→ 7/1/2019

→ Modern Physics = Special Theory of Relativity

Reference books

1) Special Relativity. — Resnick (Relativity for the mechanical systems)

2) Electrodynamics — D. J. Griffiths.

3) Spacetime — Taylor and wheelar

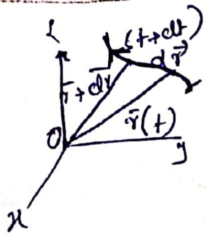
Relativity: It connects observations of a physical phenomenon observed by two or more observers.

Physical observation — (Point) of a particle under gravity → eg: free fall (car moving) → Propagation of light in free space

by an observer needs a referring frame (reference frame) (3-D-co-ordinate frame) position + a clock.

eg: Clock

Device of uniform periodicity in motion of some objects in it.



$\bar{r}$ = Position

$\bar{p} = m\bar{v}$

m = mass of the particle

$\dot{\bar{r}}$ = Velocity = $\frac{d\bar{r}}{dt}$

Mechanical State of Motion: A point particle is moving. → mechanical object.

$\bar{r}, \dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}} \ldots\ldots \to \frac{d^4\bar{r}}{dt^4}$

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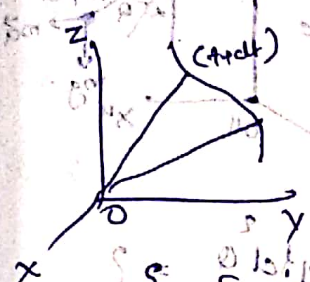
State of motion of a particle (Point Particle) w.r.t. the OXYZ ref-frame is $\bar{r}, \dot{\bar{r}}, \ddot{\bar{r}}, \dots$

$$\frac{d^2\bar{r}}{dt^2} = \frac{d^2\bar{r}(t)}{dt^2} = \ddot{\bar{r}}(t)$$

$$\frac{d^3\bar{r}}{dt^3} = \dddot{\bar{r}}(t)$$

(Before Newton) (It was wrong)



Initial conditions

$\bar{r}(t=0)$

$\dot{\bar{r}}$

$(\bar{r}(t=0), P(t=0))$

is much needed $\{\bar{r}, \dot{\bar{r}}\}$ + after newton $\rightarrow$ Measurements

$\{\bar{r}, \bar{P}\}$ or $\dot{\bar{r}}$

$\bar{r}(t=0), P(t=0)$ simultaneously is impossible for any sub atomic particles.

$$\frac{d\bar{P}}{dt} = f(\bar{r}, P, t)$$

Newton's equation of motion (II^{nd} law)

(Newton equation can't use here.)

$$m\frac{d^2\bar{r}}{dt^2} = \bar{f}(\bar{r}, \bar{P}, t) \quad \boxed{2^{nd} \text{ order d.E.}}$$

(we need two initial conditions to solve).

- or boundary condition to solve

For this kind of systems Newton equation of motion (2^{nd} law of motion) is not useful

$\rightarrow$ Here we need Quantum Mechanics.

$\rightarrow$ Locally inertial frame

The equation of motion needs a locally inertial frame (OXYZ).

$$m\frac{d^2\bar{r}}{dt^2} = \bar{f}(\bar{r}, P, t).$$

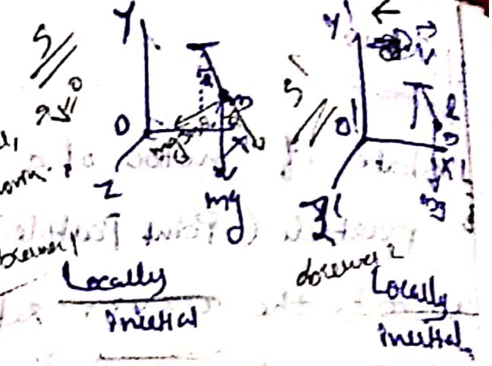
$\downarrow$ Solution

$$\bar{r} = \bar{r}(t)$$

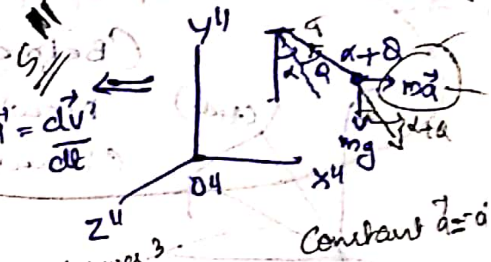
$\downarrow$

$$\frac{d\bar{r}}{dt} = \frac{d\bar{r}(t)}{dt} = \dot{\bar{r}}(t)$$

→ All the refferrence frame in the universe are locally inertial frames



(Free fall (we haven't apply any external force ∴ constant motion))



$\vec{a} = \frac{d\vec{v}}{dt}$

→ Einstein's equivalence Principle

→ If state of motion (momentum of a body) is a frame is unchanged in absence of any external (motion) forces. then

→ the frame is an inertial frame.

→ Newton's 1st law of motion.

If two inertial frames are not moving.

↓

Locally inertial not truly inertial

→

In (1) $\frac{ml\,d^2\theta}{dt^2} = ?$

In (2) $\frac{ml\,d^2\theta}{dt^2} = ?$

In (3) $\frac{ml\,d^2\theta}{dt^2} = ?$

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In 1st case

Torque $= I\alpha = \frac{ml\,d^2\theta}{dt^2}$

$= mg \sin\alpha \times -l$

In 2nd case

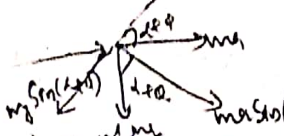
$ml^2\frac{d^2\theta}{dt^2} = -l \times mg \sin\alpha$

$s^{\prime\prime}$: is moving w.r.to the frame S with uniform velocity $\vec{v} = \vec{v}_1$.

In 3rd Case

$$m l \frac{d^2\theta}{dt^2} = \cancel{mg\cos(\alpha+\theta)}$$

$$ma\cos(\alpha+\theta) \quad + \cancel{ma\sin(\alpha+\theta)}$$



$$= -(mg\sin(\alpha+\theta) - ma\cos(\alpha+\theta))$$

$ma\sin(\alpha+\theta) + mg\cos(\alpha+\theta)$.

$\Rightarrow$ It is moving w.r.to the frame S. with the constant acceleration $\vec{a} = -a\hat{i}$.

If $\theta = 0$

$$0 = mg\sin\alpha - ma\cos\alpha$$

tnT!!

$$g\tan\alpha = a$$

$$\tan\alpha = \frac{a}{g}$$

$$\alpha = \tan^{-1}(a/g)$$

$$T = mg\cos(\alpha+\theta) + ma\sin(\alpha+\theta)$$

→ Oscillation frequency would be different w.r.to that in S and $s^{\prime}$.

(S'' is not inertial when comparing with 1st 2 frames and vice versa.)

→ Frame $s^{\prime\prime}$ is locally inertial is non-inertial w.r.to S and $s^{\prime}$.

→ S and $s^{\prime}$ are noninertial w.r.to $s^{\prime\prime}$ too.

→ Inertial frame:

If motion (momentum) of a particle does not change in a frame, then the frame is called inertial frame.

Newton's 1st law of motion

Conclusion: S and $s^{\prime}$ belong to the same category of reference frames.

→ All the frames are locally inertial (S, $s^{\prime}$, $s^{\prime\prime}$), w.r.to their respective ~~frames~~ observers. That is they are inertial frames w.r.to each other.

→ otherwise locally inertial (not noninertial)

present of constant g

↓

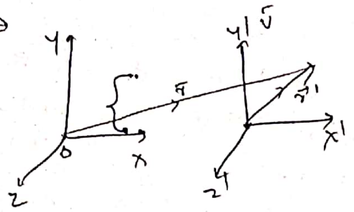
local inertial

→ presence of gravitational field or other fields or pseudoforce (constant) locally inertial.

within the locality of interest of the observer.

Causing constant acceleration due to gravity.

If it is not constant a then it is not locally inertial.

→ 

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→ 1) Galilean Relativity (1638)

In time takes by the particle to reach the ground t.

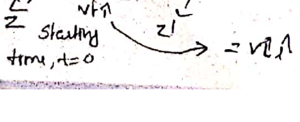
In time taken t floor of a smoothly moving ship

$t = t'$

(Galileo's observation.)

Laws of ~~motion~~ (Mechanics) must be the same in the uniformly moving frames

Uniformly moving frames are called (mutually) inertial frames.



→ All the spatial points $(\vec{r}, \vec{r}')$ are common to both observers.

→ (Both are at same locality of space)

→ The observers are within locality of space.

→ From usual vector addition. (classical)

$$\vec{r} = \vec{r}' + \vec{v}t \quad (\text{old})$$

→ $x = x' + vt$.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r}' = x'\hat{i} + y'\hat{j} + z'\hat{k}$.

→ $\hat{i}' = \hat{i}$

→ $\hat{j}' = \hat{j}$

→ $\hat{k}' = \hat{k}$

Galilean transformation of position (and time) co-ordinates.

$x' = x - vt$

$y' = y$

$z' = z$

Inverse Galilean transformation.

→ Laws of mechanics are same in all the (mutually) inertial frames which are moving uniformly with r.t. to each other

Principle of relativity.

Newtonian Relativity (1687)

Deals with the transformation of ~~co-ordinates~~ components of velocity and acceleration. $(V_{(t)})$

$\vec{u} = \frac{dx}{dt}$ = velocity of particle observed by the observer 'O'.

$\bar{u}' = \frac{d\bar{x}'}{dt} \qquad t = t'$

→ $\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k} = \frac{d\bar{r}}{dt}$

→ $\bar{u}' = u_{x'}\hat{i} + u_{y'}\hat{j} + u_{z'}\hat{k} = \frac{d\bar{r}'}{dt}$

$\bar{r} = \bar{r}' + \bar{v}t$

$u_x = u_x' + v$ (bez motion is only along x-axis.)

$u_y = u_y'$

$u_z = u_z'$.

Transformation of the velocity components. according to Newtonian relativity, on top of the Galilean transformation.

→ $\bar{a} =$ acceleration of the particle by the observer "O".

$\bar{a}' =$ acceleration of the particle by the observer "O'". $t = t'$.

Then $\bar{a} = \frac{d\bar{u}}{dt} = \frac{d^2\bar{r}}{dt^2}$

$\bar{a}' = \frac{d\bar{u}'}{dt} = \frac{d^2\bar{r}'}{dt^2}$

$\bar{a} = \frac{d}{dt}(\bar{u}' + \bar{v})$

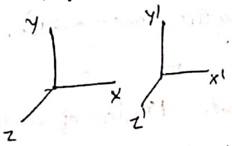
$= \bar{a}' + 0$

$a = \bar{a}'$

$m\vec{a}$ = force acting on the particle as observed by O'.

$m\bar{a}' = \vec{F}' = m\bar{a} = m\bar{a}$ as by 'O'.

→ Laws of motion are same to the inertial reference frames.



Force acting on the body is not function of

→ Space is homogeneous along x-axis. Time is also homogeneous.

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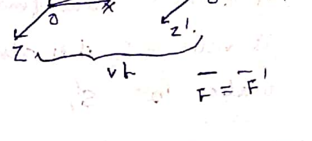
→ Newtonian Relativity (Continued)



$\bar{F} = \bar{F}'$

→ If $\vec{F}=0$ (Space is homogeneous and isotropic).

→ Time is also homogeneous

(space
↓
Allows us to consider an infinitely large no. of. Inertial frames)

→ Effects of motion of the frame (s') on physics (mechanics).

1) Newton's 1st law of motion

$\vec{F}=\vec{F}'=m\vec{a}=m\vec{a}'=0.$

$\vec{u}'=\frac{d\vec{r}'}{dt}$

$\vec{u}'=\frac{d\vec{r}'}{dt}$

$\vec{a}'=\frac{du}{dt}$

$\vec{a}=\vec{a}'$ $\quad \vec{a}'=\frac{du'}{dt}$

So, whatever happened in the frames won't affect.

→ If the 1st law of motion is valid in S, it is also valid in S'.

So the motion of s not giving any force to the particle in its s' frame

So the motion of s' with velocity $\bar{v}$ with s does not give any force on the particle at $\vec{r}'$.

2) Newton's 2nd law of motion

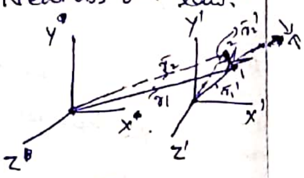
$\vec{F}=\vec{F}'=m\vec{a}=m\vec{a}'$

force observed by O.

force observed by O'.

→ Motion of s' has no effects in the force acting on the particle at $\vec{r}'$.

3) Newton's 3rd law.



$F_{21}=-F_{12}$

↓

→ force acting in particle 1. given by particle 2.

= " " on 2 by 1

$\bar{F}_{21}'=-\bar{F}_{12}'$

$\bar{F}_{21}=\bar{F}_{21}'$, $\bar{F}_{12}=+\bar{F}_{21}'$

→ If Newton's 3rd law of motion is valid in S, it will be valid in s'.

(we can check
→ Conservation of mass
→ Conservation of momentum
→ Conservation of energy.)

→ 4). Conservation of physical quantities

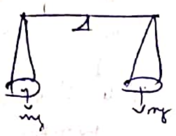
a) Conservation of mass.

$m_1+m_2=m_1'+m_2'$

Mass observed by the observer "O"

Mass of particle observed by 'O''.

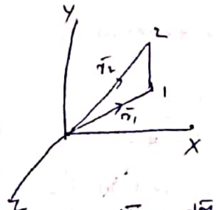
" by balance methods "

→ 

motion of

→ Motion of s' has no effects on g, on any acceleration.

→ b) Conservation of momentum



$\bar{v}_1=\frac{d\bar{r}_1}{dt}$

$\bar{v}_2=\frac{d\bar{r}_2}{dt}$

In moving frame,

$\bar{v}_1'=\bar{v}_1-\bar{v}$

$v_1=d(\bar{r}_1+\bar{v}t)$

$v_1=v_1'+\bar{v}$

$v_1'=\bar{v}_1-\bar{v}$

→ In 'O''; the total momentum.

In S $\quad m_1\bar{v}_1+m_2\bar{v}_2=c$ (constant) is conserved.

↓

In s' in $\quad m_1\bar{v}_1'+m_2\bar{v}_2'$

$=m_1(\bar{v}_1-\bar{v})+m_2(\bar{v}_2-\bar{v})$

$=m_1\bar{v}_1+m_2\bar{v}_2-\bar{v}(m_1+m_2)$

$= C - (m_1+m_2)\bar{v}$

$=$ Constant — Another constant as $\bar{v}$ is not changing (m_1, m_2) also not changing.

If momentum is conserved in S, then it is also conserved in S'.

28/1/2019

Energy conservation

(for elastic collision.)

Kinetic energy in S.

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = E$ (Constant).

Kinetic energy in S'.

$\frac{1}{2}m_1u_1'^2 + \frac{1}{2}m_2u_2'^2$

$= \frac{1}{2}m_1\bar{u}_1'\cdot\bar{u}_1' + \frac{1}{2}m_2\bar{u}_2'\cdot\bar{u}_2'$

$= \frac{1}{2}m_1(\bar{u}_1-\bar{v})(\bar{u}_1-\bar{v}) + \frac{1}{2}m_2(\bar{u}_2-\bar{v})(\bar{u}_2-\bar{v})$

$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_1v^2 - m_1\bar{u}_1\cdot\bar{v} + \frac{1}{2}m_2\bar{u}_2^2 + \frac{1}{2}m_2v^2 - m_2\bar{u}_2\cdot\bar{v}$

$= \underbrace{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2}_{E}$

$+ \underbrace{\frac{1}{2}(m_1+m_2)v^2}_{\text{Constant.}}$

$-(m_1\bar{u}_1 + m_2\bar{u}_2)\cdot\bar{v}$

∴ E is conserved in S.

∴ mass is conserved and v is constant.

momentum is conserved and entire thing is constant (v is uniform.)

If energy is conserved in S, so as in S' is an moving frame.

Conclusion:

Laws of mechanics remain same in the inertial reference frames (like S and S').

(like S' moving uniformly w.r.t. S).

→ Any effects on the motion of the particles.

→ Observer can no way to determine his/her own velocity w.r.to another frame.

i.e. there is no preferred inertial reference frames.

→ i.e. All the inertial frames are equivalents (when laws of mechanics are same).

Principle of Relativity.

Electromagnetism

Maxwell's equations determine speed of light.

e.m. field is freespace or vacuum.

The laws are

$\vec{E} = E(\bar{r}, t)$ = electric fields

$\vec{B} = B(\bar{r}, t)$ = magnetic field.

Maxwell equations for

1) $\nabla\cdot E = \rho/\varepsilon_0$ (~~Gauss's law~~). charge density

Coulomb's law in Gauss's law.

2) $\nabla\cdot B = 0$

3) $\nabla\times E = \frac{-\partial B}{\partial t}$ (Faraday's law.) for e.m. induction.

4) $\nabla\times\bar{B} = \mu_0\bar{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ (Ampere's law with Maxwell's correction.)

current density

freespace permeability.

$\sigma(\vec{E}+(\vec{v}\times\vec{B}))$

$\mu_0\sigma(\vec{E}+(\vec{v}\times\vec{B})) + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

5) $J = \sigma(\vec{E} + \vec{v}\times\bar{B})$ Ohm's law.

6) $F = q(\vec{E} + (\vec{v}\times\vec{B}))$ + Lorentz force law

Speed of light in free space:

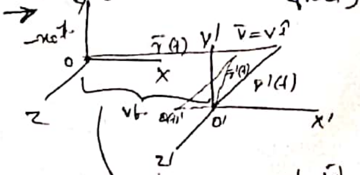
$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \simeq 3\times10^8$ m/s

(Maxwell 1864)

29/1/2019

Speed of light.

A thought experiment (electromagnetic field).



$\bar{u} = \frac{d\bar{r}}{dt}$

$\bar{u}' = \frac{d\bar{r}'}{dt}$

$\bar{r} = \bar{r}' + \bar{v}t$.

→ Speed of light in the rest frame is $c = |u|$

Speed of light in S' frame

$c' = |\bar{u}'|$

$= c - v$

$\boxed{c' = c - v}$ ✗

As in reality it's absolutely wrong

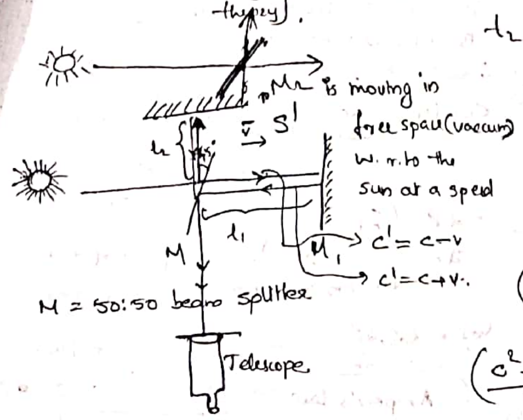
In reality $c' = c$

⇒ The classical velocity addition formula ($\vec{u}' = \vec{u} + \vec{v}$) as well as Galilean transformations

$\vec{r}' = \vec{r}' - \vec{v}t$ is questionable.

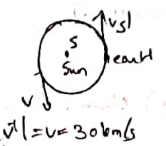
Michelson – Morley (1881) falsified this by setting up an experiment.

(we cannot prove one theory's wrong using another theory).



M = 50:50 beam splitter

time for light to reach to earth = 8 min.



$|\vec{v}| = v = 30\,km/s$

Assume the sun is at rest in vacuum or free space

c = speed of light in vacuum.

t_1 = time taken for light from M to M_1 and M_1 to M.

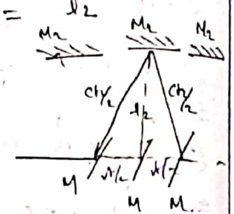
$$= \frac{l_1}{c - v} + \frac{l_1}{c + v}$$

$$t_1 = \frac{l_1 \times 2c}{c^2 - v^2}$$

$$= \frac{2l_1}{c(1 - v^2/c^2)}$$

t_2 = time taken for light from M to M_2' and M_2 to M:

$$t_2 = \frac{l_2}{}$$



$$\left(\frac{ct_2}{2}\right)^2 = l_2^2 + \left(\frac{vt_2}{2}\right)^2$$

$$\left(\frac{c^2 - v^2}{4}\right) t_2^2 = l_2^2$$

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}}$$

$$t_2 = \frac{2l_2}{c\sqrt{1 - v^2/c^2}}$$

$$t_2 = \frac{l_2}{\sqrt{c^2 - v^2}} + \frac{l_2}{\sqrt{c^2 - v^2}}$$

$$= \frac{2l_2}{c\sqrt{1 - v^2/c^2}}$$

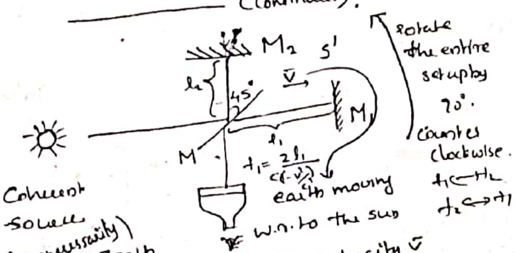
Transit time:

$\Delta t = t_2 - t_1$.

Interference spectrum

~~yesterday~~ 30/1/2019

Michelson – Morley Experiment (1887) (Continued).



Coherent source (monochromaticity)

and $|\vec{v}| = v = 30\,km/s$

Earth S' ↓ Locally inertial frame.

transit time — time for light to go and come back to original position.

$$t_1 = \frac{2l_1}{c(1 - v^2/c^2)}$$

$$t_2 = \frac{2l_2}{c\sqrt{1 - v^2/c^2}}$$

$\Delta t = t_2 - t_1$ difference of the transit times.

$$\Delta t = \frac{2}{c}\left[\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{(1 - v^2/c^2)}\right]$$

$\Delta t' = t_2' - t_1'$

= difference of the transit times after rotation.

Mirror M_1 moving ⊥ to $\vec{v}$

Mirror M " " opposite to $\vec{v}$.

t_1 will be value of t_2.

$$\Delta t' = \frac{2l_2}{c^2 - v^2} - \frac{2l_1}{c\sqrt{1 - v^2/c^2}}$$

→ MO moving opposite to v but while returning, o it will be to the same direction of v. So change is cancelling

$$\frac{l_2}{c+v} + \frac{l_2}{c-v} = \frac{2l_2}{c^2 - v^2}$$

Δt cause a interference pattern in the telescope !!!

$$\Delta t' = \frac{2l_2}{c(1 - v^2/c^2)} - \frac{2l_1}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t' = \frac{2}{c}\left[\frac{l_2}{(1 - v^2/c^2)} - \frac{l_1}{\sqrt{1 - v^2/c^2}}\right]$$

phase difference = $\omega(\Delta t)$

$\omega(\Delta t)$ = phase difference. ω angular v of light (em wave).

Scanned by CamScanner

Phase difference b/w 2 types of light came to the ~~[illegible]~~ telescope

$\omega(\Delta t)$ = phase difference of wavelengths coming to telescope via M_1 and M_2.

Δt — 2π phase difference (1 fringe shift).

$\Delta t'$ — → shift in the fringes. expected shifted fringe pattern.

$f = \frac{1}{T}$ = frequency of the em wave.

$= \frac{c}{\lambda}$ — c → speed of em wave. λ → wavelength of the em wave

$c = \nu\lambda$.

→ If T is the time phase difference is "2π"

→ $\Delta t'$ — Δt .... $1 \to 2\pi$

=

$\Delta t' - \Delta t$ — causes N fringe shift.

$$N = \frac{\Delta t' - \Delta t}{T} = \frac{2}{cT}\left[\frac{l_1 + l_2}{\sqrt{1 - v^2/c^2}}\right]$$

$T \to 1$;

N [illegible] → $\frac{N}{T}$ time

1 time

$$= \frac{2}{c}\left[\frac{l_2}{(1 - v^2/c^2)} - \frac{l_1}{\sqrt{1 - v^2/c^2}} - \frac{l_2}{\sqrt{1 - v^2/c^2}} + \frac{l_1}{1 - v^2/c^2}\right]$$

$$= \frac{2}{c}\left[\frac{l_1 + l_2}{(1 - v^2/c^2)} - \frac{(l_1 + l_2)}{(\sqrt{1 - v^2/c^2})}\right]$$

$$= \frac{2}{c}\left[\right.$$

$\frac{v}{c} \approx \frac{30 \times 10^3}{3 \times 10^8} = 10^{-4} \ll 1$

$$= \frac{2(l_1 + l_2)}{c}\left[\cancel{1} \cancel{+} \cancel{1}\right]$$

$$= \frac{2(l_1 + l_2)}{c}\left[-1 - \frac{1}{2}\frac{v^2}{c^2} + 1 + \frac{v^2}{c^2} + O\left(\frac{v}{c}\right)^4\right]$$

$(1 - x^2)^{-1/2}$

$1 + \frac{x^2}{2}$

$$= \frac{2(l_1 + l_2)}{c}\left[\frac{v^2}{2c^2}\right] = \frac{(l_1 + l_2)v^2}{c^3 T}$$

$$= \frac{2(l_1 + l_2)}{cT}\left(\frac{v^2}{2c^2}\right)$$

$$N = \frac{(l_1 + l_2)}{\lambda}\left(\frac{v^2}{c^2}\right)$$

$cT = \lambda$

$l_1 + l_2 = 22\,m$

$\lambda \simeq 5500\,Å$ $= 5.5 \times 10^{-7}\,m$ for yellow light.

$$N = \frac{22}{5.5 \times 10^{-7}} \times \frac{30 \times 10^3}{9 \times 10^{16}}$$

$$= \frac{22}{5.5 \times 10^{-7}} \times 10^{-8}$$

$= 0.4$ (theoretical prediction expt result)

$N = \frac{\Delta t' - \Delta t}{T} = \frac{l_1 + l_2}{\lambda}\left(\frac{v}{c}\right)^2$ — no. of fringes shifted.

$= 0.4$. (theoretical prediction) based on classical velocity addition formula as well as Galilean transformation

$N = 0$ (Michelson-Morley expt result) nullifies the old concept of ether

$$N = \frac{\Delta t' - \Delta t}{T} = \frac{2(l_1 + l_2)}{\lambda}\left[\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

$cT = \lambda$.

$c = \lambda/T$.

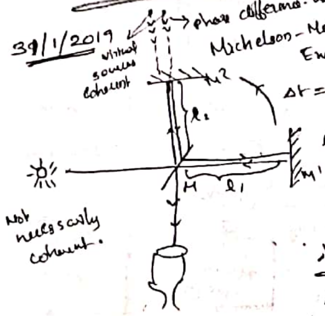
$T = \lambda/c$

$$\Delta t' = \frac{2}{c}\left[\frac{l_2}{1 - v^2/c^2} - \frac{l_1}{\sqrt{1 - v^2/c^2}}\right]$$

$$\Delta t = \frac{2}{c}\left[\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2}\right]$$

31/1/2019

phase difference: w.r.t Michelson-Morley Expt.



$\Delta t = t_2 - t_1$

$\Delta t' = t_2' - t_1'$

$l_1 + l_2 = 22\,m$

$\lambda = 5500\,Å$.

$v/c \approx 10^{-4} \ll 1$

→ $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{k}{e}}$

$= \sqrt{}$

e → density of ether.

k → Bulk modulus of ether

B → Bulk modulus of ether

→ Ether (if really exists) would be very light and very hard.
↓
If ether exists we would get $N = 0.4$

$$\frac{\frac{l_2\sqrt{x} - l_1 x}{\cancel{x\sqrt{x}}}}{\cancel{\frac{l_2 x - l_1\sqrt{x}}{x\sqrt{x}}}} = \frac{l_2 - l_1\sqrt{x}}{l_1\sqrt{x} - l_1} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{l_2\sqrt{x} - l_1 x}{l_2 x - l_1\sqrt{x}}$$

→ To preserve the concept of ether even after Michelson-Morley expt
to Lorentz-Fitzgerald

→ $\vec{v}$ → was believe to be at rest to ether.
moving in ether medium with velocity $\vec{v}$:

Lorentz-Fitzgerald Contraction Hypothesis was proposed (~~1892-1893~~) (1888-1892)

→ and l_2 be multiplied by $\sqrt{1 - v^2/c^2}$ in $\Delta t'$.
and l_1 .... $\sqrt{1 - v^2/c^2}$ in Δt.

$\Delta t = \Delta t' \Rightarrow N = 0.$

→ While rotating the system $l_2 \to$ left arm mirror will move towards interimediator, length will be reduced

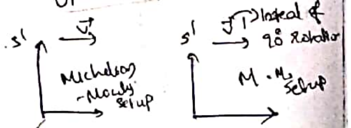
→ $l_1 \longrightarrow l_1\sqrt{1 - v^2/c^2}$ → length is contracted in moving frame

length (MM_1) when the mirrors are moving through the ether medium with velocity v.

→ No change for MM_2 because no component of it is on horizontal MM'.
(frame and MM_1 moving in same direction.)

Kennedy et al's expt. (1932)

to falsify the Lorentz-Fitzerald Contraction hypothesis.



$$\Delta t = \frac{2}{c}\left[\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1\sqrt{1 - v^2/c^2}}{1 - v^2/c^2}\right]$$

If there is no rotation.

$$\Delta t' = \frac{2}{c}\left[\frac{l_2}{\sqrt{1 - v'^2/c^2}} - \frac{l_1\sqrt{1 - v'^2/c^2}}{1 - v'^2/c^2}\right]$$

$$= \frac{2}{c}(l_2 - l_1)\left(\frac{1}{\sqrt{1 - v'^2/c^2}}\right)$$

$$= \frac{2}{c}(l_2 - l_1)\left(\frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

$$N = \frac{\Delta t' - \Delta t}{T} = \frac{2}{cT}(l_2 - l_1)\left[\frac{1}{\sqrt{1 - v'^2/c^2}} - \frac{1}{\sqrt{1 - v^2/c^2}}\right]$$

$$= \frac{2(l_2 - l_1)}{\cancel{cT}}$$

$$= \frac{2(l_2 - l_1)}{\lambda}\left[1 + \frac{v'^2}{2c^2} - 1 - \frac{v^2}{2c^2} + O(v/c)^4\right]$$

$$\left[N = \frac{(l_2 - l_1)}{\lambda c^2}(v'^2 - v^2) \neq 0\right]$$

Theoretical prediction within classical velocity addition formula
$N \neq 0$.
$N = 0$ (expt result)
So this theory wanted to remove).

4/2/2019

Questioning length contraction hypothesis while the set up moves through ether.

Aether (Ether) drag hypothesis was proposed

Earth is dragging the aether.
So that $v = v' = 0$.
$\Rightarrow N = 0$ in M-M expt Kennedy et al's expt

Bradley's expt (~1750) for stellar aberration.

A distant star (is not moving w.r.to the sun)

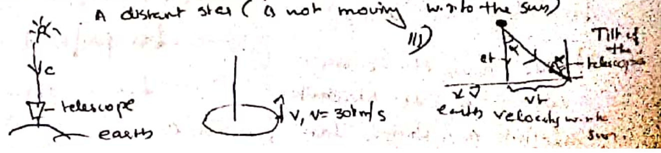


Figure 11

t → time taken by light to reach the earth from the star.

$\tan\alpha = \frac{vt}{ct} = \frac{v}{c} \approx 10^{-4}$

$\alpha = \tan^{-1}(10^{-4})$

$\alpha \longrightarrow \approx 20.5$ (seconds)

total till 10 a years: 41 seconds = 2α

$v = 30$ km/s.

→ $2\alpha = 41$ seconds (expt result.)

$2\alpha = 0$ (according to → ether drag hypothesis $\because v = 0$) (Kelvin's 3 famous predicted themselves that only cover in physics science. no need to come to science)

→ Summary

→

1) Laws of mechanics are covariant (same) in all the inertial (reference) frame.

2) No. 1 is compatible with the Galilean transformation ($\vec{r} - \vec{r}' + vt$) as well as with the classical velocity addition formula ($\vec{u} = \vec{u}' + v$)

3) No. 1 and No. 2 were consistent with Newtonian relativity and Newton's laws of motion as well as Newtonian relativity ~~[illegible]~~ is not was verified with Newton's laws of motion.

4) There is no preferred inertial frames as far as mechanics is concerned.

Principle of relativity.

5) Laws of electromagnetism (maxwell's equations) came into the picture from this speed of light is constant (c) in vacuum

6) ~~If it is possible~~ If Galilean ~~[illegible]~~ transformation as well as classical velocity addition formula is obey ~~[illegible]~~ then if it is possible to determine velocity of a inertial reference frame w.r.to another by performing an expt within the inertial frame.

⇒ There exists a set of preferred inertial frame if

Maxwell's equations and Galilean transformation are combined together.

3/2/2019

→ Summary (Continued).

→ Laws of electromagnetism are not covariant under Galilean transformations or classical velocity addition formula

7) Point 4 & 6 fails after Michelson - Morley expt, Kennedy - Thorndike expt, Bradley expt.

→ If laws of electromagnetism are correct, then we should not apply Galilean transformation to show to covariance

Null shift of fringes says that laws of electromagnetism are covariant in all the inertial frames.

→ Question → What is the appropriate transformation (other than Galilean transformation) for which laws of electromagnetism are covariant (same) in all the inertial reference frames?

Answer was given by Lorentz (1904) by a transformation named after him.

Question → It is possible to get a unique transformation of space and time (x,y,z) (t) co-ordinate for which laws of physics (including ~~and laws of~~ mechanics and electromagnetism) are covariant in all the inertial reference frames?

Answer was given by A. Einstein. (1905), He found the transformation to be the Lorentz transformation.

→ (Unification example is law of electromagnetism. not seperate rules are there for electricity and magnetism)

Einstein's theory of Special Relativity. (1905)

⇒ It doesn't deal with non-inertial frames

→ (It only deals with inertial frames). (not acceleration).

→ It does not deal with the gravitational field or any pseudo force (for which the frames should be non-inertial. (Locally the frame is inertial).

General theory of relativity.

→ Special theory of Relativity

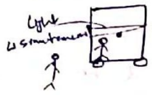
→ STR is a physics of Space and time.

→ Einstein's theory on special relativity revolutionized the concept of space and time and their transformation. (till this time wasn't changing, but now time is changing.)

(time is changing-relatively

→ In Newtonian physics, time wasn't changing)

Now time is a coordinate (b/c changing) not a parameter as people thought before.



Einstein Fundamental postulates of STR

1) Laws of physics are covariant to all the inertial reference frames.

2) Speed of light (e.m. wave) is same to all the inertial frames moving to all the directions.

11/2/2019

Special theory of (Einstein's) Relativity: Geometric point of view.

→ Theories are provable + Mathematically proved statements.

(Postulates can't be proved laws - more powerful than postulates.

1)

→ Fundamental Postulates

1) Laws of physics are covariant in all the inertial reference frames

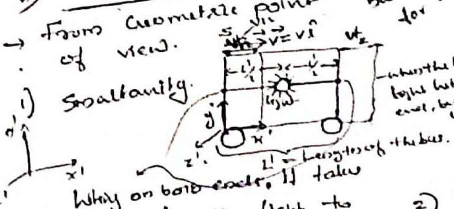
2) Speed of light is same in all the inertial frames.

3) Consequences

→ From Geometric point of view.

bulb switch on for a moment and switch off.

1) Simultaneity.



Why on both ends, [illegible] It takes Same time for the light to hit the two ends.

→ Hitting the two ends are the two events. These are simultaneous in S' frame

t_2 = time taken for the light to hit the right end

t_1 = time taken for the light to hit the left end.

S frame

$t_2 > t_1$.

→ Observation in frame 'S'.

→ The two events are not simultaneous in S.

Conclusion:

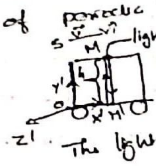
If two events are simultaneous in an Inertial frame, they are not necessarily simultaneous in another inertial frame.

According to the postulate - 2

2) Time Dilation.

clock: Device of uniform periodicity in motion and some objects in it.

Clock is an arrangement of periodic motion

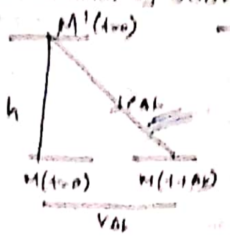


The light source and the two mirrors M and M' are making an (optical) clock

Observer S and S' are using the same clock in the bus.

→ Half of the time period are observed in S' is $\Delta t' = \frac{h}{c}$

Observation by S observer



$(c\Delta t)^2 = (v\Delta t)^2 + h^2$

$\sqrt{c^2 - v^2}$ $\quad \Delta t = \frac{h}{c\sqrt{1-v^2/c^2}}$

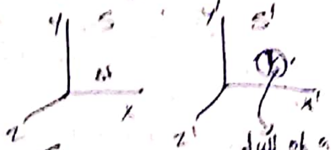
$\gamma = \sqrt{1-v^2/c^2}$ $\quad \Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}}$

$\Delta t' = \sqrt{1-v^2/c^2}\,\Delta t$

$\Delta t' = \frac{\Delta t}{\gamma}$ $\quad \Delta t > \Delta t'$

moving long, running slow.
(The clock already hitting on plane (slow less time).
The ½ time taken to hit the mirror while moving is greater than the ½ time need for when system is at rest.
Moving clock is running slow.
Time taken for to complete the ½ of the period is more in a moving clock than that in a rest frame.

17/4/2019



full of a period $\Delta t'$; 1 min. for S' observer.

I saw the unit time interval in your clock when you were at rest then I saw the unit time interval as Δt in your clock while you are in motion.

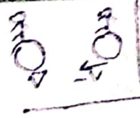
Δt = time for 1 full rotation of the rod noted by S observer.

$\Delta t = \frac{1}{\sqrt{1-v^2/c^2}}\,\Delta t'$ is $\frac{\Delta t'}{\Delta t} \leq 1$

$\sqrt{1-v^2/c^2} \leq 1$

When $\bar{v} = 0$
$\Delta t = \Delta t'$
clock is at rest w.r.to S' observer and is in motion w.r.to the S observer.
→ I found $\Delta t > \Delta t'$ as if when you are moving, your clock were slowing down
(It takes more time to complete full period.).
i.e $\Delta t = \Delta t'$.

→ Common clock for both the observers.
t' = reading of the clock noted down by S' observer.
→ Reading of the clock noted by S observer (t).



Lorentz Contraction
(Length contraction).
(Geometrical Point of view.) with a thought (Gedanken) expt.

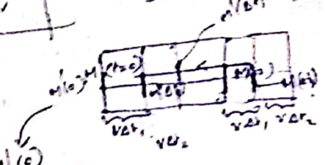
Δt = time for the round trip of the light signal from M' to M and M to M' as measured by S' observer.

L' → length of bus measured by the S' observer.

$$\boxed{\Delta t' = \frac{2L'}{c}}$$

S' → rest observer w.r.to bus.

What S observer will see?



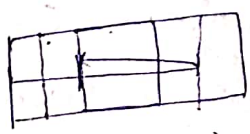
$M'(t)$
$= M'(t=0)$

$\Delta t = \Delta t_1 + \Delta t_2$

→ time for the round trip as noted by the S-observer.
→ Call L = length of the bus as observed by the S observer

$\Delta t_1 = \frac{L + v\Delta t_1}{c}$ (according to 2nd postulate)

$\Delta t_2 = \frac{L - v\Delta t_2}{c}$.



$(\Delta t_1)c = L + v(\Delta t_1)$

$(\Delta t_1)(c - v) = L$

$\Delta t_1 = \frac{L}{c-v}$

$\Delta t_2 = \frac{L}{c+v}$

$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v}$

$= L\left[\frac{2c}{c^2 - v^2}\right]$

$= 2L\left[\frac{c}{c^2(1 - v^2/c^2)}\right] = \frac{2L}{c(1-v^2/c^2)}$

$\frac{2L'}{c(1-v^2/c^2)}$

① — $\Delta t' = \sqrt{1-v^2/c^2}\,\Delta t$ (time dilation formula).

$\Delta t' = \frac{2L'}{c} \Rightarrow \Delta t' = \Delta t\sqrt{1-v^2/c^2}$

$\Delta t = \Delta t' \frac{L}{L'(1-v^2/c^2)}$

$= \frac{2}{L(1-v^2/c^2)}\sqrt{1-v^2/c^2}$

$= \frac{2}{L\sqrt{1-v^2/c^2}}$

$\frac{\Delta t}{\Delta t'} = \frac{1}{\sqrt{1-v^2/c^2}}$

$\Delta t' = \frac{2L'}{c} \Rightarrow$

$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2L}{c}$

$\Rightarrow \Delta t = \Delta t' \frac{L}{L'(1-v^2/c^2)}$

① — $\Delta t' = \sqrt{1-v^2/c^2}\,\Delta t$

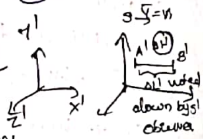
$\Delta t' = \frac{2L'}{c}$

$\Delta t = \frac{2L \cdot L'}{L'\,c\,(1-v^2/c^2)}$

$\Delta t = \frac{\Delta t' \cdot L}{L(1-v^2/c^2)}$ — ②

$\Delta t = \Delta t'$

18/2/2019



Compare ① and ②,

$\frac{1}{\sqrt{1-v^2/c^2}} = \frac{L}{L'(1-v^2/c^2)}$

Lorentz contraction

$L = L'\sqrt{1-v^2/c^2}$ } = As if when the bus moves, it seems contracted.

Length of the bus as noted down by S' observer

$L' \geq L$ ↳ length of the bus when it was moving as observed by S-observer

Length of the bus when it was not in motion w.r.to S observer

The rod is at rest w.r.to the S observer

ΔL (length) of the same rod in S' noted down by S observer. This rod is moving w.r.to the S observer.

$\Delta L \leq \Delta L'$

$\Delta L = \Delta L'$ if $v=0$

Apply 2nd postulate

$\Delta L = \sqrt{1-v^2/c^2}\,\Delta L'$

Lorentz contraction of length.

$\Delta t = \frac{1}{\sqrt{1-v^2/c^2}}\Delta t'$

$\Delta t \geq \Delta t'$

Moving clock: seen slow: $\Delta t'$ = time interval for a full rotation of the 2nd hand.

$\Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}}$

$\Delta t \geq \Delta t'$

For Galilean transformation

$t' = t$

$\vec{r}' = \vec{r} - vt$

$t' = t \Rightarrow \Delta t' = \Delta t$.

$\Delta x' = \Delta x - v\Delta t$ ($\Delta t \to 0$) (both the ends of length s have to be measured at same time).

Galilean transformation (No time dilation and length contraction)

$\Rightarrow (\Delta x' = \Delta x)$

$\Rightarrow \Delta L' = \Delta L$

If there are time dilation

$\Delta t' = \sqrt{1-v^2/c^2}\,\Delta t$

Length contraction.

$\Delta L' = \frac{1}{\sqrt{1-v^2/c^2}}\Delta L$

→ Then the corresponding coordinate transformation must not be Galilean transformation.

→ Q. What is the corresponding co-ordinate transformation.

A. Then it is Lorentz transformation.

How to correct it ($\vec{r} = \vec{r}' + vt$) to get length contraction and time dilation.

→ This length is moving w.r.to S observer (vt)

$\vec{r}' = \vec{r} - vt$.

$(\vec{r}' \cdot \hat{i}) = (\vec{r} - vt)\hat{i}$ (along x-direction)

→ $\Delta L' = \frac{1}{\sqrt{1-v^2/c^2}}\Delta L$.

$(\vec{r}' \cdot \hat{i}) = (\vec{r} - vt)\hat{i} \times \frac{1}{\sqrt{1-v^2/c^2}}$ — Ⓐ

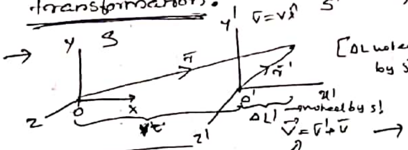
$(\vec{r}' \cdot \hat{i})$: rest length w.r.to S' observer

$(\vec{r} - vt)\hat{i}$: moving length w.r.to S observer

→ $\vec{r}'$ not moving w.r.to S' observer

→ length of $\vec{r}'$ along x axis noted down by S' observer.

→ length of $\vec{r}'$ along x axis as noted down by S observer

→ 

$\vec{V} = \vec{V}' + \vec{v}$

$\vec{r} = \vec{r}' + v\Delta t$. (From classical velocity addition formula or from Galilean transformation)

↓

Not going to explain length contraction and time dilation. →

→ $\vec{r}'$ is not moving along y and z axis. (v is on x axis)

$\Rightarrow$ $\vec{y}' = y$ ($V_y = V_z = 0$)

$\vec{z}' = z$

→ No length contraction.

From (A),

$$x' = (x - vt) \times \frac{1}{\sqrt{1 - v^2/c^2}}$$

at $t=0$, $OO' = 0$.

$\bar{r} = \bar{r}'$ at $t = 0$.

$\Rightarrow t' = t$ (when it is not moving $OO'=0$),

$\Rightarrow$ at $t=0$, $t'=0$

at $OO'=0 \rightarrow$ two readings are synchronized

$$\underbrace{x - vt}_{\Delta L} = \underbrace{x'}_{\Delta L'}\sqrt{1 - v^2/c^2} \quad \text{— (B)}$$

$\Delta L \le \Delta L'$

Consider the inverse transformation now.

$\rightarrow \vec{v} \rightarrow -\vec{v}$, $x \rightarrow x'$, $t \rightarrow t'$

$\bar{r}$ is moving w.r.to S' observer

$$\underbrace{x' + vt'}_{\tilde{\Delta L}'} = \underbrace{x}_{\Delta L}\sqrt{1 - v^2/c^2} \quad \text{— (C)}$$

(ΔL is moving w.r.to S').

$\Delta L > \Delta L'$

Combining B and C

$$\underbrace{x' + vt'}_{\tilde{\Delta L}'} = x\sqrt{1 - v^2/c^2}$$
$$\downarrow$$
$$\tilde{\Delta L}$$

$\tilde{\Delta L}' \le \tilde{\Delta L}$

is moving w.r.to S' observer

$$x' = \frac{1}{\sqrt{1 - v^2/c^2}}(x - vt) \quad \text{— (1)}$$

$$x' = x\sqrt{1 - v^2/c^2} - vt'$$

$$x\sqrt{1 - v^2/c^2}\,(x - vt')$$

$$= \frac{1}{\sqrt{1 - v^2/c^2}}(x - vt)$$

$\Rightarrow x(1 - v^2/c^2) - v\sqrt{1 - v^2/c^2}\,t' = x - vt$

$$-x\frac{v^2}{c^2} - v\sqrt{1 - v^2/c^2}\,t' = -vt$$

$$\rightarrow t' v\sqrt{1 - v^2/c^2} = vt - \frac{xv^2}{c^2}$$

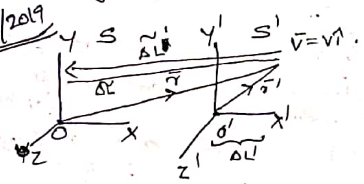
$\Rightarrow \div$ by v

$$\frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}} = t'.$$

$y' = y$ — (3)

$z' = z$. — (4)

19/2/2019



ΔL is moving w.r.to S.

$\Rightarrow$ ~~$t' = t$~~, Lorentz factor Gamma ($\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$)

Lorentz factor.

$$ct' = \frac{ct - xv/c}{\sqrt{1 - v^2/c^2}}$$

$$= (ct - xv/c)\,\gamma$$

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

Call
$ct = x^0$
$x = x^1$
$y = x^2$
$z = x^3$

$ct' = x^{0'}$

$x' = x^{1'}$

$y' = x^{2'}$

$z' = x^{3'}$ Call $v/c = \beta$.

$$x^{0'} = \gamma(x^0 - \beta x^1)$$

$$\rightarrow x^{0'} = \gamma(x^0 - \beta x^1)$$

$$\rightarrow x^{1'} = \gamma(x^1 - \beta x^0)$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

This is called Lorentz transformation.

$\frac{1}{\sqrt{1 - v^2/c^2}} \ge 1$

For inverse Lorentz transformation $\beta = \frac{v}{c} \le 1$

$v \rightarrow -v$ ($\beta \rightarrow -\beta$)

$$x^0 = \gamma(x^{0'} + \beta x^{1'})$$

$$x^1 = \gamma(x^{1'} + \beta x^{0'}) \qquad x^1 \text{ Prime } = x^{1'}$$

$$x^2 = x^{2'}$$

$$x^3 = x^{3'}$$

$\rightarrow$ For $\frac{v}{c} \rightarrow 0$, $\gamma \rightarrow \frac{1}{\sqrt{1-0}} = 1$

$t' = t$

$x' = (x - vt)$ [$\gamma = 1$, $c/c \rightarrow$ cancel]

we are getting Galileo transformation (back to classical mechanics.)

$\rightarrow$ (we are getting) mechanics. If this is not $v/c \rightarrow 0$ (relativistic mechanics)

$\rightarrow$ Lorentz transformation in matrix form.

$$\begin{pmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{Lorentz transformation matrix}} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$\Lambda =$ Lorentz transformation matrix

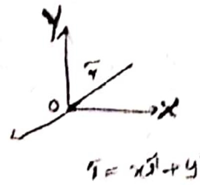
$\Lambda = $ " "

('large λ')

Column matrix. representation of a (position-time) vector.

Space time vector

4-dimentional vector



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Euclidean space

(Cartesian space

(lies in 3-D cartesian co-ordinate of

4-dimensional vector Euclidean space.

→ This cannot be accomodated in 3-D euclidean space.

It lies in 4-D Minkowski Space.

3 — position coordinates

1 time

→ Rotate the vector

So that $\vec{r} \to \vec{r}'$

$|\vec{r}'| = \sqrt{x^2 + y^2 + z^2}$

$|\vec{r}'| = x'^2 + y'^2 + z'^2$

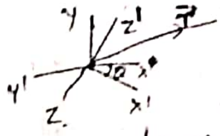
$\sqrt{x^2 + y^2 + z^2} = \sqrt{x'^2 + y'^2 + z'^2}$

—— length of the vector.

→ It is invariant quantity (~~constant~~ under rotation.

↳ typical example. $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

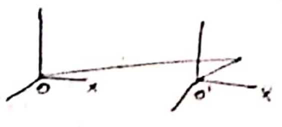
$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & \end{pmatrix}$



different from Lorentz transformation matrix.

21/02/2019

Lorentz transformation.



4 - position vector (ct, x, y, z) of an ~~point~~ event (position, time)

4 - dimensional Minkowski space

$-(ct')^2 = -\gamma^2 (ct - \beta x)^2$

$+x'^2 = +\gamma^2 (-ct\beta + x)^2$

$+y'^2 = +y^2$

$+z'^2 = +z^2$

$-(ct')^2 + x'^2 + y'^2 + z'^2$

$= \gamma^2 \left[-(ct - \beta x)^2 + (x - \beta ct)^2 + y^2 + z^2 \right]$

$= \gamma^2 \left[-c^2t^2 + 2ct\beta x - \beta^2 x^2 + c^2t^2\beta^2 + x^2 - 2x\beta ct + y^2 + z^2 \right]$

$= -\gamma^2 c^2 t^2 \left[1 - \beta^2 \right] + \gamma^2 x^2 \left[1 - \beta^2 \right] + y^2 + z^2$

$\left(\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \right.$

$= \frac{-c^2t^2 \left[1 - v^2/c^2 \right]}{(1 - v^2/c^2)} + \frac{x^2 (1 - v^2/c^2)}{(1 - v^2/c^2)} + y^2 + z^2$

$= -c^2t^2 + x^2 + y^2 + z^2 = -(ct)^2 + x^2 + y^2 + z^2$

$-(ct')^2 + x'^2 + y'^2 + z'^2 = -(ct)^2 + x^2 + y^2 + z^2$ | $c^2t'^2 + x'^2 + y'^2 + z'^2 \neq c^2t^2 + x^2 + y^2 + z^2$

→ $-(ct)^2 + x^2 + y^2 + z^2 \neq z^2$ does not change under Lorentz transformation. Lorentz confirmation does not represent rotation of the 4-position vectors in Euclidean space (Minkowski) (If $t, t' \neq 0$)

$\tau = \tau^2 = -c^2t^2 + x^2 + y^2 + z^2$ is called Lorentz invariant (transformation point of view).

$\tau = \tau = \sqrt{-c^2t^2 + x^2 + y^2 + z^2}$ is the length of the position vector in Minkowski space.

↓ can be complex no.

→ To avoid the problems always deal with τ^2.

$\tau^2 = -c^2t^2 + x^2 + y^2 + z^2$.

If, $\tau^2 < 0$.

If

→ If $\tau^2 < 0$, then the event is called time like event — It is forward

when $x^2 + y^2 + z^2 < c^2t^2$

If $\tau^2 > 0$, then event is called space like event.

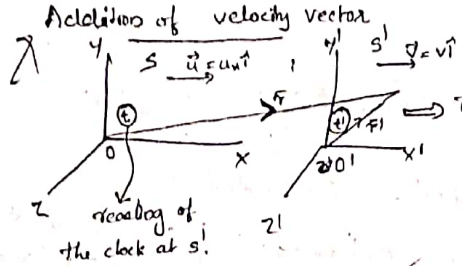
If $\tau^2 = 0$, then event is called light like event. $\tau = 0$.

→ Lorentz transformation

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$$ → does not represent a rotation rather represents a boost along x-direction with velocity $\vec{v} = v\hat{i}$.

Addition of velocity vector



→ To the S' observer, velocity of the particle is $\vec{u}' = \frac{d\vec{r}'}{dt'} = u_x'\hat{i} + 0\hat{j} + 0\hat{k}$

ordinary velocity

$$\vec{u} = \frac{d\vec{r}}{dt} = u_x\hat{i} + 0\hat{j} + 0\hat{k}$$

$$u_x' = \frac{dx'}{dt'} = \frac{\frac{d}{dt}(-\gamma\beta ct + \gamma x)}{d(\gamma ct - \gamma\beta x)/c}$$

$$= \frac{\beta c \, d\left(-\gamma ct + \frac{\gamma}{\beta}x\right)}{d\left(\frac{\gamma}{\beta}ct - \gamma\beta x\right)}$$

γ cancelled

$$= \frac{-\beta c\frac{dt}{dt} + \frac{dx}{dt}}{\left(c\frac{d(t)}{dt} - \beta\frac{dx}{dt}\right)/c} \qquad \beta = \frac{v}{c}, \quad \beta c = v$$

$$= \frac{-v + u_x}{1 - \frac{v}{c^2}\cdot u_x} \qquad \beta = v/c$$

$$= \frac{-\beta c + u_x}{1 - \beta u_x} = \frac{-v + u_x}{1 - \frac{v}{c^2}u_x}$$

$$u' = \frac{\bar{u} - \bar{v}}{1 - \bar{u}\bar{v}/c^2}$$

$$u_x' = \frac{u_x - v}{1 - u_x \cdot v/c^2}$$ → generalize it for all the components.

$$\boxed{u' = \frac{\bar{u} - \bar{v}}{1 - \bar{u}\bar{v}/c^2}}$$ → relative velocity w.r.to S' observer

Any two vectors, ordinary velocity vectors v_1 and v_2 are added.

$$\bar{u} = \frac{\bar{u}_1 + \bar{u}_2}{1 + \bar{u}_1\bar{u}_2/c^2}$$ → Einstein's velocity addition formula for two ordinary vectors, v_1 and v_2, u_1 and u_2

If $\bar{u}_1 = c\hat{i}$, (after adding more velocity)

$u_2 = u_2\hat{i}$

$$\bar{u} = \frac{c + u_2}{1 + \frac{cu_2}{c^2}} = \frac{c + u_2}{1 + u_2/c} \Rightarrow c$$

⇒ a particle velocity (speed) can never exceed c

→ 25/2/2018

Application of Lorentz Transformation

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Lorentz Boost along } x\text{-axis}} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

(cartesian space – infinite vectors)

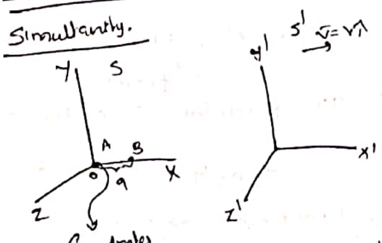
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, $\beta = \frac{v}{c}$, velocity $\vec{v} = v\hat{i}$.

4-position vector in Minkowski space.

4-position vector in the same Minkowski space.

Example 1.

Simultaneity.

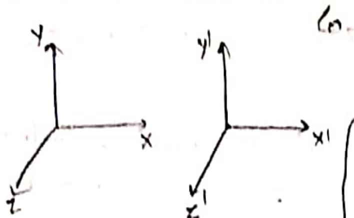


at t=0, A and B they are simultaneous events in S frame

Coordinates for the event A according to S-observer are

$A \equiv (0, 0, 0, 0)$ [ct (t=0)] $B \equiv (ct, x, 0, 0)$

$B \equiv (0, a, 0, 0)$



Co-ordinate for the event A and B in S' frame

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ \end{bmatrix}$$

If ct, x, y, z are zero- then $ct' = 0$.

$ct' = \gamma ct - \gamma\beta x + 0.y + 0.z = 0$ ~~$- \gamma\beta + 0 + 0$~~.

$x' = -\gamma\beta.ct + \gamma.x + 0.y + 0.z$
$= 0$

$y' = 0$
$z' = 0$

Co-ordinates for the event.

$A' \equiv (0,0,0,0)$

$B' \equiv (-a\gamma\beta, a\gamma, 0, 0)$.

~~$ct' = 0 - a\gamma\beta + 0 + 0$~~
~~$= -a\gamma\beta$~~.

For event B

$ct' = \gamma. ct - \gamma\beta x + 0.y + 0.z$ $\quad x' =$
$= 0 - a\gamma\beta + 0 + 0$
$= -a\gamma\beta$

$x' = -\gamma\beta.0 + \gamma.a + 0 + 0$
$= a\gamma$

$y' = 0+0+0+0 = 0$

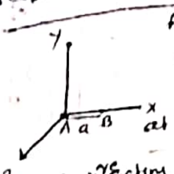
$z' = 0+0+0.1.0 = 0$

Two events in S' are spacially seperated in S frame. Then they are not simultaneous in S' frame.

Bcz they are occurring in S' frame. They are spacially seperated in

though 2 clocks are synchronized at $t=0$,
$x=0$, $y=0$, and $z=0$

Newton's 3rd law, Every action, there is an equal and opposite reaction at same time



A is acting on B at time $t=0$ (action) at position ($x=0, y=0, z=0$).

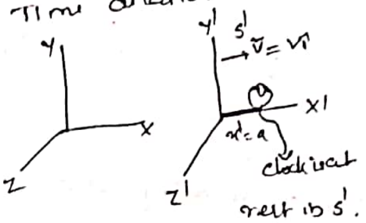
B is reacting on A at time $t=0$ (reaction) and position ($x=a, y=0, z=0$)

A and B are simultaneous in S but not in S'.

So the Newtons 3rd law is not valid in S'.

Example 2

Time dilation



$ct' = \gamma ct - \gamma\beta x$

$c\Delta t' = \gamma c\Delta t - \gamma\beta\Delta x$.
$\Delta x \neq 0$.

not telling us when the clock is at rest in S'.

Take inverse Lorentz transformation,

$$\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \gamma & +\gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix}$$

$ct = \gamma ct' + \gamma\beta x'$

$c\Delta t = \gamma c\Delta t' + \gamma\beta\Delta x'$ → so bcz clock is not moving w.r.t. to S' observer.

x' → coordinate for clock.

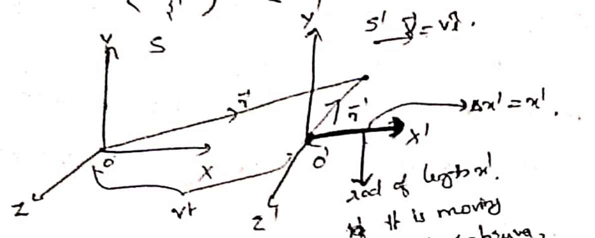
→ $\Delta t = \gamma\Delta t'$

$\Delta t' = \frac{1}{\gamma}\Delta t = \sqrt{1 - v^2/c^2}\,\Delta t$

$\Delta t' \leq \Delta t$.

26/2/2018

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$



Ex-3

Lorentz contraction of length.

$L' \leq \sqrt{1 - v^2/c^2}\, L$.

$x' \leq \sqrt{1 - v^2/c^2}\, x$.

$$\cancel{x' = \frac{-vt}{\sqrt{1 - v^2/c^2}}} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$x' = -\gamma\beta ct + \gamma x$ $\qquad \beta = v/c$

$\Delta x' = -\gamma\beta c\Delta t + \gamma\Delta x$ ↳ length of the moving rod as measured by the S observer

$\Delta x'$ ↓ Length of rod. is measured by S' observer

Δt ↓ 0 — both ends should be measured at same time (no time difference) → 0 → necessary requirement for measuring length.

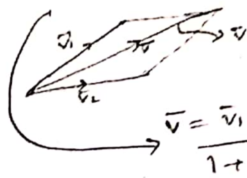
$\Rightarrow \Delta x' = \gamma\Delta x$

$\Delta x = \frac{1}{\gamma}\Delta x'$

$\Rightarrow \Delta x = \underbrace{\sqrt{1 - v^2/c^2}}_{<1}\,\Delta x'$

$\Rightarrow \boxed{\Delta x \leq \Delta x'}$ Moving length (w.r.to S) is contracted.

Einstein's velocity addition formula



$\vec{v} = \vec{v}_1 + \vec{v}_2$ (classical velocity addition formula).

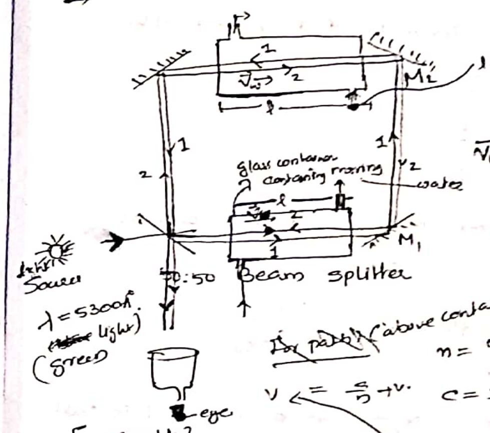
$$\vec{v} = \frac{\vec{v}_1 + \vec{v}_2}{1 + \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}}$$

[Einstein's velocity addition formula is ordinary velocity vectors $\vec{v}_1$ and $\vec{v}_2$).

Fizeau's Light Drag Experiment (~~1851~~) (1851)

for light drag by a moving medium

→



$|v_w| = 7$ m/s.

$\vec{v}_w$ = Velocity of water, it is a moving a moving medium for light to propagate in it.

For path 1 (above container) n = refractive index of water $= 1.33$.

$v \Leftarrow = \frac{c}{n} + v$. $\quad c = 3 \times 10^8$ m/s,

$v = \frac{c}{n}$

$v = \frac{3 \times 10^8}{1.33}$ = Speed of light in water when it is at rest

For path 2 (bottom container) (lower)

$\xrightarrow{v_w}$

$\xleftarrow{c/n}$

resultant speed of light in moving water $= v_2 = \frac{c}{n} - v_w$. → 100% drag (if it moves, it drags some more light) of light by the moving water

→ $\frac{c}{n} - dv_w$: d is the % of dragging. (call it drag coefficient) (d)

In upper container

path 2, c/n

V_w

Speed of light in moving water is

$= V_2 = \frac{c}{n} - dV_w$

For path 1 (upper container) and in lower container (same v)

V_w

Light and water moving in same direction with speed

c/n

$V_1 = \frac{c}{n} + dV_w$

For path 2: time spent by light in both the containers $= t_2 = \frac{2l}{V_2}$

$= \frac{2 \cdot l}{\frac{c}{n} - dV_w}$

In path 1: time spent by light in both the container,

$t_1 = \frac{2l}{\frac{c}{n} + dV_w}$

ct = optical length in free space

Difference in optical lengths in path 2 and in path 1 is $\Delta x = c(t_2 - t_1) = c\Delta t$ → cause shift of the fringes.

How many fringes should be shifted

ΔN = No. of fringes shifted

$= \frac{\Delta t}{\text{time period}} = \frac{\Delta t}{T}$

$\rightarrow T = \lambda/c$

$= \frac{\Delta t}{\lambda/c} = \frac{c}{\lambda}(\Delta t) = \frac{c}{\lambda}(t_2 - t_1)$

$$\Delta N = \frac{c}{\lambda}\left(\frac{2l}{\frac{c}{n} - dV_w} - \frac{2l}{\frac{c}{n} + dV_w}\right)$$

n = 1.3
λ = 1.5
λ = 5500 Å
dV_w = 7 m/s.

d = ?

$\Delta N = 0.23$
(expt observation)

$0.23 = \frac{3 \times 10^8 \cdot 3}{5.5 \times 10^{-7}}\left(\frac{3}{\frac{3 \times 10^8}{1.3} - d \cdot 7} - \frac{3}{\frac{3 \times 10^8}{1.3} + d \cdot 7}\right)$

$1.4 \times 10^{-16} = \left(\frac{14d}{5 \times 10^{16} - 49d^2}\right)$

$7 - 49d^2 \times 1.4 \times 10^{-16} = 14d$

$7 - 6.88 \times 10^{-15} d^2 = 14d$

$d = \frac{7}{14} = 0.5$

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$$\Delta N = \frac{\Delta t}{T} = \frac{c(\Delta t)}{\lambda} = \frac{c}{\lambda} \cdot \frac{2l}{c/n}\left[\frac{1}{1 - \frac{dnV_w}{c}} - \frac{1}{1 + \frac{dnV_w}{c}}\right] \quad \frac{V_w}{c} << 1$$

no. of fringes shifted for speed of water changed from 0 to V_w.

Binomial expn. $(1 - \frac{d}{c}nV_w)^{-1} - (1 + \frac{d}{c}nV_w)^{-1}$ ⇒ expand

$$\Delta N = \frac{2ln}{\lambda}\left[\left(1 + \frac{dnV_w}{c} + \dots\right) - \left(1 - \frac{dnV_w}{c} + \dots\right)\right]$$

$$\Delta N = \frac{2ln}{\lambda} \cdot \frac{2dnV_w}{c} = \frac{4ln^2V_w d}{c\lambda}$$

ie, $\Delta N = \frac{4ln^2V_w}{c\lambda} d$

$\Rightarrow d = \frac{(\Delta N)c\lambda}{4ln^2V_w}$ = Fresnel – Fizeau experiment 1817 formula for light drag.

expt observations $\Delta N = 0.23$

expt parameters = l = 1.5 m
n = 1.33
V_w = 7 m/s
$\lambda = 5300 \times 10^{-10}$ m
$c = 3 \times 10^8$ m/s.

d = 0.49

⇒ Laue's explanation (1907) (Theoretical expansion)

Attack from Einstein's velocity addition formula (1905)

$$V_1 = \frac{\frac{c}{n} + V_w}{1 + \frac{\frac{c}{n} V_w}{c^2}}$$

$$V_2 = \frac{\frac{c}{n} - V_w}{1 - \frac{c V_w}{n c^2}}$$

Ruling out the concept of light drag

$$t_1 = \frac{2l}{V_1} = \frac{2l\left(1 + \frac{c V_w}{n c^2}\right)}{\frac{c}{n} + V_w}$$

$$t_2 = \frac{2l\left(1 - \frac{c V_w}{n c^2}\right)}{\frac{c}{n} - V_w}$$

$$\Delta N = \frac{\Delta t}{T} = \frac{c(t_2 - t_1)}{T}$$

$$\Delta N = \frac{c}{\lambda} \cdot \frac{2l}{c/n}\left[\frac{1 - \frac{c V_w}{nc}}{1 - \frac{V_w n}{c}} - \frac{1 + \frac{c V_w}{nc}}{1 + \frac{V_w n}{c}}\right]$$

By Binomial expn for $\frac{V_w}{c} << 1$

$$\Delta N = \frac{n 2 l}{\lambda}\left[(1 - c V_w)\left(1 + \frac{V_w n}{c}\right) - (1 + c V_w)\left(1 - \frac{V_w n}{c}\right) + O(V_w^2)\right]$$

$$= \frac{2nl}{\lambda}\left[\left(1 + \frac{V_w n}{c} - \frac{V_w}{nc}\right) - \left(1 - \frac{V_w n}{c} + \frac{V_w}{nc}\right) + O(V_w^2)\right]$$

of the order of

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$O(x^2)$ neglected terms

$$= \frac{2nl}{\lambda}\left[\frac{V_w n}{c} - \frac{V_w}{nc}\right]$$

$$= \frac{4 l l}{\lambda} \frac{V_w}{nc}\left[n^2 - 1\right]$$

$n^2 << 1$

$$\Delta N = \frac{4 l V_w}{\lambda c}[n - 1] = \frac{4 l V_w n^2}{\lambda c}\left[1 - \frac{1}{n^2}\right]$$

$$\Delta N = \frac{4 l n^2 V_w}{\lambda c} d \quad [\text{Fresnel- Fizeau formula}] \quad \text{— (1)}$$

$$\Delta N = \frac{4 l V_w n^2}{\lambda c}\left(1 - \frac{1}{n^2}\right) \quad \text{— (2)}$$

from Einstein Velocity addition formula

Compare (2) & (1). ⇒ $\boxed{d = 1 - \frac{1}{n^2}}$ ← as well as experimental Fresnal's formula (1817)

⇒ $d = 1 - \frac{1}{n^2} = 0.43$

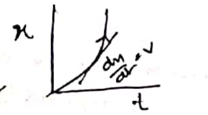
6/3/2019

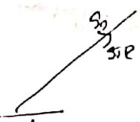
Space-Time Diagram

Consider a particle moving along x-axis with an acceleration f, such that

$$x = \frac{1}{2} f t^2$$

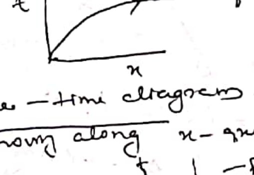
Time-space diagram is a graphical representation for the motion of a particle w.r.to time, relating position with time.



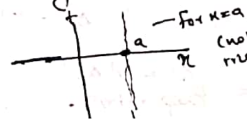


t → independent
x → dependent

In special theory of relativity (STR), we represent trajectory of a particle in space-time diagram.



In STR, Space-time diagram for a particle moving along x-axis.



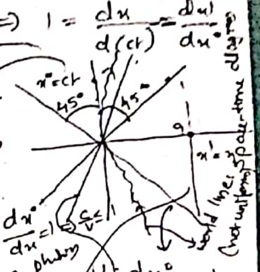
$\tan 30 = \frac{1}{\sqrt{3}}$

$\tan 60 = \sqrt{3}$

Space time diagram for a particle at rest at $x = a$.

For photon, $c = \frac{dx}{dt}$

$$\Rightarrow 1 = \frac{dx}{d(ct)} = \frac{dx^1}{dx^0}$$



Motion of all the particles are restricted within the wedge.

$\frac{dx^0}{dx} = 1$ $\frac{c}{v} < 1$ for a photon moving along x-axis

$\frac{dx^0}{dx} = -1$ for a photon moving along $-x$ axis.

world lines (not uniform) space-time diagram

Space time

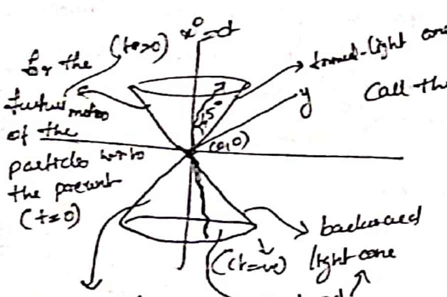
For particle: $v = \frac{dx}{dt} \le c$ moving along x? Constant

$$\frac{dx^0}{dx^1} = \frac{c\,dt}{dx} = \frac{c}{v} \ge 1.$$

Corresponds to straight line in space-time diagrams.

For a particle moving uniformly along x axis.

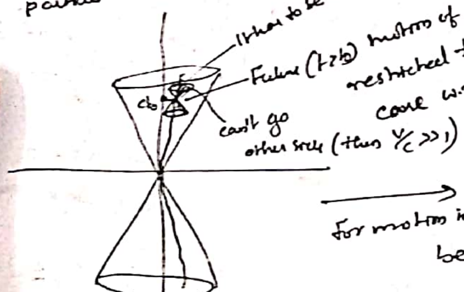
Consider 2-D motion.



Call the motion restricted within this cone

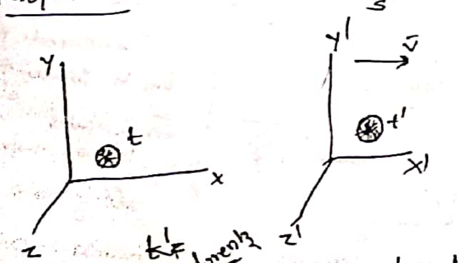
For the past (t<0) motion of the particle

Backward restricted within the forward and backward light cone



For motion in 2D; the cone is to be replaced by hyperbola

Proper time



Inverse lorentz

$ct = \gamma ct' - \gamma \beta x'$

$c\Delta t = \gamma c \Delta t' - \gamma \beta \Delta x'$

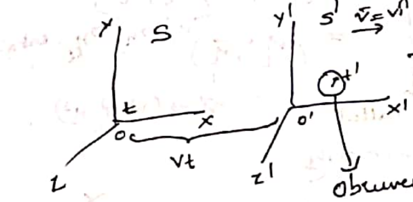
$\Delta t = \gamma \Delta t'$ → the clock is at rest w.r.to S' frame

$\Delta t'$ → is called Proper time.

Proper time → BCZ, the clock is not moving (Invariant) w.r.to S' observer.

→ 7/3/2019

Relativistic Mechanics — Proper time and Proper velocity



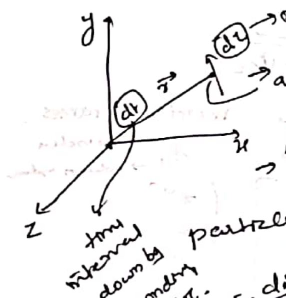
This is S' observer, clock. It is not moving w.r.to the S' observer

Observer S is observing reading (t) at this clock.

$\Delta t' = \sqrt{1 - v^2/c^2}\, \Delta t$

$\Delta t'$ → Proper time interval: the clock is not moving w.r.to S' observer ($\Delta \tau$)

Δt → Not proper time. ∴ Clock is moving w.r.to S observer



→ a particle is moving in S with ordinary velocity $u = \frac{d\vec{x}}{dt}$

→ Assume that there is a clock attached to this particle. (That is the clock is moving w.r.to S observer along with the particles).

$u = \frac{d\vec{x}}{dt}$; dt is noted down by the S observer by looking at the clock attached to the particle.

dt = not proper time interval.

Let us define proper velocity

$\frac{d\vec{x}}{d\tau} = \vec{\eta}$ → proper velocity.

$d\vec{r}$ → Noted down by S observer

dt → not noted down by S observer

$(d\tau$ is observed by body fixed clock)

→ Proper velocity $\eta^{\mu} = (\eta^0, \eta^1, \eta^2, \eta^3)$

$\mu = 0, 1, 2, 3.$

$= (\eta^0, \vec{\eta})$

$\vec{\eta} \Rightarrow (\eta^1, \eta^2, \eta^3)$

also called 4-velocity

Proper velocity / ordinary velocity.

$\vec{\eta} = \frac{d\vec{r}}{d\tau} = \frac{dt}{d\tau} \cdot \frac{d\vec{r}}{dt} = \frac{dt}{d\tau}\vec{u}$

$\frac{dr}{dt} = \vec{u}$

$d\tau = \sqrt{1 - u^2/c^2}\, dt$

$$\boxed{\vec{\eta} = \frac{1}{\sqrt{1 - u^2/c^2}}\vec{u}}$$

Define: $\eta^0 = \frac{d(ct)}{d\tau} = \frac{c\,dt}{d\tau}$

$\eta^0 = \frac{dx^0}{d\tau} = \frac{d(ct)}{d\tau} = \frac{c\,dt}{d\tau} = \frac{c}{\sqrt{1 - u^2/c^2}}.$

$d\tau$ = Same in all the inertial frames.

Λ^0_1 → 0 → row index, 1 → column index

Λ = Lorentz transformation matrix

$$\begin{bmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

Lorentz transformation matrix (Λ)

For Lorentz boost along x-axis with velocity $\vec{v} = v\hat{i}$

belongs to 4-D Minkowski space

$$\boxed{x'^{\mu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu}} \Rightarrow \eta'^{\mu} = \frac{dx'^{\mu}}{d\tau} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} \frac{dx^{\nu}}{d\tau}$$

$\eta'^{\mu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} \eta^{\nu}$

The way the space time co-ordinates in Minkowski transform the same way the components of the 4-velocity transform.

Hence the 4-velocity is also a vector in the Minkowski space

$\vec{u}' = \frac{\vec{u} - \vec{v}}{1 - \frac{\vec{u}\cdot\vec{v}}{c^2}}$ according to Einstein's velocity addition formula (ordinary v can't be a vector / coordinate)

$u'^1 = \frac{u^1 - v}{1 - \frac{u^1 v}{c^2}}$

$u'^2 = u^2$

$u'^3 = u^3$

So this does not transform according to Lorentz transformation.

Hence $\vec{u}$ though is a vector in 3-D Euclidean space, is not a vector in 4-D Minkowski space.

$m.\vec{u}$ = ordinary momentum of the particle.

m → Mass of the particle; $\vec{u}$ → ordinary velocity of a particle

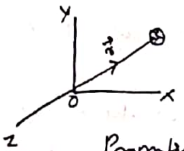
Relativistic momentum $p = \gamma m v$

$= p^{\mu} = (p^0, p^1, p^2, p^3)$

$= (p^0, \vec{p}).$

11/3/2019

Relativistic mechanics



$\vec{u} = \frac{d\vec{r}}{dt}$ → A vector in 3-D Euclidean space

Proper time interval

$$d\tau = \sqrt{1-u^2/c^2}\, dt$$

$\eta = \frac{d\vec{r}}{d\tau}$ → measured by S-observer; dt → not measured by S-observer

Relativistic formalism is not possible with the ordinary velocity u.

$\vec{u} = \frac{d\vec{r}}{dt}$ (is not a vector in Minkowski space.) (u-does have many errors)

$\eta^\mu = (\eta^0, \vec{\eta})$ is proper velocity (4-velocity). η^1, η^2, η^3 It is a vector in Minkowski space.

p^μ = relativistic 4-momentum.

$$\vec{\eta} = \frac{1}{\sqrt{1-u^2/c^2}}\vec{u}$$

$$\vec{p} = \frac{1}{\sqrt{1-u^2/c^2}}(m_0\vec{u})$$ → ordinary momentum.

relativistic momentum

Relativistic momentum

$p^\mu = m_0\eta^\mu\,(p^0, \vec{p})$ → mass of particle; (p^1, p^2, p^3)

$m_0\frac{d\vec{r}}{d\tau} = \vec{p}$

$m_0\vec{\eta} = \vec{p} = (p^1, p^2, p^3)$ → relativistic momentum.

p → must be vector in Minkowski space

4-force or Minkowski force is defined as

$$K^\mu = (K^0, K^1, K^2, K^3) = (K^0, \vec{K}) = \frac{dp^\mu}{d\tau}$$

$\vec{K} = \frac{d\vec{p}}{d\tau} = \frac{d\vec{p}}{dt}\cdot\frac{dt}{d\tau}$ → relativistic momentum $\qquad \frac{d\vec{p}}{dt}\cdot\frac{dt}{d\tau}$

$$\vec{K} = \frac{d\vec{p}}{d\tau} = \frac{d\vec{p}}{dt}\cdot\frac{dt}{d\tau} = \frac{\vec{F}}{\sqrt{1-u^2/c^2}}$$

$\vec{K} = \frac{\vec{F}}{\sqrt{1-u^2/c^2}}$ → relativistic force $\qquad \vec{F} = \frac{d\vec{p}}{dt}$ = ordinary force.

$\vec{p} \Rightarrow$ Generalized momentum.

(Inner product) Scalar product (or inner product) of two vectors in Minkowski space is defined as. (x_1^μ, x_2^μ)

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$.
$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$.
$\vec{r}_1\cdot\vec{r}_2 = x_1x_2 + y_1y_2 + z_1z_2$. in 3-D Euclidean space

Similarly, in Minkowski space is defined (r_1^μ, r_2^μ)

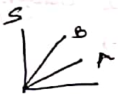
$r_1^\mu = (ct_1, x_1, y_1, z_1)$

$r_2^\mu = (ct_2, x_2, y_2, z_2)$

$r_1^\mu r_2^\mu = ct_1t_2 + x_1x_2 + y_1y_2 + z_1z_2$

→ If $r_1^\mu = r_2^\mu = r^\mu$

$r^\mu r^\mu = c^2t^2 + x^2 + y^2 + z^2$ (This is not Lorentz invariant. Hence it is not the proper definition of scalar product)



$\vec{A}\cdot\vec{B} = AB\cos\theta$. (Same for scalar product in any system) → invariant under Co-ordinate transformation (say rotation)

Proper definition of the scalar product needs a contravariant 4-vector.

eg: $x^\mu = (ct, x_1, x_2, x_3)$

and a covariant 4-vector

eg. $y_\mu = (-ct, y_1, y_2, y_3)$
↳ -ve sign (covariant vector)

Scalar product of x^μ and y^μ is defined

$$x^\mu y_\mu = -c^2 t_1 t_2 + x_1 y_1 + x_2 y_2 + x_3 y_3$$

→ If $x^\mu = y^\mu = (ct, x_1, x_2, x_3)$

Thus $x^\mu x_\mu = -c^2 t^2 + x_1^2 + x_2^2 + x_3^2$
↳ Lorentz invariant.

Relativistic Mechanics

→ Energy and Momentum.

Let us take a free particle. (no force acting on it.)

4-momentum of the particle is

$$p^\mu = (p^0, p^1, p^2, p^3) = (p^0, \bar{p})$$

⟹ p^μ must be a conserved quantity.

→ $\bar{p} = m_0 \eta = \dfrac{m_0 \vec{u}}{\sqrt{1-u^2/c^2}}$ is a conserved quantity as $\vec{u}$ is not changing with time.

↳ relativistic momentum

$$p^0 = m_0 \eta^0 = m_0 \frac{d(ct)}{d\tau} = m_0 c \frac{dt}{d\tau} = \frac{m_0 c}{\sqrt{1-u^2/c^2}}$$

$$p^0 = \frac{m_0 c}{\sqrt{1-u^2/c^2}} \quad (u \to \text{same})$$

m_0 and $\vec{u}$ not changing with time

p^0 is also a conserved quantity as m_0 and u are not changing with time

$$P_0 = \frac{m_0 c}{\sqrt{1-u^2/c^2}}$$

$\dfrac{P_0}{c} = m$ is called relativistic mass

$$m = \frac{m_0}{\sqrt{1-u^2/c^2}}$$

$m = m_0$ (If $u=0$). ($m_0 \to$ rest mass)

Thus m_0 is called rest mass.

→ $P_0 c$ is also a conserved quantity.

→ $P_0 c = \dfrac{m_0 c^2}{\sqrt{1-u^2/c^2}}$ ↓ Take $\frac{u}{c} \ll 1$ $\quad p = E/c$

$$P_0 c = m_0 c^2 (1 - u^2/c^2)^{-1/2}$$

$$= m_0 c^2 \left(1 - \frac{u^2}{2c^2} + \dots\right)$$

$$P_0 c = \underbrace{m_0 c^2}_{\text{Constant}} + \underbrace{\tfrac{1}{2} m_0 u^2}_{K.E} + \dots \quad (\text{constant} + K.E = K.E)$$

⟹ $P_0 c$ is energy.

Thus we call it as relativistic energy.

→ 12/3/2019

$$p^\mu = (p^0, p^1, p^2, p^3) = (p^0, \bar{p})$$

↳ 4-momentum $\qquad$ $\bar{p}$ ↳ relativistic momentum

$\bar{p} = \dfrac{m_0 \vec{u}}{\sqrt{1-u^2/c^2}}$ → ordinary momentum

$p^0 = \dfrac{m_0 c}{\sqrt{1-u^2/c^2}}$ (Conserved quantity for a free particle)

$$<p^0 = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} \xrightarrow[\text{limit } u/c<<1]{\text{non relativistic}} = m_0c^2 + \frac{1}{2}m_0u^2 + O(u^4)$$

$<p^0 = m_0c^2 + K.E$ → not P.E (because, no force acting on free particle so $-\frac{dU}{dr} = 0$, V is a constant.

It is constant. but not P.E

→ Thus $P_0 c = \frac{m_0c^2}{\sqrt{1-u^2/c^2}}$ is defined as the relativistic energy (E)

$$\Rightarrow E = P^0 c = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}}$$

$m = \frac{m_0}{\sqrt{1-u^2/c^2}}$ → relativistic mass (P_0/c).

m_0 → rest mass.

$$\boxed{E = mc^2}$$

→ Relativistic energy for a free particle.

When $u=0$, $E = m_0c^2$. (when particle doesn't move, it has relativistic energy which is called rest energy).

→ $p^\mu p_\mu$ is (Lorentz) invariant quantity.

⟹ $p^\mu p_\mu = p'^\mu p'_\mu$ → scalar product of p^μ and p_μ.

→ $p^\mu p_\mu = (p^0, p^1, p^2, p^3) \cdot (-p^0, p^1, p^2, p^3)$

$= -p^{0^2} + p^{1^2} + p^{2^2} + p^{3^2}$ ← for free particle

$p^0 = E/c$

$= -\left(\frac{E}{c}\right)^2 + |\vec{p}|^2$

$= -\left(\frac{E}{c}\right)^2 + p^2$

$$E = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}}$$

$$\left(\frac{E}{c}\right)^2 = \left(\frac{m_0 c}{\sqrt{1-u^2/c^2}}\right)^2$$

$$\left(\frac{E}{c}\right)^2 = \frac{m_0^2 c^2}{1-u^2/c^2} = \frac{m_0^2(u^2 + (c^2-u^2))}{c^2(1-u^2/c^2)}$$

$$p = \frac{m_0 u}{\sqrt{1-u^2/c^2}}$$

$$p^2 = \frac{m_0^2 u^2}{(1-u^2/c^2)}$$

$$\left(\frac{E}{c}\right)^2 = \frac{m_0^2 u^2}{(1-u^2/c^2)} + \frac{m_0(c^2-u^2)}{1-u^2/c^2}$$

$$= p^2 + m_0^2 c^2$$

$$\left(\frac{E}{c}\right)^2 = p^2 + m_0^2c^2$$

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$ → For a particle in a potential $V(\vec{r})$.

$(E - V(\vec{r}))^2 = p^2c^2 + m_0^2c^4$ ↓ If the potential is zero.

$\Rightarrow p^2 - \left(\frac{E}{c}\right)^2 = -m_0^2c^2$

$\Rightarrow p^\mu p_\mu = p^2 - \left(\frac{E}{c}\right)^2 = -m_0^2c^2$

↓ Repeated index ⟹ Summation over the index

$$\boxed{p^\mu p_\mu = -m_0^2 c^2}$$

Lorentz invariant.

→ Consider the following collision process.

m_0 m_0

$\vec{u}$ $\vec{u}$

Completely inelastic before collision.

No external force is acting on these two particle.

→ The two particles as a whole is a free particle.

after collision.

their rest mass M_0

Conserved quantities

p^μ ↓ $(p, \vec{p})$ (p^0, p^1, p^2, p^3)

for the joint system.

→ $p^\mu = p'^\mu = (p'^0, \vec{p}')$

Zeroth component of momentum is G

$\vec{P} = \vec{P}_1 + \vec{P}_2$

$= \frac{m_0 \vec{u}_1}{\sqrt{1-u^2/c^2}} + \frac{m_0 u}{\sqrt{1-u^2}} = 0$

Net momentum = 0.

(Joint system is not moving.) $\bar{p}'=0. = \dfrac{M_0 c}{\sqrt{1-u'^2/c^2}}$; $\xrightarrow{E'/c}$

$$\vec{p}' = 0 = \frac{M_0 \vec{u}'}{\sqrt{1-u'^2/c^2}}$$

Combined relativistic momentum after collision.

$$p^0 = p_1^0 + p_2^0$$

$$= \frac{m_0 c}{\sqrt{1-u^2/c^2}} + \frac{m_0 c}{\sqrt{1-u^2/c^2}}$$

this is not a vector

$\Rightarrow \bar{u}' = 0.$

$$= \frac{2m_0 c}{\sqrt{1-u^2/c^2}}$$

$$= p^{0\prime} = \frac{M_0 c}{\sqrt{1-u'^2/c^2}}$$

$\Rightarrow \cancel{2M_0}$

$$\frac{2m_0 c}{\sqrt{1-u^2/c^2}} = M_0 c$$

$$\frac{2m_0}{\sqrt{1-u^2/c^2}} = M_0$$

$$\frac{2m_0}{\sqrt{1-u^2/c^2}} = M_0$$

after collision

active energy converted into rest mass.

$\Rightarrow \boxed{M_0 \geq 2m_0.}$

So, Rest mass is increased $(M_0 - 2m_0)$

increased from the KE of the two particles before collision.

→ ~~Relativistic mass~~

Rest mass is not a conserved quantity in collision process

but relativistic mass is a conserved quantity.

Tutorial

1) $\dfrac{\Delta t'}{\Delta t} = \dfrac{1}{2} = \sqrt{1-v^2/c^2}$

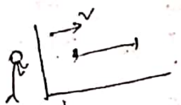
$$\frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\frac{v}{c} = \frac{\sqrt{3}}{2}$$

2). $L = 0.5L'$

$$\frac{1}{2} = \sqrt{1-v^2/c^2}$$



$$\Delta L' = \frac{1}{\sqrt{1-v^2/c^2}} \Delta L$$

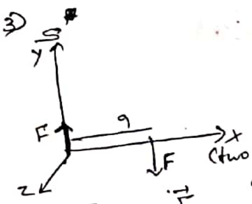
$$\frac{\Delta L}{\Delta L'} = \frac{1}{2} = \sqrt{1-v^2/c^2}$$

$$\Rightarrow 1 - v^2/c^2 = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\boxed{v = \sqrt{\tfrac{3}{2}}\, c}$$

3)

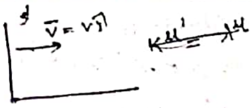


(two equal and opposite forces)

$$\bar{K} = \frac{\vec{F}}{\sqrt{1-u^2/c^2}}$$

→ when $\bar{u} = 0$

$$\boxed{\bar{K} = F}$$



$$K^0 = \frac{dE}{d(ct)} = 0$$

No change in Energy.

Nothing is moving.

$$K^{\mu\prime} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} K^{\nu} \qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$\beta = v/c$$

$$\begin{bmatrix} K^{0\prime} \\ K^{1\prime} \\ K^{2\prime} \\ K^{3\prime} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K^0 \\ K^1 \\ K^2 \\ K^3 \end{bmatrix}$$

$K^0 = 0.$

$K^{1\prime} = F$ (F is acting on x is zero) $= 0$

$K^{2\prime} = F = K^2$

$K^{3\prime} = 0$ (no force on z axis)

S' frame is then

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$ct' = \gamma ct - \gamma\beta x$$

$$x' = -\gamma\beta ct + \gamma x$$

$$y' = y$$

$$z' = z$$

when $t=0$.

$ct' = 0 - 0$ (For $\vec{F}$)

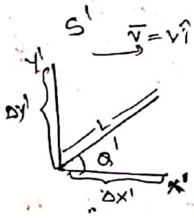
$x' = 0 + 0 = 0$.

$ct' = 0 - \gamma\beta a$
$x' = 0 + \gamma a$ } for $-\vec{F}$.

} The two forces equal and opposite ind, are not acting simultaneously. (Hence Newton's 3rd law is not valid.

2. B)

S' $\vec{v} = v\hat{i}$



$\tan\theta = \frac{\Delta y}{\Delta x}$ $\quad \tan\theta' = \frac{\Delta y'}{\Delta x'}$.

$$\Delta x' = \frac{1}{\sqrt{1 - v^2/c^2}} \Delta x,$$

$$\Delta y' = \Delta y'.$$

$$\tan\theta' = \frac{\Delta y'}{\Delta x\sqrt{1 - v^2/c^2}} = \frac{\Delta y}{\Delta x\sqrt{1 - v^2/c^2}}$$

$$\tan\theta' = \frac{\tan\theta}{\sqrt{1 - v^2/c^2}}$$

$$\theta = \tan^{-1}\left(\frac{\tan\theta}{\sqrt{1 - v^2/c^2}}\right)$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

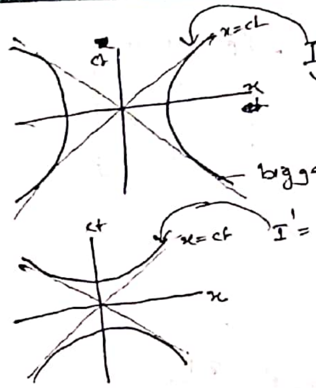
$$L = \sqrt{\Delta x^2 + \Delta y^2}$$

$$L = \sqrt{\left(\Delta x' \sqrt{1 - v^2/c^2}\right)^2 + (\Delta y')^2}$$

$$= \sqrt{L'^2\left(1 - v^2/c^2\right)\cos^2\theta + L'^2\sin^2\theta}$$

$$L = L'\sqrt{1 - \frac{v^2}{c^2}\cos^2\theta}$$

4. D.



$I = x^2 - c^2t^2 > 0$

→ Constant → Hyperbola

$I' = x^2 - c^2t^2 < 0$

19/3/2017

4. A

$$x^\mu \xrightarrow[\vec{v} = v\hat{i}]{} x'^\mu \xrightarrow[\vec{v} = v\hat{i}]{} x''^\mu$$

$$\begin{bmatrix} x^{0''} \\ x^{1''} \\ x^{2''} \\ x^{3''} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{bmatrix}$$

$$\begin{bmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \end{bmatrix}$$

$$\begin{bmatrix} x^{0''} \\ x^{1''} \\ x^{2''} \\ x^{3''} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \end{bmatrix}$$

$$= \begin{bmatrix} \gamma^2 + \gamma^2\beta^2 & -2\gamma^2\beta & 0 & 0 \\ -2\gamma^2\beta & \gamma^2 + \gamma^2\beta^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \end{bmatrix}$$

$$\begin{bmatrix} x^{0''} \\ x^{1''} \\ x^{2''} \\ x^{3''} \end{bmatrix} = \begin{bmatrix} \gamma' & -\gamma'\beta' & 0 & 0 \\ -\gamma'\beta' & \gamma' & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow \gamma' = \dfrac{1}{\sqrt{1 - v'^2/c^2}}$

$\rightarrow \beta' = v'/c$

$\rightarrow \gamma' = \gamma^2(1+\beta^2)$, $\gamma'\beta' = 2\gamma^2\beta$ $\Rightarrow \beta' = \dfrac{2\beta}{1+\beta^2}$

$\bar{v}' = \bar{v} + \bar{v} = \dfrac{\bar{v}+\bar{v}}{1 + \bar{v}\cdot\bar{v}/c^2}$ — Einstein

$\dfrac{v'}{c} = \dfrac{2v/c}{1 + v^2/c^2} \Rightarrow v' = \dfrac{2v}{1+v^2/c^2}$

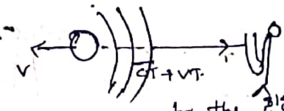
$x' \longrightarrow x'^{\mu} \longrightarrow x''^{\mu}$

$\bar{v}_1 = v\hat{i}$, $\bar{v}_2 = v_2\hat{i}$

$$\begin{bmatrix} x^{0''} \\ x^{1''} \\ x^{2''} \\ x^{3''} \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma & 0 & -\gamma\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\gamma\beta & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\Lambda}\begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

for boost along x-axis.

Q.5.)



T = time period for the signal, as noted by the observer.

$\lambda = cT + vT$
(length b/w two wavefronts)

$\nu\lambda = \Rightarrow \nu = \dfrac{c}{\lambda} = \dfrac{c}{(c+v)T}$

$\nu = \dfrac{1}{T}$ (when at rest)

$\rightarrow$ Time period of source's clock: $T' = \dfrac{1}{\nu'}$

$T' = \sqrt{1 - v^2/c^2}\, T$

$\nu = \dfrac{c\sqrt{1-v^2/c^2}}{(c+v)T'} = \dfrac{\nu' c\sqrt{1-v^2/c^2}}{c(c+v)}$

$= \dfrac{\nu'\sqrt{c^2-v^2}}{(c+v)} = \nu'\sqrt{\dfrac{c-v}{c+v}}$

$\nu \leq \nu'$ → freq. for yellow light.

$\rightarrow$ (Red shift). appear as red.

Q6). $\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$

$u = \dfrac{\Delta x}{\Delta t} = \left(\dfrac{\Delta x}{\Delta \tau}\right)\sqrt{1 - u^2/c^2}$

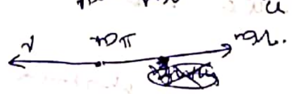
$\Delta\tau = \sqrt{1 - v^2/c^2}\,\Delta t$.

$\dfrac{u}{\sqrt{1-u^2/c^2}} = \dfrac{800}{2\times10^{-6}}$

ν, find u.

6.B

Assume $m_\nu \approx 0$.



(Conservation of energy) $m_\nu \approx 0$.

$m_\pi c^2 = P_\nu c + \left(\dfrac{m_\mu}{\sqrt{1-u^2/c^2}}\right)c^2$ —①

of neutrino

relativistic mass

$\rightarrow E^2 = p^2c^2 + m^2c^4$

$E = pc$ if $m = 0$.

for photon also $m = 0$.

momentum: $0 = -P_\nu + \dfrac{m_\mu u}{\sqrt{1-u^2/c^2}}$ —②

relativistic momentum

$$\underbrace{m_\pi c^2 \quad , \quad 0}$$

From ① and ②.

$$u = \left(\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}\right)c$$

For $P_\mu = \dfrac{m_\mu u}{\sqrt{1-u^2/c^2}}$

$$P_\mu = \frac{m_\mu}{\sqrt{1-u^2/c^2}}\left(\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}\right)c$$

$$P_\mu = \frac{c\,(m_\pi^2 - m_\mu^2)}{2m_\pi}.$$

Q

$\bar{F} = \dfrac{d\bar{P}}{dt}$

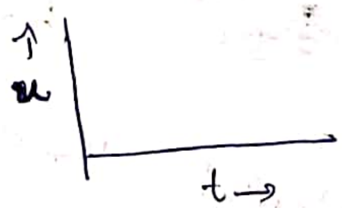
Constant

$$\Rightarrow \bar{P} = \int_0^t \bar{F}\,dt$$

$$\bar{P} = \bar{F}t$$

$$\rightarrow \quad P_0 = \frac{m_0 \bar{u}}{\sqrt{1-u^2/c^2}} = Ft,$$

$$\Rightarrow \left(\frac{mu}{Ft}\right)^2 = 1 - u^2/c^2 \quad — ①$$



when $t \to \infty$,
L.H.S of ① → 0
⇒ $u = c$

→