Lecture Notes For Time, Speed & Distance

Introduction

Time, Speed and Distance is an important chapter for the purpose of the Maths section in aptitude exams. The basic concepts of Time, Speed and Distance are used in solving questions based on straight-line motion, relative motion, circular motion, problems based on trains, problems based on boats, clocks, races, etc.

Time, Speed and Distance is a situation related to the motion of a body. If a person is moving from point 'x' to point 'y', this journey is described by three variables and every Time, Speed and Distance question has only 3 variables in it (time, speed and distance).

Time, Speed & Distance formula :

(a) Distance = Speed × Time
(b) Time = Distance/Speed
(c) Speed = Distance/Time

Units:

Speed: m/sec, km/hr and in some case, you will see km/min, m/min, feet/sec and feet/hr.
Time: min, hour and sec
Distance: km, meter and miles
Whenever you will use Speed × Time = Distance formula, units of all three Time, Speed and Distance should be consistent with each other, which means if speed is in kmph(km/hr), you can't take time in sec or min, time will have to be in "hour" and distance will have to be in "km".

Conversion:

1 km = 1000 meters = 0.6214 mile
1 mile = 1.609 km
1 hr = 60 min = 60*60 seconds = 3600 seconds
1 km/hr = 5/18 m/s
1 m/s = 18/5 km/hr
1 km/hr = 5/8 miles/hour

A car is travelling at 40 kmph from point 'x' to observer 'o' for a distance of 80 km.

40 kmph can be described as the rate at which a car is approaching the observer. So, every hour the car will keep coming 40 km closer to the observer.
If a journey is of 80km, so the car will take 2 hr to reach the observer.
Another way of looking it is;

The rate at which the car is moving away from the observer. And in this case, the car will reach the point x in 2hrs if the speed and distance are kept the same

The proportionality in the TSD equation:

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1. **s ∝ d** if time is constant.
2. **t ∝ d** if speed is constant.
3. **s ∝ 1/t** if the distance is constant.

1. s ∝ d if time is constant.

In the first proportionality, time should be constant in both motions, whether the two bodies are moving or two different journeys by the same car. After observing both the motions, if the time required is the same for both of them then, you can say that this is a constant time situation.
In time constant proportionality, if the speed increases then distance also increases in the same manner.

For example:
If train 1 starts from X and train 2 starts from Y and they start moving towards each other at the same time. They meet at a point somewhere in between.

Solution:
Let's say they start at 1 pm and meet at 3 pm.
So, here we can see that there are two motions and for these motions, the value of time is 2 hours.
Let say Sx and Dx be the speed and time respectively for train 1.
& Sy and Dy be the speed and time respectively for train 2.
In this case, the following ratio will be valid:

$$\frac{Sx}{Dx} = \frac{Sy}{Dy}$$

2. **t ∝ d if speed is constant.**

Example:

A car moves for 4 hours at a speed of 25 kmph and another car moves for 5 hours at the same speed. Find the ratio of distances covered by the two cars.

Solution:

Since the speed is constant, we can directly conclude that time ∝ distance.

Hence $\frac{Ta}{Tb} = \frac{Da}{Db}$

Since the times of travel are 2 and 3 hours respectively, the ratio of distances covered is also 4/5.

3. **s ∝ 1/t if distance is constant.**

Example:

A man goes from Delhi to Karnal and Comes back. In this case distance for Delhi to Karnal and Karnal to Delhi is the same i.e distance is constant. Hence, the speed will be inversely proportional to the time.

If the distance is constant it is also a product constancy situation ($s \times t$ = constant). Hence you can use any of the product constancy structures.

In this case, the following ration will be valid;

$\frac{Sa}{Sb} = \frac{Tb}{Ta}$

Problem Based On Proportionality:

Problem 1:

Abhishek walks at 3/4th of his normal speed and he is 16 minutes late in reaching office. Find his normal time of reaching office.

Solution:

Let S1 = s and T1= t be its normal speed and time respectively.

And S2 = $3/4 \times s$ and T2 = t+16.

Here distance is the same i.e distance constancy situation.

Speed from 's' to $3/4 \times s$ i.e. speed is reduced by 1/4th and time from 't' to t+16 i.e. time would be increased by 1/3rd as speed is reduced by ¼.

($s \times t$ = constant, is 's' reduced by 1/4 then 't' increased by 1/3)

time from ‘t’ to t+16 i.e. time is increased by 1/3rd means 1/3rd of normal time ‘t’ = 16 min
Therefore, Normal time = $16 \times 3 = 48$ min.

2nd method:
We know ratio;
$$\frac{S1}{S2} = \frac{T2}{T1}$$
$$T1 = \frac{S1}{S2} \times T2$$
$$t = \frac{3}{4} \times (t+16)$$
Therefore, the normal time ‘t’ = 48 min.

Problem 2:
Two people X and Y travelled the same distance at speeds of 6 kmph and 10 kmph respectively. If X takes 1 hour longer than Y then, what is the distance being travelled?

Solution:
Lets ‘t’ be the time taken by Y. So, time taken by X is t+1.
Speed of X = 6 kmph and speed of Y = 10 kmph.
We can solve this problem by following methods:
Method 1:
Here given that;
Difference of time = 1
d/6 - d/10 = 1
10d - 6d = 60, d = 15 km.
Therefore distance travelled = 15 km

Method 2:
Distance is constant so;
$S1 \times t1 = S2 \times t2$
6(t+1) = 10t
t = 3/2 hr
Therefore distance = speed × time
= $10 \times 3/2$ = 15 km

Problem 3:
Rohit walks at speed of 12 kmph and he reaches the railway station 10 min after the train has gone and by walking at 15 kmph, he reaches at railway station 10 min before the train has gone. Find the distance from his home to the railway station.

Solution:
Let original time of reaching = t min
We have;
$S1 = 12$ kmph and $S2 = 15$ kmph
$t1 = t+10$ min and $t2 = t - 10$ min

Method 1:
Here distance is constant so;
$S1 \times t1 = S2 \times t2$
$12(t+10) = 15(t-10)$
$15t - 12t = 120 + 150$
$t = 90$ min
Therefore distance = speed × time
= 12(90+10)/60 = 20 km

Method 2:
Difference between time = 20 min
$d/12 - d/15 = 20/60$
$5d - 4d = 20$, $d = 20$ km
Therefore distance = 20 km.

Concept Of Relative Speed:

We already discussed the movement of a body with respect to a stationary point. And now, we need to determine the movement and its relationships with respect to a moving point/body. In such situations, we have to take into account the movement of the body w.r.t. which we are trying to determine relative motion.
"Relation motion of a body is the motion of one body/point with respect to other body/point"

Case 1: Two cars C1 & C2 are moving in opposite directions. C1 moving at S1 kmph and C2 moving at S2 kmph.
So, Relative speed $S = S1+S2$.

Some problems based on Case 1:
Problem 1:
Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. After how much time they will meet?

Solution:

$S1 = 50$ kmph

$S2 = 30$ kmph

The speed with which they are approaching $S = S1+S2$

$$S = 50+30 = 80 \text{ kmph}$$

They have to approach each other and reach the meeting point.

So, approaching distance/Relative distance= 280 km

Hence, Relative Speed × Time = Relative Distance

$80 \times t = 280$

$t = 3.5$ hours.

Therefore; they will meet after 3.5 hours.

Problem 2:

Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. When will the distance between them next become 280 km?

Solution:

1st they have to approach each other and reach the meeting point.

So, approaching distance/Relative distance= 280 km

From the meeting point, they again have to separate by 280 km.

So, Separating Distance = 280 km.

Hence, net distance covered = 280+280 = 560 km

$S1 = 50$ kmph

$S2 = 30$ kmph

The speed with which they are approaching $S = S1+S2$

$$S = 50+30 = 80 \text{ kmph}$$

So, Relative Speed × Time = Relative Distance

$80 \times t = 560$

$t = 7$ hours.

Therefore, the distance between them next becomes 280 after 7 hours.

Problem 3:

Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. They start at 1 pm, after some time it was found that the distance between them was 200 km, then at what time could it be?

Solution:

$S1 = 50$ kmph

$S2 = 30$ kmph

The speed with which they are approaching $S = S1+S2$

$$S = 50+30 = 80 \text{ kmph}$$

Initial distance between them = 280 km

Final distance between them = 200 km

So, the total approaching Distance = 280 - 200 = 80 km

Relative Speed × Time = Relative Distance

$80 \times t = 80$

$t = 1$ hour.

Therefore, this situation will happen at 2 pm.

But in this question, it is not saying that they have not met.

The initial distance between them = 280 km

After the meeting point, they have to cover a distance of 200 km.

So, total distance = 280+200 = 480 km

Relative Speed × Time = Relative Distance

$80 \times t = 480$

$t = 6$ hours.

Therefore, this situation will happen at 7 pm.

Case 2: Two bodies are moving in the same direction.

So, the Relative Speed $S = S1 - S2$

Problem 1:

Two cars C1 & C2 are moving in the same direction at a speed 50 kmph and 30 kmph respectively from the same point and they start moving at 2 pm. After how many hours will C1 be 140 km ahead of C2?

Solution:

$S1 = 50$ kmph

$S2 = 30$ kmph

The Relative Speed $S = S1-S2$

$S = 50 - 30 = 20$ kmph

Relative Distance = 140 km

So, Relative Speed × Time = Relative Distance

$20 \times t = 140$

t = 7 hours.
Therefore, after 7 hours C1 ahead 140 km of C2.

Problem 2:
Two cars C1 & C2 are moving in the same direction. Car C2 going at 30 kmph and C1 catching up at 50 kmph, starting distance between them is 120 km. In how many hours does C1 catch C2?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Relative Distance = 120 km
So, Relative Speed × Time = Relative Distance
20 × t = 120
t = 6 hours.
Therefore, in 7 hours C1 catches C2.

Problem 3:
Two cars C1 & C2 are moving in the same direction. Car C2 is going at 30 kmph and C1 is catching up at 50 kmph, starting distance between them is 120 km. When will the next time they will be at 120 km distance?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Approaching distance = 120 km
Separating distance = 120km
Hence, net distance = 120+120 = 240 km.
So, Relative Speed × Time = Relative Distance
20 × t = 240
t = 12 hours.
Therefore, after 12 hours they will be at 120 km again.

Question-Based On Relative Motion:

Type 1: Policeman and theft question

Problem 1:

The theft is committed at 2 A.M and the thief after committing the theft starts escaping at a speed of 80 kmph. The theft is discovered at 6 A.M and the policeman gives pursuit of the thief at 100 kmph. Find at what time the policeman will catch the thief?

Solution:

Speed of Thief = 80 kmph
Speed of policeman = 100 kmph
According to question,
Distance between thief and policeman after 4 hours (2 A.M to 6 A.M) = $80 \times 4 = 320$ km.
Speed at which policeman approaches thief = $100 - 80 = 20$ kmph
So, Relative Speed × Time = Relative Distance
$20 \times t = 320$
$t = 16$ hours
Therefore, police caught thief at 10 P.M (6 A.M + 16 hours =10 P.M)

Problem 2:

At what distance from the original point did the thief get caught?

Solution:

To answer this question we have to find out the policeman's journey.
Speed of policeman = 100 kmph
Time taken by the policeman to catch the thief = 16 hours.
So, distance = $100 \times 16 = 1600$ km.

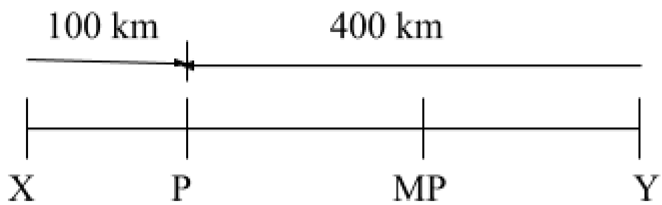
Type 2: Train question

Problem 1:

Two trains T1 and T2, T1 starting from point X to Y and T2 starting from point Y to X 2 hours later. T1 moving at 50 kmph and T2 at 30 kmph. Distance between point X and Y is 500 km. Find the distance from X, after which they will meet.

Solution:

Speed of T1 = 50 kmph
Speed of T2 = 30 kmph
Distance between X and Y = 500 km
Train T1 starts 2 hours before train T2.
Distance covered by T1 in 2 hours = 50 × 2 = 100 km
Let us say T1 reaches at point P in 2 hours.



Distance left from point P to Y = 500 - 100 = 400 km
Speed at which they approaching = 50+30 = 80 kmph
Approach required to get the meeting point (MP), the total distance they have to approach together = 400 km
So, Relative Speed × Time = Relative Distance
80 × t = 400
t = 5 hours
So, distance from P to MP = 50 × 5 = 250 km
Therefore, they will meet 350 km (100 + 250 = 350) from point X.

Concept Of Circular Motion:

The movement of an object along a circle is called circular motion. When we talk about circular motion, there are 3 variables inside the questions. 1. Speed 2. Circumference 3. Time.

Units:

Speed: m/sec, kmph or it can also be measured in %/sec,%/min and %/hr.
Circumference: meter, km or % (if circle as 100%)

Problem 1:
Three people A, B and C running around a circle, whose circumference is 100 km. Speed of A is 20 kmph and the speed of B is 15 kmph and speed of C is 12 kmph.

a. After how much time they will meet at the starting point.
b. How many rounds were done by A?
c. The time required for the first meeting at any point.

Solution:
Speed of A = 20 kmph
Speed of B = 15 kmph
Speed of C = 12 kmph
Circumference = 100 km

(a) Let Ta, Tb and Tc be the time taken by A, B and C respectively to cover the circle.
So, Ta = 100/20 = 5 hours
Tb = 100/15 = 20/3 hours
Tc = 100/12 = 25/3 hours
Time required to meet at starting point = LCM(Ta,Tb,Tc)
We know, LCM of fraction = LCM of numerator / HCF of denominator
= LCM (5,20/3,25/3) = 100/1 = 100 hours
Hence, they meet at the starting point after 100 hours.

(b) A done one round in 5 hours.
So, In 100 hours A done = 100/5 = 20 rounds.

(c) A is fastest, A would be overlapping each of B & C after some time.
Let Tab and Tac be the time in which A overlap B and C respectively.
The time required for the first meeting at any point = LCM(Tab, Tac)
Relative speed between A and B 'Sab' = 20-15 = 5 kmph
Relative speed between A and C 'Sac' = 20-12 = 8 kmph
So, Tab = 100/5 = 20 hours and Tac = 100/8 = 12.5 hours.
LCM (20,25/2) = 100 hours
Hence, they will meet at any point after 100 hours.

Some questions for practice:

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1. Ram starts walking from a place at a uniform speed of 2 km/h in a particular direction. After half an hour, Shyam starts from the same place and walks in the same direction as Ram at a uniform speed and overtakes Ram after 1 hour 48 minutes. Calculate the speed of Shyam.
   Ans: 23/9 kmph.

2. Shubham and Navneet travel the same distance at the rate of 6 km per hour and 10 km per hour respectively. If Shubham takes 30 minutes longer than Navneet, the distance travelled by each is
   Ans: 7.5 km.

3. Two trains for Kota leave Mumbai at 6: 00 a.m. and 6: 45 am and travel at 100 kmph and 136 kmph respectively. How many kilometres from Mumbai will the two trains be together?
Ans: 283.33 km.

4. Walking at 3/4 of his normal speed, a man takes 2(1/2) hours more than the normal time. Find the normal time.
Ans: 7.5 hours.

5. A motor car does a journey in 17.5 hours, covering the first half at 30 km/h and the second half at 40 km/h. Find the distance of the journey.
Ans: 600 km.