Lecture Notes for Percentages

Percentages is an important chapter. As a chapter, not only from the point of view of its own questions but apart from that, percentages is a base chapter for a lot of other chapters. E.g. profit and loss, interest these two chapters are completely built on percentages and chapters like time and work, time speed and distance are exclusively built on percentages plus ratio.
Percentage and ratio & proportion are two chapters which will be the base of all these 4 chapters. If you can master these 6 chapters together you build up an ability to handle aptitude exams easily.

What is the percentage?

Basic definition of percentage is essentially out of 100. Percentage is derived from French word 'cent'. Meaning of 'cent' in French is 100.
Percentage is used to compare data and numbers.
For example:
(a) If there are 5 (A,B,C,D,E) students who have taken the 12th board exam from five different boards. The percentages they get is a defined thing i.e. comparison between 5 diverse students in 5 diverse boards.

A	B	C	D	E
86%	92%	94%	78%	52%

By seeing the percentage of these students we can compare which student is better.

(b) GDP defines how the world is doing in terms of Global world economies. GDP compares different countries' economies in terms of their percentage.

Mathematically;
Any ratio if you multiply by 100, it gives you its percentage value. Percentage is denoted by a sign "%".
Why when the ratio is multiplied by 100, gives you a percentage value? You can see that from the unitary method.
Unitary method: It is a method which talks about a situation where two variables are moving linearly w.r.t. each other.

For example:

1. You bought 10 bananas for 30 rupees then, how many rupees will you need to buy 15 bananas?

Solution: let x rupees you will need to buy 15 bananas.

10 bananas = 30Rs

15 bananas = x Rs

Cross multiply and equate;

$10*x = 15 \times 30$

$x = 45.$

So, 45 rupees is the amount that you will need to buy 15 bananas.

2. You scored 10 out of 20 in a quiz and you want to put it in % then, how much out of 100 did you score?

Solution:

10 out of 20.

x out of 100.

So, by unitary method;

$20 \times x = 10 \times 100$

$x = (10/20) \times 100$

$x = 50\%$

NOTE: Any fraction multiplied by 100 gives its percentage value.

Concept of percentage change:

Percentage always happens when you go from one number to the next number.

Basic structure of percentage change will always be in the situation, where you are talking about the difference between two numbers.

Let say we have number x becoming y. The percentage change between x to y.

$$x \longrightarrow y$$

Formula for percentage change:

Percentage change = (change/original value)*100

1st you have to identify which number is the original number that depends on which direction you are looking at percentage change. So, percentage change is always a **directional input**.
If x changes to y the percentage change going from x to y, will be having x as the original value.

$$x \longrightarrow y$$

If y changes to x. So, in this situation the percentage change will have to be seen from y to x and will be having y as the original value.

$$y \longrightarrow x$$

For example;
If you have two numbers 20 & 40. So, going from 20 to 40.

$$20 \longrightarrow 40$$

Here change = 40 - 20 = 20, and original value = 20.
% change = (20/20) × 100 = 100%
The change is +ve. So, % change is increasing by 100%.

20 to 40 have different % change than coming from 40 to 20.

$$20 \longleftarrow 40$$

Here change = 20 - 40 = -20, and original value = 40.
% change = (-20/40) × 100 = -50%
So, % change is decreasing by 50%.

NOTE: 1. In percentage change there should be two numbers.
2. You need to understand which number is the original number.

People make a very common mistake in % change calculation.
In the question given that 50 to 75, instead of this they calculated 75 to 50. Because the language of % change can get complex sometimes, where language structures are used especially in DI.

Percentage Change Graphics(PCG):

It is an important concept in percentage change and important for chapters like interest, profit and loss etc. As the name suggests, percentage change graphics means the graphical method of doing the percentage change.

Basics of percentage change:

1. 100% of a number is a number itself.
2. 10% of a number is a shift of 1 decimal point on the number towards left.
3. 1% of a number is a shift of 2 decimal points on the number towards left.
4. 0.1% of a number is a shift of 3 decimal points on the number towards left and so on….

For example:
Let say a number N=52123.
100% of the number N is 52123
10% of the number N is 5212.3
1% of the number N is 521.23
0.1% of the number Nis 52.123

PCG has two structures:

Structure 1:
Given the starting value and the ending value. You have to calculate:
1. Absolute change (below the arrow).
2. % change(above the arrow).

$$S \xrightarrow[\text{Absolute change}]{\text{\% change}} E$$

Example:
Let us say, 40 changing into 52.

$$40 \longrightarrow 52$$

Absolute change = 52 - 40 = +12. Absolute change is +ve that means an increase in % change.
10% of the number 40 is 4, and the number 12, is 3 times the number 4 which means that the percentage increases by 30% (3 * 10% = 30%).

$$40 \xrightarrow[+12]{30\% \uparrow} 50$$

Examples:

(1). 50 changing into 70.

Absolute change = 70 - 50 = +20. Absolute change is +ve that means an increase in % change.

10% of the number 50 is 5, and the number 20, is 4 times the number 5 which means that the percentage increases by 40%(4 * 10% = 40%).

$$50 \xrightarrow[+20]{40\% \uparrow} 70$$

(2). 60 changing into 42.

Absolute change = 42 - 60 = -18. Absolute change is -ve that means a drop in % change.

10% of the number 60 is 6, and the number 18, is 3 times the number 6, which means that the percentage decreases by 30%(3 * 10% = 30%).

$$60 \xrightarrow[-18]{30\% \downarrow} 42$$

(3). You can have situations which are a little bit more calculative than this.

E.g. 33 changes into 47.

$$33 \xrightarrow{\hspace{4cm}} 47$$

Absolute change = 47 - 33 = +14.

10% of 33 is 3.3. If you keep adding 3.3, 4 times you get 13.2.(3.3+3.3+3.3+3.3=13.2).

In 14, you definitely have 40% of 33 included.

14 = 13.2 + 0.8.

0.8 as the percentage of 33. As we know, 1% of 33 is 0.33.

If you add 0.33, 2 times the value you get lies under 0.8 that means 2% increment further on the 40%.

Hence, the % change is in between 42 to 43%.

$$33 \xrightarrow[+14]{42\text{-}43\% \uparrow} 47$$

Structure 2:

1. Starting value is given to you,
2. Percentage change is given to you.
3. Absolute change you need to calculate.
4. And calculating the ending value.

Example1:
There is a number 40 that has to be increased by 30%.

$$40 \xrightarrow{30\%\uparrow}$$

Solution:
We were doing this problem by unitary method.
40 is 100%
x is 130%
Cross multiply and equate;
x = (40 × 130)/100.
Rather than this, much easy calculation is done through percentage change graphics.

$$40 \xrightarrow{30\%\uparrow}$$

10% of 40 is 4. 30% increase means adding 4, 3 times. 4+4+4 = 12 i.e adding 12 in 40 so the ans is 52.

$$40 \xrightarrow[4+4+4=12]{30\%\uparrow} 52$$

Example2:
A number 37 has to be increased by 13%.

$$37 \xrightarrow{13\%\uparrow}$$

Solution: In this question you have to build up 13% by;
10% of 37 is 3.7
1% of 37 is 0.37
1% of 37 is 0.37
1% of 37 is 0.37
So, 13% of 37 is 4.81(3.7+0.37+0.37+0.37 = 4.81), adding 4.18 in 37. So, the answer is 41.81

$$37 \xrightarrow[+4.81]{13\%\uparrow} 41.81$$

PCG applied to percentage change:

The 1st structure under which you can use the percentage change in quantitative aptitude is product change situation.

Example1:

Let say a product $x \times y$. x is increased by 20% and y is increased by 30%. You want to find out what is the % change in the product?

Solution:

x would become x(1+ (20/100)) = $x \times 1.2$

y would become y(1+ (30/100)) = $y \times 1.3$

So, in product; $x \times 1.2 \times y \times 1.3$= 1.56xy. This means, 56% change.

Same question can be done by PCG. If you assume your original product to be 100. And this product will go through two changes, 20% increase in x and 30% increase in y. You have to put two arrows,

One for ‘x’ and other for ‘y’.

$$100 \xrightarrow[\text{‘x’}]{20\%} \qquad \xrightarrow[\text{‘y’}]{30\%}$$

If x increases by 20% the product also increases by 20% and then if y increases by 30% the product also increases by 30%.

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+36]{30\%} 156$$

i.e. 56% increase in the product.

Example2:

A product $x \times y \times z$. Where x increases by 10%, y increases by 20% and z decreases by 30%. What is the % change in product?

Solution:

Let us assume the original product is 100.

$$100 \xrightarrow[+10]{10\%\uparrow} 110 \xrightarrow[+22]{20\%\uparrow} 132 \xrightarrow[-39.6]{30\%\downarrow} 92.4$$

10% increase in 100 = 110 then, 20% increase in 110 = 132 and then, 30% decrease in 132 = 92.4.

92.4 - 100 = -7.6 i.e 7.6% decrease in product.

NOTE: You can order the arrow according to what you want.

For this situation;

$$100 \xrightarrow[-30]{30\%\downarrow} 70 \xrightarrow[+14]{20\%\uparrow} 84 \xrightarrow[+8.4]{10\%\uparrow} 92.4$$

92.4 - 100 = -7.6 i.e 7.6% decrease in product.

Some question for practice:

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1. A product $x \times y \times z$. Where x increases by 5%, y increases by 20% and z decreases by 10%. What is the % change in product?
   Ans: 13.4% increase.
2. A product $x \times y \times z$. Where x decreases by 30%, y increases by 20% and z increases by 20%. What is the % change in product?
   Ans: 0.8% increase.
3. A product $x \times y \times z$. Where x decreases by 20%, y increases by 30% and z increases by 30%. What is the % change in product?
   Ans: 35.2% increase.

Problems on percentage change:

Area and volume based problem:

Problem 1:

The length of a rectangle goes up by 30% and the breadth of the rectangle comes down by 10%. What is the percentage change in area?

Solution:

Area = $l \times b$ and now it becomes a product change situation.

Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{‘l’}]{30\%\uparrow} \quad \xrightarrow[\text{‘b’}]{10\%\downarrow}$$

$$100 \xrightarrow[+30]{30\%\uparrow} 130 \xrightarrow[-13]{10\%\downarrow} 117$$

Hence 17% is the increase in the area of the rectangle.

Problem 2:

The length of a rectangle is decreased by 20% and the breadth of the rectangle is increased by 23%. What is the percentage change in area?

Solution:

Area = $l \times b$ and now it becomes a product change situation.

Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{‘b’}]{23\% \uparrow} \qquad \xrightarrow[\text{‘l’}]{20\% \downarrow}$$

For easy calculation we put breadth on the 1st arrow and length on the 2nd arrow.

$$100 \xrightarrow[+23]{23\% \uparrow} 123 \xrightarrow[-24.6]{20\% \downarrow} 98.4$$

Hence, 1.6% is the decrease in the area of the rectangle.

We can do the same problem with the help of following formula;

Percentage change = (a + b + ab/100)

Let us say, x increases by 20% and y increases by 10%. Then the percentage change;

Percentage change = 20 + 10 + (20 × 10)/100

= 32%.

But rather than this PCG is a more easy way to solve this problem.

One other problem to this formula, if $x \times y \times z$ situation occurs then the formula can not take a change 3 components of the product. PCG is always better for these problems.

Problem 3:

Length, Breadth and Height of a cuboid are decreased by 30%, increased by 20% and increased by 20% respectively. What is the percentage change in the volume of cuboid?

Solution :

Volume of cuboid = $l \times b \times h$

Assume the original volume = 100.

$$100 \xrightarrow[-30]{30\% \downarrow} 70 \quad \xrightarrow[+14]{20\% \uparrow} 84 \quad \xrightarrow[+16.8]{20\% \uparrow} 100.8$$

0.8% increment in the volume of cuboid.

Expenditure and revenue problem:

Problem 1:

Price of a commodity has gone up by 20% and a person reduces its consumption by 10%. What is the % change in the expenditure?

Solution :

Price $\times$ consumption = expenditure.

Assume the original expenditure = 100. Makes two arrows one for price and other is for consumption.

$$100 \xrightarrow[\text{`P'}]{20\% \uparrow} \quad \xrightarrow[\text{`C'}]{10\% \downarrow}$$

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108$$

Hence, 8% is the increase in the expenditure of the commodity.

Problem 2:

A shopkeeper selling chairs, reduces the price of chairs by 20% due to which he gets an increment of 60% in the sale. What is the percentage change in the revenue?

Solution :

Price $\times$ sale = revenue.

Assume the original revenue = 100. Makes two arrows one for price and other is for sale.

$$100 \xrightarrow[\text{`P'}]{20\% \downarrow} \quad \xrightarrow[\text{`S'}]{60\% \uparrow}$$

$$100 \xrightarrow[-20]{20\%\downarrow} 80 \xrightarrow[+48]{60\%\uparrow} 128$$

Hence, 28% is the increment in the revenue.

Problem 3:

Speed of a car increases by 30%, and the time for which it travels is increased by 40%. How much percent is the increment in the distance?

Solution :

Speed $\times$ Time = Distance.

Assume the original distance = 100. Makes two arrows one for speed and other is for time.

$$100 \xrightarrow[\text{'S'}]{30\%\uparrow} \quad \xrightarrow[\text{'T'}]{40\%\uparrow}$$

$$100 \xrightarrow[+30]{30\%\uparrow} 130 \xrightarrow[+52]{40\%\uparrow} 182$$

Hence, 82% is the increment in the distance.

NOTE: Anywhere you have a product relationship between two variables, you will always be able to use PCG on it.

PCG applied to product constancy:

Product constancy is after the series of changes, you need to come back to the original value. Product constancy is applied in a lot of questions directly.

$$100 \longrightarrow \quad \longrightarrow \quad \longrightarrow 100$$

Problem 1:

Price of a commodity has gone up by 25% and the consumption is reduced such that the expenditure remains constant.

Solution :

Price $\times$ consumption = expenditure.

Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.

After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\% \uparrow} 125 \xrightarrow[-25]{20\% \downarrow} 100$$

25% increase in price is offset by 20% decrease in consumption to keep expenditure constant.

Problem 2:

The length of a cuboid has increased by 20%, the breadth has increased by 50%. How much should you reduce the height to keep the volume constant?

Solution :

Volume = $l \times b \times h$

After 20% and 50% increment in length and breadth respectively, volume becomes 180. So, 180 should be reduced by 80 to keep volume constant i.e height dropped by 44.44%.

Drop in height = (80/180) $\times$ 100

= (4/9) $\times$ 100 = 44.44%

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+60]{50\% \uparrow} 180 \xrightarrow[-80]{44.4\% \downarrow} 100$$

Ratio-percentage equivalence:

1/2 = 50%	1/11 = 9.09%
1/3 = 33.3%	1/12 = 8.33%
1/4 = 25%	1/13 = 7.69%
1/5 = 20%	1/14 = 7.14%
1/6 = 16.67%	1/15 = 6.66%
1/7 = 14.28%	1/16 = 6.25%
1/8 = 12.5%	1/17 = 5.88%
1/9 = 11.11%	1/18 = 5.55%
1/10 = 10%	1/19 = 5.26%
	1/20 = 5%

These percentage values you should have to remember to make the calculations easier and faster.

Product constancy table:

Product = $x \times y$

To make the product constant one component increases then, the other component should be decreased.

	‘x’ increases(%)	**‘y’ decreases(%)**
Standard Value 1	9.09	8.33
Standard Value 2	10	9.09
Standard Value 3	11.11	10
Standard Value 4	12.5	11.11
Standard Value 5	14.28	12.5
Standard Value 6	16.66	14.28
Standard Value 7	20	16.66
Standard Value 8	25	20
Standard Value 9	33.33	25
Standard Value 10	50	33.33
Standard Value 11	60	37.5
Standard Value 12	66.66	40
Standard Value 13	75	42.85
Standard Value 14	100	50

For example:

Standard value 7 ;

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-20]{16.67\% \downarrow} 100$$

The fractional view to the product constancy table :

There is a fractional view of the product constancy table also.

Product = $x \times y$

To make the product constant one component increases then, the other component should be decreased.

‘x’ increases	**‘y’ decreases**
1/2 ↑	1/3 ↓
1/3 ↑	1/4 ↓

1/4 ↑	1/5 ↓
1/5 ↑	1/6 ↓
1/7 ↑	1/8 ↓
1/9 ↑	1/10 ↓
1/11 ↑	1/12 ↓
1/12 ↑	1/13 ↓ …...and so on.

Decrease in 'y' means the denominator of the fractional part of 'y' is more than the denominator of the fractional part of 'x'.

Generic form of this table, If 'x' increases by **1/a**, then 'y' will drop by **1/(a+1)**.

The advantage of a fractional table is, it is easier to remember than the % view of the product constancy table.

For example:

If I gave you any non standard value. Such as 1/26 is growth, you know that 1/27 has to be dropped.

It is not necessary that the fraction would be in 1/a form.

For example:

x increased by 3/14 then y will decrease by 3/17.

It is not 1/a or 1/(1+a).

If 'x' increases by **x/a**, 'y' will drop by **x/(a+x)**.

Problem 1:

A man travelling over a certain distance. He is going from Delhi to Chandigarh at a certain speed and takes a certain time to cover the distance. If speed increases by 3/23. What will be the reduction of time so that distance remains constant?

Solution :

$Distance = speed \times time$

Speed is increased by 3/23. So, for distance being constant time will reduce by 3/26.

Problem 2:

A man is going at a certain speed and he takes 260 min to reach his destination. If he increases the speed by 3/23, how much time will he take to cover the journey?

Solution :

$Distance = speed \times time$

Speed is increased by 3/23. So, for distance being constant time will reduce by 3/26.

Original time = 260min and it is reduced by 3/26. So, time = $(3/26) \times 260$ = 30min. Time will drop by 30min. Hence, he will take 230min to cover the same journey.

Problems on product constancy :

Already we have discussed what is product constancy and we did some standard problems based on product constancy. Some more problems on product constancy are:

Problem 1: The price of a commodity has gone up by 25%. To keep the total expenditure on the commodity constant, by what percentage you have to reduce consumption?
Solution :
Price $\times$ consumption = expenditure.
Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.
After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\% \uparrow} 125 \xrightarrow[-25]{20\% \downarrow} 100$$

25% increase in price is offset by 20% decrease in consumption to keep expenditure constant.

Problem 2: Speed of a car has gone up by 50%. How much would the time come down to covering the same distance?
Solution :
$Distance = speed \times time$
Let 100 be the original distance.

$$100 \xrightarrow[+50]{50\% \uparrow} 150 \xrightarrow[-50]{33.33\% \downarrow} 100$$

Reduce in time = 50/150 = 1/3, fraction 1/3 is equivalent to 33.33%.
After 50% increment in speed, distance becomes 150. So, 150 should be reduced by 50 to keep distance constant i.e time reduced by 33.33%.

Problem 3: In a triangle the length has increased by 40% and you want to restrict breadth percentage change, such that increase in area of the triangle is limited to 60%. What is the maximum percentage change in the breadth?
Solution :

Area of triangle = $1/2 \times l \times b$

Let 100 be the original area of the triangle and area of the triangle limited to 60% that means the final area would be 160.

$$100 \xrightarrow[\substack{+40 \\ \text{'l'}}]{40\% \uparrow} 140 \xrightarrow[\substack{+20 \\ \text{'b'}}]{14.28\% \uparrow} 160$$

Here the last part is 160. This can be described as targeted % change in the product.
% change in breadth = 20/140 = 1/7, fraction 1/7 is equivalent to 14.28%.
After a 40% increment in length, there is an 14.28% increment in breadth to restrict the area 160.

Problem 4: Price of a commodity has gone up by 40% and Shubham wants to limit his expenditure increase to 5%. What is the reduction in consumption, so that expenditure increases 5%?

Solution :

Price $\times$ consumption = expenditure.

Let 100 be the original expenditure and expenditure limited to 5% that means the final expenditure would be 105.

$$100 \xrightarrow[+40]{40\% \uparrow} 140 \xrightarrow[-35]{25\% \downarrow} 105$$

% change in consumption = 35/140 = 1/4, fraction 1/4 equivalent to 25%.
Hence consumption dropped by 25%, so that expenditure was limited to a 5% increase.

PCG applied on successive percentage change :

Successive percentage change use of PCG is structurally very similar to product change use of PCG. One small difference is that in product change we have seen that the arrows are interchangeable w.r.t. each other but in successive percentage change use of PCG we can not interchange the arrows because sometimes we need intermediate value, if we interchange the arrows then we do not get the exact intermediate value. You can understand that difference through some examples/problems.

Problem 1:

Population of the town goes up by 20% in 1st year, comes down by 10% in 2nd year and goes up by 5% in 3rd year. What is the % change in population after 3 years ?

Solution :

Let the population of the town is 100. Population after one year becomes 120 with an increase of 20%. Population after 2 year will become 108 and after the 3rd year the population will become 113.4.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108 \xrightarrow[+5.4]{5\% \uparrow} 113.4$$

% change in the population after 3 years is 13.4%. But the intermediate value is important, if anyone asks what is the % change in population after 2 years.

If you interchange the arrows e.g 10% is placed on the last arrow.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+6]{5\% \uparrow} 126 \xrightarrow[-12.6]{10\% \downarrow} 113.4$$

Final value does not make a difference. But after two year the population value is wrong. If the question is built on intermediate value then you will go wrong if you do not keep the arrow constant as they are, that is the only difference in this.

Problem 2:

A shopkeeper successively marks his goods by 20% increase, 30% increase and 50% increase and then gives a discount of 10% to his customers. What is the percentage profit to the shopkeeper?

Solution :

Let the original cost is 100. He is marking up 3 times successively, 1st mark up on 1st arrow,2nd markup on 2nd arrow, 3rd markup on 3rd arrow and discount on the 4th arrow.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+36]{30\% \uparrow} 156 \xrightarrow[+78]{50\% \uparrow} 234 \xrightarrow[-23.4]{-10\% \downarrow} 210.6$$

% profit of the shopkeeper is 110.6%.

Problem 3:

A product $a \times b \times c \times d$, a increased by 20%, b is increased by 30%, c is increased by 50% and d decreased by 10%. What is the percentage change in the product?

Solution :

Let the original product is 100. So;

$$100 \xrightarrow[\substack{+20 \\ \text{'a'}}]{20\% \uparrow} 120 \xrightarrow[\substack{+36 \\ \text{'b'}}]{30\% \uparrow} 156 \xrightarrow[\substack{+78 \\ \text{'c'}}]{50\% \uparrow} 234 \xrightarrow[\substack{-23.4 \\ \text{'d'}}]{-10\% \downarrow} 210.6$$

% change in product is 110.6%.

Problems on successive percentage change :

We have already discussed successive markup and discount problems and also discussed population problems.

Problem 1:

A man's salary is 100Rs out of his salary he spends 20% on food, 30% of the remaining on household expenses, 10% of total on entertainment and saves the rest.

(1). What percentage of income does he save?

(2). What percentage of income does he spend?

(3). What is the ratio of household expense to the entertainment expense?

Solution :

Total salary = 100.

Food expense is 20% of 100 = 20Rs.

Remaining salary = 100 - 20 = 80Rs

Household expense is 30% of remaining salary i.e. 30% of 80 = 24Rs

Entertainment expense is 10% of 100 = 10Rs.

Total expense = 20+24+10 = 54Rs

Saving = 100 - 54 = 46.

(1). Saving percentage = 46%

(2). % of income he spend = 54%

(3). Ratio of household expense to the entertainment expense = 24/10 = 12:5.

If a question is asked on absolute value, you can not be answering that because you do not know any value in this situation.

Let's say he save rupees 9200

The connection between the value in the left box and value in the right box, 46 becoming 9200. Give a multiplier of **200** (9200/46 = 200),then you allow to multiply any of the other numbers in the left box by 200 to answer any question asked about absolute value.

	Left box	**Right box** (Actual value)
Income	100	………

Food expense	20	………
Household expense	24	…….....
Entertainment expense	10	…........
Saving	46	9200

If you have one connector value then you can find any value.
For example here:
Total income = $100 \times 200 = 20000$.
Food expense = $20 \times 200 = 4000$.
Household expense = $24 \times 200 = 4800$.
Entertainment expense = $10 \times 200 = 2000$.

Problem 2:
A machine depreciates in value by 10% every year for 3 years consecutively before repair and maintenance in the 4th year increases the value by 10%. What is the final value of the machine after the 4th year?
Solution :

$$100 \xrightarrow[-10]{10\%\downarrow} 90 \xrightarrow[-9]{10\%\downarrow} 81 \xrightarrow[-8.1]{10\%\downarrow} 72.9 \xrightarrow[+7.29]{10\%\uparrow} 80.19$$

Final value of machine after 4th year = 80.19
If, the actual value of the machine after two year was seen as 162000 then;
(1). What is the original value?
(2). What is the value after 1st year?
(3). What is the value after 3 years?
(4). What is the value at the end of 4th year?

Ans: Here, 81 becomes 162000. 162000/81 gives a multiplier of 2000.
(1). Original value = $100 \times 2000 = 200000$.
(2). Value after 1st year = $90 \times 2000 = 180000$.
(3). Value after 3 years = $72.9 \times 2000 = 145800$.
(4). Value at the end of 4th year = $80.19 \times 2000 = 160380$.

Problem 3:
A shopkeeper gives 3 successive discounts of 20%, 30% and 50%. What is the equivalent total single discount?

Solution :

Let the original value of markup price of products is 100.

$$100 \xrightarrow[-20]{20\% \downarrow} 80 \xrightarrow[-24]{30\% \downarrow} 56 \xrightarrow[-28]{50\% \downarrow} 28$$

Final price of the product is 28 when its original markup price is 100. 100 coming down to 28. Hence, equivalent single discount = 100 - 28 = 72%.

A to B to A problems (compare two numbers) :

Very often we face a situation, where we compare two numbers, say A and B. In such cases, if we are given % change from A to B, then the reverse relationship can be determined by using PCG in the same way as the product constancy.

Problem 1:

A’s salary is 25% more than B’s salary. By what percent is B’s salary less than A’s salary?

Solution :

Let B’s salary = 100.

$$100(B) \xrightarrow[+25]{25\% \uparrow} 125(A) \xrightarrow[-25]{20\% \downarrow} 100(B)$$

A drop of 25 on 125 gives a 20% drop.

Hence B’s salary is 20% less than A’s.

NOTE: Product constancy table is also useful for this situation.

Problem 2:

B gets 20% more marks than A and C gets 50% more marks than B, then how much % less than C does A get?

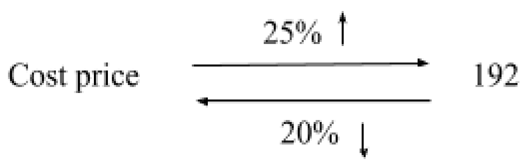
Solution :

Lets A’s marks = 100.

$$\underset{\text{‘A’}}{100} \xrightarrow[+20]{20\% \uparrow} \underset{\text{‘B’}}{120} \xrightarrow[+60]{50\% \uparrow} \underset{\text{‘C’}}{180}$$

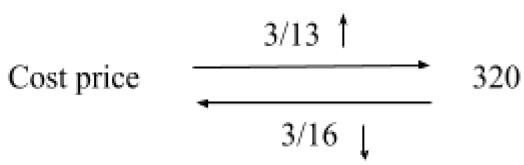
Coming back from C to A, a drop of 80 on 180 i.e 80/180 = 4/9. The fraction 4/9 is equivalent to 44.44%. Hence, A gets 44.44% marks less than C.

Problem 3:
A shopkeeper marks up his goods by 25% and the selling price for his goods is 192. What was its cost price?
Solution :
So, come back from 192 to the cost price and we know standard pair 25% increase means 20% decrease.



20% drop on 192 = - (192 × 20)/100 = -38.4.
Hence, cost price = 192 - 38.4 = 153.6.

Problem 4:
A shopkeeper increases the price of his goods by 3/13 and the selling price for his goods is 320. What was its cost price?
Solution :
We know the fractional implication of the product constancy table. If we go from A to B 3/13 increase, we will have to come back with a 3/16 decrease.



1/16 of 320 = 20 and 3/16 of 320 = 60.
Hence, cost price = 320-60 = 260.

Some questions for practice :

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1. Mr. Navdeep is worried about the balance of his monthly budget. The price of petrol has increased by 40%. By what percent should he reduce the consumption of petrol so that he is able to balance his budget?
   Ans : 28.56%.

2. In Question 1, if Mr. Navdeep wanted to limit the increase in his expenditure to 5% on his basic expenditure on petrol then what should be the corresponding decrease in consumption so that expenditure exceeds only by 5%?
   Ans : 25%.

3. Ram sells his goods 25% cheaper than Shyam and 25% dearer than Ghanshyam. How much percentage of Ghanshyam's goods are cheaper than Shyam's?
   Ans : 40%.
4. In an election between 2 candidates, Shubham gets 65% of the total valid votes. If the total votes were 6000, what is the number of valid votes that the other candidate Anjali gets if 25% of the total votes were declared invalid?
   Ans : 1575.
5. In a medical certificate, by mistake a candidate gave his height as 25% more than normal. In the interview panel, he clarified that his height was 5 feet 5 inches. Find the percentage correction made by the candidate from his stated height to his actual height.
   Ans : 20%.
6. Arjit generally wears his father's coat. Unfortunately, his cousin Shaurya poked him one day that he was wearing a coat of length more than his height by 15%. If the length of Arjit's father's coat is 120 cm then find the actual length of his coat.
   Ans : 104.34.
7. A number is mistakenly divided by 5 instead of being multiplied by 5. Find the percentage change in the result due to this mistake.
   Ans : 96%.
8. If 65% of x = 13% of y, then find the value of x if y = 2000.
   Ans : 400.
9. In a mixture of 80 litres of milk and water, 25% of the mixture is milk. How much water should be added to the mixture so that milk becomes 20% of the mixture?
   Ans : 20L.
10. 50% of a% of b is 75% of b% of c. Which of the following is c?
    Ans : 0.667a.