Capacitated Vehicle Routing Problem with Time Window - Single Depot.

Parameters:

C_{ij} = cost to visit node j from node i

t_{ij} = travel time to visit node j from node i

d_i = demand at node i

q_γ = Capacity of Vehicle γ

(a_i, b_i) = time window of customer i

t_s = service time at each customer = 20 mins

f_γ = fixed cost of vehicle $\gamma = 2 \cdot q_\gamma$

O_γ = variable cost of vehicle γ (per km)

$$= \left[\frac{2000 - q_\gamma}{1000} \right]$$

Sets:

N = Set of orders including source and sink as depot.

$C = \{1, 2, \ldots k\}$, $N = \{0, 1, 2 \ldots k, k+1\}$

$0, k+1 \Rightarrow$ depot

V = Vehicle fleet : $\{V_1, V_2 \ldots V_r\} \rightarrow$ Ids

$\{q_1, q_2 \ldots q_r\} \rightarrow$ Capacity

$(i, j) \quad i \neq j \;,\; i \neq k+1 \;,\; j \neq 0$

⇒ Decision Variables:

$$x_{ij\gamma} = \begin{cases} 1 & \text{, if vehicle } \gamma \text{ drives from } i \text{ to } j \\ 0 & \text{, else.} \end{cases}$$

$$x_{ij\gamma} \in \{0,1\} \quad , \forall\, i,j \in N \quad , \forall\, \gamma \in V$$

S_{ir} = service time at Customer i

i.e. time Vehicle γ, starts to service customer i.

l_b = 480 mins.

Cat = Continuous.

$$I_\gamma = \begin{cases} 1 & \text{, if vehicle } \gamma \text{ is used} \\ 0 & \text{, else.} \end{cases}$$

⇒ Objective:

Obj① : Minimize Total Cost:

$$\text{Min.} \sum_{\gamma \in V} I_\gamma f_\gamma + \sum_{\gamma \in V} O_\gamma \sum_{i \in N/Sink} \sum_{j \in N/Sorce} C_{ij}\, x_{ij\gamma}$$

obj② : Minimize Total Travel Distance

$$\text{Min.} \sum_{\gamma \in V} \sum_{i \in N/sink} \sum_{j \in N/source} C_{ij}\, x_{ij\gamma}$$

obj③ : Minimize the Number of Vehicles used

$$\text{Min} \sum_{\gamma \in V} I_\gamma$$

→ Constraints:

Start from depot

$$\sum_{j \in N/source} x_{source,j,\gamma} = I_\gamma \quad \forall \gamma \in V$$

End at depot:

$$\sum_{i \in N/sink} x_{i,sink,\gamma} = I_\gamma \quad \forall \gamma \in V$$

Flow balancing {After a vehicle arrives at a Customer it has to leave for another destination}

$$\sum_{i \in N/sink} x_{ih\gamma} - \sum_{j \in N/source} x_{hj\gamma} = 0 \quad \forall h \in C, \forall \gamma \in V$$

Each Customer is Visited Exactly once:

$$\sum_{\gamma \in V} \sum_{j \in N/sink} x_{ij\gamma} = 1 \quad \forall i \in C$$

Vehicle Capacity Constraint:

$$\sum_{i \in C} d_i \sum_{j \in C} x_{ijr} \leq q_r I_r \quad \forall r \in V$$

Time Window Constraints:

$$a_i \leq S_{ir} \leq b_i \quad \forall i \in N, \forall r \in V$$

$$S_{jr} \geq S_{ir} + t_s + t_{ij} - M(1 - x_{ijr})$$

$$\forall r \in V, j \in N/\text{source}$$
$$i \in N/\text{sink}$$

Vehicle Capacity

$$x_{ijr} \leq I_r \quad \forall i,j \in N, \forall r \in V$$

else $x_{ijr} = 0$

Linking Constraint:

$$x_{ijr} \leq I_r \quad \forall i,j \in N, r \in V$$

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