Lecture Notes For Application Of TSD:

Application 1: Trains

Problems based on trains are a special case in questions related to time, speed and distance because they have their own theory and distinct situations.
Problems based on trains are of two types. One is a train crossing an object having no length and another is a train crossing an object having a length.

For example:

a. A train crossing a man, tree or pole is considered as "no length" objects. In this case, the distance travelled by train would be equal to the length of the train.
b. A train crossing a bridge, a platform of another train is considered as an object having a length. In this case, the distance travelled by train would be the sum of the length of the object and the length of the train.

For each of the following situations we have to use some specific notations:

S_t = Speed of train S_o = Speed of object t = time
L_t = Length of train L_o = Length of the object

a. Train crossing a stationary object with zero length:
   $S_t \times t = L_t$

b. Train crossing a moving object with zero length:
- In opposite direction: $(S_t + S_o) \times t = L_t$
- In the same direction: $(S_t - S_o) \times t = L_t$

c. Train crossing a stationary object with length
   $S_t \times t = L_t + L_o$

d. Train crossing a moving object with length:
- In opposite direction: $(S_t + S_o) \times t = L_t + L_o$
- In the same direction: $(S_t - S_o) \times t = L_t + L_o$

Problem 1:
A train crosses a pole in 10 seconds. If the speed of the train is 18 kmph, find the length of the train.

Solution:
This is the case when a train crosses a stationary object without length.
Here, S_t = 18 kmph, t = 10 sec
So, $S_t \times t = L_t$
$18 \times 5/18 \times 10 = L_t$
$L_t = 50$ m.
Therefore, the length of the train is 50 meters.

Problem 2:
Two trains are moving towards each other having a ratio of length 4:3. One train is travelling at 54 kmph, other is travelling at 72 kmph. They cross each other in 30 sec. Find the length of two trains individually.

Solution:
Here, S_t = 54 kmph = $54 \times 5/18$ = 15 m/sec.
S_o = 72 kmph = $72 \times 5/18$ = 20 m/sec.
t = 30 sec.
So, $(S_t + S_o) \times t = L_t + L_o$
$(15+20) \times 30 = L_t + L_o$
$L_t + L_o = 1050$
And given that, $L_t : L_o = 4:3$
So, $L_t = 1050 \times 4/7$ = 600 meters and $L_o = 1050 \times 3/7$ = 450 meters.

Problem 3:
Two trains T1 and T2, T1 is travelling at 54 kmph and T2 is coming from the opposite direction at 72 kmph. A man is sitting in T2. T1 crosses the man in 10 sec. Find the length of train T1.

Solution:
This is the case in which a train crosses a moving object with zero length.
Here, S_t = 54 kmph = $54 \times 5/18$ = 15 m/sec.
S_o = 72 kmph = $72 \times 5/18$ = 20 m/sec.
t = 30 sec.
So, $(S_t + S_o) \times t = L_t$
$(15+20) \times 10 = L_t$

L_t = 350 meters.
Therefore the length of the train T1 is 350 meters.

Application 2: Boats and Streams

The problems of boats and streams are also dependent on the Speed, Time and distance formula:
Speed × Time = Distance
S_B = Speed of the boat in still water.
S_S = Speed of stream.

a. In still water, the speed of movement is S_B .
b. While moving upstream (against the flow of the water), the speed of movement is:
   $S_U = S_B - S_S$
c. While moving downstream (with the flow of the water), the speed of movement is:
   $S_D = S_B + S_S$

NOTE: Speed of Boat is an average of S_U & S_D .

Problem 1:
A boat whose downstream speed is 10 kmph and upstream speed is 6 kmph. Find the speed of Boat and Stream.

Solution:
We know,
$S_B = (S_D + S_U) / 2$
S_B = (10 + 6)/2 = 8 kmph.
$S_S = S_D - S_B$
S_S = 10 - 8 = 2 kmph.

Application 3: Clocks

Problems on clocks are based on the relative movement between the minute hand and the hour hand. You can think hands in the clock as two runners (minute hand and hour hand), the minute hand is running at a speed of 360^o per hour (here we assume distance in degree) and the hour hand is running at a speed of 30^o per hour.
Since the minute hand and the hour hand both are moving in the same direction, the relative speed of the minute hand w.r.t the hour hand is 330^o per hour. In one hour the minute hand either is approaching the hour hand or it is leaving it behind (separation).

Some useful information:

1. In every hour there are two instances of right angles when the hands of the clock are at right angles.
2. There are two instances on the clock when the hands of the clock make a straight line. It happens whether the hands are coinciding or pointing opposite to each other.

Problem 1:
At what time between 1 to 2 the hands of the clock will form a straight line?

Solution:
At 1'oclock situation hour hand 30^o ahead of the minute hand. The minute hand has to approach the hour hand by 30^o i.e. distance is 30^o
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed $\times$ time = distance
$330^o \times t = 30^o$
t = 30/330 = 1/11 hours.
So, the number of hours required to form a straight line will be 1/11 hours.

Convert into minutes:
1 hour = 60 minutes
1/11 hours = 60 $\times$ 1/11
60/11 minutes = 5(5/11) minutes.
Into seconds:
1 minute = 60 seconds
5/11 minutes = 60 $\times$ 5/11 seconds
300/11 seconds = 27.27 seconds
Hence, the required answer is 1:05:27.27 seconds.

Problem 2:
At what time between 4 to 5 the hands of the clock will form a straight line?

Solution:
At 1'oclock situation hour hand 120^o ahead of the minute hand. The minute hand has to approach the hour hand by 120^o i.e. distance is 120^o
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed $\times$ time = distance

$330^o \times t = 120^o$
t = 120/330 = 4/11 hours.
So, the number of hours required to form a straight line will be 4/11 hours.
Convert into minutes:
1 hour = 60 minutes
4/11 hours = 60 × 4/11
240/11 minutes = 21(9/11) minutes.
Into seconds:
1 minute = 60 seconds
9/11 minutes = 60 × 9/11 seconds
540/11 seconds = 49.09 seconds
Hence, the required answer is 4:21:49.09 seconds.

NOTE: Straight line happens at 0^o, 180^o behind and 180^o ahead. In an hour you will get only 2 cases.

If a right angle form, the distance either 90^o ahead, 90^o behind, 270^o ahead or 270^o behind, only these 4 cases will happen.

Problem 3:
At what time between 4–5 is the 1st and 2nd right angle formed by the hands of the clock?

Solution:
At 4'oclock the minute hand behind the hour hand by 120^o. The minute hand is going to approach the hour hand.
Also, the 1st right angle between 4-5 is formed when the minute hand is 90^o behind the hour hand.
So, the minute hand has to cover 30^o.
We know the relative speed between the hour hand and the minute hand is 330^o per hour.
So,
Relative speed × time = distance
$330^o \times t = 30^o$
t = 30/330 = 1/11 hours.

So, the number of hours required to form 1st right angle will be 1/11 hours.
Convert into minutes:
1 hour = 60 minutes
1/11 hours = 60 × 1/11

60/11 minutes = 5(5/11) minutes.
Into seconds:
1 minute = 60 seconds
5/11 minutes = 60 × 5/11 seconds
300/11 seconds = 27.27 seconds
Hence, the 1st right angle is formed at 4:05:27.27 seconds.

Also, the 2nd right angle between 4-5 is formed when the minute hand is 90^o ahead the hour hand.
So, the minute hand has to cover 210^o.
$330^o \times t = 210^o$
t = 210/330 = 7/11 hours.

So, the number of hours required to form 2nd right angle will be 7/11 hours.
Convert into minutes:
1 hour = 60 minutes
7/11 hours = 60 × 7/11
420/11 minutes = 38(2/11) minutes.
Into seconds:
1 minute = 60 seconds
2/11 minutes = 60 × 2/11 seconds
120/11 seconds = 10.90 seconds
Hence, the 2nd right angle is formed at 4:38:10.90 seconds.

Application 3: Races & Games of skill

These questions are completely based on the unitary method.

Problem 1:
In a 200 meter race, A can give B a start of 20 meters and B can give C a start of 30 meters. In a 1 km race, how much of a start can A give C?

Solution:
A gives B a start of 20 meter means when A does 200 m, B does 180 m.
B gives C a start of 30 meter means when B does 200 m, C does 170 m.
When B at 200, C will be at 170.
When B at 1, C will be at 170/200.

When B at 180, C will be at 180 × 170/200 = 153 m
Hence, when A is doing 200 m, C is doing 153 m.
In 200 m race A beats C by 47m
So, in 1000 m race A beats C by 1000 × 47/200 = 235 m.
Therefore, A can give C 235 m.

Problem 2:
In a 200 meter race, A beats B by 20 meters and in 300-meter race B beats C by 30 meters. In a 1 km race, how much A beats C?

Solution:
In a 200 meter race, when A does 200 m, B does 180 m.
In a 300 meter race, when B does 300 m, C does 270 m.
When B does 1 m, C does 270/300.
When B does 180 m, C does 180 × 270/300 = 162 m.
Hence, when A is doing 200 m, C is doing 162 m.
In 200 m race A beats C by 38 m
So, in 1000 m race A beats C by 1000 × 38/200 = 190 m.
Therefore, A beats C by 190 m.

Problem 3:
In a game of billiards, A can give B 20 points in 200. While B can give C 30 points in 300. How much A can give C in a 600 point game?

Solution:
In a 200 point game, when A does 200 points, B does 180 points
In a 300 points game, when B does 300 points, C does 270 points
When B does 1 point, C does 270/300 points.
When B does 180 points, C does 180 × 270/300 = 162 ponits

Hence, when A does 200 points, C is doing 162 points
In the 200 points game, A gives C 38 points.
So, in 600 points game, A gives C 600 × 38/200 = 114 points
Therefore, A gives C 114 points.

Some question for practice:

1. A train crosses a pole in 8 seconds. If the length of the train is 200 metres, find the speed of the train.
   Ans: 90 kmph.
2. A train crosses a man travelling in another train in the opposite direction in 8 seconds. However, the train requires 25 seconds to cross the same man if the trains are travelling in the same direction. If the length of the first train is 200 metres and that of the train in which the man is sitting is 160 metres, find the speed of the first train.
   Ans: 59.4 kmph.

3. A boat sails down the river for 10 km and then up the river for 6 km. The speed of the river flow is 1 km/h. What should be the minimum speed of the boat for the trip to take a maximum of 4 hours?
   Ans: 4 kmph.

4. At what time between 2–3 p.m. is the first right angle in that time formed by the hands of the clock?
   Ans : 2:27:16.36 seconds.

5. Vinay runs 100 metres in 20 seconds and Ajay runs the same distance in 25 seconds. By what distance will Vinay beat Ajay in a hundred-metre race?
   Ans: 25 m.

(Reference: https://schoolbag.info/mathematics/cat/16.html)