The Inattentive Consumer: Sentiment and Expectations

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Motivation

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- The workhorse approach to modeling beliefs has been full-information rational expectations (FIRE)
- Survey-based measures of beliefs systematically deviate from FIRE
- So how do agents form their economic beliefs? The answer is crucial to understanding macroeconomic dynamics and policy-making

  [W]e need to know more about the manner in which inflation expectations are formed and how monetary policy influences them. - Janet Yellen

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  - Inflation is negatively related to sentiment
- Empirics suggest a fundamental deviation from FIRE

My Contribution: Consumer Rational Inattention Model

Overview:

- Uncertainty about fundamentals and costly information

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Model Setups:

- Static (intuition)
- Two period (nests FIRE)
- Dynamic (IRFs)

Literature

Expectation formation

- Deviations from FIRE (Coibion, Gorodnichenko, Kamdar 2018)
- Personal experience affects expectations (Malmendier and Nagel 2016)

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- Partial equilibrium firm problem (Maćkowiak and Wiederholt 2009)
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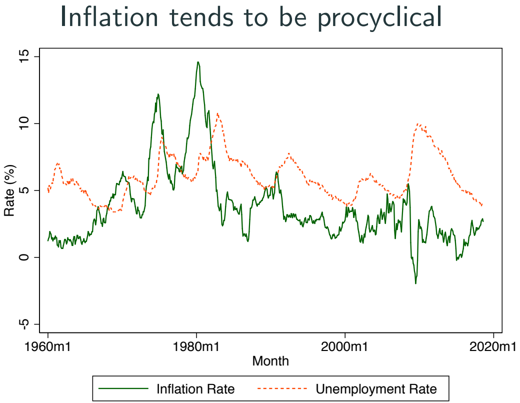
Monetary Policy

- Inflation expectations as a policy tool (Coibion et al. 2018)

My contribution: desired demand effects are attenuated

Historical Inflation and Unemployment Rates



Notes: The civilian unemployment rate and the y-o-y percent change in the consumer price index are plotted. For 1985 onwards, the correlation between the series is -0.2.

rolling reg

Survey Data

Michigan Survey of Consumers (MSC)

- Monthly, consumer survey of ≈ 500
- Rotating panel (up to 2x)
- 1978-present

NY Federal Reserve's Survey of Consumer Expectations (SCE)

- Monthly, consumer survey of $\approx 1,300$
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Survey of Professional Forecasters (SPF)

- Quarterly, professional survey of ≈ 40
- Rotating panel
- 1968-present

Consumer Survey Questions

Inflation

- *Michigan:* “By about what percent do you expect prices to go (up/down) on average, during the next 12 months?”
- *NY Fed:* “What do you expect the rate of (inflation/deflation) to be over the next 12 months?”

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Unemployment

- *Michigan:* "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?"
- *NY Fed:* "What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?"

Inflation and Unemployment Expectations: Michigan Survey

Consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
More unemployment	1.590***			
	(0.031)			
Less unemployment	-0.677***			
	(0.033)			
Time FE	N			
Consumer FE	N			
Minimum Surveys				
R-squared	0.019			
N	240356			

Sample: January 1978 - May 2017

time series

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	(1)	(2)	(3)	(4)
More unemployment	1.590***	1.268***		
	(0.031)	(0.029)		
Less unemployment	-0.677***	-0.618***		
	(0.033)	(0.032)		
Time FE	N	Y		
Consumer FE	N	N		
Minimum Surveys				
R-squared	0.019	0.116		
N	240356	240356		

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	(0.031)	(0.029)	(0.032)	
Less unemployment	-0.677***	-0.618***	-0.651***	
	(0.033)	(0.032)	(0.034)	
Time FE	N	Y	Y	
Consumer FE	N	N	N	
Minimum Surveys			> 1	
R-squared	0.019	0.116	0.057	
N	240356	240356	165900	

Sample: January 1978 - May 2017

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More unemployment	1.590***	1.268***	1.183***	0.408***
	(0.031)	(0.029)	(0.032)	(0.044)
Less unemployment	-0.677***	-0.618***	-0.651***	-0.277***
	(0.033)	(0.032)	(0.034)	(0.048)
Time FE	N	Y	Y	Y
Consumer FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.116	0.057	0.343
N	240356	240356	165900	165900

Sample: January 1978 - May 2017

time series

By Education: Michigan Survey

Across education groups, consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)
Max degree	none	high-school	college
More unemployment	0.634**	0.467***	0.282***
	(0.254)	(0.062)	(0.055)
Less unemployment	-0.811***	-0.267***	-0.191***
	(0.309)	(0.071)	(0.059)
Time FE	Y	Y	Y
Consumer FE	Y	Y	Y
R-squared	0.292	0.345	0.349
N	11979	85322	61502

Sample: January 1978 - May 2017

By Income: Michigan Survey

Across income groups, consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
Income quartile	1 (low)	2	3	4 (high)
More unemployment	0.571***	0.604***	0.272***	0.320***
	(0.138)	(0.105)	(0.095)	(0.074)
Less unemployment	-0.512***	-0.431***	-0.190*	-0.223***
	(0.159)	(0.125)	(0.103)	(0.080)
Time FE	Y	Y	Y	Y
Consumer FE	Y	Y	Y	Y
R-squared	0.301	0.353	0.344	0.380
N	27613	26359	25686	32156

Sample: January 1978 - May 2017

By Age: Michigan Survey

Across age groups, consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)
Age	<40	40 to 60	>60
More unemployment	0.492***	0.332***	0.379***
	(0.079)	(0.064)	(0.088)
Less unemployment	-0.293***	-0.221***	-0.247***
	(0.084)	(0.077)	(0.092)
Time FE	Y	Y	Y
Consumer FE	Y	Y	Y
R-squared	0.355	0.361	0.292
N	63261	57717	41880

Sample: January 1978 - May 2017

time series by birth year

Inflation and Unemployment Expectations: NY Fed Survey

Across surveys, consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

	(1)	(2)	(3)	(4)
$\mathbb{E}_{j,t}(Prob(\Delta Unemp_{t+12} > 0))$	0.070***	0.069***	0.066***	0.034***
	(0.003)	(0.003)	(0.003)	(0.003)
Time FE	N	Y	Y	Y
Consumer FE	N	N	N	Y
Minimum Surveys			> 1	> 1
R-squared	0.019	0.022	0.021	0.396
N	50660	50660	49172	49172

Sample: June 2013-August 2016

Inflation and Unemployment Expectations: Professionals

In contrast to consumers, professionals believe inflation is procyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+4}$

	(1)	(2)	(3)
$\mathbb{E}_{j,t}[\Delta Unemp_{t+4}]$	-0.416***	-0.496***	-0.377***
	(0.071)	(0.069)	(0.066)
Time FE	N	Y	Y
Professional FE	N	N	Y
R-squared	0.010	0.730	0.796
N	4852	4852	4829

Sample: 1981 Q3 - 2017 Q4

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- Use component analysis, to understand driver(s) of consumer beliefs:
  - Number of important component(s)
  - Characteristics of the key component(s) and loading(s)
- Factor models:
  - NY Fed Survey (numeric answers), use PCA
  - Michigan Survey (categorical answers), use MCA

First Component Loadings: Michigan Survey MCA

Unemployment* (next year)

Personal finances* (last year)

Personal finances* (next year)

Economic policy* (now)

Personal real income* (next year)

Inflation$^{\diamond}$ (next year)

First Component Loadings: Michigan Survey MCA

1st dimension explains 76% of the variance

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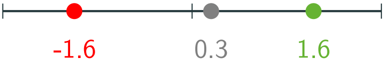

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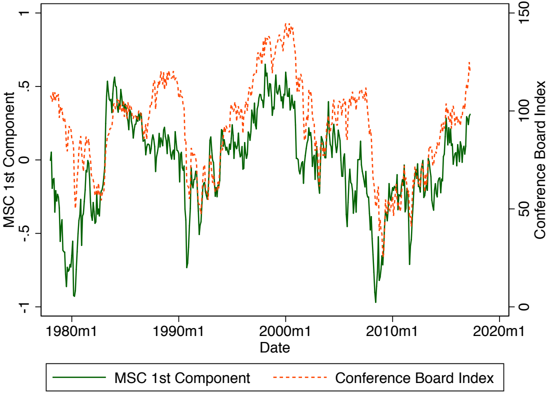
$\diamond$ $\pi_{t+1} > 4$, $0 < \pi_{t+1} \leq 4$, $\pi_{t+1} \leq 0$

First Component and Sentiment

The first component is similar to a popular measure of sentiment



Notes: MSC 1st component = the average of the MCA first components, across consumers for a given month.

ny fed compare actions

First Component Loadings: NY Fed Survey PCA

	(1)	(2)	(3)
Inflation rate will be:			
	-0.22		
% chance unemployment will rise:			
	-0.11		
% chance stock market will rise:			
	0.43		
Will you be financially better off:			
	0.54		
% change in average home price will be:			
N:	49977		
Restrictions:	n/a		
Variance explained:	0.30		

First Component Loadings: NY Fed Survey PCA

	(1)	(2)	(3)
Inflation rate will be:			
	-0.22	-0.23	
% chance unemployment will rise:			
	-0.11	-0.15	
% chance stock market will rise:			
	0.43	0.42	
Will you be financially better off:			
	0.54	0.55	
% change in average home price will be:			
		0.03	
N:	49977	36625	
Restrictions:	n/a	home-own	
Variance explained:	0.30	0.26	

First Component Loadings: NY Fed Survey PCA

	(1)	(2)	(3)
Inflation rate will be:			
	-0.22	-0.23	-0.25
% chance unemployment will rise:			
	-0.11	-0.15	-0.02
% chance stock market will rise:			
	0.43	0.42	0.44
Will you be financially better off:			
	0.54	0.55	0.50
% change in average home price will be:			
		0.03	-0.15
N:	49977	36625	13307
Restrictions:	n/a	home-own	non-own
Variance explained:	0.30	0.26	0.25

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2. The economic beliefs of consumers are driven by a single component: sentiment
3. Inflation expectations and sentiment negatively co-move

What Can Explain the Stylized Facts?

- FIRE
- Sticky Information
- Learning
- Rational Inattention

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- The consumer chooses the form of their signal
  - Economizing on information, optimal signals are linear combinations of fundamentals
- The consumer decides to be best informed along the dimension most costly to misunderstand $\rightarrow$ rationally obtained sentiment
- Optimal information gathering results in covariances of beliefs that can differ from the covariances of the underlying DGP

Consumer Problem: Static Setup

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- For each unit of labor, the consumer is paid wage $\frac{W}{\Theta}$
    - where Θ is labor market slackness and W is a base nominal wage
    - normalize the base wage: $W = 1$
- Assume the consumer knows the base wage, but faces uncertainty about labor market slackness and the price index
    - slackness and price are assumed to be independent

Consumer Problem: Static Setup Continued

- The consumer may obtain optimal signal(s) about the unknowns
  - signal(s) can be any combination of the unknowns plus noise
  - signal(s) are costly ($\lambda \times$ Shannon mutual information, $\lambda \geq 0$)

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- Utility:

$$U\Bigg(L\Big(\mathbb{F}[\Theta], \mathbb{F}[P]\Big), \Theta, P\Bigg)$$

Log-Quadratic Approximation

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- Let $\tilde{u}$ be the 2nd-order Taylor approx. of $\hat{u}$ around the steady state:

$$\tilde{u}(l, \theta, p) \approx \underbrace{\hat{u}_1}_{=0} l + \frac{1}{2} \underbrace{\hat{u}_{11}}_{<0} l^2 + \hat{u}_{12} l\theta + \hat{u}_{13} lp$$

where subscripts on $\hat{u}$ denote derivates w.r.t. the input order variable at the steady state

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- Under full-information, the utility maximizing labor is:

$$l^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}\theta + \frac{\hat{u}_{13}}{|\hat{u}_{11}|} p$$

Utility Function

- What are the optimal labor weights on slackness and price?

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- Assume the utility function:

$$U(L, \Theta, P) = \frac{C^{1-\varphi}}{1-\varphi} - \frac{L^{1+1/\eta}}{1+1/\eta} \text{ where } C = \frac{L}{\Theta P}$$

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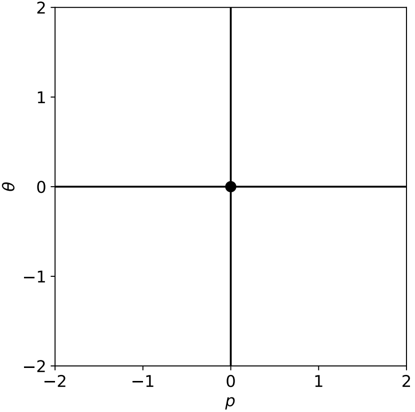
- Then the optimal labor weights are equal:

$$\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = \frac{\hat{u}_{13}}{|\hat{u}_{11}|}$$

Intuition



Optimal labor choice:

$$l^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}\theta + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p$$
$$= .5\theta + .5p$$

Then:

$p = 0$ and $\theta = 0 \Rightarrow l^* = 0$

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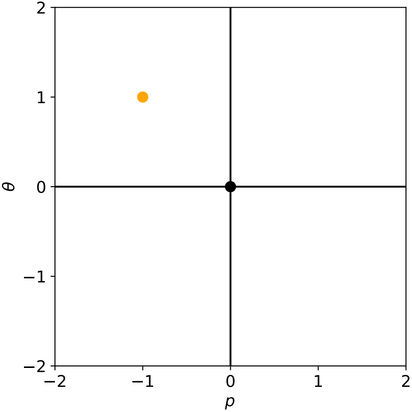

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$$
\begin{aligned}
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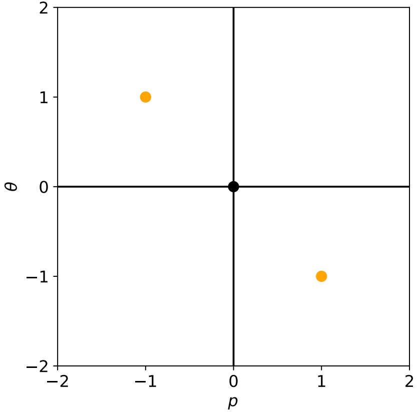

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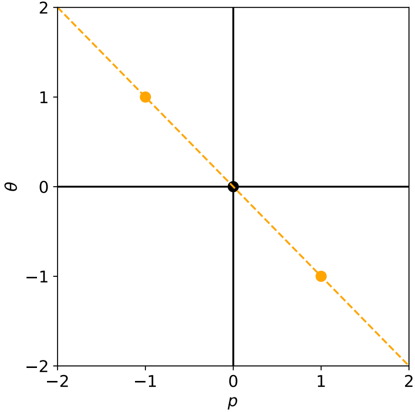

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$$l^* = \frac{\hat{u}_{12}}{|\hat{u}_{11}|}\theta + \frac{\hat{u}_{13}}{|\hat{u}_{11}|}p$$
$$= .5\theta + .5p$$

Then:

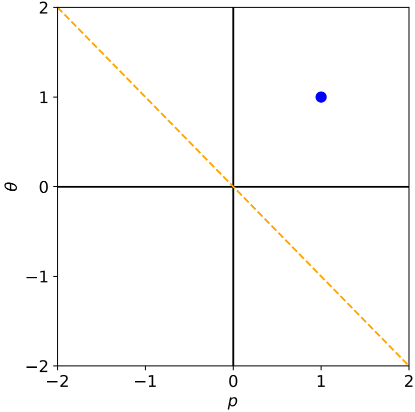
$p = 0$ and $\theta = 0 \Rightarrow l^* = 0$

$p = -1$ and $\theta = 1 \Rightarrow l^* = 0$

$p = 1$ and $\theta = -1 \Rightarrow l^* = 0$

$l^* = 0$ along the -45° line

Intuition



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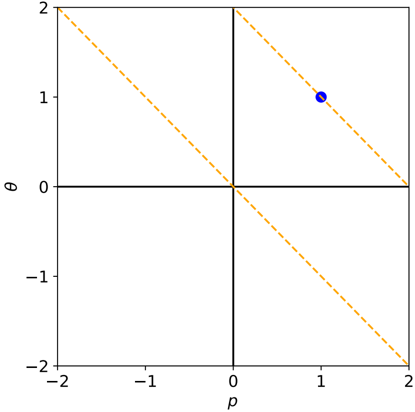
\end{aligned}
$$

Then:

$p = 1$ and $\theta = 1 \Rightarrow l^* = 1$

Intuition



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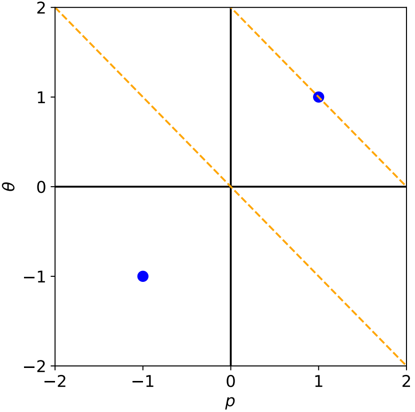

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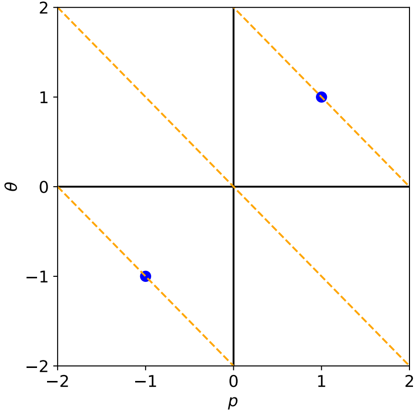

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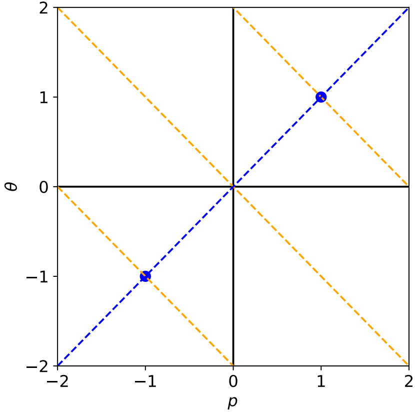

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The agent wants to learn along the 45° line!

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For the consumer problem: one eigenvalue is zero and one is nonzero
$\Rightarrow$ *at most, the consumer gets one signal*

Covariance of Posterior Slackness and Price Beliefs

What is the covariance of the posterior beliefs of slackness and price?

$$cov(\tilde{\theta}, \tilde{p}) = \begin{cases} & \text{if } \sigma_0^2 \leq \lambda|\alpha| \\ & \text{if } \sigma_0^2 > \lambda|\alpha| \end{cases}$$

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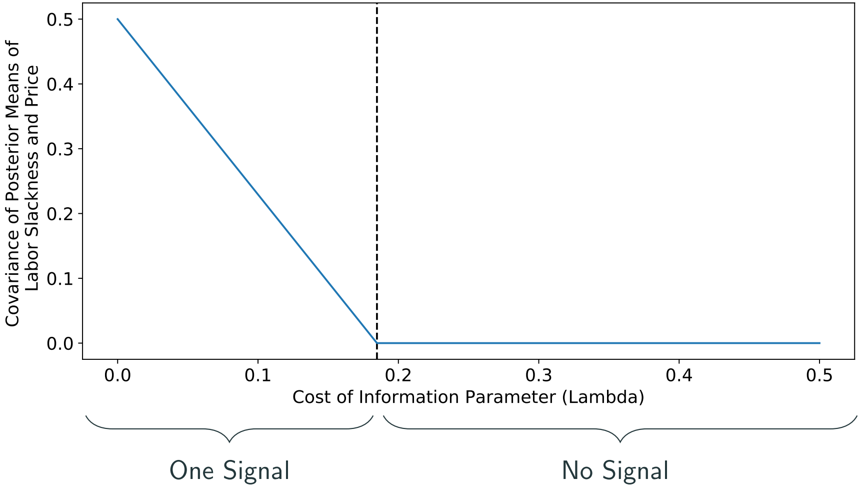
- **When the consumer gets one signal:**

$$cov(\tilde{\theta}, \tilde{p}) > 0$$

Covariance of Posterior Beliefs By Information Cost



Notes: Parameters: $\varphi = 1.4$, $\eta = 3$, $\sigma_0^2 = 1$

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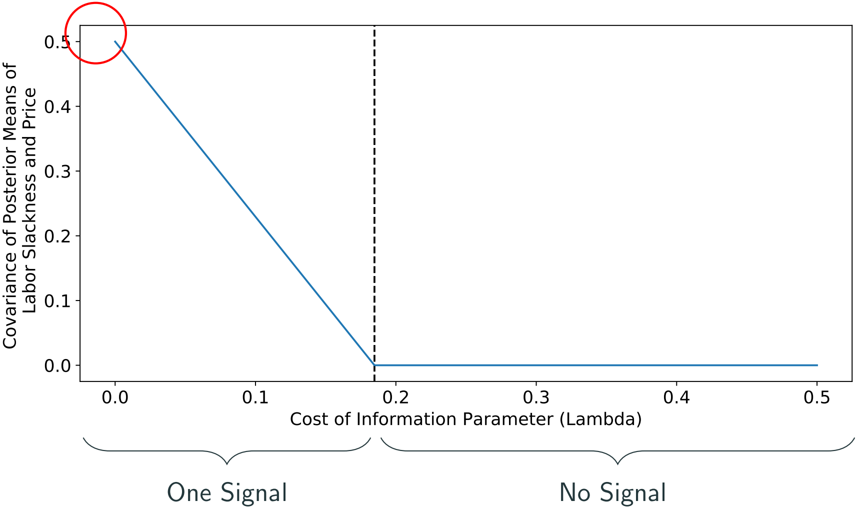

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Static Model: Takeaways

1. The consumer reduces the dimensionality of the problem, receiving one signal but facing two unknowns

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2. The optimal information gathering results in a positive covariance between consumer beliefs about prices and labor market slackness

Extension: Two Periods, Two Choices, Two States

First period:

- Do not know labor market slackness or price, obtain optimal signal(s) about them
- Choose first period labor and savings
- Consume according to, $PC_1 = L/\Theta - S$
- Assume the consumer knows the interest rate

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Second period:

- No labor
- Assume the price index is the same both periods
- Consume savings, $C_2 = \frac{(1+R)S}{P}$

Extension: Two Periods, Two Choices, Two States

- Assume the utility function:

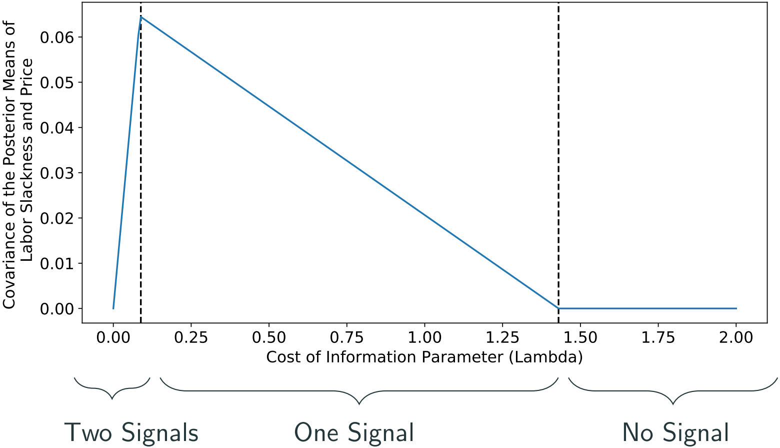
$$U(L, C_1, C_2) = \frac{C_1^{1-\varphi}}{1-\varphi} - \frac{L^{1+1/\eta}}{1+1/\eta} + \beta \frac{C_2^{1-\varphi}}{1-\varphi}$$

- Budget constraints:

$$C_1 = \frac{L/\Theta - S}{P} \text{ and } C_2 = \frac{(1+R)S}{P}$$

- Take the log-quadratic approximation around the steady state

- Solve as before



Notes: Parameters: $\varphi = 1.4$, $\eta = 3$, $\sigma_0^2 = 1$, $R = .05$, $\beta = .95$

Two-Period Model: Takeaways

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1. For low costs of information, the consumer will choose to obtain two signals
2. As information costs approach zero, the consumer smoothly learns the values of slackness and price
3. A comparison of professional forecasters (low information costs) to consumers (intermediate information costs) is possible and consistent with the empirical findings

Inflation Expectations as a Policy Tool

- *With nominal short-term interest rates at ... their effective lower bound in many countries, the broader question of how expectations are formed has taken on heightened importance ... [C]entral banks have sought additional ways to stimulate their economies, including adopting policies that are directly aimed at influencing expectations of ... inflation.*

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- *The first element [of quantitative easing] was to dispel people's deflationary mindset and raise inflation expectations.*

  \- Haruhiko Kuroda

Monetary Policy Implications

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  - Higher inflation expectations result in stronger demand today
- However based on my empirical results and rational inattention framework:
  - Consumers associate higher inflation with recessionary outcomes
  - So the increase in demand could be attenuated or even counteracted
- Rather than manipulating inflation expectations:
  - Manage wage inflation or unemployment expectations

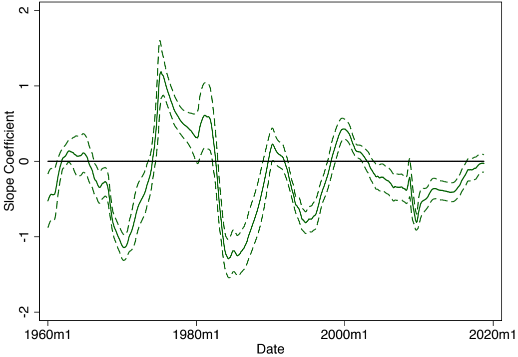
Conclusion

- Using survey data:
    - Consumers' expectations are driven by sentiment
    - ... believe inflation is countercylical
- In a rational inattention model:
    - Consumers learn about a combination of fundamentals $\Rightarrow$ rationally obtained sentiment
    - ... leading to countercylical price beliefs
- Monetary policy implications:
    - Raising inflation expectations may inadvertently cause consumers to become pessimistic

beliefs to actions | dynamics

Inflation and Unemployment Rates: Rolling Regression

$$\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$$

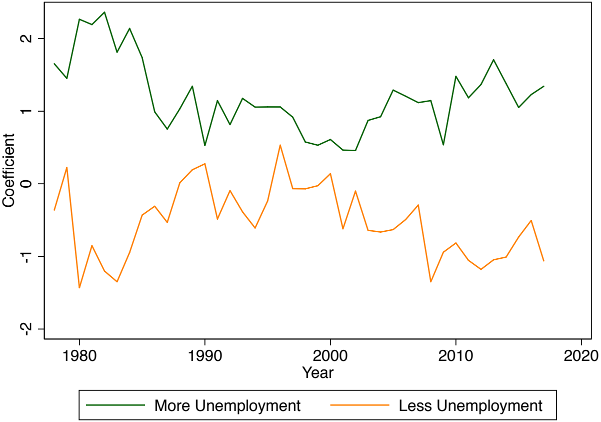


Notes: Dotted lines represent the 95% confidence interval. Ten year rolling window slope regression coefficient is plotted. The sample end date is on the x-axis.

back

Inflation and Unemployment: Consumers (Michigan Survey)

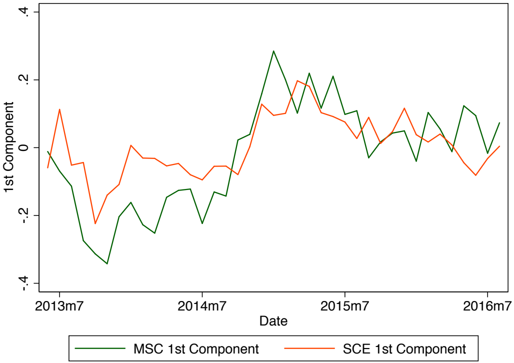
$$\mathbb{E}_{j,t}\pi_{t+1} = \alpha_t + \beta_t^{more} D_{j,t+1}^{more} + \beta_t^{less} D_{j,t+1}^{less} + \epsilon_{j,t}$$



back

First Component: Michigan and NY Fed

- MSC 1st Component =
  Michigan's MCA 1st component, averaged across consumers by month
- SCE 1st Component =
  NY Fed's PCA 1st component, averaged across consumers by month



back

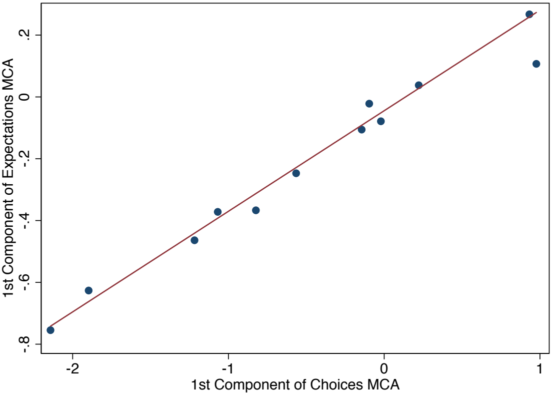
First Component and Actions

Optimistic consumers (those with a large first component), are more likely say it is a good time to make purchases

back conclusion

First Component of Beliefs vs. First Component of Actions

The first component of belief-related questions (the baseline) is highly correlated with the first component of action-related questions



back conclusion

Inflation and Unemployment, By Age

By age group: $\mathbb{E}_{j,t}\pi_{t+1} = \alpha_t + \beta_t^{more} D_{j,t+1}^{more} + \beta_t^{less} D_{j,t+1}^{less} + \epsilon_{j,t}$

back

By Birth Year: Michigan Survey

Across years of birth, consumers believe inflation is countercyclical

Dependent variable: $\mathbb{E}_{j,t}\pi_{t+12}$

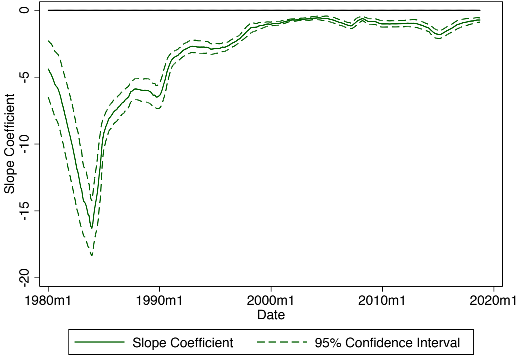
	(1)	(2)	(3)	(4)
Birth year	$<$1930	1930-1950	1950-1970	$>$1970
More unemployment	0.397***	0.377***	0.393***	0.581***
	(0.140)	(0.073)	(0.066)	(0.121)
Less unemployment	-0.137	-0.304***	-0.293***	-0.224*
	(0.150)	(0.086)	(0.073)	(0.127)
Time FE	Y	Y	Y	Y
Consumer FE	Y	Y	Y	Y
R-squared	0.280	0.381	0.350	0.308
N	23921	52103	71282	17372

Sample: January 1978 - May 2017

back

Japan: Rolling Regression

$$\pi_t = \alpha + \beta \text{unemployment}_t + \epsilon_t$$



Notes: Dotted lines represent the 95% confidence interval. Ten year rolling window slope regression coefficient is plotted. The sample end date is on the x-axis.

back

Re-express LQ Utility Approximation

- To utilize the solution methodology of Kőszegi and Matějka (2018), one must be maximizing a quadratic function of the form:

$$-\mathbf{y}'\mathbf{D}\mathbf{y} + \mathbf{x}'\mathbf{B}\mathbf{y}$$

where $\mathbf{D}$ is symmetric and positive-semidefinite

- Re-express the LQ approximation of utility

$$\begin{aligned}\tilde{u}(l,\theta,p) &= \frac{1}{2}\hat{u}_{11}l^2 + \hat{u}_{12}l\theta + \hat{u}_{13}lp \\ &= -\mathbf{y}'\mathbf{D}\mathbf{y} + \mathbf{x}'\mathbf{B}\mathbf{y}\end{aligned}$$

where

$$\mathbf{D} = \frac{|\hat{u}_{11}|}{2}, \ \mathbf{B} = \begin{bmatrix} \hat{u}_{12} \\ \hat{u}_{13} \end{bmatrix}, \ \mathbf{y} = l \text{ and } \mathbf{x} = \begin{bmatrix} \theta \\ p \end{bmatrix}$$

Transform the Problem

- For tractability, solve a transformed problem that is a function of:
  1. misperceptions about the state
  2. the cost of information
- This takes three steps:
  1. find the action $\mathbf{y}$ chosen given some posterior mean of the state $\tilde{\mathbf{x}}$
  2. express the utility function as function of the posterior mean of the state, $\tilde{\mathbf{x}}$, rather than action $\mathbf{y}$
  3. quantify the cost of information in terms of the posterior variance-covariance $\boldsymbol{\Sigma}$

back

Transformed Problem

- Let the prior variance-covariance be $\mathbf{\Gamma}$ and the posterior variance-covariance be $\mathbf{\Sigma}$
- The choice variable is the posterior variance-covariance matrix
- Transformed problem (let $\mathbf{\Omega} \equiv \frac{\mathbf{BD}^{-1}\mathbf{B}'}{4}$):

$$\max_{\mathbf{\Gamma} \geq \mathbf{\Sigma}} \underbrace{-\mathbb{E}\Big[(\tilde{\mathbf{x}} - \mathbf{x})'\mathbf{\Omega}(\tilde{\mathbf{x}} - \mathbf{x})\Big]}_{\text{expected utility}} + \underbrace{\frac{\lambda}{2} log|\mathbf{\Sigma}|}_{\text{cost of info}}$$

$$\max_{\mathbf{\Gamma} \geq \mathbf{\Sigma}} - Tr(\mathbf{\Omega\Sigma}) + \frac{\lambda}{2} log|\mathbf{\Sigma}|$$

- The restriction $\mathbf{\Gamma} \geq \mathbf{\Sigma}$ means $\mathbf{\Gamma} - \mathbf{\Sigma}$ is positive semidefinite

back

Eigen-decomposition

The loss matrix can be eigen-decomposed as:

$$\boldsymbol{\Omega} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}'$$

where:

- $\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues with $\Lambda_1 = 0$ and $\Lambda_2 = \frac{1}{2|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]$
- **V** is the matrix of the normalized eigenvectors. The eigenvectors are, respectively:

$$\begin{bmatrix} -\frac{\hat{u}_{13}}{\hat{u}_{12}} \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} \frac{\hat{u}_{12}}{\hat{u}_{13}} \\ 1 \end{bmatrix}$$

back

Rotated Posterior Variance-Covariance

- Solve for posterior variance-covariance in rotated space

$$\boldsymbol{J} = \boldsymbol{V}'\boldsymbol{\Sigma}\boldsymbol{V}$$

- Kőszegi and Matějka (2018) find the general solution:

$$J_{ij} = 0 \text{ for all } i \neq j$$
$$J_{ii} = min\left(\sigma_0^2, \frac{\lambda}{2\Lambda_i}\right)$$

back

Consumer Problem Solution

- For the consumer problem, the solution for (diagonal) **J** is:

$$J_{11} = \sigma_0^2$$

$$J_{22} = \frac{\lambda}{\frac{1}{|\hat{u}_{11}|}[\hat{u}_{12}^2 + \hat{u}_{13}^2]}$$

- Notice that households will learn about at most one direction:
  - $\Lambda_1 = 0 \implies$ households do not update from the prior in the first eigenvector direction
  - If $\sigma_0^2 > \frac{\lambda}{2\Lambda_2} \implies$, households obtain information on the 2nd eigenvector direction
- The solution for the posterior variance-covariance:

$$\boldsymbol{\Sigma} = \boldsymbol{VJV}'$$

back

Dynamic Model: Setup

- Assume slack and price log-deviations follow independent AR(1) processes

$$\theta_t = \phi_\theta \theta_{t-1} + \gamma_\theta \epsilon_t^\theta$$
$$p_t = \phi_p p_{t-1} + \gamma_p \epsilon_t^p$$

conclusion

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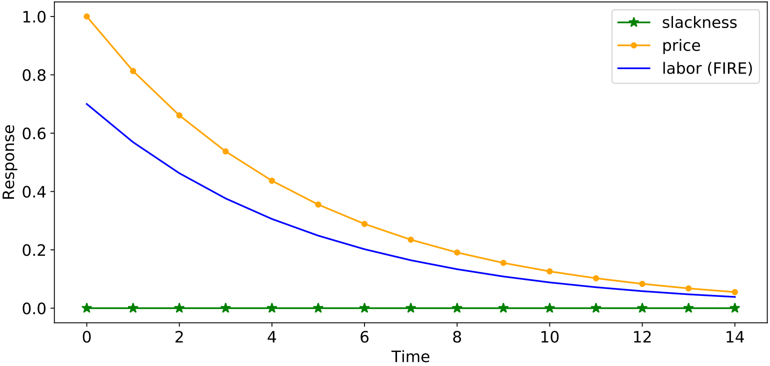
- Maćkowiak, Matějka, and Wiederholt (2018) show that the consumer will optimally choose one signal, a linear combination of current slackness and price

$$\text{signal}_t = h_1 \theta_t + h_2 p_t + \epsilon_t$$

conclusion

Impulse Response: One Standard Deviation Shock to Price

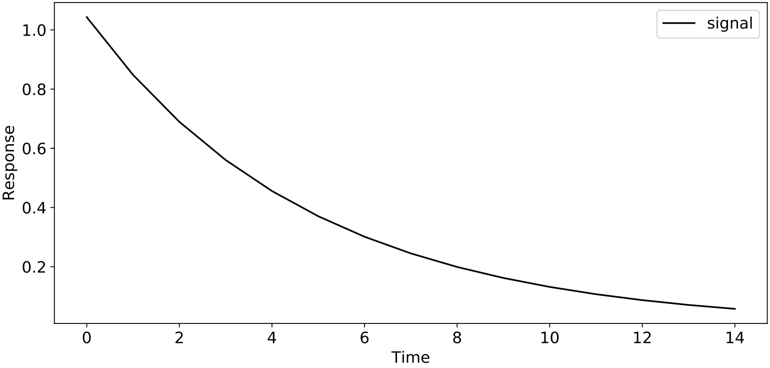


Notes: Parameters: $\phi_\theta = .715$, $\phi_p = .813$, $\gamma_\theta = \gamma_p = \lambda = 1$, $\frac{\hat{u}_{12}}{|\hat{u}_{11}|} = .7$, and $\frac{\hat{u}_{13}}{|\hat{u}_{11}|} = .7$. The resulting optimal signal weights are $h_1 = 1$ and $h_2 = 1.04$

conclusion

Impulse Response: One Standard Deviation Shock to Price

$$\text{signal}_t = \theta_t + 1.04 p_t + \epsilon_t$$



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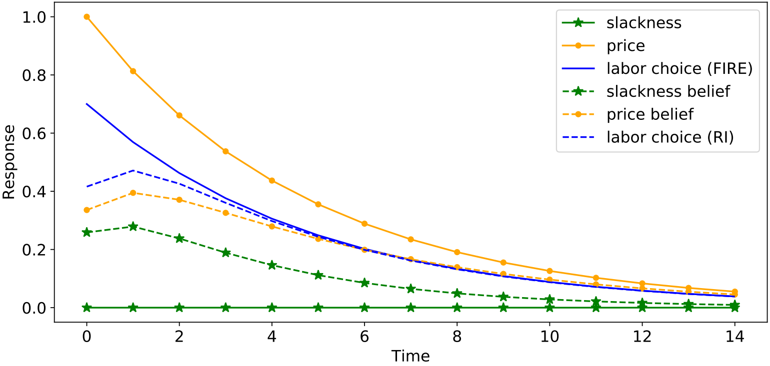
Impulse Response: One Standard Deviation Shock to Price

Labor and price beliefs under-react and slackness beliefs over-react



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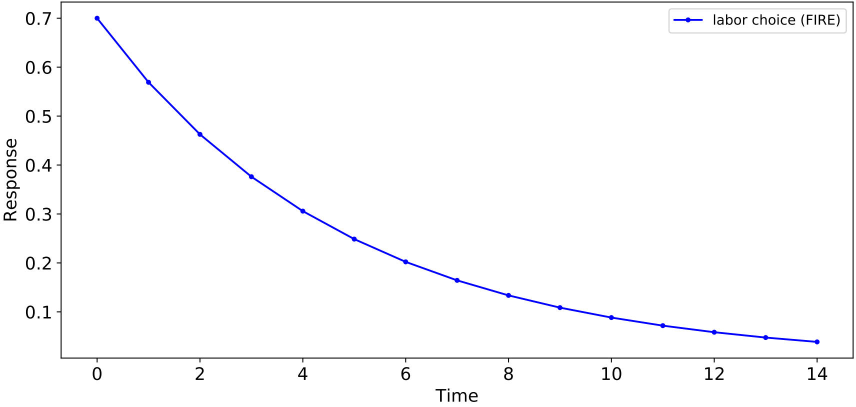
Impulse Response: Vary Information Cost

As information costs decline ($\lambda \to 0$), the consumer learns the optimal labor choice perfectly



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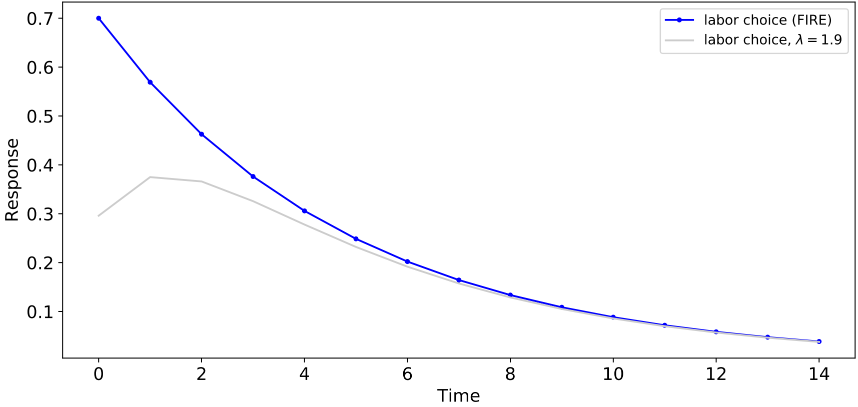

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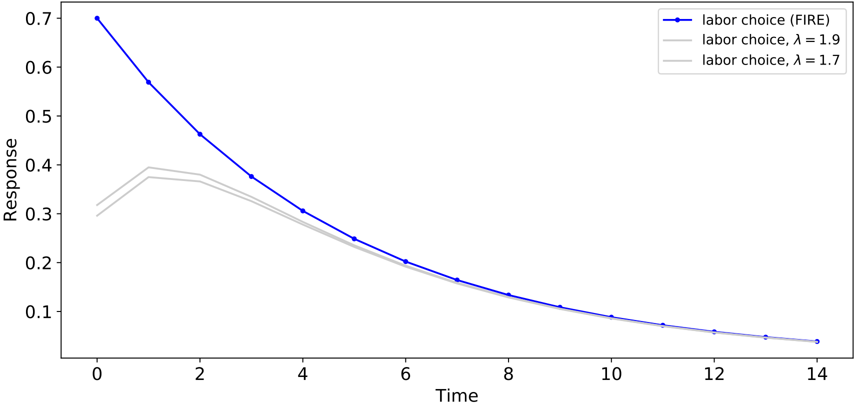

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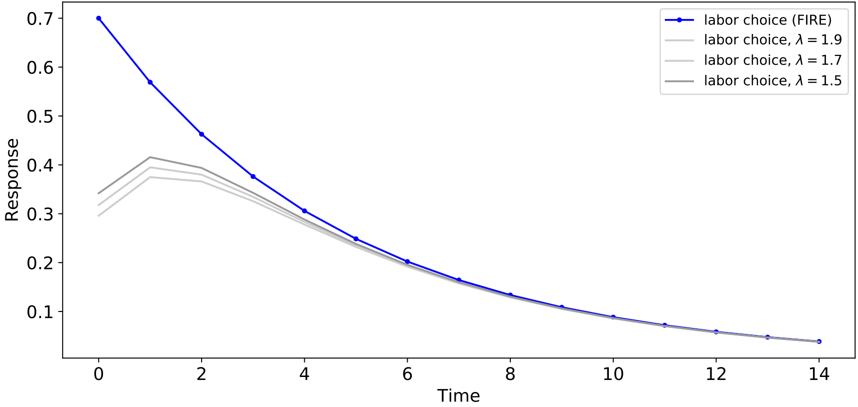

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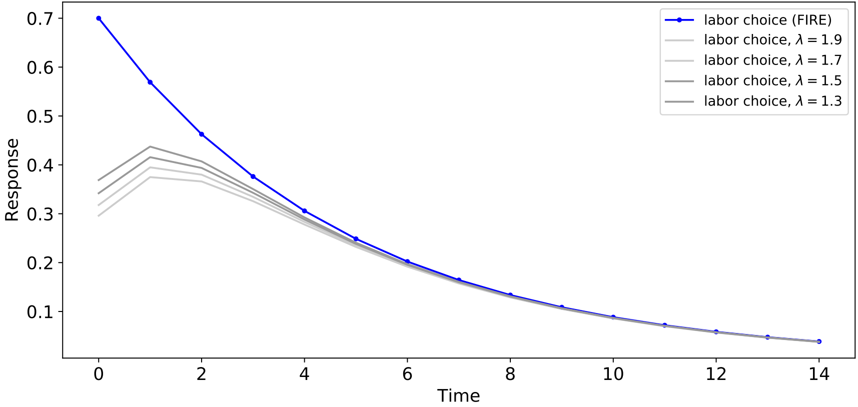

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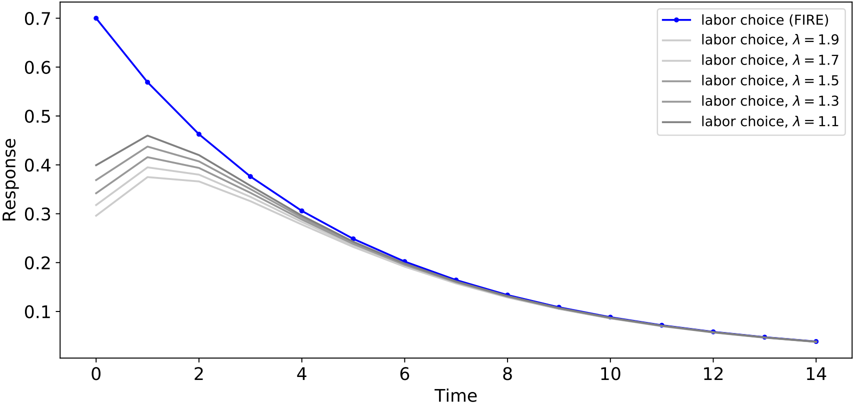

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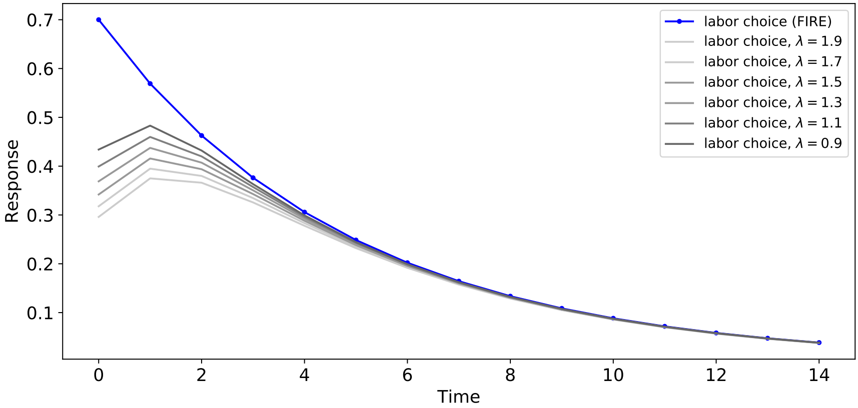

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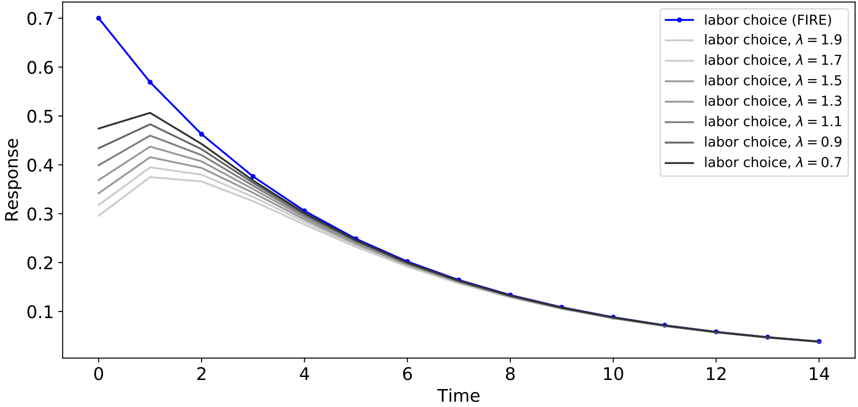

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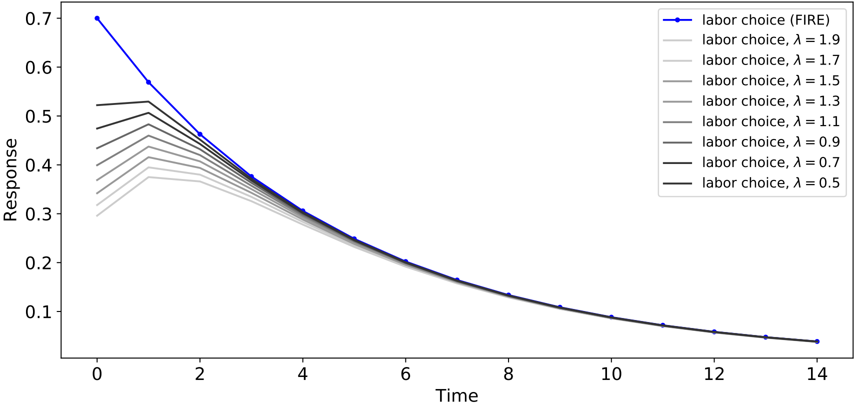
Impulse Response: Vary Information Cost

As information costs decline ($\lambda \to 0$), the consumer learns the optimal labor choice perfectly



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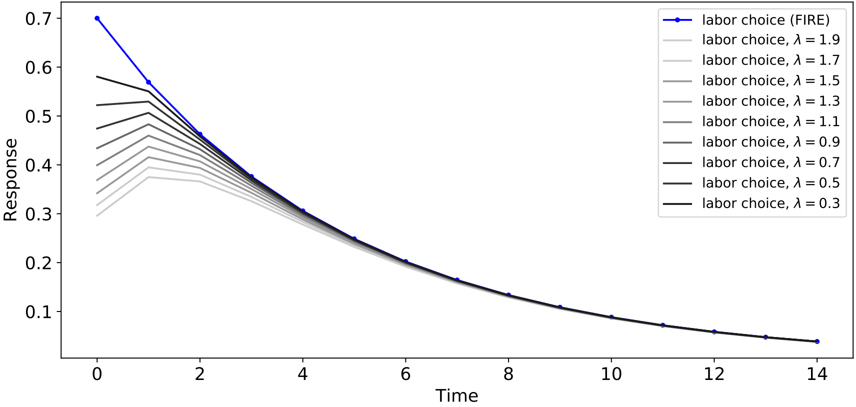

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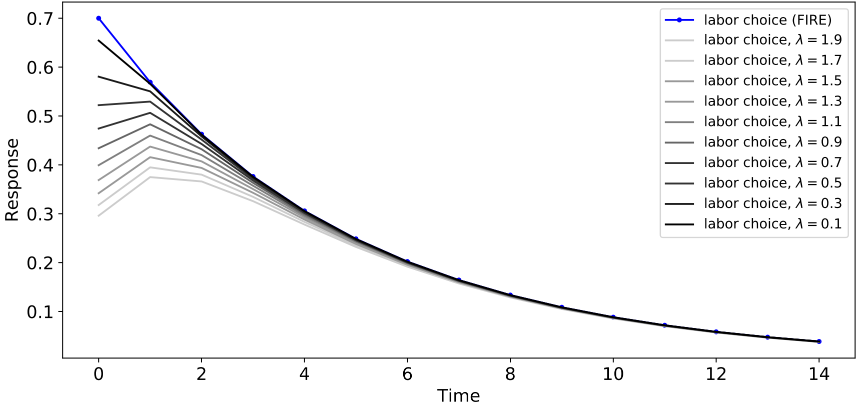

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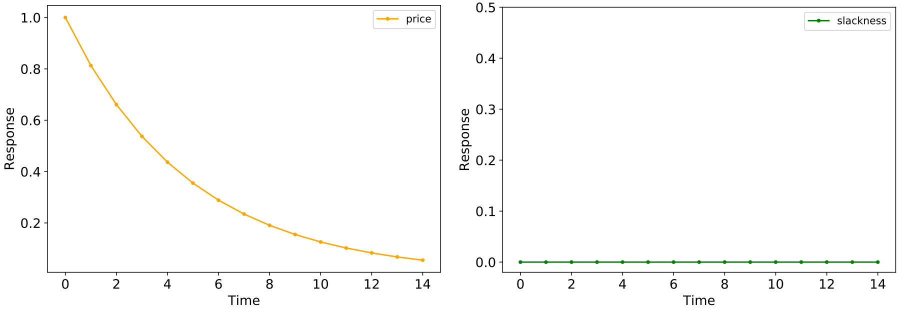
As information costs decline ($\lambda \to 0$), the consumer does not perfectly learn the true values of slackness and price



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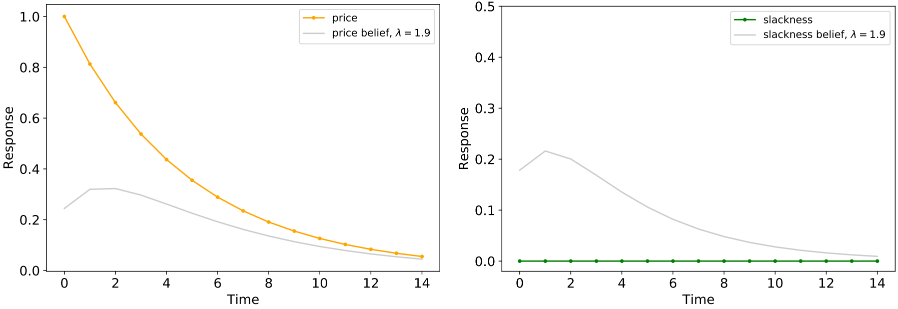

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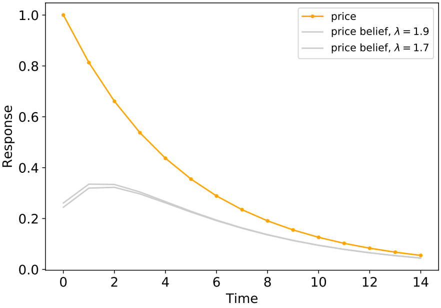

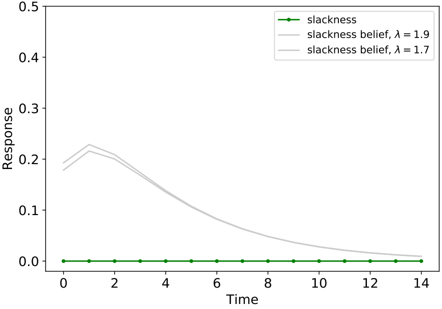

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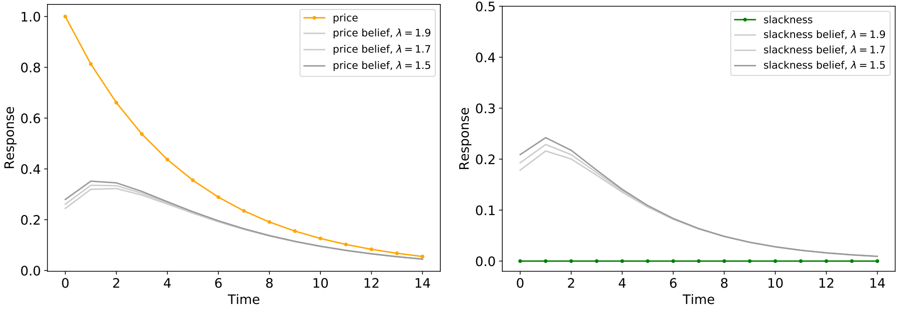

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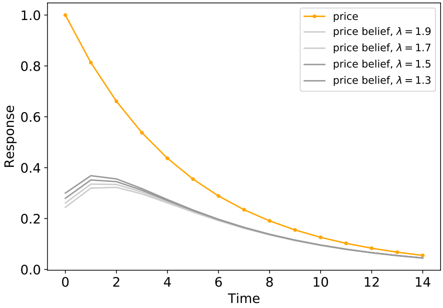

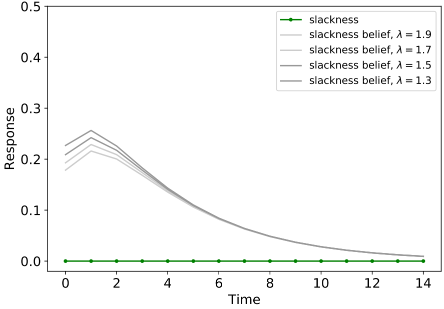

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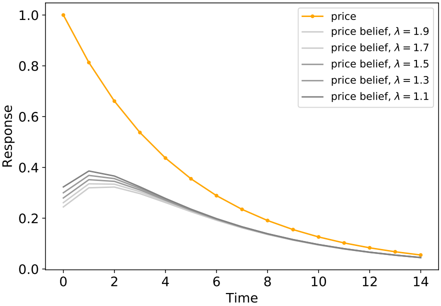

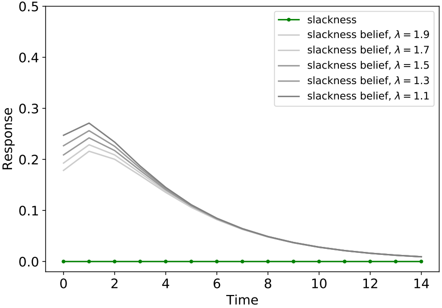

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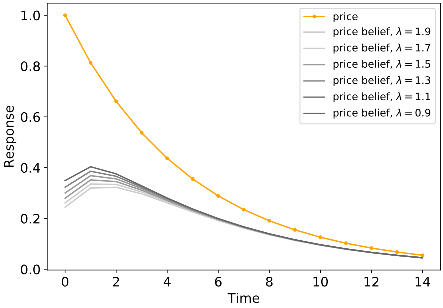

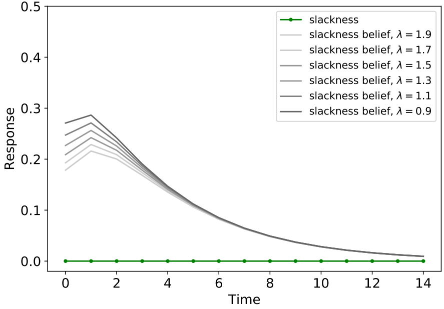

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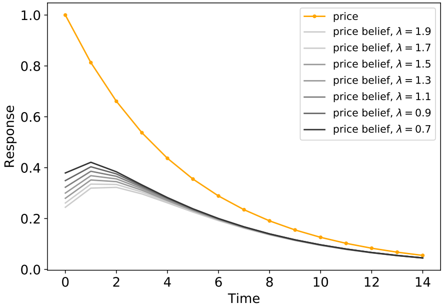

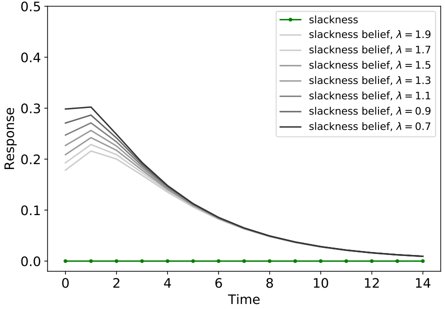

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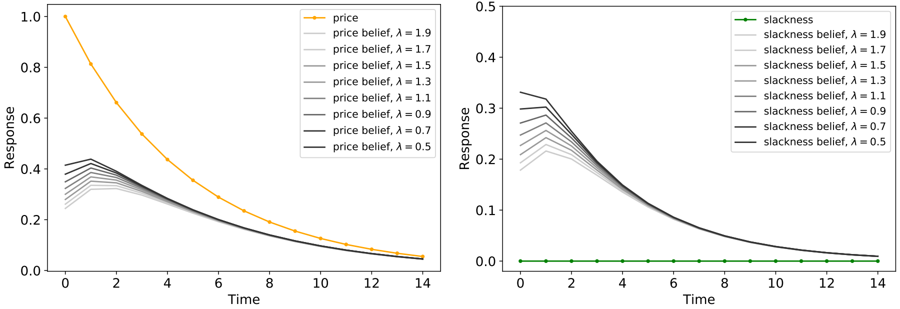

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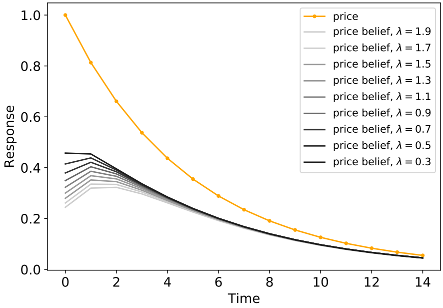

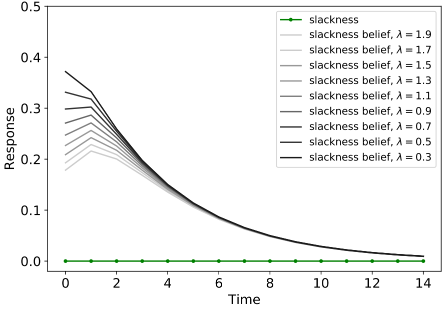

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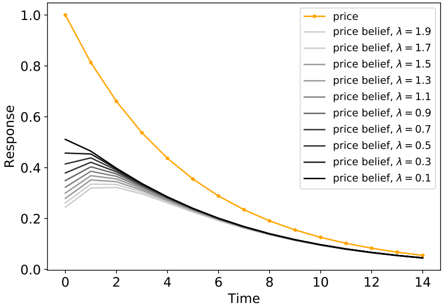
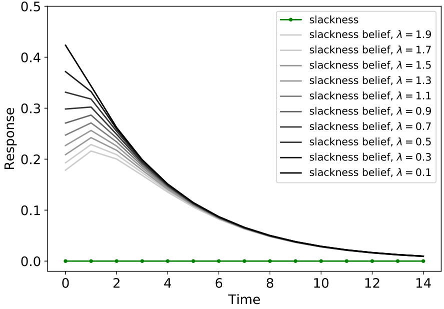
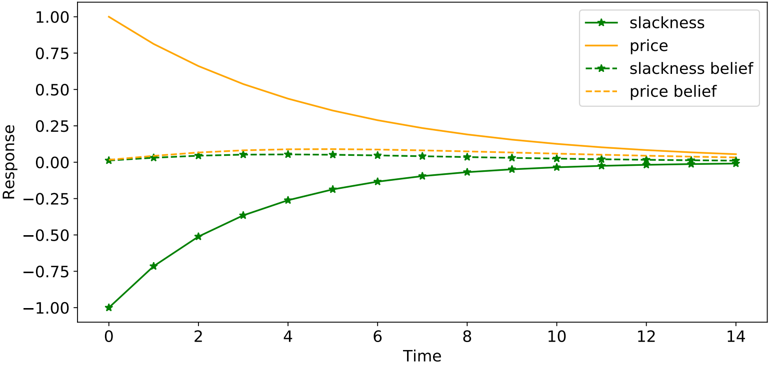




conclusion

Positive Demand Shock

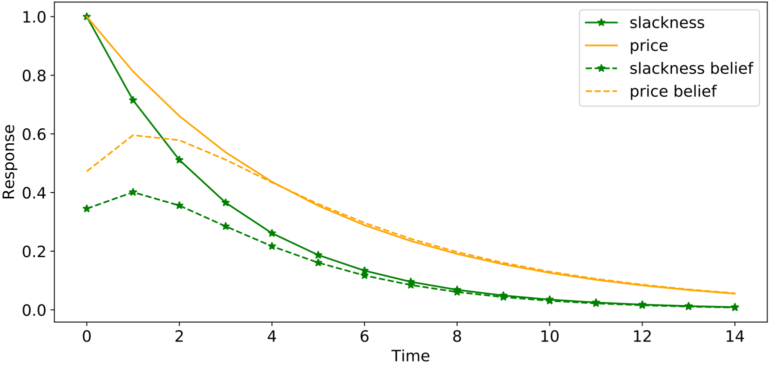
In reaction to demand shocks, the response of beliefs is muted



conclusion

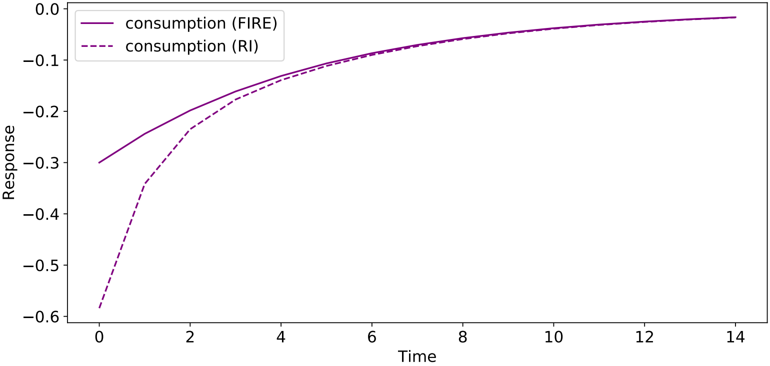
Negative Supply Shock

In reaction to supply shocks, the response of beliefs is strong, although still an under-reaction relative to actual values



conclusion

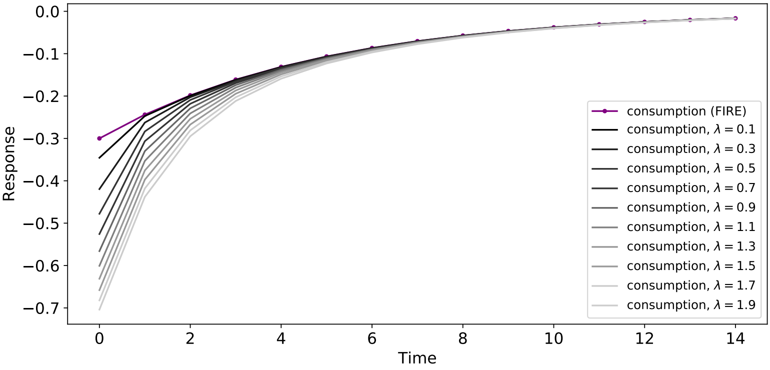
Consumption Response: One Standard Deviation Shock to Price



conclusion

Consumption Response: Vary Information Cost

As information costs decline ($\lambda \to 0$), the consumer learns the optimal labor choice perfectly and therefore obtains the optimal consumption



conclusion