Mathematical Method for Physics :-

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① $z = \sin(x^2y + y^2)$

$$\frac{\partial z}{\partial x} = \cos(x^2y + y^2) \times 2xy$$

$$\frac{\partial z}{\partial y} = \cos(x^2y + y^2) \times (x^2 + 2y)$$

② $x^2 + y^2 + z^2 = C$

$2x + 2y + 2z.$

$x^2 + a^2 + z^2 = C.$

$$2x + 2z\frac{dz}{dx} = 0.$$

③ $x^2 + xy^2 + 3xy = C$

$$2x + y^2 + 2xy\frac{dy}{dx} + 3y + 3x\frac{dy}{dx}$$

④ $z = e^{-x^2 + 2xy + 3y^2}.$

$$\frac{dz}{dx} = e^{-x^2 + 2xy + 3y^2} \times \left(-2x + 2y + 2x\frac{dy}{dx} \cancel{6y}\right).$$

⑤ $xy + y\cancel{z} + 3xyz = 1.$

$$y + x\frac{dy}{dx} + \frac{dy}{dx} \times \cancel{dz} + y\left(\frac{dz}{dx}\right) + 3yz + 3x\frac{dy}{dx}z + 3xy\frac{dz}{dx} = 0.$$

$$\frac{\partial z}{\partial x} = \left(e^{-x^2 + 2xy + 3y^2}\right)(2y - 2x).$$

$$\frac{\partial z}{\partial y} = \left(e^{-x^2 + 2xy + 3y^2}\right)(2x + 6y)$$

Arf Ken.

Schuam Series

$z = f(q)$. $q(x,y)$. ① Eg: $z = \sqrt{q}$.

$q = x^2 - 3xy + 2y^2$.

$\Rightarrow z = \sqrt{x^2 - 3xy + 2y^2}$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2-3xy+2y^2}} \times (2x - 3y)$$

$$\frac{\partial z}{\partial y} = (-3x+4y) \times \frac{1}{2\sqrt{x^2-3xy+2y^2}}$$

$$\frac{\partial z}{\partial q} = \frac{1}{2} \frac{1}{\sqrt{q}}$$

$$\frac{\partial q}{\partial x} = 2x - 3y. \qquad \frac{\partial q}{\partial y} = -3x + 4y.$$

② $z = \sqrt{x^2+y^2}$.

$$\frac{dz}{dx} = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \times (\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}}$$

③ $u = r^2$ $\qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial u}{\partial x} = \cancel{\frac{(2x)}{2\sqrt{x^2+y^2+z^2}}} \quad \frac{d(x^2)}{dx} = 2x$$

④ $z = f(x,y)$. $\qquad x = t^2$, $y = 3\sin t$

$z = 3x^2 - 2xy + 3y^2 = 3t^4 - 2t^2(3\sin t) + 3(9\sin^2 t)$

$$\frac{\partial z}{\partial t} = \frac{d}{dt}(3t^4 - 6t^2\sin t + 27\sin^2 t)$$

$= 3 \times 4t^3 - 6(2t\sin t + t^2\cos t) + 27 \times 2\sin t\cos t$

$= 12t^3 - 12t\sin t - 6t^2\cos t + 54\sin 2t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$= 12t^3 - 12t\sin t - 6t^2\cos t + 54\sin(2t)$.

$6x \times \frac{dx}{dt} - 2y\frac{dx}{dt} = 12t^3 - (2t)6\sin t$

$-2x + 6y \times \frac{dy}{dt} = -2t^2 \times 3\cos t$

$+ 18\sin t \times 3\cos t$

$= -2t^2 + 54\sin t\cos t$

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(5) $y = 3\sin(kx)$.

- $z = f(x(t,s), y(t,s))$ $\quad\Big|\quad x = g(t,s)$, $y = h(t,s)$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\times\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\times\frac{\partial y}{\partial t}.$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\times\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\times\frac{\partial y}{\partial s}.$$

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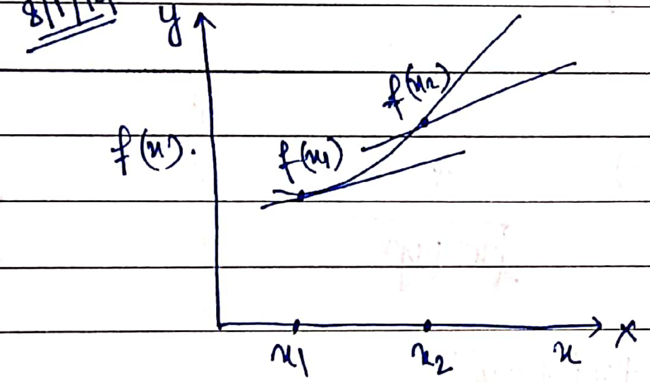
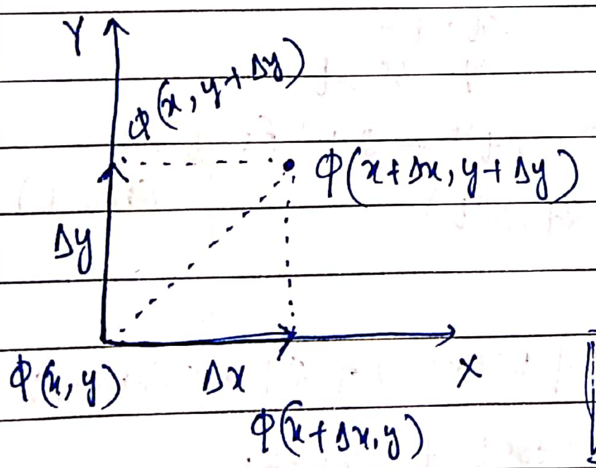


$$\left.\frac{dy}{dx}\right|_{x=x_2} = \left.\frac{df(x)}{dx}\right|_{x=x_2}$$

$z = x^2 - 3xy + 2y^2$ (x, y are independent)

$$\frac{dz}{dx} = 2x - \left(3y + 3x\cancelto{0}{\frac{dy}{dx}}\right) + 4y\cancelto{0}{\frac{dy}{dx}}$$



$$\Delta\phi = \phi(x+\Delta x, y+\Delta y) - \phi(x,y).$$

$$\boxed{d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy}$$

$$\boxed{\Delta\phi = \frac{\partial\phi}{\partial x}\Delta x + \frac{\partial\phi}{\partial y}\Delta y + \epsilon_1 + \epsilon_2}$$

$\Delta x \to 0$

$\Delta y \to 0$

1. $f(x,y,z) = x^2 - 3xy + 2y^2 + 4xyz$.

$$\frac{\partial u}{\partial x} = 2x - 3y + 4yz, \quad \frac{\partial u}{\partial y} = -3x + 4y + 4xz$$

$$\frac{\partial u}{\partial z} = 4xy.$$

$u = f(g) \qquad g(x,y).$

2. $u = \sin(x^2y + xy^2)$

$$\frac{\partial u}{\partial x} = (\cos(x^2y + xy^2)) \times (2xy + y^2)$$

$$\frac{\partial u}{\partial y} = (\cos(x^2y + xy^2)) \times (x^2 + 2xy)$$

1. x is independent variable
$y = f(x)$.
2. x, y, z are independent
$u = f(x,y,z)$
3. $u = f(g)$, $g(x,y)$.
4. $u = f(x,y)$ $x(t), y(t)$.
i.e., $v = f(x(t), y(t))$

3. $u = x^2 - 3xy + y^2 \quad , y = \sin bt \quad , x = e^{-at}$

$$\frac{du}{dt} = \left(2x \cdot \frac{dx}{dt} - x'3y\right) + \left(3xy' + 2y\frac{dy}{dt}\right)$$

$$= (2x - 3y)(-ae^{-at}) + (3x + 2y)(b\cos bt)$$

$$= -ae^{-at}(2e^{-at} - 3\sin bt) + b\cos bt\,(2\sin bt - 3e^{-at}).$$

9/1/18 (1) $z = x^2 - 3xy + 2y^2. \qquad x = \sin at \quad , y = \cos bt$

$$z = \sin^2 at - 3\sin(at)\cos(bt) + 2\cos^2 bt$$

$$\frac{dz}{dt} = 2\sin(at)\cos(at)\cdot a - [3a\cos(at)\cos(bt) + 3\sin at(\sin bt)b]$$
$$+ 4\cos bt(-\sin bt)\cdot b.$$

$$= 2a\sin(at)\cos(at) - 3a\cos(at)\cos bt + 3b\sin(at)\sin bt$$
$$- 4b\sin(bt)\cos(bt).$$

$$= a\sin(2at) - 2b\sin(2bt) - 3[a\cos(at)\cos(bt) + b\sin at\sin bt].$$

Ex (2) $u = x^2 - 2xy + 3y^2 + xz^2$

$x = \sin at$
$y = \cos bt$
$z = e^{-ct}$.

$$\frac{du}{dt} = \frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial t}$$

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$z = f(x,y)$, where, $x = g(t,s)$, $y = h(t,s)$.

eg. $z = xy^2 + x^2y - 3xy$. | $x = 2s - t^2$, $y = s^2 - 2t$

Find:

$$\frac{\partial z}{\partial t} = \left(y^2 \cdot \frac{dx}{dt} + 2xy \cdot \frac{dx}{dt} - 3\frac{dx}{dt}\right) + \left(2xy \cdot \frac{dy}{dt} + x^2\frac{dy}{dt} - 3x\frac{dy}{dt}\right)$$

$$= \cancel{y^2(-2t)2t} = (y^2 + 2xy - 3)(-2t) + (2xy + x^2 - 3x)(-2).$$

$$\frac{\partial z}{\partial s} = (y^2 + 2xy - 3)\,2 + (2xy + x^2 - 3x)(2s).$$

① $u = 3x^2 + 2xy + z^2 + y^2$. | $x = r\sin\beta\cos\theta$, $y = r\sin\beta\sin\theta$, $z = r\cos\theta$.

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial r}$$

Q. $x^2y + 3xy + 2xy^2 = C$.

$$2xy + x^2\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} + 2y^2 + 2x(2y)\frac{dy}{dx} = 0.$$

$$\Rightarrow 2xy + 3y + 2y^2 + (x^2 + 3x + 4xy)\frac{dy}{dx} = 0.$$

$$\Rightarrow dy/dx = -\left(\frac{2xy + 3y + 2y^2}{x^2 + 3x + 4xy}\right).$$

Implicit Partial Differentiation:

① $xy + yz + zx = C$ (x, y are independent variables)

(A) $y + \frac{\partial z}{\partial x}\cdot y + \frac{\partial z}{\partial x}x + z = 0$.

$$\Rightarrow \frac{\partial z}{\partial x}(x+y) = -(z+y).$$

$$\frac{\partial z}{\partial x} = \frac{-(z+y)}{(x+y)}$$

(B) $x + y\frac{\partial z}{\partial y} + z + x\frac{\partial z}{\partial y} = 0$.

$$\Rightarrow \frac{\partial z}{\partial y}(x+y) = -(z+x)$$

$$\partial z/\partial y = \frac{-(z+x)}{(x+y)}$$

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② $u = x^2 - 3xy + yz$.

$x = e^{2r}\sin\theta$.
$y = e^{3r}\cos\theta$.

$\Rightarrow u = e^{4r}\sin^2\theta - 3e^{5r}\cos\theta\sin\theta + e^{6r}\cos^2\theta$.

$$\frac{\partial u}{\partial r} = 4e^{4r}\sin^2\theta - 3\times 5 e^{5r}(\cos\theta\sin\theta) + 6e^{6r}\cos^2\theta$$

$$\frac{\partial u}{\partial \theta} = e^{4r}\,2\sin\theta\cos\theta - 3e^{5r}\cos 2\theta + e^{6r}\,2\cos\theta(-\sin\theta).$$

$$= e^{4r}\sin 2\theta - 3e^{5r}\cos 2\theta - e^{6r}\sin 2\theta.$$

③ Treating x, y as independent variable, find $\frac{\partial r}{\partial x}$, $\frac{\partial r}{\partial y}$, $\frac{\partial \theta}{\partial x}$ & $\frac{\partial \theta}{\partial y}$, given, $x = e^{2r}\sin\theta$
$y = e^{3r}\cos\theta$.

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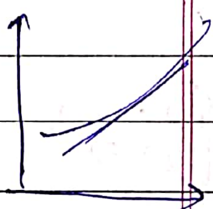
Total Differential of $u = f(x,y,z)$

$$du = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy + \left(\frac{\partial f}{\partial z}\right)dz.$$

$$= \left(\frac{\partial f}{\partial x}\hat{e}_x + \frac{\partial f}{\partial y}\hat{e}_y + \frac{\partial f}{\partial z}\hat{e}_z\right)\cdot\left(dx\,\hat{e}_x + dy\,\hat{e}_y + dz\,\hat{e}_z\right)$$

$\frac{df}{dx} \rightarrow$ slope or gradient.



$$|\nabla\phi|\cdot d\vec{r}.$$

$$\vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi.$$

gradient :- $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

$$\vec{\nabla} r = \left(\frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \hat{k}\frac{\partial r}{\partial z}\right)$$

$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

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$$\frac{\partial r}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2+z^2}} = \frac{x}{r}.$$

$$\frac{\partial r}{\partial y} = \frac{y}{r} \qquad \frac{\partial r}{\partial z} = \frac{z}{r}.$$

$$\therefore \vec{\nabla} r = \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r} = \frac{\vec{r}}{r} = \hat{r}$$

① Evaluate $\vec{\nabla} f(x)$. Here, $f(r)$ is a radial func^n.
$f(r, \theta, \phi)$ $\quad$ $r(x, y, z)$

$$\nabla = \sum_{i=1}^{n} \hat{e}_i \frac{\partial f(x)}{\partial x_i}$$

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$\vec{\nabla} \rightarrow$ Gradient $\rightarrow$ of $\rightarrow \phi \rightarrow$ Scalar field. — Represent distribution of some quantity in space
(eg. $\vec{E}$). $\quad$ $\left.\begin{matrix}\vec{A}\\ \vec{V}\end{matrix}\right\}$ Vector field. $\longrightarrow$

$f(r) \Rightarrow V = \frac{kq}{r}$, $\vec{\nabla} f(r) = ?$.

$$\vec{\nabla} V = -\frac{kq}{r^2}$$

Gradient of a radial func^n :-

$$\vec{\nabla} f(r) = \hat{i}\frac{\partial f(r)}{\partial x} + \hat{j}\frac{\partial f(r)}{\partial y} + \hat{k}\frac{\partial f(r)}{\partial z}.$$

$\vec{\nabla} \times \vec{A} \rightarrow$ Curl of $\vec{A}$

$\vec{\nabla} . \vec{A} \rightarrow$ divergence of $\vec{A}$.

① find $\vec{\nabla}.\vec{r} = \frac{\partial}{\partial x}(x\hat{i} + y\hat{j} + z\hat{k}) + \cdots$
$= 3$

② $\vec{\nabla} \cdot f(r)\hat{r}$

$f(r) \cdot \hat{r} \rightarrow$ Central force field. or, Radial force field.

15/1/19. operator $\vec{\nabla}$

Scalar field $\phi(x, y, z)$

Vector field $\vec{V}, \vec{A}, \vec{B}$ etc.

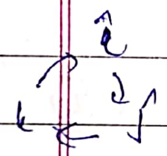
1. $\vec{\nabla}\phi$. Combination with vector ?,

$$\vec{\nabla} = \hat{e}_1 \frac{\partial}{\partial x_1} + \hat{e}_2 \frac{\partial}{\partial x_2} + \hat{e}_3 \frac{\partial}{\partial x_3} = \sum_i \hat{e}_i \frac{\partial}{\partial x_i}$$

$$\vec{\nabla}\phi = \hat{i}\,\partial_x \phi + \hat{j}\,\partial_y \phi + \hat{k}\,\partial_z \phi.$$

2. $$\vec{\nabla} \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z.$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$



$$= \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right)\hat{i} + \hat{j}\left(\cancel{\frac{\partial}{\partial x} A_z} + \frac{\partial}{\partial z} A_x\right) + \hat{k}\left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right)$$

$$= \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right)\hat{i} + \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z\right)\hat{j} + \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right)\hat{k}$$

3. $$\vec{B} \cdot (\vec{\nabla} \times \vec{A}) = B_x\left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y\right) + B_y\left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z\right) + B_z\left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x\right)$$

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$$= \begin{vmatrix} B_x & B_y & B_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

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Differential operators

$\vec{\nabla}$ and ϕ $\qquad$ $\vec{A}, \vec{V}, \vec{B}$, etc.

We constructed the following $\vec{\nabla}\phi$

$\vec{\nabla}\phi$, $\vec{\nabla}\cdot\vec{A}$, $\vec{\nabla}\times\vec{A}$ $\qquad$ Physical significance.

Further combinations b/w.

$\vec{\nabla}$ and $\vec{\nabla}\phi$, $\vec{\nabla}\cdot\vec{A}$, $\vec{\nabla}\times\vec{A}$

(1) (2) (3) (4)

b/w (1) & (2) $\quad \vec{\nabla}\cdot\vec{\nabla}\phi$

$\vec{\nabla}\times\vec{\nabla}\phi$

(1) & (3) $\quad \vec{\nabla}(\vec{\nabla}\cdot\vec{A})$

(1) & (4) $\quad \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{A})$

$$\vec{\nabla}\cdot\vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right)$$

$$= \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$ Laplacian.

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$$\vec{\nabla}\times\vec{\nabla}\phi = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix}$$

$$= \cancel{\hat{i}\ \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right) - \hat{j}\ \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}\right) + \hat{k}\ \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial y}\right)}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial y}\right)\right] - \hat{j}\left(\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial x}\right)\right)$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right)\right]$$

17/1/19. Prove the identities : $g(x,y,z)$, $h(x,y,z)$ are scalar fields, $\vec{A}(x,y,z)$, $\vec{B}(x,y,z)$ are vector fields.

$$\boxed{\vec{\nabla}(gh) = g(\vec{\nabla}h) + h(\vec{\nabla}g)}$$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(gh) = \hat{i}\frac{\partial}{\partial x}(gh) + \hat{j}\frac{\partial}{\partial y}(gh) + \hat{k}\frac{\partial}{\partial z}(gh)$$

$$= \hat{i}\,g\frac{\partial h}{\partial x} + \hat{j}\,g\frac{\partial h}{\partial y} + \hat{k}\,g\frac{\partial h}{\partial y} + \hat{i}\,h\frac{\partial g}{\partial x} + \hat{j}\,h\frac{\partial g}{\partial y}$$

$$+ \hat{k}\,h\frac{\partial g}{\partial z} = g(\vec{\nabla}h) + h(\vec{\nabla}g)$$

central (radial) force field : $f(r)\,\hat{r}$.

central potential : $f(r)$

find: $\vec{\nabla}f(r)$

$\vec{\nabla}\cdot f(r)\,\hat{r}$

$\vec{\nabla}\times f(r)\,\hat{r}$.

$$\frac{\partial r^n}{\partial x} = \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cdot 2x$$

$$= n\,r^{n-2}\cdot x.$$

$$\left(r = \sqrt{x^2+y^2+z^2}\right).$$

$$\frac{\partial r^n}{\partial x} = n r^{n-1}\cdot\frac{x}{r}$$

$$\left(\frac{\partial r^n}{\partial r}\cdot\frac{\partial r}{\partial x}\right).$$

$g(x,y,z)$ | ① $\vec{\nabla}(gh) = g\vec{\nabla}h + h\vec{\nabla}g$

$\vec{A}(x,y,z)$ | ② $\vec{\nabla}\cdot(g\vec{A}) = g(\vec{\nabla}\cdot\vec{A}) + (\vec{A}\cdot\vec{\nabla})g$

③ $\vec{\nabla}\times(g\vec{A}) = \vec{\nabla}\times(\hat{i}gA_x + \hat{j}gA_y + \hat{k}gA_z)$

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$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ gA_x & gA_y & gA_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial gA_z}{\partial y} - \frac{\partial gA_y}{\partial z}\right) - \hat{j}\left(\frac{\partial gA_z}{\partial x} - \frac{\partial gA_x}{\partial z}\right) + \hat{k}\left(\frac{\partial gA_y}{\partial x} - \frac{\partial gA_x}{\partial y}\right)$$

$$= \left(\frac{\partial gA_z}{\partial y} - \frac{\partial gA_y}{\partial z}\right)\hat{i} + \left(\frac{\partial gA_x}{\partial z} - \frac{\partial gA_z}{\partial x}\right)\hat{j} + \left(\frac{\partial gA_y}{\partial x} - \frac{\partial gA_x}{\partial y}\right)\hat{k}$$

$$= \left(g\frac{\partial A_z}{\partial y} + A_z\frac{\partial g}{\partial y} - g\frac{\partial A_y}{\partial z} - A_y\frac{\partial g}{\partial z}\right)\hat{i} + \left(g\frac{\partial A_x}{\partial z} + A_x\frac{\partial g}{\partial z} - g\frac{\partial A_z}{\partial x} - A_z\frac{\partial g}{\partial x}\right)\hat{j} + \left(g\frac{\partial A_y}{\partial x} + A_y\frac{\partial g}{\partial x} - g\frac{\partial A_x}{\partial y} - A_x\frac{\partial g}{\partial y}\right)\hat{k}$$

$$= g\left\{\hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\right\} +$$

$$\left\{\left(A_z\frac{\partial}{\partial y} - A_y\frac{\partial}{\partial z}\right)\hat{i} + \left(A_x\frac{\partial}{\partial z} - A_z\frac{\partial}{\partial x}\right)\hat{j} + \left(A_y\frac{\partial}{\partial x} - A_x\frac{\partial}{\partial y}\right)\hat{k}\right\}g$$

$$= g(\vec{\nabla}\times\vec{A}) - (\vec{A}\times\vec{\nabla})g$$

$$(\vec{\nabla}\cdot\vec{A})\vec{B} = \left(\frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z\right)\vec{B}$$

$$= \frac{\partial}{\partial x}A_xB + \frac{\partial}{\partial y}A_yB + \frac{\partial}{\partial z}A_zB.$$

$$= \left(A_x\frac{\partial}{\partial x}B + A_y\frac{\partial B}{\partial y} + A_z\frac{\partial B}{\partial z}\right) + \left(B\frac{\partial A_x}{\partial x} + B\frac{\partial A_y}{\partial y} + B\frac{\partial A_z}{\partial z}\right)$$

$$= \left(A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}\right)B + B\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)$$

$$= (\vec{A}\cdot\vec{\nabla})\vec{B} + B(\vec{\nabla}\cdot\vec{A})$$

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$\vec{\nabla} \times \vec{A} \times \vec{B} = (\vec{\nabla} \cdot \vec{B}) \vec{A} - (\vec{\nabla} \cdot \vec{A}) \vec{B}$.

$\vec{\nabla} f(r) = \frac{\partial}{\partial x} f(r) \hat{i} + \frac{\partial}{\partial y} f(r) \hat{j} + \frac{\partial}{\partial z} f(r) \hat{k}$

$= \frac{\partial f(r)}{\partial r} \cdot \frac{\partial r}{\partial x} \hat{i} + \frac{\partial f(r)}{\partial r} \cdot \frac{\partial r}{\partial y} \hat{j} + \frac{\partial f(r)}{\partial r} \cdot \frac{\partial r}{\partial z} \hat{k}$.

22/01/19

Q. Find the gradient $f(x,y,z) = 3x^2y - 2y^2z + 4xyz^2$.
Find the divergence & curl of $g(x,y,z) = \hat{i}\, x^2y + \hat{j}\, y^2zx - \hat{k}\, z^2yx$.

* If $\vec{\nabla} \cdot \vec{B} = 0$ everywhere, then, $\vec{B}$ is called Solenoidal (rotational) ||
If $\vec{\nabla} \times \vec{A} = 0$ everywhere, then, $\vec{A}$ is called Irrotational ||.

$\vec{\nabla} f(x,y,z) = (6xy + 4yz^2)\hat{i} + (3x^2 - 4yz + 4xz^2)\hat{j} + (2y^2 + 8xyz)\hat{k}$.

$\vec{\nabla} \cdot \vec{g}(x,y,z) = 2xy + 2yzx - 2zy^2x$.

$D_2 = \{\hat{x}, \hat{j}\} \rightarrow$ basis vector (Unit Vectors).
$D_3 = \{\hat{i}, \hat{j}, \hat{k}\}$

In general, $\boxed{P_n = \sum_n a_n \phi_A}$ → basis vector.

$\{\phi = x^0, x^1, x^2, x^3, \dots x^n\} \rightarrow$ This basis can be used to expand any funcn.

23/01/19

INNER PRODUCT

Given two funcns over a range (a,b), an inner product is defined as,

$$\boxed{\langle f|g \rangle = \int_a^b f^*(s) \cdot g(s)\, w(s)\, ds}$$

where, $w(s)$ is a weight funcn.

eg:- The dot product in ordinary vector space.

$\hat{i}\cdot\hat{i}=1 \quad \hat{j}\cdot\hat{j}=1 \quad \hat{k}\cdot\hat{k}=1$

$\hat{i}\cdot\hat{j}=0. \quad \hat{j}\cdot\hat{k}=0 \quad \hat{k}\cdot\hat{i}=0.$

$\longrightarrow \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$

* $\{\hat{i}, \hat{j}, \hat{k}\}$ form a basis in ordinary 3-D vector space.

set of all vectors in 3-D ordinary space.

Any vector $\vec{g} = a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3, \cdots + a_n\hat{e}_n$

$= \sum_{i=1}^{n} a_i \hat{e}_i$ in this space can be expanded.

$\{\hat{i}, \hat{j}, \hat{k}\} \Rightarrow$ Orthonormal Basis ✓

The basis vectors together with Inner product

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0.$$

define or form a 3-D vector space (ordinary).

No. of basis vectors = ~~No~~ Dimension.

In a vector space, the minimum set of linearly independent vectors form the basis vector in that space.

In 2-D vector-space, all linear combination are possible.

$S = \{f(x), g(x), h(x) - - - \}$

$S = \{x^0, x, x^2, x^3, P_1(x), P_2(x), P_3(x), - - - \}$

24/01/19. SCALAR PRODUCT over some interval (a,b) and weight func^n $W(s)$.

$$\langle f | g \rangle = \int_a^b f^*(s)\, g(s)\, W(s)\, ds. \quad \equiv \vec{A}\cdot\vec{B}.$$

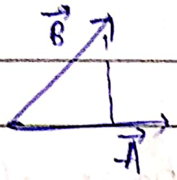
Scalar product is like dot product in ordinary space.

can be a complex/real number.

$\vec{A}\cdot\vec{B} \equiv \langle f/g \rangle$ | $|A|^2 \equiv \langle f/f \rangle$

$\vec{A}\cdot\vec{A} \equiv \langle f/f \rangle$ | $|A| \equiv \langle f/f \rangle^{1/2}$ is the length of the vector also denoted as, $\|f\|$

A vector space in which scalar product is defined for all pairs of vectors f, g, is called a Hilbert space.



ABcosθ

$\vec{A}\cdot\vec{B}$ $(\vec{A}\cdot\vec{B})^2 \leq |A|^2|B|^2$

$$\boxed{|\langle f/g \rangle|^2 \leq \langle f/f \rangle \langle g/g \rangle}$$

$\Downarrow$

$\boxed{\text{Schawartz's Inequality}}$.

Q: find the inner product of between all the elements of the set $\{1, x, x^2\}$ using $\langle f/g \rangle = \int_{-1}^{1} f^*(x)g(x)\,dx$.

$$\langle 1/x \rangle = \int_{-1}^{1} 1\cdot x\,dx = \left[x^2/2\right]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

$$\langle 1/x^2 \rangle = \int_{-1}^{1} 1\cdot x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\langle x/x^2 \rangle = \int_{-1}^{1} x\cdot x^2\,dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$$

$$\langle 1/1 \rangle = \int_{-1}^{1} 1\cdot 1\,dx = \left[x\right]_{-1}^{1} = 1 + 1 = 2.$$

$$\langle x/x \rangle = \int_{-1}^{1} x\cdot x\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = 2/3$$

$$\langle x^2/x^2 \rangle = \int_{-1}^{1} x^2\cdot x^2\,dx = \left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{1}{5} + \frac{1}{5} = 2/5.$$

Vector Space $\xrightarrow[\text{Scalar Product}]{\text{Well defined}}$ $\longrightarrow$ Hilbert space.

$(f+g)+h = f+(g+h)$

These are set with special properties.

① closure wrt addition : $h = f + g = g + f$.

② wrt multiplication, $kf = h$.

$$k(f+g) = kf + kg.$$

$$k_1(k_2 f) = k_1 k_2 f.$$

One can find a set of linearly independent vectors which forms the basis in this vector space.

All vectors in this space can be written as,

$$\boxed{f = \sum_n a_n f_n}.$$

A well defined scalar product exists for all pairs of vectors in this space.

$$\langle f|g\rangle = \int_a^b f^*(s)\, g(s)\, ds.$$

Scalar product with following properties:-

$\langle g|f\rangle = \langle f|g\rangle^*$

$\langle f|f\rangle \geq 0$ and is 'zero' only if $f=0$.

$\vec{A}\cdot\vec{A} = |A|^2 = (\text{length})^2$

$\langle f|kg\rangle = k\langle f|g\rangle.$

$\langle kf|g\rangle = k^*\langle f|g\rangle.$

$$\langle f|(k_1 g + k_2 h)\rangle = \langle f|k_1 g\rangle + \langle f|k_2 g\rangle.$$
$$= k_1\langle f|g\rangle + k_2\langle f|g\rangle$$

$$\langle k_1 f + k_2 g|h\rangle = \langle k_1 f|h\rangle + \langle k_2 g|h\rangle$$
$$= k_1^*\langle f|h\rangle + k_2^*\langle g|h\rangle$$

$\langle f|f\rangle^{1/2} = \|f\|.$

$z = a+ib$

$z^* = a-ib.$

$zz^* = |z|^2$