

# Dot charge in the 1st Even Excited State: is it the same as the dot charge in the Odd Ground State?

## SUMMARY

Here I “check” Rok’s statement that the dot charge in the 1st Even Excited State is the same as the dot charge in the Odd Ground State.

By “check” I mean I look at the case of the uncoupled dot-island system, that is,  $\Gamma_{\text{tun}} = 0$ .

My result is positive, Rok’s statement is correct in this special case. This is shown by the thick black solid line and the thick black dashed line running parallel on my last plot below.

## DETAILS

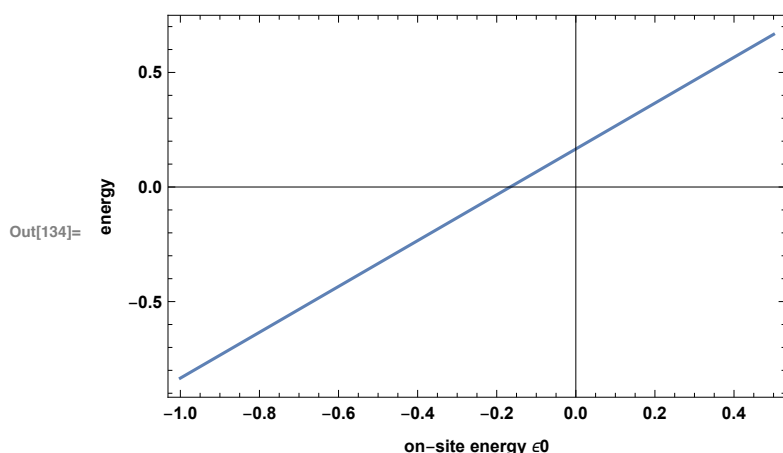
Let’s keep track of Even and Odd states up to 2 electrons on the dot and 2 quasiparticles on the island.

Here, first number is  $n_d$ , dot occupation; second number is  $n_q$ , quasiparticle occupation.

```
In[129]:= EvenStates = {{0, 0}, {0, 2}, {1, 1}, {2, 0}, {2, 2}};
In[130]:= OddStates = {{1, 0}, {0, 1}, {1, 2}, {2, 1}};
In[131]:= States = Join[EvenStates, OddStates]
Out[131]= {{0, 0}, {0, 2}, {1, 1}, {2, 0}, {2, 2}, {1, 0}, {0, 1}, {1, 2}, {2, 1}}
In[132]:= energy[{nd_, nq_}] :=  $\epsilon_0 * nd + (1/2) U * nd * (nd - 1) + \Delta * nq$ ;
In[133]:= RokParameters = {U  $\rightarrow$  0.333,  $\Delta \rightarrow$  0.166}
Out[133]= {U  $\rightarrow$  0.333,  $\Delta \rightarrow$  0.166}
```

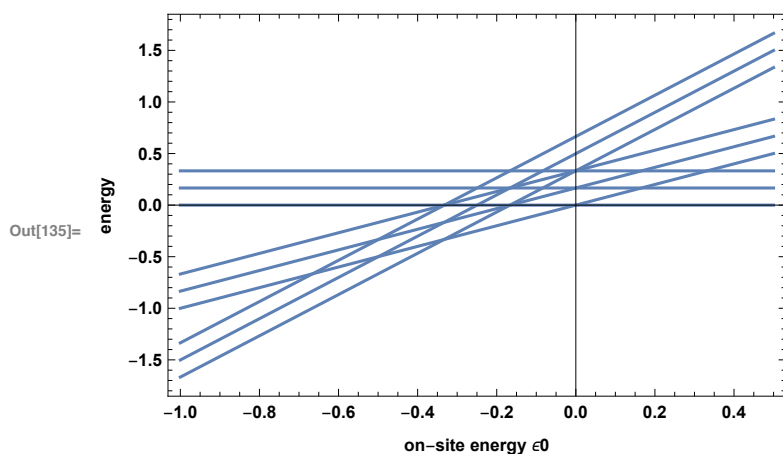
Plot the dispersion of one state:

```
In[134]:= Plot[energy[EvenState[[3]]] /. RokParameters, { $\epsilon_0$ , -1, 1/2},
  Frame  $\rightarrow$  True, FrameLabel  $\rightarrow$  {"on-site energy  $\epsilon_0$ ", "energy"}]
```



Plot the dispersions of all states:

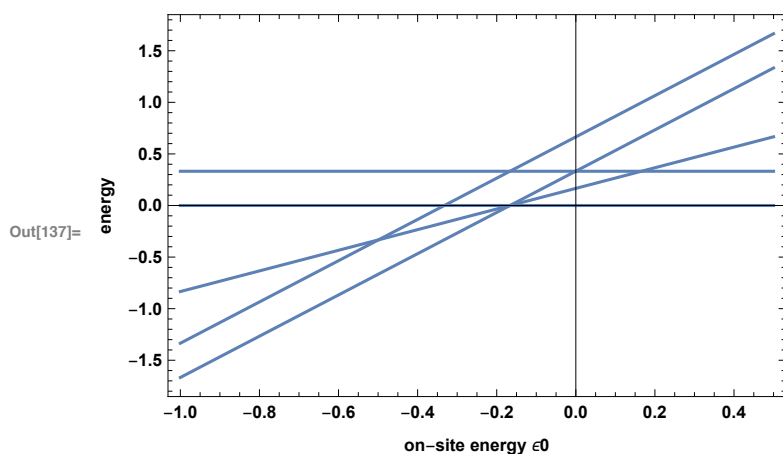
```
In[135]:= Plot[(energy[#] /. RokParameters &) /@ States, {ε0, -1, 1/2},
  Frame → True, FrameLabel → {"on-site energy ε0", "energy"}]
```



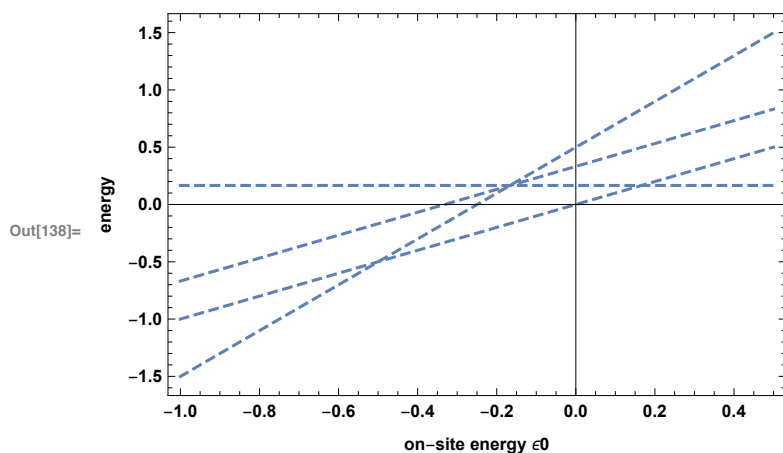
```
In[136]:=
```

**Plot only Even states as solid and Odd states as dashed:**

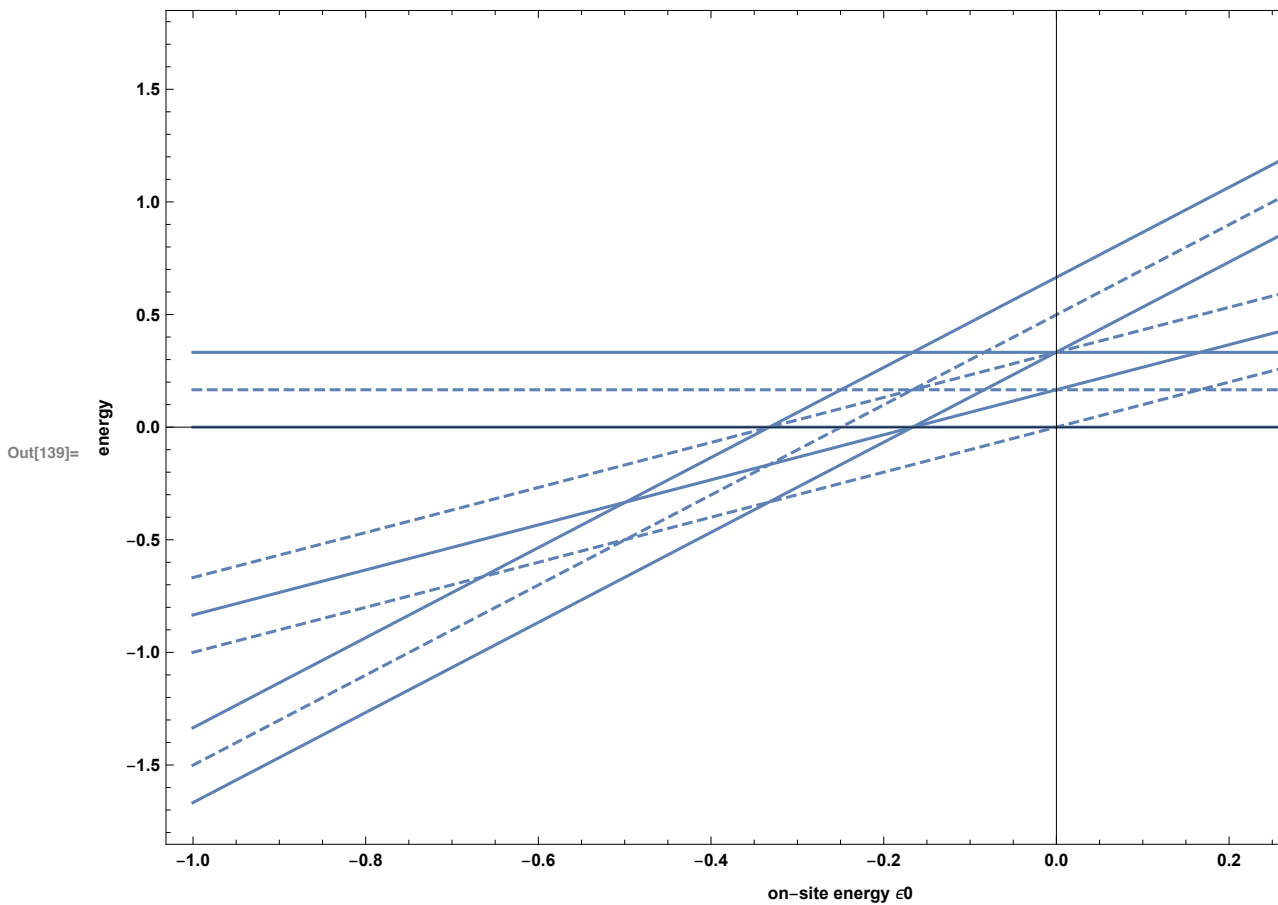
```
In[137]:= PlotEven = Plot[(energy[#] /. RokParameters &) /@ EvenStates, {ε0, -1, 1/2},
  Frame → True, FrameLabel → {"on-site energy ε0", "energy"}]
```



```
In[138]:= PlotOdd = Plot[(energy[#] /. RokParameters &) /@ OddStates, {ε0, -1, 1/2},
  PlotStyle → Dashed,
  Frame → True, FrameLabel → {"on-site energy ε0", "energy"}]
```



In[139]:= Show[PlotEven, PlotOdd]



Superimpose the even ground state on the previous plot as a red, thick line.

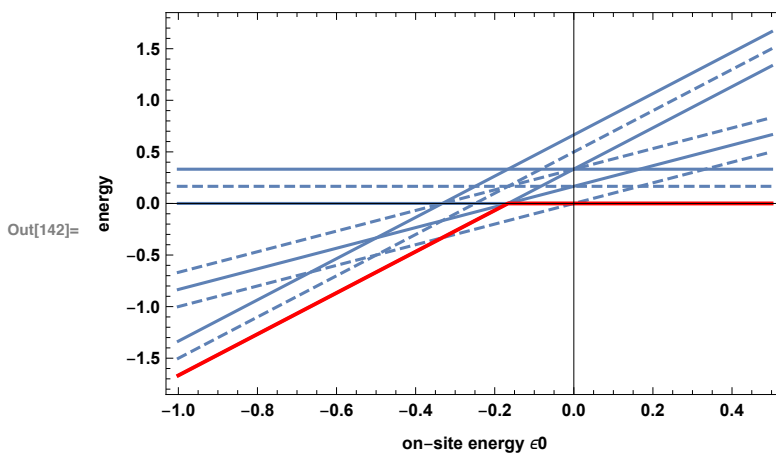
In[140]:= (energy[#] /. RokParameters &) /@ EvenStates

Out[140]= {0, 0.332, 0.166 +  $\epsilon_0$ , 0.333 + 2  $\epsilon_0$ , 0.665 + 2  $\epsilon_0$ }

In[141]:= PlotEvenGS =

Plot[(energy[#] /. RokParameters &) /@ EvenStates // Min, { $\epsilon_0$ , -1, 1/2},  
PlotStyle → {Red, Thick},  
Frame → True, FrameLabel → {"on-site energy  $\epsilon_0$ ", "energy"}];

In[142]:= Show[PlotEven, PlotOdd, PlotEvenGS]



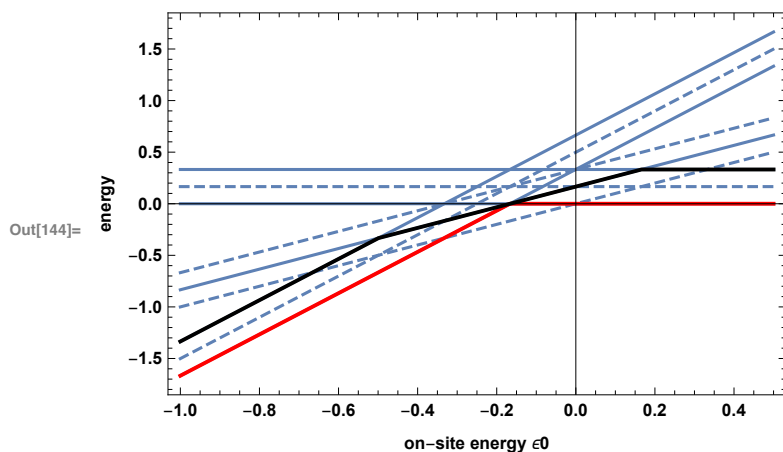
Superimpose the first excited state in the even sector on the previous plot as a black, thick line.

```

In[143]:= PlotEvenES = Plot[(energy[#] /. RokParameters &) /@EvenStates // Sort // #[[2]] &,
    {ε0, -1, 1/2},
    PlotStyle → {Black, Thick},
    Frame → True, FrameLabel → {"on-site energy ε0", "energy"}];

In[144]:= Show[PlotEven, PlotOdd, PlotEvenGS, PlotEvenES]

```



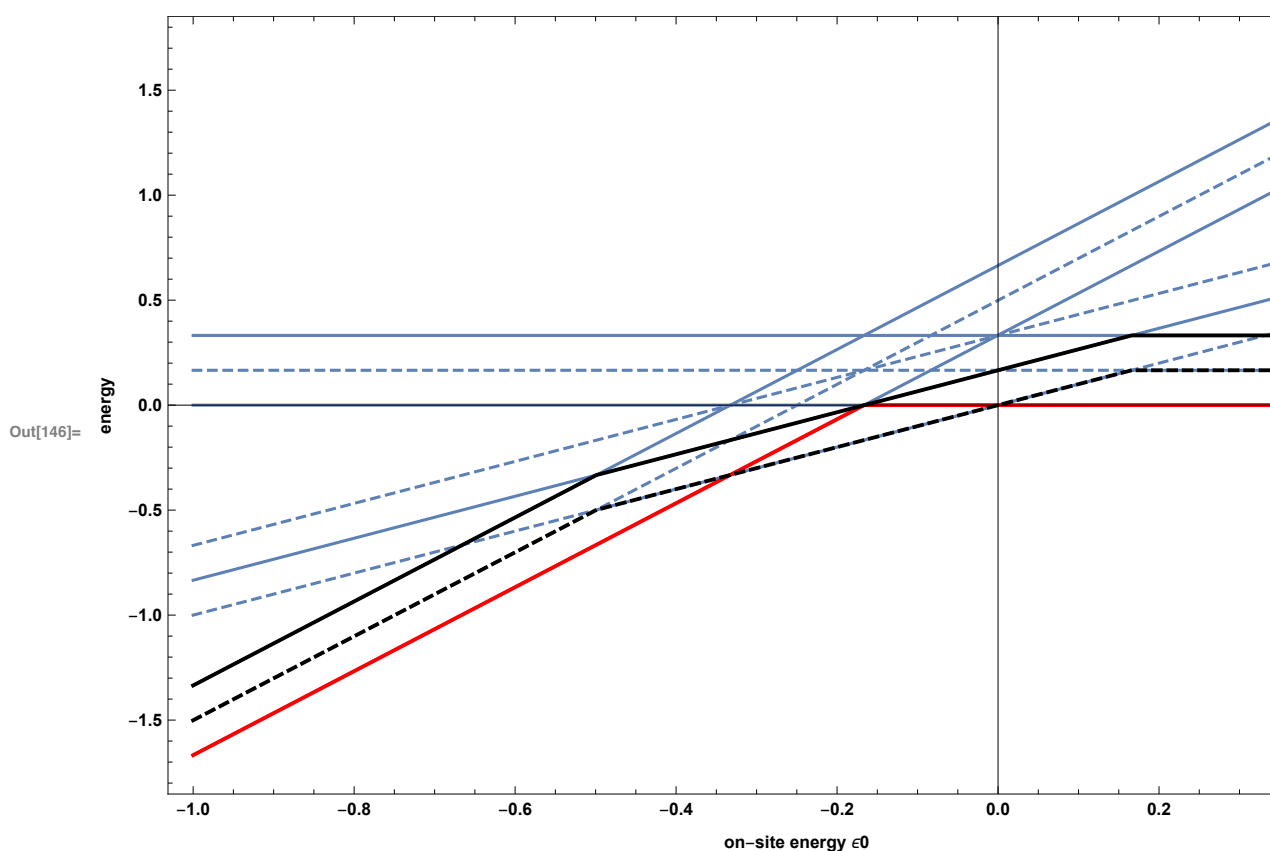
Superimpose the odd ground state on the previous plot as a black, thick, dashed line.

```

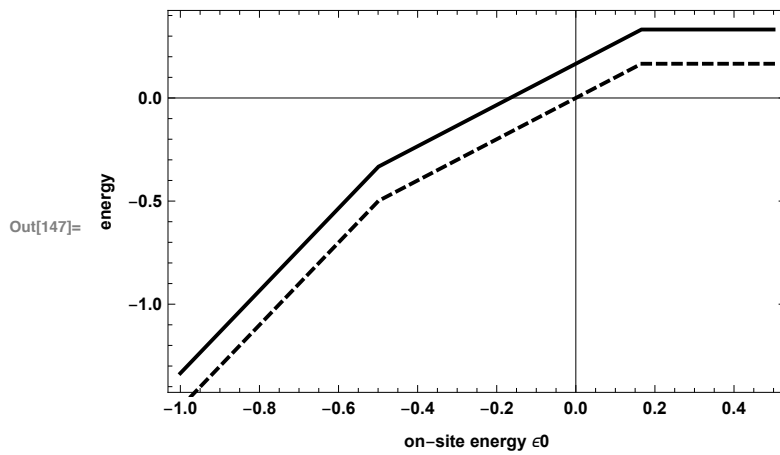
In[145]:= PlotOddGS = Plot[
    (energy[#] /. RokParameters &) /@OddStates // Sort // #[[1]] &, {ε0, -1, 1/2},
    PlotStyle → {Black, Thick, Dashed},
    Frame → True, FrameLabel → {"on-site energy ε0", "energy"}];

In[146]:= Show[PlotEven, PlotOdd, PlotEvenGS, PlotEvenES, PlotOddGS]

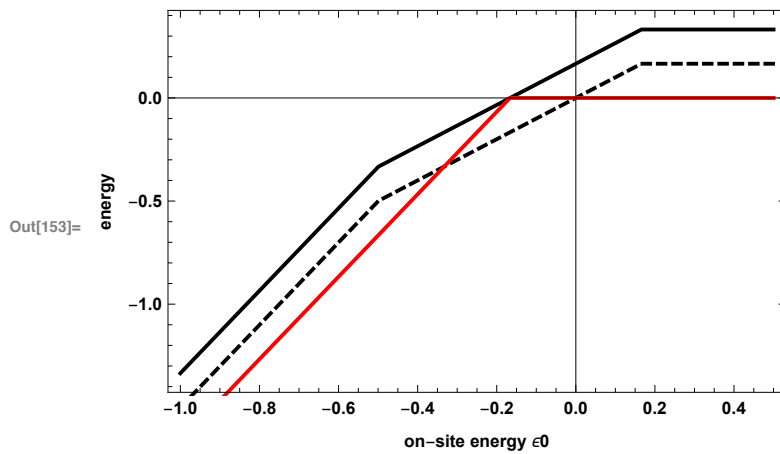
```



In[147]:= Show[PlotEvenES, PlotOddGS]



In[153]:= Show[PlotEvenES(\*solid\*), PlotOddGS(\*dashed\*), PlotEvenGS(\*red\*)]



**Calculate  $\epsilon_q$ ,  $\epsilon_{tun}$ ,  $R_{sis}$  for  $\epsilon_0 = -0.4$**