Lecture Notes For Alligations

Concept of alligation is closely related to the weighted average.
Alligations is a visual approach to solve weighted averages, involving the mixing of two groups.

For example:
Two varieties of rice at 50 per kg and 80 per kg are mixed together in the ratio 3 : 7. Find the average price of the resulting mixture.
Solution : By using weighted average formula; $A_w = (n_1A_1 + n_2A_2) / (n_1+n_2)$

$$\text{Average price} = (3 \times 50 + 7 \times 80) / (3 + 7)$$
$$= 710 / 10$$
$$= 70.$$

Weighted average approach is slightly slower than, if we see the same situation through alligations. Alligations is a faster approach.

Mathematical formula for alligation:
In the case of a situation where just two groups are being mixed, we can write weighted average formula:

$$A_w = (n_1A_1 + n_2A_2) / (n_1 + n_2)$$

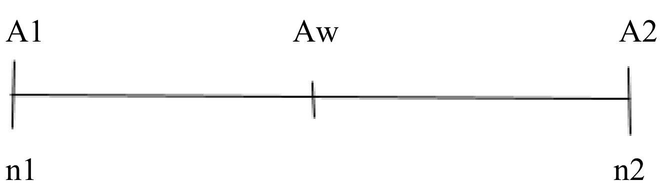
Here, we have 2 groups with averages A1, A2 and having n1 and n2 elements respectively.
Rewriting this equation we get:

$$(n_1 + n_2) A_w = n_1A_1 + n_2A_2$$
$$n_1(A_w - A_1) = n_2 (A_2 - A_w) \text{ or}$$
$$n_1/n_2 = (A_2 - A_w)/(A_w - A_1) \text{ ……….. The alligation equation.}$$

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages,
we get $A_1 < A_w < A_2$.

Straight line approach:

Positions of A1, A2, Aw, n1 and n2 on number line are;

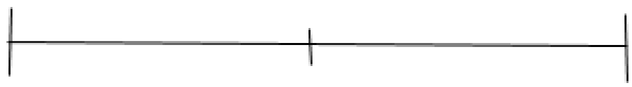


On the number line the points A1, Aw and A2 are in order from left to right because we have the condition: A1 < Aw < A2

Now, according to the given alligation equation; n1/n2 = (A2 – Aw)/(Aw – A1):
(a) n2 is responsible for the distance between A1 and Aw or n2 corresponds to Aw – A1
(b) n1 is responsible for the distance between Aw and A2. or n1 corresponds to A2 – Aw
(c) (n1 + n2) is responsible for the distance between A1 and A2. or (n1 + n2) corresponds to A2 – A1.

Solving the above example of rice mixing by this approach;

50 Aw 80

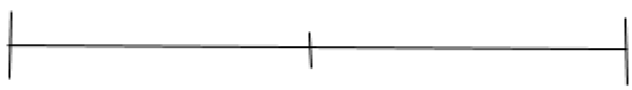


3 7

Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7 parts are 9 and 21 respectively.
As, the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are 21. So;
I.e. Aw - 50 = 21 ≫ Aw = 71.
OR the distance between 50 and Aw is corresponding to n1(i.e. 3) and 3 parts are 9. So;
80 - Aw = 9 i.e . Aw = 71.

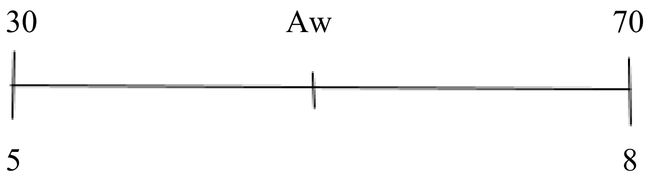
For practice, try yourself:

30 Aw 70



n1	n2	**Answers**
1	3	60
3	1	40
3	5	55
5	3	45
7	1	35

By this approach you can handle fraction situations too:
For example;



Since the total distance = (70 - 30) = 40. If we split 40 into 5:8,

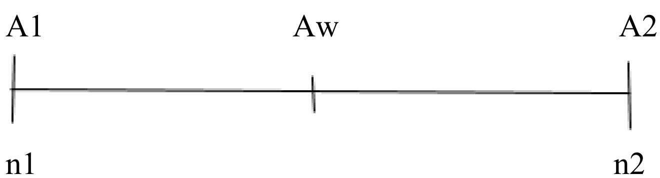
Aw - 30 = (8/13) × 40

Aw - 30 = 24.62;

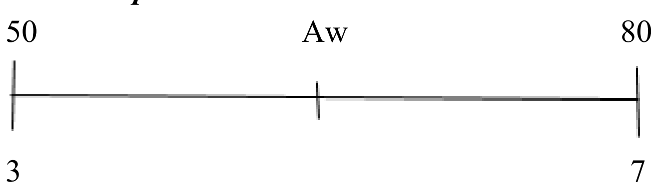
So, Aw = 54.62.

Alligation has essential three situations:

Situation 1: When A1, A2, n1 and n2 are known and Aw is unknown.



For example:

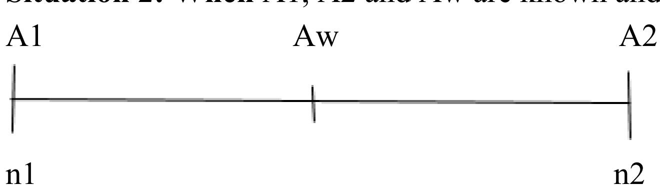


Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7parts are 9 and 21 respectively.

Thus the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are equal to 21.

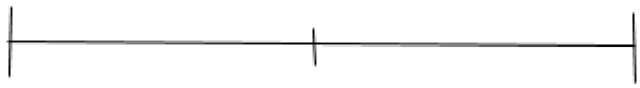
I.e. Aw - 50 = 21 ≫ Aw = 71.

Situation 2: When A1, A2 and Aw are known and n1: n2 is unknown.



For example:

70 80 84



n1 n2

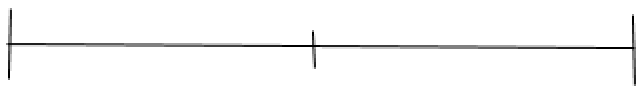
By using alligation equation,

n1/n2 = (A2 – Aw)/(Aw – A1)

n1:n2 = 4:10 or 2:5.

Situation 3: When A1, Aw and n1:n2 are known and A2 is unknown.

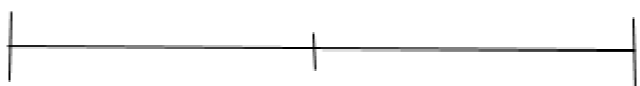
A1 Aw A2



n1 n2

For example:

30 70 A2



3 5

By using alligation equation,

n1/n2 = (A2 – Aw)/(Aw – A1)

3/5 = (A2 - 70)/(70-30)

A2 = 94.

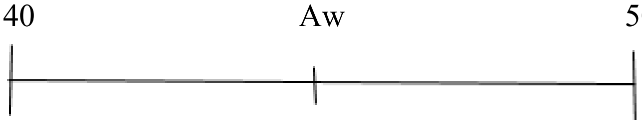
Problems Where You Can Use Alligations-1

Example 1:

Two varieties of rice at 40 per kg and 50 per kg are mixed together in the ratio 2 : 3. Find the average price of the resulting mixture.

Solution :

40 Aw 50

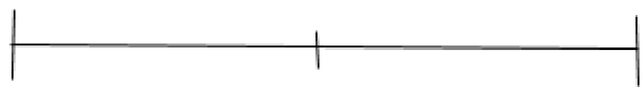


2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2 (i.e. 3) and 3 parts are equal to 6.
I.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average price of the resulting mixture is at 46 per kg.

Example 2:
A man has driven a car at 40kmph and 50kmph. He has driven for 2 hours and 3 hours respectively. Find the average speed of a car?
Solution :

40 Aw 50



2 3

Here, Aw is the average speed of the car.
Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average speed of the car is 46kmph.

These two questions are on the surface different from each other, the first one was talking about average price and the other is talking about the average speed, But structurally both are the same.
Equation in 1st question ;

Average price = (n1A1 +n2A2) / (n1 + n2).

Here, n1= 2kg, n2= 3kg , A1= 40per kg, A2= 50 per kg.
So,

Average price = (2*40 + 3*50)/(2+3)

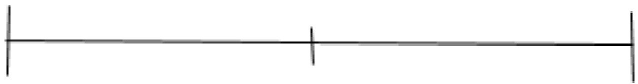
Equation in 2nd question ;

Average speed = (t1S1 +t2S2) / (t1 + t2).

Here t1= 2hr, t2= 3hr , S1= 40kmph, S2= 50kmph.
So,

Average speed= (2*40 + 3*50)/(2+3)

By looking at these two equations you will observe that these both are the same, only difference is in variables.

Example 3:
Class1 has 20 students having average marks of 60 and class2 has 30 students having average marks of 70. Find the average marks of two classes combined ?
Solution :
60 Aw 70

20 30
In weighted average and in alligation we take ratio of the quantities. So, n1 : n2 is 2 : 3.
Since the total distance = (70 - 60) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 60 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw - 60 = 6
≫ Aw = 66.
Hence average marks of two classes is 66.

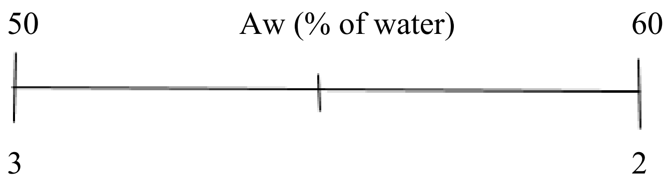
Problems Where You Can Use Alligations-2

Example 1:
We have two mixtures of milk and water, the 1st mixture contains 40% milk & 60% water and the 2nd mixture contains 50% milk & 50% water. These two mixtures are mixed in ratio 2:3, then find the % of milk in the mixture?

Solution : Using milk %
40 Aw (% of milk) 50

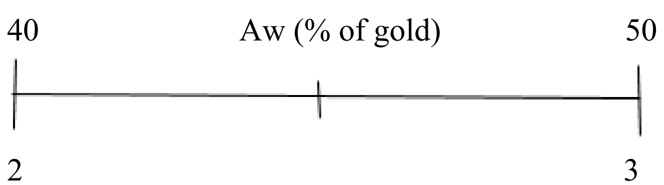
2 3
Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw (% of milk) = 46%.
Another way to solve this question is by using water %
The 1st mixture has 60% water and the 2nd mixture has 50% water.

According to convention, we need A1< Aw < A2 and the ratio of 1st mixture to 2nd mixture is 2:3, this will be inverted here because we have to flip the % here to make it according to the given convention.



Since the total distance = (60 - 50) = 10. If we split 10 into 3:2, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 50 is corresponding to n2(i.e. 2) and 2 parts are equal to 4.
i.e. Aw-50 = 4
≫ Aw (% of water) = 54%.
Thus; % of milk = 100 - 54 = 46%.

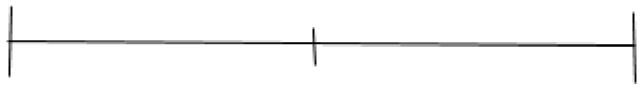
Example 2:
Anjali mixes 2 alloys of gold and copper in ratio 2:3. The 1st alloy contains 40% gold and the 2nd alloy contains 50% gold. Find the gold % in the mixture?
Solution :



Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw (% of gold) = 46%.
Another way to solve this question is by using copper %.

Example 3:
Two alloys of gold and copper mix in ratio 2:3. The 1st alloy contains gold and copper in ratio 2:3 and 2nd alloy contains gold and copper in ratio 1:1. What is the ratio of gold and copper in the final mixture?
Solution :
Ratio 2:3 means 40% gold and 60% copper. & ratio 1:1 means 50% gold and 50% copper.

40 Aw (% of gold) 50



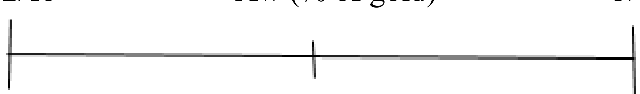
2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6..
I.e. Aw-40 = 6
≫ Aw (% of gold) = 46%. So, % of copper = 54%
Hence, the ratio of gold and copper in the mixture is 46:54 i.e. 23:27.

The ratio in a particular question can be converted into percentage composition only if the percentages are easy to get.

Example 4:
Two varieties of gold and copper alloy mixed in ratio 3:5. The 1st alloy contains gold and copper in ratio 3:8 & 2nd alloy contains gold and copper in ratio 2:13. What is the final ratio of gold and copper in the mixture?
Solution :
In this situation if you try to put this into alligation, the numbers will not support you because the ratio 2:13 gets converted into % is difficult and putting on a number line is also difficult.
If you still want to do this question through alligation, you will do it. The thought is that the fraction of gold in 1st alloy is 3/11 and in 2nd alloy is 2/15.
Since, 2/15< 3/8 , so the ratio will flip.

2/15 Aw (% of gold) 3/8



5 3

Finding the 3 parts and 5parts of total distance is not going to be a very easy calculation and hence alligation in this situation structurally does apply but it's not a good approach for such a type of question.

So, what you have to do in this type of question;

Alloy 1		**Alloy 2**
3	:	5

G:C	**G:C**
3:8	2:13

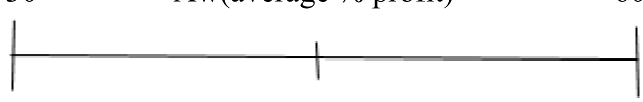
1st you have to take LCM of (3+8) = 11 and (2+13)= 15. Thus , LCM (11&15) = 165.
This means 165 kg of 1st alloy and 165 kg of 2nd alloy in 1 part.

Alloy 1		**Alloy 2**
3	:	5
165 × 3= 495	:	165 × 5= 825
G:C		G:C
3:8		2:13

You have 3:8 divisions of 495kg & 2:13 divisions of 825kg. So,
Total gold in the mixture = (3/11) × 495 + (2/15) × 825
= 135 + 110 = 235.
Total mixture = 495 +825 = 1320.
Total copper in the mixture = 1320 - 235 = 1075.
Final ratio of gold and copper in the mixture will be 235:1075 i.e. 49:215.

Problems Where You Can Use Alligations-3

Example 1:
A shopkeeper sold chairs and tables. The ratio of cost price of chair and table is 1:2. He sold chairs at 30% profit and tables at 60% profit. What is the average % profit?
Solution :
30 Aw(average % profit) 60



1 2

Since the total distance = (60 - 30) = 30. If we split 30 into 1 : 2, the value of 1 part and 2 parts are 10 and 20 respectively.
Thus the distance between Aw and 30 is corresponding to n2(i.e. 2) and 2 parts are equal to 20.
I.e. Aw-30 = 20
≫ Aw (average % profit) = 50%.

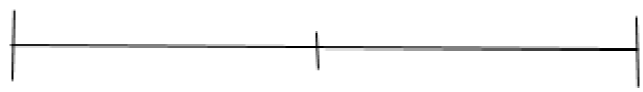
Example 2:

A shopkeeper sold chairs and tables. He sold tables at 20% profit and chairs at 30% loss. Thereby he made no profit or no loss in the transaction. What is the cost price ratio of table to chair?

Solution :

Chair **Table**

-30 0 20



n1 n2

n1/n2 = (A2 – Aw)/(Aw – A1)

Here A1= -30, A2= 20, Aw= 0.

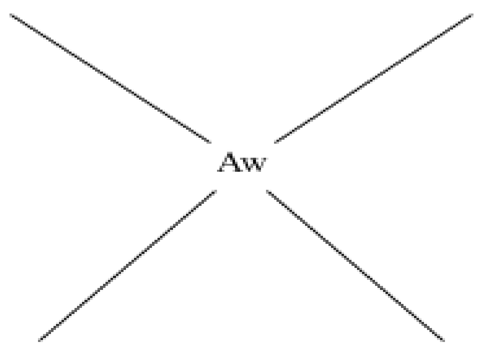
n1/n2 = (20 – 0)/(0 – (-30))

n1/n2 = 20/30 i.e. n1:n2 = 2:3.

Thus, table to chair cost price ratio = 3:2.

Cross diagram approach:

A1 A2



A2-Aw : Aw-A1

n1 : n2

Note : That the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.

As we have seen, there are five variables in the alligation equation.

The three averages A1,A2 and Aw. and the two weights n1 and n2.

Example 1:

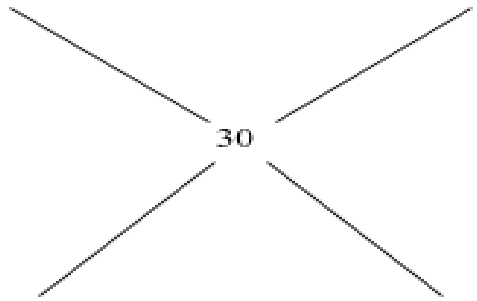
On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) The ratio of students in the classes

(b) The number of students in the first class if the second class had 30 students.

Solution :

25 40



A2-Aw : Aw-A1

10 : 5

(a) The ratio of students in class is 10:5 i.e 2:1.

(b) If the ratio is 2 : 1 and the second class has 30 students, then the first class will have 60 students.

NOTE: 1. A1, A2 and Aw are always rate units, while n1 and n2 are quantity units.

2. All percentage values represent the average values.

Some questions for practice:

---

1. If 5 kg of salt costing 5/kg and 3 kg of salt costing 4/kg are mixed, find the average cost of the mixture per kilogram.
   Ans:4.625kg.
2. Two types of oils having the rates of 4/kg and 5/ kg respectively are mixed in order to produce a mixture having the rate of 4.60/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?
   Ans:60kg.
3. How many kilograms of sugar worth 3.60 per kg should be mixed with 8 kg of sugar worth 4.20 per kg, such that by selling the mixture at 4.40 per kg, there may be a gain of 10%?
   Ans:4kg.
4. Ravi lends 3600 on simple interest to Harsh for a period of 5 years. He lends a part of the amount at 4% interest and the rest at 6% and receives 960 as the amount of interest. How much money did he lend on a 4% interest rate?
   Ans:1200.
5. 400 students took a mock exam in Delhi. 60% of the boys and 80% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?

Ans:100 girls.