The Inattentive Consumer: Sentiment and Expectations

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Abstract

Expectations play a crucial role in macroeconomic models and are commonly assumed to be full-information rational. However, information is vast, costly to obtain, and difficult to understand. Using survey data, I show that consumer beliefs about economic variables are driven by a single component: sentiment. When consumers are "optimistic" (have positive sentiment), they expect the economy to expand but inflation to *decline*. This correlation stands in contrast to recent U.S. experience. I explain these stylized facts with a model of a rationally inattentive consumer who faces uncertainty about fundamentals. To economize on information costs, the consumer chooses to reduce the dimensionality of the problem and obtain a signal that is a linear combination of fundamentals. Optimal information gathering results in covariances of beliefs that differ from the underlying data-generating process, and in particular leads to countercyclical price beliefs. Thus, monetary policies that aim to stimulate the economy by raising inflation expectations can have counterproductive consequences.

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1 Introduction

Nearly all economic decisions are based on agents' perceptions about the current economy and expectations about future economic outcomes. The workhorse approach to modeling these beliefs has been full-information rational expectations (FIRE), which posits that agents not only understand the data-generating process but also know all relevant information, past and present. While useful from a modeling perspective, these assumptions are clearly heroic. Moreover, survey-based measures of perceptions and expectations throw water on the FIRE by deviating from the full-information framework in systematic ways.

If not via FIRE, then how do agents form their economic beliefs? Answering this question is at the heart of understanding macroeconomic dynamics and crafting optimal policy. For instance, in the wake of the Great Recession many central banks ran out of standard ammunition to stimulate the economy. Policymakers turned to the management of expectations, and inflation expectations in particular, through unconventional policies. For example, Governor of the Bank of Japan, Haruhiko Kuroda emphasized, "the first element [of QE] was to dispel people's deflationary mindset and raise inflation expectations."[1] However, the way in which agents form beliefs is far from well understood. As former Fed Chair Janet Yellen has stated, "we need to know more about the manner in which inflation expectations are formed and how monetary policy influences them."[2]

In this paper, I contribute to the discussion on expectation formation by documenting new stylized facts in U.S. consumer surveys. I begin by documenting a surprising correlation: consumers who believe unemployment will rise (fall) also expect higher (lower) inflation on average. While there have been historical periods of positive correlation between inflation and unemployment rates (in particular the "stagflation" period of the 1970s), experience since the mid-1980s has been marked by a negative co-movement between inflation and unemployment rates.

The positive correlation of consumer inflation and unemployment expectations is a robust feature in the cross-section and across the time-series. The result is not driven by periods like stagflation; the positive correlation holds for each year from the late 1970s to the present. Neither is the result due to certain subsets of consumers; the positive correlation holds across the distribution of education and income, as well as across age and birth year.

What drives consumers to misinterpret the co-movement of inflation and unem-

[1]Speech at the Research Institute of Japan on 8/1/2014.

[2]Speech at the Federal Reserve Bank of Boston's 60th Economic Conference on 10/14/2016.

ployment? To better understand consumer expectation formation, I conduct factor analyses on a broad set of survey responses. The survey questions run the gamut from macroeconomic forecasts (not only unemployment or inflation but also business conditions and economic policy) to personal financial conditions. Assessing the number of key component(s) and their characteristics sheds light on what drives consumer beliefs. The factor analysis shows that a single component explains the bulk of survey responses. This suggests consumers are compressing information to inform their beliefs.

Moreover, I demonstrate that this driver of consumer beliefs is sentiment. That is, at any point in time, a consumer can range on a spectrum of being optimistic to pessimistic. When consumers are optimistic, they expect typical expansionary outcomes (such as falling unemployment and improving business conditions) as well as improving personal financial conditions. If consumers were simply forecasting booms and busts, otherwise optimistic individuals should predict inflation will *rise*. However, optimistic consumers expect lower inflation.

Why do otherwise optimistic individuals expect inflation to fall, an outcome generally observed in recessions? One reason is that there is widespread consumer contempt for inflation. For example, Shiller (1996) documents that consumers worry that inflation will lower their standards of living by increasing costs without commensurate increases in income. Hence, optimistic consumers who expect that inflation will fall is consistent with sentiment-driven beliefs.

But why would unidimensional sentiment be a reasonable way to form beliefs? If consumers had perfect access to information, even a strong distaste for inflation should not lead them to misunderstand the interaction between inflation and the business cycle. Thus, consumers must be facing a friction that prevents them from obtaining full-information. Furthermore, the robustness of my empirical results suggests that the misunderstanding must be driven by something fundamental about how people form beliefs, not one-off mistakes, or errors that get corrected over time.

Many commonly used frameworks for modeling beliefs are unable to explain my findings that: (i) consumers believe inflation is countercylical in contrast to recent experience and (ii) consumers' beliefs are effectively driven by one principal component. For example, FIRE is inadequate because consumers clearly do not understand the data-generating process. Sticky information is also insufficient since it has no predictions about the dimensionality of information driving beliefs. Further still, models of learning will also not do as the stylized facts are robust across time, age, and year of birth. However, a model featuring rational inattention is able to match the stylized facts. This approach is also appealing because rational inattention is not ad-hoc; agents behave optimally in the face of information constraints.

I develop several models (static, two-period, and dynamic) of a rationally inattentive consumer. The consumer faces uncertainty about the fundamentals in the economy. To obtain information about fundamentals is costly, but doing so helps the consumer make better choices about how much to work and consume. Instead of obtaining *independent* signals about each fundamental, the consumer economizes on information costs and reduces the dimensionality of the problem. That is, the consumer optimally chooses to get a signal about a *linear combination* of fundamentals.[3] The consumer decides to learn about things most useful to him rather than acquiring all information.

After receiving a signal, the consumer updates his beliefs about the fundamentals and decides on the value of the choice variable(s). In line with the empirical stylized facts, I show the covariance of the fundamental beliefs can have a sign that is inconsistent with the underlying data-generating process. Furthermore, the consumer's signal can be viewed as rationally-obtained sentiment. From this perspective, sentiment is driven by optimal signal choice rather than amorphous animal-spirits.

In Section 3, I develop a static, partial-equilibrium model of a rationally inattentive consumer. It is purposefully stylized in order to obtain analytical results and to develop the intuition of how consumers decide on the form of their signals. The hand-to-mouth consumer chooses labor to trade-off the disutility of labor and the utility of consumption (where consumption is determined by the budget constraint). The unknown fundamentals the consumer faces are his wage and price index. The consumer is allowed to obtain noisy signal(s) on any combination of the fundamentals, but this information comes at a cost. Rather than receiving noisy independent signals on each unknown, the consumer optimally decides to learn about a linear combination of wage and price that is the most useful to know. In this setup, the consumer chooses to learn about the difference of (log) wage and price; that is, his real wage.[4]

It may not be surprising that hand-to-mouth consumers want to know about their real wage (rather than wage or price independently or some other combination of wage and price) to inform their labor choice. However, what may be unforeseen, are the implications of the information acquisition strategy for the covariance of wage and price beliefs. Suppose that the consumer receives a signal that suggests his real wage

[3]The number of signals chosen by the consumer will vary between zero, one and two, across models setups and parameters.

[4]At high information costs or for a precise prior, the consumer may opt to obtain no information. See Section 3 for the static model setup and solution.

is high. The consumer does not know whether that is due to high nominal wage, low price, or a high draw of noise. In response, the consumer adjusts his posterior beliefs about wage up slightly and price down slightly. This results in a negative covariance of wage and price beliefs.[5] Viewing wage as a measure the economic strength over the business cycle, consumers have countercylical price beliefs, in line with the empirical results.

In this static model, if the consumer knew his real wage exactly, he would also know the optimal labor choice that maximizes utility. Therefore, as the cost of information declines, the consumer will decide to obtain more precise signals on the real wage but never wants a signal on any other linear combination of wage and price. Only when information is completely free to obtain will the agent be *indifferent* to collecting more information and understanding the true values of wage and price. Collecting any information beyond a noiseless signal on real wage will not improve the consumer's utility. This result is the product of the consumer having fewer choice variables (one) than unknowns (two).

To address this limitation of the static model, I propose a two-period model with two choice variables and two unknowns in Section 4. The consumer decides how much to work in the first period and how much to save for his second period "retirement." As before, there are still two unknown fundamentals, wage and price. The consumer's consumption each period will be determined as the residual of the budget constraint; this ensures that the budget constraints hold in realizations of the fundamentals. Now, depending on the cost of information, the consumer may choose to get (i) one signal on the real wage or (ii) one signal on the real wage and one signal on wage plus price. As information costs approach zero, the agent's beliefs smoothly approach FIRE.

Section 5 extends the baseline static model to include dynamics. The consumer is assumed to be hand-to-mouth and chooses how much labor to supply. Wage and price are assumed to follow independent AR(1) processes. The consumer, as in the static model, chooses a one dimensional signal that is a linear combination of fundamentals. The dynamic model allows for investigations into how one-time shocks propagate in beliefs. For example, suppose the price level experiences a one time positive shock and wage is unaffected (i.e., a surprise expansion of the money supply with sticky wages). The labor choice response is delayed and muted in comparison to the response under full-information. The hump-shaped response to a shock is a common implication of models with rational inattention. But beyond this, I find that beliefs about price

[5]The consumer uses Bayesian updating to adjust beliefs when faced with new information.

and wage move in opposite directions. The consumer's beliefs about price increase on impact, but his beliefs about wage drop. This is due to the consumer optimally selecting a signal format that best informs him about his one choice variable, labor, rather than learning about fundamentals individually. The findings suggest that policies that seek to raise inflation expectations may perversely result in consumers becoming more pessimistic, and their expectations of other macroeconomic outcomes may deteriorate. I conclude in Section 6 with a summary and avenues for further work.

This paper contributes to three literatures: (i) empirical investigations into how agents form expectations, (ii) models of rationally inattentive agents, and (iii) the use of inflation expectations as a policy tool. First, I add to the large and growing literature that uses survey-based expectations to study how agents form expectations. Coibion et al. (2018a) provide a history of how survey-based measures of beliefs have been used to document deviations from FIRE. Recent papers have proposed lived experiences affect expectations (Malmendier and Nagel (2016) and Kuchler and Zafar (2015)) or that agents have time-varying concerns for model misspecification (Bhandari et al. (2016)). Related research has found that consumers do not understand basic macroeconomic relationships such as the income Fisher equation, the Taylor rule, or the Phillips curve (Dräger et al. (2016) and Carvalho and Nechio (2014)). I contribute to this literature by proposing and documenting that consumers' sentiment drives their perceptions and expectations about all economic variables. This approach to forming beliefs can lead to a covariance of beliefs inconsistent with the underlying data-generating process.

Second, I contribute to the rational inattention literature by developing partial-equilibrium consumer models that allow for multi-dimensional signals. I focus on the sign of the covariance of posterior means, which is under-explored in the literature, and match it to stylized facts documented in consumer surveys. The models in this paper build upon the work of Sims (2003) which began the literature on rational inattention, Maćkowiak and Wiederholt (2009) which formulates a partial equilibrium firm problem, Kőszegi and Matějka (2018) that allow for multi-dimensional signals, and Maćkowiak et al. (2018) that develop analytical results for dynamic rational inattention problems.

Third, I contribute to the literature that investigates using inflation expectations as a policy tool. I empirically document that consumers associate inflation with recessionary outcomes and explain that this correlation can be the result of a rational and optimal information acquisition strategy. Therefore monetary policies that aim to stimulate the economy by raising inflation expectations can have attenuated or even counterproductive effects. As discussed in Coibion et al. (2018c), research into how expectations influence actions is limited. In the context of the firm, higher inflation expectations have been shown to have small and short-lived effects on pricing decisions (Coibion et al. (2018b)). One explanation proposed for this limited pass-through is that higher inflation expectations lead firms to become more pessimistic about other economic outcomes. Coibion et al. (2018d) show that higher inflation expectations are associated with firms becoming more pessimistic about business conditions, more concerned about credit accessibility, and more uncertain. This is the firm counterpart to my finding that consumers associate inflation with bad outcomes.

2 Empirics

I begin this section with a discussion of the two consumer surveys utilized. In both surveys, I document a positive correlation between inflation and unemployment expectations, which stands in contrast to recent U.S. experience. Why do consumers have these odd beliefs? Using a component analysis, I show that consumers' beliefs are effectively based on one principal component. I argue that this key component is sentiment. Overall, the findings suggest consumers form a large portion of their expectations based on their general sentiment. Optimistic consumers tend to expect typical expansion-period outcomes such as unemployment declines and improved business conditions, while also (surprisingly) expecting lower inflation. Shiller (1996) documented that consumers dislike inflation; therefore, optimistic consumers believing inflation will fall in consistent with sentiment driven beliefs.

In macroeconomic models, expectations about future economic outcomes influence choices today through intertemporal substitution. For example, the consumer Euler equation suggests consumption today is a function about expectations about future outcomes. However, one may wonder if survey-based expectations inform real-world choices of the respondents in the same way expectations affect choices in a model. There are three strands of literature that suggest expectations solicited through surveys are informative of actions. First, survey-based confidence indices contain information about the future aggregate consumer expenditure (Carroll et al. (1994), Bram and Ludvigson (1998), and Ludvigson (2004)). Second, self-reported expectations influence savings decisions (Arnold et al. (2014)) and choices in a financially-incentivized experiments (Armantier et al. (2015)). Third, inflation expectations have an effect on household's spending decisions, but the direction of the relationship has varied across environments and individuals studied (e.g., Bachmann et al. (2015), D'Acunto et al. (2016), and D'Acunto et al. (2018)).[6] Therefore the sentiment-based consumer expectations that I document are likely to affect consumers' real-world actions. Appendix A.2 presents additional evidence that survey-based expectations are correlated with purchasing attitudes.

I conclude this section with an analysis of a professional forecasters' expectations. In contrast to the consumer survey results, I show that professional forecasters correctly understand the correlation of unemployment and inflation expectations to be negative. Their first principal component, like with consumers, appears to be a measure of sentiment; however, professionals get the sign on inflation correct. That is, optimistic professionals expect higher inflation along with other expansion-period outcomes.

2.1 Consumer Survey Data

I use two consumer surveys: the Michigan Survey of Consumers (MSC) and the Federal Reserve Bank of New York's Survey of Consumer Expectations (SCE). Both are monthly surveys where some participants get resampled. They differ in their sample size, with the MSC surveying approximately 500 consumers and the SCE surveying approximately 1,300. The MSC has a long time series having begun in 1978, whereas the SCE only began in 2013.

The MSC and SCE ask comparable, but not identical questions. Their questions differ in phrasing and/or the types of responses allowed (categorical versus continuous). The MSC tends to ask questions that allow categorical responses, while the SCE tends to ask questions that allow continuous responses. Given the differences in answer types, the MSC and SCE empirical analysis approaches need to be slightly different. I note the differences as they arise.

To get a sense of the question format, I discuss the two most relevant survey questions. First, the inflation questions in the MSC and SCE differ only in the phrasing used. The MSC asks, "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?" The SCE asks, "What do

[6] D'Acunto et al. (2018) show high-IQ individuals with high inflation expectations are more likely to say it is a good time to spend; however, low-IQ individuals with high inflation expectations are more likely to say it is a bad time to spend. Bachmann et al. (2015) show that higher inflation expectations are associated with a small decrease in readiness to spend. D'Acunto et al. (2016) use an announced future increase in the German VAT as an exogenous source of variation in inflation expectations in Germany relative to other European countries. This change in policy was particularly salient and easy to understand for the consumer. In response to having higher inflation expectations, German consumers reported a higher willingness to spend relative to their European counterparts.

you expect the rate of (inflation/deflation) to be over the next 12 months?" Both questions solicit the consumers' expected inflation rate, in percent, over the next year. Second, the unemployment rate questions differ in the phrasing used and the type of response requested. The MSC asks a categorical question on the expected change in the unemployment rate, "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?" The SCE solicits a numerical answer on the probability of unemployment rising with, "What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?"

2.2 Inflation and Unemployment

The New Keynesian Phillips curve is the benchmark structural relationship between inflation and output gap (or more generally, a measure of economic slack). Equation (1) is the New Keynesian Phillips curve where π_t is inflation, $\mathbb{E}_t[\pi_{t+1}]$ is the time t FIRE expectation of $t+1$ inflation, X_t is the output gap, β is the discount rate, and κ is related to the parameters of the model. As shown in Galí (2008), reasonable parameterizations will result in $\kappa > 0$, such that a higher output gap is associated with higher inflation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa X_t \tag{1}$$

Although, the New Keynesian Phillips curve contains the output gap, it can more generally be estimated using measures of economic slack. The less slack in the economy (e.g., high output gap, low unemployment), the New Keynesian model predicts higher inflation.

Historical experience generally confirms model predictions of a negative correlation between economic slack and inflation. In fact, the original "Phillips curve" was an empirical negative correlation between inflation and unemployment in the United Kingdom (Phillips (1958)). Recent experience in the U.S. also suggests inflation and unemployment have a negative correlation. Figure 1 plots the time series of the inflation and unemployment rate in the U.S. Visually, the series appear to negatively co-move in recent years. To get a better sense of the time variation of the correlation of inflation and unemployment rates, Figure 2 plots the slope coefficient for $\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$ for a ten year rolling-window regression. For the most part, the past 40 years have been characterized by a negative correlation of inflation and unemployment rates. The one era marked by a positive correlation of inflation and unemployment rates was the stagflation period of the late 1970's.

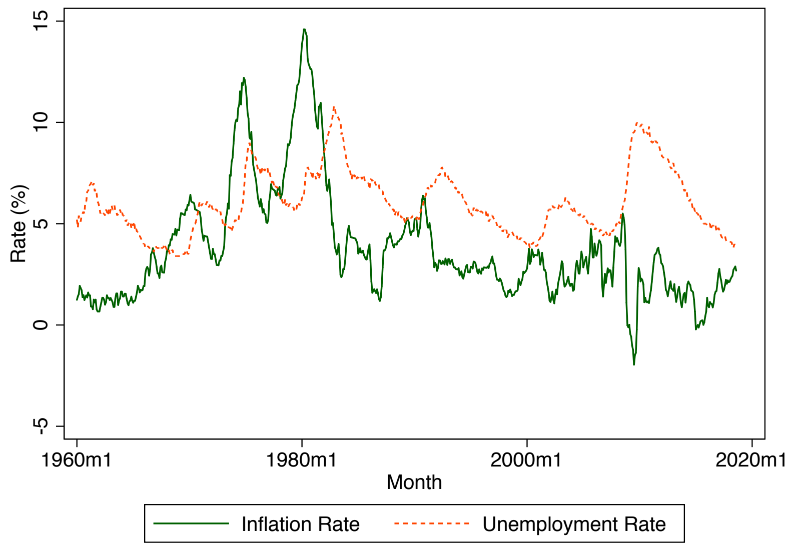


Figure 1: Inflation and Unemployment Rates

Notes: Data are from FRED. The inflation rate is the year-over-year percent change in the consumer price index for all urban consumers.

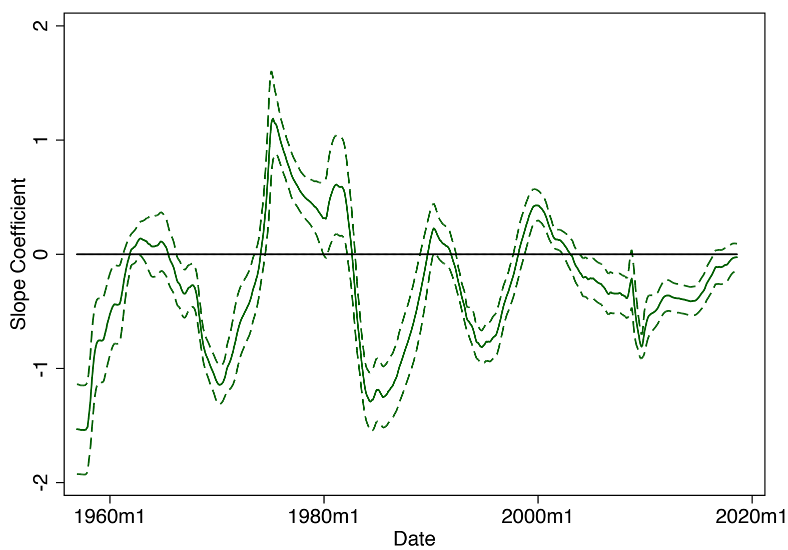


Figure 2: Correlation of Inflation and Unemployment Rates

Notes: Data are from FRED. The inflation rate is the year-over-year percent change in the consumer price index for all urban consumers. Dotted lines represent the 95% confidence interval. Ten year rolling window slope regression coefficient of $\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$ is plotted on the y-axis. The end date of the rolling regression sample is on the x-axis.

In contrast to recent experience, consumer survey-based expectations of inflation and unemployment are positively correlated. Figure 3 uses MSC data and plots, for each year, the difference in inflation expectations relative to consumers that believe unemployment will stay the same. Consumers that expect unemployment to rise have higher inflation expectations, on average, compared to those that say unemployment will stay the same or decrease, for all periods. Conversely, consumers that expect unemployment will fall have lower inflation expectations on average.

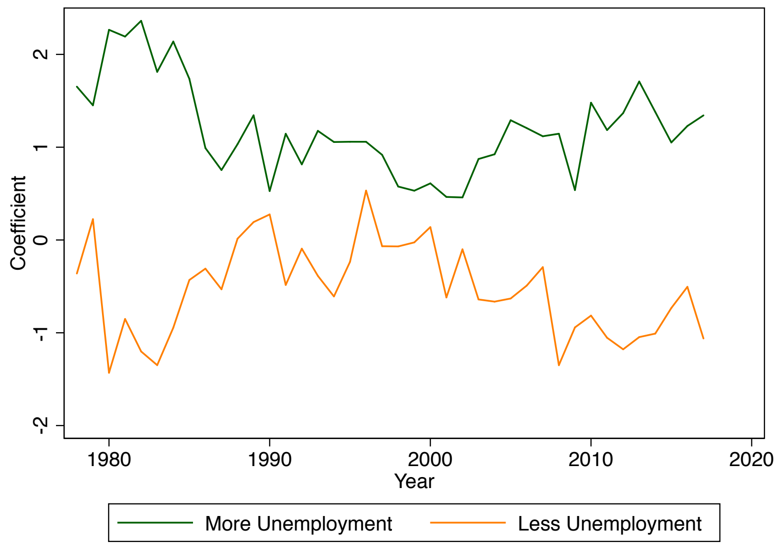


Figure 3: Unemployment and Inflation Expectations (MSC)

Notes: Data are from the MSC. The regression coefficients of $\mathbb{E}_{j,t}\pi_{t+1} = \alpha_t + \beta_t^{more} D_{j,t+1}^{more} + \beta_t^{less} D_{j,t+1}^{less} + \epsilon_{j,t}$ are plotted across t. Subscripts j and t denote consumer and year respectively. $D_{j,t+1}^{less}$ is a dummy for if consumer j stated there would be less unemployment in 1 year. $D_{j,t+1}^{more}$ is a dummy for if consumer j stated there would be more unemployment in 1 year. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall).

A regression of expected inflation on the indicator of expected change in unemployment is shown in Table 1. In comparison to consumers that expect unemployment will stay roughly the same over the next year, consumers who expect unemployment will rise expect higher inflation and consumers who expect unemployment will fall expect inflation will be lower. Using the panel structure of the survey, I can absorb time fixed effects and/or consumer fixed effects. Column (2) adds time fixed effects to remove any effects from aggregate fluctuations. The qualitative results remain.

Before adding consumer fixed effects, column (3) runs the regression from the previous column on the sample of consumers that were surveyed more than once (this is the sample that will not get absorbed into consumer fixed effects). The sample restriction does not qualitatively change the regression coefficients or significance. Column (4) includes time fixed effects and consumer fixed effects. The coefficients decrease in magnitude but remain significant. Why did the inclusion of household fixed effects attenuate the coefficients? Note that in the MSC, respondents that are re-sampled are surveyed a total of twice. The initial survey and another survey six months later. Because of the tight re-sampling window, the addition of household fixed effects removes long-term experience-based explanations. Suppose living through a high-inflation period permanently makes a consumer expect higher inflation. This effect would get absorbed into the consumer fixed effects. Accounting for personal experiences is plausibly what attenuates the coefficients, but what remains cannot be explained by experience-based stories.

Furthermore, Appendix A.1 demonstrates that across the education, income, age, and birth-year distributions, consumers believe inflation is countercylical. The magnitudes of the coefficients are somewhat attenuated at higher education and income levels, but they never flip signs and remain significant.

A similar result is found in the SCE as shown in Table 2. The SCE unemployment question is a continuous measure of the consumer's percent chance unemployment will be higher in one year. Consumers that assign a higher probability to unemployment rising have higher inflation expectations. The inclusion of time fixed effects in column (2) does not change the qualitative findings. Column (3) restricts the sample to consumers that were surveyed more than once and column (4) has both time and consumer fixed effects. The significant positive coefficient remains, although the coefficient is attenuated with the addition of consumer fixed effects.

Despite macroeconomic theory and recent U.S. experience suggesting inflation is procyclical, consumers believe inflation will be higher when unemployment rises. Section 2.5 conducts a similar exercise for professional forecasters and finds that forecasters have expectation correlations consistent with theory and recent U.S. experience.

2.3 Component Analysis

What is driving the surprising correlation between inflation expectations and unemployment expectations in consumer surveys? The surveys contain a number of other questions and utilizing them in a component analysis sheds light on what is occurring. Both consumer surveys' have a first component that explains a large portion of the variation in responses and resembles a measure of sentiment. I discuss the results for

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
More unemployment	1.590***	1.268***	1.183***	0.408***
	(0.031)	(0.029)	(0.032)	(0.044)
Less unemployment	-0.677***	-0.618***	-0.651***	-0.277***
	(0.033)	(0.032)	(0.034)	(0.048)
Time FE	N	Y	Y	Y
Consumer FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.116	0.057	0.343
N	240356	240356	165900	165900

Table 1: Positive Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Regression results from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta^{more} D^{more}_{j,t+12} + \beta^{less} D^{less}_{j,t+12} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D^{less}_{j,t+12}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D^{more}_{j,t+12}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. Columns (3) and (4) restrict the sample to households surveyed more than once. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
$\mathbb{E}_{j,t}(Prob(\Delta Unemp_{t+12} > 0))$	0.070***	0.069***	0.066***	0.034***
	(0.003)	(0.003)	(0.003)	(0.003)
Time FE	N	Y	Y	Y
Consumer FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.022	0.021	0.396
N	50660	50660	49172	49172

Table 2: Positive Correlation of Inflation and the Probability of Unemployment Rising (SCE)

Notes: Data are from the SCE. Regression results from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta \mathbb{E}_{j,t}(Prob(\Delta Unemp_{t+12} > 0)) + \mu_t + \mu_i + \epsilon_{i,t}$ are reported. Subscripts j and t denote consumer and month respectively. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

each survey in turn, because the question types (categorical vs. continuous) requires differential treatment.

First, let us consider the MSC. It tends to ask categorical questions. The responses are coded as numeric values; however, the value nominal in nature, and the distance between the values does not hold any meaning. Accordingly a multiple correspondence analysis (MCA), the categorical analog of principal component analysis (PCA), is appropriate. MCA addresses the nominal nature of the survey data, by transforming the data into an indicator matrix. The rows represent an individual's responses and the columns are indicators for each category of variables.

I include forward-looking variables (over the next year personal financial conditions, personal real income, expected rates, expected business conditions, expected unemployment, expected inflation) and backward-looking variables (last year's change in personal financial conditions, last year's change in business conditions, current government policy) in the MCA. All answers are originally categorical, with the exception of expected inflation, which I bin into three categories (deflation, inflation between 0% and 4%, inflation above 4%). The first component alone accounts for an extraordinary 76% of the variation in consumer expectations and perceptions.

What is this important first component? Although a component analysis cannot tell us the meaning of the component, the first component loadings are consistent with a measure of sentiment. The ordering of the first component loadings for all variables is such that a pessimistic expectation has a negative loading whereas an optimistic expectation has a positive loading. Table 3 presents the MCA results.

Let us consider the question on unemployment which asks, "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?" In the MCA the first component loadings are -1.54 (more unemployment), .485 (same unemployment), and 1.62 (less unemployment). Consistent with the sentiment ordering, the pessimistic opinion that unemployment will rise has the smallest loading; whereas the optimistic opinion that unemployment will fall has the largest loading.

Alternatively, consider the inflation question, "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?"[7]. What response is pessimistic or optimistic is not immediately obvious. Shiller (1996) provides insight into what inflation outcomes consumers prefer. He documents through a series of surveys that consumers dislike inflation because they believe inflation lowers their standard of living. When the surveyor pointed out that nominal incomes would

[7]Recall for the MCA I binned the responses into deflation, inflation between 0% and 4%, inflation above 4%

rise to match inflation, respondents often stated their concern about when and if their nominal incomes would adjust sufficiently to match inflation. This suggests that the consumers that are pessimistic will report high inflation while those that are optimistic will report low inflation or deflation. The MCA first component loadings are -0.80 (inflation above 4%), 0.43 (inflation between 0% and 4%), and 0.80 (less than 0%). Consistent with the sentiment ordering, the pessimistic opinion of high inflation has the smallest loading; whereas, the optimistic opinion of deflation has the largest loading.

Second, I conduct a component analysis for the SCE as a robustness check to the MSC component analysis results. The SCE's questions most commonly solicit numeric responses. Accordingly I can use PCA, rather than MCA, to find the first component. I include forward-looking questions (expected inflation, chance unemployment rises, chance savings rate rises, chance stock market rises) and one backward-looking question (last year's change in personal financial conditions) in the PCA.

The resulting first component loadings are in Table 4 column (1). The signs of the loadings for all questions is such that a pessimistic expectation has a negative loading whereas an optimistic expectation has a positive loading. This is consistent with the MSC findings, and suggests the first component in consumer expectations is sentiment. The first component explains approximately 30% of the variance in the responses. This is lower than the MSC findings because the SCE solicits continuous responses resulting in more variation in the data.

To further test if the first component is a measure of sentiment, I compare home-owners and non-home-owner expectations about average home price appreciation. One would expect home-owners to enjoy home price appreciation as their asset gains value, and rent-paying non-home-owners to dislike home price appreciation. In Table 4 columns (2) and (3), the PCA sample is restricted to home-owners only and non home-owners, respectively. The expectation of average home price appreciation has a positive first component loading for home-owners and a negative first component loading for non-home-owners.

2.4 Sentiment Index

How does the first component compare to commonly used measures of sentiment such as the Conference Board Consumer Confidence Index and the Michigan Survey of Consumer's Sentiment Index? I have argued that the signs of the loadings in the component analyses suggest that the first component is a measure of sentiment;

	(1) "optimistic"	(2) "same"	(3) "pessimistic"
Unemployment will:			
	decrease	same	increase
	1.62	0.485	-1.54
Inflation will be:			
	$\leq 0\%$	$> 0\%$ and $\leq 4\%$	$\geq 4\%$
	0.80	0.43	-0.80
Personal financial conditions will:			
	improve	same	decline
	1.04	-0.15	-2.40
Real income will:			
	increase	same	decrease
	1.44	0.46	-1.27
Rates will:			
	decrease	same	increase
	0.13	0.31	-0.23
Business conditions will:			
	improve	same	decline
	1.38	0.05	-2.15
Personal financial conditions have:			
	improved	same	declined
	.95	-.10	-1.22
Business conditions have:			
	improved	same	declined
	1.22	0.11	-1.20
Economic policy is:			
	good	fair	poor
	1.60	0.25	-1.56

Table 3: 1st Dimension Loadings for an MCA on MSC

Notes: Data are from the MSC. Multiple correspondence analysis' first component loadings are reported. Forward looking questions compare the 12 month expectation to the present. Backward looking questions compare the present to 12 months ago. The inflation response is a continuous measure; however, for the MCA I bucket the values.

however, a direct comparison to confidence indices is another way to assess the claim.

To begin, I create an aggregate time series for the first component of both (i) the Michigan Survey of Consumers' MCA and (ii) the Survey of Consumer Expectations' PCA. The first component is found for each respondent and averaged across respondents for a given response month within a survey. Recall that the MSC starts in 1978 and the SCE began in 2013. The SCE is so recent that it has not even

	(1)	(2)	(3)
Inflation rate will be:			
	-0.2228	-0.2320	-0.2463
% chance unemployment will rise:			
	-0.1094	-0.1496	-0.0210
% chance savings rate will rise:			
	0.4132	0.3840	0.4617
% chance stock market will rise:			
	0.4298	0.4172	0.4411
Will you be financially better off:			
	0.5394	0.5501	0.4999
Have you become financially better off:			
	0.5404	0.5465	0.5079
% change in average home price will be:			
		0.0336	-0.1523
N:	49977	36625	13307
Restrictions:	n/a	home-own	non-home-own
Variance explained:	0.2971	0.2608	0.2470

Table 4: 1st Dimension Loadings for a PCA on SCE

Notes: Data are from the SCE. Principal component analysis' first component loadings are reported. Forward looking questions have a 12 month horizon. The one backward looking question compares the present to 12 months ago. The responses for financial condition vary from 1 indicating much worse off to 5 indicating much better off.

experienced a whole business cycle, and so comparisons to the sentiment indexes will focus of the MSC. For completeness sake, Figure 4 plots the MSC and SCE first component for the time period both are available. Notice they track each other closely, despite being based on different surveys.

Next, I compare the first component time series to popular sentiment indices, the Conference Board Consumer Confidence Index and the Michigan Survey of Consumer's Sentiment Index. The Conference Board index relies on their own internal survey of consumers. The Michigan Sentiment Index does rely on the same underlying survey as the calculated first component; however, the questions relied upon and methodology to construct the indices is different.[8] The first component is compared to the Conference Board index and the Michigan index in Figures 5 and 6, respectively.

[8] To construct the Consumer Sentiment Index, the MSC considers five categorical questions. For each question, the relative score is calculated as the percent of consumers giving favorable responses minus the percent giving unfavorable responses, plus 100 and rounded to the nearest whole number. The five relative scores are added together, divided by the 6.7558 for the base year of 1966, and a constant is added to correct for sample design changes.

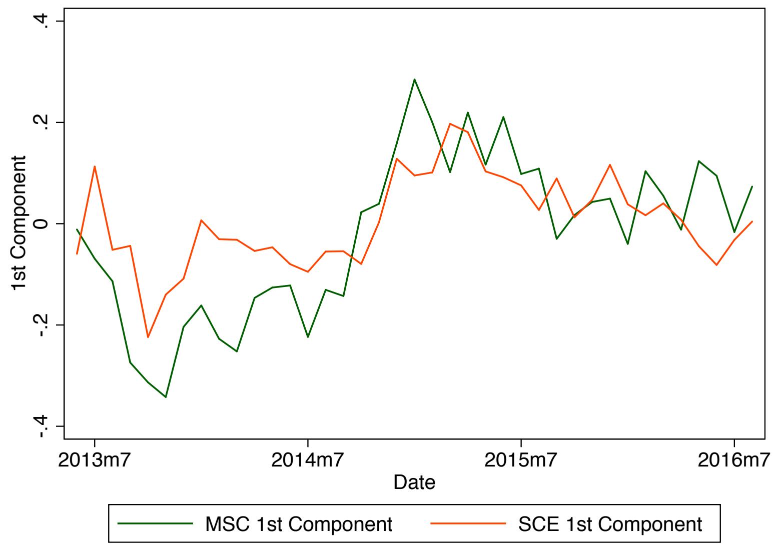


Figure 4: Comparison of 1st Components of MSC and SCE

Notes: Data are from the MSC and SCE. The MSC 1st component is based on a multiple correspondence analysis. The SCE 1st component is based on a principal component analysis. The aggregate time series are calculated as the average of 1st component values for a given month.

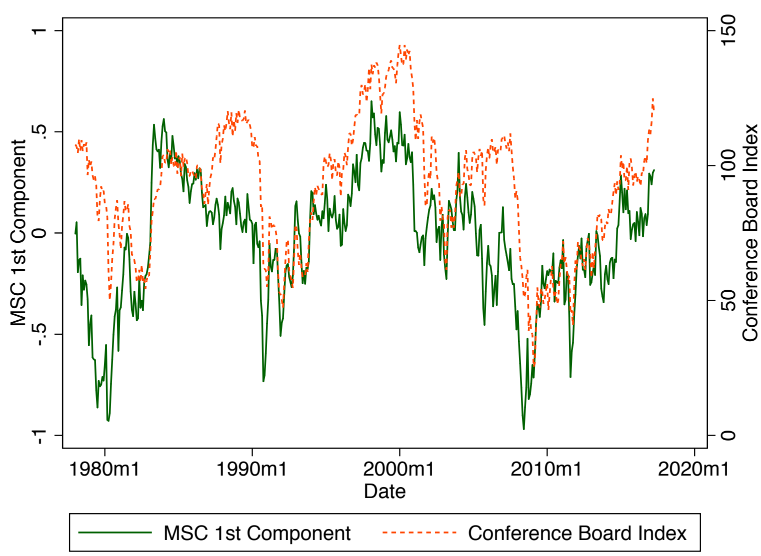


Figure 5: Comparison of MSC's 1st Component and Conference Board Confidence Index

Notes: Data are from the MSC and the Conference Board. The MSC 1st component is based on a multiple correspondence analysis. The aggregate time series is calculated as the average of 1st component values for a given month.

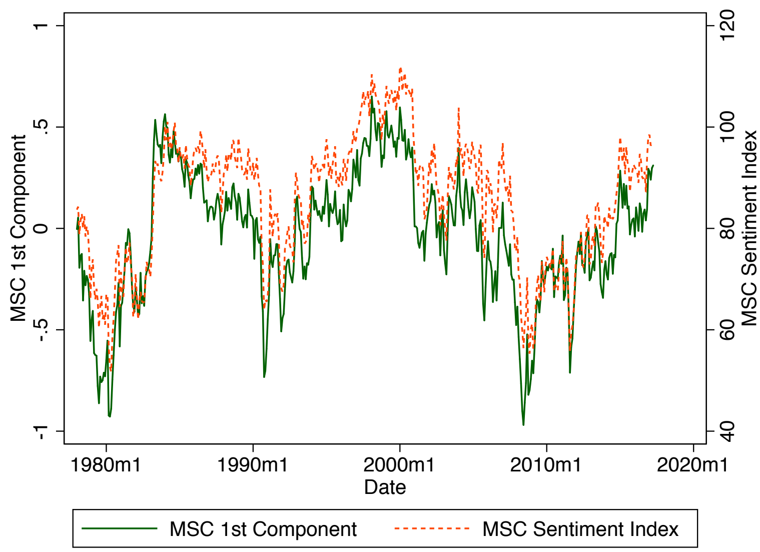


Figure 6: Comparison of MSC's 1st Component and MSC Sentiment Index

Notes: Data are from the MSC. The MSC 1st component is based on a multiple correspondence analysis. The aggregate time series is calculated as the average of 1st component values for a given month.

Indeed, the first component series looks very similar to both of the commonly used measures of sentiment, supporting the hypothesis that the MCA's first component is a measure of sentiment.

2.5 Professional Forecasters

The empirics so far have focused on consumers expectations. However, it is interesting to assess how and if the stylized facts documented for consumers differ for professional forecasters. Professional forecasters may have a lower cost of acquiring information or a more precise prior. If so, forecasters may get the sign of the variance of the posterior beliefs of inflation and unemployment correct. I use the Survey of Professional Forecasters (SPF) to test this hypothesis.

The SPF began running quarterly surveys in 1968. The first year with both inflation and unemployment questions was 1981. The number of responses vary, but recent surveys have approximately 40 responses. Some respondents are repeatedly sampled resulting in a panel structure. The respondents' forecasts are often based on a combination of models, experience, and intuition (Stark (2013)).

Table 5 contains the results of regressing inflation expectations on unemployment expectations for the SPF. The coefficient is positive, but time fixed effects are added in column (2) and the sign becomes negative.[9] The coefficient remains negative with the addition of respondent fixed effects. In line with standard macro-models, higher unemployment expectations is associated with lower inflation expectations for professional forecasters.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+4}$

	(1)	(2)	(3)
$\mathbb{E}_{j,t}Unemp_{t+4}$	0.153***	-0.443***	-0.327***
	(0.015)	(0.056)	(0.052)
Time FE	N	Y	Y
Professional FE	N	N	Y
R-squared	0.033	0.732	0.796
N	4853	4853	4830

Table 5: Negative Correlation of Inflation and Unemployment Expectations (SPF)

Notes: Data are from the SPF. Regression results from $\mathbb{E}_{j,t}\pi_{t+4} = \alpha + \beta\mathbb{E}_{j,t}Unemp_{t+4} + \mu_t + \mu_j + \epsilon_{j,t}$. Subscripts j and t denote forecaster and quarter respectively. The dependent variable is the average of the annualized forecast for CPI inflation for the next four quarters. The independent variable, $\mathbb{E}_{j,t}Unemp_{t+4}$, is the average of the forecast for unemployment for the next four quarters. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Recall that the consumer surveys regressed inflation expectations on beliefs about the change in unemployment. To compare the SPF results directly to the consumer survey results, I construct a measure of the change in unemployment expectations. This independent variable is the respondents' average of the next four quarters unemployment rate minus the current quarter belief for the unemployment rate. The coefficient is negative both with and without quarter and respondent fixed effects. Professional forecasters who believe unemployment is going to increase have a lower inflation rate expectation on average. This stands in contrast to consumer surveys but is in line with U.S. experience and standard macro-models. From the perspective of the rational inattention models developed later, this may be because professionals have a lower cost of information or a more precise prior than consumers.

A principal component analysis on SPF data on annual expectations and current-quarter perceptions of consumer price index, core consumer price index, personal consumption expenditures, real gross domestic product, unemployment rate, housing

[9]The positive coefficient in Table 5 column (1) is due to the stagflation expectations in the early 80's. If the sample is restricted to 1985 onwards (rather than 1981 onwards) the regression coefficient is negative without any fixed effects.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+4}$

	(1)	(2)	(3)
$\mathbb{E}_{j,t}[\Delta Unemp_{t+4}]$	-0.416***	-0.496***	-0.377***
	(0.071)	(0.069)	(0.066)
Time FE	N	Y	Y
Professional FE	N	N	Y
R-squared	0.010	0.730	0.796
N	4852	4852	4829

Table 6: Negative Correlation of Inflation and Change in Unemployment Expectations (SPF)

Notes: Data are from the SPF. Regression results from $\mathbb{E}_{j,t}\pi_{t+4} = \alpha+\beta\mathbb{E}_{j,t}[\Delta Unemp_{t+4}]+\mu_t+\mu_j+\epsilon_{j,t}$. Subscripts j and t denote forecaster and quarter respectively. The dependent variable is the average of the annualized forecast for CPI inflation for the next four quarters. The independent variable, $\mathbb{E}_{i,t}\Delta Unemp_{t+4} = \mathbb{E}_{i,t}Unemp_{t+4} - \mathbb{E}_{i,t}Unemp_t$, is the average of the forecast for unemployment for the next four quarters minus the current quarter belief about unemployment. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

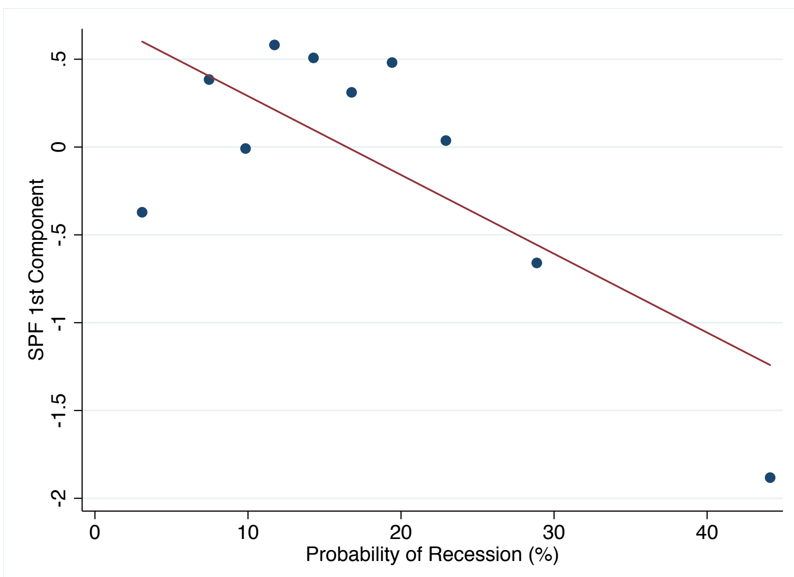


Figure 7: Binscatter of Professional's 1st Components and Their Probabilities of Recession

Notes: Data are from SPF. The first component is from a principal component analysis that contains annual expectations and current-quarter perceptions of consumer price index, core consumer price index, personal consumption expenditures, real gross domestic product, unemployment rate, housing price inflation, and nominal gross domestic product. The probability of recession is the average of the probabilities of recession for the next four quarters.

price inflation, and nominal gross domestic product was conducted.[10] It has a negative loading on unemployment and positive loadings on inflation and the other variables. The signs of the loadings suggest the first component is sentiment; however, unlike with consumers, professionals associate other expansionary outcomes with inflation.

The first component, like in the consumer surveys, appears to be a measure of sentiment or a business-cycle measure. In fact, the SPF has a question that solicits respondents to provide their subjective probability of recession in future quarters. The first component of the PCA is negatively correlated with the probability of recession, as shown in Figure 7.

3 Static Rational Inattention Model

What modeling approach can capture the stylized facts that (i) inflation and unemployment expectations have positive covariance and (ii) consumers have one principal component that effectively drives their perceptions and expectations. Common approaches to modeling beliefs such as FIRE, sticky information, or learning will not suffice. FIRE assumes that consumers understand the "model"; however, consumers consistently do not understand the role of inflation. In sticky information models, when an agent updates their information set they achieve FIRE. There are no implications about the dimensionality of information that informs consumer beliefs. Furthermore, the empirical results are stable across time, ages and birth years; this suggests learning is also not the correct theoretic underpinning.

A model of a rationally inattentive consumer is capable of matching the documented stylized facts. This section develops a static, partial equilibrium model for a consumer facing costly information acquisition. The model is stylistic to obtain an analytical solution and to clearly develop the intuition for how information is optimally gathered. The consumer has one choice variable and faces two unknown fundamentals. He is allowed to obtain costly, noisy signal(s) that are any linear combination of the state variables. It turns out the consumer optimizes by choosing one signal that is a linear combination of fundamentals. The model is similar in approach to that of Maćkowiak and Wiederholt (2009)'s for the firm and the rational inattention solution is from Kőszegi and Matějka (2018).

[10] All variables are only available for 2007 and after. The first component explains a large 41% of the variation in beliefs

3.1 Consumer Problem

The agent consumes and supplies labor. For now the model is static so the consumer is hand-to-mouth and uses all earnings to purchase the consumption good. For each unit of work, the consumer is paid wage MW, where M is a measure of labor market tightness and W is a base nominal wage. This captures that concept that when the labor market is tight (strong labor demand relative to labor supply, as in business cycle booms) workers are paid more. Crucially, I assume the consumer knows the base wage, but knows neither the labor market tightness nor the price index. So, the consumer faces two unknowns: the tightness of the labor market and the price index. Furthermore, since the wage earned is labor market tightness times base wage, I can normalize either input in the product; I normalize the base wage to $W = 1$.

The consumer can obtain costly, noisy signal(s) about any linear combination of the unknowns. If the variance of the signal noise is low, then the signal is more costly. Section 3.2 discusses the information cost in further detail. The static problem is broken into three sequential steps: (i) obtain noisy signal(s); (ii) commit to amount of labor supplied L; and (iii) consume so that the budget constraint ($CP = LM$) binds. The timing implies the budget constraint will hold in the realizations of the unknowns, not just in expectations. That is, the consumer makes a labor choice depending his *beliefs* about M and P. But how much he gets to consume will be the residual of the budget constraint that depends on his labor choice and *actual realizations* of M and P.

The consumers' utility can be expressed as $U(L, M, P)$ rather than a direct function of labor and consumption. Here labor is a choice and a function of the consumers' beliefs about the wage and the price index (where wage is the product of unknown labor market tightness and known base wage). Let $\mathbb{E}$ be the expectation operator conditioned on the consumer's information set. The consumer seeks to maximize:

$$\max_{L} U\left(L\Big(\mathbb{E}[M], \mathbb{E}[P]\Big), M, P\right)$$

I remain agnostic about the exact specification of the utility function for now to derive general results, but Section 3.6 will put structure on it.

3.2 Information Cost

The cost of information is the friction that prevents rationally inattentive agents from achieving FIRE. In the rational inattention literature, the cost of information is commonly measured using Shannon mutual information. Mathematically, the Shannon mutual information is the expected reduction of entropy (a measure of uncertainty) from the prior to the posterior. Intuitively, the more precise the posterior, the higher the Shannon mutual information.

For flexibility, the cost of information I use is Shannon mutual information times a scaling parameter, $\lambda \in \mathbb{R}_+$. If $\lambda = 0$, information is free, and the agent can costlessly obtain FIRE. If λ is very high, the agent chooses not to get a signal, and accordingly not update from their prior beliefs about the state.[11] The interesting cases arise for intermediate values of λ, where the consumer collects some but not all information.

3.3 Non-Stochastic Steady State

There exists a non-stochastic steady state, where $M = \bar{M}$ and $P = \bar{P}$. The labor supplied by each consumer solves the first order condition below. All consumers pick the same labor for a symmetric equilibrium $\bar{L}$ such that the first-order condition is satisfied:

$$U_1(L, \bar{M}, \bar{P}) = 0$$

Subscripts on the utility function denote derivates with respect to the input order variable. The "1" subscript above denotes the derivative with respect to the first input (labor).

3.4 Second-Order Approximation

Next, I take the log-quadratic approximation of the utility function around the non-stochastic solution. Small deviations from the steady state are well approximated by a log-quadratic approach. Furthermore, quadratic approximations are commonly used in the rational inattention literature because quadratic problems featuring Gaussian uncertainty result in the optimal signal(s) being Gaussian.

Denote log-deviations with lower case variables (e.g., $p = \ln P - \ln \bar{P}$). Assume p and m are drawn from independent Gaussian distributions with mean zero and variance σ_0^2. So the model's true data-generating process is such that the labor market tightness and the price index have zero correlation.

Let $\hat{u}$ be the utility function expressed in terms of log-deviations $\hat{u}(l, m, p) = U(\bar{L}e^l, \bar{M}e^m, \bar{P}e^p) = U(L, M, P)$. And let $\tilde{u}$ denote the second-order Taylor approxi-

[11] Not getting a signal is equivalent to obtaining a signal where the noise has infinite variance

mation of $\hat{u}$ at the steady state:

$$\tilde{u}(l,m,p) = \hat{u}_1 l + \frac{1}{2}\hat{u}_{11}l^2 + \hat{u}_{12}lm + \hat{u}_{13}lp + \text{terms independent of labor}$$

Subscripts on $\hat{u}$ denote derivatives with respect to the input order variable, evaluated at the non-stochastic steady state. For example, $\hat{u}_1$ is the derivate of $\hat{u}$ with respect to labor, evaluated at the non-stochastic steady state. Since labor is the choice variable, $\hat{u}_1 = 0$. Additionally, assume standard convexity in the choice variable such that $\hat{u}_{11} < 0$. The consumer cannot affect the "terms independent of labor". Therefore these additional terms that simply act as a level shifter to utility and can be ignored.

Suppose the consumer had full-information and thus knew the values m and p. How much labor would the consumer choose to supply? Let l^* be the utility-maximizing labor choice under perfect information. Taking the first-order condition of $\tilde{u}$ with respect to labor and setting it to zero, he would choose l^* according to:

$$l^* = \frac{1}{|\hat{u}_{11}|}(\hat{u}_{12}m + \hat{u}_{13}p)$$

What if the consumer does not have perfect information? He must calculate the expectation of the optimal labor choice given his information set, $l^\diamond = \mathbb{E}[l^*|\mathcal{I}]$.

3.5 Solution

In the consumer problem, there is one choice variable (labor) and two unknown fundamentals (labor market tightness and price index). Let y be the choice variable and x be the vector of unknown fundamentals:

$$y = l \quad \text{and} \quad x = \begin{bmatrix} m \\ p \end{bmatrix}$$

Recall the consumer wants to maximize his log-quadratic approximation to utility, $\tilde{u}(l,m,p) = \frac{1}{2}\hat{u}_{11}l^2 + \hat{u}_{12}lm + \hat{u}_{13}lp + \text{terms independent of labor}$. This objective function can be re-written as $-y'Dy + x'By$ where D and B are:

$$D = \frac{|\hat{u}_{11}|}{2} \quad \text{and} \quad B = \begin{bmatrix} \hat{u}_{12} \\ \hat{u}_{13} \end{bmatrix}$$

The consumer problem now takes the form of the objective function in Kőszegi and

Matějka (2018). And so, their solution methodology is applicable.[12]

Rather than solving the maximization problem directly, it is more tractable to solve a transformed problem that is a function of (i) misperceptions about the fundamentals and (ii) the cost of information. The transformation takes three steps. First, find the action, y, the agent would choose given some posterior mean of the fundamentals, $\tilde{x}$. Re-arranging the utility function as follows:

$$\begin{aligned} U(y,x) &= -y'Dy + x'By \\ &= -\left(y - \frac{D^{-1}B'}{2}x\right)' D \left(y - \frac{D^{-1}B'}{2}x\right) + \frac{x'BD^{-1}B'x}{4} \end{aligned}$$

What action maximizes utility? If the consumer's best guess of x is $\tilde{x}$, he would choose action $y = \frac{D^{-1}B'}{2}\tilde{x}$ to maximize expected utility. Notice the second term in the summation ($\frac{1}{4}x'BD^{-1}B'x$) only contains the true fundamentals and parameters. It cannot be affected by the consumer's choice; it is a level shift in the consumer's utility. Therefore this term can be dropped from the consumer's optimization problem and is so going forward.

Second, express the utility as a function of the posterior mean of the fundamentals, $\tilde{x}$, rather than the action, y. Substitute $y = \frac{D^{-1}B'}{2}\tilde{x}$ into the utility function as follows:

$$\begin{aligned} \tilde{U}(\tilde{x},x) &= -\left(\frac{D^{-1}B'}{2}\tilde{x} - \frac{D^{-1}B'}{2}x\right)' D \left(\frac{D^{-1}B'}{2}\tilde{x} - \frac{D^{-1}B'}{2}x\right) \\ &= -\left(\tilde{x} - x\right)' \Omega \left(\tilde{x} - x\right) \end{aligned}$$

where:

$$\Omega \equiv \frac{BD^{-1}B'}{4} = \frac{1}{2|\hat{u}_{11}|}\begin{bmatrix} \hat{u}_{12}^2 & \hat{u}_{12}\hat{u}_{13} \\ \hat{u}_{12}\hat{u}_{13} & \hat{u}_{13}^2 \end{bmatrix}$$

Now utility is expressed as a function of the agent's misperceptions about the fundamentals, $\tilde{x} - x$. How severe the utility loss is for misperceptions is governed by the positive semidefinite matrix Ω. This matrix can be viewed as the "loss matrix" and will be key in determining what the consumer chooses to pay attention to. He wants to learn about things that are most useful in maximizing utility.

Third, I quantify the cost of information. As previously discussed, I assume

[12] Kőszegi and Matějka (2018) provide the solution for a multi-dimensional rational inattention problem where the objective takes the form $-y'Dy + x'By$ and D is symmetric and positive-semidefinite. In the consumer problem, there is only one choice variable so D is one number and clearly symmetric. Furthermore the assumed convexity of the utility function in labor guarantees that D is positive.

the cost of information is a scaling parameter, λ, times the Shannon mutual information. The Shannon mutual information is the change in entropy from the prior to the posterior. The consumer's prior and posterior are both Gaussian, and any n-dimensional vector distributed as multivariate normal $N(mean, var)$ has entropy $\frac{n}{2} + \frac{n}{2}log(2\pi) + \frac{1}{2}log|var|$.[13] So the only term in the Shannon mutual information that the consumer's choices can affect is $\frac{1}{2}log|\Sigma|$, where Σ is the posterior variance-covariance.

Therefore, the consumer's maximization problem can be expressed as the sum of (i) the expected utility (a function of misperceptions about the fundamentals) and (ii) the cost of information:

$$\max_{\Gamma \geq \Sigma} -\mathbb{E}\Big[(\tilde{x} - x)'\Omega(\tilde{x} - x)\Big] + \frac{\lambda}{2}log|\Sigma| \tag{2}$$

The consumer's choice variable is Σ, the posterior variance-covariance matrix. So he is picking the precision of his posterior. Let $\Gamma = \sigma_0^2 I$ be the prior variance-covariance. The restriction of $\Gamma \geq \Sigma$ implies $\Gamma - \Sigma$ must be positive semidefinite. This forces the prior to be no more precise in any dimension than the posterior. Intuitively, the agent is not allowed to forget information in their prior, in exchange for more information in a dimension the agent cares more about.

Next, with the consumer optimization in-hand, I walk through the intuition for what the consumer will choose to do (Appendix B.1 has the complete proof). The loss matrix, Ω, governs how misperceptions about the fundamentals are translated to utility losses. An eigen-decomposition of Ω results in (i) eigenvectors which dictate what the consumer cares about and (ii) eigenvalues which measure how much he cares about each direction. Recall that the loss matrix is positive semidefinite. Therefore its eigenvectors are orthogonal, and the "directions" the consumer may choose to get a signal are independent. Let the matrix of eigenvectors be V. Each eigenvector, has a corresponding eigenvalue (Λ_1 and Λ_2).[14] The eigenvalues are a measure of the consumer's value of information on the corresponding eigenvector.

The consumer problem described in Equation (2) is one where the consumer is picking his posterior variance-covariance. As shown in Kőszegi and Matějka (2018), there is a simple solution in the rotated space defined by the eigenvectors of Ω. Let $J = V'\Sigma V$ be the posterior variance-covariance in the basis of the eigenvectors of Ω.

[13]The consumer faces a quadratic problem and Gaussian uncertainty. Accordingly, he will choose a Gaussian signal and have a Gaussian posterior.

[14]The eigenvalues will be non-negative because Ω is positive semidefinite.

Then the analytical solution for J is:

$$J_{ij} = 0 \text{ for all } i \neq j$$

$$J_{ii} = \min\left(\sigma_0^2, \frac{\lambda}{2\Lambda_i}\right)$$

With the solution of J, the agent's choice for Σ simply involves rotating back to the original basis.

In the consumer problem, $\Lambda_1 = 0$ and $\Lambda_2 = \frac{1}{2|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]$. Intuitively, information on the first eigenvector has no value for the consumer. He therefore does not collect costly information on this dimension ($J_{11} = \sigma_0^2$); opting to stay with his prior. Along the second dimension, the consumer wants a signal only if $\sigma_0^2 > \frac{\lambda}{2\Lambda_2}$. A signal is worthwhile if his prior variance is particularly noisy (σ_0^2 large), information is especially cheap (λ small), or if misperceptions in this direction are associated with large losses in utility (Λ_2 large).

What does the consumer's choice of the posterior-covariance imply for the consumer's posterior beliefs about the fundamentals? Let S be the realized signal that the consumer gets and Σ_ϵ be the variance-covariance of the signal error. Upon receipt of his signal, the consumer updates as a Bayesian and gets his posterior mean:[15]

$$\tilde{x} = \Gamma(\Gamma + \Sigma_\epsilon)^{-1} S$$

In the empirics section, I documented a positive covariance of consumer beliefs about unemployment and inflation. How does this map into the model? Suppose there were several consumers. Each solves the consumer optimization problem, gets their own signal, and reaches a posterior belief about the fundamentals (labor market tightness and price). The model's counterpart to empirical findings, is the negative[16] of the covariance of the posterior means. The variance-covariance of the posterior beliefs is:

$$\begin{aligned} var(\tilde{x}) &= var(\Gamma(\Gamma + \Sigma_\epsilon)^{-1} S) \\ &= \Gamma(\Gamma + (\Sigma^{-1} - \Gamma^{-1})^{-1})^{-1} \Gamma' \end{aligned}$$

The covariance between labor market tightness and price beliefs, the element of

[15] Assume the consumer's prior about all fundamentals is zero.
[16] Labor market tightness is the opposite of unemployment.

interest, for the consumer problem is:

$$cov(\tilde{m}, \tilde{p}) = \frac{\hat{u}_{12}\hat{u}_{13}\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{\hat{u}_{12}^2+\hat{u}_{13}^2}\right)}{\hat{u}_{12}^2 + \hat{u}_{13}^2} \tag{3}$$

What is the sign of the posterior means' covariance term? Everything is known to be positive with the exception of $\hat{u}_{12}\hat{u}_{13}$. The sign of the covariance will be the sign of $\hat{u}_{12}\hat{u}_{13}$. Clearly, this will depend on the functional form of the utility function.

3.6 Utility Function

So far I have been agnostic about the utility function to develop general results, but now I assume a functional form. This allows me to: (i) discuss the economic interpretation of the signal the consumer chooses; and (ii) determine the sign of the covariance of posterior beliefs of labor market tightness and price. I assume the canonical utility function:

$$U(C, L) = \frac{C^{1-\varphi}}{1-\varphi} - \frac{L^{1+1/\eta}}{1+1/\eta} \tag{4}$$

Parameter φ is the constant of relative risk aversion and η is the Frisch labor supply elasticity. Substituting the budget constraint $C = \frac{ML}{P}$ into the utility function, I remove consumption:

$$U(L, M, P) = \frac{\left(\frac{ML}{P}\right)^{1-\varphi}}{1-\varphi} - \frac{L^{1+1/\eta}}{1+1/\eta}$$

And the utility function in log-deviations is:

$$\hat{u}(l, m, p) = \frac{\left(\frac{\bar{M}e^m\bar{L}e^l}{\bar{P}e^p}\right)^{1-\varphi}}{1-\varphi} - \frac{(\bar{L}e^l)^{1+1/\eta}}{1+1/\eta}$$

For this utility function, what does the consumer choose to learn about and does it have any economic significance? The eigenvectors of the loss matrix are $(1, 1)$ and $(1, -1)$ and the eigenvalues are zero and non-zero, respectively. So, the consumer will never choose to get a signal on $m + p$. He may (depending on the cost of information, the variance of the prior, and the non-zero eigenvalue) choose to get a noisy signal on $m - p$. This is the real wage.

Why does the consumer only care about real wage? Suppose the consumer knew the real wage perfectly; he would be able to pick the optimal labor choice.[17] Knowing any extra information is unnecessary; it would neither change his optimal labor choice or his utility, yet obtaining the extra information would be costly. Thus, a consumer choosing what information to obtain, optimally pick to obtain a noisy signal on real wage. How noisy the signal is depends on the parameterization of the problem.

Next, let us consider the covariance of the posterior beliefs about labor market tightness and price under this utility function. As already shown, the sign of the covariance of labor market tightness and price beliefs will take the sign of $\hat{u}_{12}\hat{u}_{13}$. For any parametrization where $\varphi \neq 1$, indeed $\hat{u}_{12}\hat{u}_{13}$ is negative. In fact, as shown in Appendix B.2, $\hat{u}_{12} = -\hat{u}_{13}$. The covariance of the posterior labor market tightness and price beliefs, when the agent chooses to get one signal, expressed in Equation (3) can be thus be simplified to:

$$cov(\tilde{m}, \tilde{p}) = -\frac{1}{2}\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{2\hat{u}_{12}^2}\right) < 0$$

Thus optimal information gathering strategies can lead to consumers thinking prices are negatively correlated with labor market tightness. That is, when prices are high (survey data: high inflation) the labor market has slack (survey data: high unemployment).

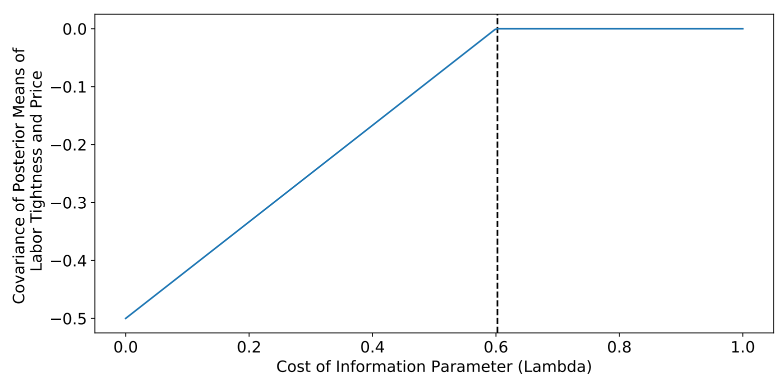


Figure 8: Covariance of Posterior Means, Static Model

Notes: The covariance of the posterior means of labor market tightness and price are plotted for varying information costs. For high information costs (λ large), the consumer gets no signals and the covariance is zero. For low information costs (λ small), he gets one signal and the covariance is negative. The prior variance-covariance is assumed to be $\sigma_0^2 I$. Parameterization values for the plot are: $\eta = 3$, $\varphi = .5$, $\sigma_0^2 = 1$.

[17] Recall optimal labor under FIRE is $l^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}m + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p$. For this utility function, $\hat{u}_{12} = -\hat{u}_{13}$. So, if the consumer knows his real wage, he also knows the optimal labor choice.

How do information costs affect the covariance? Figure 8 plots the covariance of the posterior labor market tightness and price across information costs. Recall that the true underlying data-generating process has zero covariance between labor market tightness and price. At high information costs, the agent receives no signals and the posterior covariance between labor market tightness and price is the same as the prior covariance (assumed to be zero). However when information costs are sufficiently low, the agent decides to collect one signal and the posterior covariance between labor market tightness and price becomes negative. Optimal signal collection results in price beliefs that are countercylical, consistent with survey-data but in contrast to recent experience.

As information costs approach zero, the covariance approaches -.5, rather than the zero covariance of the underlying data-generating process.[18] This is driven by the fact the agent has one choice variables and faces two unknown state variables. At zero cost of information, the consumer can perfectly learn the optimal labor choice through one signal with zero noise. With one signal, the consumer will not perfectly know the labor market tightness and price. However, the consumer has no incentive to gather more information as doing so will not increase their utility.

In the next section, I develop a two-period model where the consumer has two choice variables and faces two unknowns. In this extension, at sufficiently low information costs, the agent obtains two orthogonal signals. The consumer's covariance of posterior labor market tightness and price beliefs will approach the true underlying data-generating process' covariance as information becomes costless.

4 Two-Period Model

In the static model presented in the previous section, the household had one choice variable and faced two unknowns. As the information cost went to zero, the agent perfectly learned about their optimal choice by receiving one signal on the real wage. Regardless of how low the information cost gets, the agent never needed a second signal (the eigenvalue is zero). Therefore the covariance of the posterior means of labor market tightness and price was negative whenever the agent obtained a signal.

This may be unsatisfying in that, as information costs go to zero, one may want the agent to (i) obtain full information about all variables and (ii) have the covariance of the posterior means of labor market tightness and price smoothly approach the

[18] When the agent obtains one signal, the posterior covariance of wage and price beliefs is $-.5\left(\sigma_0^2 - \frac{\lambda|\hat{u}_{11}|}{2\hat{u}_{12}^2}\right) = -.5$ when $\sigma_0^2 = 1$ and $\lambda = 0$.

underlying data-generating covariance of zero. In this section, I develop a two-period model with two actions and two unknown fundamentals. At high levels of information costs, the consumer gets no signal (as before). At intermediate values of information costs, he obtains one signal along the eigenvector associated with the real wage. With low information costs, he chooses to obtain two signals, one along each of the orthogonal eigenvectors. In the limit of costless information, the consumer wants to learn about both unknowns perfectly.

There are two periods. In the first period, the consumer chooses how much labor to supply (L) and how much to save (S) for his second period "retirement". As in the static model, the consumer is paid MW per unit of labor, where the base wage W is normalized to one. He does not know the labor market tightness (M) or the price index (P) but may obtain costly signals about them. First period consumption (C_1) is the value that makes the budget constraint bind ($P_1C_1 = ML - S$). In the second period, the consumer consumes all of their savings, which have grown at rate R. Assume that the price index in both periods are the same ($P_1 = P_2 = P$) and the consumer understands this. Further assume the consumer knows their discount rate (β) and the savings interest rate (R). These simplifying assumptions are made so that the consumer has the same number of choices as unknowns. This is what delivers the consumer approaching FIRE as information costs fall.

To summarize, the consumer has two choice variables (labor and savings) and two unknown state variables (price index and labor market tightness). The present value of the consumer's utility is:

$$U(L, C_1, C_2) = u(C_1) - v(L) + \beta u(C_2)$$

Assume utility from consumption and disutility from labor take the forms $u(C) = \frac{C^{1-\varphi}}{1-\varphi}$ and $v(L) = \frac{L^{1+1/\eta}}{1+1/\eta}$, respectively.

Each period's budget constraint must bind: $PC_1 = ML - S$ and $PC_2 = (1 + R)S$. Substituting in these constraints, obtain utility as a function of the two choice variables and the two unknowns:

$$U(L, S, M, P) = u\left(\frac{ML - S}{P}\right) - v(L) + \beta u\left(\frac{(1 + R)S}{P}\right)$$

As in the static model, let lower case variables denote log-deviations from steady state, $\hat{u}$ be the utility function expressed in terms of log-deviations from steady state, and $\tilde{u}$ be the second order approximation at the steady state:

$$\begin{aligned}\tilde{u}(l,s,m,p) = \;& \hat{u}_1 l + \hat{u}_2 s + \frac{1}{2}\hat{u}_{11} l^2 + \frac{1}{2}\hat{u}_{22} s^2 + \hat{u}_{12} ls + \hat{u}_{13} lm + \hat{u}_{14} lp + \\ & \hat{u}_{23} sm + \hat{u}_{24} sp + \text{terms independent of labor and savings}\end{aligned}$$

Subscripts on $\hat{u}$ indicate derivates with respect to the input variable, evaluated at the steady state. Optimality of the labor and savings choices implies that $\hat{u}_1 = 0$ and $\hat{u}_2 = 0$. The non-stochastic steady state is found by normalizing the steady states of labor market tightness and the price index to one ($\bar{M} = \bar{P} = 1$), and then solving for the steady state of savings ($\bar{S}$) and labor ($\bar{L}$) so that $\hat{u}_1 = 0$ and $\hat{u}_2 = 0$.

The log-quadratic utility can be expressed as $-y'Dy + x'By$ where:

$$y = \begin{bmatrix} l \\ s \end{bmatrix}, \; x = \begin{bmatrix} m \\ p \end{bmatrix}, \; D = -\frac{1}{2}\begin{bmatrix} \hat{u}_{11} & \hat{u}_{12} \\ \hat{u}_{12} & \hat{u}_{22} \end{bmatrix}, \text{ and } B = \begin{bmatrix} \hat{u}_{13} & \hat{u}_{23} \\ \hat{u}_{14} & \hat{u}_{24} \end{bmatrix}$$

Again, the consumer problem can be solved using the methodology of Kőszegi and Matějka (2018). In the static model, the loss matrix due to misperceptions had one zero and one nonzero eigenvalue. However, in this two-period model, the loss matrix due to misperceptions has two nonzero eigenvalues. The agent will, depending on the information cost, either obtain (i) no signal and stay with their prior, (ii) one signal along the eigenvector direction with the higher eigenvalue, or (iii) two signals, one along each eigenvector direction. As information costs approach zero, the consumer will get closer to knowing his optimal labor and optimal savings choice. Furthermore, the consumer learns more about both labor market tightness and price.

The covariance of the posterior mean of labor market tightness and price, the model analog to taking the covariance of beliefs in survey data, will vary across information costs. For high information costs (λ large), the consumer gets no signals and the covariance is their prior (assumed to be zero). For intermediate information costs, the consumer gets one signal and the covariance of the posterior wage and price beliefs is negative. For low information costs (λ small), he obtains two signals and the covariance is negative; however, in the limit the covariance smoothly approaches zero. Therefore when information is costless, the covariance of the posterior labor market tightness and price beliefs matches the zero covariance in the true underlying data-generating process. Figure 9 plots the covariance of the posterior means of labor market tightness and price across information costs for a parameterization.

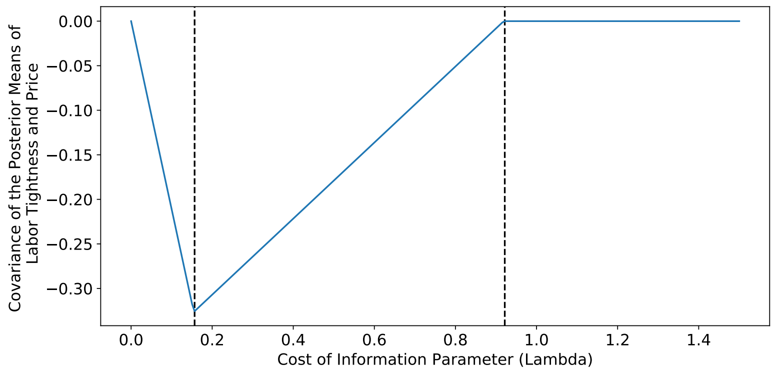


Figure 9: Covariance of Posterior Means, Two-Period Model

Notes: The covariance of the posterior means of labor market tightness and price are plotted for varying information costs. For high information costs (λ large), the consumer gets no signals and the covariance is zero. For intermediate information costs, he gets one signal and the covariance of the beliefs is negative. For low information costs (λ small), he obtains two signals and the covariance is negative; however, in the limit it smoothly approaches zero. The prior variance-covariance is assumed to be $\sigma_0^2 I$. Parameterization values for the plot are: $\eta = 3$, $\varphi = .5$, $\beta = .95$, $R = .05$, $\sigma_0^2 = 1$.

5 Dynamic Rational Inattention Model

This section extends the static consumer problem (labor market tightness and price unknown fundamentals, and labor is the choice variable) to a dynamic setup. I solve the model numerically, using the approach and findings of Maćkowiak et al. (2018). Then, I calibrate the model and use it to investigate the impulse response functions of beliefs to a shock to the price level.

5.1 Setup

Let time be discrete and denote it with t. As before, the agent consumes and supplies labor. He does not know the labor market tightness or the price index, but may obtain optimal signal(s) about them. Every period is broken into three sequential steps: (i) obtain noisy signal(s) (ii) commit to amount of labor supplied L_t and (iii) consume so that the budget constraint binds. The timing forces the budget constraint to hold in realization and not in the consumer's expectations. Notice, as in the static model, the consumer is not allowed to hold savings. The consumer can update his labor choice each period.

Unlike in the static approach, a dynamic approach requires a specification of how the fundamentals evolve. Let the log-deviations from steady state of the labor market tightness and price index follow the AR(1) processes in equations (5) and (6). The errors, ϵ_t^m and ϵ_t^p, are independent and drawn from a standard normal distribution.

$$m_t = \phi_m m_{t-1} + \theta_m \epsilon_t^m \tag{5}$$

$$p_{j,t} = \phi_p p_{j,t-1} + \theta_p \epsilon_{j,t}^p \tag{6}$$

Signals can be any linear combination of the log-deviations of current or past period wages ($\{m_t, ..., m_{t-N}\}$), prices ($\{p_t, ..., p_{t-N}\}$), wage errors ($\{\epsilon_t^m, ..., \epsilon_{t-N}^m\}$), and price errors ($\{\epsilon_t^p, ..., \epsilon_{t-N}^p\}$), where N is arbitrarily large. The consumer chooses the weights to put on each and the standard deviation of the signal error, to optimally learn about their best labor choice subject to information costs. The precise objective function is to minimize the present value of the expected mean-squared error between the optimal labor choice and their belief about the optimal labor choice plus the information cost. Subsection 5.3 discusses the objective in further detail.

Searching over the large set of possible weightings and signal error variance is time-intensive. Fortunately, the results of Maćkowiak et al. (2018) show that (in the setup used here) the consumer will optimally choose to get one signal and it will be a linear combination of current labor market tightness and price. This restriction significantly reduces the computational time needed to solve the model. Optimal signals will be of the form in Equation (7) where $S_t \in \mathbb{R}$ is the signal, $h_1 \in \mathbb{R}$ and $h_2 \in \mathbb{R}$ are signal weights, $\epsilon_t \in \mathbb{R}$ is Gaussian white noise, and $\sigma_\epsilon^2 \in \mathbb{R}_+$ is the variance of the signal error. The consumer will pick their optimal signal weights (h_1 and h_2) and variance of the signal error (σ_ϵ^2).

$$S_t = h_1 m_t + h_2 p_t + \epsilon_t \tag{7}$$

5.2 Information Set and Costs

In the static setup, a signal informs the agent about the current state. However in a dynamic model, the current signal serves two purposes. It informs the agent about the current state and stays forever in the agent's information set possibly informing the agent about future states. The information set at time t contains the current signal (S_t), all previous signals ($\{S_1, ..., S_{t-1}\}$), and the initial information set ($\mathcal{I}_0$):

$$\mathcal{I}_t = \mathcal{I}_0 \cup \{S_1, ..., S_t\} \tag{8}$$

The information set at time 0 is an infinite set of signals so that the agent's conditional variance-covariance of the true state is not time-dependent.

In line with the static model, the dynamic information cost is the Shannon mutual information scaled by λ. The information cost, $\frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right)$, is derived in Lemma 2 of Maćkowiak et al. (2018).[19]

5.3 Consumer Problem and Solution

If the consumer had full information about labor market tightness and price, he would chose optimal labor ($l_t^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}m_t + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p_t$) every period. Without full information, the consumer seeks to minimize the present value of the expected mean-squared error between the optimal labor choice and their belief about optimal labor, $\mathbb{E}[\sum_{t=1}^{\infty}\beta^t(l_t^* - \mathbb{E}(l_t^*|\mathcal{I}_t))^2]$, plus the present value of the information cost, $\mathbb{E}[\sum_{t=1}^{\infty}\beta^t\frac{\lambda}{2}\log_2(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2})]$.

The discount factor, $\beta \in (0,1)$, is assumed to be known by the consumer. Furthermore, as discussed above, the consumer's expected mean-squared error of his optimal labor choice is not time-independent. The form of the signal is also the same across time, so the information cost is constant across periods. Together, this implies that the loss function is proportional to $\mathbb{E}[(l_t^* - \mathbb{E}(l_t^*|\mathcal{I}_t))^2 + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right)]$.

Putting everything together, the consumer minimizes the per-period expected mean-squared error of optimal labor plus the information cost (Equation 9) subject to the labor market tightess and price AR(1) processes (Equations 5 and 6), the signal form (Equation 7), and the information set (Equation 8).

$$\min_{h,\sigma_\epsilon} \ \mathbb{E}\left[(l_t^* - \mathbb{E}(l_t^*|\mathcal{I}_t))^2\right] + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right) \tag{9}$$

I numerically solve the consumer problem using the algorithm discussed in detail in Appendix B.3. First, I find the consumer's optimal signal (signal weights and variance of the signal error) that minimizes the mean-squared error of the labor choice plus the information cost. Second, I use standard recursive Kalman filter updating to determine how the consumer will update his beliefs of labor, price, and labor market tightness in response to signals.

5.4 Impulse Response Functions

The model contains seven parameters to calibrate. Four parameters are associated with the AR(1) processes for log-deviations in labor market tightness and price: ϕ_m,

[19] Σ_1 is the limit as t approaches infinity of the variance-covariance of $\begin{bmatrix} m_t \\ p_t \end{bmatrix}$ given the information set at $t-1$.

θ_m, ϕ_p, and θ_p. Two parameters are associated with the utility function, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$. The cost of information scaling factor, λ, adds one last parameter.

I estimate the AR(1) coefficients for log-deviations in price and wage using quarterly, seasonally-adjusted data on the consumer price index and average weekly real earnings for full-time employees from 1980 onwards.[20] AR(1) processes are estimated on the cyclical component of each series obtained using a Hendrick-Prescott filter with a smoothing parameter of 1600. The autoregressive coefficients are $\phi_m = .600$ and $\phi_m = .813$. I set $\theta_m = 1$ and $\theta_p = 1$.

The weights on labor market tightness and price log-deviations in optimal labor log-deviations are $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$, respectively. Assume the utility function in Equation (4) and $\varphi \neq 1$. Then, as shown in Appendix B.2, the weights must be equal in magnitude but of opposite signs. The precise values will depend on the steady state values of labor market tightness, price, labor, and the values of φ and η in the utility function. Assume the steady state values for labor market tightness and price are 1, $\varphi = .5$, and $\eta = 3$. Then, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$.[21] I set $\lambda = 1$ for the baseline results, but later will vary it to assess the impact of scaling the cost of information.

With the parameters set, consider the effects of an exogenous, positive, one standard-deviation shock to the log-deviation in price. This shock can be interpreted as a positive money supply shock. The impulse response functions of the signal, the true log-deviations of labor market tightness, price and optimal labor, as well as the beliefs about the log-deviations of labor market tightness, price, and optimal labor are plotted in Figure 10. The true value of the log-deviation of price jumps up on impact and reverts back to steady-state following its AR(1) process. Log-deviations of labor market tightness are not affected by the price shock so the log-deviation of tightness stays at zero. Optimal labor's log-deviation falls by .4 on impact because tightness was unaffected, price rose by 1 log-deviation, and the coefficient on price on optimal labor is -.4.

Panel A shows the evolution of the signal in response to a one standard deviation shock to price. The optimal signal weights for this calibration are 1 and -1.23 for labor market tightness and price log-deviations, respectively. Therefore the shock to price results in a simultaneous signal of -1.23. The signal then reverts back to zero. Reversion speed is dependent on the signal weight on price log-deviations and the AR(1) process that governs the return of price to steady state.

What happens to labor beliefs (and thus the consumer's labor choice) in response

[20] Data from FRED.

[21] At the steady state, $\hat{u}_1 = 0$.

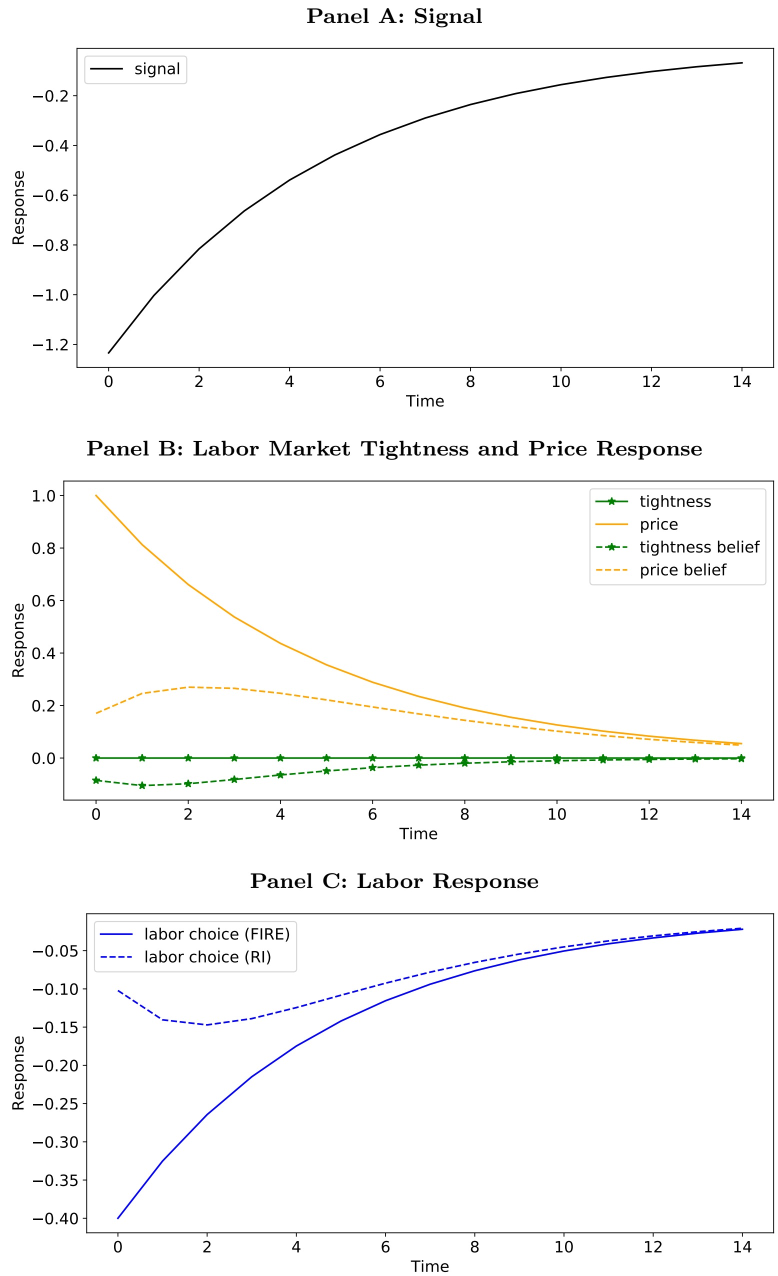
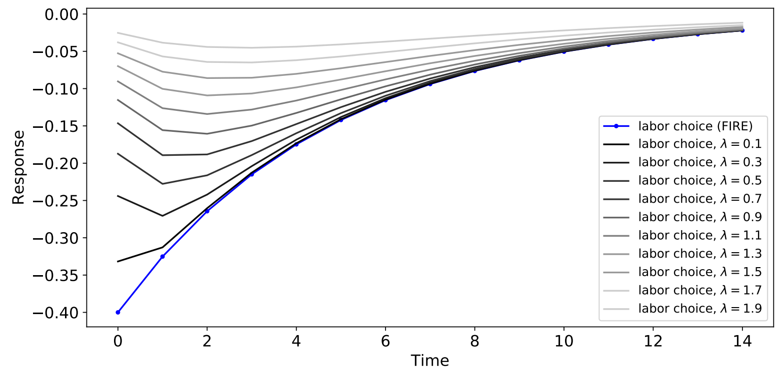


Figure 10: Impulse Responses After a Standard Deviation Shock to Price

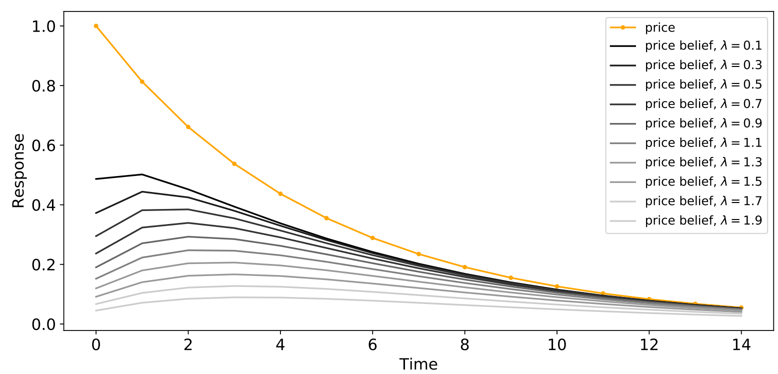
Notes: Signal and response of log-deviations in labor market tightness, price, and optimal labor (actual and beliefs) to one standard deviation shock to price. Parameter values are: $\phi_m = .600$, $\theta_m = 1$, $\phi_p = .813$, $\theta_p = 1$, $\lambda = 1$, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$. For this calibration, the optimal signal weights are $h_1 = 1$ and $h_2 = -1.23$.

to this money supply shock? The consumer does not fully believe the signal because he understands the signal is noisy. Accordingly, he uses recursive Kalman filter updating to form his labor belief and under-reacts to shocks on impact. The consumer's labor choice falls less than the optimal labor choice, in response to an expansion of the money supply. This under-reaction to shocks is typical of rational inattention models.

Panel A: Labor Response



Panel B: Price Response



Panel C: Labor Market Tightness Response

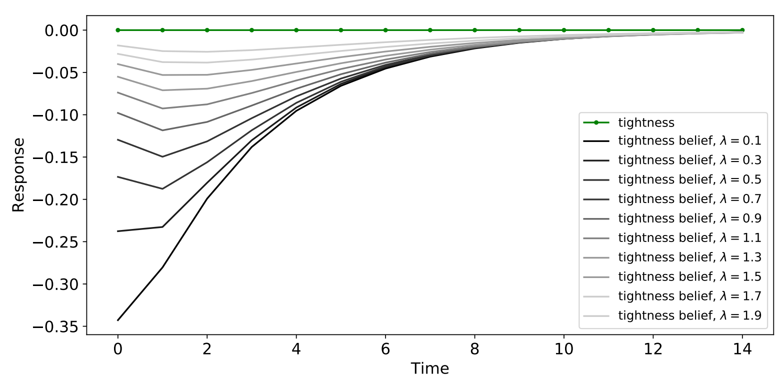


Figure 11: Varying the Cost of Information, Impulse Responses After a Standard Deviation Shock to Price

Notes: Response of labor beliefs for different information costs to one standard deviation shock to price. Parameter values are: $\phi_m = .600$, $\theta_m = 1$, $\phi_p = .813$, $\theta_p = 1$, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .4$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = -.4$

The consumer optimized his signal so as to minimize the mean-squared error of their labor choice. He did not care about the labor market tightness or price independently, and only cared about these unknowns to the extent they entered the optimal labor choice. Upon getting a negative signal due to a money supply shock, he is not sure if it came from a negative labor market tightness shock or a positive price shock (or even noise for that matter). The consumer, if asked to provide his best estimates of tightness and price, would use recursive Kalman filter updating to form their beliefs about tightness and price. His price belief jumps up (less than the price shock) and his tightness belief jumps down (despite wage still being at steady state) on impact. As time passes and he recursively updates, tightness and price beliefs approach true tightness and true price, respectively. Note that if the consumer had obtained independent signals on tightness and price, tightness beliefs would not have reacted to the price shock.

How does varying the information cost affect the impulse response functions? The cost of information influences the consumer's choice of optimal signal and thus their response to shocks. Figure 11 plots the impulse response functions for labor, labor market tightness, and price in response to a price shock, for varying costs of information. As the cost of information decreases (λ declines), the consumer chooses to have less noise in their signal. In the limit, the consumer learns the exact optimal labor choice. The consumer's price and labor market tightness beliefs, in contrast to their labor beliefs, do not reach the true values as information costs go to zero. This is because the consumer does not care about price or tightness independently; they only seek to know their optimal labor choice.

Notice that as information costs fall, the consumer's price beliefs rise (closer to the true price) and tightness beliefs decrease (further from the true labor market tightness). Why is the consumer's belief about labor market tightness getting further from the truth? As the information cost goes to zero, the consumer optimally decreases noise in his signal, and increasingly believes the signal. Due to the signal being one dimensional, the consumer does not know if a low (high) signal should be attributed to a negative tightness shock or a positive price shock (positive tightness shock or negative price shock). He therefore updates his beliefs about both state parameters, despite the fact, that in reality there was only a shock to one state parameter.

6 Concluding Remarks

Although full-information rational expectations has served macroeconomics well, mounting survey-based evidence suggests agents deviate from FIRE in systematic ways. These differences are essential to document and incorporate into the canon, as they will affect macroeconomic dynamics and optimal policies.

This paper documented new stylized facts about consumer perceptions and expectations. Consumer beliefs about economic variables are driven by a single component: sentiment. When consumers are "optimistic" (have positive sentiment), they expect the economy to expand (e.g., unemployment to decline, business conditions to improve, and personal financial conditions to strengthen) but inflation to *decline*. This correlation stands in contrast to recent experience, but is robust across the distributions of age, education, age, and birth year.

I developed static, two-period, and dynamic models of a rationally inattentive consumer that explain the stylized facts. The consumer has uncertainty about fundamentals and faces information costs. He economizes on these costs by reducing the dimensionality of the problem and optimally choosing to obtain a signal about a *linear combination* of fundamentals. In particular, he wants to learn about his real wage first (rather than wage or price independently). This information acquisition strategy leads to countercylical price beliefs. The models: (i) generate the observed counter-intuitive covariance of expectations; and (ii) demonstrate why and how consumers reduce the dimensionality of their information acquisition problem.

The findings suggest central bankers should use caution when attempting to use inflation expectations as a policy tool. Consumers associate inflation with recessionary outcomes, both in survey-data and in rational inattention models. This suggests monetary policies that aim to stimulate the economy by raising inflation expectations can have attenuated or even counterproductive effects.

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Appendix A Empiric Robustness

A.1 Inflation and Unemployment Expectations

Consumers believe inflation is countercylical across incomes, education achieved, ages and birth years. Table 7 demonstrates that consumers that expect unemployment will rise have higher inflation expectations on average, across highest educational degree achieved. Column (1) uses consumers without a degree, column (2) uses consumers whose highest degree is high-school and column (3) uses consumers who hold a college degree. The coefficients' magnitudes decline as education levels increase, however they remain significant and do not flip signs.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)
More unemployment	0.634**	0.467***	0.282***
	(0.254)	(0.062)	(0.055)
Less unemployment	-0.811***	-0.267***	-0.191***
	(0.309)	(0.071)	(0.059)
Time FE	Y	Y	Y
Consumer FE	Y	Y	Y
Highest Degree	none	high-school	college
R-squared	0.292	0.345	0.349
N	11979	85322	61502

Table 7: By Education Level, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Regression results, by highest degree obtained groups, from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta^{more}D^{more}_{j,t+12} + \beta^{less}D^{less}_{j,t+12} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D^{less}_{j,t+12}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D^{more}_{j,t+12}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Table 8 shows that regardless of income quartile, consumers believe inflation is countercylical. A consumer's income quartile is based on their income relative to the distribution of incomes reported for that month in the MSC. The number of observations is not constant due to bunching at the cutoffs between quartiles. The coefficients are slightly attenuated for higher incomes, but the qualitative takeaways are the same across quartiles.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
More unemployment	0.571***	0.604***	0.272***	0.320***
	(0.138)	(0.105)	(0.095)	(0.074)
Less unemployment	-0.512***	-0.431***	-0.190*	-0.223***
	(0.159)	(0.125)	(0.103)	(0.080)
Time FE	Y	Y	Y	Y
Consumer FE	Y	Y	Y	Y
Income Quartile	1 (poor)	2	3	4 (rich)
R-squared	0.301	0.353	0.344	0.380
N	27613	26359	25686	32156

Table 8: By Income Quartile, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Income quartiles are based on the consumer's reported income relative to the distribution of incomes reported that month. Regression results, by income quartile, from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta^{more} D^{more}_{j,t+12} + \beta^{less} D^{less}_{j,t+12} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D^{less}_{j,t+12}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D^{more}_{j,t+12}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

Across ages, consumers who expect higher unemployment over the next year have higher inflation expectations and vis versa. Table 9 presents regression results for consumers under 40, between 40 and 60, and above 60. The age is determined by the consumer's age at the time of the survey. There is no evidence that age-based learning attenuates the effects.

Lived-experience of high inflation has been shown to result in higher inflation expectations (Malmendier and Nagel (2016)). More generally, ones life experience can affect their expectations. So does the covariance of inflation and unemployment expectations differ across lived experience? Table 10 regresses inflation on unemployment expectations by groups according to birth years, with time and consumer fixed effects. Across cohorts, agents believe unemployment and inflation are positively correlated.

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)
More unemployment	0.492***	0.332***	0.379***
	(0.079)	(0.064)	(0.088)
Less unemployment	-0.293***	-0.221***	-0.247***
	(0.084)	(0.077)	(0.092)
Time FE	Y	Y	Y
Consumer FE	Y	Y	Y
Age	<40	40 to 60	>60
R-squared	0.355	0.361	0.292
N	63261	57717	41880

Table 9: By Age, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Age is the consumer's age at the time of the survey. Regression results, by age, from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta^{more}D^{more}_{j,t+12} + \beta^{less}D^{less}_{j,t+12} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D^{less}_{j,t+12}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D^{more}_{j,t+12}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

A.2 Expectations to Actions

As discussed in Section 2, consumer beliefs affect their actions in macroeconomic-models. The empirical literature also suggests survey-based consumer expectations predict outcomes such as savings decisions and contain information on future aggregate outcomes. This paper does not focus on investigating the empirical relationship between survey-based expectations and actions since the relationship is already well-established, and the surveys used in this paper (MSC, SCE, SPF) do not contain direct data on the respondent's actions or choices.

There is however data that may be correlated related to choices. That is, the MSC asks three questions (listed below) on if it is a good or bad time to buy a home, durable household goods, and vehicles. It is plausible to expect that people that say it is a good time to buy an item are more likely to buy that item.

- "Generally speaking, do you think now is a good time or a bad time to buy a house?"

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
More unemployment	0.397***	0.377***	0.393***	0.581***
	(0.140)	(0.073)	(0.066)	(0.121)
Less unemployment	-0.137	-0.304***	-0.293***	-0.224*
	(0.150)	(0.086)	(0.073)	(0.127)
Time FE	Y	Y	Y	Y
Consumer FE	Y	Y	Y	Y
Birth Year	<1930	1930-1950	1950-1970	>1970
R-squared	0.280	0.381	0.350	0.308
N	23921	52103	71282	17372

Table 10: By Birth Year, Correlation of Inflation and the Change in Unemployment Expectations (MSC)

Notes: Data are from the MSC. Regression results, by birth year, from $\mathbb{E}_{j,t}\pi_{t+12} = \alpha + \beta^{more} D^{more}_{j,t+12} + \beta^{less} D^{less}_{j,t+12} + \mu_t + \mu_j + \epsilon_{j,t}$ are reported. Subscripts j and t denote consumer and month respectively. $D^{less}_{j,t+12}$ is a dummy for if consumer j stated there would be less unemployment in 12 months. $D^{more}_{j,t+12}$ is a dummy for if consumer j stated there would be more unemployment in 12 months. The expectation of the change in unemployment MSC question allows for categorical answers (unemployment will rise, stay the same, or fall). The omitted group are those who responded unemployment will stay the same. Robust standard errors are in parenthesis. ***, **, * denotes statistical significance at 1, 5 and 10 percent levels.

- "About the big things people buy for their homes – such as furniture, a refrigerator, stove, television, and things like that. Generally speaking, do you think now is a good or a bad time for people to buy major household items?"
- "Speaking now of the automobile market – do you think the next 12 months or so will be a good time or a bad time to buy a vehicle, such as a car, pickup, van or sport utility vehicle?"

The baseline MCA empirical results did not utilize these questions. This Appendix incorporates the questions in three ways. First, the consumers who state it is a good time to buy items have, on average, a higher first dimension component in the baseline MCA. Second, the addition of the 'choice-related' questions to the baseline MCA does not alter the qualitative takeaways. Third, the first component of an MCA with only the 'choice-related' questions is highly correlated with the first component of the baseline MCA. Taken together, these findings suggest that consumers' choices are related to their expectations, and 'optimistic' consumers are more likely to purchase a home, vehicle, or household durable because they think it is a good time to do so.

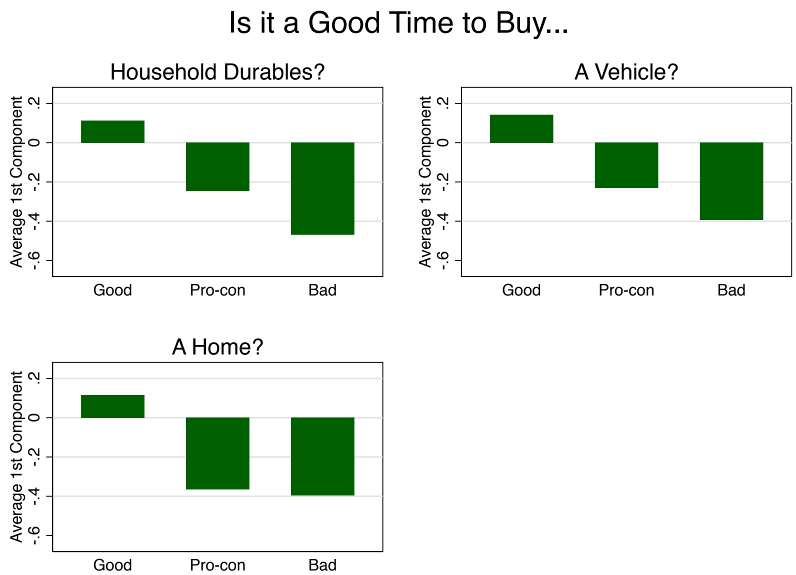

Figure 12: Average 1st Components by Response to Choice-Related Questions

Notes: Data are from MSC. The y-axis is the average first component of the MCA containing questions about expectations and perceptions, across consumers that responded to the choice-related questions

To begin, I plot the average 1st dimension component from the baseline MCA, calculated across responses to the 'choice-related' questions in Figure 12. Consumers who respond it is a good time to buy household durables, vehicles, and cars have a higher average 1st component (i.e. are relatively optimistic). Whereas consumers who respond it is a bad time to buy these items have a lower average 1st component (i.e. are relatively pessimistic).

Table 11 presents the first component loadings for an MCA with all baseline questions and the three 'choice-related' questions. As with the baseline results, the signs of the loadings are such that a 'pessimistic' belief has a negative loading whereas a 'optimistic' belief has a positive loading.[22] For example, the 'optimistic' belief that it is a good time to purchase a home, vehicle, or household durable all have positive loadings in the first component.

Next, I conduct an MCA on only the 'choice-related' questions. The first component of the choice-related questions and the first component of the baseline MCA are plotted in Figure 13's binscatter. The first component of choice-related-questions is

[22]The one question that deviates from the pattern is the respondent's expectation on how rates will change. Responding decrease or increase has a small negative loading and responding stay the same has a positive loading. This could be because whether rate changes are 'good' or 'bad' for a respondent depends on their savings and debt.

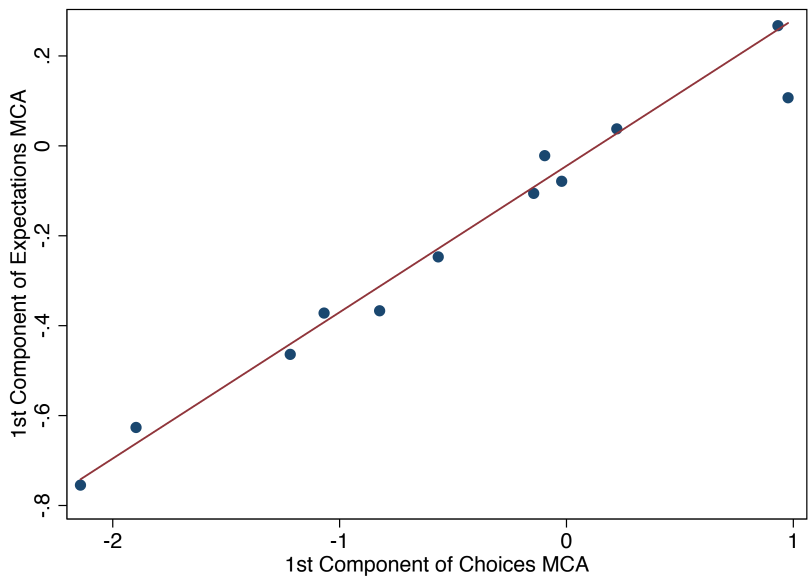

Figure 13: Binscatter of 1st Component of Expectation Questions and 1st Component of Choice-Related Questions (MSC)

Notes: Data are from MSC. The x-axis is the first component of an MCA containing three questions: is it a good or bad time to buy a vehicle, home, household durable. The y-axis is the first component of the MCA containing questions about expectations and perceptions.

strongly correlated with the first component of expectation-questions.

In summary, responses to choice-related questions are highly correlated to expectation-related questions, with 'optimistic' consumers saying it is a good time to make purchases. Assuming a respondent who says it is a good time to buy a home, vehicle, or household durable is more likely to do so, then expectations will be correlated to real actions.

	(1) "optimistic"	(2) "same"	(3) "pessimistic"
Unemployment will:			
	decrease	same	increase
	1.60	0.55	-1.63
Inflation will be:			
	$\leq 0\%$	> 0 and $\leq 4\%$	≥ 4
	0.68	0.57	-0.88
Personal financial conditions will:			
	improve	same	decline
	1.03	-0.14	-2.44
Real income will:			
	increase	same	decrease
	1.46	0.45	-1.28
Rates will:			
	decrease	same	increase
	-.17	0.33	-0.08
Business conditions will:			
	improve	same	decline
	1.36	0.08	-2.22
Personal financial conditions have:			
	improved	same	declined
	.98	-0.07	-1.31
Business conditions have:			
	improved	same	declined
	1.33	0.18	-1.31
Economic policy is:			
	good	fair	poor
	1.65	0.26	-1.61
Good/bad time to buy a house:			
	good time	pro-con	bad-time
	0.55	-1.10	-1.31
Good/bad time to buy household durables:			
	good time	pro-con	bad-time
	0.53	-0.75	-1.57
Good/bad time to buy a vehicle:			
	good time	pro-con	bad-time
	0.64	-0.77	-1.30

Table 11: 1st Dimension Loadings for an MCA on MSC, Includes Choice-Related Questions

Notes: Data are from the MSC. Multiple correspondence analysis' first component loadings are reported. Forward looking questions compare the 12 month expectation to the present. Backward looking questions compare the present to 12 months ago. The inflation response is a continuous measure; however, for the MCA I bucket the values. The rates expectation question is colored in gray as it is the only question whose coefficients are not ordered from largest to smallest going from answers that are optimistic to those that are pessimistic.

Appendix B Proofs

B.1 Static Model Solution

This section provides the proofs to achieve the solution of the static consumer problem. I begin from the consumer's maximization problem in Equation (2). The first term in the consumer's objective can be simplified to $-Tr(\Omega\Sigma)$ because Σ is the posterior variance-covariance:

$$\max_{\Gamma \geq \Sigma} -Tr(\Omega\Sigma) + \frac{\lambda}{2} log|\Sigma|$$

Let v^1 and v^2 be an orthonormal basis of eigenvectors of the loss matrix Ω (which is positive semidefinite). Let the matrix consisting of columns v^1 and v^2 be called V. The eigenvalue corresponding to v^i is Λ_i. Let Λ be the matrix with Λ_i elements on the diagonal and 0 entries elsewhere. Decomposing the loss matrix, Ω, into its eigenvalues and eigenvectors results in $\Omega = V\Lambda V'$. Note that because Ω is symmetric, the eigenvectors will be orthogonal. The consumer problems' eigenvalues of Ω are $\Lambda_1 = 0$ and $\Lambda_2 = \frac{1}{2|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]$. The corresponding eigenvectors and the resulting matrix of the orthonormal basis of eigenvectors are:

$$v^1 = \frac{1}{\sqrt{1 + \frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}} \\ 1 \end{bmatrix} \text{ and } v^2 = \frac{1}{\sqrt{1 + \frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} \begin{bmatrix} \frac{\hat{u}_{12}}{\hat{u}_{13}} \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}} \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{\hat{u}_{12}}{\hat{u}_{13}} \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \\ \frac{1}{\sqrt{1+\frac{\hat{u}_{13}^2}{\hat{u}_{12}^2}}} & \frac{1}{\sqrt{1+\frac{\hat{u}_{12}^2}{\hat{u}_{13}^2}}} \end{bmatrix}$$

The agent will not update along the first eigenvector since it has an eigenvalue of zero. However, the agent may choose to get a signal along the second eigenvector. Intuitively, the agent is transforming the problem into "directions" and choosing a signal in a direction that is important to him. Notice that the second eigenvector multiplied by x, is the direction of optimal labor under perfect information.

Let $J = V^{-1}\Sigma V = V'\Sigma V$ be the variance-covariance of the posterior in the basis of the eigenvectors of Ω. Note since V is orthogonal its inverse is the same as its transpose. Once J is determined, Σ can be found by rotating back into the original basis. See Kőszegi and Matějka (2018) for the proof of the general solution that:

$$J_{ij} = 0 \text{ for all } i \neq j$$

$$J_{ii} = \min\left(\sigma_0^2, \frac{\lambda}{2\Lambda_i}\right)$$

With V and J determined, the posterior variance-covariance that the consumer chooses is simply $\Sigma = VJV'$.

B.2 Utility Function: Second Derivatives

This appendix demonstrates that whenever $\varphi \neq 1$ (i) the sign of $\frac{\hat{u}_{13}}{\hat{u}_{12}} = -1$ and (ii) the weights on labor market tightness and price log-deviations in optimal labor are equal, but opposite signs. First, recall the utility function:

$$U(C, L) = \frac{C^{1-\varphi}}{1-\varphi} - \frac{L^{1+1/\eta}}{1+1/\eta}$$

Substituting the budget constraint $C_j = \frac{W_j L_j}{P}$ into the utility function results in:

$$U(L, M, P) = \frac{\left(\frac{ML}{P}\right)^{1-\varphi}}{1-\varphi} - \frac{L_j^{1+1/\eta}}{1+1/\eta}$$

The utility function written in log-deviations is:

$$\hat{u}(l, m, p) = \frac{\left(\frac{\bar{M}e^m \bar{L}e^l}{\bar{P}e^p}\right)^{1-\varphi}}{1-\varphi} - \frac{(\bar{L}e^l)^{1+1/\eta}}{1+1/\eta}$$

For $\varphi \neq 1$, the second order partial derivative of $\hat{u}$ with respect to labor and labor market tightness evaluated at the steady state is:

$$\hat{u}_{12} = (1-\varphi)\left(\frac{\bar{M}\bar{L}}{\bar{P}}\right)^{-\varphi+1}$$

Similarly, for $\varphi \neq 1$, the second order partial derivative of $\hat{u}$ with respect to labor and price evaluated at the steady state is:

$$\hat{u}_{13} = (\varphi-1)\left(\frac{\bar{M}\bar{L}}{\bar{P}}\right)^{-\varphi+1}$$

The ratio of the second order partial derivatives is negative one when $\varphi \neq 1$:

$$\frac{\hat{u}_{12}}{\hat{u}_{13}} = -1$$

Recall that the weights on labor market tightness and price log-deviations on optimal labor log-deviations were, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}$ and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$, respectively. If $\varphi \neq 1$, then the weights will be of equal magnitude but have opposite signs.

$$\hat{u}_{12} = -\hat{u}_{13} \Rightarrow \frac{\hat{u}_{12}}{|\hat{u}_{11}|} = -\frac{\hat{u}_{13}}{|\hat{u}_{11}|}$$

B.3 Dynamic Model Solution Algorithm

This appendix explains the numerical solution of the dynamic rational inattention model. I begin with notation and the Kalman filter equations. Then, I proceed to describe how to obtain the format of the optimal signal and the variance of the signal error.

Equation (10) is the state-space representation of the AR(1) processes that govern labor market tightness and price. Equation (11) is the period t signal. The signal error, ϵ_t, is normally distributed with mean zero and standard deviation σ_ϵ.

$$\xi_{t+1} = F\xi_t + \epsilon^{\xi}_{t+1} \tag{10}$$

$$S_t = h'\xi_t + \epsilon_t \tag{11}$$

$$\text{where } \xi_t \equiv \begin{bmatrix} m_t \\ p_t \end{bmatrix}, \; F \equiv \begin{bmatrix} \phi_m & 0 \\ 0 & \phi_p \end{bmatrix}, \; \epsilon^{\xi}_{t+1} \equiv \begin{bmatrix} \theta_m \epsilon^m_t \\ \theta_p \epsilon^p_t \end{bmatrix}, \; h \equiv \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

Let $\Sigma_{t|t-1}$ and $\Sigma_{t|t}$ be the variance-covariance matrices of ξ_t conditional on $\mathcal{I}_{t-1}$ and $\mathcal{I}_t$, respectively. Define $\Sigma_1 \equiv \lim_{t\to\infty} \Sigma_{t|t-1}$ and $\Sigma_0 \equiv \lim_{t\to\infty} \Sigma_{t|t}$. Let Q be the variance-covariance matrix of ϵ^{ξ}_{t+1}. From the Kalman filter equations (e.g., Hamilton (1994) and Bougerol (1993)), Equations (12) and (13) govern how the conditional variance-covariance matrices update.

$$\Sigma_{t+1|t} = F\Sigma_{t|t}F' + Q \tag{12}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}h\left(h'\Sigma_{t|t-1}h + \sigma^2_\epsilon\right)^{-1} h'\Sigma_{t|t-1} \tag{13}$$

Taking limits, Σ_1 and Σ_0 results in:

$$\Sigma_1 = F\Sigma_0 F' + Q$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 h\left(h'\Sigma_1 h + \sigma^2_\epsilon\right)^{-1} h'\Sigma_1$$

Recall from the paper that the consumer wants to minimize Equation (14).

$$\min_{h,\sigma_\epsilon} \; \mathbb{E}\left[(l_t^* - \mathbb{E}(l_t^*|\mathcal{I}_t))^2\right] + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right) \tag{14}$$

The following two points allow the minimization problem in Equation (14) to be re-written as Equation (15). First, for a given prior variance-covariance ($\Sigma_{t|t-1}$), the posterior variance-covariance ($\Sigma_{t|t}$) evolves according to the Kalman filter dynamic equations above, and converges to limiting conditional variance-covariance Σ_0. Since the consumer at time zero has received an infinite set of signals, their posterior variance-covariance after time zero does not change and remains at Σ_0. Second, recall that optimal labor is a linear combination of labor market tightness and price ($l^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}m + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p$). So the conditional variance-covariance of l^*, is the conditional variance of $\frac{\hat{u}_{12}}{|\hat{u}_{11}|}m + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p$.

$$\min_{h,\sigma_\epsilon} \; \mathbb{E}[w'\Sigma_0 w] + \frac{\lambda}{2}\log_2\left(\frac{h'\Sigma_1 h}{\sigma_\epsilon^2}\right), \text{ where } w = \begin{bmatrix} \frac{\hat{u}_{12}}{|\hat{u}_{11}|} \\ \frac{\hat{u}_{13}}{|\hat{u}_{11}|} \end{bmatrix} \tag{15}$$

Now, finding the signal weights and the variance of the signal error that optimize the objective function is straightforward. It amounts to searching over signal weights h and signal variance σ_ϵ to minimize Equation (15). For any combination of h and σ_ϵ, Σ_1 can be solved by iterating Equation (16) to a fixed point. Once Σ_1 is found, Equation (17) solves for Σ_0.

$$\Sigma_1 = F\left(\Sigma_1 - \Sigma_1 h(h'\Sigma_1 h + \sigma_\epsilon^2)^{-1}h'\Sigma_1\right)F' + Q \tag{16}$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 h(h'\Sigma_1 h + \sigma_\epsilon^2)^{-1}h'\Sigma_1 \tag{17}$$