Lecture Notes for Ratio, Proportion And Variation - 2

We have already discussed the theory of this chapter and did some problems based on that, now we will go further with some standard problems.

Problems On Ratio-1:

Problem 1:
Divide rupees 252 amongst A, B and C such that 1/3rd of what A gets is equal to 1/5th of what B gets is equal to 1/4th of what C gets. How much A, B and C will get individually?

Solution :
Here 2 equations are formed;
$A + B + C = 252$ ………(1)
$A/3 = B/5 = C/4$ ……….(2)

Let's say, eq (2) is equal to k.
$A/3 = B/5 = C/4 = k$, $A = 3k$, $B = 5k$, $C = 4k$
Put value of A,B and C in eq(1) we get;
$3k+5k+4k = 252$, $12k = 252$ and $k = 252/12 = 21$.
Hence the numbers are; $A = 3 \times 21 = 63$, $B = 5 \times 21 = 110$ and $C = 4 \times 21 = 84$.

Problem 2:
Divide rupees 517 amongst A, B and C such that 1/3rd of what A gets is equal to 2/5th of what B gets is equal to 3/7th of what C gets. How much A, B and C will get individually?

Solution :
$A + B + C = 517$ ………(1)
$A/3 = B2/5 = C3/7$ …….(2).
You can get a direct ratio from eq(2).
A:B:C = 3:5/2:7/3

Whenever you have a ratio which itself has its component in the fractions, you should multiply the ratio by the denominator LCM.
LCM (2,3) = 6. Multiply A:B:C by 6 you will get a proper ratio.

So; A:B:C = 18:15:14
So, Sum of the component of ratio = 18+15+14 = 47.
$47 \equiv 517$ that means the multiplier would be 11.

Hence the numbers are; A = $18 \times 11 = 198$, B = $15 \times 11 = 165$ and C = $14 \times 11 = 154$.

Problem 3:
3 monkeys have bananas in ratio 1/3:1/5:1/7. The total number of bananas with 3 monkeys is 284. How many bananas does each monkey have?

Solution :
A + B + C = 284
A:B:C = 1/3:1/5:1/7
Whenever you have a ratio which itself has its component in the fractions, you should multiply the ratio by the denominator LCM.

LCM (3,5,7) = 105. Multiply A:B:C by 105 you will get a proper ratio.
So; A:B:C = 35:21:15

Some of the components of ratio = 35+21+15 = 71.
$47 \equiv 284$ that means the multiplier would be 4
Hence the numbers are; A = $35 \times 4 = 140$, B = $21 \times 4 = 84$ and C = $15 \times 4 = 60$.

Problem 3:
Divide rupees 500 amongst A, B, C and D such that A&B gets 3 times what C&D gets, B gets 4 times what C gets, C gets 1.5 times what D gets. How much B&C gets individually?

Solution :
A + B + C + D = 500 ………(1)
A + B = 3(C + D) ……….(2)
B = 4C ………(3)
C = 1.5D ……...(4)

Solving this question from these 4 equations is a very tedious process. Instead of this, you can simply do this question in following way;

Divide 500 amongst A, B, C&D.
From the given statement A&B gets 3 times of C & D. The sum of **A&B has to be 375** and the sum of **C&D has to be 125**.

The logic behind this;
If you divide 500 into 2 parts such that one part is 3 times of another part i.e in ratio 3:1.
Divide 500 in ratio 3:1 i.e one part = 375 and other part = 125.
A + B = 375 and C + D = 125.

From the given statement C gets 1.4 time D i.e C:D = 1.4:1or 3:2
Divide C + D = 125 in ratio 3:2. Sum the of ratio = 3+2 = 5.
$5 \equiv 125$ that means the multiplier would be 25

Hence C = $3 \times 25 = 75$ and D = $2 \times 25 = 50$.
And B gets 4 times C. hence B = $4 \times 75 = 300$.
A + B + C + D = 500. Thus; A = 75.

Problems On Ratio-2:

Problem 1:
Anjali has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and 2nd mixture has milk to water in ratio 2:7. She mixes equal quantities of these mixtures. What is the ratio of milk to water in the final mixture?

Solution :

Mixture 1	Mixture 2
M:W = 3:8	M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Take 99L for both mixtures because mix equal quantities of mixture.

Mixture 1	Mixture 2
99L	99L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

$3+8 = 11 \equiv 99$	$2+7 = 9 \equiv 99$
11 being 99 so; multiplier would be 9.	9 being 99 so; multiplier would be 11.
Hence M = $3 \times 9 = 27$ & W = $8 \times 9 = 72$.	Hence M = $2 \times 11 = 22$ & W = $7 \times 11 = 77$

Thus; total milk = 27+ 22 = 49 and total water = 72 + 77 = 149.

Hence the final mixture has milk to water ratio = 49:149.

Problem 2:
Shubham has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and the 2nd mixture has milk to water in ratio 2:7. He is mixing these mixtures in 2:3. What is the ratio of milk to water in the final mixture?

Solution :

Mixture 1	Mixture 2
M:W = 3:8	M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Here you can not take 99L for each mixture because the question is not talking about equal quantities.
Mixture 1 = 99L and mixture2 = 99L, To make both the mixture in 2:3. Then;

Mixture1 = 99×2 = 198L and Mixture2 = 99×3 = 297L

Mixture 1	**Mixture 2**
198L	297L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

$3+8 = 11 \equiv 198$	$2+7 = 9 \equiv 297$
11 being 198 so; multiplier would be 18.	9 being 297 so; multiplier would be 33.
Hence M $=3 \times 18= 54$ & W $=8 \times 18=144$	Hence M $= 2 \times 33 = 66$ & W $=7 \times 33=231$

Thus; total milk = 54 + 66 = 120 and total water = 144 + 231 = 375.
Hence the final mixture has milk to water ratio = 120:375.

Problems On Ratio-3:

Problem 1:
The income of P & Q is in ratio 1:2 and expenditure of P & Q is in ratio 1:3. If each saves 500 of their income. Find the P's income.

Solution :
Lets P's income = x and Q's income = 2x.
P's expenditure = y and Q's expenditure = 3y.
And we know;
Saving = Income - Expenditure

Saving for P; x - y = 500 …………(1)

And Saving for Q; $2x - 3y = 500$ ………..(2)
Solving eq(1) and (2) we get;
$x = 1000$ and $y = 500$.
Hence P's income = 1000.

Problem 2:
Rupees 232 is to be divided among 150 girls and boys, such that each girl gets Rs 1 and each boy gets Rs 2. Find the number of boys and girls.

Solution :
Let the number of girls = G and number of boys = B
$G + B = 150$ ………(1)
$G + 2B = 232$ ……….(2)
Solving eq (1) and (2) we get;
$G = 68$ & $B = 82$.

Problem 2:
In a zoo, there are Ducks and Rabbits. If heads are counted, there are 80 heads and if legs are counted there are 212 legs. How many ducks are there?

Solution :
The equation for heads;
$D + R = 80$ …………(1)
The equation for legs;
$2D + 4R = 212$ ……….(2)
You can solve this question from these equations, but there is another way to solve the question; Suppose there are all the ducks then total no of heads are 80 and total no of legs are 160.

Now, since 52 (212-160) legs are extra, it means there will be 26 (52/2) rabbits. As we know a rabbit has two extra legs than that of a duck.
Therefore, the number of rabbits = 26
and number of ducks = 80 - 26 = 54

Problem 3:
In a bag, there are coins of 1 Rs, 2 Rs and 5 Rs in the ratio of 5: 15: 12. If there is Rs. 95 in all, how many 5 Rs coins are there?

Solution :
In this question, there are 3 variables. One variable is the number of coins, 2nd variable is the value of coins and the 3rd variable is the total value of the coins.
Value of per coins $\rightarrow$ 1:2:5

Number of coins $\rightarrow$ 5:15:12

The total value of coins = value of per coin $\times$ no of coins

The total value of coins $\rightarrow$ 5:30:60

Divide Rs 95 in 5:30:60, the multiplier would be 1.

Therefore; 5 Rs in 1 Rs coin, 30 Rs in 2 Rs coin & 60 Rs in 5 Rs coin.

Hence 5 Rs coins = 60/5 = 12.

Variation and its 3 types :

Variation is an important concept in mathematics. To understand variation first you need to understand 3 kinds of variation.

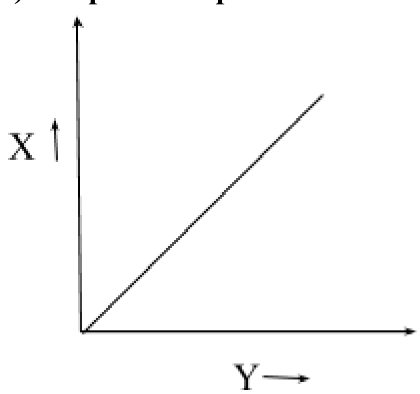
1. **Direct variation :**
   x varies directly as y or x is directly proportional to y.
   Mathematically; **x α y**.

(a) Logical implication: When x increases y increases. And if x decreasing y also decreases

(b) Calculation implication: If x increases by 20%, y will also increase by 20%.

(c) Ratio : If x is increasing by 1/5 then y will also increase by 1/5.

(d) Graphical implications: The following graph is representative of this situation.



(e) Equation implication: The ratio x/y is constant i.e x = ky (where k is a constant)

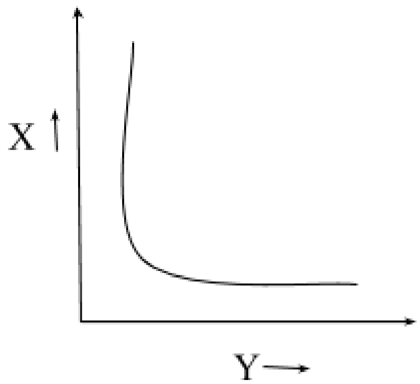
2. **Inverse variation :**
   X is inversely proportional to y or x varies inversely as y or product of x and y is constant.
   Mathematically; **x α 1/y**.

(a) Logical implication: When x increases y decreases and vice versa.

(b) Percentage implication: If x increases by 25% then y decreases by 20%.

(c) Ratio implication: If x increases by 1/4 then y decreases by 1/5.

(d) Graphical implications: The following graph is representative of this situation.



(d) Equation implication: The product $x \times y$ is constant.

3. **Joint variation :**

   If x varies jointly as y & z or $\mathbf{x\,\alpha\,(y \times z)} \Rightarrow \mathbf{x = k(y \times z).}$ Or if x varies as y when z is constant and x varies as z when y is constant.

   Mathematically; $\mathbf{x\,\alpha\,(y \times z)}$

Problems On variation :

Problem 1:

Given that, x directly varies with y and x is 18 when y is 7. Find x when y is 21?

Solution :

x directly varies with y i.e $x\,\alpha\,\mathbf{y}$ **or** $x = ky$ ……….(1)

Replace x and y with their respective values. So; eq (1) becomes

$18 = k \times 7 \Rightarrow k = 18/7$.

When y =21 the value of x is;

from(1); $x = 18/7 \times 21 = 54$.

Problem 2:

The duration of a railway journey varies as the distance and inversely as the velocity, while velocity varies as the square root of quantity of the coal used and inversely as the number of carriages in the train. In the journey of 50 km in half an hour with 18 carriages, 100 kg of coal is required. How much coal will consume in a journey of 42km in 28 minutes with 16 carriages?

Solution :

There are 5 variables.

Assume duration = T, distance = D, velocity = V, quantity of coal = Qc and No. of carriage = N.
According to question;

$T \ \alpha \ \frac{D}{V}$ ....(1) and $V = \frac{\sqrt{Qc}}{N}$ ....(2)

From (2) put value of V in (1);

$T \ \alpha \ \frac{D \times N}{\sqrt{Qc}}$ or $T = \frac{k \times D \times N}{\sqrt{Qc}}$ ....(3)

Put value of T = 30min, D = 50km, N = 18 and Qc = 100 kg in (3);

$30 = \frac{k \times 50 \times 18}{\sqrt{100}} \Rightarrow k = 1/3.$

Now from eq (3);

$T = \frac{1}{3} \frac{D \times N}{\sqrt{Qc}}$ ....(4)

Therefor for the T = 28min, D = 42km, N = 16 ; the value of coal required is,

$28 = \frac{1}{3} \frac{42 \times 16}{\sqrt{Qc}} \Rightarrow \sqrt{Qc} = 8$; Hence Qc = 64 kg.

Problem 3:

A precious stone of weight 35 gm worth rupees 12250 is accidentally dropped and breaks into 2 pieces. The weight of the pieces in ratio 2:5. If the piece of the stone varies as the square root of weight. What is the amount of loss incurred due to the breakage? What is the value of loss?

Solution :

Assume the price of stone = P, Weight = W
According to question;

$P \ \alpha \ W^2$ or $P = k\,W^2$ ....(1)

Given P = 12250 and W = 35gm. Put value of P and W in eq (1)

$12250 = k\,(35)^2 \Rightarrow k = 12250/1225 = 10.$

Now from eq (1)

$P = 10 \times W^2$ ....(2)

35 breaks in 2:3. So; one piece of 10gm and another is 25gm.
W1 = 10 gm and W2 = 25 gm.

Value of 1st piece P1 = $10 \times (10)^2 = 1000$.
Value of 1st piece P2 = $10 \times (25)^2 = 6250$.
Total price of two pieces = 1000+6250 = 7250.
Therefore; loss incurred = 12250 - 7250 = 5000 Rs.

Problem 4:
A train without its compartments runs at a speed of 42kmph. The reduction in speed of the train is directly proportional to the square of the numbers of compartments attached. When 9 compartments were attached the speed of the train was 24kmph. What are the maximum numbers of compartments with which the train can just move?

Solution :
Assume the speed of the train = S, Reduction in speed = Sr and Number of compartments = N
According to question;
S = 42 - Sr ………(1)
Sr $\alpha\ \sqrt{N}$ or Sr = $k\sqrt{N}$ ……….(2)

Using eq (2) in (1);
S = 42 - $k\sqrt{N}$ ……….(3)

Given S = 24kmph and N = 9. So; from (3)
$24 = 42 - k\sqrt{9} \Rightarrow 3k = 18$ or $k = 6$.

Put the value of k in eq (3)
S = 42 - $3\sqrt{N}$ …………(4)

Interpretation of that situation here is that if you assume the value of N in such a way that S becomes zero. Speed zero means the train would not move.

It is clear that when N = 49 in eq (4), then S = 0.
RHS of eq (4)
RHS = $42 - 6\sqrt{49} = 42 - 6 \times 7 = 0$

Therefore with 49 compartments, the train would not move. So; 1 less 49 i.e with 48 compartments the train would just move.

Some Questions For Practice :

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1. 5783 is divided among Anjali, Shubham, and Navdeep in such a way that if 28, 37 and 18 be deducted from their respective shares, they have money in the ratio 4: 6: 9. Find Anjali's share.
   Ans: 1228.

2. If 10 persons can clean 10 floors by 10 mops in 10 days, in how many days can 8 persons clean 8 floors by 8 mops?
   Ans: 10 days.

3. Three quantities A, B, C are such that AB = KC, where K is a constant. When A is kept constant, B varies directly as C; when B is kept constant, A varies directly C and when C is kept constant, A varies inversely as B.
   Ans: 8.33.

4. If $x/y = 3/4$, then find the value of the expression, $(5x – 3y)/(7x + 2y)$.
   Ans : 3/29.

5. 3650 is divided among 4 engineers, 3 MBAs and 5 CAs such that 3 CAs get as much as 2 MBAs and 3 Engineers as much as 2 CAs. Find the share of an MBA.
   Ans: 450.

(Ref: Quantitative Aptitude by
Arun Sharma)