

Lecture Notes for HCF and LCM

Shortcut for finding factors of a number

For finding the factors of a number, first, we must know about the prime numbers.

Prime Number: *A number which is greater than unity and only has two divisors; itself and 1.*

NOTE: 1 is not a prime number.

Some properties of Prime numbers

- 2 is the lowest prime and even number.
- The lowest odd prime number is 3.
- When a prime number $p \geq 5$, is divided by 6, it gives remainder is 1 or 5. But, vice versa is not true, i.e. if a number is divided by 6 and gives remainder 1 or 5 then, it need not be prime. For example, 65 is a number which when divided by 6 gives remainder 5, but, 65 is not a prime number. Thus, this can be referred to as a necessary condition but not a sufficient condition.

What do you mean by the factor of a number?

The factor of a number is nothing but the divisor of a number.

Let us take a number as our example,

Number = 20

The factors of 20 are 1, 2, 4 and 5. Like this, if you find out the factors of the numbers 210, 159, 253 etc. it will take a lot of time to do it this way.

Thus, performing the same task in a good way can be:

To find factors of 20:

$$\begin{array}{l} 1 \times 20 \\ 2 \times 10 \\ \text{and } 4 \times 5 \end{array}$$

i.e. By finding the factors in pairs, *The discovery of one factor will automatically find out another factor*, for example, here, finding out 4 as a factor of 20, it will automatically give you 5 as another factor.

Now, let us take a look again at the “factor pairs” in the example above. If you compare the values in each pair with the square root of 20 (i.e. 4....) you will find that for each pair the number in the left side is lower than the square root of 20, while the number in the right side is higher than the square root of 20. This is always true for all the numbers.

Thus one need not make any effort to find the factors of a number above the square root of the number; these will come automatically. All you need to do is to find the factors below the square root of the number.

Why do we need a factor of a number?

Suppose you are facing a question like this;

The product of two numbers is 192 and their difference is 26. Then find these two numbers?

Sol:

To solve this question we have two approaches. One is a mathematical way of doing and other is a quantitative or logical way of doing things.

Mathematically,

$$a \times b = 192 \dots\dots\dots(1)$$

$$\text{And } a - b = 26 \dots\dots\dots(2)$$

Here we have two variables and two equations. These two equations lead to a quadratic equation & now, we solve the quadratic equation to get the numbers. But it will take a lot of time.

Logically, make factor pairs of 192,

$$1 \times 192$$

$$2 \times 96$$

$$3 \times 64$$

$$4 \times 48$$

$$6 \times 32$$

$$8 \times 24$$

$$12 \times 16$$

According to the second statement, the difference between the two numbers was 26.

You can easily spot your answer from the factors. A factor of pair 6×32 gives the difference between two numbers ($32 - 6 = 26$). Thus two numbers are 6 & 32.

If you find out the factor of 192 like this 1,2,3,4,6.....so on. You can't easily spot your answer and then you have to go through the quadratic equation approach, which is time-consuming.

Shortcut for finding the factors of a number:

Let's take a number: 192

Left side \times Right side

$$1 \times 192$$

$$2 \times 96$$

$$\begin{array}{l}
 3 \times 64 \\
 4 \times 48 \\
 6 \times 32 \\
 8 \times 42 \\
 12 \times 16
 \end{array}$$

How to approach?

$$1 \times 192$$

When 192 is divided by 2 you get 96.

$$2 \times 96$$

Now, from this point when you see 3, you have two option for getting ($3 \times \dots$) the right side number. One is, 192 divided by 3 giving you 64, and if you see the previous pair you had 2×96 , so from this pair when you see 96 (96 can be broken down into 3×32), if we think of 192 like $2 \times 3 \times 32$ then you can rewrite it as 3×64 . It is easier than 192 divided by 3.

$$3 \times 64$$

After that when you go to 4, you see 4 which is 2 times “2”, So you should have half of 96 on the right side. It is better than dividing 192 by 4.

$$4 \times 48$$

As you keep going you will keep getting prime and composite numbers on the left side. When you see the **prime number** on the left side then you try to look at the **larger number of the previous pair** to get the right side of the current pair and **when you see the composite number on the left side, you try to bring the right side from one of the previous numbers on the left side.**

Thus, after 4 you have 5, which is a prime number & 5 does not divide 48 (larger in the previous pair), so 5 is not a factor of 192.

After that 6 is a composite number, from left side 6 is double of 3 & half of 64 is 32 in the left side, thus you get another factor is

$$6 \times 32$$

Using this approach you can get all the factors.

$$8 \times 42$$

$$12 \times 16$$

NOTE: 1. The number on the left side is always less than the square root of the given number.

Let's take a number of 148, then factors are

$$1 \times 148$$

$$2 \times 74$$

$$4 \times 37$$

So, here, 37 is the prime number, thus no further factor of 148.

NOTE: 2. Once you get a prime factor, then no further factor is possible.

NOTE: 3. When we have to check whether a number N is prime or not, we need to only check for its divisibility by prime factors; below the square root of N.

Quant shortcut strategies & Finding Prime number

To build up the speed in aptitude, you have to do these three things:

1. Superior reaction
2. Superior equation solving process
3. Superior calculation

The superior reaction through which you can build up your speed is to improve your reaction to the question situation.

Superior equation solving process, in this you need to know that aptitude mathematics is dominated by linear equations, at max, they become quadratic.

Most of the time you can avoid quadratic equations by just thinking of the number you are dealing with. Eg, $a \times b$ followed by $a + b$

$a \times b$ followed by $a - b$

$a \times b$ followed by $a : b$

Here, you have a list of complete factor pairs then you can easily choose your answer from the list. Thus, somebody solving this problem mathematically can't match your speed.

The superior calculation is to improve your calculation speed as well as accuracy. In this, to speed up the calculations you need to memorize multiplication tables, at least upto 20 and you should use some tricks like, “multiply in parts” ($3 \times 74 = 3 \times 70 + 3 \times 4$), “it’s ok to approximate”(if options are far apart) etc. The accuracy can only be improved by a lot of practice.

To check whether a number is Prime or not?

Shortcut method :

To check whether a number N is Prime or not, follow these steps:

1. Take the square root of the number.

2. Consider the lower integer after taking the square root. Say this number is x . For example, if you have to check for 241, its square root will be 15.52. Hence, the value of x , in this case, will be 15.
 3. Check for divisibility of the number N by all prime numbers below x . If there is no prime number below the value of x which divides N , then the number N will be prime.
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For example :

Let the number is 241,

1. $\sqrt{241}$ is 15.52.
2. The value of $\sqrt{241}$ lies between 15 to 16. Hence, take the value of x as 15 (lower integer).
3. Prime numbers less than 15 are 2,3,5,7,11 and 13. 241 is not divisible by any of these. Hence, you can conclude that 241 is a prime number.

NOTE: 1. While checking “Is the number N prime?” We have two things to keep in mind; first, the number N should be odd and second, the number should not end with 5 (e.g. 25 is an odd number but it ends by 5, so it is automatically divisible by 5).

Some important shortcut points:

1. For number below 49,
The only number you would need to check for divisibility with is number “3”.
2. A number between 49 and 121,
You need to check divisibility by “3” and “7” only.
3. A number between 121 and 169,
You need to check divisibility by “3”, “7” and “11” only.

Basic concept of HCF

When you talk about HCF, you are always thinking about two or more than two numbers. Similarly in LCM, you also talk about two or more numbers.

HCF (Highest common factor) is also called GCD (Greatest common divisor).

What is HCF?

Let us consider two numbers 20 & 30.

First, write the factors of 20 & 30

Factors of 20

$$1 \times 20$$

$$2 \times 10$$

$$4 \times 5$$

Factors of 30

$$1 \times 30$$

$$2 \times 15$$

$$3 \times 10$$

$$5 \times 6$$

Between 20 & 30 you can observe some factors which are common between the two. So, from the list of the factors, you can see 1, 2, 5, 10 are available in both the factor list of 20 as well as 30. Thus, the common factors between 20 & 30 are 2, 5 & 10. Among these factors, the largest/highest one is 10, which is referred to as the HCF.

NOTE: In factors of a number you have prime & composite numbers (Every composite number written as the product of their prime number)

For example, prime factors of 80 and 224 are

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$$

Rules for Finding the HCF of Two Numbers a & b

1. Write down the **prime factors** of the given numbers.
2. Write down the **prime factors** which are common to both.
3. And products of the common factors will give you HCF of the numbers.

Illustration: Find the HCF of 150 & 375.

Step 1: Write down the prime factors of the given numbers.

$$150 = 2 \times 3 \times 5 \times 5$$

$$375 = 3 \times 5 \times 5 \times 5$$

Step 2: Write down the prime factors which are common to 150 & 375.

3, 5 & 5.

Step 3: Products of the common factors are $3 \times 5 \times 5$

Hence, HCF = 75.

How do you find out the HCF of more than two numbers?

Let us take three numbers a,b & c.

To find their HCF, what you need to do is, first find out the prime factors of each of the numbers.

Say,

$$a = 2^3 \times 3^4 \times 5^1 \times 11^2$$

$$b = 2^5 \times 3^5 \times 5^2 \times 7^3$$

$$c = 2^6 \times 3^4 \times 5^3 \times 7^2$$

HCF (a,b,c) → All common prime factors with their *lowest* available power.

Thus, HCF of a,b,c will be

$$\text{HCF} = 2^3 \times 3^4 \times 5^1$$

NOTE: Finding LCM is a very similar process to finding the HCF.

LCM(Lowest Common Multiple) of a,b,c → All Prime factors with their *maximum* available power.

Thus, LCM of a,b,c will be

$$\text{LCM} = 2^6 \times 3^5 \times 5^3 \times 7^3 \times 11^2$$

Shortcut of HCF

Finding the HCF of a set of numbers is however extremely difficult and time taking, through the prime number approach (that was discussed previously). So, we adopt some shortcut for finding the HCF of a set of numbers.

What is the shortcut of HCF?

Let's take two numbers 46 & 76.

Now, the question is "*which are the numbers that would leave the same remainder when they divide both 46 & 76*"?

NOTE: This question statement is useful for shortcuts of HCF.

Properties of remainder:

Let us say two numbers 22 & 39.

Find out the remainder of these two numbers when divided by 7?

$22 \div 7$ gives a remainder 1.

$39 \div 7$ gives a remainder 4.

This remainder "4" can be found by another method. As, $39 = 22 + 17$; so, $(22 + 17) \div 7$ when we look at individual remainders $22 \div 7$ has 1 & $17 \div 7$ has 3, So the total remainder is 4.

Now, let us try to answer the above question "*which are the numbers that would leave the same remainder when they divide both 46 & 76*"? using the remainder property. So, we have these two numbers 46 & $46 + 30$.

Say, you divide these numbers by a divisor D.

Now,

$46 \div D$. Suppose, this gives remainder 'r'

According to question statement if $(46 + 30) \div D$, you want a remainder 'r' only.

The key element for being overall remainder 'r' will depend on whether $(30 \div D)$ will give remainder "zero" or not, because only in that case, you will get the remainder 'r'.

If $(30 \div D)$ gives you remainder "zero", it is quite obvious that the value of 'D' has to be a factor of 30.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30. The value of D will be any of these numbers.

If we take $D = 5$ then,

$(46 \div 5)$ will give you a remainder 1. & $(46 + 30) \div 5$ will give you a remainder $(1 + 0)$. No number is possible outside factors of 30, that can have this property.

For example, if you divide 46 & 76 by 7, which is not the factor of 30, then you will get different remainders from 46 and 76.

How does this approach connect to the HCF?

When you talk about the common factor of two numbers X & Y . then the common factor has to leave the same remainder “zero”. Which means

Let two numbers X & $X+12$, the only numbers that will have the possibility of leaving the same remainder zero would be factors of 12.

$$1 \times 12$$

$$2 \times 6$$

$$3 \times 4$$

All the common factors of these two numbers would come in the factors of 12, they can't come from any outer range. And hence, if all the common factors of X & $X+12$ are inside the factors of 12, So the HCF of X & $X+12$ would also come from the factors of 12. Which means HCF of X & $X+12$, can only be one of (1,2,3,4,6 & 12) these numbers.

For example,

Find the HCF of 38 & 50?

Now, you would do HCF without doing prime factors. You would do the HCF by difference method i.e by finding out the factors of difference between the two numbers.

$50-38=12$, factor of 12 are 12,6,4,3,2&1.

12 → Does Not divide 38, so this is not HCF of these two numbers.

6 → Does Not divide 38, so this is not HCF of these two numbers.

4 → Does Not divide 38, so this is not HCF of these two numbers.

3 → Does Not divide 38, so this is not HCF of these two numbers.

2 → Divide 38, so this is HCF of these two numbers.

Then it is obvious it will divide $38+12$ and hence HCF is 2.

NOTE: HCF may vary according to the numbers, but it will always come from the difference.

What about more than two numbers?

Let us consider the numbers are x , $x+12$, y , z .

For finding HCF of these numbers, take the differences between the numbers. Here, many differences are possible, but you have to choose the smallest difference between any pair of these numbers.

Write the factors of that number and HCF of all these numbers would be from the factor list.

Sometimes you might want to go for prime number difference instead of the smallest difference, For example, suppose the numbers are 44,56 & 93.

$$\text{So, } 56 - 44 = 12$$

$$93 - 56 = 37$$

$$93 - 44 = 49$$

Here, a better difference to take here is 37 because 37 is a Prime number, then the factors of 37 are either 1 or 37. So, HCF, in this case, is either 1 or 37. 37 does not divide any number, so, the HCF=1.

Question:

A nursery has 363, 429 and 693 plants respectively of 3 distinct varieties. It is desired to place these plants in straight rows of plants of 1 variety only, so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

Sol:

The size of each row would be the HCF of 363, 429 and 693.

Difference between 363 and 429 = 66.

Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.

66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693.

Hence, 33 plants is the correct answer for the size of each row.

For the number of rows that would be required = Minimum number of rows required = $363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45$ rows.

Problems on HCF

Problem 1: The sides of a pentagonal field are 918, 2160, 2244, 2358 & 1431 meters. Find the greatest length of tape that would be able to exactly measure each of these sides?

Sol: Here in the question, it is required to find the largest tape that would be able to measure all the sides exactly, which means it is talking about HCF.

Sides of pentagons are 918, 2160, 2214, 2358 & 1431.

Step1. Find the smallest difference from all the pairs of numbers. So, the smallest difference comes out between 2214 & 2160, which is 54, as all the other differences are greater than 54.

Step2. Factors of 54 are

$$1 \times 54$$

$$2 \times 27$$

$$3 \times 18$$

$$6 \times 9$$

54 → Does Not divide 1431

27 → Does Not divide 2358

No need to check for 18.(it will not divide odd numbers)

9 will divide all these numbers. Thus, the HCF of these numbers is 9.

And hence, the largest length of the tape that can measure all the sides exactly is 9.

Problem 2: A milkman has the milk of three varieties. He has 403L, 465L & 651L of the three varieties of milk with him.

- What is the largest size of bottle in which he can bottle each of the three types of milk completely without mixing the milk?
- What is the minimum number of bottles required?
- How many different sizes of bottles (with the integral number of litres) can be used in order to bottle all the three varieties of milk?

Sol:

- Three varieties of milk are 402L, 465L & 651L.
Largest bottle size means HCF of these numbers.

Step1: Find the smallest difference between all the pairs of numbers. So, the smallest difference comes out between 465 & 403, which is 62, as all other differences are greater than 62.

Step2: Factors of 62 are

$$1 \times 62$$

$$2 \times 31$$

31 is a Prime number so no further factor is possible.

62 → Does Not divide 403. 31 will divide all the three numbers.

Thus, the largest bottle size is 31L.

b. Let a bottle size of 'b' L.

$$\text{Minimum No. of bottle required} = 403/b + 465/b + 651/b \dots\dots\dots(1)$$

To minimise this equation 'b' should be maximised. So, maximum value of b = 31L.

Thus,

$$\begin{aligned} \text{Minimum no. of bottle required} &= 403/31 + 465/31 + 651/31 \\ &= 13+15+21 \\ &= 49L. \end{aligned}$$

c. This part is basically asking how many factors 31 has. Since, 31 is a prime number, so, it has only two factors 31 & 1. Hence, 2 sizes of bottle can be used.

Problem 3: What is the largest number that would leave the same remainder when it divides 283, 411 & 475.

Sol: To solve this type of question you have to follow the following steps:

1. Arrange all the numbers in increasing order.
2. Then take the difference, pairwise (linked pair i.e. for numbers a,b,c, & d, the differences are (b-a),(c-b) & (d-c)). Let us say that the differences of these pairs are p,q & r respectively.
3. Find the common factors of p,q & r. (common factor of p,q & r gives you the same remainder in all the three cases.)
4. The largest number that leaves the same remainder will then become the HCF of p, q & r.

Some questions for practice:

1. Two equilateral triangles have the sides of lengths 34 and 85 respectively.
(a) The greatest length of tape that can measure both of them exactly is:
(b) How many such equal parts can be measured?

Ans: (a) 17.

(b) 21.

2. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of the number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The minimum number of rows that the forester will require to plant trees are?

Ans: 10.

3. Find the HCF of
(a) 420 and 1782 (b) 36 and 48
(c) 54, 72, 198 (d) 62, 186 and 279

Ans: (a) 6 (b) 12 (c) 18 (d) 31.

Concept and shortcuts of LCM

LCM: Lets two natural numbers be a & b . *The smallest natural number which will be completely divisible by a & b is called LCM (lowest common multiplication) of a & b .*

For example:

Let's have two natural numbers 4 & 6. Here, 12 is the smallest natural number which will be completely divisible by 4 & 6.

Thus, $\text{LCM}(4,6) = 12$.

Procedure for finding LCM of two numbers:

Step1: Find the prime factor of two numbers a & b .

Step2: Write down all the prime factors that appear at least once in the numbers a & b .

Step3: Write all the prime factors with their highest power.

Step4: Products of all the prime factors with their highest power will give you LCM of a & b .

For example: Let's have two numbers 12 & 80.

Step1: List the prime factors

$$12 = 2 \times 2 \times 3$$

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

Step2: Write down all the prime factors that appear, at least once in the numbers: 2,3,5.

Step3: Write all the prime factors with their highest power: $2^4 \times 3^1 \times 5^1$

Step4: The $\text{LCM} = 2^4 \times 3^1 \times 5^1$
 $= 240$.

Let's take two numbers 4 & 6. When you see the multiples of 4 & 6.

Multiples of 4: 8,12,16,20,.....

Multiples of 6: 6,12,18,24,.....

Now, the common multiples between 4 & 6 are: 12,24,36,.....

Thus, the smallest multiple in the common multiple list is called LCM.

NOTE: Multiplication table of LCM would essentially give the common multiples of all the numbers.

For example, you have three numbers a , b & c and their LCM is 12, then 12, 24, 36, 48, all these multiples of 12, would be multiples of a , b & c .

Shortcut for finding the LCM:

As you saw LCM is the product of the highest power of all the prime factors, but that process would be very tedious, especially when the numbers are small.

When the numbers are small the logic of LCM builds around the **Co-prime numbers**.

Co-prime Number: Two numbers are Co-prime to each other when they have no common factor among each other.

For example: (6, 13), (7, 11), (9, 19) etc.

Three numbers are Co-prime to each other when pairwise, each pair is Co-prime.

For example: Three numbers be a, b and c are Co-prime when,

a, b are Co-prime,

a, c are Co-prime,

& b, c are Co-prime.

All three pairs should be Coprime to each other, only then, a, b and c will be Co-prime.

NOTE: When a & b is Co-prime then the HCF should be 1.

Some important points about the Co-prime numbers:

- (i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 etc.)
- (ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 etc.)
- (iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)
- (iv) One prime number and another composite number (such that the composite number is not a multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note that 17 and 51 are not co-prime, as 51 is a multiple of 17)

Shortcut for LCM:

Step1: When the numbers are co-prime, then LCM is simply their product.

So, 7, 9 and 11 are co-prime, The LCM is $7 \times 9 \times 11$.

Step2: What to do when you have a mix of prime and Co-prime.

For example, four numbers 42, 44, 18, 25.

(i) If you see any co-prime put them down in your LCM. Here you can see 18 & 25 are Co-prime (and 25, 42; 25, 44 are also Co-prime).

(ii) LCM of these numbers starts with $18 \times 25 \times \dots$ And

(iii) Now the logic of LCM should contain all the other numbers from the given numbers.

(iv) Out of the LCM, you should be able to construct 42 and 44 also.

(v) The factor of $42 = 2 \times 3 \times 7$. Inside 18 you have 2 & 3, But you don't have 7 in 25 and 18. To construct 42, you should have a 7 in your LCM. ($LCM = 18 \times 25 \times 7 \dots$)

(vi) The factor of $44 = 2 \times 2 \times 11$. Inside 18, you have one 2, but there is no 11 and other 2 in this LCM; so, to construct 44 you need to introduce 2 & 11 into the LCM.

So, LCM will be $= 18 \times 25 \times 7 \times 2 \times 11$.

NOTE: (i). LCM has to be the multiple of HCF.

(ii). For any two numbers a & b, Product of two numbers $(a \times b) = LCM \times HCF$
(this formula is valid for two numbers)

HCF & LCM of a Fraction:

HCF of a Fraction:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

LCM of a Fraction:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

For example: LCM & HCF of $\frac{1}{2}$, $\frac{5}{7}$ and $\frac{8}{11}$ are:

$$\text{LCM} = \text{LCM}(1, 5, 8) / \text{HCF}(2, 7, 11)$$

$$\text{HCF} = \text{HCF}(1, 5, 8) / \text{LCM}(2, 7, 11)$$

So,

$$\text{LCM} = 40/1$$

$$\text{HCF} = 1/(2 \times 7 \times 11)$$

Standard questions on LCM

Type 1: Based on the formula: Product of two numbers = HCF \times LCM.

Question 1: HCF of two numbers is 75 & their LCM is 1800. If one of the numbers is 600, then what will be the other number?

Sol: Let the required number is b , and the given question says $a=600$.

So, According to formula:

$$\text{HCF} \times \text{LCM} = a \times b$$

$$75 \times 1800 = 600 \times b$$

$$b = 225. \text{ This is the required number.}$$

But be careful of traps in this question.

For example:

HCF of two numbers is 75 & their LCM is 900. If one of the numbers is 600 then, what will be the other number?

Sol:

Let the required number is b , and the given question says $a=600$.

So, According to the formula:

$$\text{HCF} \times \text{LCM} = a \times b$$

$$75 \times 900 = 600 \times b$$

$$b = 112.5.$$

So, 600 & 112.5 don't have an HCF of 75 and LCM of 900.

Another trap in this type of question is:

For example:

HCF of two numbers is 75 & their LCM is 400. If one of the numbers is 100 then, what will be the other number?

Sol:

Let the required number is b , and the given question says $a=100$.

So, According to the formula:

$$\text{HCF} \times \text{LCM} = a \times b$$

$$75 \times 400 = 100 \times b$$

$$b = 300.$$

So, 100 & 300 don't have an HCF of 75 and LCM of 400. Also in this question, LCM is not a multiple of HCF.

So, be careful of these types of traps.

Type 2: Bell tolling question

Question 1: 4 Bells toll together at 9:00 A.M. They toll at an interval of 7, 8, 11 and 12 seconds respectively. After 9:00 A.M., at what time will these bells toll together for the first time & how many times will they toll together again in the next 3 hours?

Sol: 4 Bells toll at an interval of 7, 8, 11 & 12 sec.

7-sec bell tolls at multiples of 7; 8-sec bell tolls at multiples of 8; 11-sec bell tolls at multiples of 11 and 12-sec bell tolls at multiples of 12.

To find, at what time will these bells toll together the first time after 9 AM, you need to find the LCM of these intervals.

So, $\text{LCM}(7, 8, 11 \text{ \& } 12) = 1848 \text{ sec.}$

Thus, after 9 AM bells toll together for the first time at 9:30:08.

In 3 hr i.e. 10800 sec:

Number of times the bells toll together in the next 3 hours = $10800/1848$
 $= 5.84 \approx 5 \text{ times.}$

Type 3:

Question: What is the smallest number greater than 1, that leaves a remainder 1, when divided by 1, 2, 3, 4, 5, 6, 7, 8, 9 & 10.

Sol:

To understand the logic of this question first you have to understand this logic:

For example: What is the smallest number greater than 1, that leaves a remainder 1, when divided by 3 & 4.

LCM of 3 & 4 is 12. And $12+1=13$ is the smallest number.

So, in this type of question, the logic is **LCM+Remainder**

So, now come to the question

LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9 & 10 is 2520. And the smallest number that leaves a remainder 1, is $\text{LCM}+1$ i.e. $2520+1=2521$.

Some question for practice:

Question1: Find the HCF of

- (a) 420 and 1782 (b) 36 and 48
(c) 54, 72, 198 (d) 62, 186 and 279

Ans: (a) 6 (b) 12 (c) 18 (d) 31.

Question2: Find the LCM of

- (a) 13, 23 and 48 (b) 24, 36, 44 and 62
(c) 22, 33, 45, and 72 (d) 13, 17, 21 and 33

Ans: (a) 14352 (b) 24552 (c) 3960 (d) 51051.

Question3: The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is?

Ans: 52.

Question 4: Three runners running around a circular track can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?

Ans: 44.

Question 5: The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. The other number is?

Ans: 132.