the unit mass of final goods, and N_t is hours worked. $\rho > 0$ is the discount rate while ζ_t is an aggregate preference shock whose law of motion is described below. $\Phi > 1$ is the elasticity of substitution between goods, γ is the elasticity of intertemporal substitution, and φ is the inverse of the Frisch elasticity of labor supply. Note that the households' expectation is the standard full-information rational expectations operator.

In each period, the household faces the flow budget constraint

$$\int_0^1 p_{j,t} c_{j,t} dj + B_t = P_t w_t N_t + P_t D_t - P_t T_t + (1 + \iota_{t-1}) B_{t-1} \tag{28}$$

where $p_{j,t}$ is the nominal price of good j, B_t is the household's bond position, ι_t is the nominal bond return, w_t is the real wage, D_t is real non-labor income, T_t is a real lump sum tax, and P_t is the aggregate nominal price level. The household chooses paths of consumption of each final good, labor supply, and bond positions to maximize her utility subject to the sequence of flow budget constraints, an initial bond position B_{-1}, and a no-Ponzi condition.

Optimization yields the demand curves and price index,

$$c_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\Phi} C_t, \quad P_t = \left(\int_0^1 p_{j,t}^{1-\Phi} dj \right)^{\frac{1}{1-\Phi}}, \tag{29}$$

the consumption-labor trade-off

$$C_t^{1/\gamma} N_t^{\varphi} = w_t, \tag{30}$$

and the consumption Euler equation

$$\mathbb{E}_t\left[\Lambda_{t+1}\frac{1+\iota_t}{1+\pi_{t+1}}\right]=1, \tag{31}$$

where $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate and $\Lambda_{t+1} = \frac{1}{1+\rho}\frac{\zeta_{t+1}}{\zeta_t}(C_t/C_{t+1})^{1/\gamma}$ is the stochastic discount factor.

Firms Each good $j \in [0,1]$ is produced by a monopolistically competitive firm operating a constant returns to scale production function $y_{j,t} = a_t n_{j,t}$. a_t is total factor productivity (TFP), and is subject to aggregate shocks. $n_{j,t}$ is the labor demand of firm j in period t.

Each firm chooses its path of prices to maximize the discounted sum of profits subject to quadratic price adjustment costs (Rotemberg, 1982):

$$\max_{\{p_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty}\prod_{s=0}^{t}\Lambda_s\left(\frac{p_{j,t}}{P_t}y_{j,t} - (1-\tau_n)\,w_t\frac{y_{j,t}}{a_t} - \frac{\xi}{2}\left(\frac{p_{j,t}}{p_{j,t-1}}-1\right)^2 Y_t\right) \tag{32}$$

subject to

$$y_{j,t} = (p_{j,t}/P_t)^{-\Phi}\,Y_t \tag{33}$$

$$p_{j,0} = P_0. \tag{34}$$

Here, τ_n is a labor subsidy that is financed by the lump sum tax on households, and $\xi \geq 0$ is the Rotemberg price adjustment parameter.

As is standard, we focus on the symmetric equilibrium in which $p_{j,t} = P_t$ and $y_{j,t} = Y_t$ for all $j \in [0,1]$. In this case, inflation satisfies the firm first order condition, which defines the New Keynesian Phillips Curve,

$$\xi\pi_t(1+\pi_t) = 1 - \Phi + \Phi(1-\tau_n)w_t/a_t + \xi\mathbb{E}_t\left[\Lambda_{t+1}\pi_{t+1}(1+\pi_{t+1})Y_{t+1}/Y_t\right] \tag{35}$$

The aggregate dividend in period t is given by

$$D_t = Y_t\left(1-(1-\tau_n)\,w_t/a_t - \xi\pi_t^2/2\right) \tag{36}$$

Fiscal Policy The fiscal authority finances the labor subsidy via a lump sum tax on households, and is subject to the flow budget constraint

$$P_tT_t + B_t = \tau_n P_t w_t N_t + (1+\iota_{t-1})\,B_{t-1} \tag{37}$$

where we have already imposed the bond market clearing condition. We note that Ricardian Equivalence holds, so that the path of debt $\{B_t\}_{t=0}^{\infty}$ will be indeterminate in equilibrium.

In order to fully offset the monopoly distortion in the equilibrium with flexible prices, we follow the New Keynesian literature and assume that $\tau_n = 1/\Phi$. This guarantees that monetary policy does not need to substitute for missing tax instruments, making the efficient policy benchmark cleaner to interpret since price stability will prevail as the optimal policy under complete information (Correia et al., 2008).

Aggregate Shocks We assume that the preference shocks and aggregate TFP follow AR(1) processes in logs,

$$\log \zeta_t = \delta_\zeta \log \zeta_{t-1} + e_{\zeta,t} \tag{38}$$

$$\log a_t = (1-\delta_a)\log a + \delta_a \log a_{t-1} + e_{a,t} \tag{39}$$

where $e_{\zeta,t} \sim N\left(0, \sigma_\zeta^2\right)$ and $e_{a,t} \sim N\left(0, \sigma_a^2\right)$.[20]

Market Clearing In the symmetric equilibrium, final good and labor market clearing are given by

$$C_t = Y_t(1 - \xi\pi_t^2/2) \tag{40}$$

$$Y_t/a_t = N_t \tag{41}$$

A.1 Log-Linear Derivations

We now use first order approximations to describe key features of the equilibrium conditional on a path of nominal rates that describe the conduct of monetary policy. The following analysis follows very closely the textbook exposition found in Galí (2015).

Flexible Price Benchmark In the absence of price-adjustment frictions ($\xi = 0$), monetary policy is neutral, and the equilibrium is Pareto efficient. Optimal monetary policy will try to attain this allocation when prices are sticky.

In order to characterize efficient equilibrium dynamics, we adopt a log-linear approximation of the economy around its deterministic steady state in which $\zeta_t = 1$ and $a_t = a$. Let

[20] We assume aggregate shocks are independent to clearly deliver the main insights for optimal monetary policy. Correlated shocks would result in the monetary authority's limited information about one shock to effect her beliefs about both shocks.

$\hat{y}_t^* = \log Y_t^* - \log Y$ denote the log deviation of efficient output from its deterministic steady state value, and define r_t^* as the efficient real interest rate.

In the flexible price equilibrium, $w_t = a_t$. Combining with (30), (40), and (41), yields $\hat{y}_t^* = \frac{1+\varphi}{1/\gamma+\varphi}\hat{a}_t$. Log-linearizing (31), substituting $\hat{y}_t^* = \hat{c}_t$, and defining $\hat{\rho}_t = -(1-\delta_\zeta)\log\zeta_t$, we obtain $r_t^* = \rho + \hat{\rho}_t - \frac{1+\varphi}{1+\gamma\varphi}(1-\delta_a)\hat{a}_t$. Note that $\hat{\rho}_t$ follows an AR(1) process with persistence $\delta_\rho = \delta_\zeta$ and shock variance $\sigma_\rho^2 = (1-\delta_\zeta)^2\sigma_\zeta^2$. Since $\hat{\rho}_t$ is sufficient to characterize demand shocks, we refer to $\hat{\rho}_t$ instead of ζ_t in the main analysis.

Sticky Price Equilibrium We describe the equilibrium with sticky prices ($\xi > 0$) in terms of deviations from the efficient benchmark. To this end, define $\hat{y}_t = \log Y_t - \log Y$ as the log deviation of equilibrium output, and denote $\tilde{y}_t = \hat{y}_t - \hat{y}_t^*$ as the output gap: the log deviation of equilibrium output from its efficient level.

Log-linearizing (35) around the deterministic steady state yields

$$\pi_t = \frac{\Phi-1}{\xi}(\hat{w}_t - \hat{a}_t) + \frac{1}{1+\rho}\mathbb{E}_t[\pi_{t+1}]. \tag{42}$$

Use log-linear approximations of (30), (40), and (41) to substitute for $\hat{w}_t$ and $\hat{a}_t$ and obtain

$$\pi_t = \frac{\Phi-1}{\xi}\left(\frac{1}{\gamma}+\varphi\right)\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t[\pi_{t+1}] \tag{43}$$

so that $\varphi_y = \frac{\Phi-1}{\xi}\left(\frac{1}{\gamma}+\varphi\right)$. The Euler equation follows from log-linearizing (31) and substituting for $\hat{y}_t$ using $\tilde{y}_t$.

B Solution to the Optimal Monetary Policy Problem

We begin by deriving the per-period household utility loss ℓ_t. Let

$$U_t = \zeta_t\left(\frac{C_t^{1-1/\gamma}}{1-1/\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \tag{44}$$

denote the undiscounted flow utility to the household in period t. A second order approximation around the deterministic steady state yields

$$\begin{aligned} U_t \approx U + C^{1-1/\gamma}&\left(\hat{c}_t\left(1+\hat{\zeta}_t\right) + \frac{1-1/\gamma}{2}\hat{c}_t^2 - \hat{n}_t\left(1+\hat{\zeta}_t\right) - \frac{1+\varphi}{2}\hat{n}_t^2\right) \\ &+ \left(\frac{C^{1-1/\gamma}}{1-1/\gamma} - \frac{N^{1+\varphi}}{1+\varphi}\right)\left(\hat{\zeta}_t + \frac{1}{2}\hat{\zeta}_t^2\right) \end{aligned} \tag{45}$$

Take a second order approximation of (40) to obtain $\hat{c}_t \approx \hat{y}_t - \frac{\xi}{2}\pi_t^2$. Substitute for $\hat{c}_t$ and $\hat{n}_t$ using this and log-linear version of (41) to obtain

$$\begin{aligned} U_t \approx U + C^{1-1/\gamma}\left(-\frac{\xi}{2}\pi_t^2 - \frac{1}{2}\left(\frac{1}{\gamma}+\varphi\right)\hat{y}_t^2 + (1+\varphi)\,\hat{y}_t\hat{a}_t + \hat{a}_t\left(1+\hat{\zeta}_t\right) - \frac{1+\varphi}{2}\hat{a}_t^2\right) \\ + \left(\frac{C^{1-1/\gamma}}{1-1/\gamma} - \frac{N^{1+\varphi}}{1+\varphi}\right)\left(\hat{\zeta}_t + \frac{1}{2}\hat{\zeta}_t^2\right) \end{aligned} \tag{46}$$

Now substitute using $\hat{y}_t^* = \frac{1+\varphi}{1/\gamma+\varphi}\hat{a}_t$ and rearrange to get

$$\frac{U_t - U}{C^{1-1/\gamma}} \approx -\frac{1}{2}\left(1/\gamma+\varphi\right)\left(\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2\right) + \text{t.i.p.} \tag{47}$$

where t.i.p. are terms independent of policy. Therefore, the per-period undiscounted utility loss is proportional to

$$\ell_t = \frac{1}{2}\left(\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2\right)$$

Optimal Discretionary Monetary Policy and Proof of Proposition 1 We now turn to the choice of nominal interest rate ι_t. The policy maker solves

$$\min_{\iota_t} \frac{1}{2}\mathbb{E}_{M,t}\left[\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2\right] \tag{48}$$

subject to

$$\pi_t = \varphi_y \tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t\pi_{t+1} \tag{49}$$

$$\mathbb{E}_t\tilde{y}_{t+1} - \tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_t\pi_{t+1} - r_t^*\right) \tag{50}$$

where $\mathbb{E}_t\pi_{t+1}$ and $\mathbb{E}_t\tilde{y}_{t+1}$ are taken as given by the policy maker. The Lagrangean is

$$\begin{aligned} L = \frac{1}{2}\mathbb{E}_{M,t}\Big[\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2 + \lambda_{1,t}\left(\pi_t - \varphi_y\tilde{y}_t - \frac{1}{1+\rho}\mathbb{E}_t\pi_{t+1}\right) \\ + \lambda_{2,t}\left(\mathbb{E}_t\tilde{y}_{t+1} - \tilde{y}_t - \gamma\left(\iota_t - \mathbb{E}_t\pi_{t+1} - r_t^*\right)\right)\Big] \end{aligned}$$

Noting that ι_t is measurable with respect to $\mathbb{E}_{M,t}$, the FOCs are

$$\begin{gathered}
\tilde{y}_t - \frac{1}{2}\lambda_{1,t}\varphi_y - \frac{1}{2}\lambda_{2,t} = 0 \\
\frac{\xi}{1/\gamma + \varphi}\pi_t + \frac{1}{2}\lambda_{1,t} = 0 \\
\mathbb{E}_{M,t}\lambda_{2,t} = 0 \\
\pi_t = \varphi_y \tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t \pi_{t+1} \\
\mathbb{E}_t \tilde{y}_{t+1} - \tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_t \pi_{t+1} - r_t^*\right)
\end{gathered}$$

Applying the policy maker's expectation operator to each equation yields

$$\begin{gathered}
\mathbb{E}_{M,t}\tilde{y}_t - \frac{1}{2}\varphi_y \mathbb{E}_{M,t}\lambda_{1,t} - \frac{1}{2}\mathbb{E}_{M,t}\lambda_{2,t} = 0 \\
\frac{\xi}{1/\gamma + \varphi}\mathbb{E}_{M,t}\pi_t + \frac{1}{2}\mathbb{E}_{M,t}\lambda_{1,t} = 0 \\
\mathbb{E}_{M,t}\lambda_{2,t} = 0 \\
\mathbb{E}_{M,t}\pi_t = \varphi_y \mathbb{E}_{M,t}\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_{M,t}\pi_{t+1} \\
\mathbb{E}_{M,t}\tilde{y}_{t+1} - \mathbb{E}_{M,t}\tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_{M,t}\pi_{t+1} - \mathbb{E}_{M,t}r_t^*\right)
\end{gathered}$$

where we have used the law of iterated expectations (and the fact that the policy maker's information set is contained in the full information set) to simplify $\mathbb{E}_{M,t}\mathbb{E}_t\pi_{t+1} = \mathbb{E}_{M,t}\pi_{t+1}$ and $\mathbb{E}_{M,t}\mathbb{E}_t\tilde{y}_{t+1} = \mathbb{E}_{M,t}\tilde{y}_{t+1}$. This system has solution

$$\mathbb{E}_{M,t}\tilde{y}_t = \mathbb{E}_{M,t}\lambda_{1,t} = \mathbb{E}_{M,t}\pi_t = \mathbb{E}_{M,t}\lambda_{2,t} = 0 \tag{51}$$

$$\iota_t = \mathbb{E}_{M,t}r_t^* \tag{52}$$

Hence the optimal policy is implemented using $\iota_t = \mathbb{E}_{M,t}r_t^*$, thus proving Proposition 1.

Equilibrium Dynamics In order to solve the rational inattention problem, we must first characterize the equilibrium stochastic processes for $\tilde{y}_t$, π_t, and ι_t.

We begin with the nominal interest rate, which we know satisfies

$$\iota_t = \rho + \mathbb{E}_{M,t}\hat{\rho}_t - \frac{1+\varphi}{1+\gamma\varphi}\left(1-\delta_a\right)\mathbb{E}_{M,t}\hat{a}_t$$

Because demand and supply shocks follow independent AR(1) processes, we can apply

Proposition 3 in Maćkowiak and Wiederholt (2009) to evaluate the expectations:

$$\mathbb{E}_{M,t}\hat{\rho}_t = \sum_{s=0}^{\infty}\left(\delta_\rho^s - \frac{1}{2^{2\kappa_\rho}}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s\right)e_{\rho,t-s} + \sum_{s=0}^{\infty}\sqrt{\frac{1}{2^{2\kappa_\rho}}\frac{2^{2\kappa_\rho}-1}{2^{2\kappa_\rho}-\delta_\rho^2}}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s\sigma_\rho u_{t-s} \tag{53}$$

$$\mathbb{E}_{M,t}\hat{a}_t = \sum_{s=0}^{\infty}\left(\delta_a^s - \frac{1}{2^{2\kappa_a}}\left(\frac{\delta_a}{2^{2\kappa_a}}\right)^s\right)e_{a,t-s} + \sum_{s=0}^{\infty}\sqrt{\frac{1}{2^{2\kappa_a}}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}-\delta_a^2}}\left(\frac{\delta_a}{2^{2\kappa_a}}\right)^s\sigma_a v_{t-s} \tag{54}$$

where u and v are independent standard Normal random variables. Combining with r_t^* yields the following expression for the interest rate gap

$$\begin{aligned}\iota_t - r_t^* =& -\frac{1}{2^{2\kappa_\rho}}\sum_{s=0}^{\infty}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s e_{\rho,t-s} + \sqrt{\frac{1}{2^{2\kappa_\rho}}\frac{2^{2\kappa_\rho}-1}{2^{2\kappa_\rho}-\delta_\rho^2}}\sum_{s=0}^{\infty}\left(\frac{\delta_\rho}{2^{2\kappa_\rho}}\right)^s\sigma_\rho u_{t-s}\\ &-\frac{1+\varphi}{1+\gamma\varphi}(1-\delta_a)\left(-\frac{1}{2^{2\kappa_a}}\sum_{s=0}^{\infty}\left(\frac{\delta_a}{2^{2\kappa_a}}\right)^s e_{a,t-s} + \sqrt{\frac{1}{2^{2\kappa_a}}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}-\delta_a^2}}\sum_{s=0}^{\infty}\left(\frac{\delta_a}{2^{2\kappa_a}}\right)^s\sigma_a v_{t-s}\right)\end{aligned} \tag{55}$$

It is now useful to define the auxiliary stochastic processes

$$\begin{aligned}\tilde{\rho}_t &= \tilde{\delta}_\rho\tilde{\rho}_{t-1} + e_{\rho,t}\\ \tilde{u}_t &= \tilde{\delta}_\rho\tilde{u}_{t-1} + \sigma_\rho u_t\\ \tilde{a}_t &= \tilde{\delta}_a\tilde{a}_{t-1} + e_{a,t}\\ \tilde{v}_t &= \tilde{\delta}_a\tilde{v}_{t-1} + \sigma_a v_t\end{aligned}$$

where $\tilde{\delta}_\rho = \delta_\rho/2^{2\kappa_\rho}$ and $\tilde{\delta}_a = \delta_a/2^{2\kappa_a}$. Then the interest rate gap can be expressed as

$$\iota_t - r_t^* = -\frac{1}{2^{2\kappa_\rho}}\tilde{\rho}_t + \sqrt{\frac{1}{2^{2\kappa_\rho}}\frac{2^{2\kappa_\rho}-1}{2^{2\kappa_\rho}-\delta_\rho^2}}\tilde{u}_t + \frac{1+\varphi}{1+\gamma\varphi}(1-\delta_a)\frac{1}{2^{2\kappa_a}}\tilde{a}_t - \frac{1+\varphi}{1+\gamma\varphi}(1-\delta_a)\sqrt{\frac{1}{2^{2\kappa_a}}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}-\delta_a^2}}\tilde{v}_t$$

Since the output gap and inflation must satisfy the linear system defined by (49) and (50), we guess and verify a solution of the form

$$\pi_t = f^\pi\tilde{\rho}_t + g^\pi\tilde{u}_t + b^\pi\tilde{a}_t + c^\pi\tilde{v}_t \tag{56}$$

$$\tilde{y}_t = f^y\tilde{\rho}_t + g^y\tilde{u}_t + b^y\tilde{a}_t + c^y\tilde{v}_t \tag{57}$$

The coefficients satisfy the conditions

$$f^{\pi} = \frac{\gamma\varphi_y}{2^{2\kappa_\rho}} \frac{1}{\left(1 - \frac{\tilde{\delta}_\rho}{1+\rho}\right)\left(1 - \tilde{\delta}_\rho\right) - \gamma\varphi_y\tilde{\delta}_\rho} \tag{58}$$

$$g^{\pi} = -\gamma\varphi_y \sqrt{\frac{1}{2^{2\kappa_\rho}} \frac{2^{2\kappa_\rho} - 1}{2^{2\kappa_\rho} - \delta_\rho^2}} \frac{1}{\left(1 - \frac{\tilde{\delta}_\rho}{1+\rho}\right)\left(1 - \tilde{\delta}_\rho\right) - \gamma\varphi_y\tilde{\delta}_\rho} \tag{59}$$

$$b^{\pi} = -\frac{1+\varphi}{1+\gamma\varphi}\left(1 - \delta_a\right)\frac{\gamma\varphi_y}{2^{2\kappa_a}} \frac{1}{\left(1 - \frac{\tilde{\delta}_a}{1+\rho}\right)\left(1 - \tilde{\delta}_a\right) - \gamma\varphi_y\tilde{\delta}_a} \tag{60}$$

$$c^{\pi} = \gamma\varphi_y \frac{1+\varphi}{1+\gamma\varphi}\left(1 - \delta_a\right)\sqrt{\frac{1}{2^{2\kappa_a}} \frac{2^{2\kappa_a} - 1}{2^{2\kappa_a} - \delta_a^2}} \frac{1}{\left(1 - \frac{\tilde{\delta}_a}{1+\rho}\right)\left(1 - \tilde{\delta}_a\right) - \gamma\varphi_y\tilde{\delta}_a} \tag{61}$$

$$f^{y} = f^{\pi}\frac{1}{\varphi_y}\left(1 - \frac{\tilde{\delta}_\rho}{1+\rho}\right) \tag{62}$$

$$g^{y} = g^{\pi}\frac{1}{\varphi_y}\left(1 - \frac{\tilde{\delta}_\rho}{1+\rho}\right) \tag{63}$$

$$b^{y} = b^{\pi}\frac{1}{\varphi_y}\left(1 - \frac{\tilde{\delta}_a}{1+\rho}\right) \tag{64}$$

$$c^{y} = c^{\pi}\frac{1}{\varphi_y}\left(1 - \frac{\tilde{\delta}_a}{1+\rho}\right) \tag{65}$$

Optimal Information Allocation We can now solve for the optimal choices of κ_ρ and κ_a. Using the equilibrium dynamics, the ex-ante expected utility loss is given by

$$\begin{aligned}
\frac{1}{2}\mathbb{E}\left[\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2\right] = &\frac{1}{2}\left(\left(\frac{1}{\varphi_y}\left(1 - \frac{\delta_\rho/2^{2\kappa_\rho}}{1+\rho}\right)\right)^2 + \frac{\xi}{1/\gamma+\varphi}\right)\times \\
&\left(\frac{\gamma\varphi_y}{\left(1 - \frac{\delta_\rho/2^{2\kappa_\rho}}{1+\rho}\right)\left(1 - \delta_\rho/2^{2\kappa_\rho}\right) - \gamma\varphi_y\delta_\rho/2^{2\kappa_\rho}}\right)^2 \frac{\sigma_\rho^2}{2^{2\kappa_\rho} - \delta_\rho^2} + \\
&\frac{1}{2}\left(\left(\frac{1}{\varphi_y}\left(1 - \frac{\delta_a/2^{2\kappa_a}}{1+\rho}\right)\right)^2 + \frac{\xi}{1/\gamma+\varphi}\right)\times \\
&\left(\frac{\frac{1+\varphi}{1+\gamma\varphi}\left(1-\delta_a\right)\gamma\varphi_y}{\left(1 - \frac{\delta_a/2^{2\kappa_a}}{1+\rho}\right)\left(1 - \delta_a/2^{2\kappa_a}\right) - \gamma\varphi_y\delta_a/2^{2\kappa_a}}\right)^2 \frac{\sigma_a^2}{2^{2\kappa_a} - \delta_a^2}
\end{aligned} \tag{66}$$

which can be minimized numerically by choosing (κ_ρ, κ_a) subject to $\kappa_\rho + \kappa_a \leq \kappa_M$.

C Proofs for Section 3

The results in Section 3 follow from imposing Assumption 1 in Appendix B.

Proof of Proposition 2

Set $\delta_\rho = \delta_a = 0$ in (53) and (54) to obtain the expressions in text.

Proof of Proposition 3

Set $\delta_\rho = \delta_a = 0$ in (58) to (65), and substitute into (56) and (57), noting that $\tilde{\rho}_t = \hat{\rho}_t$, $\tilde{u}_t = u_t$, $\tilde{a}_t = \hat{a}_t$, and $\tilde{v}_t = v_t$ under Assumption 1.

Proof of Proposition D

We use the expressions for output and inflation to write

$$\frac{1+\varphi}{1+\gamma\varphi}\hat{a}_t = \frac{\hat{y}_t - \frac{1}{\varphi_y}\pi_t}{\gamma}$$

and

$$\hat{\rho}_t = 2^{2\kappa_\rho}\left(1 - \frac{1}{2^{2\kappa_a}}\right)\frac{1}{\gamma\varphi_y}\pi_t + 2^{2\kappa_\rho}\sqrt{\frac{2^{2\kappa_\rho}-1}{2^{4\kappa_\rho}}\sigma_\rho^2}u_t + \frac{2^{2\kappa_\rho}}{2^{2\kappa_a}}\frac{1}{\gamma}\hat{y}_t - 2^{2\kappa_\rho}\frac{1+\varphi}{1+\gamma\varphi}\sqrt{\frac{2^{2\kappa_a}-1}{2^{4\kappa_a}}\sigma_a^2}v_t$$

Substituting these into the equation for the nominal interest rate yields

$$\iota_t = \rho + 2^{2\kappa_\rho}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}}\frac{1}{\gamma\varphi_y}\pi_t + \frac{2^{2\kappa_\rho}-2^{2\kappa_a}}{2^{2\kappa_a}}\frac{1}{\gamma}\hat{y}_t - 2^{2\kappa_\rho}\frac{1+\varphi}{1+\gamma\varphi}\sqrt{\frac{2^{2\kappa_a}-1}{2^{4\kappa_a}}\sigma_a^2}v_t + 2^{2\kappa_\rho}\sqrt{\frac{2^{2\kappa_\rho}-1}{2^{4\kappa_\rho}}\sigma_\rho^2}u_t$$

Hence

$$\iota_t = \rho + 2^{2\kappa_\rho}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}}\frac{1}{\gamma\varphi_y}\pi_t^o + \frac{2^{2\kappa_\rho}-2^{2\kappa_a}}{2^{2\kappa_a}}\frac{1}{\gamma}\hat{y}_t^o$$

where

$$\pi_t^o = \pi_t - \gamma\varphi_y\frac{1+\varphi}{1+\gamma\varphi}\sqrt{\frac{1}{2^{2\kappa_a}-1}\sigma_a^2}v_t$$

$$\hat{y}_t^o = \hat{y}_t + \gamma\frac{2^{2\kappa_a}}{2^{2\kappa_\rho}-2^{2\kappa_a}}\sqrt{\left(2^{2\kappa_\rho}-1\right)\sigma_\rho^2}u_t$$

Proof of Proposition 5

Under Assumption 1, (66) simplifies to an expression proportional to $\frac{\sigma_\rho^2}{2^{2\kappa_\rho}} + \left(\frac{1+\varphi}{1+\gamma\varphi}\right)^2\frac{\sigma_a^2}{2^{2\kappa_a}}$, which must be minimized subject to $\kappa_a + \kappa_\rho \leq \kappa_M$. The Kuhn-Karush-Tucker conditions yield the solution.

Proof of Proposition 6

Rewrite the path of nominal rates as

$$\begin{aligned}
\iota_t = \rho &+ 2^{2\kappa_\rho}\frac{2^{2\kappa_a}-1}{2^{2\kappa_a}}\frac{1}{\gamma\varphi_y}\pi_t + \frac{2^{2\kappa_\rho}-2^{2\kappa_a}}{2^{2\kappa_a}}\frac{1}{\gamma}\tilde{y}_t \\
&+ \frac{2^{2\kappa_\rho}-2^{2\kappa_a}}{2^{2\kappa_a}}\frac{1+\varphi}{1+\gamma\varphi}\hat{a}_t - 2^{2\kappa_\rho}\frac{1+\varphi}{1+\gamma\varphi}\sqrt{\frac{2^{2\kappa_a}-1}{2^{4\kappa_a}}}\sigma_a^2 v_t + 2^{2\kappa_\rho}\sqrt{\frac{2^{2\kappa_\rho}-1}{2^{4\kappa_\rho}}}\sigma_\rho^2 u_t
\end{aligned}$$

Combining this with

$$\pi_t = \varphi_y\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\mathbb{E}_t\left[\tilde{y}_{t+1}\right] - \tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^*\right)$$

yields a three equation system. Following the steps in Galí (2015), and using Proposition 5, this system is determinate when

$$\kappa_M > \log_2\left(\frac{1}{1+\rho}\frac{\sigma_\rho}{\left(\frac{1+\varphi}{1+\gamma\varphi}\right)\sigma_a} + \left(\frac{\rho}{1+\rho}+\gamma\varphi_y\right)\frac{\left(\frac{1+\varphi}{1+\gamma\varphi}\right)\sigma_a}{\sigma_\rho}\right)$$

D Mark Up Shocks

Let $\mu_t \sim N(0, \sigma_\mu^2)$ denote an i.i.d. mark up (cost push) shock that enters the New Keynesian Phillips curve, following Galí (2015). To maintain tractability, we drop TFP shocks, and assume that demand shocks are also i.i.d. Hence $r_t^* = \rho + \hat{\rho}_t$. The policy maker solves

$$\min_{\iota_t}\frac{1}{2}\mathbb{E}_{M,t}\left[\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2\right] \tag{67}$$

subject to

$$\pi_t = \varphi_y\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t\pi_{t+1} + \mu_t \tag{68}$$

$$\mathbb{E}_t\tilde{y}_{t+1} - \tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_t\pi_{t+1} - r_t^*\right) \tag{69}$$

where $\mathbb{E}_t \pi_{t+1}$ and $\mathbb{E}_t \tilde{y}_{t+1}$ are taken as given by the policy maker. The Lagrangean is

$$L = \frac{1}{2}\mathbb{E}_{M,t}\Big[\tilde{y}_t^2 + \frac{\xi}{1/\gamma+\varphi}\pi_t^2 + \lambda_{1,t}\left(\pi_t - \varphi_y \tilde{y}_t - \frac{1}{1+\rho}\mathbb{E}_t\pi_{t+1} - \mu_t\right)$$
$$+ \lambda_{2,t}\left(\mathbb{E}_t\tilde{y}_{t+1} - \tilde{y}_t - \gamma\left(\iota_t - \mathbb{E}_t\pi_{t+1} - r_t^*\right)\right)\Big]$$

Noting that ι_t is measurable with respect to $\mathbb{E}_{M,t}$, the FOCs are

$$\tilde{y}_t - \frac{1}{2}\lambda_{1,t}\varphi_y - \frac{1}{2}\lambda_{2,t} = 0$$
$$\frac{\xi}{1/\gamma+\varphi}\pi_t + \frac{1}{2}\lambda_{1,t} = 0$$
$$\mathbb{E}_{M,t}\lambda_{2,t} = 0$$
$$\pi_t = \varphi_y\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_t\pi_{t+1} + \mu_t$$
$$\mathbb{E}_t\tilde{y}_{t+1} - \tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_t\pi_{t+1} - r_t^*\right)$$

Applying the policy maker's expectation operator to each equation yields

$$\mathbb{E}_{M,t}\tilde{y}_t - \frac{1}{2}\varphi_y\mathbb{E}_{M,t}\lambda_{1,t} - \frac{1}{2}\mathbb{E}_{M,t}\lambda_{2,t} = 0$$
$$\frac{\xi}{1/\gamma+\varphi}\mathbb{E}_{M,t}\pi_t + \frac{1}{2}\mathbb{E}_{M,t}\lambda_{1,t} = 0$$
$$\mathbb{E}_{M,t}\lambda_{2,t} = 0$$
$$\mathbb{E}_{M,t}\pi_t = \varphi_y\mathbb{E}_{M,t}\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_{M,t}\pi_{t+1} + \mathbb{E}_{M,t}\mu_t$$
$$\mathbb{E}_{M,t}\tilde{y}_{t+1} - \mathbb{E}_{M,t}\tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_{M,t}\pi_{t+1} - \mathbb{E}_{M,t}r_t^*\right)$$

where we have used the law of iterated expectations (and the fact that the policy maker's information set is contained in the full information set) to simplify $\mathbb{E}_{M,t}\mathbb{E}_t\pi_{t+1} = \mathbb{E}_{M,t}\pi_{t+1}$ and $\mathbb{E}_{M,t}\mathbb{E}_t\tilde{y}_{t+1} = \mathbb{E}_{M,t}\tilde{y}_{t+1}$. Simplifying yields

$$\mathbb{E}_{M,t}\tilde{y}_t = -\varphi_y\frac{\xi}{1/\gamma+\varphi}\mathbb{E}_{M,t}\pi_t$$
$$\mathbb{E}_{M,t}\pi_t = \varphi_y\mathbb{E}_{M,t}\tilde{y}_t + \frac{1}{1+\rho}\mathbb{E}_{M,t}\pi_{t+1} + \mathbb{E}_{M,t}\mu_t$$
$$\mathbb{E}_{M,t}\tilde{y}_{t+1} - \mathbb{E}_{M,t}\tilde{y}_t = \gamma\left(\iota_t - \mathbb{E}_{M,t}\pi_{t+1} - \mathbb{E}_{M,t}r_t^*\right)$$

so that

$$\mathbb{E}_{M,t}\pi_t = -\varphi_y^2 \frac{\xi}{1/\gamma + \varphi}\mathbb{E}_{M,t}\pi_t + \frac{1}{1+\rho}\mathbb{E}_{M,t}\mathbb{E}_{M,t+1}\pi_{t+1} + \mathbb{E}_{M,t}\mu_t$$

Guess a solution of the form $\mathbb{E}_{M,t}\pi_t = c_\pi \mathbb{E}_{M,t}\mu_t$, and use the the fact that μ_t is i.i.d. to set $\mathbb{E}_{M,t}\mathbb{E}_{M,t+1}\mu_{t+1} = 0$. Then we can solve for $c_\pi = \frac{1}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}$, so that

$$\mathbb{E}_{M,t}\pi_t = \frac{1}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}\mathbb{E}_{M,t}\mu_t$$

$$\mathbb{E}_{M,t}\tilde{y}_t = -\frac{\varphi_y \frac{\xi}{1/\gamma+\varphi}}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}\mathbb{E}_{M,t}\mu_t$$

Substituting into the Euler equation yields the optimal monetary policy

$$\iota_t = \rho + \mathbb{E}_{M,t}\hat{\rho}_t + \frac{1}{\gamma}\frac{\varphi_y \frac{\xi}{1/\gamma+\varphi}}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}\mathbb{E}_{M,t}\mu_t$$

Under perfect information, we recover the textbook monetary policy found in Galí (2015). The remainder of the analysis is analogous to the case with TFP shocks.

Optimal Expectations When shocks are i.i.d., the policy maker's expectations satisfy

$$\mathbb{E}_{M,t}\left[\hat{\rho}_t\right] = (1 - 1/2^{2\kappa_\rho})\hat{\rho}_t + (\sqrt{2^{2\kappa_\rho}-1}/2^{2\kappa_\rho})\sigma_\rho u_t \tag{70}$$

$$\mathbb{E}_{M,t}[\mu_t] = (1 - 1/2^{2\kappa_\mu})\mu_t + (\sqrt{2^{2\kappa_\mu}-1}/2^{2\kappa_\mu})\sigma_\mu w_t \tag{71}$$

where u_t and w_t are i.i.d. standard normal random variables.

The optimal path of nominal (equal to real) rates satisfies

$$\begin{aligned}\iota_t = \rho &+ (1 - 1/2^{2\kappa_\rho})\hat{\rho}_t + (\sqrt{2^{2\kappa_\rho}-1}/2^{2\kappa_\rho})\sigma_\rho u_t \\ &+ \frac{1}{\gamma}\frac{\varphi_y \frac{\xi}{1/\gamma+\varphi}}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}((1 - 1/2^{2\kappa_\mu})\mu_t + (\sqrt{2^{2\kappa_\mu}-1}/2^{2\kappa_\mu})\sigma_\mu w_t).\end{aligned} \tag{72}$$

Equilibrium Dynamics The optimal paths for output and inflation are

$$
\begin{aligned}
\hat{y}_t = {} & \gamma(\hat{\rho}_t/2^{2\kappa_\rho} - (\sqrt{2^{2\kappa_\rho}-1}/2^{2\kappa_\rho})\sigma_\rho u_t) \\
& - \frac{\varphi_y \frac{\xi}{1/\gamma+\varphi}}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}((1-1/2^{2\kappa_\mu})\mu_t + (\sqrt{2^{2\kappa_\mu}-1}/2^{2\kappa_\mu})\sigma_\mu w_t),
\end{aligned}
\tag{73}
$$

$$
\begin{aligned}
\pi_t = {} & \varphi_y \gamma(\hat{\rho}_t/2^{2\kappa_\rho} - (\sqrt{2^{2\kappa_\rho}-1}/2^{2\kappa_\rho})\sigma_\rho u_t) \\
& \frac{1}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}\mu_t + \frac{\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}(\mu_t/2^{2\kappa_\mu} - (\sqrt{2^{2\kappa_\mu}-1}/2^{2\kappa_\mu})\sigma_\mu w_t).
\end{aligned}
\tag{74}
$$

Implementation The optimal monetary policy can be implemented using the rule

$$
\iota_t = \rho + 2^{2\kappa_\rho}\frac{1}{\gamma}\frac{\varphi_y \frac{\xi}{1/\gamma+\varphi}\left(1-\frac{1}{2^{2\kappa_\mu}}\right)}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}}\pi_t^o + \frac{1}{\gamma}\left(\frac{2^{2\kappa_\rho}\left(1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}1/2^{2\kappa_\mu}\right)}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}} - 1\right)\hat{y}_t^o \tag{75}
$$

where π_t^o and $\hat{y}_t^o$ are noisy observations of output and inflation that satisfy

$$
\pi_t^o = \pi_t + \frac{1}{\sqrt{2^{2\kappa_\mu}-1}}\sigma_\mu w_t, \tag{76}
$$

$$
\hat{y}_t^o = \hat{y}_t + \frac{\sqrt{2^{2\kappa_\rho}-1}}{\frac{1}{\gamma}\left(\frac{2^{2\kappa_\rho}\left(1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}1/2^{2\kappa_\mu}\right)}{1+\varphi_y^2 \frac{\xi}{1/\gamma+\varphi}} - 1\right)}\sigma_\rho u_t. \tag{77}
$$

Optimal Information Allocation The ex-ante expected utility loss is given by

$$
\mathbb{E}\ell(\kappa_\rho, \kappa_\mu) = \frac{1}{2}\left(\gamma^2\left(1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}\right)\frac{\sigma_\rho^2}{2^{2\kappa_\rho}} + \left(1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}\frac{1}{2^{2\kappa_\mu}}\right)\frac{\frac{\xi}{1/\gamma+\varphi}}{1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}}\sigma_\mu^2\right) \tag{78}
$$

Minimizing this loss subject to $\kappa_\rho + \kappa_\mu \leq \kappa_M$ yields the following solution.

$$
\kappa_\rho = \begin{cases}
0 & \text{if} \quad \log_2\left(\gamma\frac{1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}}{\frac{\xi\varphi_y}{1/\gamma+\varphi}}\frac{\sigma_\rho}{\sigma_\mu}\right) \leq -\kappa_M \\
\frac{1}{2}\kappa_M + \frac{1}{2}\log_2\left(\gamma\frac{1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}}{\frac{\xi\varphi_y}{1/\gamma+\varphi}}\frac{\sigma_\rho}{\sigma_\mu}\right) & \text{if} \quad \log_2\left(\gamma\frac{1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}}{\frac{\xi\varphi_y}{1/\gamma+\varphi}}\frac{\sigma_\rho}{\sigma_\mu}\right) \in (-\kappa_M, \kappa_M) \\
\kappa_M & \text{if} \quad \log_2\left(\gamma\frac{1+\frac{\xi\varphi_y^2}{1/\gamma+\varphi}}{\frac{\xi\varphi_y}{1/\gamma+\varphi}}\frac{\sigma_\rho}{\sigma_\mu}\right) \geq \kappa_M
\end{cases} \tag{79}
$$

and $\kappa_\mu = \kappa_M - \kappa_\rho$.

Equilibrium Determinacy The optimal equilibrium can be implemented uniquely when

$$\kappa_M > \log_2 \frac{\frac{1}{1+\rho}\varphi_y \left(\gamma \frac{\sigma_\rho}{\sigma_\mu}\right)\left(1 + \frac{\xi\varphi_y^2}{1/\gamma+\varphi}\right) + \left(\frac{\rho}{1+\rho} + \gamma\varphi_y\right)\frac{\xi\varphi_y}{1/\gamma+\varphi}\frac{1}{\gamma}\frac{\sigma_\mu}{\sigma_\rho}}{\left(\frac{\rho}{1+\rho} + \frac{\xi\varphi_y^2}{1/\gamma+\varphi}\right)}. \tag{80}$$

E An Alternative Information Cost

Our benchmark analysis assumes that mutual information is a good model of the information processing frictions faced by the policy maker. In this section, we show that our main results continue to hold when we consider an alternative information cost function proposed by Hébert and Woodford (2020). The authors propose a family of "neighborhood-based" information cost functions. In contrast to mutual information, this family of cost functions allows for the notion that certain subgroups of hidden states are easier to distinguish than others. For example, it may be less costly for the policy maker to distinguish between negative and positive shocks, than to distinguish between positive shocks of different magnitude. Hébert and Woodford (2020) argue that this feature allows such cost functions to make accurate predictions about behavior in perceptual experiments, where mutual information cannot. In our linear-quadratic setting with normal shocks, we can use the result in Hébert and Woodford (2020) that the cost function takes the form of the average Fisher information to retain tractability.

Consider the case of i.i.d. aggregate shocks (impose Assumption 1). Applying the same steps as before, it is straightforward to establish that Proposition 1 continues to hold, and that the information constraint can be expressed as

$$\underbrace{I^F\left(\mathbb{E}_{M,t}\left[\hat{\rho}_t\right];\hat{\rho}_t\right)}_{\kappa_\rho^F} + \underbrace{I^F\left(\mathbb{E}_{M,t}\left[\hat{a}_t\right];\hat{a}_t\right)}_{\kappa_a^F} \le \kappa_M^F$$

where I^F is the average Fisher information cost function, and κ_ρ^F and κ_a^F denote the information allocation under the Fisher cost.

We now use the result in Hébert and Woodford (2020) that when the underlying shocks are normal, it is optimal for the policy maker to form normally distributed conditional expectations.[21] Given this, we can exploit the fact that the average Fisher information cost

[21] Technically, Hébert and Woodford (2020) show that the optimal signal structure with be normal, which implies that the updated expectations will also be normal.

generated by two scalar normal random variables X and Y is given by

$$I^F(X;Y) = \int \phi(x) \left(\left(\frac{\partial \mu_{Y|X=x}}{\partial X} \right)^2 / \sigma^2_{Y|X=x} \right) dx$$

where $\mu_{Y|X=x}$ is the mean of Y conditional on $X = x$, $\sigma^2_{Y|X=x}$ is the variance of Y conditional on $X = x$, and $\phi(x)$ is the normal density function of X. Applying this result allows us to compute the expectations (the analogous result to Proposition 2).

Proposition 7. *Let Assumption 1 hold, and suppose that* $I^F(\mathbb{E}_{M,t}[\hat{a}_t]; \hat{a}_t) = \kappa_a^F$ *and* $I^F(\mathbb{E}_{M,t}[\hat{\rho}_t]; \hat{\rho}_t) = \kappa_\rho^F$. *Then, the expectations satisfy*

$$\mathbb{E}_{M,t}[\hat{a}_t] = \frac{\kappa_a^F}{\kappa_a^F + 1/\sigma_a^2}\hat{a}_t + \frac{\sqrt{\kappa_a^F}}{\kappa_a^F + 1/\sigma_a^2}v_t$$

$$\mathbb{E}_{M,t}[\hat{\rho}_t] = \frac{\kappa_\rho^F}{\kappa_\rho^F + 1/\sigma_\rho^2}\hat{\rho}_t + \frac{\sqrt{\kappa_\rho^F}}{\kappa_\rho^F + 1/\sigma_\rho^2}u_t$$

where v_t *and* u_t *are i.i.d. standard normal random variables.*

Comparing this result to Proposition 2 demonstrates the key effect that the Fisher information cost specification has on the equilibrium. Specifically, we see that an increase in the variance of a fundamental shock increases the correlation between the policy maker's expectation of the shock and the shock itself, all else equal. This contrasts with the mutual information case, in which the correlation did not directly depend on the variance.

This difference stems from the "neighborhood" feature of the Fisher information cost. When the variance of a fundamental shock increases, it becomes less costly for the policy maker to discriminate between nearby states at every point in the state space. Therefore, for a given information allocation, the policy maker can make more accurate forecasts of the current state in each period. Given these expectations, the construction of the equilibrium follows the same steps as before. As a result, the main qualitative features are unchanged from the mutual information cost model. The economy features stochastic fluctuations driven by endogenous monetary policy shocks. The optimal equilibrium can be implemented using a Taylor rule based on noisy observations of output and inflation, and it may not be determinate.