Lecture Notes For Set Theory, Mensuration & Logarithms

Intro To Logs:

Questions based on this chapter are not so frequent in aptitude exams. You will find some questions based on logs, to solve those questions you have to learn some basic formulae.

Definition of "log":

Let 'a' be a positive real number and $a^b = $ c. then 'b' is called the logarithm of 'c' to the base 'a' and written as $log_a c$ and vice versa, if $log_a c = $ b, then $a^b = $ c.

NOTE: Log of a negative base is not defined.

$log_a c = $ b is possible if and only if a>0 and c>0.

Formulae for log:

1. $log_b a + log_b c = log_b(a \times c)$
2. $log_b a - log_b c = log_b \frac{a}{c}$
3. $log_a 1 = 0$ for all a > 0
4. $log_a a = 1$ for all a > 0
5. $log_c a^b = $ b $log_c a$

Base change rule:

Till now all the formulae are in logarithm with the same base. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different bases. Those situations are:

1. $log_y x = log_z x / log_z y$
2. $log_y x = log_x x / log_x y = 1 / log_x y$
3. $log_{(y^z)} x = $ (1/z) $log_y x$

Logs Problem Solving

Problem 1:

$log_3 x = log_{12} y$ = a, where x,y are real positive numbers. If G is the geometric mean of x and y. What is the value of $log_6 G$?

Solution:

From the statement, $log_3 x = log_{12} y$ = a, we have

$log_3 x$ = a and $log_{12} y$ = a

By definition of the log;

$log_3 x$ = a, x = 3^a and $log_{12} y$ = a, y = 12^a

G is the geometric mean of x and y. So, G = $\sqrt{xy}$

G = $\sqrt{3^a.12^a}$ = $\sqrt{36^a}$ = 6^a

Now; $log_6 G = log_6 6^a$ = a $log_6 6$ = a

Hence, $log_6 G$ = a.

Problem 2:

X is a real number such that $log_3 5 = log_5 (2+x)$, which of the following is true.

a. 0<x<3
b. 23<x<30
c. x>30
d. 3<x<23

Solution:

Given, $log_3 5 = log_5 (2+x)$ ……….(1)

We know;

$log_3 3 = 1$, $log_3 9 = 2$

So, we can conclude that the value of $log_3 5$ lies between 1 and 2.

Hence, $log_3 5 = 1.46$

So, from eq(1)

$log_5 (2+x) = 1.46$ …………(2)

Now, $log_5 (2+x)$

If x = 2. Then, $log_5 (2+5) = log_5 5 = 1$

If x = 23. Then, $log_5 (2+23) = log_5 25 = 2$.

But from eq(2) it is clear that $log_5 (2+5)$, can not be 2. Hence, x should be greater than and less than 23.

Hence, option (d) is the answer.

Problem 3:

$\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, which of the following is correct;

a. xyz = 1
b. $x^a y^b z^c = 1$
c. $x^{b+c} y^{c+a} z^{a+b} = 1$
d. All are correct

Solution:

Let $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = \text{k}$

$\frac{\log x}{b-c} = \text{k}$, $\frac{\log y}{c-a} = \text{k}$ and $\frac{\log z}{a-b} = \text{k}$

$\log x = \text{k(b-c)}$ and $\log y = \text{k(c-a)}$ and $\log z = \text{k(a-b)}$ and

x = $10^{k(b-c)}$ y = $10^{k(c-a)}$ z = $10^{k(a-b)}$

Now,

xyz = $10^{k(b-c)} \times 10^{k(c-a)} \times 10^{k(a-b)}$

xyz = $10^{k(b-c+c-a+a-b)} = 10^0 = 1$

Hence, xyz = 1.

Problem 4:

$\frac{1}{\log_1 n} + \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + ... + \frac{1}{\log_{43} n} = ?$

Solution:

Use base change rule:

$\log_n 1 + \log_n 2 + \log_n 3 + ... + \log_n 43 = \log_n(1.2.3.....43)$

$= \log_n 43!$

Hence, $\frac{1}{\log_1 n} + \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + ... + \frac{1}{\log_{43} n} = \log_n 43!$

Problem 5:

Find the minimum value of 2 $\log_{10} x$ - $\log_x(0.01)$, if x>1.

Solution:

Given statement; 2 $\log_{10} x$ - $\log_x(0.01)$

$\log_x(0.01) = \log_x 10^{-2} = -2 \log_x 10 = -2/\log_{10} x$ (using base change rule)

Now given statement becomes 2 $\log_{10} x + 2/\log_{10} x = 2(\log_{10} x + 1/\log_{10} x)$

Since, x>1, we can conclude that the minimum value of this expression would come when x = 10.

$2(\log_{10}10 + 1/\log_{10}10\ ) = 2(1+1) = 4.$
If we try any value of x other than 10, we will always get a value greater than 4.

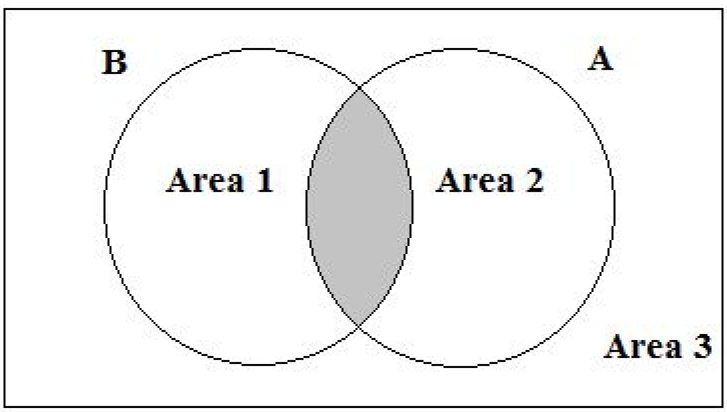
Set theory

Set theory is important both from a mathematical point of view as well as a reasoning point of view. You will see a lot of questions based on set theory in a lot of aptitude exams. Set theory questions have two ways of solving.

1. Formula approach.
2. Venn Diagram approach.

Two attributes situation:

Let's have a situation where two attributes A and B. A refers to those people who passed Physics and B refers to those people who passed Chemistry.



The rectangular box represents a universal set.
Area 1: People who passed only Physics.
Area 2: People who passed only Chemistry.
Area 3: People who passed neither Physics nor Chemistry.

Formula: $A \cup B = A + B - A \cap B$.

Problem 1:
In a school of 350 students, 100 are in the Band, 200 are in the Sports team and 50 are in both Band and Sports team.

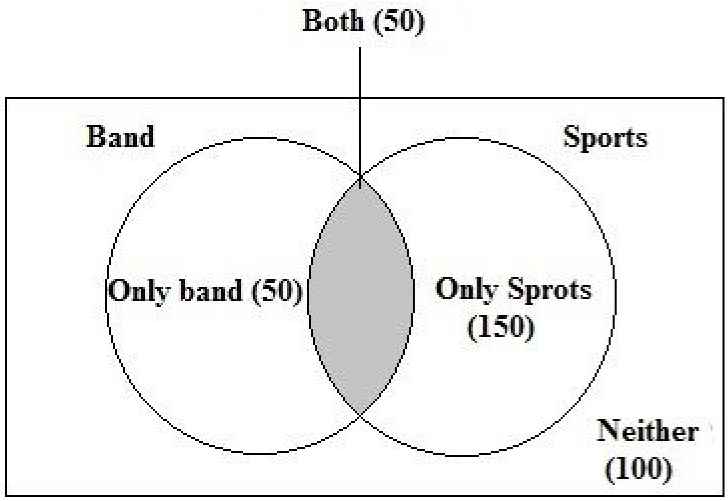
1. How many students are involved neither in Band nor in Sports?

2. How many people involved at least one of the two?
3. What is the ratio of people who participate only in the band to only in sports?

Solution:

50 students are in both Band and Sports. So, 100 - 50 = 50 students are in Band only and 200 - 50 = 150 students are in Sports only.

Total students 350 and 350 - 250 = 100 students are neither in Band nor in Sports.



1. Students are involved neither in Band nor in Sports = 100.
2. Students involved at least one of the two = 50+50+150 = 250.
3. Students only in Band = 50 and students only in Sports = 150
   Hence, the Ratio of students only in the band to only in sports = 50:150 = 1:3.

Problem 2:

There are 60 students in a class, 60% fail in English and 30% pass in Maths and 20% pass in both English and Maths. How many students fail in either of 2 subjects or at least in one subject?

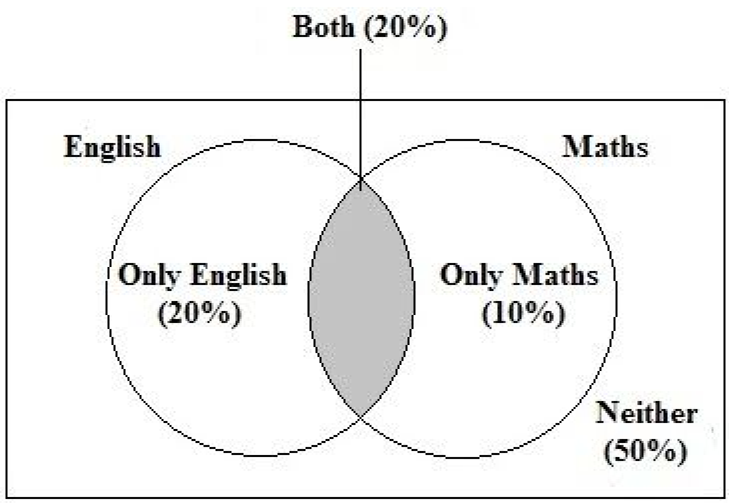
Solution:

20% of students pass in both English and Maths. So, 30% - 20% = 10% of students pass in maths only and 60% fail in english means 40% pass in english and 40% - 20% = 20% of students pass in English only.

Total students 100% and 100 - 50 = 50% of students neither pass in english nor pass in maths.

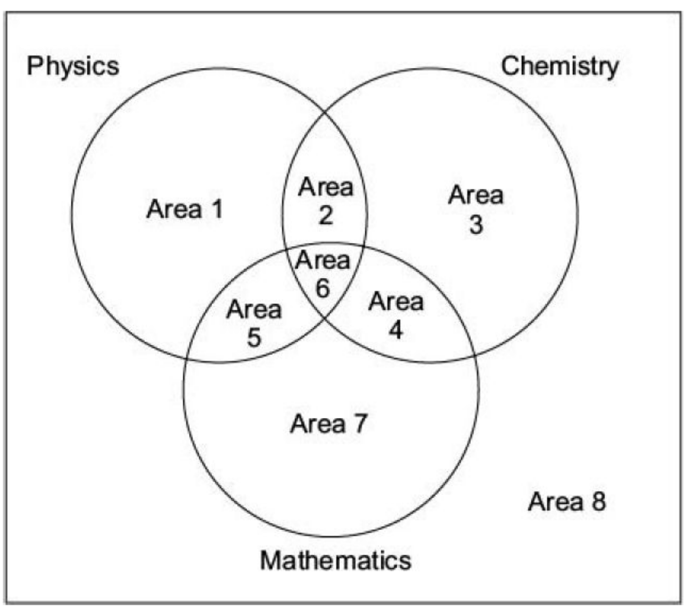
Number of students fail in either of two subjects = 20% + 10% = 30% i.e 30% of 60 = 18 students.

Number of students fail in at least one subject = 20 + 10 + 50 = 80% i.e 80% of 60 = 48 students.



Three attributes situation:

Let's have a situation where there are three attributes being measured. Suppose we are talking about people who passed Physics, Chemistry and Mathematics.



Area 1: People who passed in Physics only
Area 2: People who passed Physics and Chemistry but not Maths.
Area 3: People who passed Chemistry only

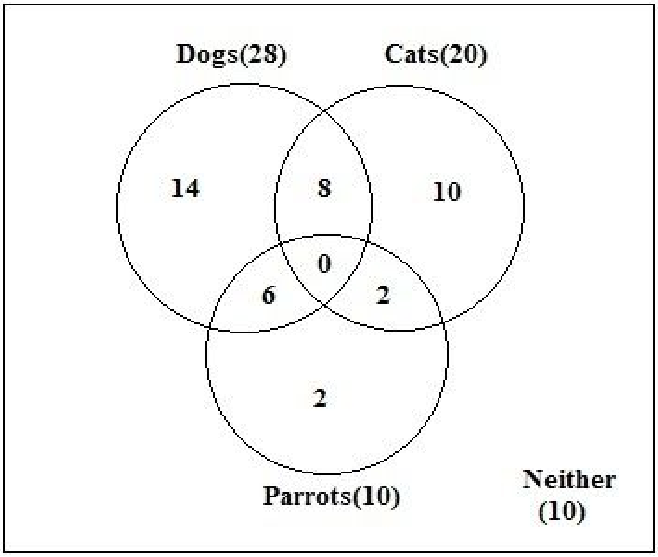
Area 4: People who passed Chemistry and Maths but not physics.
Area 5: People who passed Physics and Maths but not in Chemistry.
Area 6: People who passed Physics, Chemistry and Maths
Area 7: People who passed Maths only
Area 8: People who passed in no subjects.

People passing Physics and Chemistry: Represented by the sum of areas 2 and 6
People passing Physics and Maths: Represented by the sum of areas 5 and 6
People passing Chemistry and Maths: Represented by the sum of areas 4 and 6
People passing Physics: Represented by the sum of the areas 1, 2, 5 and 6
People passing at least 2 subjects = area 6 + area 2/4/5
People passing exactly 2 subjects: represented by area 2,4 and 5.

Problem 1:
A veterinary doctor surveyed 52 people. He discovered that 28 have dogs, 20 have cats and 10 have parrots, 8 have dogs and cats, 6 have dogs and parrots and 2 have cats and parrots. No one has all three pets.

1. How many people have only a dog?
2. How many people have at least 2 pets among dogs, cats and parrots?
3. How many people have none of the 3 pets?

Solution:
8 people have dogs and cats, 6 people have dogs and parrots. 28 - (8+6) = 14 people have only dogs.
8 people have dogs and cats, 2 people have cats and parrots. 20 - (8+2) = 10 people have only cats.
6 people have dogs and parrots, 2 people have cats and parrots. 10 - (6+2) = 2 people have only parrots.



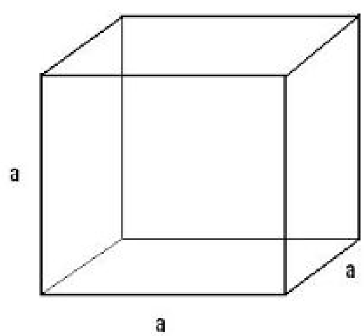
1. People have only a dog = 14.
2. People have at least 2 pets = 6+8+2= 16.
3. People have none of the 3 pets = 10.

Cubes And Cuboids

Cubes and cuboids are a chapter of mensuration. Mensuration is a measurement of 2-D and 3-D figures. Cubes and cuboids are 3-D shapes which consist of 6 faces, 8 vertices and twelve edges.

Cube:

Cube is a 3-D shape. It consists of 6 faces, 8 vertices and twelve edges. All faces of the cube are square-shaped and have equal dimensions.

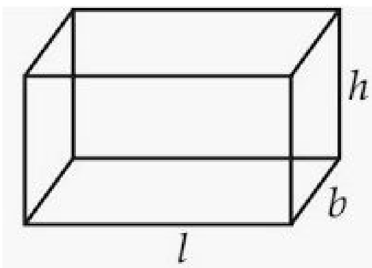


Some formulae of cube:

1. The total surface area of cube = $6a^2$
2. The volume of cube = a^3
3. Length of diagonal = $\sqrt{3}a$
4. Perimeter of cube = 12a

Cuboid:

The cuboid is also a 3-D shape. It consists of 6 faces, 8 vertices and twelve edges. All faces are not equal in dimensions.



Some formulae of cuboid:

1. The total surface area of cuboid = $2[\, l \times b + b \times h + l \times h \,]$
2. The volume of cube = $l \times b \times h$
3. Length of diagonal = $\sqrt{l^2 \times b^2 \times h^2}$
4. The perimeter of cube = $4[\, l + b + h \,]$

Problem 1:

The surface area of a cube is 216 cm^2 . Find its volume?

Solution:

The surface area of cube = 216

$$6a^2 = 216$$

$$a = 6.$$

So, the volume of the cube = a^3 = 6^3 = 216 cm^3 .

Problem 2:

3 cubes of volume 1 cm^3 , 8 cm^3 and 27 cm^3 . These are melted to form a new cube. Find the side of a new cube.

Solution:

Volume of a new cube = sum of volume of 3 cubes

$$= 1+8+27 = 36\,cm^3$$

Volume of cube = $a^3 = 36$
Hence, side of new cube = $\sqrt[3]{36}\ cm$

Problem 3:
A room with sides $6\,cm$, $4\,cm$ and $3\,cm$. What is the longest length of stick that can placedt in this room?

Solution:
Length (l)= 6 cm
Breadth (b)= 4cm
Height (h)= 3 cm
The longest stick that can be placed will be along the diagonal of the room.

Length of the Diagonal $= \sqrt{l^2 \times b^2 \times h^2}$

$= \sqrt{6^2 \times 4^2 \times 3^2} = \sqrt{36 \times 16 \times 9} = \sqrt{61}$ cm
Hence, the length of the longest stick that can be placed = $\sqrt{61}$ cm

Problem 4:
4 equal squares are placed side by side in a row. Find the ratio of the total surface area of the resulting cuboid and to the sum of the surface area of all cubes.

Solution:
Let the side of a cube = a
Total surface area of one cube = $6a^2$
Total surface area of 4 cubes = $4 \times 6a^2 = 24a^2$

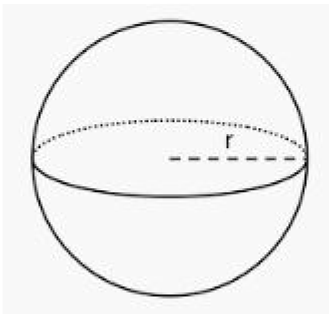
If 3 cubes are placed side by side, dimension of resulting cuboid is
length = 4a
breadth = a
height = a
Total surface area of cuboid =2[$l \times b + b \times h + l \times h$]
$= 2(4a \times a + a \times a + a \times 4a)$
$= 2(4a^2 + a^2 + 4a^2)$
$= 2 \times 9a^2$
$= 18a^2$
Ratio = $\frac{total\ surface\ area\ of\ cuboid}{total\ surface\ area\ of\ cubes} = 18/24 = 3:4$

Hence, the ratio is 3:4.

Sphere And Cylinder

Sphere:

A sphere is solid as a ball with radius ‘r’.



Basic measurement in sphere:

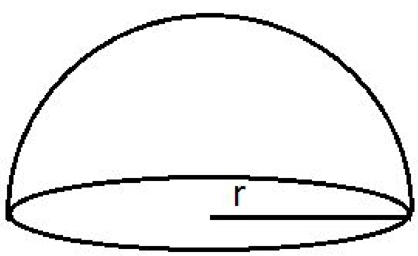
1. Radius of sphere.
2. The surface area of the sphere.
3. The volume of the sphere.

Formulae:

1. Surface area of Sphere = $4\pi r^2$
2. Volume of Sphere = $4/3(\pi r^3)$

Hemisphere:

When a plane cuts across the sphere at the centre then it forms a hemisphere.

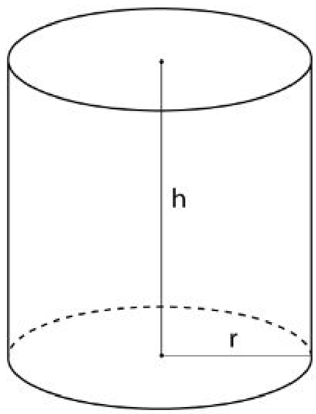


Formulae:

1. Surface area of hemisphere = $2\pi r^2$
2. Surface area of top = πr^2
3. Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Cylinder:

A cylinder is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base ‘r’ and the height ‘h’.



Formulae:

1. Volume of cylinder = $\pi r^2 h$
2. Total surface area of cylinder = $2\pi r h + 2\pi r^2 = 2\pi r$ (r+h)
3. The curved surface area of cylinder = $2\pi r h$

Problem 1:

From a sphere of radius 2 cm, how many spheres of radius 0.2 can be made?

Solution:

Volume of original sphere = $4/3(\pi r^3)$

$= 4/3(\pi 2^3) = 32/3(\pi)\ cm^3$

Volume of new sphere = $4/3(\pi r^3)$

$= 4/3(\pi (0.2)^3) = 0.032/3(\pi)\ cm^3$

Number of sphere can be formed = $\frac{volume\ of\ original\ sphere}{volume\ of\ new\ sphere}$

$= \frac{32/3(\pi)}{0.032/3(\pi)} = 1000$ spheres.

Problem 2:

Sphere has the same volume as the cylinder of height 10cm and radius 4 cm. Find the radius of the sphere.

Solution:

Volume of the sphere = $4/3(\pi\ r^3)$
Volume of the cylinder = $\pi\ r^2 h$

$= \pi\ 4^2 \times 10 = 160\pi$

According to question;
Volume of sphere = volume of cylinder

$4/3(\pi\ r^3) = 160\pi$

$r^3 = 120$

Hence, radius of sphere = $\sqrt[3]{120}$ cm.

Problem 3:
Two right circular cylinders have equal volume and their height are in ratio 1:2. What is the ratio of their radii?

Solution:
Let r_1 and r_2 be the radii of two cylinders. And h_1 and $2h_1$ be their heights.(ratio of height is 1:2 given)
Volume of both the cylinders are equal

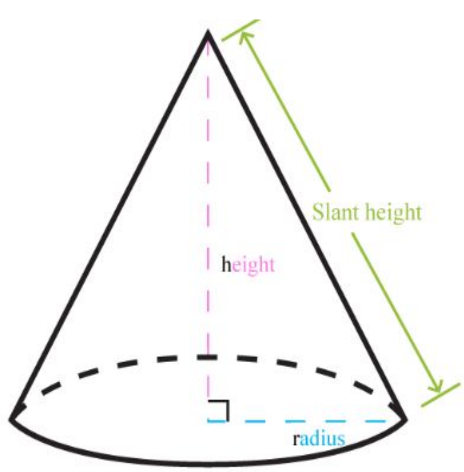
$\pi\ r_1^{\ 2} h_1 = \pi\ r_2^{\ 2} \times 2h_1$

$r_1^{\ 2} / r_2^{\ 2} = 2$

$r_1 : r_2 = \sqrt{2} : 1$

Cones, Prisms and Pyramids

Cone:
Cone is an object of circular base and its lateral sides converse to a single point at the top.



Measurement in cone:

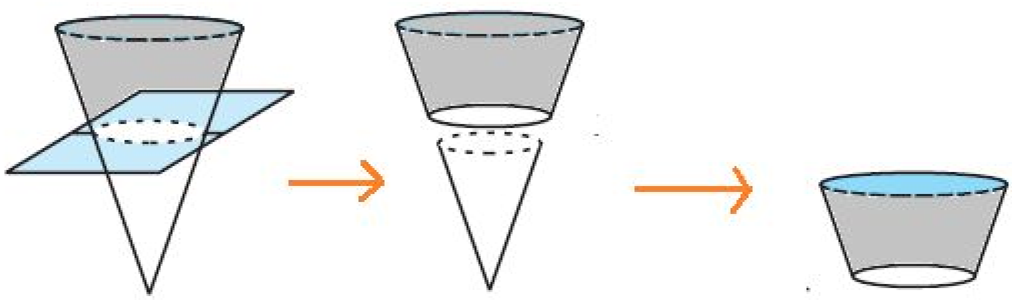
1. The diameter of the base.
2. Height of the cone.
3. Slant height of the cone.
4. Volume of the cone.
5. Curved surface area of cone.
6. Total surface area of cones.

Formulae:

1. Curved surface area of cone = πrl
2. Total surface area of cone = $\pi rl + \pi r^2 = \pi r(r + l)$
3. Volume of cone = $1/3(\pi r^2 h)$

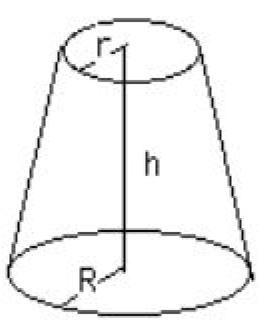
Frustum of a cone:

When a cone is cut by a parallel plane is called the frustum of a cone.



A cone sliced by a plane parallel to base **The two parts separated** **Frustum of a cone**

Formulae:



1. Slant surface of the frustum of a cone = π (r + R)l; where l is the slant height.
2. Volume of the frustum of a cone = $\frac{1}{3}\pi h(r^2 + rR + R^2)$

Similarity concept:

Cone on top which we will cut out and the cone which was originally, because of their angle, they are similar to each other.

Let say we cut the cone from half of the height i.e. h/2
In the similarity concept, all the ratios have to be similarly maintained. This tells us the two cones are similar and hence,

1. Area is **"Square of the length"**.
2. Volume is **"cube of the length"**.

***For example*:**
When we cut a cone from between.
Let the surface area of the original cone is 'x'.
Hence, the surface area of cut out = Area × square of the length

$= x \times (1/2)^2 = (1/4)x.$

Now, slant area of frustum = **surface area of the original cone - surface area of cut out**

$= x - (1/4)x = (3/4)x \ cm^2$

Similarly, volume of frustum = **volume of original cone - volume of cut out**

Prism:

A prism is solid and has the same geometrical shape as polygon at both its ends. Its dimensions are defined by the dimensions of the polygon at its ends and its height.

1. Lateral surface area of a prism = Perimeter of base × height
2. Volume of a prism = area of base × height

Pyramids:

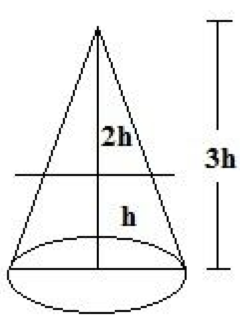
A pyramid is a solid which can have any polygon as its base and its outer surfaces are triangular and converge to a single point at the top.

1. Slant surface of a pyramid = 1/2 × Perimeter of the base × slant height
2. Whole surface of a pyramid = Slant surface + area of the base
3. Volume of a pyramid = $(\frac{Area\ of\ base}{3}) \times height$

Problem On Cones And Pyramids

Problem 1:
A cone is cut at 1/3rd of height from the base to make a frustum. What is the ratio of original cone to the frustum of the cone?

Solution:
Let take the original height = 3h and the cone cut 1/3rd from the bottom.
The ratio of the height of the original cone and the cut out cone = 3:2



All length ration should be 3:2. Hence, by similarity concept;
The ratio of the volume of the original cone to cut out cone = $3^3 : 2^3 = 27:8$

Let the volume of the original cone = 27x and volume of the cutout cone = 8x
Hence, volume of the frustum of cone = 27x - 8x = 19x.
Therefore, the ratio of the original cone volume to the frustum of cone = 27:19.

Problem 2:
The radius of a cone is 7 cm, slant height is 25 cm. Find the volume of the cone.

Solution:
The volume of the cone = $1/3(\pi r^2 h)$
We have, r = 7cm and l = 25 cm.
Height of the cone 'h' = $\sqrt{l^2 + r^2}$

$$h = \sqrt{25^2 + 7^2} = 24 \text{ cm}$$

Hence, volume of the cone = $1/3(\pi 7^2 24) = 392\,\pi\, cm^3$

Problem 3:

Height and radius of the conical vessel is ‘h’ and ‘r’ respectively. Vessel has a capacity of 10 litres of water. What is the capacity of a cylinder having the same height ‘h’ and radius ‘r’?

Solution:

Volume of the cone = $1/3(\pi r^2 h)$

Volume of the cylinder = $\pi r^2 h$

Given, the volume of the cone = 10

$$1/3(\pi r^2 h) = 10$$

$$\pi r^2 h = 30$$

Hence, volume of a cylinder = 30 cm^3

Problem 4:

A map of a country of height 6 feet requires 10 litres of paints. How much paint would be required for a map of the same country of height 12 feet?

Solution:

Heights are in the ratio of 6:12 or 1:2

Therefore area will be in the ratio of 1:4

Therefore, to paint a map of 12 feet height $= 4 \times 10 = 40$ litre

Problem 5:

A circular iron ball of radius 1 m and weight 20 kg. Find the weight of another iron ball of radius 3 m.

Solution:

Weight of a ball is a function of volume.

Radius is in the ratio of 1:3

Area will be in the ratio of 1:9

Therefore, volume in the ratio = 1:27

Hence, weight of iron ball of radius 3m $= 20 \times 27 = 540$ kg.

Some questions for practice

---

1. Calculate: log2 (2/3) + log4 (9/4) = log2
   Ans : 0.

2. If log102 = 0.301 find log10125.
   Ans: 2.097.

3. If log10a = b, find the value of 103b in terms of a.
   Ans: a^3 **.**

4. In the Fun club, all the members participate either in the Tambola or the Fete. 320 participate in the Fete, 350 participate in the Tambola and 220 participate in both. How many members does the club have?
   Ans: 450.

5. A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.
   Ans: 266.11 cm^2

(Ref: Quantitative Aptitude by
Arun Sharma)