ECE 2045: Statistical Signal Processing

Lab 4: Spectral Estimation Method - Welch's Method

An improved estimator of the PSD is the one proposed by Welch. The method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. The result is Welch's PSD estimate.

Welch's method is implemented in the Signal Processing Toolbox by the `spectrum.welch` object or `pwelch` function. By default, the data is divided into four segments with 50% overlap between them. A Hamming window is used to compute the modified periodogram of each segment.

The averaging of modified periodograms tends to decrease the variance of the estimate relative to a single periodogram estimate of the entire data record. Although overlap between segments tends to introduce redundant information, this effect is diminished by the use of a nonrectangular window, which reduces the importance or *weight* given to the end samples of segments (the samples that overlap).

However, as mentioned above, the combined use of short data records and nonrectangular windows results in reduced resolution of the estimator. In summary, there is a tradeoff between variance reduction and resolution. One can manipulate the parameters in Welch's method to obtain improved estimates relative to the periodogram, especially when the SNR is low. This is illustrated in the following example.

Consider an original signal consisting of 301 samples:

```
• randn('state',1)
• fs = 1000;                % Sampling frequency
• t = (0:0.3*fs)./fs;       % 301 samples
• A = [2 8];                % Sinusoid amplitudes
(row vector)
• f = [150;140];            % Sinusoid frequencies
(column vector)
• xn = A*sin(2*pi*f*t) + 5*randn(size(t));
• Hs = spectrum.periodogram('rectangular')
• psd(Hs,xn,'Fs',fs,'NFFT',1024);
```

We can obtain Welch's spectral estimate for 3 segments with 50% overlap with

```
• Hs = spectrum.welch('rectangular',150,50);
• psd(Hs,xn,'Fs',fs,'NFFT',512);
```

In the periodogram above, noise and the leakage make one of the sinusoids essentially indistinguishable from the artificial peaks. In contrast, although the PSD produced by Welch's method has wider peaks, you can still distinguish the two sinusoids, which stand out from the "noise floor."

However, if we try to reduce the variance further, the loss of resolution causes one of the sinusoids to be lost altogether:

- `Hs = spectrum.welch('rectangular',100,75);`
- `psd(Hs,xn,'Fs',fs,'NFFT',512);`