Blood relations

Introduction to Blood relations

Blood relation is one of the most important topics of logical reasoning and found its importance in almost every entrance exam. This topic tests the analytical skills of the students and their solution approach. The questions asked in this chapter depend upon 'Relations'. You should have a sound knowledge of the blood relation in order to solve the questions.

To remember easily, the relation may be divided into two forms:

Relation of the paternal side

Father's father	Grandfather
Father's mother	Grandmother
Father's brother	Uncle
Father's sister	Aunt
Children of uncle	Cousin
Wife of uncle	Aunt
Children of aunt	Cousin
Husband of aunt	Uncle

Relation of the maternal side

Mother's father	Maternal Grandfather
Mother's mother	Maternal Grandmother
Mother's brother	Maternal Uncle
Mother's sister	Aunt
Children of maternal uncle	Cousin
Wife of maternal uncle	Maternal Aunt
Children of the maternal aunt	Cousin
Husband of the maternal aunt	Maternal Uncle

Others

Son's wife → Daughter-in-law
Daughter's husband → Son-in-law
Husband's (or) wife's father → Father-in-law
Husband's (or) wife's mother → Mother-in-law
Husband's (or) wife's brother → Brother-in-law
Husband's (or) wife's sister → Sister-in-law
Sister's husband → Brother-in-law
Brother's (or) sister's son → Nephew
Brother's (or) sister's daughter → Niece

Son’s wife	Daughter-in-law
Daughter’s husband	Son-in-law
Husband’s (or) wife’s father	Father-in-law
Husband’s (or) wife’s mother	Mother-in-law
Husband’s (or) wife’s brother	Brother-in-law
Husband’s (or) wife’s sister	Sister-in-law
Sister’s husband	Brother-in-law
Brother’s (or) sister’s son	Nephew
Brother’s (or) sister’s daughter	Niece

Relations from one generation to other

Generation1: Grandfather, Grandmother, Maternal grandfather, Maternal grandmother
Generation 2: Mother, Father, Uncle, Aunt, Maternal uncle, Maternal aunt
Generation 3: Self, Sister, Sister-in-law, Brother, Brother-in-law
Generation 4: Son, Daughter, Nephew, Niece

Symbols

1. ‘+’ for male 2. ‘-’ for female 3. ‘ ⇔’ for couples

Types of problem statements

Type 1: Statement based relationship questions

Problem 1:
Pointing to a lady on the stage, Sonali said, “She is the sister of the son of the wife of my husband.” How is the lady related to Sonali?
Solution:
My husband = Sonali’s husband
Wife of my husband = is me = Sonali
Son of the wife of my husband = My Son
Sister of the Son of the wife of my Husband = My Son’s Sister = My daughter
So, the lady on the stage is Sonali’s daughter.

Problem 2:
Eeshas father was 34 years of age when she was born. Her younger brother, Shashank, now that he is 13, is very proud of the fact that he is as tall as her, even though he is three years younger than her. Eeshas mother, who is shorter than Eesha, was only 29 when Shashank was born. What is the sum of the ages of Eeshas parents now? **(asked in TCS)**

a) 92
b) 76
c) 66
d) 89

Answer: a) 92
Solution:Let Eesha's present age be x.
Eesha's father's present age = x + 34
Shashank's age = 13
Eesha's present age = 13 + 3 = 16
Eesha's mother's present age = 29 + 13 = 42
Sum of the ages of Eeshas parents now = 42 + 16 + 34 = 92

Problem 3:
Pointing to a lady a man said, “Her husband is the only son of my mother”. How is the lady related to the man?
Solution:

My mother's only son = is me (man)
Her husband = is me
So, the lady is Man's wife.

Problem 4:
Pointing to Alex, Lita says, "I am the daughter of the only son of his grandfather." How Lita is related to Alex? **(Asked in Sapient)**

a) Niece
b) Daughter
c) Sister
d) Cannot be determined

Answer: C) Sister
Solution:
Lita is the daughter of the only son of Alex's grandfather. Hence, it's clear that Lita is the sister of Alex.

Problem 5:
Pointing to a man Manisha said, "He is the youngest son of my father-in-law's only son". How is Manisha related to this youngest son's father?

a) Sister
b) Sister-in-law
c) Wife
d) Mother

Solution:
Manisha's father in law's only son = Manisha's husband
The youngest son of my father-in-law's only son is my husband's son = My son = Manisha's son
So, Manisha is the **wife** of the youngest son's father

Type 2: Puzzle type questions with a family relationship component

Problem 1:

A family consists of a husband and wife, their three sons and two daughters, three wives of three sons. How many females are in this family? **(Wipro hiring 2018)**

Solution:

Husband wife (female)

Three sons = S1 S2 S3 and two daughter = D1 D2

Son's wives = W1 W2 W3

So, the total number of females = wife + D1 + D2 + W1 + W2 + W3 = 6 females.

Directions for problem 2 to 6:

If a + b means, a is the daughter of b,

a - b means, a is the husband of b,

a×b means, a is the brother of b.

Problem 2:

What does the relation p×q - r show?

(a) p is the son-in-law of r
(b) p is the brother of r
(c) r is the wife of p
(d) None of these

Solution:

p×q means p is the brother of q

q - r means, q is the husband of r i.e.

p is the brother-in-law of r or r is the sister-in-law of p.

So the answer to this question is an option (d).

Problem 3:

If h+i×j+k×l+m×n, then what is the present generation of h. Assume that the oldest generation of this group is 1st generation.

(a) 2nd (b) 3rd
(c) 1st (d) None of these

Solution:

Here symbol '+' is for a generation change.

m×n = m is the brother of n

l+m = l is the daughter of m (1st generation)

k×l = k is the brother of l

j+k = j is the daughter of k (2nd generation)

$i \times j$ = i is the brother of j
h+i = h is the daughter of i (3rd generation)
Hence, present generation of 'h' = 3rd generation i.e. option (c)

Problem 4:
Which of the following options does not hold?
(a) a+b×c
(b) a-b×c
(c) a+b+c
(d) a+b-c

Solution:
(a) a+b×c, here 'b' is the brother of 'c' i.e 'b' is a male and 'a' is the daughter of 'b'. This option is correct.
(b) a-b×c, here 'b' is the brother of 'c' i.e 'b' is a male and 'a' is the husband of 'b' This option can not hold. 'a' can't be the husband of 'b', because 'b' comes out a male.

Problem 5:
From the statement a×b×c×d, which of the following statements is not necessarily true?
(a) 'b' is the brother of 'a'
(b) 'c' is the brother of 'a'
(c) 'd' is the brother of 'c'
(d) a,b,c are male

Solution:
a×b×c×d, here 'c' is the brother of 'd', 'b' is the brother of 'c' and 'a' is the brother of 'b'
So, here a,b,c are males.
Option (c) 'd' is the brother of 'c' is not necessarily true because we don't know whether 'd' is male or not.

Problem 6:
From the statement p-q+r×s, how is 'q' related to 's'?
(a) Niece
(b) Sister
(c) Daughter
(d) Brother

Solution:

r×s = ‘r’ is the brother of ‘s’ (‘r’ is male)

q+r = ‘q’ is the daughter of ‘r’(‘q’ is a female)

p-q = ‘p’ is the husband of ‘q’

So from the above conclusion, ‘q’ is the niece of ‘s’ i.e. option (a) is the correct answer.

Directions for questions 7 to 8.

a*b means ‘a’ is the brother of ‘b’

a@b means ‘a’ is the daughter of ‘b’

a$b means ‘a’ is the sister of ‘b’

Problem 7:

Which of the following show the relationship ‘p’ is the paternal uncle of ‘c’?

(a) n $ o @ p
(b) n @ o $ p
(c) n @ o * p
(d) None of these

Solution:

(a) n $ o @ p

o @ p = ‘o’ is the daughter of ‘p’ and n $ o = ‘n’ is the sister of ‘o’

So, here ‘p’ is either the father or the mother of ‘n’.

(b) n @ o $ p

o $ p = ‘o’ is the sister of ‘p’ and n @ o = ‘n’ is the daughter of ‘o’

So, ‘p’ is either uncle or aunt of ‘n’ because the gender of p can not be determined.

Hence, the answer will be an option (d).

Problem 8:

abc@d@e*f*g, then how many males and females are there respectively?

(a) 4,3
(b) 3,4
(c) 5,2
(d) Can’t be determined

Solution:

f*g = ‘f’ is the brother of ‘g’ (i.e. ‘f’ is a male)

e*f = ‘e’ is the brother of ‘f’ (i.e. ‘e’ is a male)

d@e = ‘d’ is the daughter of ‘e’ (i.e. ‘d’ is a female)

c@d = ‘c’ is the daughter of ‘d’ (i.e. ‘c’ is a female)

b$c = ‘b’ is the sister of ‘c’ (i.e. ‘b’ is a female)

a$b = ‘a’ is the sister of ‘b’ (i.e. ‘a’ is a female)

Here we can not find the gender of ‘g’.

Here 4 women and 2 men but we can't find the gender of one person.

So, the answer is can’t be determined, option(d)

Calendar Problems

Introduction to Calendar problems

The calendar is a small chapter but an important chapter of the reasoning part. Questions in calendars come from time to time for you in your exams.
A calendar is a series of pages that contains days, weeks, and months of a particular year and gives information.

Normal year: Any year which contains 365 days is called a normal year.
Leap year: Any year which contains 366 days is called a leap year.
Odd days: those number of available days from which we can't complete a week are called odd days.

A normal year has 365 days. In which there are 52 complete weeks and the last day would be an odd day. It would shift the calendar ahead or behind by a certain day.

Suppose in a normal year you start 1st January of the year on Monday, then 30th Dec of that year would be a Sunday and 31st Dec being a Monday and hence, the 1st Jan of the next year will skip the calendar forward by one day.

A leap year has 366 days. If 1st Jan starts with Monday of leap year then 29th Dec would be the last Sunday of that year. 30th Dec will again Monday and 31th Dec will be Tuesday. Hence, 1st Jan of the next year will skip by 2 days.

The number of odd days in different months of a calendar

MONTHS	NUMBER OF ODD DAYS
JANUARY	3
FEBRUARY(normal/leap)	0/1
MARCH	3
APRIL	2
MAY	3
JUNE	2
JULY	3
AUGUST	3
SEPTEMBER	2
OCTOBER	3
NOVEMBER	2
DECEMBER	3

NOTE:

1. The number of odd days in the first 100 consecutive years is 5.
2. The number of odd days in the first 200 consecutive years is 3.
3. The number of odd days in the first 300 consecutive years is 1.
4. The number of odd days in the first 400 consecutive years is 0.

Example 1:
11 August 2019 is a Sunday, what day was on 11 August 1983?
Solution:
To find the day on 11 August 1983, you have to count the number of odd days.
From 1983 to 2019 there are 36 years. This means 36 odd days and now count how many leap years or 29th Feb will appear.

So, 29th Feb would appear in 1984,1988,1992,1996,2000,2004,2008,2012,2016. So, 9 leap years means 9 further odd days.

Hence, the total number of odd days = 36+9=45 days
45 days have 6 complete weeks and 3 odd days left out.
Going behind 3 odd days from Sunday. Hence, 11 August 1983 would be a Thursday.

Example 2: What was the day of the week on 13th April 1723?

(a) Monday
(b) Tuesday
(c) Wednesday
(d) Thursday

Answer:b)

Solution: No. of odd days in 1700=5(1600=0+ 100=5)
No of odd days in 22 years=5(leap years)*2+17(normal years)=27mod7=6
No. of odd days in Jan, Feb, and march=3+0(1723 is not a leap year)+3=6
No. of odd days in 13 days=6
Total odd days =23mod7=2
Thus 13th April 1723 is Tuesday

OR

No. of odd days in 1700 = 5 (1600 = 0 + 100 =5)
No of odd days in 22 year = 5(leap years) * 2 + 17 (normal years) = 27mod7 = 6
No. of odd days in Jan, Feb, March and 13 days of April = 31 + 28(not leap year) + 31 + 13 = 103mod7 = 5
Total odd days = 5+6+5 = 16mod7 = 2
Thus 13th April 1723 is Tuesday.

Cause and effect

Introduction to Cause and effect problems

The main aim of cause and effect questions is to derive the relationship between two given statements. The relationship that needs to be tested is whether the statement is causally related to each other. This means we need to find out whether one of the statements is a direct cause of the other or conversely whether one of the statements is a direct effect of the other.

The cause is an event that leads to another event, which in turn is called the effect of the triggering event.

If there is a sequence of events, the event which is an **effect** shall always be preceded by an event which was its triggering cause event.

Example:

1. The Australian Cricket Team worked hard for four years to rebuild itself after all the key senior players retired.
2. The Australian team won the Cricket World cup after their consistent efforts.
3. The Australian team reached number 1 ranking after the huge success.

If we consider statements 1 and 2, we can clearly see that statement 2 is the result of statement 1. Hence, statement 1 is the cause for the effect in statement 2.
Similarly, if statements 2 and 3 are analyzed in a pair, you will see that statement 2 was the cause for the effect in statement 3.

Hence, any particular statement may play the role of both cause and effect, depending on the other statements in whose respect this particular statement is being analyzed.

Sufficient & necessary conditions

In a cause and effect question, the cause is the sufficient condition. It is assumed that the sufficient condition of the occurrence of the events must include the different necessary conditions as well.

A **necessary** condition is one that must be satisfied for the occurrence of an event.

Example: You must adhere to the deadline to get your work appraised.

This means if you get your work appraised you have to adhere to the deadline. Or, if you do not adhere to the deadline, you do not get your appraised.

A condition is called a sufficient condition if in a certain event, you are satisfied with the results.

Example: Being human is sufficient for being a mammal.

The act of being a human is not possible unless one is also a mammal. But it is not necessary that being human is a necessary condition for being a mammal.

Types of causes

1. **Immediate cause**: It immediately precedes the effect. This cause shares the closest proximity with the effect with relation to time.

2. **Principal cause**: The most important reason behind the effect. The immediate cause can be the principal cause and vice versa.

3. **Independent cause**: there is no relationship between the cause and the given effect.

Format of cause and effect questions

(a) If statement I is the cause and statement II is its effect;

(b) If statement II is the cause and statement I is its effect;

(c) If both the statements I and II are independent causes;

(d) If both the statements I and II are effects of independent causes; and

(e) If both the statements I and II are effects of some common cause

Example 1 :
I – The committee appointed by the Government on the fee structure of the professional courses has drastically reduced the fees of various courses in comparison to those charged in the last year.

II – The parents of aspiring students seeking admission to professional courses had launched a severe agitation protesting against the high fees charged by the professional institutes and the admission process was delayed considerably.

a. Statement I is the cause and statement II is its effect.
b. Statement II is the cause and statement I is its effect.
c. Both statements I and II are independent causes.
d. Both statements I and II are effects of independent causes.
e. Both statements I and II are effects of some common cause.

Answer: b) Statement II is the cause and statement I is its effect.
Explanation: Since the parents of aspiring students seeking admission to professional courses had launched a severe agitation protesting against the high fees charged by the professional institutes.
So, the committee was appointed by the Government on the fee structure of professional courses has drastically reduced the fees of the previous courses.
Hence the correct answer is B.

Example 2:
I – Police authority has recently increased vigil during the evening hours in the locality.
II – There has been a considerable reduction in the incidents of petty crimes in the locality.

a. Statement I is the cause and statement II is its effect.
b. Statement II is the cause and statement I is its effect.
c. Both statements I and II are independent causes.
d. Both statements I and II are effects of independent causes.
e. Both statements I and II are effects of some common cause.

Answer: a) Statement I is the cause and statement II is its effect.
Explanation: Since the police authority has recently increased vigil during the evening hours in the locality, therefore, the petty crimes have reduced considerably.

Clock problems

Introduction to clock problems

A clock is a complete circle having 360 degrees. It is divided into 12 equal parts i.e. each part is 360/12 = 30°.

As the minute hand takes a complete round in one hour, it covers 360° in 60 minutes. In 1 minute it covers 360/60 = 6°/ minute.

Also, as the hour hand covers just one part out of the given 12 parts in one hour. This implies it covers 30° in 60 minutes i.e. ½° per minute.

This implies that the relative speed of the minute hand is 6 - ½ = 5 ½ degrees.
We will use the concept of relative speed and relative distance while solving problems on clocks.

Some facts about clock problems

- Every hour, both hands coincide once. In 12 hours, they will coincide 11 times. It happens due to only one such incident between 12 and 1'o clock.
- The hands are in the same straight line when they are coincident or opposite to each other.
- When the two hands are at a right angle, they are 15-minute spaces apart. In one hour, they will form two right angles and in 12 hours there are only 22 right angles. It happens due to right angles formed by the minute and hour hand at 3'o clock and 9'o clock.
- When the hands are in opposite directions, they are 30-minute spaces apart.

- If both the hour hand and minute hand move at their normal speeds, then both the hands meet after $65\frac{5}{11}$ minutes.

Types of clock problems

Type 1: Finding the time when the angle between the two hands is given.
When the angle between the hands is not perfect angles like 180°, 90°, or 270°, the solving of the questions becomes difficult and time-consuming at the same time. The logic below provides a trick to address problems involving angles of hands for other than standard aspects.

*T = 2/11 [H*30±A]*

Where:

1. T stands for the time at which the angle formed.
2. H stands for an hour, which is running.

(If the question is for the duration between 4 o'clock and 5 o'clock, it's the 4th hour which is running hence the value of H will be '4'.)

3. A stands for the angle at which the hands are at present.

(The value of A is provided in the question generally)

The clock is divided into two parts: 1st and 2nd half as shown above

If the time given in the question lies in the first half, then the positive sign is considered while evaluating the time else, then the negative sign is used.

Example: At what time between 3 and 4 o'clock, the hands make an angle of 10 degrees?
Solution: Given: H = 3 , A = 10
Since both three and four lies in the first half considered a positive sign.
T = 2/11 [H*30±A]
T = 2/11 [3*30+10]
T = 2/11 [90+10]
T = 2/11 [100]

T = 200/11
T =18 2/11

The answer indicates that the hands of a clock will make an angle of 10 between 3 and 4 o'clock at exactly 3:18:2/11 (3' o clock 18 minutes and 2/11 of minutes = 2/11*60 = 10.9 seconds)

Type 2: Finding the angle between the two hands at a given time.

Example:
The angle between the minute hand and the hour hand of a clock when the time is 4:20 is:
Answer: 10 degrees.
Solution: At 4:00, the hour hand was at 120 degrees.

Using the concept of relative distance, the minute hand will cover = $\frac{20 \times 11}{2}$ =110 degrees
The angle between the hour hand and the minute hand is = 120-110 = 10 degrees.

Type 3: Questions on clocks gaining/losing time.
If a watch indicates 9.20, when the correct time is 9.10, it is said to be 10 minutes too fast. And if it indicates 9.00, when the correct time is 9.10, it is said to be 10 minutes too slow.
Such kinds of problems appear in exams very often, when a clock runs faster or slower than the expected pace.

The clock is running fast: It is also referred to as gaining time i.e. when a normal clock covers 60 minutes, a faster clock will cover more than 60 minutes.

The clock is running slow: It is also referred to as losing time i.e. when a normal clock covers 60 minutes, a slower clock will cover less than 60 minutes.

Example: A watch gains 5 minutes in one hour and was set right at 8 AM. What time will it show at 8 PM on the same day?
Answer: 9 pm
Solution: A correct clock would have completed 12 hours by 8 pm. But the faster clock actually covers 5 min. extra in one hour. So, it will cover 12 x 5 = 60 minutes extra. Therefore, when the correct clock would show 8 pm, the faster clock will show 60 minutes extra i.e. 9 pm.

Coding decoding problems

Introduction to coding-decoding problems

A code is 'a system of signals'. Coding is, therefore, a method of transmitting a message between sender and receiver which cannot be understood or comprehended by a third person. The Coding and Decoding test is mainly to judge the test-taker's ability to decipher a particular word/message by breaking the code or decoding the same.

Codes based on English alphabets

Alphabets	Code
A	1
B	2
C	3
D	4
.	.
.	.
.	.
Z	26

Series of opposite alphabets

A	B	C	D	E	F	G	H	I	J	K	L	M
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

Types of coding decoding problems

1. **Letter coding**

   Case - I: To form the code for another word (Coding)

   Example: In a certain code, TEACHER is written as VGCEJGT. How is CHILDREN written in that code?
   (a) EJKNEGTP
   (b) EGKNFITP
   (c) EJKNFGTO
   (d) EJKNFTGP
   Answer: d) EJKNFTGP
   Solution: Each alphabet in the word "TEACHER " is moved two steps forward to obtain the corresponding alphabet of the code.
   T E A C H E R=V G C E J G T (Each alphabet is increasing by 2)
   Similarly, we have
   C H I L D R E N=E J K N F T G D

   Case - II: To find the word by analyzing the given code (Decoding)

   Example: In a certain code language, the word ROAD is written as WTFI. Following the same rule of coding, what should be the word for the code GJFY?
   (a) REAP
   (b) TAKE
   (c) BEAT
   (d) LATE
   Answer: c) BEAT
   Solution: Each alphabet of the word is five steps behind the corresponding alphabet of the given code word.
   Hence, BEAT is coded as GJFY.

2. **Substitution**

   Example: If 'sky' is called as 'star', 'star' is called as 'cloud', 'cloud' is called as 'earth', 'earth' is called as 'tree', and 'tree' is called as 'book', then where do the birds fly?

(a) Cloud
(b) Sky
(c) Star
(d) Data Inadequate
Answer: c) star
Solution: Birds fly in the sky. The code for sky is star. Therefore, birds fly in the 'star'.

3. **Deciphering message word code**

Example: In a certain language, 'sun shines brightly' is written as 'ba lo sul', 'houses are brightly lit' as 'kado ula ari ba' and 'light comes from sun' as 'dopi kup lo nro'. What is the code for sun and brightly?
(a) ba sul
(b) sul lo
(c) lo ba
(d) ba nro
Answer: c)lo ba
Solution: In the first and third statements, the common word is 'sun' and the common codeword is 'lo'. So, 'lo' is the code for 'sun'. In the first and second statements, the common word is 'brightly' and the common code word is 'ba'. So, 'ba' is the code for 'brightly'. Hence, the answer is (c).

4. **Numerical code values assigned to words**

Example: If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, then what will be the code for SEARCH?
(a) 246173
(b) 214673
(c) 214763
(d) 216473
Answer: b) 214673
Solution: The alphabets are coded as shown:
ROSECHAIR=682173456
Therefore, in SEARCH, S is coded as 2, E is coded as 1, A is coded as 4, R is coded as 6, C is coded as 7, H is coded as 3. Thus, the code for SEARCH is 214673.

Data sufficiency problems

Introduction to data sufficiency problems

Data sufficiency questions test your knowledge of basic math facts and skills along with reasoning, analytical, and problem-solving abilities. Each data sufficiency item presents you with a question. You do not actually have to find the answer to the problem; instead, your challenge is to decide whether or not the information presented along with the question would be sufficient to allow you to answer the question.

Data sufficiency means you need to check whether the data given in the statements are sufficient to answer the question asked or not. You need to find a unique answer to the question asked. More than one answer is not allowed.

How to answer data sufficiency questions

First of all, you need to read the directions of a particular Data Sufficiency question very carefully as the examiner can change the directions, and even after solving all the questions correctly, you mark the wrong answers.

You need to remember the steps involved in solving a particular Data Sufficiency question and follow them in this particular order:
Check A(first statement)
Then check B(the second statement)
And lastly, if required combine the two statements to get the answer.

Do not make any assumptions while solving Data Sufficiency problems

Example 1: Three packages have a combined weight of 48 pounds. What is the weight of the heaviest package?

A. One package weighs 12 pounds.

B. One package weighs 24 pounds.

1. Statement A alone is sufficient to answer this question, but statement B alone

is not sufficient.

2. Statement B alone is sufficient to answer this question, but statement A alone is not sufficient.

3. Both statements together are needed to answer this question, but neither statement alone is sufficient.

4. Either statement by itself is sufficient to answer this question.

5. Not enough facts are given to answer the question.

Answer: 2) Statement B alone is sufficient to answer this question, but statement A alone is not sufficient.

Solution: Statement A is not sufficient to determine the weight of the heaviest package. It implies only that the combined weight of the other two packages is 36 pounds. (Eliminate options 1 and 4). Statement B alone is sufficient for it implies that the combined weight of two of the packages is only 24 pounds. Since the weight of the 24 -pound packages is equal to the combined weight of the other two packages, the heaviest package must weigh 24 pounds. (Eliminate options 3 and 5). Since statement B alone is sufficient to answer the question but statement A alone is not, answer this question as option 2.

Example 2: How many books are there on a certain shelf?

A. If four books are removed, the number of books remaining on the shelf will be less than 12.

B. If three more books are placed on the shelf, the total number of books on the shelf will be more than 17.

1. Statement A alone is sufficient to answer the question, but statement B alone is not sufficient.

2. Statement B alone is sufficient to answer the question, but statement A alone is not sufficient.

3. Both statements together are needed to answer the question, but neither statement alone is sufficient.

4. Either statement by itself is sufficient to answer the question.

5. Not enough facts are given to answer the question.

Answer: 3. Both statements together are needed to answer the question, but neither statement alone is sufficient.

Solution: Neither statement alone is sufficient to answer the question asked. Statement A alone implies only that the number of books on the shelf is 15 or fewer, and statement B alone implies only that the number of books on the shelf is 15 or more. (Eliminate options 1, 2, and 4). But the two statements taken together are sufficient to answer the question, for they imply that the number of books on the shelf is 15. (15 is the only integer that satisfies both statements A and B). Since neither statement alone is sufficient, but the two statements together are, answer this question as option 3.

Example 3: Directions for data sufficiency questions (3 and 4):

A. If data in statement I alone is sufficient to answer the question.
B. If data in statement II alone is sufficient to answer the question.
C. If data either in statement I alone or statement II alone are sufficient to answer the question.
D. If data given in both I & II together are not sufficient to answer the question.
E. If data in both statements I & II together are necessary to answer the question.

Question 3. Who is taller among P, Q, R, S & T?

I. S is shorter than Q. P is shorter than only T.
II. Q is taller than only S. T is taller than P and R.

Answer: C.

Solution:From I : P is shorter than only T, this means that P is taller than all Q, R & S, so T is tallest.

From II : Q only taller than S, so S is shortest, and Q is second shortest, Now T taller than P and R both, So tallest of all.

Question 4. What is the distance between point P and point Q?

I. Point R is 10 m west of point P and point S is 10 m north of point P.
II. Point Q is 10 m south-east of point R. Point S is 20 m north-west of point Q.

Answer: D)

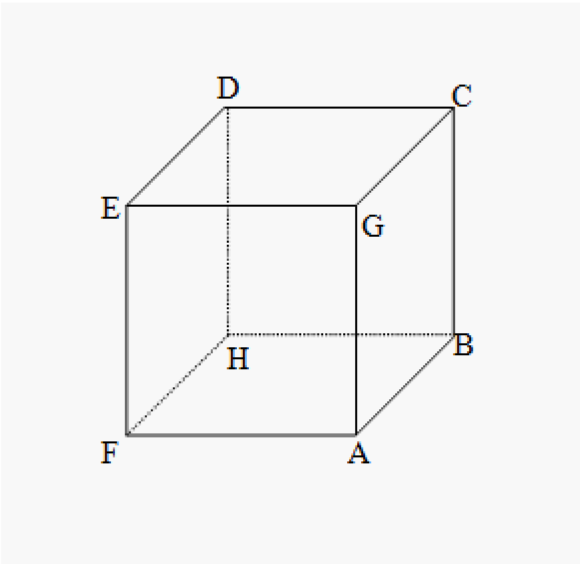
Solution:From I : No relation between points P and Q

From II : In this since we don’t know the angles between sides of triangle forming with points PQS and PQR, PQ cannot be determined.

Dices problems

Introduction to Dices problems

A Dice is a cube. In a cube, there are six faces. The six faces in the cube are– ABCG, GCDE, DEFH, BCDH, AGEF, and ABHF.



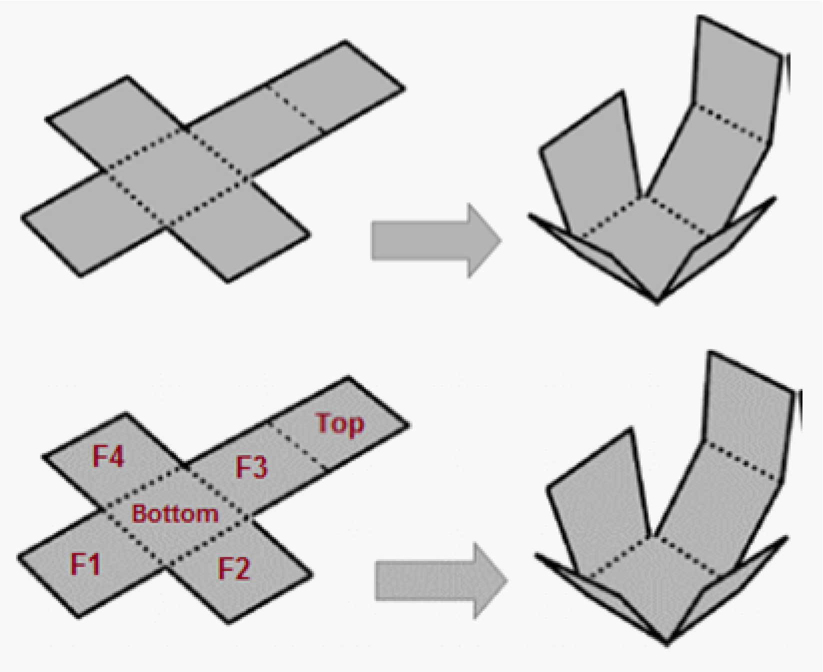
1. Four faces are adjacent to one face
2. There are pairs of opposing faces e.g. Opposite of DEFH is ABCG and so on.
3. CDEG is the upper face of the cube
4. ABHF is the bottom face of the cube.

Important facts

1. A cube has 6 square faces or sides
2. A cube has 8 points(vertices)
3. A cube has 12 edges
4. Only 3 sides of a cube are visible at a time and these sides can never be on the opposite side of each other
5. Things that are shaped like a cube are often referred to as cubic
6. Most dice are cube-shaped, with the numbers 1 to 6 on the different faces.

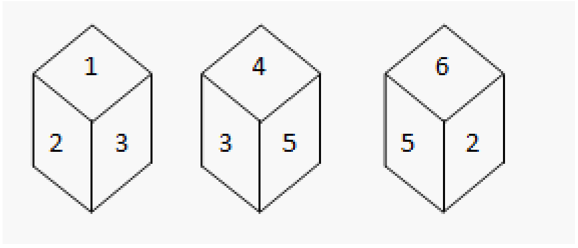
Deconstruction of a cube(flat view)

When we are given a dice it is somewhat difficult to visualize it in 3-D so what we do is that flatten the cube. We can form a cube that has been flattened where we can visualize, the square at the farthest end will give the top of the cube and the square that is the middle will form the base of the given dice. The given figures below can help you understand the theory stated above.



The rest of the square will give the adjacent sides of the dice. Note that we have to clearly visualize the adjacent sides and we have to figure out what exactly the question is asking. The flattening of dice is the easiest way that we can use to solve the dice problems.

Example: 1. Six faces of dice are marked with six numbers 1, 2, 3, 4, 5, and 6. This die is rolled three times and it shows three places:

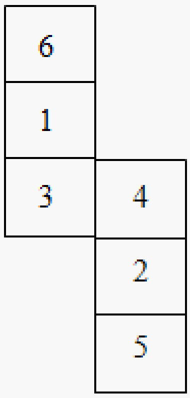


a) 2

b) 6
c) 5
d) 4

Answer: c) 5
Solution: 2 and 3 are adjacent to 1. So for the number opposite to 1, it should be the same. We observe from the second and third figures that 2 and 3 are adjacent to 5 . therefore the answer is 5.

Example: When the given figure is folded to form a cube then which face is opposite to the face with 2?



a) 2
b) 4
c) 1
d) 6

Answer: c) 1
Solution: Now, this is a peculiar type of flattened cube. However, the rules to solve the questions remain the same. Now, if you want to find the face that is opposite to the face with 2, you should consider it as the base. So, the top face (opposite face) will be the 1.

Embedded images Problems

Introduction to Embedded images problems

Embedded figures are the figures hidden inside another figure. In other words, a figure (x) is said to be embedded inside figure (y), if figure (y) contains figure (x) in it.

Embedded figure questions consist of a unique figure which is hidden or embedded in one of the four option figures. In such questions, all the options look the same and confusing. So candidates need to be careful while attempting such questions. Anyhow with practice, we can master these embedded figures. We must basically understand the structure of the given question figure and then proceed to find the correct answer figure.

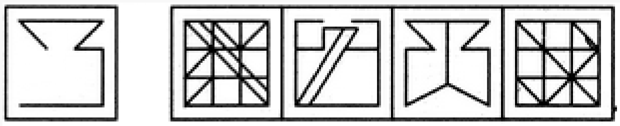
The embedded Images test is to assess how quickly a candidate can recognize a figure that is hidden among other figures. In each question, there is a model figure and four answer figures. The candidate has to look for the model from the answer figures.

Types of embedded image problems

Type 1: Complex answer figure

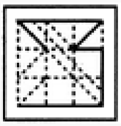
A question figure (x) is given followed by four complex option figures in such a way that the question figure (x) is embedded in only one of the given four options. The candidate must identify the correct figure in which figure (x) is hidden.

Example 1: Find out the alternative figure which contains figure (X) as its part.



a) 1
b) 2
c) 3
d) 4

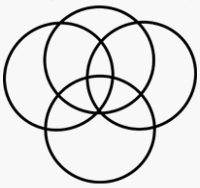
Answer: a) 1

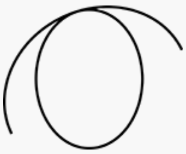
Solution: 

Type 2: Complex question figure

A complex question figure (x) is given followed by four option figures in such a way that only one of the option figures is embedded inside the given question figure. The candidate must identify the correct option figure which is hidden inside the question figure (x).

Example 2: In the given question a complex figure is given. Find out which of the simple figure given in the alternatives is hidden in the complex figure?



a) 

b) 

c) 

d) 

Answer: b)

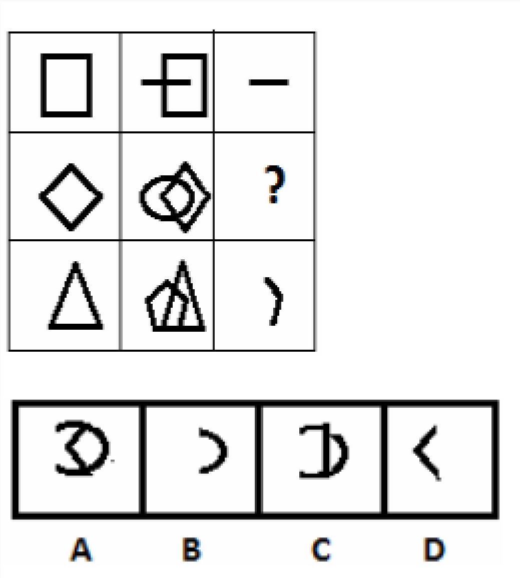
Solution: From the figure, the intersection part of right-most and bottom circles contains two curve lines inside the intersection part which looks the same as it is given in figure B, and also the part of the bottom figure which is extended towards the right.

Figure matrix

Introduction to Embedded images problems

Figure Matrix questions contain a grouping of diagrams or figures in the shape of a rectangular matrix. This arrangement of diagrams in the form of a matrix forms the Figure Matrix. Each diagram in the figure matrix is there as a result of some rule. You will have to figure out this rule and make necessary decisions using this knowledge.

Example: Select a suitable figure from the four alternatives that would complete the figure matrix.

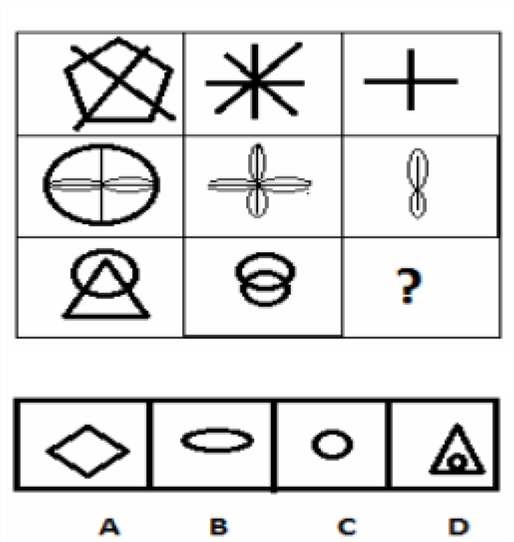


a) A
b) B
c) C
d) D

Answer: b) B

Solution: From the figure, we observe that the last column of the matrix contains the shape inside the second column image. Therefore the missing figure will be a semicircle.

Example 2: Select a suitable figure from the four alternatives that would complete the figure matrix.



a) A
b) B
c) C
d) D

Answer: c) C
Solution: The third column contains the figure which is in the second column but not in first. Therefore the missing figure will be a circle.

Input/output problems

Introduction to Embedded images problems

In such a type of reasoning-based question, you are given a word and number arrangement. With each subsequent operation, the arrangement of the words and numbers changes following a particular pattern. These operations are performed until a final arrangement is reached or is performed in a loop. You are required to identify the hidden pattern in the rearrangement and apply it to the questions.

Types of input-output problems

Ordering

Either the words are arranged alphabetically (forward/ reversed) or numbers are arranged in ascending/ descending order. The arrangement is usually based on the first letter of every word. Sometimes it is based on the last letter of every word. Both words and numbers could be arranged individually or simultaneously in each step. The rearrangement can start from the left or right side of the sentence and sometimes even simultaneously from both ends. The rearrangement could either start with a word or a number etc.

Example: Input: Cat Rail Snow Moon Fear
Step 1: Snow Cat Rail Moon Fear
Step 2: Snow Rail Cat Moon Fear
Step 3: Snow Rail Moon Cat Fear
Step 4: Snow Rail Moon Fear Cat
This is the final arrangement and STEP IV is the last step for this input.
What will be the last step of the following input?
INPUT: Care Steel Brick Nap Bomb Cry
Solution: The given rearrangement has a pattern that can be followed from the input step to the final step, which is Step IV. Observe carefully. The rearrangement follows the following patterns:

- The rearrangement is taking place from left to right.

- The rearrangement is taking place one word at a time.
- The rearrangement is done on the basis of descending alphabetical order based on the first letter of the word.

Mathematical Operations

Some mathematical operations (like squaring the number, adding the digits within the number, some common number added/subtracted/multiplied/divided to each number, etc.) are applied to the numbers in each step.

Example: Input : 26 34 56 78 63 99
Step 1: 20 34 56 78 63 90
Step 2: 20 30 56 78 60 90
Step 3: 20 30 50 70 60 90
Step III is the final step. Explain the operation.

Solution: In this example, the unit's digits of the left-most and right-most numbers are simultaneously being subtracted from the numbers themselves. This is followed by the number to the right of the left-most one, and to the left of the right-most one.

Shifting or interchanging

In this type the positions of characters/alphabets/words etc., in the input changes according to questions, following a particular pattern which repeats itself e.g. 'shift 1st character to last' or 'interchange 1st & last' etc.

Example: A computer rearranges a particular input using some operations 01, 02, 03, and 04.
Input: I am not coming home for dinner
Step 01: dinner not am coming home for I
Step 02: not dinner coming am home I for
Step 03: for coming dinner am home I not
Step 04: coming for am dinner home not IIf
Step 4 gives "I know you will not come back" what step will have "you back I come not will know"
Solution: Since the words remain unchanged here, so this is a case of either

rearrangement or shifting. Let us number each word for our convenience.
I = 1, am = 2, not = 3, coming = 4, home = 5, for = 6, dinner = 7
Input : 1 2 3 4 5 6 7
Step 01: 7 3 2 4 5 6 1
Step 02: 3 7 4 2 5 1 6
Step 03: 6 4 7 2 5 1 3
Step 04: 4 6 2 7 5 3 1
So, the logic being followed is:-
Step 01 = Swap 1st & last; 2nd & 3rd

Step 02 = Swap 1st & 2nd, last two & 3rd and 4th

Step 03 = Swap 1st & last, 2nd & 3rd.

Step 04 = 1st & 2nd, last two, & 3rd and 4th.

Since after two steps, the operation is repeated, hence you can guess the 5th, 6th, 7thsteps.

Mirror/ water images problem

Introduction to mirror/water images

The **Mirror Image** which we see in a plane mirror is a reflection of an object which will appear almost identical to the object but will be reversed in the manner in such a way that the image is exactly perpendicular to the plane mirror.

In the above image, if you closely look at the lady, you can understand that the left hand of the lady will appear to be her right hand and her right hand will appear as the left

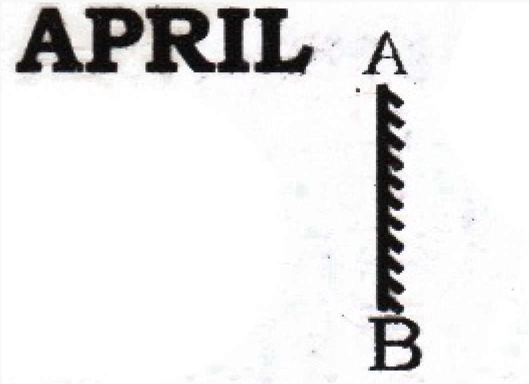
A reflection of a particular object in the water is what we call the **water image**.

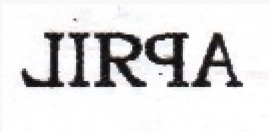
The above image is an example of a water image.

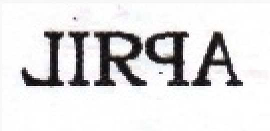
Difference between a mirror image and a water image

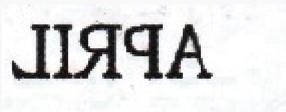
Water images in non-verbal reasoning are the same as the mirror images method. Basically, water image is just a reflection where the top and bottom part of the image changes where the left and right side of the image remains the same. In a mirror image, the left side and right side changes vice versa where the top and bottom remain the same.

Example: which of the given answer figures will be the correct image of the question figure?



a) 

b) 

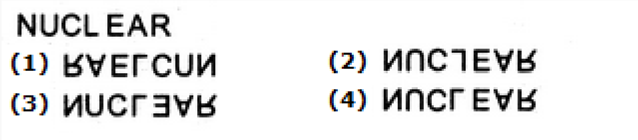
c) 

d) 

Answer: c)

Solution: Since the mirror is placed to the right of the word, we observe that option c can be the only answer.

Example: Choose the alternative which closely resembles the water-image of the given combination.



a) 1
b) 2
c) 3
d) 4

Answer: d)
Solution: Since the problem is asking for a water image, option d is the only answer possible.

Odd one out problems

Introduction to the odd one out problems

In this, out of the given options, you have to choose, which one is different or odd one out, i.e. one which is not related to others. These problems are considered easier than the problems.
The idea is to solve it with the help of basic common sense

Example 1: 10, 25, 45, 54, 60, 75, 80

a) 10
b) 45
c) 54
d) 75

Answer: c)
Solution: Each of the numbers except 54 is a multiple of 5.

Example 2: In each of the following questions, four words have been given of which three are alike in some way and one is different. Choose the odd one out.

1. Dollar

2. Peso

3. Ounce

4. Euro

Answer: 3)

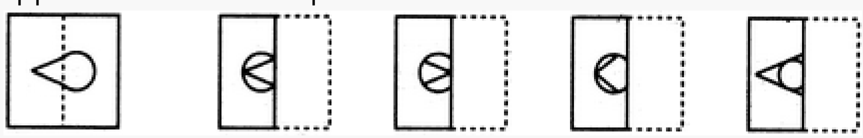
Solution: An ounce is a unit of weight

Paper folding

Introduction to paper folding

In this section, a sheet of paper is folded in a given manner, and cuts are made on it. A cut may be of varying designs. We have to analyze how this sheet of paper will look when the paper is unfolded. Note that when a cut is made on folded paper, the designs of the cut will appear on each fold. In each of the following examples, figures A and B show a sequence of folding a square sheet. Figure C shows the manner in which folded paper has been cut. You have to select the appropriate figure from alternatives that would appear when the sheet is opened.

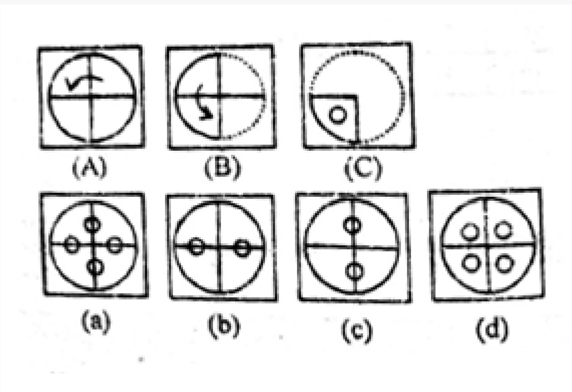
Example 1: Find out from amongst the four alternatives as to how the pattern would appear when the transparent sheet is folded at the dotted line



(X) (1) (2) (3) (4)

Answer: 1)

Solution: Simple observation



Example 2:

Answer: d)

Solution: Here, a circular cut is made on the quarter circle. Hence, this sheet, when completely unfolded, will contain a small circle on each quarter, and will appear as option d.

Ordering and ranking

Introduction to ordering and ranking

In Ordering and Ranking questions, the rank or position of a person from right/left, top or bottom of a class, or a row is determined. Also, a position or rank of the total number of people is to be calculated. Also, you may be asked to calculate the floor on which a person lives from the data given.

Points to remember while solving problems

1.The total number of a person/objects in a group or class is equal to one less than the sum of the positions of the same person from both the ends (either right and left or top and bottom). Since the same person is counted twice in the sum, the final answer is one less than the total sum.
Total number of objects/persons = [(sum of positions of the same person/object from both sides) – 1]

Example: In a row of persons, the position of Saket from the left side of the row is 27th, and the position of Saket from the right side of the row is 34th. Find the total number of students in the row? **(IBPS PO 2019)**

a) 60
b) 61
c) 62
d) 59

Answer: a) 60
Solution: Total number of students
= (Position of Saket from left + Position of Saket from right) -1
Total number of students = (27 + 34) – 1 = 61 – 1 = 60.
Hence the correct answer is option A.

2. The total number of persons/objects in a group is the sum of before or after the given person in a row and the position of the same person from the other side.
Total no. of persons/objects = No. of persons/objects before or after the given person in a row + Position of the same person from the other side.

Example: In a row of persons, the position of Aparna Nair from the left side of the row is 27th and there are 5 persons after her in the row. Find the total no. of persons in the row?
Solution: No. of persons in the row = Position of Aparna from left + No. of persons after Aparna
⇒ Total no. of persons = 27 + 5 = 32

3. If the positions of two objects/persons are given from the opposite ends and also the total number of persons/objects, then the problem can be addressed in two different ways to determine the number of persons between these two persons/objects.

Case 1: Overlapping

The total number of objects or persons in a group is always lesser than the addition of the position of two objects or persons from ends.

Example: The number of objects between two different persons = Total number of books – (Sum of positions of two different persons from opposite sides)
There are 24 students in the dance class, and the teacher is planning for an arrangement of students on stage. Samantha is 9th from the left side of the row and Supreetha is 22nd from the right side of the row. Find the number of dancers standing between the sisters Sampratha and Supreetha? **(Asked in bank exams)**

a) 4
b) 5
c) 6
d) 7

Answer:b) 5
Solution:Adding the position of Sampratha and Supreetha we get:
= 9 + 22 = 31
The result '31' is greater than the total number of students in a dance class.
Therefore the number of dancers standing between the sisters will be = [(Position of Sampratha from left + Position of Supreetha from right) – Total number of dancers – 2]
The number of dancers between Sampratha and Supreetha
= (9+22) – 24 – 2 = 31 – 24 – 2 = 5.
Hence the correct answer is option B.

Case 2: Non-Overlapping

The total number of objects or persons in a group is always greater than the addition of the position of two objects or persons from ends.

Example: There are 64 history books arranged in a row at central library Bangalore. Ancient history is 25th from the left side of the row and Medieval history is 30th from the right side of the row. What is the total number of books between Ancient and Medieval history?

a) 6
b) 7
c) 8
d) 9

Answer: d) 9
Solution: Adding the position of ancient and medieval history books, we get:
Ancient history Medieval history = 25 + 30 = 55
Hence the number '55' should be less than the total number of books.
∴ The number of books between ancient and medieval history = Total number of books – (Place value of Ancient history book from left + Place value of Medieval history from right)
The number of books between ancient and medieval history = 64 – (25+30) = 64 – 55 = 9
Hence the correct is option D.

4. If the data in the question provides only information of position of different objects or persons then it is impossible to find the total number of objects or people in a group or class. As the cases can either be overlapping or non-overlapping. In such a situation, the final answer will always be found. Save time by not trying to solve these types of questions.

Example: Deepavali or Diwali a festival lights in India. One can find the row of lamps in every house these days. Chaitra lights a row of the lamp in her home. A square-shaped lamp is at 18th from left, and a circular-shaped lamp is at 25th position in a row from right. Find the total number of lamps Chaitra had lit?**(Infosys 2019)**

a) 27
b) 30
c) 43
d) Can't be determined

Answer: d) Can't be determined
Solution: The scenario can be either be of Overlapping or non-overlapping one.
Hence the correct answer is option D.

5. Swapping of position to find the order/ ranking
In this section, the placement or the position of the two objects/persons are interchanged. The position of the two people or objects is examined before and after the interchanged.
The place value or the position of the second person from the same side as before interchanging
= Position of 2nd person from the same side before interchanging + (Position of 1st person after interchanging – position of 1st person before interchanging from the same side)

Example: Soldiers Punita and Mitali and are standing in a row of female soldiers. Punita is 18th from the left side of the row, and Mitali is 24th from the right side of the row. If they interchange their positions, Punita becomes 31st from left. Find:

1. The new position of Mitali from the right side
2. The total number of female soldiers in a row
3. Number of soldier between Punita and Mitali

Solution 1: The new position of Mithali from right side = Position of Mithali from the right side before interchanging + (Position of Punita from the left side after interchanging – Position of Punita from the left side before interchanging)
New position of Mithali from right side = 24 + (31 – 18) = 24 + 13 = 37
The new position of Mithali is 37th.
Solution 2: Total no. of persons = (B's position from right after interchanging + A's the position from left before interchanging) – 1
The Total number of female soldiers = (Mithali's position from right before interchanging + Punita's position from left before interchanging) – 1
= 37+18 -1 = 54.
Solution 3: To find the total number of people between any two persons.
No. of persons between A & B = (Position of A from left after interchanging– Position of A from left before interchanging) – 1
The total numbers of soldiers between Punita & Mithali = (Position of Punita from left after interchanging– Position of Punita from left before interchanging) – 1
= (31 – 18) – 1 = 13 – 1 = 12

6. If the positions of two objects from opposite sides of the row are known there is a third object right in the middle of the two, then the total number of objects can be evaluated based on the position of the third object.

Example: There is a pride of lions and their cubs in a row, the position of eldest lioness from the left side of the row is 9th & position of youngest lioness from the right side of the row is 8th. If the newborn cub is sitting just in the middle of eldest & youngest and position of cub from the left side of the row is 15th. Find the total number of lions in the row ? **(Asked in Google)**
Solution: The position of a cub from left is 15th, and the eldest lioness from left is 9th so there are 15 – 9 – 1 = 5 lions are sitting between eldest and youngest lioness. As the cub is sitting in the middle of the eldest and youngest lioness so there must also be 5 persons sitting between the youngest lioness and a cub.

Thus the position of a cub from right =
Position of youngest from right + 5 + 1 == 8 + 6 = 14
Total number of lions = (Sum of positions of cubs from both sides – 1)
= (15 + 14) – 1 = 29 – 1 = 28

Seating Arrangement

Introduction to Seating Arrangement

The questions on seating arrangement are regular features of almost every competitive examination. In these questions, you have to arrange a group of persons fulfilling certain conditions. This is also written as sitting arrangement or sitting arrangement reasoning at some places.

Types of Seating Arrangement

1. **Linear Arrangement:** Here the arrangement of the persons is linear i.e. you have to arrange them in a line. Here generally a single row of arrangement is formed.

2. **Double row arrangement:** In these questions, there will be two groups of persons. You have to arrange one group in one row and the other group in other rows. The persons in these rows normally face each other.

3. **Circular arrangement:** In the circular seating arrangement questions, you have to arrange the persons around a circular table, etc. fulfilling certain conditions.

4. **Rectangular arrangement:** These arrangements are almost similar to the circular arrangements; the only difference is that the persons are sitting around a rectangular table.

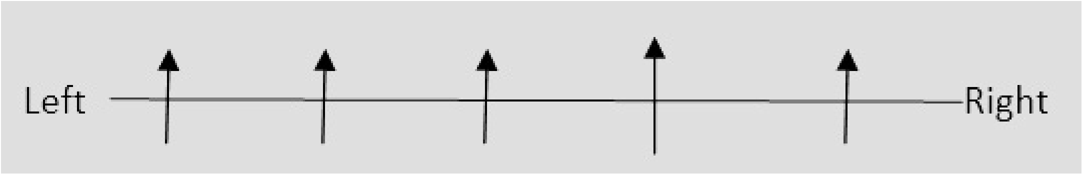
How to solve Seating Arrangement Problems

Questions on seating arrangement are generally asked in blocks of 4 – 5 questions. You are given some information and then there will 4 -5 questions based on the information. These questions have two types of information:

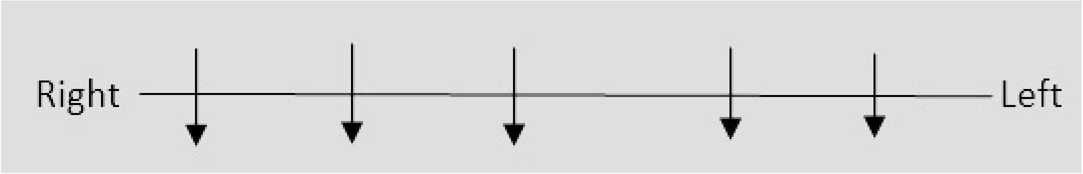
1. **Direct information:** This is the information that is clearly mentioned in the statement of the question. This is the information that you will use when you start solving the questions.

2**. Indirect information:** After filling in the direct information you will look for the connection between different parts of the information. These connections form indirect information.

While arranging the persons, the direction to which the persons are facing is very important.
Let us take the case of linear arrangements. Here if it is stated that there are five persons sitting facing North then the arrangement will be like

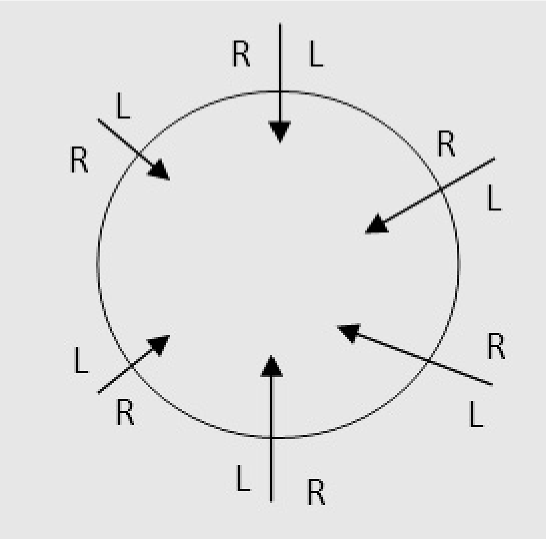


On the other hand, if these persons are sitting facing South then the arrangement will be like

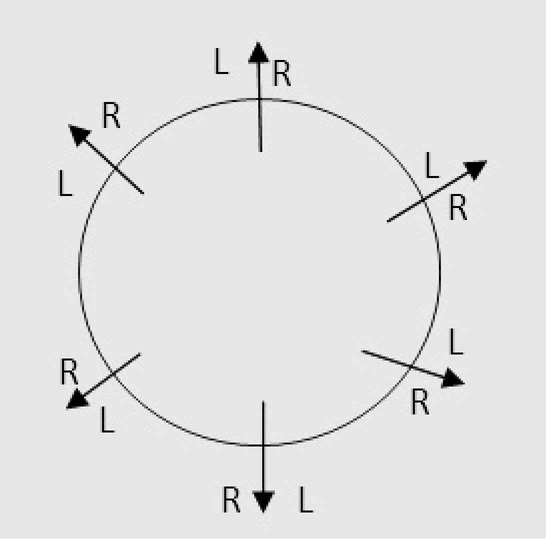


Similarly, if the arrangement is a double row arrangement, then one group of people will be facing north and the second will face south and the directions will be taken as similar to the above figures.

In the case of circular arrangements questions, or rectangular arrangements, the persons may be facing the center of the circle or they may be looking away from the center. If they are looking towards the center, then the right-hand side will be in the anticlockwise direction and the left-hand side will be in the clockwise direction as shown below:



If the persons are looking away from the center then the right-hand side will be in the clockwise direction and the left-hand side will be in the anti-clockwise direction as shown below:



The same concept of directions follows if the persons are sitting around a rectangular table.

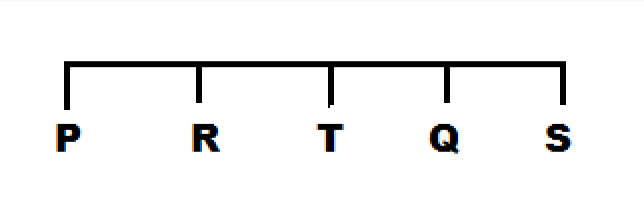
Next, while solving the questions related to linear arrangements or double row arrangements, the information regarding the position of the persons is very important. If it is written that A is sitting next to B, then it means that A and B are sitting together. B may be to the right or left of A. Further, if it is given that B is sitting to the right/left of A, then it does not mean that B is sitting immediately right/left of A. There may be some other persons sitting between A and B. If B is sitting immediate right/left of A then it will be mentioned in the statement of the question.

Example

Linear Arrangement:

Example 1:1. There are five students P, Q, R, S, and T who are sitting on a bench. T & Q are sitting together, T & R are sitting together, P is on the extreme left, Q is second from the extreme right. Who is sitting between P &Q? (SBI
(
A)Q & R
B)R & T
C)R & P
D)R & S

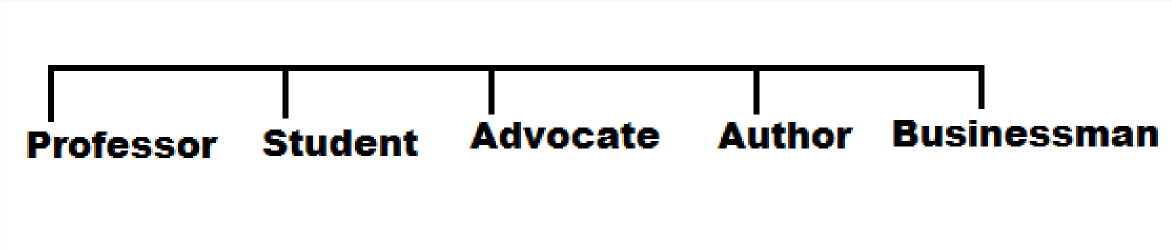
Answer:B)R & T

Solution: 

Example 2: Five persons are standing in one line. One of the two persons at extreme end is a professor and the other is a businessman. An advocate is to right of a student. An author is to the left of the businessman. What is the position of advocate from the left?

A) 2nd
B) 3rd
C) 4th
D) 1st

Answer: B)3rd
Solution:



Double row arrangement:

Example: Twelve persons were seated in two parallel rows containing six persons each, in such a way that there is an equal distance between adjacent persons. In row 1 – A, B, C, D, E, and F were seated and all of them are facing south and in row 2 – P, Q, R, S, T, and U were seated and all of them were facing north.**(IBPS 2018)**

P was seated second from the end. One person was seated between P and the one who faces A. B was seated to the immediate right of A. C was seated third to the right of D. Q faces the one who sits second to the left of B. One of the immediate neighbors of D faces S. T was seated second to the right of S. One person was seated between E and the one who faces U.

1. Who was seated second to the left of U?
    A) S
    B) P
    C) Q
    D) R
    E) T

Answer: C) Q

Solution: P was seated second from the end. One person was seated between P and the one who faces A. B was seated to the immediate right of A.
(Started with the person/variable whose position is fixed. Here P's position is second from the end. So, we need to find more information about P)

Case 1:

Row 1 (s)	___	___	B	A	___	___
Row 2 (n)	___	P	___	___	___	___

Case 2:

Row 1 (s)	___	B	A	___	___	___
Row 2 (n)	___	___	___	___	P	___

2. How many persons were seated between Q and the one who faces C?
   A) 3
   B) More than 3
   C) 1
   D) none
   E) 2

Answer: E) 2
Solution: C was seated third to the right of D. Q faces the one who sits second to the left of B.

Case 1:

Row 1 (s)	____	C	B	A	D	____
Row 2 (n)	____	P	____	____	Q	____

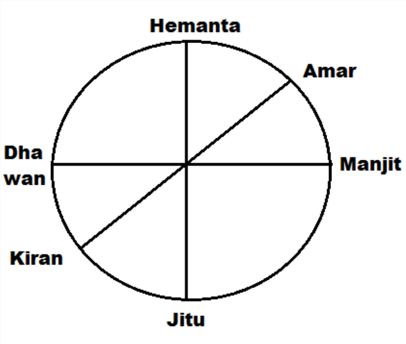
Case 2:

Row 1 (s)	C	B	A	D	____	____
Row 2 (n)	____	____	____	Q	P	____

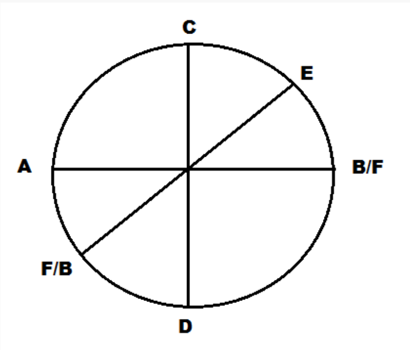
Circular arrangement:

Example1: Six friends were sitting around a circular table facing at the center Amar, Kiran, Jeetu, Hemanth, Dhawan, and Manjeet. Jeetu is sitting 2 places to the left of Amar and opposite to Kiran. If Dhawan and Manjeet are opposite to each other. Who is sitting left of Jeetu?

A)Dhawan
B)Manjeet
C)Kiran
D)Hemant
Answer: C)Kiran

Solution: 

Example 2: A, B, C, D, E, and F are sitting in a circular manner facing at the center. D is between F and B, A is second to the left of D, and second to the right of E. Who is facing A? **(Wipro hiring 2017)**
A)B
B)F
C)D
D)B or F
Answer: D)B or F

Solution: 

Rectangular arrangement:

Example 1: Study the following information and answer the questions given below. There are eight people viz. D, E, R, N, P, T, V, and A sitting around a square table. They have a different profession viz. Engineer, Soldier, Teacher, Pilot, Artist, Doctor, Politician, and Player but not necessarily in the same order. Four of them sit on the middle of the four sides while four of them sit on the four corners of the square table. All persons who sit at the four corners are facing the center except one, while those who sit in the middle of the sides are facing outward the center except one. T is neither Politician nor a Pilot. E is immediately left to the Player. N is not facing towards the center. Soldiers and Engineers are the neighbors of the Doctor. D is a Teacher but not facing towards the center. The Politician is the neighbor of both E and Soldier. The doctor is not facing towards the center. R and A are facing each other but none of them is in the middle of the sides. The Player is facing towards the center but not sitting in the middle of any side of the table. E sits second to the right of P, who is a Soldier. Z is not the neighbor of either E or P and sits second to the right of N.V is between Artist and teacher. **(SBI 2016)**

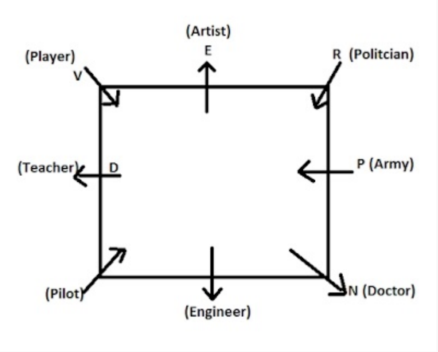
1. Who among the following is a Pilot?
    A) T
    B) N
    C) R
    D) Z

Answer: D) Z

2. The profession of N is
    A) Artist
    B) Pilot
    C) Doctor
    D) Teacher

Answer: A) Artist

Solution 1 & 2:



Shape Construction

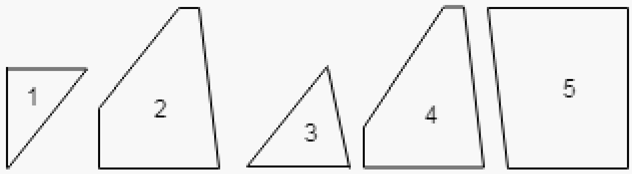
Introduction to Shape Construction

In this type of reasoning problem, you have to construct the shape by joining three pieces out of the five pieces given in the question.

Types of shape construction problems

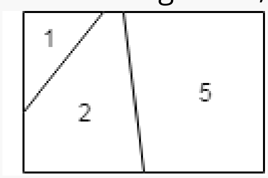
1. **Polygon forming**

Example1: Select three figures out of the following five figures which when fitted into each other would form a rectangle?(**IBPS 2017)**

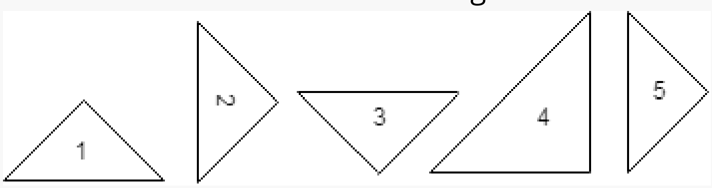


A) 1,4,5
B) 1,2,5
C) 1,3,4
D) 3,4,5

Answer: B) 1,2,5
Solution: Figures 1, 2, and 5 will form the rectangle as shown below:

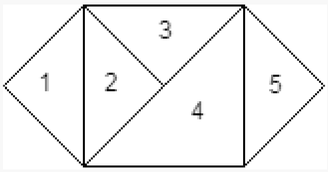


Example 2: Select three shapes out of the following five shapes which when fitted into each other would form a hexagon?



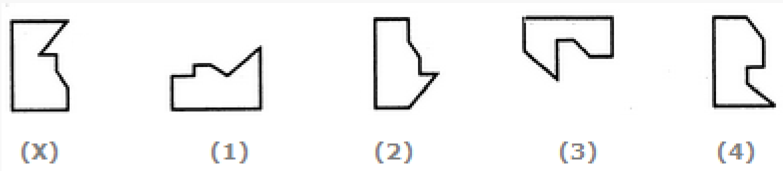
A) 2, 3, 4
B) 3, 4, 5
C) 1 3 5
D) All the figures

Answer: D) All the figures
Solution:All of the given shapes are required to form the hexagon, as shown below:



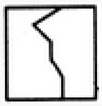
2. Shape Fitting:

Example 1: Select a figure from the given four alternatives which fits exactly into Figure-X to form a complete square. **(SBI PO 2018)**



A) 1
B) 2
C) 3
D) 4

Answer: B) 2

Solution: 

Example 2: Select a figure from the given four alternatives which fits exactly into Figure-X to form a complete square

A) 1
B) 2
C) 3
D) 4

Answer: D) 4

Solution: 

Statement and Assumption

Introduction to Statement and Assumption

In a statement and assumption question, a statement is given in the question followed by a few assumptions made on the basis of them. Candidates need to pick the assumption which most appropriate and logically is correct.

Understanding the terms

The first step to solve questions based on Statement, Assumption is to acquaint yourself with terms associated with such questions.

Statement Anything one says is a statement. But, in logic, a statement is either a meaningful declarative sentence that is either true or false or that which a true or false declarative sentence asserts. There may be several ways of framing the same statement. For instance, ‘It is desirable to put a child in school around 3 or so.’

Assumption An assumption is an unstated premise that we take for valid or granted or that supports the conclusion. The assumption is stated implicitly and needs to be identified. For example, in the above statement, that ‘ it is desirable to put a child in school around 3 or so’, the assumption is that ‘At that age, the child reaches appropriate level of growth and development and is ready to learn.’

How to solve Statement and assumption problems

Given below is a list of tips to solve the statement and assumption questions:

- Read the statement with an approach that the assumptions would be true with regard to the statement
- Do not go too logical with the statements. Analyze the information given and the assumption must only be made based on the information in the statement. Do not overcomplicate it
- Common assumptions can always be followed but other than that do not align the statement with General Knowledge or other facts
- Use the elimination method if you are unable to apprehend the answer. Read the statement and then the assumptions given in the options, you shall notice that a few of them will most definitely not follow. Eliminate them and then choosing from lesser options may prove to be more convenient

- One thing to make a note of is that the assumption is something that the Author believes to be true so while choosing the correct option, keep this thought in mind as well. If any option contradicts the statement, then that assumption will not follow

Example 1: **Statement:** Food poisoning due to the consumption of liquor is very common in rural areas
Assumption I: There are more illegal and unauthorised shops selling liquor in villages and rural areas
Assumption II: The ratio of people drinking liquor in villages is much more than that in towns

1. Both assumptions I and II follow
2. Neither assumption I nor II follows
3. Only assumption I follows
4. Assumptions II follows but assumptionI does not follow
5. Either assumption I or assumption II follows

Answer: Only assumption I follows
Solution:The statement is talking about food poisoning due to liquor so the number of people consuming liquor in towns or villages is not the main concern here. Which is why the only assumption I follow

Example 2: **Statement:** In an election conducted in Village X, only 20% of the total number of women in the village came to vote.
Assumption I: The number of men in the village is more than the number of women in the village X
Assumption II: Women had to cook food and could not come to vote

1.Both assumptions I and II follow
2.Neither assumption I nor II follows
3.Only assumption I follow
4.Only assumption II follows
5. Either assumption I or assumption II follows

Answer: Neither assumption I nor II follows
Solution:The statement clearly indicates that out of the total number of women in the village only 20% came around to vote so the ratio between the number of men and women is not applicable here and the second assumption is not applicable as well.

Example 3: **Statement:** "You are hereby appointed as a programmer with a probation period of one year and your performance will be reviewed at the end of the period for confirmation." - A line in an appointment letter.
Assumption 1: The performance of an individual generally is not known at the time of

appointment offer.
Assumption 2: Generally an individual tries to prove his worth in the probation period.

1. Only assumption I is implicit
2. Only assumption II is implicit
3. Either I or II is implicit
4. Neither I nor II is implicit
5. Both I and II are implicit

Answer: 5. Both I and II are implicit
Solution: The performance of the individual has to be tested over a span of time as the statement mentions. So, I is implicit. The statement mentions that the individual's worth shall be reviewed (during the probation period) before confirmation. So, II is also implicit.

Statement and Conclusion

Introduction to statement and conclusion

A statement is a group of words arranged to form a meaningful sentence. A conclusion is a judgment or decision reached after consideration of the given statement.

A conclusion is an opinion or decision that is formed after a period of thought or research on some facts or sentence stated by someone. A consequent effect has always to be analyzed before reaching the final result or conclusion of a given premise. This requires a very very systematic and logical approach.

How to solve statement and conclusion problems

- If there are two or more sentences that are used to frame a statement, then, the sentences must be interrelated, and mutual contradiction should be there.
- Do not look for truthful notions. The information provided in the statement is the only requirement for a candidate to answer the question. No assumptions must be made.
- Read the statement carefully and look for keywords that are common between the statement and the conclusions
- If there is more than one conclusion that is applicable to the statement, candidates must ensure that the conclusions they opt for have some relation with each other.
- Do not go by the length of the statement or statements. Make sure that you read the statement carefully before you make a conclusion.
- Candidates happen to lose a lot of marks in negative marking in such questions. So ensure that you do not guess the answers in this topic.

Example1 : **Statements:** The best way to escape from a problem is to solve it

Conclusions:

1. Your life will be dull if you don't face a problem.
2. To escape from problems, you should always have some solutions with you

A. Only conclusion 1 follow
B. Only conclusion 2 follow
C. Either conclusion 1 or 2 follows
D. Neither conclusion 1 nor 2 follows
E. Both conclusions 1 and 2 follows

Answer: D. Neither conclusion 1 nor 2 follows
Solution:Clearly, both I and II do not follow from the given statement

Example 2: **Statements:** Irregularity is a cause for failure in exams. Some regular students fail in the examinations
Conclusions:

1. All failed students are regular
2. All successful students are not regular

A. Only conclusion 1 follow
B. Only conclusion 2 follow
C. Either conclusion 1 or 2 follows
D. Neither conclusion 1 nor 2 follows
E. Both conclusions 1 and 2 follows

Answer: D. Neither conclusion 1 nor 2 follows
Solution: The given statement clearly implies that all irregular and some regular students fail in the examinations. This, in turn, means that all successful students are regular but not all regular students are successful. So, neither I nor II follows.

Example 3: **Statements:** The XYZ Medical College has started a cell which will conduct counselling workshops in the field of stress management to patients and general public
Conclusions:

1. The hospital has needed resources to start such activity
2. Patients and general public feel a need to have such cell in the hospital.

A. Only conclusion 1 follow
B. Only conclusion 2 follow
C. Either conclusion 1 or 2 follows
D. Neither conclusion 1 nor 2 follows
E. Both conclusions 1 and 2 follows

Answer: E. Both conclusions 1 and 2 follows

Solution: Since the hospital has started the activity, it must have been well-equipped for the same. So, I follows. Also, any new activity is started keeping in mind the need for it. So, II also follows

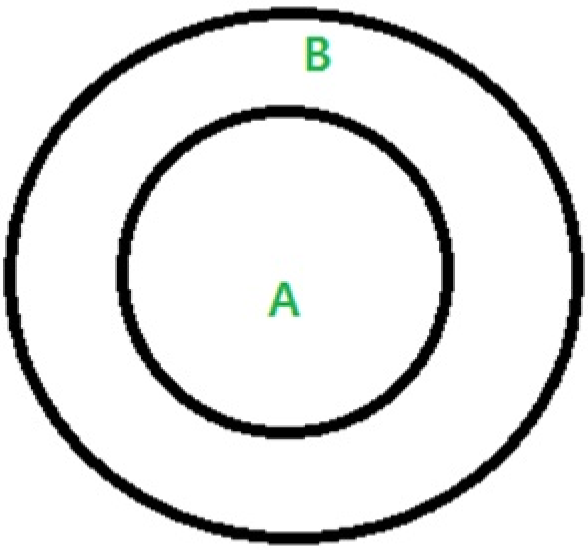
Syllogism

Introduction to Syllogism

Syllogism comes under the Verbal Reasoning Section and is a very important topic which is frequently asked in almost all the competitive exams. These types of questions are very simply framed. They contain generally two or more statements. These statements are then followed by a number of conclusions. Based on the statements, you have to find the authenticity of the conclusions. In simple words you have to find that from the given statements.which conclusions logically follows them. The most widelu used approach in solving these types of questions is the Venn diagram approach.

Types of Syllogism problem

1. **Type: All A are B**
   In this type of questions, first element is the subset of the second element.Representing it by Venn diagram, the pattern consists of a circle representing A, lying within a circle representing B.



Conclusions we get from the above pattern

- Some B are A.
- Some A are B.

Coding Ninjas Be A Ninja!

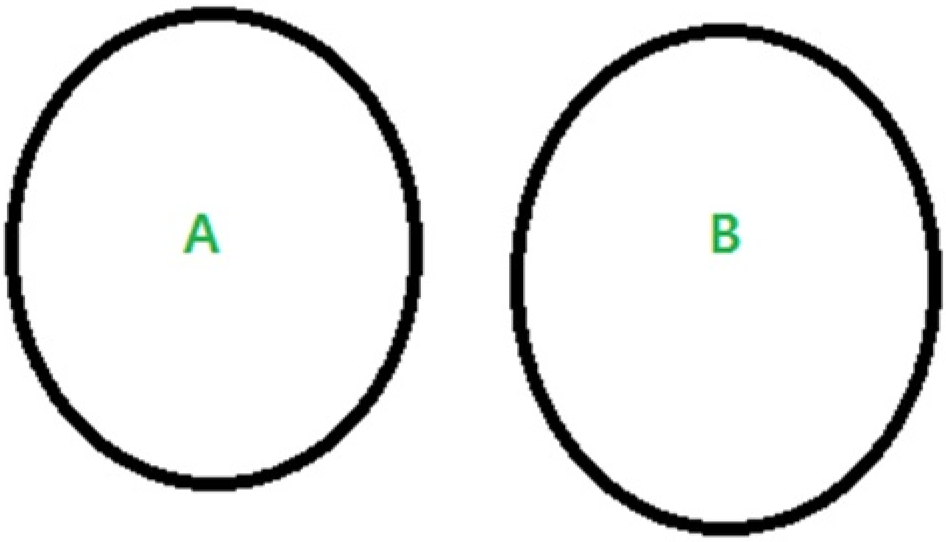
Example: All cats are animals.
Conclusion 1: Some animals are cats
Conclusion 2: Some animals are cats

1. Only conclusion 1 follows
2. Only conclusion 2 follows
3. Either 1 or 2 follows
4. Neither 1 nor 2 follows
5. Both 1 and 2 follows

Answer: Both conclusions follows

2. Type2: No A is B.

In this type of questions, first element is the not at all associated with the second element.Representing it by Venn diagram, the pattern consists of a circle representing A not intersecting the circle representing B.

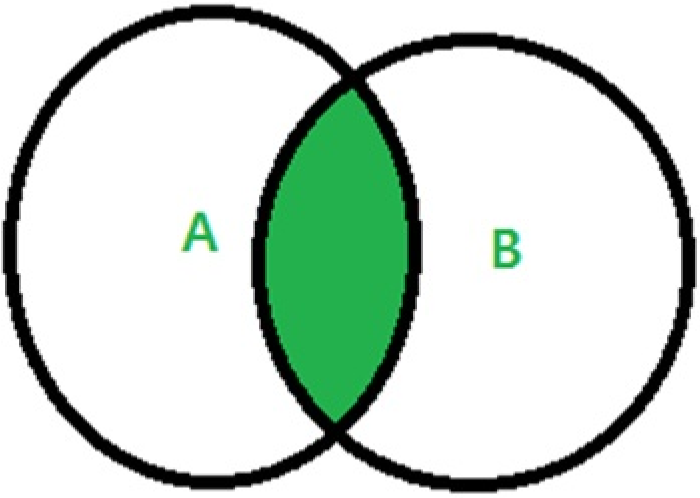


Conclusions we get from the above pattern:
No B is A

Example: No cats are animals.
Conclusion : No animals are cats.
Answer: The conclusion is correct

3. Type 3: Some A are B (Possibility Case)

In this type of questions, first element is having some part in common with the second element.Representing it by Venn diagram, the pattern consists of a circle representing A patially overlapping the circle representing B. The remaining portion of circle A is uncertain whether this portion touches B or not.



Conclusions we may get from the above pattern are based on possibilty and only one or a few out of them will be following the statement.

- Some A are not B
- All A are B.
- All B are A.
- All A are B and All B are A.

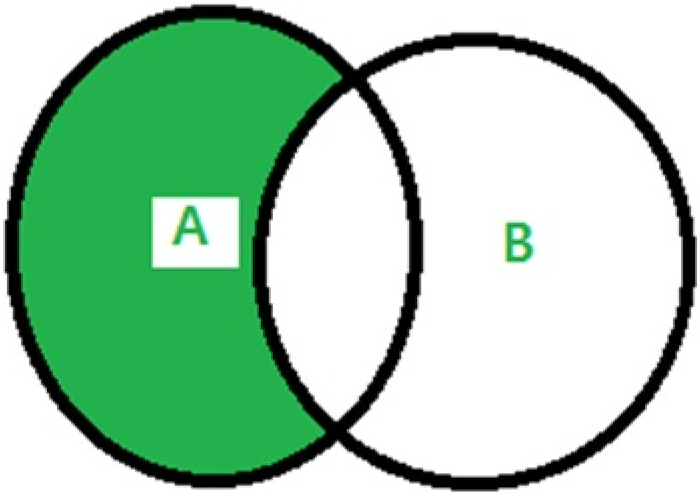
Example: Some bats are cats.
Conclusions:

1. Some bats are not cats
2. All bats are cats
3. All cats are bats
4. All bats are cats and All cats are bats

Answer: All of them

4. Type 4: Some A are not B.

In this type of questions, first element is having some part which is not common with the second element.Representing it by Venn diagram, the pattern consists of a circle representing A having atleast some part that is not overlapping the circle representing B. The remaining portion of circle A is uncertain whether this portion touches B or not.



Conclusions we may get from the above pattern are based on possibilty and only one or a few out of them will be following the statement.

- Some A are not B
- All A are B.
- All B are A.
- All A are not B and All B are not A.

Example: Some bats are not cats.
Conclusions:

1. Some bats are cats
2. All bats are not cats
3. All cats are not bats
4. All bats are not cats and All cats are not bats

Answer: All the conclusions follow

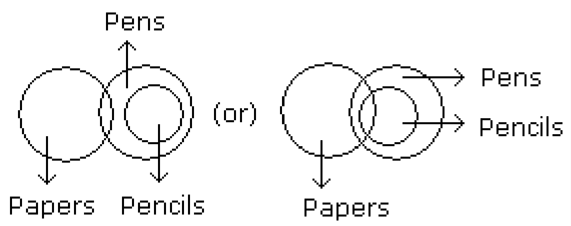
Example1: Statement: Some papers are pens. All the pencils are pens.
Conclusions:

1. Some pens are pencils
2. Some pens are papers

A. Only (1) conclusion follows
B. Only 2 conclusion follows
C. Either 1 or 2 conclusion follows
D. Neither 1 nor 2 conclusions follow
E. Both 1 and 2 follows

Answer: E.

Solution:



Both (1) and (2) follow.

Example2: Statement: Some papers are pens. All the pencils are pens.

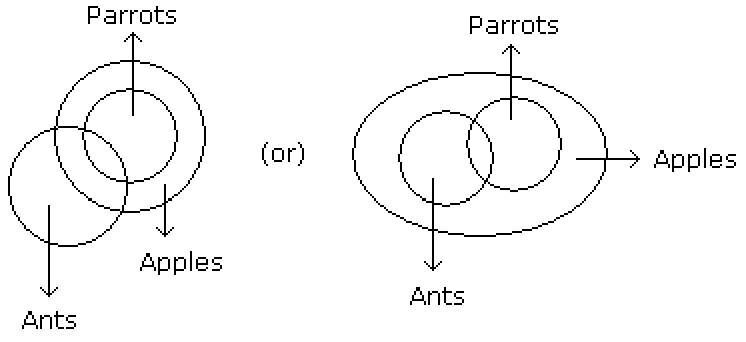
Conclusions:

1. Some pens are pencils
2. Some pens are papers

A. Only (1) conclusion follows
B. Only 2 conclusion follows
C. Either 1 or 2 conclusion follows
D. Neither 1 nor 2 conclusions follow
E. Both 1 and 2 follows

Answer: B

Solution:



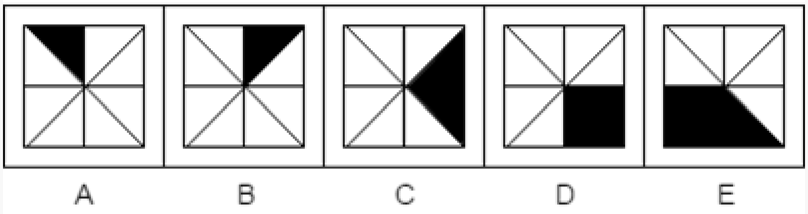
Only (2) follow.

Picture series and sequences

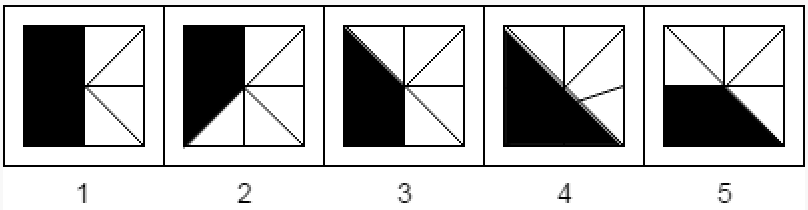
Introduction to picture series and sequences

This type of reasoning problem is based on a series of images, i.e. a question has five figures in a sequence; marked A, B, C, D, and E. These figures are called problem figures which depict a change step by step. The problem figures are followed by five answer figures; marked 1,2,3,4 and 5. You have to choose one figure out of five answer figures which will continue the series established by the problem figures.

Example: Which answer figure will continue the same series as established by the problem figures?

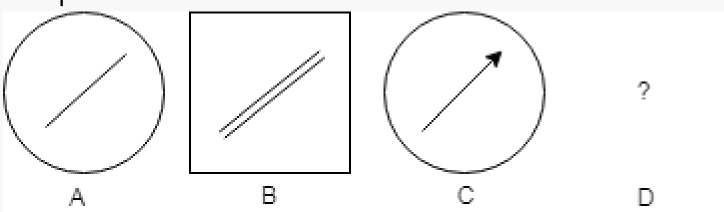


Options:

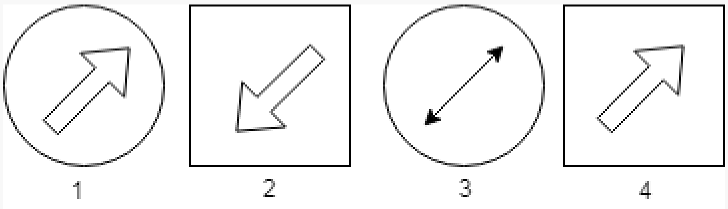


Answer: 3.
Solution: The shaded portion moves one step ahead clockwise. Also, an extra portion gets shaded after one step. So, the answer in figure 3 will continue the series.

Example 2: Which of the answer figures should be placed in the problem figure ?D? to complete the series?



Options:



Answer: 4.

Solution: In the series of problem figures, a circle alternates with a square, and a single line alternates with a single line arrow. So the answer figure 4 should complete the problem series, as it is a square and also contains a double line arrow.

HCF and LCM

Introduction to HCF and LCM

HCF:
The greatest number which divides each of the two or more numbers is called **HCF or Highest Common Factor**. It is also called the **Greatest Common Measure(GCM)** and **Greatest Common Divisor(GCD)**.

LCM:
Least Common Multiple(LCM) is a method to find the smallest common multiple between any two or more numbers. A common multiple is a number which is a multiple of two or more numbers.

How to find HCF

1. **By Prime factorization method:**
   i) Write down the ***prime factors*** of the given numbers.
   ii) Write down the ***prime factors*** which are common to both.
   iii) And products of the common factors will give you HCF of the numbers.

Example: Find the HCF of 150 & 375.
Answer: 75
Solution:
Step 1: Write down the prime factors of the given numbers.
$150 = 2 \times 3 \times 5 \times 5$
$375 = 3 \times 5 \times 5 \times 5$
Step 2: Write down the prime factors which are common to 150 & 375.
3,5 & 5.
Step 3: Products of the common factors are $3 \times 5 \times 5$
Hence, HCF = 75.

Note: To find HCF of more than 2 numbers
Let us take three numbers a,b & c.
To find their HCF, what you need to do is, first find out the prime factors of each of the numbers.
Say,

$$a = 2^3 \times 3^4 \times 5^1 \times 11^2$$
$$b = 2^5 \times 3^5 \times 5^2 \times 7^3$$
$$c = 2^6 \times 3^4 \times 5^3 \times 7^2$$

HCF (a,b,c) →All common prime factors with their ***lowest*** available power.
Thus, HCF of a,b,c will be

$$HCF=2^3 \times 3^4 \times 5^1$$

2. **By division method:**

If we were given two numbers, then

- First, divide the large number by a small number.
- If the remainder is left, then divide the first divisor by remainder.
- If the remainder divides the first divisor completely, then it is the HCF or highest common factor of the given two numbers.
- If the remainder does not divide the first divisor completely, then repeat the steps.

Example: What is the HCF of 120 and 100.
Answer: 20
Solution: Divide 120 by 100.
120/100 → 1 and remainder is 20
Now, divide the first divisor 100 by the first remainder 20
100/20 → 5 and remainder is 0.
Therefore, 20 is the HCF of 120 and 100.

3. **By shortcut method:**

When you talk about the common factor of two numbers X & Y. then the common factor has to leave the same remainder "zero". Which means
Let two numbers X & X+12, the only numbers that will have the possibility of leaving the same remainder zero would be factors of 12.

$$1 \times 12$$
$$2 \times 6$$
$$3 \times 4$$

All the common factors of these two numbers would come in the factors of 12, they can't come from any outer range. And hence, if all the common factors of X & X+12 are inside the factors of 12, So the HCF of X & X+12 would also come from the factors of 12. Which means HCF of X & X+12, can only be one of (1,2,3,4,6 & 12) these numbers.

Example: Find the HCF of 38 & 50?
Answer: 2
Solution:50-38=12, factor of 12 are 12,6,4,3,2&1.

12 →Does Not divide 38, so this is not HCF of these two numbers.
6→Does Not divide 38, so this is not HCF of these two numbers.
4 →Does Not divide 38, so this is not HCF of these two numbers.
3 →Does Not divide 38, so this is not HCF of these two numbers.
2 →Divide 38, so this is the HCF of these two numbers.
Then it is obvious it will divide 38+12 and hence HCF is 2.

Note: To find HCF of more than 2 number by shortcut method

Let us consider the numbers are x, x+12, y, z.
For finding the HCF of these numbers, take the differences between the numbers. Here, many differences are possible, but you have to choose the smallest difference between any pair of these numbers.
Write the factors of that number and the HCF of all these numbers would be from the factor list.

Sometimes you might want to go for prime number difference instead of the smallest difference,
For example, suppose the numbers are 44,56 & 93.

So, 56 - 44=12
93 - 56=37
93 - 44=49

Here, a better difference to take here is 37 because 37 is a Prime number, then the factors of 37 are either 1 or 37. So, HCF, in this case, is either 1 or 37. 37 does not divide any number, so, the HCF=1.

Example: A nursery has 363,429 and 693 plants respectively of 3 distinct varieties. It is desired to
place these plants in straight rows of plants of 1 variety only so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

Answer: 45 rows
Solution: The size of each row would be the HCF of 363, 429, and 693.
Difference between 363 and 429 =66.
Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.
66 need not be checked as it is even and 363 is odd. 33 divides 363, hence would automatically divide 429 and also divides 693.
Hence, 33 plants are the correct answer for the size of each row.
For the number of rows that would be required = Minimum number of rows required = 363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45 rows.

How to find LCM

1. **By prime factorization:**

Step1: Find the prime factor of two numbers a & b.
Step2: Write down all the prime factors that appear at least once in the numbers a & b.
Step3: Write all the prime factors with their highest power.
Step4: Products of all the prime factors with their highest power will give you LCM of a & b.

Example: Let's have two numbers 12 & 80.
Answer: 240
Solution: **Step1:** List the prime factors
$12=2 \times 2 \times 3$
$80=2 \times 2 \times 2 \times 2 \times 5$
Step2: Write down all the prime factors that appear, at least once in the numbers: 2,3,5.
Step3: Write all the prime factors with their highest power: $2^4 \times 3^1 \times 5^1$
Step4: The LCM = $2^4 \times 3^1 \times 5^1$
= 240.

2. **By shortcut method:**

As you saw LCM is the product of the highest power of all the prime factors, but that process would be very tedious, especially when the numbers are small.
When the numbers are small the logic of LCM builds around the ***Co-prime numbers***.
Co-prime Number: Two numbers are Co-prime to each other when they have no common factor among each other.

For example: (6, 13), (7, 11), (9, 19) etc.

Three numbers are Co-prime to each other when pairwise, each pair is Co-prime.
For example: Three numbers be a,b and c are Co-prime when,
a,b are Co-prime,
a,c are Co-prime,
& b,c are Co-prime.
All three pairs should be Coprime to each other, only then, a, b and c will be Co-prime.

NOTE: When a & b is Co-prime then the HCF should be 1.

Some important points about the Co-prime numbers:

(i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 etc.)
(ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 etc.)
(iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)
(iv) One prime number and another composite number (such that the composite number is not a
multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note
that 17 and 51 are not co-prime, as 51 is a multiple of 17)

Shortcut for LCM:
Step1: When the numbers are co-prime, then LCM is simply their product.
So, 7, 9 and 11 are co-prime, The LCM is $7 \times 9 \times 11$.

Step2: What to do when you have a mix of prime and Co-prime.

NOTE: (i). LCM has to be the multiple of HCF.
(ii). For any two numbers a & b, Product of two numbers $(a \times b) = LCM \times HCF$ (this formula is valid for two numbers)

Example: LCM of four numbers 42, 44, 18, 25.
Answer: 69300
Solution:(i) If you see any co-prime put them down in your LCM. Here you can see 18 & 25 are Co-prime (and 25,42 ; 25,44 are also Co-prime).
(ii) LCM of these numbers starts with $18 \times 25 \times$.... And
(iii) Now the logic of LCM should contain all the other numbers from the given numbers.
(iv) Out of the LCM, you should be able to construct 42 and 44 also.
(v) The factor of 42= $2 \times 3 \times 7$. Inside 18 you have 2 & 3, But you don't have 7 in 25 and 18. To construct 42, you should have a 7 in your LCM. (LCM=$18 \times 25 \times 7$....)
(vi) The factor of 44= $2 \times 2 \times 11$. Inside 18, you have one 2, but there is no 11 and other 2 in this LCM; so, to construct 44 you need to introduce 2 & 11 into the LCM.
So, LCM will be = $18 \times 25 \times 7 \times 2 \times 11$.

HCF and LCM of fraction

HCF of a Fraction:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

LCM of a Fraction:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Example: LCM & HCF of 1/2 ,5/7 and 8/11 are:
Answer: LCM= 40/1
HCF= 1/(2 x 7x 11)

Solution: LCM = LCM(1,5,8) / HCF(2,7,11)
HCF = HCF(1,5,8) / LCM(2,7,11)
So,
LCM= 40/1
HCF= 1/(2 $\times$ 7 $\times$ 11)

Example: Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

a) 4
b) 10
c) 15
d) 16

Answer: d) 16
Solution: L.C.M. of 2, 4, 6, 8, 10, 12 is 120.
So, the bells will toll together after every 120 seconds(2 minutes).
In 30 minutes, they will toll together (30/2)+1 16 times.

Averages

Introduction to Averages

An average is a number that measures the central tendency of a set of numbers. In other words, it is an estimate where the centre point of the set of numbers lies. Average is also known as the mean.

Another meaning of average is, the average is that single number, that can replace each of the given numbers present in the set with the average number and still get the same total.

For Example:

The average of 5 numbers 11, 14, 17, 18, and 20 is:

Average = $(11 + 14 + 17 + 18 + 20)/5$ = 80/5=16

This means that if you replace all the 5 numbers with 16 (average number), even then the sum will be 80, there would be no change in the total.

How to find the average

1. **Formula**

$$\text{Average} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

In mathematics, the average is equal to the sum of the set of numbers divided by the numbers of values in the sets

2. **Assumed Average Approach**

We already know that Average is that one number that can replace each of the numbers in a group of numbers and still keep the same total.

By using this concept the assumed average approach is a bypass for getting the average of the numbers.

Let us say 6, 10, 7 & 5 are the 4 numbers. So, Average is;

Average = (6 + 10 + 7 + 5)/4 = 7
7 can replace all the 4 numbers.
If you see the deviation between the numbers and their average (between left column and the right column), the direction should be: left column - right column

Left column	Right column	Deviation
6	7	- 1
10	7	+3
7	7	0
5	7	- 2

The net sum of all these deviations is 0 (-1+3+0-2 = 0). This means the average value is correct.

The following are some steps to calculate the correct average from the assumed average:
Step1. You have to assume an average.
Step2. Calculate how much the given numbers deviate from the assumed average.
Step3. Calculate the sum of all the deviations (i.e. Total deviation).
Step4. Calculate the average deviation with the help of the following formula :

$$\text{Average Deviation} = \frac{\text{Total Deviation}}{\text{Number of numbers}}$$

Step5. Now, the correct average will be equal to the sum of the assumed average and average deviation. i.e.
Correct average = Assumed average + Average Deviation.

Example: Let 37, 75, 83, 94 & 46 are 5 numbers. You don't know the average and you want to find out the average for these numbers without doing the sum of these numbers.
Answer: 67

Solution: **Step1.** For this example, assume an average of letting us say, 60.
Step2. Deviation calculation
60 to 37 there is a deviation of -23.
60 to 75 there is a deviation of +15.
60 to 83 there is a deviation of +23.

60 to 94 there is a deviation of +34.
60 to 46 there is a deviation of -14.

Step3. Total deviation = -23+15+23+34-14 = 35.
Step4. Average deviation = 35/5 = 7.
Step5. Correct average = 60+7 = 67.

You can assume any value of average, but the assumed value should be nearly equal to the one of the given value for simple calculation.

In the above example, let us say you assume the average to be 70 instead of 60.
Step1. Assumed average = 70.
Step2. Deviation calculation

70 to 37 there is a deviation of -33.
70 to 75 there is a deviation of +5.
70 to 83 there is a deviation of +13.
70 to 94 there is a deviation of +24.
70 to 46 there is a deviation of -24.

Step3. Total deviation = -33+5+13+24-24 = -15.
Step4. Average deviation = -15/5 = -3.
Step5. Correct average = 70+(-3) = 67.
So you can see that the answer will be the same irrespective of what average you take.

The benefit of the assumed average method is that it is much faster in the case when numbers are bigger and they are clustered (for example,a group of numbers between the range of 300 to 400), then your calculation is much faster than what you normally do.

NOTE : 1. Average of first n natural numbers = (n+1)/2
2. Average of first n even numbers = n+1
3. Average of first n odd numbers = n

Standard rules in average problems

1. **Standard Language In Average**

Every chapter has standard language inside it. You can also observe that there is some standard language inside the Average chapter.

You can understand the standard language on average with the help of some examples. So, here we understand the standard language with the help of the following examples:

Example 1: **Statement :** The average of 5 numbers is 12.
Explanation: When you see this statement two reactions come to mind. The 1st one is that, $5 \times 12 = 60$. and 2nd is that, add 12 five times i.e. 12 + 12 + 12 + 12 + 12 = 60. So, there are two approaches to tackle this statement.

Example 2: **Statement 1:** The average age of 24 students and principal is 15.
Solution: When you look at the statement you realize that there are 25 people with an average of 15. Your reaction is $25 \times 15 = 375$, that means the total age of 25 people is 375.

2. **Standard Situation In Averages**
   Situation 1:

This chapter is about identifying those standard situations that are generally asked in exams with the help of some examples. In the above 3 examples (example No. 2, 3 & 4) one thing is common that one new number is entering into the group.
In example 2. A Group of 24 students and principal added to it.
In example 3. Group of 9 innings and added 10th innings to it.
In example 4. It has an 11 monthly income and added 12th-month income into it.
Here the situation is entering a new number.

Example: Let us say you got 5 numbers with an average of 12 and 6th number entered and the average of all 6 numbers becomes 15. What is the 6th number?
Answer:
Solution: There are two ways of solving this type of question.

The 1st way;

6th number = Total of 6 numbers - Total of 5 numbers

= $6 \times 15 - 5 \times 12 = 30$

The 2nd way;

The addition of a 6th number increases the average by 3.

12 + 3 = 15
12 + 3 = 15
12 + 3 = 15
12 + 3 = 15
12 + 3 = 15

The +3 appearing 5 times is due to the 6th number, which is able to maintain the average of 15 first, and then 'give 3' to each of the first 5.
Hence, the 6th number in this case = **maintain + contribute**

$$= 15+3 \times 5 = 30$$

Standard situation 2:
The 2nd standard situation is about what happens if more than one number enters. This situation can also be explained with the help of examples:

Example: Let us say 8 numbers with an average of 10. Two new numbers enter due to that the average becomes 13. What is the average value of these two numbers?
Answer: 25
Solution: There are two approaches to solve this question;

1. Total difference approach:
   Total of two numbers = $10\times 13 - 8 \times 10 = 50$
   Thus, the average of two numbers = 50/2 = 25.

2. 2nd approach;
   The addition of 2 numbers, increases the average by 3.

Average of 8 numbers	Average after 2 number entry
10	13
10	13
10	13
…	…
…	…
8 times…	10times…

Average of two number = **maintain + average contribution**

= 13 + (3×8)/2

= 13 + 24/2 = 25

The contribution 24 has to be brought by these two together.
You can understand this situation like when you and your friend go to a hotel and you are going to be paid equally. Bill comes out of 24, then you will divide the bill into 2. So, each individual will pay 12.

Example: After 120 innings batsman has an average of 55. And he realizes that he is going to play 180 innings more and he wants an average of 100 runs per inning. So what should be the average of the remaining 180 innings?
Answer: 130
Solution: Average increases by 45runs.

Average in first 120 innings	Average after 300 innings
55	100
55	100
55	100
…	100
…	…
120 times…	300 times…

180 new innings maintain the average 100 and make the average contribution in 120 innings.

Average of remaining 180 innings = maintain + average contribution

$$= 100 + (45\times120)/180$$

$$= 130$$

Standard situation 3:

The 3rd standard situation that you will see in the average chapter is replacement of a number.

Example: A set of 5 numbers with an average of 13 and one number is replaced. Average is increased by 4. The outgoing number is 32, then find the replaced number?

Answer: 52

Solution: In this situation, there is an outgoing number and an incoming number and the average changes by 4 for 5 numbers. The difference in the total = (5×17) - (5×13) = 20.

Incoming number how much larger = (change in average) ×(number of numbers)

= 4 ×5 = 20

If the average increases then it is obvious that the incoming number is larger.

Incoming number - outgoing number = difference in total

Incoming number = 20 + 32 = 52.

Concept of weighted average

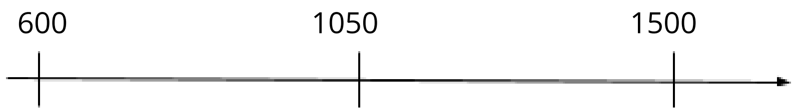
The concept of a weighted average can be understood with the help of an example.

Suppose I had to buy a T-shirt and jeans and let us say that the average cost of a T-shirt was 600, while that of jeans was 1500.

In such a case, the average cost of a T-shirt and jeans would be given by (600 + 1500)/2 = 1050.

This can be observed on the number line as:

(midpoint) = answer.



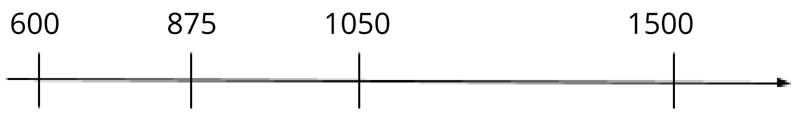
From the figure it is observed that the average occurs at the midpoint of the two numbers.

Now, let us try to modify the situation:

Suppose I had to buy 3 T-shirts and 1jeans. In such a case I would end up spending (600 + 600 + 600 +1500) = 3300 in buying a total of 4 items. So,

Average = 3300/4 = 825. Clearly, the average has shifted.

On the number line :



It is clearly visible that the average has shifted towards 600 (which was the cost price of the T-shirts, the larger purchased item.)

In a way, this shift is similar to the way a two-pan weighing balance shifts when the weights are put on it. The balance shifts towards the pan containing the larger weight.

Similarly, in this case, the correct average (875) is closer to 600 than it is to 1500. Since, this is very similar to the system of weights, we call this a weighted average situation.

Formula for weighted average:

Let say, we have k groups with averages A1, A2 ... Ak and having n1, n2 ... nk elements then the weighted average is;

$$Aw = \frac{n1A1+n2A2+n3A3+..................+nkAk}{n1+n2+n3+.........+nk}$$

Situations involving weighted average

Situation 1: Purchasing two kinds or k varieties of something and mixing them together, to form composite.

Example: Suppose I purchase 30Rs/kg rice and 70Rs/kg rice in the ratio 2:3. What is the average price of rice?

Solution : Average price = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30 , A2 = 70 , n1 = 2 , n2 = 3

Average price = (2× 30+3×70)/(2 + 3)

= 270/5 = 54Rs/kg.

Situation 2:

Example: Let's say you drive a car 30km/hr and 70km/hr and drive it for 2hr and 3 hr respectively. Find the average speed?

Solution : Average speed = (total distance)/(total time)

= (2× 30+3×70)/(2 + 3)

= 270/5 = 54 km/hr.

Situation 1 and 2 are the same but the story is different.

Situation 3:

Example: Let say you invest 2 lac and give 30% return. Investment of 3 lac rupees, give70% return. What is the average % return?

Solution : Average % return = (n1A1 +n2A2)/(n1+n2)

Here, A1= 30% , A2 = 70% , n1 = 2 Lac , n2 = 3 Lac

Average % return = (2× 30+3×70)/(2 + 3)

= 270/5 = 54 %.

Situation 4:

Example: There are two sections, in section 1 there are 20 students who scored 30 marks on an average in exam, while in section 2 there are 30 students who scored 70 marks on an average in exam. What is the average marks of both the sections?

Solution : Ratio of the quantities 20:30 = 2:3

So, Average marks = (2× 30+3×70)/(2 + 3)

= 270/5 = 54.

This situation can be modified into Boys and Girls in a class with ratio 2:3 and Boys average marks is 30 and Girls average marks is 70. So what are the average marks of the class?
Average marks of the class will be 54.

Situation 5: Alloys and Mixture

Example: Let say two water and milk solutions of 2L and 3L, In one solution milk is 30% and other solution milk is 70% respectively. Mix both the solutions then what is the % of milk in the mixture?
Solution : % of milk in the mixture = $(2\times 30+3\times70)/(2+3)$
$= 270/5 = 54\%$.
Instead of water milk solution, we can take gold and copper alloy, 2kg gold and copper alloy with 30% of gold & 3kg gold and copper alloy with 70% of gold. If both the alloys are mixed and a new alloy is formed, then what is the % of gold in the new alloy?
Solution : % of gold in the new alloy = $(2\times 30+3\times70)/(2+3)$
$= 270/5 = 54\%$.

These are some important situations that are used in weighted averages.

Alligation

Introduction to Alligation

The concept of alligation is closely related to the weighted average.

Alligations is a visual approach to solve weighted averages, involving the mixing of two groups.

For example:

Two varieties of rice at 50 per kg and 80 per kg are mixed together in the ratio 3 : 7. Find the average price of the resulting mixture.

Solution : By using weighted average formula; Aw = (n1A1 +n2A2) / (n1+n2)

$$\text{Average price} = (3\times 50+7\times 80) / (3 + 7)$$

$$= 710 / 10$$

$$= 70.$$

The weighted average approach is slightly slower than if we see the same situation through alligations. Alligations are a faster approach.

The mathematical formula for alligation:

In the case of a situation where just two groups are being mixed, we can write weighted average formula:

$$Aw = (n1A1 + n2\ A2\ ) / (n1 + n2)$$

Here, we have 2 groups with averages A1, A2 and having n1 and n2 elements respectively.

Rewriting this equation we get:

$$(n1 + n2)\ Aw = n1A1 + n2A2$$

$$n1(Aw – A1) = n2\ (A2 – Aw) \text{ or}$$

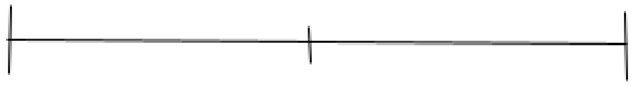
$$n1/n2 = (A2 – Aw)/(Aw – A1) \text{ .... The alligation equation.}$$

As a convenient convention, we take A1 < A2. Then, by the principal of averages, we get A1 < Aw < A2.

Situations in alligation problems

Situation 1: When A1, A2, n1, and n2 are known and Aw is unknown.

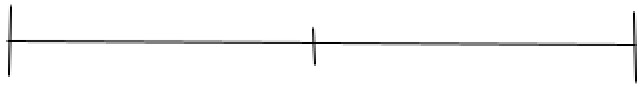
A1 Aw A2



n1 n2

For example:

50 Aw 80

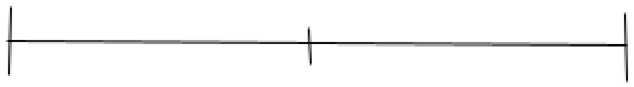


3 7

Since the total distance = (80 - 50) = 30. If we split 30 into 3:7, the value of 3 parts and 7parts are 9 and 21 respectively.
Thus the distance between Aw and 50 is corresponding to n2 (i.e. 7) and 7 parts are equal to 21.
I.e. Aw - 50 = 21 ≫ Aw = 71.

Situation 2: When A1, A2, and Aw are known and n1: n2 is unknown.

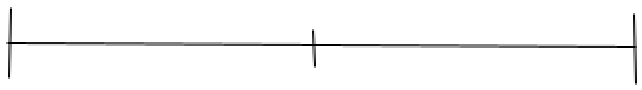
A1 Aw A2



n1 n2

For example:

70 80 84

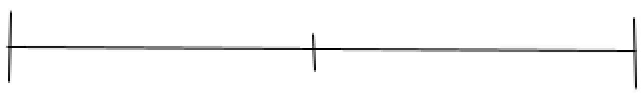


n1 n2

By using alligation equation,
n1/n2 = (A2 – Aw)/(Aw – A1)
n1:n2 = 4:10 or 2:5.

Situation 3: When A1, Aw, and n1:n2 are known and A2 is unknown.

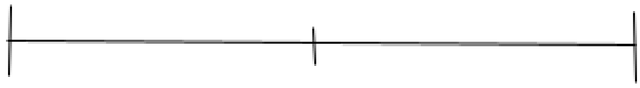
A1 Aw A2



n1 n2

For example:

30 70 A2



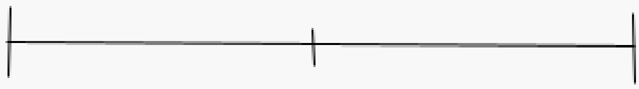
3 5

By using alligation equation,
n1/n2 = (A2 – Aw)/(Aw – A1)
3/5 = (A2 - 70)/(70-30)
A2 = 94.

Problems where we can use alligation-1

Example 1:
Two varieties of rice at 40 per kg and 50 per kg are mixed together in the ratio 2 : 3. Find the average price of the resulting mixture.
Solution :

40 Aw 50

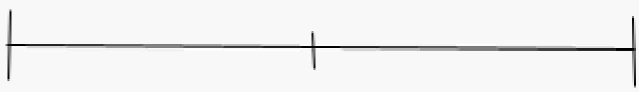


2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2 (i.e. 3) and 3 parts are equal to 6.
I.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average price of the resulting mixture is at 46 per kg.

Example 2:
A man has driven a car at 40kmph and 50kmph. He has driven for 2 hours and 3 hours respectively. Find the average speed of a car?
Solution :

40 Aw 50



2 3

Here, Aw is the average speed of the car.

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.
Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw = 46.
Hence, the average speed of the car is 46kmph.

These two questions are on the surface different from each other, the first one was talking about average price and the other is talking about the average speed, But structurally both are the same.
Equation in 1st question ;

Average price = (n1A1 +n2A2) / (n1 + n2).

Here, n1= 2kg, n2= 3kg , A1= 40per kg, A2= 50 per kg.
So,

Average price = (2*40 + 3*50)/(2+3)

Equation in 2nd question ;

Average speed = (t1S1 +t2S2) / (t1 + t2).

Here t1= 2hr, t2= 3hr , S1= 40kmph, S2= 50kmph.
So,

Average speed= (2*40 + 3*50)/(2+3)

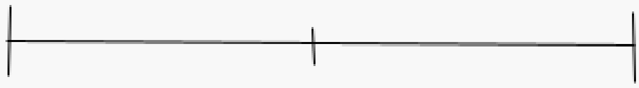
By looking at these two equations you will observe that these both are the same, only difference is in variables.

Problems where we can use alligation-2

Example 1:
We have two mixtures of milk and water, the 1st mixture contains 40% milk & 60% water and the 2nd mixture contains 50% milk & 50% water. These two mixtures are mixed in ratio 2:3, then find the % of milk in the mixture?

Solution : Using milk %

40	Aw (% of milk)	50



2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.

i.e. Aw-40 = 6

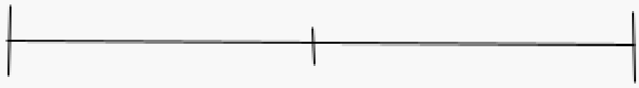
≫ Aw (% of milk) = 46%.

Another way to solve this question is by using water %

The 1st mixture has 60% water and the 2nd mixture has 50% water.

According to convention, we need A1< Aw < A2 and the ratio of 1st mixture to 2nd mixture is 2:3, this will be inverted here because we have to flip the % here to make it according to the given convention.

50 Aw (% of water) 60



3 2

Since the total distance = (60 - 50) = 10. If we split 10 into 3:2, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 50 is corresponding to n2(i.e. 2) and 2 parts are equal to 4.

i.e. Aw-50 = 4

≫ Aw (% of water) = 54%.

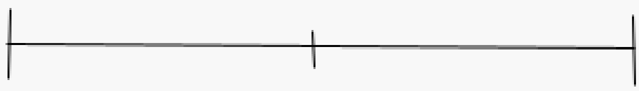
Thus; % of milk = 100 - 54 = 46%.

Example 2:

Anjali mixes 2 alloys of gold and copper in ratio 2:3. The 1st alloy contains 40% gold and the 2nd alloy contains 50% gold. Find the gold % in the mixture?

Solution :

40 Aw (% of gold) 50



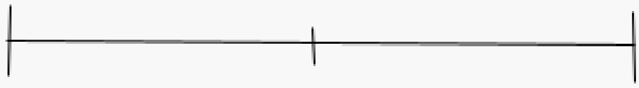
2 3

Since the total distance = (50 - 40) = 10. If we split 10 into 2:3, the value of 2 parts and 3 parts are 4 and 6 respectively.

Thus the distance between Aw and 40 is corresponding to n2(i.e. 3) and 3 parts are equal to 6.
i.e. Aw-40 = 6
≫ Aw (% of gold) = 46%.
Another way to solve this question is by using copper %.

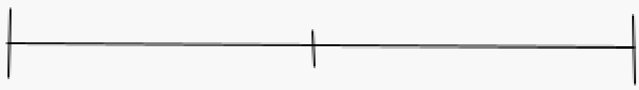
Problems where we can use alligation-3

Example 1:
A shopkeeper sold chairs and tables. The ratio of the cost price of chair and table is 1:2. He sold chairs at 30% profit and tables at 60% profit. What is the average % profit?
Solution :
30 Aw(average % profit) 60



1 2
Since the total distance = (60 - 30) = 30. If we split 30 into 1: 2, the value of 1 part and 2 parts are 10 and 20 respectively.
Thus the distance between Aw and 30 is corresponding to n2(i.e. 2) and 2 parts are equal to 20.
I.e. Aw-30 = 20
≫ Aw (average % profit) = 50%.

Example 2:
A shopkeeper sold chairs and tables. He sold tables at 20% profit and chairs at 30% loss. Thereby he made no profit or no loss in the transaction. What is the cost price ratio of table to chair?
Solution :
Chair **Table**
-30 0 20

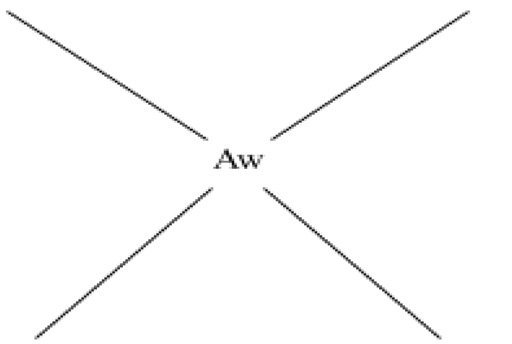


n1 n2
n1/n2 = (A2 – Aw)/(Aw – A1)
Here A1= -30, A2= 20, Aw= 0.
n1/n2 = (20 – 0)/(0 – (-30))

n1/n2 = 20/30 i.e. n1:n2 = 2:3.
Thus, table to chair cost price ratio = 3:2.

Cross diagram approach

A1 A2



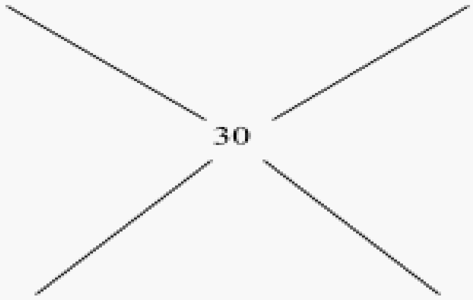
A2-Aw : Aw-A1

n1 : n2

Note: That the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.
As we have seen, there are five variables in the alligation equation.
The three averages A1, A2, and Aw. and the two weights n1 and n2.

Example 1:
On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find
(a) The ratio of students in the classes
(b) The number of students in the first class if the second class had 30 students.
Solution :

25 40



A2-Aw : Aw-A1

10 : 5

(a) The ratio of students in class is 10:5 i.e 2:1.

(b) If the ratio is 2: 1 and the second class has 30 students, then the first class will have 60 students.

Percentages

Introduction to Percentages

The basic definition of percentage is essentially out of 100. The percentage is derived from the French word 'cent'. The meaning of 'cent' in French is 100.
The percentage is used to compare data and numbers.
For example:
(a) If there are 5 (A, B, C, D, E) students who have taken the 12th board exam from five different boards. The percentages they get is a defined thing i.e. comparison between 5 diverse students in 5 diverse boards.

A	B	C	D	E
86%	92%	94%	78%	52%

By seeing the percentage of these students we can compare which student is better.

(b) GDP defines how the world is doing in terms of Global world economies. GDP compares different countries' economies in terms of their percentage.

Mathematically;
Any ratio if you multiply by 100, it gives you its percentage value. The percentage is denoted by the sign "%".

Why when the ratio is multiplied by 100, gives you a percentage value? You can see that from the unitary method.
Unitary method: It is a method which talks about a situation where two variables are moving linearly w.r.t. each other.

Example: You bought 10 bananas for 30 rupees then, how many rupees will you need to buy 15 bananas?
Solution: let x rupees you will need to buy 15 bananas.
10 bananas = 30Rs
15 bananas = x Rs
Cross multiply and equate;
$10*x = 15\times30$
$x = 45$.
So, 45 rupees is the amount that you will need to buy 15 bananas.

Example: You scored 10 out of 20 in a quiz and you want to put it in % then, how much out of 100 did you score?
Solution:

10 out of 20.

x out of 100.

So, by unitary method;

$20\times x = 10\times100$

$x = (10/20)\times100$

$x = 50\%$

NOTE: Any fraction multiplied by 100 gives its percentage value.

Concept of percentage change

Percentage always happens when you go from one number to the next number.
Basic structure of percentage change will always be in the situation, where you are talking about the difference between two numbers.
Let say we have number x becoming y. The percentage change between x to y.

$$x \longrightarrow y$$

Formula for percentage change:

Percentage change = (change/original value)*100

1st you have to identify which number is the original number that depends on which direction you are looking at percentage change. So, percentage change is always a **directional input**.
If x changes to y the percentage change going from x to y, will be having x as the original value.

$$x \longrightarrow y$$

If y changes to x. So, in this situation the percentage change will have to be seen from y to x and will be having y as the original value.

$y \longrightarrow x$

For example;
If you have two numbers 20 & 40. So, going from 20 to 40.

$20 \longrightarrow 40$

Here change = 40 - 20 = 20, and original value = 20.
% change = (20/20)×100 = 100%
The change is +ve. So, % change is increasing by 100%.

20 to 40 have different % change than coming from 40 to 20.

$20 \longleftarrow 40$

Here change = 20 - 40 = -20, and original value = 40.
% change = (-20/40)×100 = -50%
So, % change is decreasing by 50%.

NOTE: 1. In percentage change, there should be two numbers.
2. You need to understand which number is the original number.

People make a very common mistake in the % change calculation.
In the question given that 50 to 75, instead of this they calculated 75 to 50. Because the language of % change can get complex sometimes, where language structures are used especially in DI.

Percentage change graphics

It is an important concept in percentage change and important for chapters like interest, profit, and loss, etc. As the name suggests, percentage change graphics means the graphical method of doing the percentage change.

Basics of percentage change:

---

1. 100% of a number is a number itself.
2. 10% of a number is a shift of 1 decimal point on the number towards left.
3. 1% of a number is a shift of 2 decimal points on the number towards left.
4. 0.1% of a number is a shift of 3 decimal points on the number towards left and so on….

For example:
Let say a number N=52123.
100% of the number N is 52123
10% of the number N is 5212.3

1% of the number N is 521.23
0.1% of the number Nis 52.123

PCG has two structures:

Structure 1:
Given the starting value and the ending value. You have to calculate:
1. Absolute change (below the arrow).
2. % change(above the arrow).

$$S \xrightarrow[\text{Absolute change}]{\text{\% change}} E$$

Example:
Let us say, 40 changing into 52.

$$40 \longrightarrow 52$$

Absolute change = 52 - 40 = +12. Absolute change is +ve that means an increase in % change.
10% of the number 40 is 4, and the number 12, is 3 times the number 4 which means that the percentage increases by 30% (3 * 10% = 30%).

$$40 \xrightarrow[+12]{30\% \uparrow} 50$$

Structure 2:
1. Starting value is given to you,
2. Percentage change is given to you.
3. Absolute change you need to calculate.
4. And calculating the ending value.

Example1:
There is a number 40 that has to be increased by 30%.

$$40 \xrightarrow{30\% \uparrow}$$

Solution:
We were doing this problem by the unitary method.
40 is 100%

x is 130%

Cross multiply and equate;

x = (40×130)/100.

Rather than this, a much easy calculation is done through percentage change graphics.

$$40 \xrightarrow{30\% \uparrow}$$

10% of 40 is 4. 30% increase means adding 4, 3 times. 4+4+4 = 12 i.e adding 12 in 40 so the ans is 52.

$$40 \xrightarrow[4+4+4=12]{30\% \uparrow} 52$$

Example2:

The number 37 has to be increased by 13%.

$$37 \xrightarrow{13\% \uparrow}$$

Solution: In this question, you have to build up 13% by;

10% of 37 is 3.7

1% of 37 is 0.37

1% of 37 is 0.37

1% of 37 is 0.37

So, 13% of 37 is 4.81(3.7+0.37+0.37+0.37 = 4.81), adding 4.18 in 37. So, the answer is 41.81

$$37 \xrightarrow[+4.81]{13\% \uparrow} 41.81$$

PCG applied to percentage change:

The 1st structure under which you can use the percentage change in quantitative aptitude is product change situation.

Example1:
Let say a product $x \times y$. x is increased by 20% and y is increased by 30%. You want to find out what is the % change in the product?
Solution:
x would become x(1+ (20/100)) = $x \times 1.2$
y would become y(1+ (30/100)) = y ×1.3
So, in product; $x \times 1.2 \times y$ ×1.3= 1.56xy. This means, 56% change.

Same question can be done by PCG. If you assume your original product to be 100. And this product will go through two changes, 20% increase in x and 30% increase in y. You have to put two arrows,
One for 'x' and other for 'y'.

$$100 \xrightarrow[\text{'x'}]{20\%} \qquad \xrightarrow[\text{'y'}]{30\%}$$

If x increases by 20% the product also increases by 20% and then if y increases by 30% the product also increases by 30%.

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+36]{30\%} 156$$

i.e. 56% increase in the product.

Problems on percentage change:

Area and volume-based problem:

Problem 1:
The length of a rectangle goes up by 30% and the breadth of the rectangle comes down by 10%. What is the percentage change in area?
Solution:
Area = $l \times b$and now it becomes a product change situation.
Assume the original area = 100. Makes two arrows one for length and other is for breadth.

$$100 \xrightarrow[\text{'l'}]{30\% \uparrow} \qquad \xrightarrow[\text{'b'}]{10\% \downarrow}$$

$$100 \xrightarrow[+30]{30\% \uparrow} 130 \xrightarrow[-13]{10\% \downarrow} 117$$

Hence 17% is the increase in the area of the rectangle.

Problem 2:
The length of a rectangle is decreased by 20% and the breadth of the rectangle is increased by 23%. What is the percentage change in area?
Solution:
Area = $l \times b$and now it becomes a product change situation.
Assume the original area = 100. Makes two arrows one for length and the other is for breadth.

$$100 \xrightarrow[\text{'b'}]{23\% \uparrow} \qquad \xrightarrow[\text{'l'}]{20\% \downarrow}$$

For easy calculation, we put breadth on the 1st arrow and length on the 2nd arrow.

$$100 \xrightarrow[+23]{23\% \uparrow} 123 \xrightarrow[-24.6]{20\% \downarrow} 98.4$$

Hence, 1.6% is the decrease in the area of the rectangle.

We can do the same problem with the help of the following formula;
Percentage change = (a + b + ab/100)

Let us say, x increases by 20% and y increases by 10%. Then the percentage change;
Percentage change = 20 + 10 + $(20 \times 10)/100$
= 32%.

But rather than this PCG is a more easy way to solve this problem.

One other problem to this formula, if $x \times y \times z$situation occurs then the formula can not make a change 3 components of the of product. PCG is always better for these problems.

Expenditure and revenue problem:

Problem 1:
The price of a commodity has gone up by 20% and a person reduces its consumption by 10%. What is the % change in the expenditure?
Solution :
Price ×consumption = expenditure.
Assume the original expenditure = 100. Makes two arrows one for price and the other is for consumption.

$$100 \xrightarrow[\text{‘P’}]{20\% \uparrow} \quad \xrightarrow[\text{‘C’}]{10\% \downarrow}$$

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108$$

Hence, 8% is the increase in the expenditure of the commodity.

Problem 2:
A shopkeeper selling chairs, reduces the price of chairs by 20% due to which he gets an increment of 60% in the sale. What is the percentage change in the revenue?
Solution :
Price ×sale = revenue.
Assume the original revenue = 100. Makes two arrows one for price and other is for sale.

$$100 \xrightarrow[\text{‘P’}]{20\% \downarrow} \quad \xrightarrow[\text{‘S’}]{60\% \uparrow}$$

$$100 \xrightarrow[-20]{20\% \downarrow} 80 \xrightarrow[+48]{60\% \uparrow} 128$$

Hence, 28% is the increment in the revenue.

PCG applied to product constancy:

Product constancy is after the series of changes, you need to come back to the original value. Product constancy is applied in a lot of questions directly.

$$100 \longrightarrow \quad \longrightarrow \quad \longrightarrow 100$$

Problem 1:

Price of a commodity has gone up by 25% and the consumption is reduced such that the expenditure remains constant.

Solution :

Price ×consumption = expenditure.

Let 100 be the original expenditure after two change one on price and other on consumption, the expenditure should be back at 100.

After a 25% increment in price, expenditure becomes 125. So, 125 should be reduced by 25 to keep expenditure constant i.e consumption reduced by 20%.

$$100 \xrightarrow[+25]{25\% \uparrow} 125 \xrightarrow[-25]{20\% \downarrow} 100$$

25% increase in price is offset by a 20% decrease in consumption to keep expenditure constant.

Problem 2:

The length of a cuboid has increased by 20%, the breadth has increased by 50%. How much should you reduce the height to keep the volume constant?

Solution :

Volume = $l \times b \times h$

After 20% and 50% increment in length and breadth respectively, the volume becomes 180. So, 180 should be reduced by 80 to keep volume constant i.e height dropped by 44.44%.

Drop in height = (80/180)×100

= (4/9)×100 = 44.44%

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+60]{50\% \uparrow} 180 \xrightarrow[-80]{44.4\% \downarrow} 100$$

PCG applied on successive percentage change :

Successive percentage change use of PCG is structurally very similar to product change use of PCG. One small difference is that in product change we have seen that the arrows are interchangeable w.r.t. each other but in successive percentage change use of PCG we can not interchange the arrows because sometimes we need intermediate value, if we interchange the arrows then we do not get the exact intermediate value. You can understand that difference through some examples/problems.

Problem 1:
Population of the town goes up by 20% in 1st year, comes down by 10% in 2nd year and goes up by 5% in 3rd year. What is the % change in population after 3 years ?

Solution :
Let the population of the town is 100. Population after one year becomes 120 with an increase of 20%. Population after 2 year will become 108 and after the 3rd year the population will become 113.4.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[-12]{10\% \downarrow} 108 \xrightarrow[+5.4]{5\% \uparrow} 113.4$$

% change in the population after 3 years is 13.4%. But the intermediate value is important, if anyone asks what is the % change in population after 2 years.
If you interchange the arrows e.g 10% is placed on the last arrow.

$$100 \xrightarrow[+20]{20\% \uparrow} 120 \xrightarrow[+6]{5\% \uparrow} 126 \xrightarrow[-12.6]{10\% \downarrow} 113.4$$

Final value does not make a difference. But after two year the population value is wrong. If the question is built on intermediate value then you will go wrong if you do not keep the arrow constant as they are, that is the only difference in this.

A to B to A problems (compare two numbers) :

Very often we face a situation, where we compare two numbers, say A and B. In such cases, if we are given % change from A to B, then the reverse relationship can be determined by using PCG in the same way as the product constancy.

Problem 1:
A’s salary is 25% more than B’s salary. By what percent is B’s salary less than A’s salary?

Solution :

Let B’s salary = 100.

$$100(B) \xrightarrow[+25]{25\%\uparrow} 125(A) \xrightarrow[-25]{20\%\downarrow} 100(B)$$

A drop of 25 on 125 gives a 20% drop.

Hence B’s salary is 20% less than A’s.

NOTE: Product constancy table is also useful for this situation.

Problem 2:

B gets 20% more marks than A and C gets 50% more marks than B, then how much % less than C does A get?

Solution :

Lets A’s marks = 100.

$$\underset{\text{‘A’}}{100} \xrightarrow[+20]{20\%\uparrow} \underset{\text{‘B’}}{120} \xrightarrow[+60]{50\%\uparrow} \underset{\text{‘C’}}{180}$$

Coming back from C to A, a drop of 80 on 180 i.e 80/180 = 4/9. The fraction 4/9 is equivalent to 44.44%. Hence, A gets 44.44% marks less than C.

Ratio, proportions, variation

Introduction to Ratio, proportion, variation

The ratio is a method to compare quantities. When you compare the quantities the first thing that comes to mind is that the quantities should be in the same unit.

Example: 20kmph and 30kmph are the two quantities which are in the same unit.
So,
Ratio = 20/30 = 2/3
= 2:3.
If quantities are in different units, then they can't be compared.
For example:
20 km and 18Rs/kg are the two quantities in different units. So, these two quantities can't be compared.

Proportion basically equates to two or more ratios. When two ratios are equal, the four quantities composing them are said to be proportional. Thus if a/b = c/d, then a, b, c, d are proportional.
The proportion can be written as;
a:b::c:d, that means a is to b as c is to d. Also, it can be written as a:b = c:d.
NOTE: The terms a and d are called the extremes while the terms b and c are called the means.

- ➢ If four quantities are in proportion then the product of extremes and product of means are equal.
  Let a,b,c and d are in proportion. Then ; $a \times d = b \times c$ i.e, ad = bc.
- ➢ Sometimes the mean proportion is the same.
  Let say a:b::b:c is referred to as a continued proportion. Thus, the product of extremes is equal to the product of means.

$a \times c = b \times b$ i.e $b^2 = ac$ or we can say that $b = \sqrt{ac}$. So, b is called a geometric mean
between a & c.
NOTE: Mean proportion is always the geometric mean of extremes.

Example: Let us say 2:3::a:33. What is the value of a?
Solution: the product of extremes = the product of means
$2 \times 33 = 3 \times a$
a = 22.

Some properties of ratio and proportion

Ratio:

1. If we multiply the numerator and the denominator of the ratio by the same number, the ratio does not change.
   Thus, multiplying 'm' by both numerator and denominator of the same ratio gives,
   a/b = ma/mb
   For example :
   For Ratio = 3/4
   Multiply the numerator and the denominator by 6 i.e 3/4 = $(3 \times 6)/(4 \times 6)$= 18/24
   Here 3/4 is the **lowest/basic form** of a ratio. This lowest/basic form gives an infinite number of ratio values.
   For example :
   3/4=6/8=15/20=18/24=....................so on.

NOTE: In the lowest form of ratio the numerator and the denominator are always coprime numbers.

2. If we divide the numerator and the denominator of a ratio by the same number, then the ratio does not change. Thus;
   Dividing 'd', by both numerator and denominator or ratio a/b gives,
   a/b = $(a \div d)/b \div d)$

3. Dividing one ratio by another ratio can be expressed as a new ratio.
   Let the 2 ratios be 'a/b' and 'c/d'. Therefore,
   (a/b)÷(c/d) OR
   a/b:c/d = ad/bc
   For example:
   2/3:4/5 = $(2 \times 5)/(4 \times 3)$
   = 10/12.

4. The multiplication of two ratios a/b and c/d gives:
   a/b×c/d = ac/bd.

5. If a/b = c/d = e/f = k then;
   (a+c+e)/(b+d+f) = k.
   For example : 2/3 = 4/6 = 10/15 = 200/300 = k then,
   (2+4+10+200) / (3+6+15+300) = 216/324 = 2/3.

6. When numbers are added in both numerator and denominator to maintain equality, then the numbers should have the same ratio as that of the original ratio in which we are adding.
   Let say ratio = 400/800
   400/800 = (400+2)/(800+4) i.e a/b = (a + c)/(b + d) if and only if c/d = a/b.

7. In a ratio, if we add two numbers such that their ratio is larger than the original ratio, then the final ratio becomes larger.
   Let say a ratio = 400/800.
   (400+5)/(800+7).Here, ratio 5/7 is larger than the original ratio(400/800 =1/2).
   i.e c/d > a/b then (a + c)/(b + d) > a/b
   i.e. (400+5)/(800+7) > 400/800
   In case you add a smaller ratio than your final ratio will be less than the original ratio.
   Let say a ratio = 400/800.
   (400+3)/(800+7). Here, the ratio of 3/7 is smaller than the original ratio.
   i.e. c/d < a/b then (a + c)/(b + d) < a/b
   i.e. (400+3)/(800+7) < 400/800

8. If, some ratio is in fractional form, then to convert it into an integral ratio, multiply all fractions by LCM of their denominators.
   For example:
   1/2: 3/5: 7/6 to convert this ratio into integral ratio, multiply all the fractions by LCM of their denominators (2,5&6). LCM(2,5,6) = 30.
   i.e 30/2 : (3×30)/5 : (7×30)/6 = 15:18:35.

Proportions:

1. **Invertendo:** If a/b = c/d then b/a = d/c
2. **Alternando:** If a/b = c/d, then a/c = b/d
3. **Componendo:** If a/b = c/d, then (a+b)/b = (c+d)/d.
4. **Dividendo:** If a/b = c/d, then (a-b)/b = (c-d)/d.
5. **Componendo and Dividendo:** If a/b = c/d, then (a + b)/(a – b) = (c + d)/(c – d)

Chain Ratio

Chain ratio is a ratio in which one to next, next to the next, and next to next ratios are given.
Let say A: B, B: C, and C:D are chain ratios given and convert these ratios into A:B: C:D.
For example :
A:B = 3:5, B:C = 7:8 then, convert chain ratios into a single ratio A:B:C.
Here B is a common element in both the ratios. To equate 5 & 7, take LCM of 5 & 7.
LCM(5,7) = 35. To make common element 35. Multiply the ratios A: B and B: C by 7 and 5 respectively. Thus, A: B will become 21:35, and B: C will become 35:40. B is the same in both cases.
Hence A: B: C is 21:35:40.

Example: If there are 4 and 5 ratios in this case the LCM process will become tedious.
Let us say, A:B = 3:5, B:C = 7:8 and C:D = 9:13. Find A:B:C:D?
Solution :
We have already calculated A: B: C is 21:35:40 and we have C:D is 9:13. C is a common element in both the ratio. To equate 40 and 9, take LCM of 40 & 9.
LCM(40,9) = 360. To make common element 360. Multiply the ratio A: B: C and C:D by 9 and 40 respectively. Thus; A: B: C will become 189:315:360 and C:D will become 360:520. C is the same in both cases.
Hence A:B:C:D is 189:315:360:520.
If D: E is also there this will become even longer to do because you will have to take LCM 3 times.

Methods to solve chain ratio problems

Bypass method:

There is a bypass to this without doing LCM to convert it into a single ratio.
Let us say A: B is N1:D1, B: C is N2:D2, C:D is N3:D3, and D: E is N4:D4. Find A:B: C:D: E.
The value of A would correspond to the multiplication of all numerators. So, A would be N1N2N3N4.
The value of B would be D1N2N3N4.
The value of C would be D1D2N3N4.
The value of D would be D1D2D3N4.
And the value of E would be D1D2D3D4.

A B C D E

N1N2N3N4 : D1N2N3N4 : D1D2N3N4 : D1D2D3N4 : D1D2D3ND

Example: A: B is 3:5, B: C is 7:8, and C:D is 9:13. Find A:B: C:D.
Solution: A B C D
N1N2N3: D1N2N3: D1D2N3: D1D2D3
A B C D
$3 \times 7 \times 9 : 5 \times 7 \times 9 : 5 \times 8 \times 9 : 5 \times 8 \times 13$
A B C D
189 : 315 : 360 : 520

Example: There are three sections A, B, and C in a school. Section A & B have a student ratio of 5: 7. Section B & C have a student ratio of 8: 11. The number of students in section C is 154. What is the total no of students in all sections?
Solution: Given A: B is 5:7 and B: C is 8:11. A:B: C will be;
A B C
$5 \times 8 : 7 \times 8 : 7 \times 11$
A: B: C is 40: 56: 77.
The number of students in section C is 154.
Assume A= 40x, B = 56x and C=77x.
We have C = 154. Thus; 77x = 154, x = 2.
Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.
Total number of students in all sections = 80 + 112 + 154 = 346.

Multiplier logic

It is an important construct of thinking in a ratio situation.
In the last topic, we had a question about 3 sections in a class. In that, we had a ratio 40: 56: 77. And the number of students in section C was 154.
We assumed 3 numbers were 40x,56x, and 77x.
We had C = 154. Thus; 77x = 154,
x = 2. **Here x = 2 is a multiplier.**
Students in section A = 40×2 = 80. Students in section B = 56×2 = 112.
Total number of students in all sections = 80 + 112 + 154 = 346.

1st way in which a multiplier could be communicated to you:
Sometimes this multiplier will be communicated to you by giving you an individual value of one of the given numbers.
Let us say 3 children have toys in the ratio 3:4:9. The child with the largest number of toys is 36 toys.
i.e 9 is 36, Which means a multiplier of 4.
Hence, the number of toys with each child will be 3×4 = 12, 4×4 = 16 and 9×4 = 36.

2nd way in which a multiplier could be communicated to you:
Let us say the salary of three people is 5:7:13 and the total is 225.
The total ratio 5: 7: 13 is 25. And the total in the actual number running parallel to the given ratio is 225. i.e 25 is 225, which means a multiplier of 9.
Hence the numbers are $5 \times 9 = 45$, $7 \times 9 = 63$ and $13 \times 9 = 117$.

3rd way in which a multiplier could be communicated to you:
If a ratio of 5: 7: 13 is given. If the difference between the smaller two numbers is 18.
Difference between smaller two numbers = 7-5 = 2. So, 2 is 18, which means a multiplier of 9.
Hence the numbers are $5 \times 9 = 45$, $7 \times 9 = 63$ and $13 \times 9 = 117$.

Profit and Loss

Introduction to profit and loss

Profit and loss is an important topic of the arithmetic section of quantitative aptitude. You will find this chapter's application in certain DI questions as well. It is used to determine the price of a commodity in the market and understand how to profit an organization. Every product has a cost price and selling price. Based on these values we can calculate the profit and loss of a product.

Basic terms related to profit and loss

- **Cost price:** The price at which an item is purchased is called its cost price (C.P).
- **Selling price:** The price at which an item is sold is called its selling price (S.P).
- **Profit:** If the selling price of an item is more than its cost price, then there is a profit/gain on that item. i.e SP - CP = Profit/Gain.
- **Loss:** If the cost price of an item is more than its selling price, then there is a loss on that item. i.e CP -SP = Loss.
- **Marked Price:** The price that is marked on the article in shops is called as the Marked Price of that article, abbreviated as M.P.
  Between cost price and selling price, there is a % markup or markup % is defined.
  If CP = 100 and markup by 30% then MP should be 130. But when you sell you might also give a discount while selling.
- **Discount:** Discount is the amount given on the marked price by lowering the price.
  S.P = M.P – discount
  Or, Discount = M.P – S.P

Positive profit is a negative loss and negative profit is a positive loss.
For example :
If CP = 20 and And SP = 18. Then, Profit = 18 - 20 = -2. i.e negative profit is a positive loss.

NOTE: If the cost price and selling price of an item is equal then there is no loss and no profit on that item.

Basic formulas related to profit and loss

1. Profit = SP – CP
2. SP = Profit + CP
3. CP = SP – Profit
4. Percentage Profit = (Profit/CP)×100
5. SP = CP + Gain
   = CP + (Gain%/100) × CP
   = (1 + Gain%/100) × CP
6. Loss = CP – SP
7. SP = CP – Loss
8. CP = SP + Loss
9. Loss% = (Loss/CP) ×100

NOTE: Profit percent and Loss percent are always calculated on the basis of cost price (CP).

Example: A shopkeeper bought 10 mangoes for 80Rs and sold 8 mangoes for 96 Rs. What is the percentage profit?
Answer: 50%
Solution: In such a situation when the number of units bought and sell are different, then the first thing you will have to think is profit % can only be calculated when;
Number of units bought = number of units sold

For calculating profit % either calculate the selling price of 10 mangoes or you would have to look at the cost price of 8 mangoes.

10 mangoes bought for 80 Rs and 8 mangoes sell for 96 Rs.

CP of 1 mango = 8 Rs

SP of 1 mango = 12 Rs

Profit = 12 - 8 = 4 Rs/mango

% Profit = (4/8)×100 = 50% or

CP of 10 mangoes = 80 Rs

SP of 10 mangoes = 120 Rs

Profit = 120 - 80 = 40 Rs

% Profit = (40/80)×100 = 50%.

Types of questions asked in profit and loss

Type 1: Simple question based on profit and loss

Example: You bought an item of 800 Rs and you sold the item at a profit of 10%. What are the selling price and absolute profit?

Solution :
CP = 800Rs
% profit = 15. 15% of 800 = 800×15/100 = 120.

$$800 \xrightarrow[+120]{15\% \uparrow} 920$$

Hence SP = 920Rs. And absolute profit = 920 - 800 = 120Rs.

Example: A shopkeeper sold goods for 2000 at a profit of 25%. Find the cost price for the shopkeeper.
Solution :
SP = 2000Rs
%profit = 25.
% Profit = (SP - CP)/CP *100
CP = SP ×100/125.   CP = 2000×100/125 = 1600Rs.

Type 2: Problem on markup price and Discount

Example: The cost price of an article was 800 and it is sold at a discount of 10% and at a profit of 12.5%. What is the selling price and mark price?
Solution :
Using the PCG structure;

$$\begin{array}{ccccc} 800 & \xrightarrow[+100]{12.5\% \uparrow} & 900 & \xleftarrow[-100]{10\% \downarrow} & 1000 \\ \text{CP} & & \text{SP} & & \text{MP} \end{array}$$

CP = 800 , %Profit = 12.5, SP = CP + CP×% Profit , SP = 800 + 800×12.5/100 = 900.
Let Mark price = x , Discount = 10% , SP = MP - MP×Discount%
SP = x - x×10/100 = 0.9x and we have SP = 900.
Hence 900 = 0.9x , x = 1000.

Example: An item was sold at 639 after giving a discount of 10%. What is the original mark price of the item?
Solution :

$$\begin{array}{ccc} \text{MP} & \xrightarrow{10\% \downarrow} & 639 \\ & & \text{SP} \end{array}$$

This type of situation we have seen in % chapter. In PCG structure going from one side to the other side between 2 numbers. Here drop of 10% going from left to right side then there is an increment of 11.11% going from right to left.

$$\text{MP} \underset{11.11\% \uparrow}{\overset{10\% \downarrow}{\rightleftarrows}} \begin{matrix} 639 \\ \text{SP} \end{matrix}$$

11.11% equivalent to 1/9. So, 1/9 of 639 = 71.
Hence mark price = 639 +71 = 710.

Type 3:

Example: An item is sold at a profit of 16%. If it was sold at 20Rs more. The net profit would have been 20%. Find the cost price of the item?

Solution :

Let original cost price = x.
% profit = 16%.

$$\begin{matrix} x \\ \text{CP} \end{matrix} \xrightarrow[+0.16x]{16\% \uparrow} \begin{matrix} 1.16x \\ \text{SP} \end{matrix}$$

New SP = 1.16x + 20
New profit = 20%

$$\begin{matrix} x \\ \text{CP} \end{matrix} \xrightarrow[+0.2x]{20\% \uparrow} \begin{matrix} 1.2x \\ \text{SP} \end{matrix}$$

Hence 1.16x + 20 = 1.2x ,   x = 500.

2nd method;

Let CP = 100.
SP in 1st case when profit = 16%

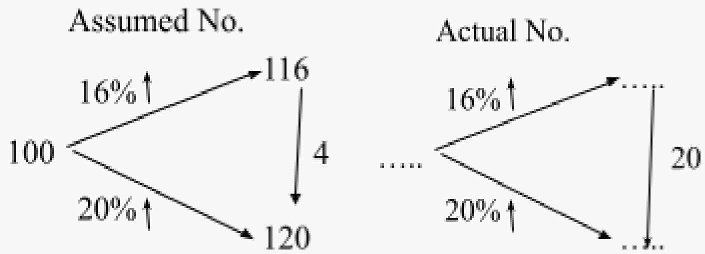
$$\begin{matrix} 100. \\ \text{CP} \end{matrix} \xrightarrow[+16]{16\% \uparrow} \begin{matrix} 116 \\ \text{SP} \end{matrix}$$

P in 2nd case when profit = 20%.

$$\begin{matrix} 100. \\ \text{CP} \end{matrix} \xrightarrow[+20]{20\% \uparrow} \begin{matrix} 120 \\ \text{SP} \end{matrix}$$

Difference between two SP = 120 - 116 = 4.

These problems always have a parallel actual set of numbers.
Parallel number to 100 which is not known.
Parallel number to 116 which is not known.
Parallel number to 120 which is also not known.
There is a parallel number to 4 which is 20.



Between 4 & 20 there is a multiplier of 5. You can apply a multiplier of 5 to any of these numbers to find which is asked.
Hence CP = 100×5 = 500.

Type 4: Multiple transaction question

Example: A manufacturer who sells his items to a wholesaler at a profit of 20% and wholesaler sells it to a shopkeeper at a profit of 20% and shopkeeper sells it to a customer at a loss of 15%. What % above the manufacturer cost were the items sold at?
Solution :
Let manufacturer CP = 100.

$$\underset{CP}{100} \xrightarrow[+10]{10\% \uparrow} 110 \xrightarrow[+22]{20\% \uparrow} 132 \xrightarrow[-19.8]{15\% \downarrow} 112.2$$

Hence, items sold 12.2% more than the manufacturer cost.
If in this question it is given that the customer bought the items for 56100.
112.2 would correspond to 56100 then, the multiplier will be 56100/112.2 = 500.
The multiplier would be constant between assumed value and actual value.
CP of manufacturer = 100×500 = 50000.

Type 5: Dishonest shopkeeper question

Example: A shopkeeper professes to sell at CP and he cheats the customer by 10% (on weight) while selling. What is % profit to the shopkeeper?
Solution :

Assume that he sells 1kg = 1000gm and the price of each gm is 1Rs. CP of 1000gm = 1000Rs. His SP for 1000gm is also 1000 Rs.
But the only problem is while selling 1000gm, he only gives 900gm because he cheats the customer by 10%.
SP of 900gm is 1000Rs.
In profit and loss problem if money is equated, Money got = Money given, **then you can use the formula for % profit;**
% Profit = (Goods left / Goods sold)×100.
Hence % profit to shopkeeper = (100/900)×100 = 11.11%. OR
CP for 1000 gm = 1000 Rs
SP for 900 gm = 1000 Rs
So, CP for 900 gm = 900 Rs.
Hence % profit to shopkeeper = (100/900)×100 = 11.11%.

Example: A man sells 2 items 1 at a profit of 20% and other at a loss of 20% and SP of both the items are equal. What is his % profit or loss?
Solution :
If a man sells two items at the same price in which he sells one at a profit of x% and the other one at a loss of x%, then **the result will always be a loss percent of [x/10]^2 %**
Here x is 20. Hence, the answer = (20/10)^2 = 4% Loss.

Introduction to Interest

Chapter of interest is an application of percentages. Interest is calculated as a percentage of a loan (or deposit) balance, paid to the lender periodically for the advantage of using their money. Interest can be calculated for periods that are longer or shorter than one year.

Interest is of two types:

1. Simple interest
2. Compound interest

The basic difference between simple interest and compound interest is the compounding factor that is often talked about in all economic and finance.

Simple interest :
Simple interest is the interest that is paid only on the amount borrowed (or invested), and not on past interest.
Compound interest :

Compound interest is the interest on capital invested as well as interest on the interest.

For example :
If you invested 100Rs @ 10% per annum on simple interest for 3 years.
Interest after 1st year = 10, after 2nd year = 10 and after 3rd year also be 10.
Amount after 1st year = 100 + 10 = 110
Amount after 2nd year = 110 + 10 = 120
Amount after 3rd year = 120 + 10 = 130

In the case of compound interest
Let say you invested 100 Rs @ 10% per annum on compound interest for 3 years.
Interest after 1st year = 10, Amount after 1st year = 100 + 10 = 110
Interest after 2nd year on 110 @ 10% = 11, Amount after 2nd year = 110 + 11 = 121
Interest after 2nd year on 121 @ 10% = 12.1, Amount after 3rd year = 121 + 12.1 = 133.1
Difference between compound interest and simple interest starts from 2nd year not from 1st year (after 1st year CI & SI both are same) it is illustrated as;

A sum of 100 at 10% per annum will have

Simple interest		Compound interest
10	After First year	10
10	After Second year	11
10	After Third year	12.1

NOTE: 1. Simple interest is generally used only on the short-term i.e duration of less than one year.
2. Compound interest is used for a longer period.

Basic terms related to interest

1. The man who borrows the money is **Debtor** and the man who lends money is the **Creditor**
2. The initially borrowed amount of money is known as the **Capital or Principal money**.
3. The extra money that will be paid or received for the use of the principal after a certain period is called the **Total interest on the capital**.
4. The sum of the principal and the interest at the end of any time is called the **Amount**.
5. The period for which money is deposited or borrowed is called **Time**.

Hence, **Amount = Principal + Total Interest.**
Rate of Interest is the rate at which the interest is calculated and it is always specified in terms of percentage.

Concept of Simple Interest

Simple interest is the interest that is paid only on the amount borrowed (or invested), and not on past interest.
The formula for simple interest:

$$I = P \times r \times t/100.$$

Here I = total interest, P = Principal amount, r = rate%, t = time period
Since the Amount = Principal + Total interest

NOTE: The half-yearly rate of interest is half the annual rate of interest.

Example: An amount of 4000 Rs is invested at a rate of 8% per annum simple interest and after a certain time period, it becomes 5920 Rs. What is the time period?
Answer: 6 years
Solution:Total amount = Principal + total interest.
5920 = 4000 + total interest
Total interest = 5920 - 4000 = 1920 Rs.
Annual interest = 8% of 4000 = 4000×8/100 = 320.
No of time period = total interest / annual interest
Total time period = 1920 / 320 = 6 years.
Hence time period = 6 years.

Example: A sum of money lends a simple interest. Sum of money after 2 years is 2394 Rs and after another 3 years is 2835 Rs. What is the sum, annual interest and the rate of interest?
Answer: Rs. 4185
Solution:Let 'i' be the interest for 1 year.
Sum of money after 2 years;
Sum = P + total interest after 2 years
2394 = P + i + i, 2394 = P +2i …………….(1).
And sum after another 3 years;
Here P = 2394 Rs
Sum = P + total interest after 3 years
2835 = 2394 + 3i

3i = 2835 - 2394 = 441, i = 441/3 = 147.
Hence annual interest = 147.
Put i = 147 in equation (1).
2394 = P +2×147, P = 2394 - 294 = 2100.
Annual rate = (interest / Principal)×100
= (147 / 2100)×100 = 7%.
Total sum = P + interest after 5 years
= 2100 + 5i
= 2100 + 5×147 = 4185 Rs.

Concept of Compound Interest

Compound interest is the interest on capital invested as well as interest on the interest.
Let say you invested 100 Rs @ 10% per annum on compound interest for 3 years.
In compound interest every year you will get the interest on the amount.
Interest after 1st year = 10, Amount after 1st year = 100 + 10 = 110
Interest after 2nd year on 110 @ 10% = 11, Amount after 2nd year = 110 + 11 = 121
Interest after 2nd year on 121 @ 10% = 12.1, Amount after 3rd year = 121 + 12.1 = 133.1

Formula :
Case 1: Let principal = P, time = ‘n’ years and rate = r% per annum and let A be the total amount at the end of n years, then
A = P$\times(1 + r/100)^t$
Let say a man invested 1000 Rs @ 20% per annum. What will be the amount in 3 years?
P = 1000 Rs, r = 20% , t = 3 years.
A = P$\times(1 + r/100)^t$
A = $1000\times(1 + 20/100)^3$ = $1000\times(1.2)^3$ = 1000×1.728
A = 1728 Rs.

Case 2: When compound interest is half-yearly then,
If the annual rate is r% per annum and is to be calculated for n years.
Here, rate = r/2 % half-yearly and time = (2n) half-years.
From the above we get
A = P$\times(1 + (r/2)/100)^t$
In case of quarterly, rate = r/4 % and time = (4n) quarter years.

Let say a man invested 1000 Rs @ 10% per 6 months. What will be the amount after 2 years?

A = P$\times(1 + (r/2)/100)^t$

Rate = 6% half-yearly, t = 2 years means 4 half years. Hence t = 4.

A = 1000$\times(1 + 10/100)^4$ = 1000$\times(1.1)^4$= 1000×1.4641

Hence amount = 1464.1 Rs.

In the given formula what you notice is that the power in the formula, if it goes to 4 or 5 it becomes slightly complex to calculate the amount because you might not know the value.

To solve this question think about PCG structure.

invested 1000 Rs @ 10% per 6 months for 2years.

$$1000 \xrightarrow[+100]{10\% \uparrow} 1100 \xrightarrow[+110]{10\% \uparrow} 1210 \xrightarrow[+121]{10\% \uparrow} 1331 \xrightarrow[+133.1]{10\% \uparrow} 1464.1$$

You should solve all compound interest questions through PCG structure.

Example: What principal amounts to 270.40 Rs in 2 years at the 4% compound interest per annum?

Solution: As we know;

A = P$\times(1 + r/100)^t$

270.40 = P$\times(1 + 4/100)^2$

270.40 = P$\times(104/100)^2$

Method of multiplying 2 numbers when they are close to 100, that is very useful in CI.

For example :

You multiply 103 and 106.

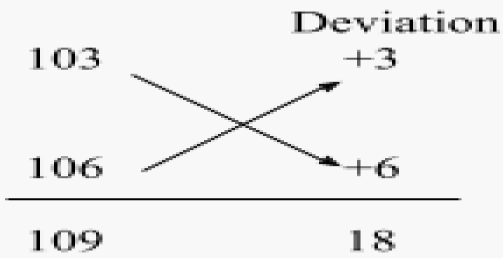
In this method, you have to take the base value as 100.

103 is a deviation of +3 from 100.

106 is a deviation of +6 from 100.

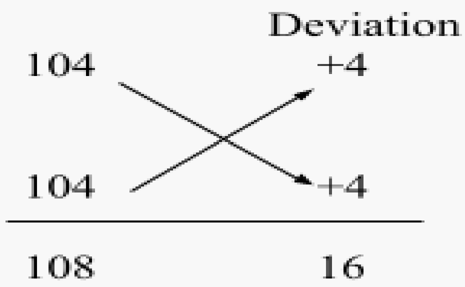
Answer to multiplication will consist of two parts; the last two digits and the starting digit.

1. The value of two digits of this multiplication is obtained by multiplying the deviation +3 and +6.
2. And across the diagonal, you will have to get the initial digits. Whether you do 103+6 or 106+3, you will get the same number in both additions.



Hence 103×106 =10918

In this question, we have 104×104



Hence 104×104 = 10816.

$270.40 = P\times(1.04)^2 = P\times1.0816$

P = 270.40/1.0816 = 250 Rs.

Here the calculation of P is not easy. So; solve these type of problems from the options given to you.

Let us say the options for this problem are

a) 220 b) 200 c) 250 d) 225

Let's say you try from 220.

$$220 \xrightarrow[+8.8]{4\%} 228.8 \xrightarrow[+9.15]{4\%} 237.95$$

Hence 220 gets rejected. 200 & 225 are also rejected because 200 is less than 220 and 225 is not far away from 220.

Now try for 250.

$$250 \xrightarrow[+10]{4\%} 260 \xrightarrow[+10.4]{4\%} 270.4$$

This is exactly what was required.

Example: On a certain principal, the compound interest is Rs 132 for the 2nd year and rate of interest 10% per annum. What was the principal?
Solution :
Solve by PCG structure; let say P is the principal

$$P \xrightarrow{10\%} x \xrightarrow[+132]{10\%}$$

After one year the amount not given to us. Let say it is x. The interest for 2nd year is 132. It is obvious interest on x at a rate of interest is 132 that means x must be 1320.
So the starting principal is ;
$P \times 1.1 = 1320$, $P = 1200$.
Hence principal amount = 1200 and the amount after two years is;

$$1200 \xrightarrow[+120]{10\%} 1320 \xrightarrow[+132]{10\%} 1452$$

Problems for practice

1. Compound interest on a sum of money for 2 years is 615. While the SI for the same period is Rs 600. Find the principal and rate of interest. (capgemini recruiting 2019)

a. Rs 6000
b. Rs 5000
c. Rs 1200
d. Rs 4000

Answer: 6000 rs
Solution: Let's say P is the principal. Here IS for 2 years is 600 that means annual interest is 300.
In the case of SI :

$$P \xrightarrow[+300]{\text{1st year}} \quad \xrightarrow[+300]{\text{2nd year}}$$

In the case of CI :

CI is 615 for 2 years. As we know CI in the 1st year is the same as the SI in the 1st year.

CI for 1st year = 300. And for 2nd year = 600 - 300 = 315.

$$P \xrightarrow[+300]{\text{1st year}} \quad \xrightarrow[+315]{\text{2nd year}}$$

Let the annual rate of interest is x%.

x% of P = 300 ....(1)

In case of CI :

x% of (P + 300) = 315

x% of P + x% of 300 = 315

300 + x% of 300 = 315 ....(2)

So; from this equation x% of 300 should be equal to 15 to satisfy the equation.

x% of 300 = 315. Hence x = 5%.

For calculation of Principal from eq (1)

5% of P = 300.

Hence P = 6000 Rs.

2. Difference between CI and SI of a certain sum of money for 2 years at 20% per annum is Rs 48. What is the sum of money? (Infosys 2018)
    a. 1200
    b. 1300
    c. 1600
    d. 1700

Answer: 1200

Solution: Let say x is the original amount.

SI @ 20% for 2 year on x = 0.2x + 0.2x = 0.4x

CI @ 20% for 2 year on x;

A = $x \times (1 + 20/100)^2$ = 1.44x

CI = 1.44x - x = 0.44x.

Difference between CI & SI = 48

0.44x - 0.4x = 48

x = 1200.

2nd method :

Assume principal= 100 Rs.

In the case of SI :

SI on 100 @ 20% for 2 year is;

I = 20 + 20 = 40.
In the case of CI :

$$100 \xrightarrow[+20]{20\%} 120 \xrightarrow[+24]{20\%} 144$$

A = 144.   CI = 144 - 100 = 44.
Difference between CI & SI = 48
44 - 40 = 48
4 ≡48.
Using the multiplier logic, 4 to 48 the multiplier is 12. Multiply all the assumed values by 12 you will get the actual value.
Hence Principal amount = 100×12 = 1200.
3rd method :
The difference can also be calculated by a formula which is $p \times (r/100)^2$. This gives you the difference between CI & SI for 2 years for a principal amount P @ a rate "r".
Difference between CI & SI = 48 = $p \times (20/100)^2$
48 = p×1/25
P = 1200.
NOTE : $p \times (r/100)^2$ This work on the difference between CI & SI for 2 year.

3. Sum of money at simple interest tripled in 6 years. In how many years would it become 12 times itself? (AWS hiring 2020)

a. 33 years
b. 31 years
c. 35 years
d. 42 years

Answer: 33 years
Solution:Let if money was 100 it has become 300 after 6 years. That means an addition of 200 in 6 years and money became 12 times itself i.e 1200.

$$100 \xrightarrow[\text{In 6 years}]{+200} 300 \xrightarrow[?]{+900} 1200$$

6 years interest is 200 and for another 6 years interest would be again 200 because annual interest is the same. Hence in every 6 years, you will add 200.
So; after 12 years the amount will become = 300+200 = 500.

After 18 years the amount will become = 500+200 = 700.
After 24 years the amount will become = 700+200 = 900.
After 30 years the amount will become = 900+200 = 1100.
Now you need 100 Rs interest more.
200 Rs interest in 6 years. So; 100 Rs Interest in 3years.
So; after 33 years the amount will become = 1100+100 = 1200
Hence 1200 will become in 33 years.

4. A lent B Rs 6000 for 2 years and to C he lent Rs 1500 for 4 years. Together he earned a total interest of Rs 900. What is the rate of interest? (Goldman Sachs campus hiring 2019)

a. 5%
b. 3%
c. 6%
d. 10%

Answer: 5%
Solution: Mathematically;
A lent B Rs 6000 for 2 years. So;
I = $P \times r \times t/100$, I = 6000×r×2/100 = 120r …………(1)
A lent C Rs 1500 for 4 years. So;
I' = 1500×r×4/100 = 60r ………...(2)
And total interest = 900 i.e I + I' = 900
120r + 60r = 900, 180r = 900
r = 5%
Hence rate of interest = 5%.
Another way to do this question;
i.e; 6000 for 2 years ≡12000 for 1 year ………..(1)
1500for 4 years ≡ 6000 for 1 year ………..(2)
Form (1) and (2);
18000 for 1 year and total interest earned is 900.
Hence annual rate of interest = (900/18000)×100 = 5%.

Ratio, proportions, variation 2

Introduction to Ratio, proportion, variation

We have already discussed the theory of this chapter and did some problems based on that, now we will go further with some standard problems.

Problems On Ratio-1:

Problem 1:
Divide rupees 252 amongst A, B, and C such that 1/3rd of what A gets is equal to 1/5th of what B gets is equal to 1/4th of what C gets. How much A, B, and C will get individually?

Solution :
Here 2 equations are formed;
A + B + C = 252 ....(1)
A/3 = B/5 = C/4 ....(2)

Let's say, eq (2) is equal to k.
A/3 = B/5 = C/4 = k , A = 3k , B = 5k , C = 4k
Put value of A,B and C in eq(1) we get;
3k+5k+4k = 252, 12k = 252 and k = 252/12 = 21.
Hence the numbers are; A = 3×21= 63, B = 5×21= 110 and C =4×21= 84.

Problem 2:
Divide rupees 517 amongst A, B, and C such that 1/3rd of what A gets is equal to 2/5th of what B gets is equal to 3/7th of what C gets. How much A, B ,and C will get individually?

Solution :
A + B + C = 517 ....(1)
A/3 = B2/5 = C3/7 ....(2).
You can get a direct ratio from eq(2).
A:B:C = 3:5/2:7/3

Whenever you have a ratio that itself has its component in the fractions, you should multiply the ratio by the denominator LCM.

LCM (2,3) = 6. Multiply A:B: C by 6 you will get a proper ratio.
So; A:B:C = 18:15:14
So, Sum of the component of ratio = 18+15+14 = 47.
47≡517 that means the multiplier would be 11.

Hence the numbers are; A = 18×11= 198, B = 15×11= 165 and C = 14×11= 154.

Problems On Ratio-2:

Problem 1:
Anjali has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and 2nd mixture has milk to water in ratio 2:7. She mixes equal quantities of these mixtures. What is the ratio of milk to water in the final mixture?
Solution :

Mixture 1	Mixture 2
M:W = 3:8	M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Take 99L for both mixtures because mix equal quantities of mixture.

Mixture 1	Mixture 2
99L	99L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

3+8 = 11≡99	2+7 = 9≡99
11 being 99 so; multiplier would be 9.	9 being 99 so; multiplier would be 11.
Hence M = 3×9 = 27 & W = 8×9 = 72.	Hence M = 2×11 = 22 & W =7×11= 77

Thus; total milk = 27+ 22 = 49 and total water = 72 + 77 = 149.
Hence the final mixture has milk to water ratio = 49:149.

Problem 2:
Shubham has 2 mixtures of milk and water. One mixture has milk to water in ratio 3:8 and the 2nd mixture has milk to water in ratio 2:7. He is mixing these mixtures in 2:3. What is the ratio of milk to water in the final mixture?

Solution :

Mixture 1 — Mixture 2

M:W = 3:8 — M:W = 2:7

Take the LCM of 3+8 = 11 and 2+7 = 9. LCM(11,9) = 99. Here you can not take 99L for each mixture because the question is not talking about equal quantities.
Mixture 1 = 99L and mixture2 = 99L, To make both the mixture in 2:3. Then;

Mixture1 = 99×2 = 198L and Mixture2 = 99×3 = 297L

Mixture 1	**Mixture 2**
198L	297L
M:W = 3:8	M:W = 2:7

Using multiplier logic;

3+8 = 11≡198 — 2+7 = 9≡297

11 being 198 so; multiplier would be 18. — 9 being 297 so; multiplier would be 33.

Hence M =3×18= 54 & W =8×18=144 — Hence M = 2×33 = 66 & W =7×33=231

Thus; total milk = 54 + 66 = 120 and total water = 144 + 231 = 375.
Hence the final mixture has milk to water ratio = 120:375.

Problems On Ratio-3:

Problem 1:
The income of P & Q is in ratio 1:2 and expenditure of P & Q is in ratio 1:3. If each saves 500 of their income. Find the P's income.

Solution :
Lets P's income = x and Q's income = 2x.
P's expenditure = y and Q's expenditure = 3y.
And we know;
Saving = Income - Expenditure

Saving for P; x - y = 500 ....(1)
And Saving for Q; 2x - 3y = 500 ....(2)
Solving eq(1) and (2) we get;
x = 1000 and y = 500.

Hence P's income = 1000.

Problem 2:
Rupees 232 is to be divided among 150 girls and boys, such that each girl gets Rs 1 and each boy gets Rs 2. Find the number of boys and girls.

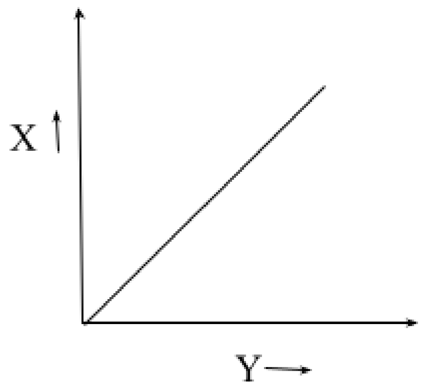
Solution :
Let the number of girls = G and number of boys = B
G + B = 150 ....(1)
G + 2B = 232 ....(2)
Solving eq (1) and (2) we get;
G = 68 & B = 82.w

Variation and its 3 types :

Variation is an important concept in mathematics. To understand variation first you need to understand 3 kinds of variation.

1. **Direct variation :**
   x varies directly as y or x is directly proportional to y.
   Mathematically; $\mathbf{x \alpha y}$.

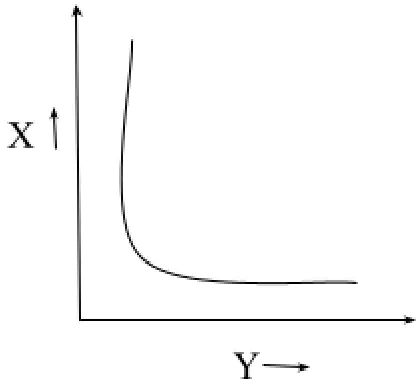
(a) Logical implication: When x increases y increases. And if x decreasing y also decreases
(b) Calculation implication: If x increases by 20%, y will also increase by 20%.
(c) Ratio : If x is increasing by 1/5 then y will also increase by 1/5.
(d) Graphical implications: The following graph is representative of this situation.



(e) Equation implication: The ratio x/y is constant i.e x = ky (where k is a constant)

2. **Inverse variation :**
   X is inversely proportional to y or x varies inversely as y or product of x and y is constant.
   Mathematically; $\mathbf{x \alpha 1/y}$.

(a) Logical implication: When x increases y decreases and vice versa.
(b) Percentage implication: If x increases by 25% then y decreases by 20%.
(c) Ratio implication: If x increases by 1/4 then y decreases by 1/5.
(d) Graphical implications: The following graph is representative of this situation.



(d) Equation implication: The product x × y is constant.

3. **Joint variation :**
   If x varies jointly as y & z or **x**α**(y×z)** ⇒**x = k(y×z).** Or if x varies as y when z is constant and x varies as z when y is constant.
   Mathematically; **x**α**(y×z)**

Problem 1:
Given that, x directly varies with y and x is 18 when y is 7. Find x when y is 21?

Solution :
x directly varies with y i.e xα**y** **or** x = ky ....(1)
Replace x and y with their respective values. So; eq (1) becomes
18 = k×7 ⇒k = 18/7.
When y =21 the value of x is;
from(1); x = 18/7×21 = 54.

Problem 2:
The duration of a railway journey varies as the distance and inversely as the velocity, while velocity varies as the square root of quantity of the coal used and inversely as the number of carriages in the train. In the journey of 50 km in half an hour with 18 carriages, 100 kg of coal is required. How much coal will consume in a journey of 42km in 28 minutes with 16 carriages?

Solution :

There are 5 variables.

Assume duration = T, distance = D, velocity = V, quantity of coal = Qc and No. of carriage = N.

According to question;

T $\alpha \frac{D}{V}$ ....(1) and V = $\frac{\sqrt{Qc}}{N}$ ....(2)

From (2) put value of V in (1);

T $\alpha \frac{D \times N}{\sqrt{Qc}}$ or T = $\frac{k \times D \times N}{\sqrt{Qc}}$ ....(3)

Put value of T = 30min, D = 50km, N = 18 and Qc = 100 kg in (3);

30 = $\frac{k \times 50 \times 18}{\sqrt{100}} \Rightarrow$ k = 1/3.

Now from eq (3);

T = $\frac{1}{3} \frac{D \times N}{\sqrt{Qc}}$ ....(4)

Therefor for the T = 28min, D = 42km, N = 16 ; the value of coal required is,

28 = $\frac{1}{3} \frac{42 \times 16}{\sqrt{Qc}} \Rightarrow \sqrt{Qc}$ = 8; Hence Qc = 64 kg.

Time and work

Introduction to time and work

Work is defined as something which has an effect or outcome. The basic concept of Time and Work is similar to that across all Arithmetic topics, i.e. the concept of Proportionality.

Method for solving time and work

1. Fraction method :

Let the total work = 1unit.
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 unit.
B's per day work = 1/15 unit.

In time & work the basic equation is;
Rate of work×Time = work done

Rate of work = 1/12 + 1/15 = 9/60 unit
9/60×t = 1. Therefore t = 60/9 = 6.66 days.
Time is reciprocal of rate of work.
It is a very combusive method. One advantage of this method is in the last step you just take the reciprocal of the value you got.

2. Percentage method :

Let the total work = 100%
A can finish the work in 12 days and B can finish the work in 15 days.
A's per day work = 1/12 i.e. 8.33%
B's per day work = 1/15 i.e. 6.66%

Rate of work×Time = work done
Rate of work = 8.33 + 6.66 = 15%
15×t = 100. Therefore t = 100/15 = 6.66%.
It is a better method than fraction, but this method has only the problem of decimal work.

For example; A can finish the work in 5 days and B can finish the work in 9 days.
A's per day work = 1/5 i.e. 20%
B's per day work = 1/9 i.e. 11.11%
Rate of work = 20+11.11 = 31.11%. So in this case numbers are not supporting you.

3. LCM method :

A can finish the work in 12 days and B can finish the work in 15 days.
Assume total work be the LCM of 12 &15.
LCM(12,15) = 60.
A's per day work = 60/12 = 5 unit.
B's per day work = 60/15 = 4 unit.
One day total work = 5+4 = 9unit.
Total time required = total work / per day work
= 60/9 = 6.33 days.
This is the better method to work upon by avoiding the use of decimal work

Types of problems in time and work

People come and go type problem :

Example: A can do a piece of work in 10 days. B can also do the same work in 12 days and C can do the same work in 15 days. A & B start the work and work for 2 days and then B leave and after 1 more day, C joins A to complete the work. In how many days will the work be completed?

Solution :
Total work = LCM(10,12,15) = 60 units.
A's per day work = 60/10 = 6 units.
B's per day work = 60/12 = 5 units.
C's per day work = 60/15 = 4 units.
A+B per day work = 6+5 = 11 units.
Work in 2 days = 11×2 = 22 units.

On the 3rd day, A is working alone and B left.
3rd work = 6 units.
Total work in 3 days = 22+6 = 28 units.
So; work left = 60-28 = 32 units. This work has to be done by A & C.
A+C per day work = 6+4 = 10 units. Therefore remaining work 32 units will take 32/10 = 3.2 days more.
Hence total days required = 3 + 3.2 = 6.2 days.

Pipe & Cistern Problem :

Example: 2 pipes A & B are filling a tank. A can fill it in 12 hours and B can fill it in 15 hours. How much time will they take to fill an empty tank?

Solution :

A can fill the tank in 12 hours and B can fill the tank in15 hours.

Assume the total capacity of the tank be the LCM of 12 &15.

LCM(12,15) = 60 L.

A's per hour filling = 60/12 = 5 L.

B's per hour filling = 60/15 = 4 L.

In one hour total filling = 5+4 = 9 L.

Total time required = total capacity / per hour filling

= 60/9 = 6.33 hours.

Time and work (man-days) :

Here we will discuss that the work is measured in terms of man-day or man-hours.

Let 20 men work on a project for 8 days. Work done can be measured in such a case, as multiplication of 20×8 and units used here man-days. i.e 20×8 = 160 man-days.

We use the concept of work equivalence in such situation means;

20 men working for 8 days is the same as 10 men working for 16 days is same as 1 man working for 160 days i.e 20×8 ≡10×16 ≡1×160.

Example: A certain number of people can complete a piece of work in 55 days. If there were 6 more men added, the work could get done in 11 days less.What is the number of men initially?

Solution :

Assume in the starting there is x number of men.

Total work is done by x men = x ×55 man-days.

6 men more join & work is done in 55 - 11 = 44 days.

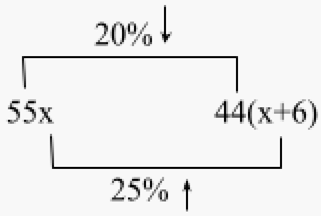
So; according to work equivalence ;

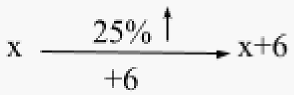
x ×55 = (x+6)×44

55x = 44x +264 ⇒x = 24 men.

We can do this question by-product constancy also.

The numerical component of the product is going down by 20% and the other component going up by 25%.





+6 present 25% increase on x. 25% is 6 and 100% is 6/25×100 = 24.
Hence the number of men = 24 men.

Example: 10 men working 6 hours a day can complete work in 18 days. In how many hours a day should 15 men work for 12 days. So that they can complete double the work?
Solution :

Original work = $10 \times 18 \times 6$ man-days.

New work = $10 \times 18 \times 6 \times 2$

Let x hours per day 15 men take.

According to work equivalence;

$10 \times 18 \times 6 \times 2 = 15 \times 12 \times x$

Therefore x = 12 hr/day

Time and work (man-days)-2 :

Example: A contractor undertakes to complete a job in 100 days and employs 200 men to complete the work. After 50 days he finds that only 40% of the work is completed. To complete the work in time how many men should he hire?
Solution :

Work to be done in 50 days = 200×50 =10000 man-days

10000 man-days are only 40% of the work.

Remaining work = 100 - 40 = 60%

40% work = 10000 man-days

60% work = (10000/40)×60 = 15000 man-days.

You have only 50 more days left. Let n be the number of men required to complete the work.
Therefore; $50 \times n = 15000$ and n = 300 men.
Hence; 300 - 200 = 100 men need to hire.

The Specific Case of Building a Wall :

Building of a wall of a certain length, breadth, and height.
In such cases, the following formula applies:
$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$
where L, B, and H are respectively the length, breadth, and height of the wall to be built, while m, t, and d are respectively the number of men, the amount of time per day, and the number of days. Further, suffix 1 is for the first work situation, while suffix 2 is for the second work situation.

Example: 12 men working 8 hours a day can completely build a wall of length 12ft, breadth 40 ft, and height 4ft in 10 days. How many days will 10 men working 6 hours a day require to build a wall of length 24ft, breadth 60ft, and height of 2ft?
Solution :
Using formula;
$\frac{M1 \times D1 \times T1}{M2 \times D2 \times T2} = \frac{L1 \times B1 \times H1}{L2 \times B2 \times H2}$

Here, L1 is 12ft	L2 is 24ft
B1 is 40ft	B2 is 60ft
H1 is 4ft	H2 is 2ft
while M1 is 12 men	M2 is 10 men
D1 is 10 days	D2 is unknown
and T1 is 8 hours a day	T2 is 6 hours a day

$\frac{12 \times 10 \times 8}{10 \times D2 \times 6} = \frac{12 \times 40 \times 4}{24 \times 60 \times 2}$
16/D2 = 2/3 ,   D2 = 24 days

Men, Women & Children :

Example: 20 women can do work in 16 days while 16 men can do it in 15 days. What is the ratio of the capacity of a man and a woman?
Solution :

Total work to be done = 20 × 16 = 320 woman-days.
or total work to be done = 16 × 15 = 240 man-days.
Since, the work is the same, we can equate 240 man-days = 320 woman-days.
Hence, 3 man-days = 4 woman-days or 1 man-day = 1.33 woman-days.
Assume total work = 12 unit
1 man-day work rate = 4 units.
1 woman-day work rate = 3 units.
Therefore the work rate of man to woman = 4:3.
The answer is not 3:4, the answer is 4:3 because 3 man-days doing the same work as 4 woman-days. So; the work rate of a man must be higher than the work rate of a woman.

Permutation and combination

Introduction to permutation and combination

Permutation and combination are all about counting and arrangements made from a certain group of data. You have a counting situation that requires formulas. If count is small you do not require formulas but if the count is large you require formulas for counting.
For example:
If you have to count 1 to 10, you can easily do this, but if you have to count up to 10255 it will require formulas.

Permutation: In mathematics, permutation relates to the act of arranging all the things of a set into some sequence or order.

Combination: Combinations can be defined as the number of ways in which ‘r’ things at a time can be selected from amongst ‘n’ things available for selection.

This chapter gives you counting situations that are mapped to the use of certain formulas and you have to know which formula is used in which situation.

Every P & C question will always end with asking you to “Find the numbers of ways?’ doing something. Whenever you identify that the question is a P & C question, you 1st ask yourself if it is a selection question, distribution question, or it is an arrangement question then you go with an appropriate formula.

This chapter splits into 3 parts:
1. Selection **2.** Distribution **3.** Arrangement

Selection:

Selection can be defined as the number of ways in which r things at a time can be selected from amongst n things available for selection.
Let say select two people for 4 people A,B,C,D and count the number of different ways i which one can make the selection.

Count physically;
1st selection is AB, 2nd selection is AC, 3rd selection is AD, 4th selection is BC, 5th selection is BD, 6th selection is CD.
Hence the number of possible selections = 6.
But if you have to select 8 people from the 16 people. You can not physically count the number of selections because there are so many possible cases which are not possible to visualize. Hence in order to handle this situation you need the **nCr** formula.

This formula tells us if you have 'n' "distinct" objects from them select 'r' objects and you want to count the number of selections.

Thus, **nCr = n! /[r! (n–r)!]** ; where n ≥ r.

Formulae For Selection

Already we have discussed two formulae for selection,

1. **nCr =** $\frac{n!}{r!(n-r)!}$
2. **nCr = nC(n-r)**
3. **Total number of selections of zero or more things out of n different things**
   nCo + nC1 + nC2 + ... + nCn
   nCo + nC1 + nC2 + ... + nCn = 2^n

Questions on selection

Problem 1:
In a room there are 8 men, and 6 women and a handshake is held between 1 man and 1 woman.What is the number of handshakes?

Solution:
To visualize this take a small case, 3 men A,B,C and 2 women D, E in a room and they start handshake with each other.

Then, 1. A handshake with D.
2. A handshake with E.
3. B handshake with D.
4. B handshake with E.
5. C handshake with D.
6. C handshake with E.

Hence the total number of handshakes is 6.

This is similar to selecting a man and a woman. Number of handshake = 3C1×2C1 = 3 ×2 = 6.
Hence, in the given questions selecting a man out of 8 men and a woman out of 6 women, then the number of handshake will be 8C1×6C1 = 8×6 = 48.

Problem 2:
In a room there are a certain number of people and everybody handshake with each other. It was found that the number of handshakes was 153. Find the number of people in the room?

Solution:
Let's say in the room there are n people and everybody handshake with each other.
Total number of handshake = nC2 = 153
n×(n-1)/2 = 153
n^2 - n - 306 = 0
Therefore n = 18,-17 but the number of people can not be -ve. So, n = 18 people.

Type 1: Question involving pre selection

Problem 1:

In a cricket team there are 16 players and select 11 players such that the captain is always selected. Find the total number of selections?

Solution:

Here given that the captain always be selected (i.e. preselected) now you have to select only 10 players from 15 players.

Therefore, selection of 10 from 15 = 15C10.

Type 2: Constraint based selection

Problem 1:

Out of 6 men and 4 women and you have to select a committee of 3 with at least one woman. In how many different ways can it be done?

Solution:

You have committee with at least 1 woman are,

1 women and 2 men or 2 women and 1 man or 3 women and no man

$4C1 \times 6C2 + 4C2 \times 6C1 + 4C3 \times 6C0$

2nd method:

Committee of all men subtracted from total number of committee i.e. 10C3 - 6C3

From 10 people if you want to draw a committee of 3, will be 10C3.

If divide 10 people into 6 men and 4 women and you have to make committee of 3 and do not given any constraint in case you decide to do this problem using how many men and how many women then you have to write all possible committee i.e.

3 men & no woman or 2 men & 1 woman or 1 man & 2 women or no man & 3 women i.e.

$6C3 \times 4C0 + 6C2 \times 4C1 + 6C1 \times 4C2 + 6C0 \times 4C3$

Distribution of identical objects

Distribution can happen of identical objects or distinct objects.

Number of ways of distributing n identical things among r persons when each person may get any number of things = (n + r – 1) C(r–1)

Problem 1:

If you have 4 identical objects to give between two friends X & Y. What are the number of distributions?

Solution:

	X	**Y**
1st distribution	4	0
2nd distribution	3	1
3rd distribution	2	2
4th distribution	1	3
5th distribution	0	4

Therefore total number of distributions = 5

According to formula;

Here n = 4 and r = 2

So, the total number of distributions = (4+2-1)C(2-1) = 5C1 = 5.

Problem 2:

If x+y+z = 20 and x,y,z are whole numbers. How many solutions does x+y+z = 20 have?

Solution:

x+y+z = 20 is the same as distributing 20 objects between x,y and z.

Here n = 20 and r = 3.

So, the total number of solutions = (20+3-1)C(3-1) = 22C2 = 231 .

If x,y,z are natural numbers, in this case this formula does not work directly because in this case zero is not allowed.

Formulae For Arrangement

1. MNP Rule

It tells us if you have 3 tasks to do and there are M ways of doing the first thing, N ways of doing the second thing and P ways of doing the third thing then there will be M × N × P ways of doing all the three things together.

This formula is used to do problems on arrangements and also used for distribution of distinct objects.

Problem 1:
Shubham wants to go from Mumbai to Pune and Pune to Delhi and Delhi to Kolkata. There are 6 trains from Mumbai to Pune, 5 trains from Pune to Delhi and 8 trains from Delhi to kolkata. Find the total number of ways of travelling?

Solution:

$$\text{Mumbai} \xrightarrow[6\ T]{} \text{Pune} \xrightarrow[5\ T]{} \text{Delhi} \xrightarrow[8\ T]{} \text{Kolkata}$$

So, total number of ways of travelling = 6× 5 × 8= 240.

2. r! Formula

If you have ‘r’ distinct things and you want to place them in ‘r’ places, then the total number of ways = **r!**

Problem 1:
6 people ABCDEF and you want to sit them on 6 chairs. Find the total number of ways of sitting?

Solution:
The 1st chair can be filled by 6 people.
The 2nd chair can be filled by 5 people.
The 3rd chair can be filled by 4 people.
The 4th chair can be filled by 3 people.
The 5th chair can be filled by 2 people.

The 6th chair can be filled by 1 person.
So the total number of ways = 6! = $6 \times 5 \times 4 \times 3 \times 2 \times 1$= 720.
r! Nothing but the MNP rule used for ‘r’ distinct objects in ‘r’ places.

3. r! modified for arrangement of identical objects

Number of arrangements of ‘n’ things out of which P1 are alike and are of one type, P2 are alike and are of a second type and P3 are alike and are of a third type and the rest are all different
= **n!/ P1! P2! P3!**

For example:
AAA BB CCC and you want to be placed in 8 places.
AAA are 3 alike things, BB are two alike things and CCC are three alike things.

So, total number of ways = 8!/3!$\times$ 2! $\times$ 3!

4. nPr formula

nPr = number of arrangements of ‘n’ distinct things taken r at a time.
nPr = n!/ (n – r)!; n ≥ r

For example:
Six people ABCDEF arrange in 3 places = 6P3 = 6!/3! = 120.
Similar situation is getting handled using the MNP rule. So, according to MNP rule, 6 people arranging in 3 places = $6 \times 5 \times 4$ = 120

The Relationship Between Permutation & Combination:

When we look at the formulae for Permutations and Combinations and compare the two we see
that,
nPr = r! × nCr
i.e. The arrangement of r things out of n is nothing but the selection of r things out of n followed by the arrangement of the r selected things amongst themselves.

Generic Questions On Arrangements

Problem 1:
In how many ways 7 people A,B,C,D,E,F,G are arranged in a straight line in 7 places such that A is always in the middle?

Solution:
Middle place is fixed by A and the remaining 6 places are filled by 6 people.
So, total number of ways = 6!.

Problem 2:
In how many ways 7 people A,B,C,D,E,F,G are arranged in 7 places such that no two of A,B,C are together?

Solution:
A,B,C in 3 places is 3! And D,E,F,G in 4 places is 4!
Total number of ways = $3! \times 4!$.

Problem 3:
In how many ways 7 people A,B,C,D,E,F,G are placed in 7 places such that A & B are together?

Solution:
A&B are together. So, A&B counted as one person and 5 people separately, effectively there are 6 people.
Arrangement of 6 people is 6! And arrangement of AB = 2!.
Therefore total number of ways = $6! \times 2!$.

Questions On Word Formation

Type 1: Word formation question
Problem 1:
How many words can be formed with the word PATNA, LUCKNOW and JAIPUR which have
1. No restrictions.
2. Total number of new words
3. Start with the first letter.
4. Start and end with vowels.

Solution:
PATNA
1. Total number of letters - P,T,N occurs once while A occurs twice.
So, the total number of words that can be formed = 5!/2! = 60

2. Total number of new words = 60 - 1 = 59.

3. We can arrange only 4 letters (as place of P is restricted) in which A occurs twice.
So, the total number of words that can be formed = 4!/2!

4. In the word PATNA in which we have 2 vowels(A,A).
So, the total number of words that start with A and end with A = 3!

LUCKNOW
1. Total number of distinct letters = 7.
So, the total number of words that can be formed = 7!

2. Total number of new words = 7! - 1.

3. We can arrange only 6 letters (as place of L is restricted)
So, the total number of words that can be formed = 6!

4. In the word LUCKNOW in which we have 2 vowels(U,O). Arrangement of two vowel = 2!
So, the total number of words that can be formed = 2!×5!

JAIPUR

1. Total number of distinct letters = 6.
So, the total number of words that can be formed = 6!

2. Total number of new words = 6! - 1.

3. We can arrange only 5 letters (as place of J is restricted)
So, the total number of words that can be formed = 5!

4. In the word JAIPUR in which we have 3 vowels(A,I,U). We have to select 2 vowels and arrange them amongst 1st and last place = 3C2×2!and also arrange 3 consonants and 1 vowel = 4!
So, the total number of words that can be formed = 3C2×2!×4!.

Type 2: Dictionary position question
Problem 1:
What is the dictionary position of the word RUPAJI that can be formed by letters of the word JAIPUR?

Solution:
1st arrange all the letters of the word JAIPUR in alphabetically order for reference.
A-I-J-P-R-U
Number of words starting with A = 5!
Number of words starting with I = 5!
Number of words starting with J = 5!
Number of words starting with P = 5!
Number of words starting with R = 5!
Number of words starting with U = 5!

You are looking for the word RUPAJI. In this word letter ‘U’ will come only after the letter ‘R’. so, the words starting with letter ‘U’ are not considered. RUPAJI one of the word inside words start with letter ‘R’

Before the words start with the letter ‘R’ we have words = 5! + 5! +5! + 5! = 480 words.

Words start with the letter ‘R’

Number of words starting with RA = 4!

Number of words starting with RI = 4!
Number of words starting with RJ = 4 !
Number of words starting with RP = 4!
Number of words starting with RU = 4!

RUPAJI one of the word inside the words start with letters ‘RU’
Before the words start with the letters ‘RU’ we have words = 480 + 4! + 4! + 4! + 4! = 480 + 96 = 576 words.

Words start with the letter ‘RU’
Number of words starting with RUA = 3!
Number of words starting with RUI = 3!
Number of words starting with RUJ = 3!
Number of words starting with RUP = 3!

RUPAJI one of the word inside the words start with letters ‘RUP’

Before the words start with the letters ‘RUP’ we have words = 480 + 96 + 18 = 594 words.
Remaining lettres A,I,J six words can be form from A,I,J

AIJ,AJI,IAJ,IJA,JAI,JIA. So out of six the 2nd word AJI will complete the word RUPAJI
Therefore the position of the word RUPAJI = 594 + 2 = 596.

Questions On Number Formation

Forming numbers with and without replacement:
Problem 1:
How many 4 digit numbers can be formed by using digit 1,2,3,4,5,6 and 7 with replacement of digit allowed?

Solution:
To forming a 4 digit number with replacement;

1st place can be filled with any of the 7 digits.
2nd place can be filled with any of the 7 digits.
3rd place can be filled with any of the 7 digits.
4th place can be filled with any of the 7 digits.
Therefore total number of ways = $7 \times 7 \times 7 \times 7 = 7^4$

Limit based question:
Problem 1:
How many 4 digit numbers can be formed by using digit 0,1,2,3,4 such that the numbers are not greater than 4000?

Solution:
In this question we can think that numbers are not greater than 4000. So, numbers are starting with digit 1,2 and 3. First place cannot be filled with zero because it makes 4 digit numbers in 3 digit numbers.

Numbers starting with 1
1st place can be filled with 1 digit i.e 1.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 2
1st place can be filled with 1 digit i.e. 2.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits
So, the number of ways = $1 \times 5 \times 5 \times 5 = 125$.

Numbers starting with 3
1st place can be filled with 1 digit i.e. 3.
2nd place can be filled with any of the 5 digits.
3rd place can be filled with any of the 5 digits.
4th place can be filled with any of the 5 digits

So, the number of ways = 1 × 5 × 5 × 5= 125.
And number 4000 itself will get counted.
Therefore total 4 digit numbers = 125+125+125+1 = 376.

NOTE : When in number formation nothing is mentioned about weather repetition allowed or not, in that case default is repetition allowed.

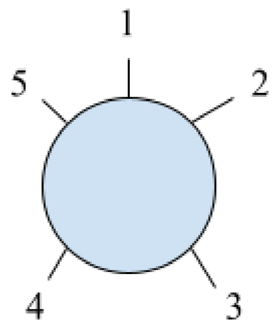
Circular Arrangements

In this chapter you just need to understand a couple of things. On a circle every position is the same, unlike straight lines every position is different.

1. **Number of ways of placing ‘r’ distinct objects on ‘r’ places is equal to (r-1)!**
2. **If there is a reference point on a circle no need to do minus 1.**

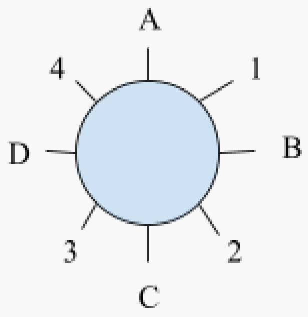
For example:
How many ways of arranging 5 people on seats in a circular table (seat 1 is a reference point)?

Solution:



Seat 1 is a reference point. So, the number of arrangements = 5!

Problem 1:
In how many ways 4 Indian and 4 European sit in alternate places around a circle?



Solution:

Let say 4 Indian sit in A,B,C,D places around a circle. Now you have a circle with a reference point.

Number of ways of arranging 4 Indian = (4-1)! = 6 and Number of ways of arranging 4 European = 4! = 24

Therefore total number of ways = 6

×24 = 144.

3. **'N' objects arrange around a circle where clockwise is equal to anticlockwise, then the number of arrangements = (n-1)!/2**

Probability

Introduction to probability

Probability is one of the most important mathematical concepts that we use in our daily life.
Probability means the possibility of something. It is a mathematical tool that deals with the occurrence of random events. The value of the probability lies between 0 and 1.

The probability of an event is defined by the number of ways in which the event occurs divided by the number of outcomes in the sample space.

P(event) = n(E)/n(S)

Sample space: The sample space of an event is the set of all possible outcomes of that event.

For example:

1. You tossed a coin. Your sample space is head or tail.
   P(H) = 1/2.
2. You throw a dice. Your sample space {1,2,3,4,5,6}
   P(6) = 1/6.
3. England and India play a one-day match.
   In this case, 3 events will happen. 1. England wins 2. India wins 3. Match tie.
   P(tie) $\neq$ 1/3 because the possible outcomes of the India Vs England match is not the sample space in this situation.

Two things happen to form a sample space;
1. Exhaustive or complete list of all possible outcomes.
2. A list to become a sample space is that the outcome should be equally likely.

So, in India Vs England match, the tie is not an equally like outcome. Hence it is not in the sample space.

1st kind of questions based on coins:

Problem 1:
A coin tossed three times. What is the probability of a) All heads? b) Exactly two heads. c) Minimum two heads. d) At Least one head.
Solution:
List of the possible outcomes {HHH,HHT,HTH,THH,TTH,THT,HTT,TTT}
Total number of outcomes = 8. i.e. n(S) = 8.
a) All heads n(E) = 1.
P(All heads) = n(E)/n(S) = 1/8.

b) exactly two heads n(E) = 3
P(Exactly two heads) = 3/8.

c) minimum two heads n(E) = 4.
P(Minimum two heads) = 4/8.

d) P(At least 1 head) = 1 - P(not heads)
= 1 - P(all tails) = 1 - 1/8 = 7/8.

NOTE: 1. None event in probability is denoted by $\overline{E}$ or E' and P(E) + P($\overline{E}$) = 1.
2. The probability of all events in a sample space is 1.

2nd method: Without forming sample space
a) All heads.
If you do not want to form a sample space, you can define this in 3 events.
Event definition: All heads.
H&H&H i.e. $1/2\times 1/2 \times 1/2 = 1/8$.

b) Exactly two heads.
Event definition: Exactly two heads.
H&H&T or H&T&H or T&H&H i.e. $1/2\times 1/2 \times 1/2 + 1/2\times 1/2 \times 1/2 + 1/2\times 1/2 \times 1/2 =$ 1/8 + 1/8 + 1/8 = 3/8.

Biased coin question:

Problem 1:
A coin toss three times, what is the probability of getting 2 Heads and 1 Tail if the probability of a head is 0.6 and tail is 0.4?
Solution:
2 Heads and 1Tail.
Event definition: 2 Heads and 1 Tail
H&H&T or H&T&H or T&H&H i.e. $0.6\times 0.6 \times 0.4 + 0.6\times 0.6 \times 0.4 + 0.6\times 0.6 \times 0.4 =$ $3(0.6\times 0.6 \times 0.4) = 3\times 0.144 = 0.432$.

Probability-based on dice

The single dice situation is very simple. Normally we get in dice question type is the 2 dice situation. In such cases normally questions are asked on the sum of the dice.

In a 2 dice situation, you need to understand that there is a certain pattern for different numbers.
For example:

Sum 2 can happen in only 1 way.(i.e. 1,1)	←Sum 12 can happen in 1 way.
Sum 3 can happen in 2 ways.	←Sum 11 can happen in 2 ways.
Sum 4 can happen in 3 ways.	←Sum 10 can happen in 3 ways.
Sum 5 can happen in 4 ways.	←Sum 9 can happen in 4 ways.
Sum 6 can happen in 5 ways.	←Sum 8 can happen in 5 ways.
Sum 7 can happen in 6 ways.	

The pair which have same number of ways;

$2 \Leftrightarrow 12$ **sum of each pair = 14.(i.e. 2+12=14).**
$3 \Leftrightarrow 11$
$4 \Leftrightarrow 10$
$5 \Leftrightarrow 9$
$6 \Leftrightarrow 8$

Problem 1:

Two dice are thrown together. Find the probability of :

1. Getting a number greater than 10.
2. Getting a sum of 5.
3. Getting a sum is prime.
4. Getting a multiple of 3 or 4.

Solution:

1. Total number of possible outcome = 36
   Getting a number greater than 10 means we want 11 or 12.

Sum 11 can happen in 2 ways or Sum 12 can happen in 1 way.
Number of events of getting number greater than 10 = 2+1=3
There for probability of Getting a number greater than 10
P(E) = 3/36 = 1/12.

2. Total number of possible outcome = 36
   Getting a sum of 5:

Sum 5 can happen in 4 ways.
A number of events of getting a sum of 5 = 4.
There for probability of getting a sum of 5
P(E) = 4/36 = 1/9.

3. Total number of possible outcome = 36

Getting a sum is prime. In this case, we will go and search situations for sum2,sum3,sum5,sum7, and sum 11.
Sum 2 can happen in only 1 way.
Sum 3 can happen in 2 ways.
Sum 5 can happen in 4 ways.
Sum 7 can happen in 6 ways.
Sum 11 can happen in 2 ways.
Number of events of getting sum is prime = 1+2+4+6+2 =15
There for probability of getting a sum is prime
P(E) = 15/36 = 5/12.

4. Total number of possible outcome = 36

Getting a sum is multiple of 3or4. Multiple of 3 or 4 is 3,4,6,8,9,12
Sum 3 can happen in 2 ways.
Sum 4 can happen in 3 ways.
Sum 6 can happen in 5 ways.
Sum 8 can happen in 5 ways.

Sum 9 can happen in 4 ways.
Sum 12 can happen in 1 way.
Number of events of getting sum is multiple of 3or4 = 2+3+5+5+4+1 = 20
There for probability of getting a sum is multiple of 3or4
P(E) = 20/36 = 5/9.

Probability-based on cards

Some basic information about cards:

1. Pack of cards = 52
2. There are 4 suites in a pack of 52 cards.(**clubs,spades,diamonds,hearts)**
3. 13 cards in each of the 4 suits.
4. Each of 4 suits has an ace,2,3,4………,10, jack, queen, king.
5. Clubs and spades are in black color.
6. Diamonds and hearts are in red color.
7. Jack is at the same time in problems also referred to as Knave.
8. Jack, Queen, and King are face cards.

Problem 1:
A card is drawn from a pack of 52 cards. Find the probability:

1. A spade.
2. A king.
3. A Black card.
4. A king or a queen.
5. A face card.
6. A king or a spade.

Solution:

1. A total number of possible outcomes = 52.

The number of events of drawing a spade = 13.
Therefore the probability of a spade
P(E) = 13/52.

2. The total number of possible outcomes = 52.

The number of events of drawing a king = 4.
Therefore the probability of a king
P(E) = 4/52.

3. The total number of possible outcomes = 52.

The number of events of drawing a black card = 26.
Therefore the probability of a black card
P(E) = 26/52.

4. The total number of possible outcomes = 52.

The number of events of drawing a king or queen = 4+4=8.
Therefore the probability of a spade
P(E) = 8/52.

5. The total number of possible outcomes = 52.

The number of events of drawing a face card = 12.
Therefore the probability of a face card
P(E) = 12/52.

6. The total number of possible outcomes = 52.

A king or a spade: there are 4 kings(among 4 kings one king of spades) out of 52 cards and 13 cards of spades.

Number of events of drawing a king or a spade = 4+12 = 16
Therefore the probability of a king or a spade
P(E) = 16/52.

Problem 2:
Two cards are drawn at random **without replacement** from a pack of 52 cards. Find the probability of:

1. 1 queen and 1 king.
2. 1 red and 1 black.

Solution:

1. **1 queen and 1 king :**

The total number of possible outcomes = 52.
From a pack of 52 cards probability of queen = 4/52.
From a pack of 52 cards probability of king = 4/52.

1 queen and 1 king :
In this case, 1st is queen & 2nd is king or 1st is king and 2nd is the queen
Q&K or K&Q i.e. $4/52 \times 4/51 + 4/52 \times 4/51 = 8/(52 \times 51)$.
Therefore P(1Q & 1K) = $8/(52 \times 51)$.

2. **1 red and 1 black :**

A total number of possible outcomes = 52.
From a pack of 52 cards probability of red = 26/52.
From a pack of 52 cards probability of black = 26/52.

1 red and 1black :
In this case, 1st is red & 2nd is black or 1st is black and 2nd is red
R&B or B&R i.e. $26/52 \times 26/51 + 26/52 \times 26/51 = 52/(52 \times 51)$.
Therefore P(1Q & 1K) = 1/51.

Probability-based on balls from boxes

Problem 1:
A box contains 10 red, 5 blue, and 1 black. All the balls are identical and 1 ball drawn at random. What is the probability that :

1. The ball is red.
2. The ball is blue.
3. The ball is black.

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.

1. **Ball is red:**

n(E) = number of ways of drawing red balls = 10.
Therefore probability of drawing red balls
p(E) = n(E)/n(S) = 10/16.

2. **Ball is blue:**

n(E) = number of ways of drawing blue balls = 4.
Therefore probability of drawing blue balls
p(E) = n(E)/n(S) = 5/16.

3. **Ball is black:**

n(E) = number of ways of drawing black balls = 1.
Therefore probability of drawing black balls
p(E) = n(E)/n(S) = 1/16.

One ball question is very simple, but the main question here draws two balls. In such cases, there are two kinds of questions.

1. Ball drawn with replacement.
2. Ball drawn without replacement.

Problem 2:
A box contains 10 red, 5 blue, and 1 black. All the balls are identical and 3 balls drawn at random one after the other with replacement. What is the probability that all 3 balls are red?

Solution:
Total number of balls = 10+5+1=16. i.e. n(S) = 16.
n(E) = number of ways of drawing red balls = 10.
Probability of a red ball = 10/16
Therefore the probability of drawing 3 red balls with replacement
1st red & 2nd red & 3rd red
10/16× 10/16 ×10/16.

Draw 1 ball from 2 boxes or 3 boxes

Problem 1:
A box contains 10 red, 5 blue and 2 black and another box contains 5 red, 7 blue and 8 black. 1 ball drawn at random from any of the 2 boxes. Find the probability that the ball is black?

Solution:
Probability of black ball from 1st box = 2/16
Probability of black ball from 2nd box = 8/20.
Selection of 1st box = 1/2
Selection of 2nd box = 1/2
P(Ball is black) = 1st box & Black ball or 2nd box & Black ball
= 1/2×2/16 + 1/2×8/20
= 1/16 + 8/40.

Word-based question on probability

Problem 1:
What is the probability that there are 53 Sundays in a normal non-leap year?

Solution:
In a non-leap year = 365 days.
365 days have 52 complete weeks and 1 day.
The 365 days calendar will start on 1st January and 1st week will end on 7th January …….
And so on …. the 52nd week will end on 30th December.

For 53 Sundays in a non-leap year, the last day of the year 31th December has to be a Sunday, and the probability of 31st Dec being a sunday = 1/7.
Hence the answer = 1/7.

Problem 2:
What is the probability that there are 53 Sundays in a leap year?

Solution:
In a leap year = 366 days.
366 days have 52 complete weeks and 2 days.

The 52nd week would end on the 364th day of the year and that day would be 29th December.
For 53 Sundays in a non-leap year, the last 2 days of the year would be

1. Sunday or Monday
2. Saturday or Sunday
3. Monday or Tuesday
4. Tuesday or Wednesday
5. Wednesday or Thursday
6. Thursday or Friday and
7. Friday or Saturday

Last 2 days of the year out of 7 cases. Out of 7 cases, only 2 cases have Sundays in them.
Hence the probability of 53 Sundays in a leap year = 1/7.

Time, speed, and distance

Introduction to time, speed, and distance

Introduction

Time, Speed, and Distance is an important chapter for the purpose of the Maths section in aptitude exams. The basic concepts of Time, Speed, and Distance are used in solving questions based on straight-line motion, relative motion, circular motion, problems based on trains, problems based on boats, clocks, races, etc.

Time, Speed and Distance is a situation related to the motion of a body. If a person is moving from point 'x' to point 'y', this journey is described by three variables, and every Time, Speed and Distance question has only 3 variables in it (time, speed, and distance).

Time, Speed & Distance formula :

(a) Distance = Speed×Time
(b) Time = Distance/Speed
(c) Speed = Distance/Time

Units:

Speed: m/sec, km/hr, and in some cases, you will see km/min, m/min, feet/sec, and feet/hr.

Time: min, hour and sec

Distance: km, meter and miles

Whenever you will use Speed×Time = Distance formula, units of all three Time, Speed and Distance should be consistent with each other, which means if speed is in kmph(km/hr), you can't take time in sec or min, time will have to be in "hour" and distance will have to be in "km".

Conversion:

1 km = 1000 meters = 0.6214 mile

1 mile = 1.609 km

1 hr = 60 min = 60*60 seconds = 3600 seconds

1 km/hr = 5/18 m/s

1 m/s = 18/5 km/hr

1 km/hr = 5/8 miles/hour

A car is travelling at 40 kmph from point ‘x’ to observer ‘o’ for a distance of 80 km.

40 kmph can be described as the rate at which a car is approaching the observer. So, every hour the car will keep coming 40 km closer to the observer.
If a journey is of 80km, so the car will take 2 hr to reach the observer.
Another way of looking it is;

The rate at which the car is moving away from the observer. And in this case, the car will reach the point x in 2hrs if the speed and distance are kept the same

The proportionality in the TSD equation:

1. **s ∝ d** if time is constant.
2. **t ∝ d** if speed is constant.
3. **s ∝ 1/t** if the distance is constant.

1. s ∝ d if time is constant.

In the first proportionality, time should be constant in both motions, whether the two bodies are moving or two different journeys by the same car. After observing both the motions, if the time required is the same for both of them then, you can say that this is a constant time situation.
In time constant proportionality, if the speed increases then distance also increases in the same manner.

For example:
If train 1 starts from X and train 2 starts from Y and they start moving towards each other at the same time. They meet at a point somewhere in between.

Solution:
Let's say they start at 1 pm and meet at 3 pm.

So, here we can see that there are two motions and for these motions, the value of time is 2 hours.
Let say Sx and Dx be the speed and time respectively for train 1.
& Sy and Dy be the speed and time respectively for train 2.
In this case, the following ratio will be valid:
$\frac{Sx}{Dx} = \frac{Sy}{Dy}$

2. **t ∝ d if speed is constant.**

Example:
A car moves for 4 hours at a speed of 25 kmph and another car moves for 5 hours at the same speed. Find the ratio of distances covered by the two cars.
Solution:
Since the speed is constant, we can directly conclude that time ∝ distance.
Hence $\frac{Ta}{Tb} = \frac{Da}{Db}$
Since the times of travel are 2 and 3 hours respectively, the ratio of distances covered is also 4/5.

3. **s ∝ 1/t if distance is constant.**

Example:
A man goes from Delhi to Karnal and Comes back. In this case distance for Delhi to Karnal and Karnal to Delhi is the same i.e distance is constant. Hence, the speed will be inversely proportional to the time.
If the distance is constant it is also a product constancy situation (s×t = constant). Hence you can use any of the product constancy structures.
In this case, the following ratio will be valid;
$\frac{Sa}{Sb} = \frac{Tb}{Ta}$

Problem Based On Proportionality:

Problem 1:
Abhishek walks at 3/4th of his normal speed and he is 16 minutes late in reaching the office. Find his normal time of reaching office.

Solution:
Let S1 = s and T1= t be its normal speed and time respectively.
And S2 = 3/4×s and T2 = t+16.
Here distance is the same i.e distance constancy situation.
Speed from 's' to 3/4×s i.e. speed is reduced by 1/4th and time from 't' to t+16 i.e. time would be increased by 1/3rd as speed is reduced by ¼.
(s×t = constant, is 's' reduced by 1/4 then 't' increased by 1/3)
time from 't' to t+16 i.e. time is increased by 1/3rd means 1/3rd of normal time 't' = 16 min
Therefore, Normal time = 16×3 = 48 min.

2nd method:
We know ratio;
$\frac{S1}{S2} = \frac{T2}{T1}$
T1 = $\frac{S1}{S2} \times$ T2
t = $\frac{3}{4} \times (t+16)$
Therefore, the normal time 't' = 48 min.

Problem 2:
Two people X and Y travelled the same distance at speeds of 6 kmph and 10 kmph respectively. If X takes 1 hour longer than Y then, what is the distance being travelled?

Solution:
Lets 't' be the time taken by Y. So, time taken by X is t+1.
Speed of X = 6 kmph and speed of Y = 10 kmph.
We can solve this problem by following methods:
Method 1:
Here given that;
Difference of time = 1

d/6 - d/10 = 1
10d - 6d = 60, d = 15 km.
Therefore distance travelled = 15 km

Method 2:
Distance is constant so;
$S1 \times t1 = S2 \times t2$
6(t+1) = 10t
t = 3/2 hr
Therefore distance = speed×time
= 10×3/2 = 15 km

Concept Of Relative Speed:

We already discussed the movement of a body with respect to a stationary point. And now, we need to determine the movement and its relationships with respect to a moving point/body. In such situations, we have to take into account the movement of the body w.r.t. which we are trying to determine relative motion.
"Relation motion of a body is the motion of one body/point with respect to other body/point"

Case 1: Two cars C1 & C2 are moving in opposite directions. C1 moving at S1 kmph and C2 moving at S2 kmph.
So, Relative speed S = S1+S2.

Problem 1:
Two cars C1 & C2 are moving towards each other. C1 at 50 kmph and C2 at 30 kmph. The initial distance between them is 280 km. After how much time they will meet?
Solution:
S1 = 50 kmph
S2 = 30 kmph
The speed with which they are approaching S = S1+S2
S = 50+30 = 80 kmph
They have to approach each other and reach the meeting point.
So, approaching distance/Relative distance= 280 km
Hence, Relative Speed×Time = Relative Distance

$80 \times t = 280$
t = 3.5 hours.
Therefore; they will meet after 3.5 hours

Case 2: Two bodies are moving in the same direction.
So, the Relative Speed S = S1 - S2

Problem 1:
Two cars C1 & C2 are moving in the same direction at a speed 50 kmph and 30 kmph respectively from the same point and they start moving at 2 pm. After how many hours will C1 be 140 km ahead of C2?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Relative Distance = 140 km
So, Relative Speed×Time = Relative Distance
$20 \times t = 140$
t = 7 hours.
Therefore, after 7 hours C1 ahead 140 km of C2.

Problem 2:
Two cars C1 & C2 are moving in the same direction. Car C2 going at 30 kmph and C1 catching up at 50 kmph, starting distance between them is 120 km. In how many hours does C1 catch C2?

Solution:
S1 = 50 kmph
S2 = 30 kmph
The Relative Speed S = S1-S2
S = 50 - 30 = 20 kmph
Relative Distance = 120 km
So, Relative Speed×Time = Relative Distance
$20 \times t = 120$

$t = 6$ hours.
Therefore, in 7 hours C1 catches C2.

Question-Based On Relative Motion:

Type 1: Policeman and theft question

Problem 1:
The theft is committed at 2 A.M and the thief after committing the theft starts escaping at a speed of 80 kmph. The theft is discovered at 6 A.M and the policeman gives pursuit of the thief at 100 kmph. Find at what time the policeman will catch the thief?

Solution:
Speed of Thief = 80 kmph
Speed of policeman = 100 kmph
According to question,
Distance between thief and policeman after 4 hours (2 A.M to 6 A.M) = $80 \times 4 = 320$ km.
Speed at which policeman approaches thief = 100 - 80 = 20 kmph
So, Relative Speed×Time = Relative Distance
$20 \times t = 320$
$t = 16$ hours
Therefore, police caught thief at 10 P.M (6 A.M + 16 hours =10 P.M)

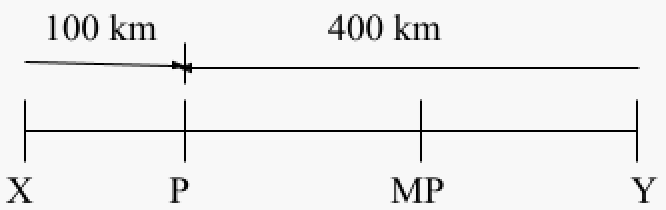
Problem 2:
At what distance from the original point did the thief get caught?

Solution:
To answer this question we have to find out the policeman's journey.
Speed of policeman = 100 kmph
Time taken by the policeman to catch the thief = 16 hours.
So, distance = $100 \times 16 = 1600$ km.

Type 2: Train question

Problem 1:
Two trains T1 and T2, T1 starting from point X to Y and T2 starting from point Y to X 2 hours later. T1 moving at 50 kmph and T2 at 30 kmph. Distance between point X and Y is 500 km. Find the distance from X, after which they will meet.

Solution:
Speed of T1 = 50 kmph
Speed of T2 = 30 kmph
Distance between X and Y = 500 km
Train T1 starts 2 hours before train T2.
Distance covered by T1 in 2 hours = 50×2 = 100 km
Let us say T1 reaches at point P in 2 hours.



Distance left from point P to Y = 500 - 100 = 400 km
Speed at which they approaching = 50+30 = 80 kmph
Approach required to get the meeting point (MP), the total distance they have to approach together = 400 km
So, Relative Speed×Time = Relative Distance
80×t = 400
t = 5 hours
So, distance from P to MP = 50×5 = 250 km
Therefore, they will meet 350 km (100 + 250 = 350) from point X.

Concept Of Circular Motion:

The movement of an object along a circle is called circular motion. When we talk about circular motion, there are 3 variables inside the questions. 1. Speed 2. Circumference 3. Time.

Units:
Speed: m/sec, kmph or it can also be measured in %/sec,%/min and %/hr.

Circumference: meter, km or % (if circle as 100%)

Problem 1:

Three people A, B, and C running around a circle, whose circumference is 100 km. Speed of A is 20 kmph and the speed of B is 15 kmph and speed of C is 12 kmph.

a. After how much time they will meet at the starting point.
b. How many rounds were done by A?
c. The time required for the first meeting at any point.

Solution:

Speed of A = 20 kmph
Speed of B = 15 kmph
Speed of C = 12 kmph
Circumference = 100 km

(a) Let Ta, Tb and Tc be the time taken by A, B and C respectively to cover the circle.

So, Ta = 100/20 = 5 hours
Tb = 100/15 = 20/3 hours
Tc = 100/12 = 25/3 hours
Time required to meet at starting point = LCM(Ta,Tb,Tc)
We know, LCM of fraction = LCM of numerator / HCF of denominator
= LCM (5,20/3,25/3) = 100/1 = 100 hours
Hence, they meet at the starting point after 100 hours.

(b) A done one round in 5 hours.

So, In 100 hours A done = 100/5 = 20 rounds.

(c) A is fastest, A would be overlapping each of B & C after some time.

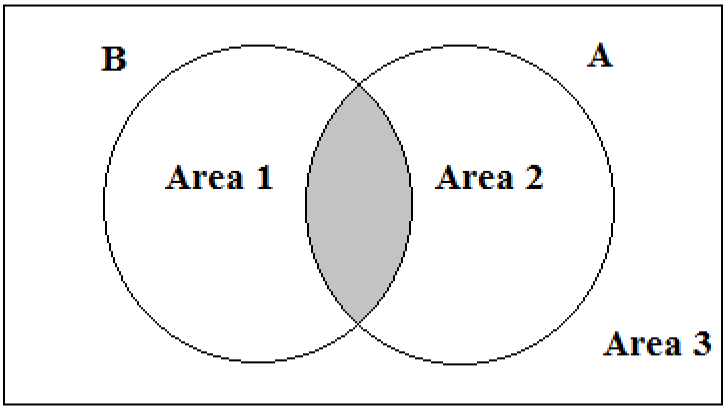
Let Tab and Tac be the time in which A overlap B and C respectively.
The time required for the first meeting at any point = LCM(Tab, Tac)
Relative speed between A and B 'Sab' = 20-15 = 5 kmph
Relative speed between A and C 'Sac' = 20-12 = 8 kmph
So, Tab = 100/5 = 20 hours and Tac = 100/8 = 12.5 hours.
LCM (20,25/2) = 100 hours
Hence, they will meet at any point after 100 hours.

Set theory

Set theory is important both from a mathematical point of view as well as a reasoning point of view. You will see a lot of questions based on set theory in a lot of aptitude exams. Set theory questions have two ways of solving.

1. Formula approach.
2. Venn Diagram approach.

Two attributes situation:
Let's have a situation where two attributes A and B. A refers to those people who passed Physics and B refers to those people who passed Chemistry.



The rectangular box represents a universal set.
Area 1: People who passed only Physics.
Area 2: People who passed only Chemistry.
Area 3: People who passed neither Physics nor Chemistry.

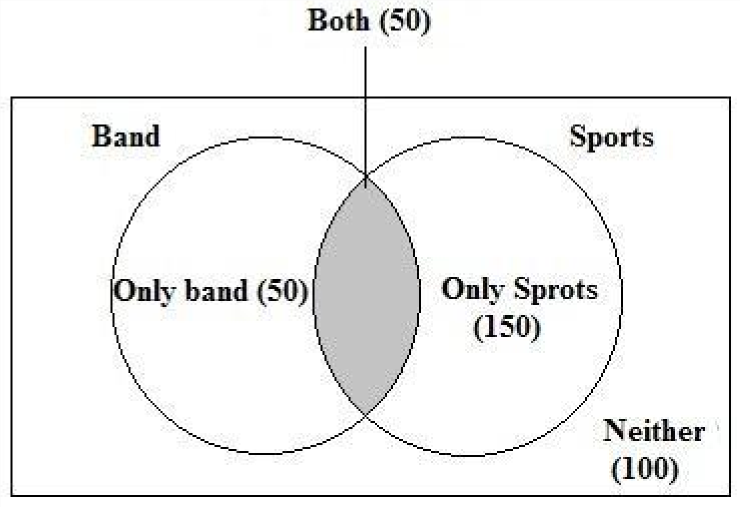
Formula: A∪B = A + B - A∩B.

Problem 1:
In a school of 350 students, 100 are in the Band, 200 are in the Sports team and 50 are in both Band and Sports team.

1. How many students are involved neither in Band nor in Sports?
2. How many people involved at least one of the two?
3. What is the ratio of people who participate only in the band to only in sports?

Solution:

50 students are in both Band and Sports. So, 100 - 50 = 50 students are in Band only and 200 - 50 = 150 students are in Sports only.
Total students 350 and 350 - 250 = 100 students are neither in Band nor in Sports.

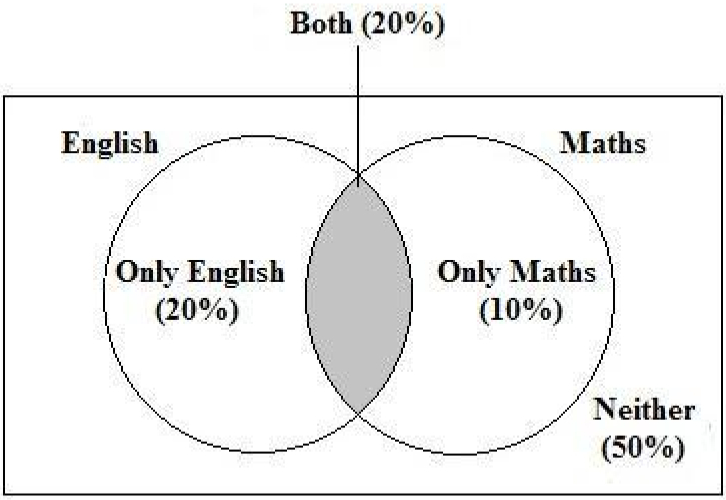


1. Students are involved neither in Band nor in Sports = 100.
2. Students involved at least one of the two = 50+50+150 = 250.
3. Students only in Band = 50 and students only in Sports = 150
   Hence, the Ratio of students only in the band to only in sports = 50:150 = 1:3.

Problem 2:
There are 60 students in a class, 60% fail in English and 30% pass in Maths and 20% pass in both English and Maths. How many students fail in either of 2 subjects or at least in one subject?

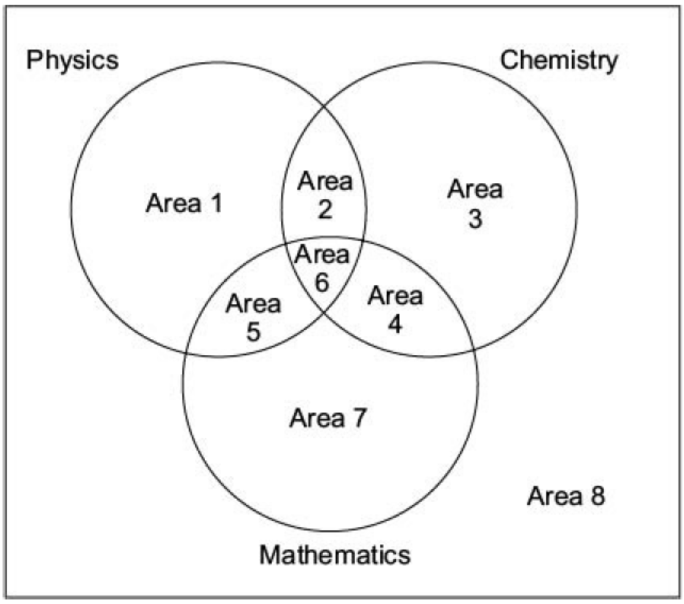
Solution:
20% of students pass in both English and Maths. So, 30% - 20% = 10% of students pass in maths only and 60% fail in english means 40% pass in english and 40% - 20% = 20% of students pass in English only.
Total students 100% and 100 - 50 = 50% of students neither pass in english nor pass in maths.

Number of students fail in either of two subjects = 20% + 10% = 30% i.e 30% of 60 = 18 students.
Number of students fail in at least one subject = 20 + 10 + 50 = 80% i.e 80% of 60 = 48 students.



Three attributes situation:

Let's have a situation where there are three attributes being measured. Suppose we are talking about people who passed Physics, Chemistry and Mathematics.



Area 1: People who passed in Physics only
Area 2: People who passed Physics and Chemistry but not Maths.
Area 3: People who passed Chemistry only
Area 4: People who passed Chemistry and Maths but not physics.
Area 5: People who passed Physics and Maths but not in Chemistry.
Area 6: People who passed Physics, Chemistry and Maths

Area 7: People who passed Maths only
Area 8: People who passed in no subjects.

People passing Physics and Chemistry: Represented by the sum of areas 2 and 6
People passing Physics and Maths: Represented by the sum of areas 5 and 6
People passing Chemistry and Maths: Represented by the sum of areas 4 and 6
People passing Physics: Represented by the sum of the areas 1, 2, 5 and 6
People passing at least 2 subjects = area 6 + area 2/4/5
People passing exactly 2 subjects: represented by area 2,4 and 5.

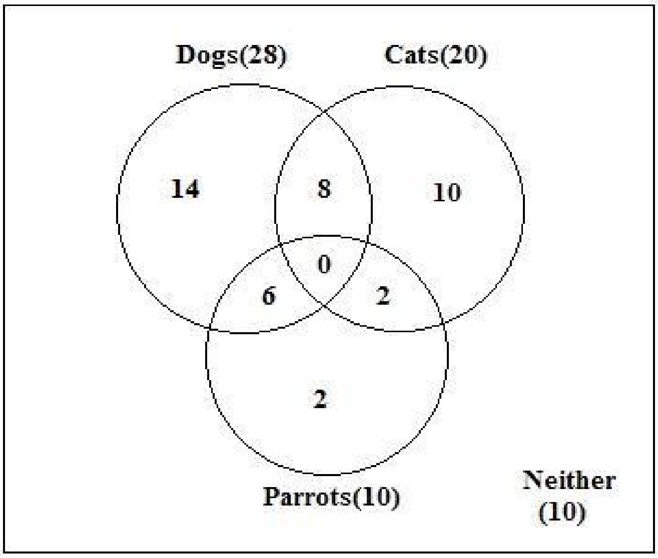
Problem 1:

A veterinary doctor surveyed 52 people. He discovered that 28 have dogs, 20 have cats and 10 have parrots, 8 have dogs and cats, 6 have dogs and parrots and 2 have cats and parrots. No one has all three pets.

1. How many people have only a dog?
2. How many people have at least 2 pets among dogs, cats and parrots?
3. How many people have none of the 3 pets?

Solution:

8 people have dogs and cats, 6 people have dogs and parrots. 28 - (8+6) = 14 people have only dogs.
8 people have dogs and cats, 2 people have cats and parrots. 20 - (8+2) = 10 people have only cats.
6 people have dogs and parrots, 2 people have cats and parrots. 10 - (6+2) = 2 people have only parrots.



1. People have only a dog = 14.
2. People have at least 2 pets = 6+8+2= 16.
3. People have none of the 3 pets = 10.

Logarithms

Introduction To Logs:

Questions based on this chapter are not so frequent in aptitude exams. You will find some questions based on logs, to solve those questions you have to learn some basic formulae.

Definition of "log":

Let 'a' be a positive real number and a^b= c. then 'b' is called the logarithm of 'c' to the base 'a' and written as $log_a c$and vice versa, if $log_a c$= b, then a^b= c.

NOTE: Log of a negative base is not defined.

$log_a c$= b is possible if and only if a>0 and c>0.

Formulae for log:

1. $log_b a + log_b c = log_b (a \times c)$
2. $log_b a - log_b c = log_b \frac{a}{c}$
3. $log_a 1$ = 0 for all a > 0
4. $log_a a$ = 1 for all a > 0
5. $log_c a^b$ = b$log_c a$

Base change rule:

Till now all the formulae are in logarithm with the same base. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different bases. Those situations are:

1. $log_y x = log_z x / log_z y$
2. $log_y x = log_x x / log_x y$= 1/$log_x y$
3. $log_{(y^z)} x$= (1/z)$log_y x$

Problem 1:

$log_3 x$= $log_{12} y$= a, where x,y are real positive numbers. If G is the geometric mean of x and y. What is the value of $log_6 G$?

Solution:

From the statement, $log_3 x$= $log_{12} y$= a, we have

$log_3 x$= a and $log_{12} y$= a

By definition of the log;

$log_3 x$= a, x = 3^a and $log_{12} y$= a, y = 12^a

G is the geometric mean of x and y. So, G = $\sqrt{xy}$

G = $\sqrt{3^a . 12^a}$ = $\sqrt{36^a}$= 6^a

Now; $log_6 G$= $log_6 6^a$= a$log_6 6$ = a

Hence, $log_6 G$= a.

Sentence Completion/Fillups

Introduction:

Fill in the blanks also in the same case is called sentence completion. It is basically a combination of both reading skills and grammar knowledge.

Sentence completion is of three types:

1. Single blank
2. Double blank
3. Cloze test

1. Single blank: It is basically one sentence with one blank that you have to fill.
2. Double blank: It is a longer sentence with two blanks that you have to fill.
3. Cloze test: It is like a paragraph having some blanks. Actually it a combination of both fill in the blanks and reading comprehension.

What all factor kept in mind:

1. First of all, you should always have a mental answer when you are trying to solve a problem.
2. With the mental answer, match it with the option skill.
3. Vocabulary should be very very strong.

4. **The idea of the sentences**:
   Every sentence has an idea and each sentence also communicates ideas.

   For example:
   Sentences are either positive or negative. If the positive sentence the blank word will be positive and if the sentence is negative the blank word will be negative.
   Whether sentences are formal or informal. Let us say friend is a formal word and pal/buddy is an informal word.

5. **Proactive solving:**
   Usually, sentences go through the option first and try to somewhat how to fit into the blanks, this way of approach is called **reactive solving** and this is likely to cause errors.
   A better way would be proactive solving means acting in anticipation. In other words, try to guess the answer without solving.

6. Identify the clues present in the sentence. A positive sentence, negative sentence, formal sentence, informal sentence these all are clues in the sentence.

7. Pay special attention to introductory and transitional words. **Introductory** means this thing or that thing is talking about one thing or many things. **Transitional** words are like, but, although, however, yet, even, in spite off, despite off, etc.

   For example:
   Ravi is a good boy but his brother is a bad boy.
   If the 1st part is positive and the 2nd part will be negative and vice versa.

8. Be sure your choice is both logically and grammatically correct. Make sure your grammar matches with the sentence, otherwise, grammar is not matching even if the meaning of the word is correct, grammatically the sentence will be wrong.

9. If you do not know words use elimination and educated guessing. This means you are able to make one or more choices that are definitely wrong or guessing from context when you know a related word.

There are several types of sentence completion:

1. Restatement
2. Comparison
3. Contrast
4. Cause and effect

1. Restatement: Restatement means repeating the same things again and again. So, if it's a positive one, it will be positive and if it's a negative one, it will be negative.

For example:
The city council formed a committee to simplify several dozen _______ city ordinances that were unnecessarily complicated and out-of-date.

a. feckless b. empirical c. byzantine d. Slovenly e. Pedantic

Answer :
Here we are talking about something which was very complex and has been simplified. So, here the answer is 'c' i.e. byzantine that means very complicated.

2. Comparison: Two things are being compared. eg. Ram is a good boy similarly his brother is also a good boy.
In this case if it is positive it will remain positive and vice versa.
Similarly, likewise, and just as etc. are used for comparison.

3. Contrast: If contrast is there then but, although, despite, however, though, or etc. words you will be seen.
eg. Ram is a good boy but Shyam is a bad boy.

4. Cause and effect: Cause and effect mean one thing is the reason for others. Words like cause, lead to because, etc. when you have these words then you know there is a **cause & effect.** Even without these words, we can have cause & effect.

For example:
After a brief and violent _______ that ousted the president, General Mosanto declared himself the dictator of the country.
a. nurance b. Coup c. solicitation d. upbraiding e. lament

Answer:
In this sentence outage is a clue. Outage means to remove. Here the answer is ‘b’ coup that means to take over any government.

Questions On Sentence Completion:

a) Single blanks question:

1. His neighbours find his _______ manner bossy and irritating and they stop inviting him to backyard barbeques.

a. insentinent b. magisterial c. reparatary
d. restorative e. modest

Answer: b.

Explanation:
Find something which talks about his manner is bossy and irritating. So, magisterial is the answer that means dominating.
Insentient - can not sense anything, Reparatory - repayment, Restorative - having the ability to restore health and modest - very humble.

2. Shubham is always ________ about showing off work because he feels that tardiness is a sign of irresponsibility.

a. legible b. Tolerable c. punctual
d. literal e. Belligerent

Answer: c.

Explanation:
Tardiness means unpunctual or lazy. So, the answer is punctual.
Legible - handwriting, Tolerable - something you can tolerate, Literal - taking words in their usual sense and belligerent - a war like happening.

3. Anjali would ________ her little sister into an argument by teasing her and calling her names.

a. advocate b. provoke c. perforate
d. lament e. Expunge

Answer: b.

Explanation:
Her sister made her angry. So, the answer is, provoke that means anger.
Advocate - incorrect, perforate - make holes, lament - very sad and expunge - remove.

Cloze test:

In the cloze test, the whole paragraph has to be taken into concentration. Sometimes clues are given later on also, so it is a good idea to read the whole paragraph and then keep filling it as and when you can.

Text 1.
Giant pandas are black-and-white Chinese beers that are on the verge of (1)_____. These large, cuddly-looking mammals have a big head, a heavy body, rounded ears, and a short tail. Most bears' eyes have round pupils. The (2)______ is the giant panda, whose pupils

are vertical slits, like cats' eyes, these unusual eyes (3) _______ the Chinese call the panda "giant cat bear."
1. A) indication B) accommodation C) extinction.
2. A) dimension B) exception C) speculation
3. A) inspired B) predicated C) reversed

Answer:
1. Extinction.
The clue is on the verge.
2. Exception.
The clue is pupils are vertical slits.
3. Inspired.
Chinese inspired by the looking of giant pandas.]

Text 2.
Although the population of England in the nineteenth century was rising at a (1) ______ rate, that of the city was increasing by leaps. This was due to the effect of the industrial revolution; people were (2)______ into the towns and cities in search of employment; for the same, it was also the call of the unknown, (3)______ and a better way of life. This period is known to be the beginning of many new things.
1. A) crepuscular B) unprecedented C) reprehensible
2. A) flocking B) abrogating C) ensconcing
3. A) escapade B) pliable C) abstruse

Answer:
1. Unprecedented
Means like never before.
2. Flocking.
Means moving. The clue is town.
3. Escapade
Means adventurous

Introduction to Vocabulary

Vocabulary is an essential part of the English language section in almost every competitive exam. Vocabulary is dependent on the individual learning process. Vocabulary requires more and more practice in day-to-day life.

Vocab-Root words:

Root words are a very easy way to learn vocabulary. These words are a part of layer words. Root words have a significant meaning and root words can come either at the middle of the word or at the start of the word or at the end of the word. When the root word comes at the start of the word, it is called a prefix and when it comes at the end of the word, it is called a suffix.
By learning one root word we can learn many words.

Words starting with BENE, BON & BOUN :(BENE, BON & BOUN = WELL & GOOD)

Benefit = An advantage; as, the employees has fringe benefits
Benefiter = One who benefits; as, the employee is the benefiter
Beneficial = Wholesome; as, bathing is beneficial
Benefactor = Once who benefits others
Benefection = A gift; a donation
Benefactress = A female benefactor
Benedict = A male name which means "Blessed"
Benediction = A blessing
Benefice = The gift of an income to a priest of a church
Benevolent = Being good hearted; a well-wisher
Bonny = Sweet and attractive; as, a bonny child. And it is also used for pretty ladies.
Bonus = Extra benefits, usually extra pay
Bonanza = As unusually rich vein of gold or silver in a mine
Bonbon = A small candy
Bonbonniere = A fancy dish or box for bonbons
Bon mot = Witty remark or repartee
Bonnily = In a bonny manner
Bounty = A reward; a gift; generosity

Bountiful = Generous; munificent; large hearted
Bountifully = Generously
Benign = Harmless; also known as cancer

Words with root word MAL :(MAL = BAD & EVIL)

Maladroit = Clumsy; awkward; inept (not efficient) & (Adroit = skill)
Malady = Sickness; diseases
Malapert = Ill-bred; impudent, (Ill-bred = not well mannered)
Malapropos = Inappropriate; Not fitting
Malapropism = Humorous misuse of words
Malaria = A diseases carried by a mosquito
Malcontent = Rebellious; discontented; bad tempered; a person who is not happy
Malediction = Slander; curse
Malefaction = An evil deed
Malefactor = One who commits an evil deed; evildoer
Malevolent = Wishing evil to others
Malevolence = Wishing evil to others
Malevolence = the state of wishing evil to others; ill will; viciousness
Malfeasance = Evil conduct; especially by a public official
Malice = Ill-will as, "with malice toward none'; something bad; something negative
Malicious = Full of ill-will; full of malice
Malpractice = Professional misconduct; something wrong
Malignant = Injurious; extremely evil; tending to produce death
Malign = to utter slander of; to defame unjustly; to speak bad about someone
Maliferous = Disease bringing; productive of evil

What is important in vocabulary:

Principle 1: 90% of the tasks to remembering a word is to remember its meaning.

Principle 2: Meanings are better remembered through experience than just going through sequences of words.(same as our mother tongue).

Principle 3: You need to crash 20 years of experience into 6 months. (20 years of knowing your mother tongue whereas 6 months of preparation for your exams in english.)

Principle 4: The power of learning through synonyms and antonyms.
Synonyms are words that are similar in meaning, not same.

Introduction to Synonyms and Antonyms:

A word or phrase that has the same or nearly the same meaning as another word or phrase in the same language is called a synonym.

A word opposite in meaning to another (e.g. *bad* and *good*) is called an antonym.

Synonyms of slow:

- CAREFUL CAUTIOUS CRAWLING DAWDLING
- DELIBERATE DELAYED DILATORY GRADUAL
- LAGGING LATE LAZY LEISURELY
- LINGERING LOITERING MEASURED PAINSTAKING
- PLODDING PROTRACTED SLOW MOVING SLUGGISH
- STEADY TARDY TORPID UNHURRIED
- UNPUNCTUAL

DAWDLING:
To spend time idly, to move lackadaisically (lackadaisically- lazy)
#Dowdle up the hill.
Careful and cautious are related to the word slow.

DELIBERATE:

1. Characterized by or resulting from careful and thorough consideration *a deliberate decision*
2. Characterized by awareness of the consequences *deliberate falsehood* (deliberate falsehood - deliberately trying to slow)
3. Slow, unhurried, and steady as though allowing time for a decision on each individual action involved *a deliberate pace*.

DILATORY:

1. Tending or intending to cause delay ‘dilatory tactics’
2. Characterized by procrastination: TARDY (procrastination - just being lazy, try to delay something)

LEISURELY:

1. Without haste: DELIBERATELY
2. Comes from the word leisure which means free time.

LOITERING:

1. To delay an activity with aimless idle stops and pauses: DAWDLE
2. To remain in an area for no obvious reason: HANG AROUND
3. To lag behind

LINGERING:

1. To be slow in parting or in quitting something: TARRY
2. To remain alive, although gradually dying
3. To remain existent although often waning in strength, importance, or influence ‘lingering doubts’
4. To be slow to act: PROCRASTINATION
5. To move slowly: SAUNTER

PAINSTAKING:

1. The action of taking pains: diligent care and efforts (diligent - hard work)

PLODDING:

1. To work laboriously and monotonously: DRUDGE
2. To walk heavily or slowly: TRUDGE
3. To processed slowly or tediously ‘the moving just plods along’
4. To tread slowly or heavily along or over

PROTRACTED:

1. Archaic: DELAY, DEFER
2. To prolong in time or space: CONTINUE
3. To extend forward or outward

SLUGGARDLY:

1. Lazily inactive

SLUGGISH:

1. Averse to actively or exertion: INDOLENT (lazy); also: TORPID
2. Slow to respond (as to simulate or treatment)
3. Markedly slow in movement, flow, or growth
4. Economically inactive or slow

TARDY:

1. Moving slowly: SLUGGISH
2. Delayed beyond the expected or proper time: LATE

TORPID:

1. Having lost motion or the power of exertion or feeling: DORMANT, NUMB (not feel any sensation)
2. Sluggish in functioning or acting 'a torpid frog' 'a torpid mind'
3. Lacking in energy or vigor: APATHETIC, DULL

Antonyms of slow and synonyms of fast:

ADVERBS:

- FAST
- AT FULL TILT: at a very high speed
- BRISKLY: very fast
- IN ON TIME: without wasting time
- POST HASTE: do very very fast, now. It is used at the end of the sentence.
- QUICKLY: fast
- RAPIDLY
- SWIFTLY: comes from a swift bird. Swift is the fastest flying bird

ADJECTIVES:

- BREAKNECK: very fast
- BRISK: walking fast
- EXPEDITE: process doing fast / increasing the speed
- EXPRESS: very fast
- HASTY: fast (using in a negative manner)
- HEADLONG: used in two senses

1. Going at the headlong (very very fast) speed
2. When someone deep in his/her work

➢ HIGH SPEED
➢ LIVELY: very excited, very energetic
➢ NIPPY: speed
    1. Nip in the bud: means finishing them off right at the start
    2. Nip in the air: means moisture in the air
    3. Nip: bite, scratch, cut
➢ PRECIPITATE: move to action
    1. Undessolve part of solute
    2. Rainfall
➢ QUICK
➢ RAPID
➢ SMART
➢ SPANKING : (at a-pace) : very fast speed
➢ SPEEDY
➢ SUPERSONIC: faster than the speed of sound
➢ SWIFT
➢ UNHESITATING

Data Interpretation

Data interpretation, as the name suggests, is all about the analysis of data. Data interpretation is the process of making sense out of the collection of data. Data may be collected in the form of bar graphs, line charts and tabular forms and hence some kind of interpretation that we need.

Introduction to data

Data is the number that comes from the occurrence of any event - physical, social, economic, graphical and other kinds of events.
A number value by itself represents nothing. Thus if we imagine a number, say 40, it means nothing by itself. The number starts to gain some significance when any unit attaches to it, say 40 crores. However, just by saying that the number represents crores does not complete the description of the number. It has to be further qualified by specific descriptions, that is the sales revenue of Coding ninjas for the year 2019-20.

Thus, three facts attached to the number:

a. The number which represents the sales revenue.
b. It refers to a company Coding ninjas.
c. In the year 2019-20.

Introduction to data interpretation

The interpretation of data is the process through which some information is drawn about the data available for analysis.
Let say Coding ninjas in 2019-20 has sales revenue of Rs 40 crores and in 2020-21 has sales revenue of Rs 50 crore.

From these two sales revenue, you get certain information:

a. Company sales have grown by 10 crores.
b. Company % growth has been 25%.
c. The ratio of sales revenue for 2019-20 to 2020-21 is 5:4.

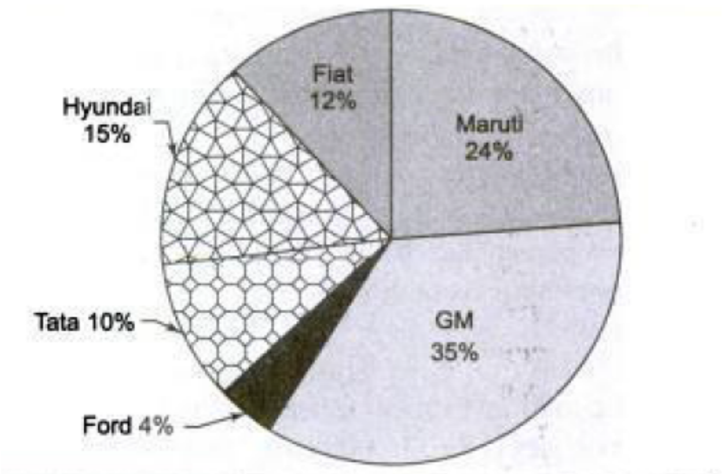
You make out these types of deduction by interpreting the data.

Data does not make any sense when it is in random form or it is difficult to draw out information from random data. So, you have to represent the data in some standard forms like a line graph, pie chart, bar chart, tables and caselet.

How To Read Pie Charts

Pie chart is a specific type of data presentation where data is presented in the form of a circle and pie charts essentially divide 100% of value within a circle. The circle is divided into various subparts. Each subpart represents a certain percentage of total. In the pie chart, the value of the individual pie chart will be an additive construct.

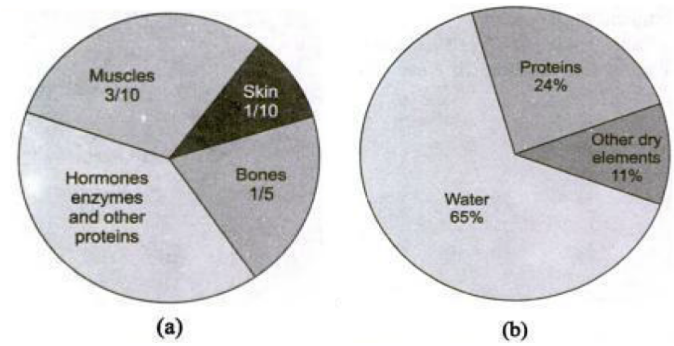
For example: A pie chart showing the distribution of car sales between six companies.



In this pie chart, Maruti has 24% of market share, while GM has 35% of market share, ford has 4% of market share, tata has 10% of market share, Hyundai has 15% of market share and fiat has 12% of market share.

The basic component here is car sales and divided into six companies. The pie chart is a circle, so it is also equal to 360^oor 100%. Thus, 1% is 3.6^oon a pie chart.

For example, The following pie chart figures (a) and (b) gives the information about the distribution of weight in the human body w.r.t. different kinds of components.



In this case, the kind of information that we can extract by interpreting what is given: Here muscles are 3/10 means 30%, the skin is 1/10 means 10%, bones 1/5 means 20% and hormones and enzymes and other proteins is 40%.

Let's say a person whose weight is 40kg. So, we can extract information about the components. Thus, weight of the muscles = 30% of 40 = 12 kg,
Weight of skin = 10% of 40 = 4 kg
Weight of bones = 20% of 40 = 8 kg
Weight of hormones and enzymes and other proteins = 40% of 40 = 16 kg
Weight of protein = 24% of 40 = 9.6 kg
Weight of other dry elements = 11% of 40 = 4.4 kg
Weight of water = 65% of 40 = 26 kg.

The question may be asked, what is the difference between water weight of a 40 and 60 kg person?
Water weight of 40 kg = 65% of 40 = 26 kg.
Water weight of 60 kg = 65% of 60 = 39 kg.
Difference between water weight = 39 - 26 = 13 kg.

In DI once you start understanding the variable, you start understanding the extraction or deduction you make. So, understanding the variable is the most important construct in DI.

How To Read Bar Charts

Data is always about variables, variables are either continuous or discrete.

For example, Sales revenue of company coding ninjas is 40 crores in the year 2019-20. Inside this statement there are few variables, which are running. The running variables are as follow:

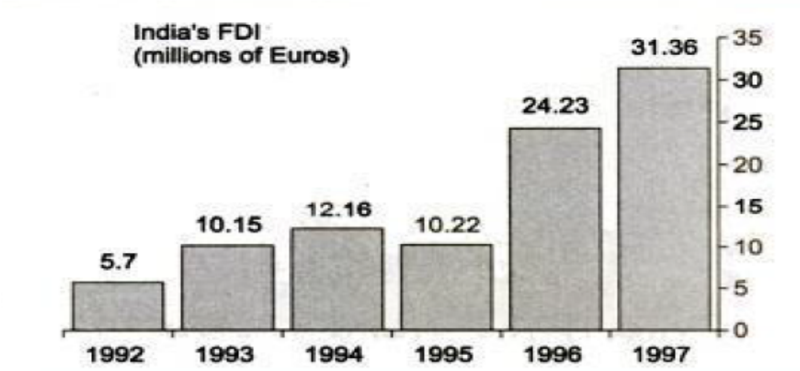
1. Number 40 crore is a sales revenue, which is a variable.
2. The year 2019-20 is also a variable because it could be 2020-21.
3. Company Coding Ninjas is also a variable.

Sales revenue is a continuous variable because it could be 40.01,40.12 etc. whereas the year 2019-20 and company coding ninjas are discrete variables.

Simple Bar Chart:

The simple bar chart is the simplest bar chart which has one continuous variable charted along with one discrete variable.

For example:

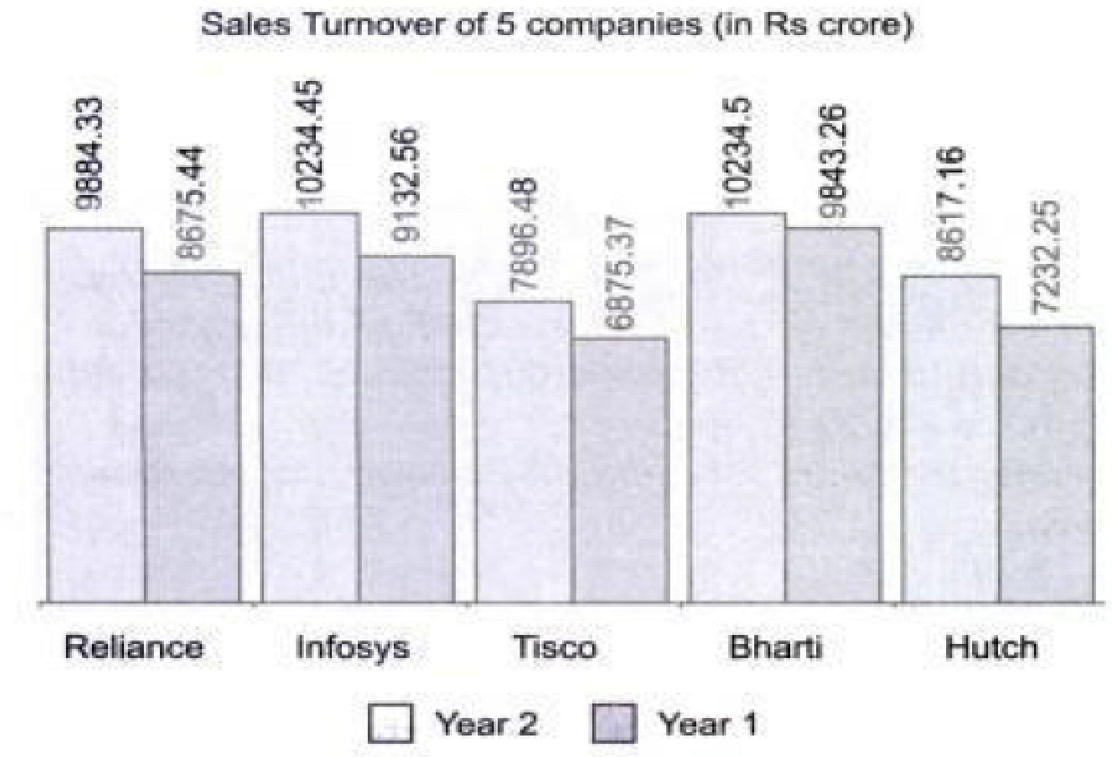


To read this Bar chart, we have to focus on the variables involved.
The year is a discrete variable.
Country India is also a discrete variable.
India's FDI is a continuous variable.
In 1992, number 5.7 meant 5.7 million euros. In this bar chart, we can see the trends of what is happening to India's FDI.

Composite Bar Charts:

In the composite Bar chart, we have two or more continuous variables that are represented.

For example: The following figure shows a Composite Bar Chart.



To read this Composite Bar chart, we have to focus on variables involved.

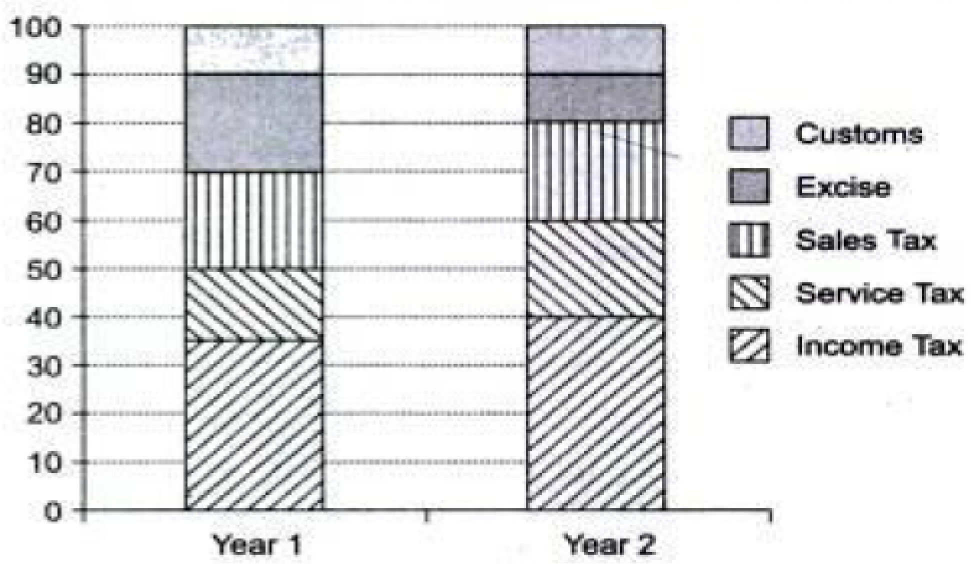
1. Year is a discrete variable.
2. Company names are also a discrete variable.
3. Sales turnover is a continuous variable.

This bar chart gives two or more information about the same discrete variable, for Reliance in Year 1 the sales turnover was 8675.44 crores and in the Year 2 was 9884.33 crores.

Stacked Bar Charts:

Stacked Bar charts represent multiple continuous variables. Sometimes stacked Bar chart can also be used to represent the break-up of some continuous variables.

For example:



Representing Percentage on Stacked Bar Chart

To read this Composite Bar chart, we have to focus on variables involved.

1. Year is a discrete variable.
4. Percentage is also a discrete variable.
5. Taxes i.e. customs, excise, sales tax, service tax and income tax are five continuous variables.

In the Stacked Bar Chart defining different types of taxes into their percentage component breakdown for Year 1 and Year 2.

How To Read Tables and X-Y charts

Tables:

Tables refer to the representation of data in form horizontal and vertical columns. Tables are one of the more versatile methods of representation of data. In tables, we can have any number of continuous variables over any number of discrete variables. The data that can be represented on any type of chart can also be represented on a table.

For example:: Representation of state-wise Literacy and Population growth on a table.

State	Percentage increase in		
	Total Literacy (From 1981 to 1991)	Female Literacy (From 1981 to 1991)	Change in % Population Growth Rate (From 1981 to 1991)
Andhra Pradesh	25.17	23.32	+ 0.09
Bihar	22.34	19.48	– 0.04
Gujarat	27.21	26.20	– 0.53
Haryana	29.19	28.67	– 0.11
Himachal Pradesh	31.06	31.00	– 0.24
Karnataka	27.52	26.63	– 0.47
Kerala	30.17	31.20	– 0.43
Madhya Pradesh	25.58	22.86	+ 0.13
Maharashtra	25.87	25.92	+ 0.10
Manipur	29.61	29.68	– 0.25

To read this Composite Bar chart, we have to focus on the variables involved.

1. Three continuous variables: (a) total literacy (b) female literacy (c) change in % population growth rate.
2. States are discrete variables.
3. Year is also a discrete variable.

Total literacy of Andhra Pradesh 25.17% (from 1981 to 1991) means literacy rate 10 years later increased by 25.17%.
Change percentage growth rate 0.09 means percentage growth rate 10 years later increased by 0.09.

Some following type of questions may arise after reading this table:

1. Which state has the highest % growth in literacy?
   Ans: % growth literacy highest for Himachal Pradesh (31.06%)
2. Which state shows the lowest % growth in female literacy?
   Ans: Bihar (19.48%)
3. How many states may have negative growth in population growth rate while having more than 20% growth in both total literacy and female literacy?
   Ans: Gujrat, Himachal Pradesh, Haryana, Karnataka, Kerala and Manipur i.e. 6 states.

Example 2: Shows courier charges (in Rs) for sending a parcel of 1 kg from one city to another city.

Courier Charges For Sending Parcel:

Cities	Allahabad	Mumbai	Kolkata	Delhi	Lucknow
Allahabad	–	10	5	15	10
Mumbai	10	–	7	25	20
Kolkata	5	7	–	20	15
Delhi	15	25	20	–	10
Lucknow	10	20	15	10	–

In this table, sending parcels from Allahabad to Mumbai costs 10 Rs. and sending parcels from Lucknow to Mumbai costs 25 Rs. Similarly, we can see the costs for other cities.

In this table what kind of question can be asked, Minimum cost, maximum cost, % difference in cost or cost of the parcel from Mumbai to Kolkata and then from Kolkata to Delhi.

In this table from Mumbai to Delhi and Delhi to Mumbai has the same courier cost and this is true for every pair.

Example 3: Employees working in various departments of Hoola Moola Boola, Inc.

Years	Departments (Number of Employees)				
	Production	Marketing	Corporate	Accounts	Research
1999	150	25	50	45	75
2000	225	40	45	62	70
2001	450	65	30	90	73
2002	470	73	32	105	70
2003	500	80	35	132	74
2004	505	75	36	130	75

Variables are; Year, Departments and Number of employees. Let's say if we want to extract in 2004 the number of employees in Research, then it is 75 employees.

X-Y Charts:

As the name itself suggests the X-Y Charts will be, in which discrete variables against the continuous variables.

X-Y charts are also useful in determining the trends, rate of change and for illustrating comparison w.r.t some time series.

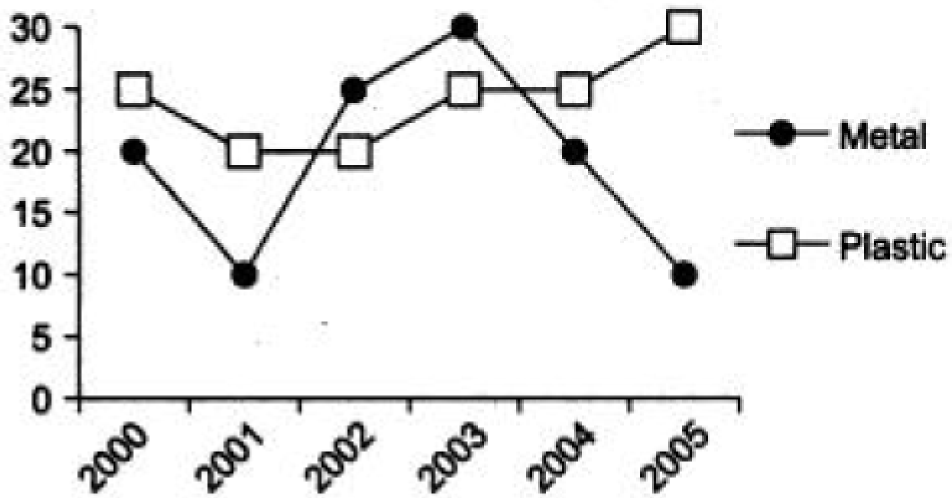
Feo example: The X-Y Chart of Consumer Price Index In 1993-94.

The continuous variable in this set is the consumer price index for the year 1993-94 and the discrete variable is the name of months.

Consumer price index in 1993-94, Jan was 335.

In X-Y charts we also have multiple continuous variables:

For example, The following graph shows the trends of consumption of metals and plastic in the production of cars between 2000-2005.



Consumption of metals versus plastic in given years for car manufacturing (in thousand tons)

In 2000 the metal used in cars was 10 k tons and plastic used in cars was 20k tons.

There is a small difference between Line cheats and X-Y charts, in Line charts we draw the lines and in X-Y charts the line will not be there, only points will be marked.