Modern Physics

- Valence Electrons
- Crystal field splitting : $d \begin{cases} t_{2g} \\ e_g \end{cases}$
- Stern and Gerlac expt. → proved the e^- spin existence Ag source.
- Non-uniform magnetic field.
- Photoelectric Effect.
- Radiation from an object.
- Spectral lines in atoms

Selection rules –

09/01/19

Max Planck's law: — Rayleigh-Jeans law
— Wein's Displacement law.

Power emitted per unit area per unit frequency

Black Body Radiation
↓
Emits EM Radiations

$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \times \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$

$\nu = \frac{c}{\lambda}$

$k_B \rightarrow$ Boltzman constant

$h \rightarrow$ Planck's constant

$c \rightarrow$ speed of light.

$\frac{\nu^3}{c^2} = \frac{c^3}{\lambda^3 . c^2}$

$= \frac{c}{\lambda^3}$

$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$

Lower frequency (larger wavelength) → Rayleigh Jeans law

Higher frequency (smaller wavelength) → Wein's Displacement law.

Rayleigh & Jeans law

$B_\lambda(T) = \frac{2 c k_B T}{\lambda^4}$

$c \rightarrow$ speed of light

$k_B \rightarrow$ Boltzman's Constant

B_λ = Spectral Radiance

$T \rightarrow$ Temperature

$B_\nu(T) = \frac{2\nu^2 k_B T}{c^2}$ → larger wavelength (lower frequency).

Ultraviolet catastrophe → Rayleigh Jeans catastrophe classical physics.

* Black body emit ~~the~~ radiation in all frequencies range.

$$\lambda = \frac{c}{\nu}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{\frac{hc}{\lambda k_B T}} \simeq 1 + \frac{hc}{\lambda k_B T}$$

$$\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} = \frac{1}{\frac{hc}{\lambda k_B T}} = \frac{\lambda k_B T}{hc}$$

Black Body radiatn is a quantum physics phenomenon.
- classical phy is not adequate
- Max planck's law is universally accepted.

$$B_\lambda(T) = \frac{2ck_B T}{\lambda^4}$$ which is identical to the classical physics rayleigh Jeans law.

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{k_B T}{h\nu} = \frac{2\nu^2 k_B T}{c^2}$$

Wien's Displacement law

$\frac{hc}{k_B T \lambda_{max}} = 4.965 \rightarrow$ $$\lambda_{max} = \frac{b}{T}$$ b = Proportionality constant

$b = 2.898 \times 10^{-3}$ m.k

λ is inversely proportional to temp (T).

ν is directly proportional to temp (T).

There is a change in λ when we change the temp (T). Displacement is $\Delta\lambda$.

10/01/19

Radiation Thermometres

1. Temperature sensor
2. Converting the same in numbers.

When we heat an iron bar, there are 3 range in colour:

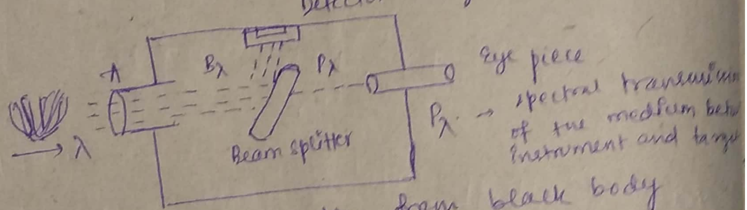
(i) Red-hot

(ii) Red-Orange

(iii) White.

Radiation thermometre
⇓
measures the thermal energy emitted by a source
⇓
using Planck's law

Temperature Measurement using Radiation

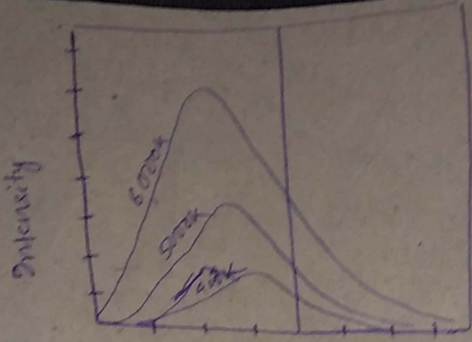


The intensity of radiation from black body is a function of

$$I_b = f(r, \theta, \phi)$$

from Planck's Law,

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$



The area under the curve grows rapidly with temperature.

Basic ideas for Radiation Thermometre

- 10.6 μm at 0°c and 1.5 μm at 200°c
- Radiation predominantly lies in visible region, sometimes near and middle infrared regions
- collects radiation from target and produces output signal (electrical)

$V \Rightarrow$ Potential
$E \Rightarrow$ Spectral Radiance

$$R_\lambda = \frac{dV}{dE_\lambda}$$

$dV \Rightarrow$ output in response to the radiant flux dE_λ.

$$dV = R_\lambda \, dE_\lambda$$

$$dV = R_\lambda I_\lambda A B_\lambda P_\lambda \, d\lambda$$

$$\boxed{V = \int_0^\infty R_\lambda I_\lambda A B_\lambda P_\lambda \, d\lambda}$$ → Radiometer measurement equation.

$$I_{b,\lambda}(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

Differentiate this w.r.t Temp T;

$$\cancel{\frac{dI}{dT} = \frac{2hc^2}{\lambda^5}}$$

$$dI_{b,\lambda}(\lambda, T) = \frac{2hc^2}{\lambda^6 T^2} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} dT$$

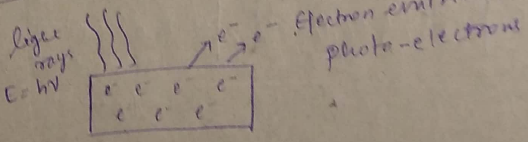
$$\frac{dI_{b,\lambda}(\lambda, T)}{I_{b,\lambda}(\lambda, T)} = \frac{hc}{k_B} \frac{dT}{\lambda T^2}$$

$$\frac{dT}{T} = \frac{dI_{b,\lambda}(\lambda, T)}{I_{b,\lambda}(\lambda, T)} \times \frac{\lambda T}{\frac{hc}{k_B}}$$

1. Wavelength
2. shortest possible wavelength
3. wavelength vs Temp.

<u>16/01/19</u>

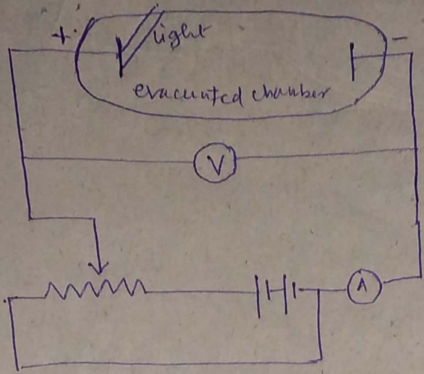
<u>PHOTOELECTRIC EFFECT</u>



- The inter-band transition exists in <u>semi-conductor</u> and <u>insulators</u>. <u>optical absorption</u>

In metals, there is no band-gap; optical conductivity due to intra-band transition.

Experimental Setup



Work function $h\nu_0$

NOTE: (I) constant P.D is not maintained between cathode and anode.

(II) Not all the e^- ejected will move to cathode as cathode is -vely charged.

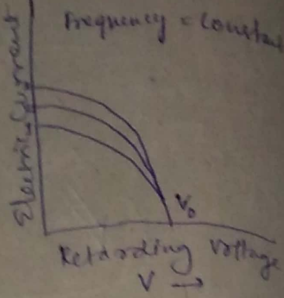
(III) When the intensity of light is increased, the probability of e^- which reaches cathode increases.

(IV) A min threshold energy called work function ($\phi = h\nu_0$) is required for an e^- to get ejected.

(V) Stopping Potential is the potential difference needed to prevent the most energetic e^- from reaching the other electrode.

(VI) Electric dipole approximation

Kinetic Energy ∝ Intensity of incident light



17/01/19

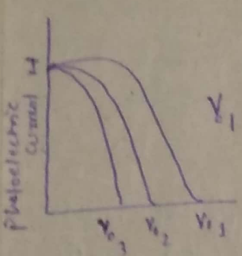
* The energy from photons are transferred to the e^- and they attain the K.E to come out of the metal surface.

Note:

(i) Large K.E then they can travel & reach the cathode

(ii) No time interval between light falling and photoelectron coming out of the surface -10^{-6} W/m²

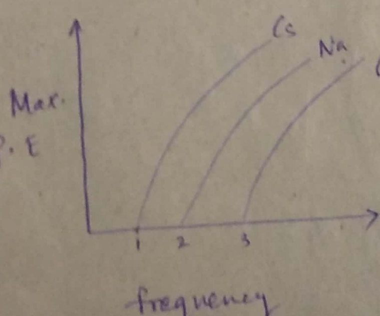
(iii) A layer of sodium 1 atom thick. Each atom releases an energy 10^{-25} W.



$\nu_1 > \nu_2 > \nu_3$

Light intensity constant

Higher the frequency, more energy of the electrons.



Characteristic frequency of a metal.

Quantum Theory of Light:

Light is not spread as wave front, small packet called photons.

- Total Energy is concentrated on photons.
- All photons have same freq. 'ν'.
  Intensity change will change the no. of e^- ejected but energy remains unchanged.
- The higher the freq ν, the greater the photon energy $h\nu$.
- Work function, $\phi = h\nu_0$
- Total Energy = $K.E_{max} + \phi$

$h\nu$ = Photo electric Energy.

$K.E_{max}$ = Maximum photoelectron energy (which is proportional to stopping potential).

ϕ = Max energy required for an e^- to leave the metal.

$$h\nu = K.E_{max} + h\nu_0$$

$$\boxed{K.E_{max} = h(\nu - \nu_0)}$$

1. What are the energy and momentum of a photon of a red light of wavelength 650 nm? What is the wavelength of a photon of energy 2.40 eV?

Ans : $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{650 \times 10^{-9}}$

$$\boxed{E = 0.031 \times 10^{-17} J.}$$

$1 J = 6.24 \times 10^{18} eV$

$0.031 \times 10^{-17} J = 0.193 \times 10 \; eV$

$= 1.93 \; eV$.

Momentum $p = \frac{E}{c} = \frac{0.031 \times 10^{-17}}{3 \times 10^{8}}$

$= 0.01 \times 10^{-9}$

$= 0.01 \; nm$

Now,

$\frac{2.4}{6.24 \times 10^{18}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\lambda = \frac{6.63 \times 3 \times 10^{-34} \times 6.24 \times 10^{18} \times 10^{8}}{2.4}$

$= 517 \; nm$.

2. The work function of tungsten metal is 4.52 eV

(a) what is the cut-off wavelength.

(b) what is the max kinetic energy of the e^- when radiation wavelength 198 nm is used?

(c) what is the stopping potential in this case?

Ans - (a) $\frac{4.52}{6.24 \times 10^{18}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\therefore \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8} \times 6.24 \times 10^{18}}{4.52}$

$\lambda = 4.4 \times 10^{-26}$ nm/eV

$= 27.45 \times 10^{-8}$ nm

$= 275$ nm.

(b) $E - \frac{4.52}{6.24 \times 10^{18}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{198 \times 10^{-9}}$

$E = (0.1 \times 10^{-17}) + (0.72 \times 10^{-18})$

$= 0.73 \times 10^{-18}$ J

$\therefore E = 4.55$ eV

(c) $V = -4.55$ eV.

8. The wavelength of the photo-electric threshold for silver is 3.250×10^{-10} m. Determine the velocity of the e^- ejected from silver surface by UV light of wavelength 2.536×10^{-10} m.

Ans - $T = h\nu - h\nu_0$

$\lambda = 2.536 \times 10^{-10}$ m

$\lambda_0 = 3.250 \times 10^{-10}$ m

$$T = \frac{6.63 \times 10^{-34}}{3 \times 10^{8}} \times \left(\frac{1}{2.536} - \frac{1}{3.250} \right) \times 10^{10}$$

$$= 6.63 \times 3 \times 9.23 \times 10^{-20}$$

$$= 183.58 \times 10^{-20}$$

$$183.58 \times 10^{-20} = \frac{1}{2} \times 9.1 \times 10^{-27} \times v^2$$

$$\therefore \quad v = \sqrt{40.34 \times 10^{7}}$$

$$\boxed{v = 20.08 \times 10^{3} \text{ m s}^{-1}}$$

24/01/19

1. An ultraviolet light of wavelength 350nm and intensity $\boxed{1.00 \text{ W/m}^2}$ is directed at a potassium surface.

E $\quad$ $= \quad 1.00 \times 10^{-4} \text{ m}^2$

(a) Find the max kinetic energy of the photoelectrons.

(b) If 0.5% of incident photons produce photo-electrons; how many are emitted per second if the potassium surface has an area $1.00\ cm^2$? $W_0 = 2.2\ eV$

Ans (a) $\frac{hc}{\lambda} = E$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{350 \times 10^{-9}}$$

$$= \frac{6.63 \times 3}{350} \times 10^{-34+8+9}$$

$$= 0.056 \times 10^{-17}\ J$$

$$0.056 \times 10^{-17}\ J = 0.056 \times 6.24 \times 10$$

$$= 0.349 \times 10$$

$$= 3.49\ eV$$

$$E = 3.5\ eV$$

$$K.E_{max} = E - h\nu_0$$ hp:

$$= 3.5 - 2.2$$

$$\boxed{K.E_{max} = 1.3\ eV}$$

(b) Total energy of Photoelectrons = $0.056 \times 10^{-17}\ J$.

No. of e^- incident at area $1 cm^2$ ($10^{-4} m^2$) $= \frac{0.056 \times 10^{-17}}{10^{-4}}$

$$= 0.056 \times 10^{-13}\ e^-$$

0.5% of incident photons $= 0.056 \times 10^{-13} \times 0.5 \times 10^{-2}$

$$= 0.056 \times 0.5 \times 10^{-15}$$

$$=$$

Electrons emitted

$\lambda = \frac{h}{p}$, $\lambda = \frac{h}{mv}$

$p^2 = 2mK$

$\frac{p^2}{2m} = K$

$p = mv$

$E = \frac{1}{2}mv^2$

$p^2 = 2mE$

2. Show that photo electric effect cannot take peace with a free e^-?

$E = h\nu$ $\quad$ $E^2 = h^2\nu^2$

$p = h\nu/c$

$E = p^2c^2$

$= \frac{h^2\nu^2}{c^2}c^2$

$= h^2\nu^2$

$\boxed{E = cp.}$

$T = h\nu$

$p = \frac{h\nu}{c}$

$T^2 = c^2p^2$

$= 2mTc^2$

$p = \sqrt{2mKE}$

$2h\nu \cdot mc^2 = 0$

$E = c\sqrt{2mKE}$

$E^2 = c^2 2mKE.$

$KE^2 = c^2 2m\cancel{KE}$

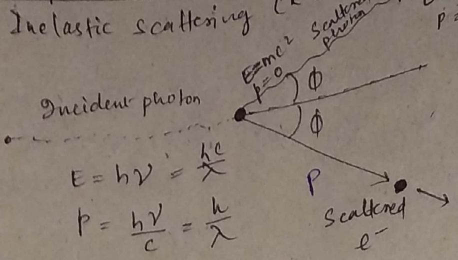
$KE = c^2 2m.$

$\boxed{K.E = 2mc^2}$

6-02-19

Compton Effect (Light-matter interaction)

Inelastic scattering (K.E is not conserved)



$E = h\nu = \frac{hc}{\lambda}$

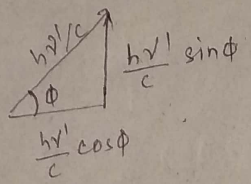
$p = \frac{h\nu}{c} = \frac{h}{\lambda}$

Arthur Holly Compton

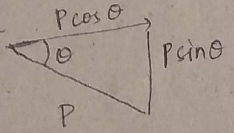
→ when the photon hits the e^-, the K.E of photon decrease and the wavelength of photon increases

Scattering of e^-

Energy - Momentum conservation



Vector diagram of the momenta and their components of incident scattered photon.



ν = Initial photon

ν' = Lower than the initial.

Loss in photon energy = Gain in electron energy

$h\nu - h\nu' = KE$ —— (1)

$E = pc \Rightarrow p = E/c = \frac{h\nu}{c}$ (Initial photon momentum)

0 and p (initial, final)

Scattered Photon Momentum $= \frac{h\nu'}{c}$

Initial Momentum = Final Momentum.

6-02-19

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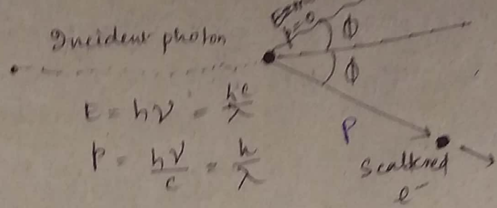

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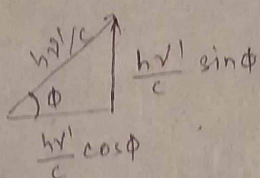

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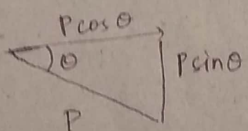

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p and p
initial final

Scattered Photon Momentum $= \frac{h\nu'}{c}$

Initial Momentum = Final Momentum.

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{--- (2)}$$

1^r direction: $0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$ --- (3)

ϕ: Angle between the direction of initial and scattered photon

θ: Angle between the direction of initial photon and scattered e^-.

Multiplying (2) and (3) by c.

(2) $\times c$: $h\nu = h\nu'\cos\phi + pc\cos\theta$

$\Rightarrow pc\cos\theta = h\nu - h\nu'\cos\phi$

(3) $\times c$: $0 = h\nu'\sin\phi - pc\sin\theta$

$\Rightarrow pc\sin\theta = h\nu'\sin\phi$

Squaring and adding,

$$p^2c^2 = h^2\nu^2 + (h\nu')^2\cos^2\phi - 2h^2\nu\nu'\cos\phi + (h\nu')^2\sin^2\phi.$$

$$p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi \quad \text{--- (4)}$$

$$E = K.E + Mc^2 \quad \text{--- (5)}$$

$$E = \sqrt{M^2c^4 + p^2c^2} \quad \text{--- (6)}.$$

We know,

$$(K.E + Mc^2)^2 = M^2c^4 + p^2c^2$$

$$p^2c^2 = K.E + 2mc^2 K.E$$

$K.E = h\nu - h\nu'$

$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu')$

Sub p^2c^2 in (4),

$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1-\cos\phi)$ — (7)

Dividing by $2h^2c^2$,

$\frac{mc}{h}(\nu/c - \nu'/c) = \frac{\nu}{c} \cdot \frac{\nu'}{c}(1-\cos\phi)$

$\nu/c = \frac{1}{\lambda}, \quad \nu'/c = \frac{1}{\lambda'}$

$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1-\cos\phi}{\lambda\lambda'}$

$\boxed{(\lambda - \lambda') = \frac{h}{mc}(1-\cos\phi)} \Rightarrow$ Compton Effect.

$\lambda \to$ Initial Wavelength

$\lambda' \to$ Scattering wavelength

$h \to$ Planck's constant

$M \to$ Mass of e^-

$c \to$ Speed of light

$\phi \to$ scattering Angle.

07/02/19

Compton scattering

$$E = \sqrt{m^2c^4 + p^2c^2}$$

Mass and Energy

'W' on an object by constant force F'.

$$W = \vec{F} \cdot s$$

$$K.E = \vec{F} \cdot s$$

F is not constant:

$$K.E = \int_0^s F \cdot ds$$

$$K.E = \frac{1}{2} m v^2$$

$p = \gamma m v$ (Relativistic Momentum)

Here $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$F = \frac{dP}{dt} = \frac{d}{dt}(\gamma m v)$$

$$K.E = \int \frac{d}{dt}(\gamma m v)\, ds = \int_0^v v\, d(\gamma v m)$$

$$= \int_0^v v\, d\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right)$$

Integration by parts:

$$K.E = \frac{mv^2}{\sqrt{1 - v^2/c^2}} - m \int_0^v \frac{v\,dv}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{mv^2}{\sqrt{1 - v^2/c^2}} + \left[mc^2 \sqrt{1 - v^2/c^2} \right]_0^v$$

$$= \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

$$\Rightarrow k.E = \gamma mc^2 - mc^2$$

$$\boxed{k.E = (\gamma - 1) mc^2}$$

Total Energy, $E = \underset{E_0}{\underset{\parallel}{mc^2}} + k.E$

$$E = E_0 + k.E$$

mc^2 = rest energy of e^-

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

Energy and momentum : $E = PC$

Total Energy, E_0 and momentum of the particle

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad E^2 = \frac{m^2c^4}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$P = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad p^2c^2 = \frac{m^2v^2c^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$E^2 = p^2c^2 = \frac{m^2c^2(c^2 - v^2)}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$E^2 = \frac{m^2c^2(c^2 - v^2)}{1 - \frac{v^2}{c^2}} = \frac{m^2c^2\cancel{(c^2 - v^2)} \times c^2}{\cancel{(c^2 - v^2)}}$$

$E^2 = m^2c^4 = (mc^2)^2$

$T.E = $ (speed + mass energy)

$E^2 - p^2c^2 = m^2c^4$

$$\boxed{E^2 = m^2c^4 + p^2c^2}$$

mass-energy equivalence

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

Initial momentum = final momentum

for perpendicular direction.

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta.$$

ϕ = angle between the incident and scattered photon.

θ = Between incident ~~photon~~ ~~electron~~ photon and recoil e^- electron.

$$pc\cos\theta = h\nu - h\nu'\cos\phi$$

$$pc\sin\theta = h\nu'\sin\phi$$

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \quad (1)$$

$$E = K.E + mc^2$$

$$E = \sqrt{m^2c^4 + p^2c^2} \qquad K.E = h\nu - h\nu'$$

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu') \quad (2)$$

Substituting (2) in (1),

$$\cancel{(h\nu)^2} - 2(h\nu)(h\nu') + \cancel{(h\nu')^2} + 2mc^2(h\nu - h\nu') = \cancel{(h\nu)^2} - 2(h\nu)(h\nu')\cos\phi + \cancel{(h\nu')^2}$$

$$\cancel{2}mc^2 (h\nu - h\nu') = \cancel{2}(h\nu)(h\nu')(1-\cos\phi)$$

Put $\frac{\nu}{c} = \frac{1}{\lambda}$, $\frac{\nu'}{c} = \frac{1}{\lambda'}$

$$\frac{mc^2}{c}\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right) = \left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)(1-\cos\phi)$$

$$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda\lambda'}(1-\cos\phi).$$

$$\frac{mc}{h}\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{1}{\lambda\lambda'}(1-\cos\phi)$$

$$\boxed{(\lambda'-\lambda) = \frac{h}{mc}(1-\cos\phi)}$$

* change in wavelength for a scattered photon
* Change is independent of the wavelength of the incident photon

$\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$. depends only on ϕ i.e angle of scattering.

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\phi)$$

$$\boxed{\lambda_e = \frac{h}{m_e c}}$$

Compton Effect - Definition

The scattering of EM radiation such as x-rays, γ-rays of wavelength λ, from certain low atomic no. material like paraffin or graphite into radiation of higher wavelength λ' is called as Compton Effect.

- The difference $\lambda' - \lambda$ is independent of incident wavelength as well as nature of scatterer.
- It depends only on angle of scattering.

Energy of Recoil-electrons

The energy transferred to the e^- by the incident photon is given as -

$$E = h\nu - h\nu' = h\nu\left(1 - \frac{\nu'}{\nu}\right)$$

$$\frac{E}{h\nu} = \left(1 - \frac{\nu'}{\nu}\right) = \frac{\lambda' - \lambda}{\lambda'}$$

we have

$$\frac{E}{h\nu} = \frac{\lambda_c (1 - \cos\phi)}{\lambda + \lambda_c (1 - \cos\phi)}$$

$$\therefore \boxed{E = \frac{h\nu}{1 + \dfrac{m_0 c^2}{h\nu (1 - \cos\phi)}}}$$

14/02/19

Problems :

1. $\lambda = 400nm$

$\theta = 180^\circ$

$E' = ?$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

$$\Delta\lambda = \lambda(1-\cos\theta)$$

$$= 2\lambda e$$

$$\Delta\lambda = \frac{6.624\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos180^\circ)$$

$$= 0.$$

$$\Delta\lambda = 4.86\ pm = 0.00486\ nm$$

$$\lambda' = \lambda - \Delta\lambda$$

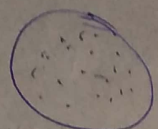
$$\lambda' = (400 - 0.00486)nm$$

$$\lambda' =$$

ATOMS

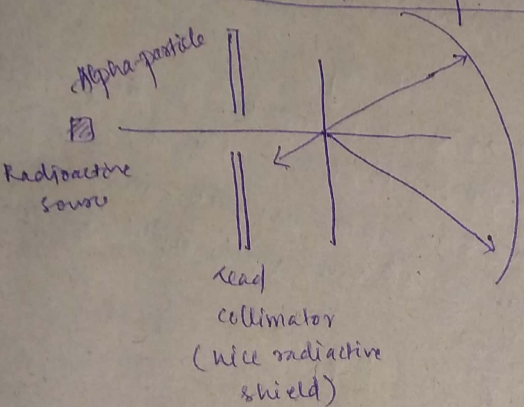
- electrons
- protons and neutrons → nucleus

Thomson's Plum Pudding Model



J. J. Thomson →

Rutherford's Gold foil Experiment



* All +ve charges are conc. at the centre of the atom, and e^- are found to revolve around the nucleus.

27.02.2019

$$N(\theta) = \frac{N_i\, n\, t\, Z^2 e^4}{(8\pi\epsilon_0)^2\, r^2\, KE^2 \sin^2(\theta/2)}$$

$N(\theta)$ → Number of α-particles per unit area that reach the screen at a scattering angle θ.

$N_i \rightarrow$ Total number of α-particles that reaches the screen.

$n \rightarrow$ No. of atoms per unit volume in the foil.

$Z \rightarrow$ atomic number.

$r \rightarrow$ distance of the screen from the foil.

K.E $\rightarrow$ K.E of the α - particle

$t \rightarrow$ foil thickness.

→ The nucleus also spins, but due to its ~~huge~~ high magnitude of mass, the spin is almost negligible. It has a non-zero spin-magnetic moment. But this magnetic moment scales with the mass

$$F_c = \frac{mv^2}{r}$$

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

The condition for dynamically stable orbit is

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}}$$

$$E = K.E + P.E$$

$$= \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

(... sign in P.E as P.E = 0 at $r = \infty$, i.e when e^- and proton are infinitely far apart)

Substitute r in this eqn,

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\boxed{E = \frac{-e^2}{8\pi\epsilon_0 r}}$$

Atomic spectra — Absorption - Dark lines

— Emission - Bright lines

* Missing of spectral lines is absorption.

Spectral lines

Balmer (Visible region of H-atom spectrum)

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5 \ldots$$

R - Rydberg constant

$= 1.097 \times 10^7 \, m^{-1}$

$= 0.01097 \, nm^{-1}$

05/03/19

Bohr's Model

1. To define the nature of orbit.

2. $E = h\nu$

Bohr's Postulates : Hydrogen atom ($Z = 1$).

(i) Electron orbit - Priviledged orbits

(ii) Origin of spectral lines

Nature of Priviledged Quantum Orbits

$x \rightarrow$ displacement

't' $\rightarrow$ time

$$x = A\sin 2\pi\nu t \longrightarrow (1)$$
$$= A\sin 2\pi\nu t$$

A = Amplitude of ν - frequency

$$K.E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$$

$m \rightarrow$ mass

$\frac{dx}{dt} \rightarrow$ Linear velocity

$$\frac{dx}{dt} = 2\pi\nu A\cos 2\pi\nu t$$

$$\left(\frac{dx}{dt}\right)_{max} = 2\pi\nu A$$

(Since the max value of $\cos 2\pi\nu t = 1$)

$$\text{The total Energy} = \frac{1}{2}m\left[\left(\frac{dx}{dt}\right)_{max}\right]^2$$

$$= \frac{1}{2}m(2\pi\nu A)^2$$

$$E = 2m\pi^2\nu^2A^2$$

Now,

$$nh\nu = 2m\pi^2\nu^2A^2$$

$$nh = 2m\pi^2\nu A^2 \longrightarrow (2)$$

p at time 't',

$$p_x = m\left(\frac{dx}{dt}\right) = m(2\pi\gamma A\cos 2\pi\gamma t)$$

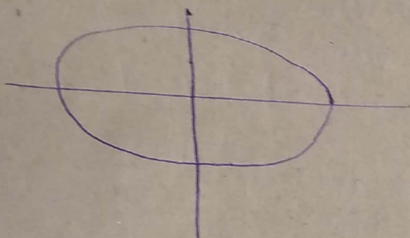
Put $m \cdot 2\pi\gamma A = B$, $p_x = B\cos 2\pi\gamma t$.

$$\frac{p_x}{B} = \cos 2\pi\gamma t.$$

From (1), $\frac{x}{A} = \sin 2\pi\gamma t$.

Squaring and adding above eq^n's.

$$\frac{x^2}{A^2} + \frac{p_x^2}{B^2} = 1.$$



$dx \rightarrow$ width of an element at distance x from origin

Let $p_x \rightarrow$ ordinate corresponding to 'x'.

$p_x\, dx \rightarrow$ phase integral.

$$\oint p_x \cdot dx = \text{area of the ellipse}$$

$$= \pi \times A \times B$$

$$= 2\pi^2 A^2 \gamma m$$

$$= nh$$

$h \rightarrow$ Planck's constant

$$p_x \rightarrow p_\phi \; ; \; dx \rightarrow d\phi$$

$$\oint p_\phi \cdot d\phi = nh$$

$$p_\phi = I\omega$$

$I \rightarrow$ Moment of Inertia of an e^-

$\omega \rightarrow$ angular velocity.

Limitations of Bohr's Model

① Higher 'Z' → Inadequate.
② Splitting of spectral lines (in presence of electric & magnetic fields)
③ No. of spectral lines
④ Intensity of spectral lines.

Bohr's Radius

M, E → Let be the mass and charge of the nucleus.

m, e, v → mass, charge, velocity of e^-

$E = Ze$; Z → atomic structure of element

$E = e$ $\quad$ $Z = 1$.

Ee/a^2 → force of attraction between nucleus & e^-.

$\dfrac{mv^2}{a}$ → centrifugal force

Now, $\dfrac{Ee}{a^2} = \dfrac{mv^2}{a}$

$$v = \frac{Ee}{am} \quad \text{—— (1)}$$

$$p_\phi = I\omega = n\left(\frac{h}{2\pi}\right)$$

$$I\omega = ma^2\omega = ma^2\left(\frac{v}{a}\right) = \frac{nh}{2\pi} = mav$$

$$v = \frac{nh}{2\pi ma} \longrightarrow (2)$$

Dividing (1) by (2),

$$v = \frac{Ee}{am} = \frac{2\pi me}{nh}$$

$$= \frac{2\pi Ee}{nh} \longrightarrow (3)$$

$$a = \frac{nh}{2\pi mv}$$ substitute value of v in (2) from (3).

$$a = \frac{nh}{2\pi m} \times \frac{nh}{2\pi Ee} = \frac{n^2h^2}{4\pi^2 Ee\, m}$$

$$a \propto n^2$$

$n = 1 \Rightarrow 0.53 \times 10^{-8}$ cm $\rightarrow 0.53 \times 10^{-10}$ m

= 53 pm Bohr's radius

13.03.19

Bohr's Radius

$$a = \frac{n^2 h^2}{4\pi^2 E_0 m}$$

$a \propto n^2$. $n = 1, 2, 3 \ldots$

0.53×10^{-8} cm (Bohr's radius)

Orbital frequency,

$$f = \frac{4\pi^2 E_0^2 e^2 m}{n^3 h^3}$$

Orbital Energy;

$W = K.E + P.E$

$$K.E = \frac{1}{2} m v^2 = \frac{Ee}{2a}$$

$$P.E = -\frac{Ee}{a}$$

$$W = \frac{Ee}{2a} - \frac{Ee}{a}$$

$$= -\frac{Ee}{2a}$$

$$W = W_n = -\frac{Ee}{2} \frac{4\pi^2 E e m}{n^2 h^2}$$

$$= -\frac{2\pi^2 m E^2 e^2}{n^2 h^2}$$

$W_n \rightarrow$ energy of e^- in the nth orbit.

Atomic Spectra < origin of spectral lines / How/why they are spaced?

Emission — Absorption.

n_i — Excited state

ΔE

n_f — lower state

Initial - Final = Photon Energy

$$E_i - E_f = h\nu$$

14.03.19

Q - Find the longest wavelength present in the Balmer series of hydrogen corresponding to the H_α line.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Quantum number of final state $n = 2$

$$= R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= R\left(\frac{9-4}{36}\right)$$

$$\frac{1}{\lambda} = \frac{5}{36}R$$

The longest wavelength in the series correspond to smallest energy diff the energy levels. Here $n_i = 3$.

$$\lambda = \frac{36}{5R}$$

$$= \frac{36}{5 \times 109677} \text{ cm}$$

$$= 6.56 \times 10^{-5} \text{ cm} = 6.56 \times 10^{-7} \text{ m}$$

656 nm

Bohr's Correspondance Principle

The greater the quantum number, the closer quantum physics approaches classical physics.

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}} \quad , \; r \rightarrow \text{radius of the orbit}$$

$$f = \frac{\text{electron speed}}{\text{orbital circumference}}$$

$$= \frac{v}{2\pi r} = \frac{e}{2\pi\sqrt{4\pi\epsilon_0 m r^3}}$$

Also, $r_n = \dfrac{n^2 h^2 \epsilon_0}{\pi m_e^2}$ $\quad \therefore f = \dfrac{me^4}{8\epsilon_0^2 h^3}\left(\dfrac{2}{n^3}\right)$

$$= -\frac{E_1}{h}\left(\frac{2}{n^3}\right)$$

$n_i \rightarrow n_j$

$$\nu = \frac{-E_1}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$n \rightarrow$ Initial Q.N n_i

$$\nu = -\frac{E_1}{h}\left(\frac{1}{(n-p)^2} - \frac{1}{n^2}\right)$$

$n-p$ $(p = 1, 2, \cdots)$ for final Q.N n_f

$$= \frac{-E_1}{h}\left[\frac{2np - p^2}{n^2(n-p)^2}\right]$$

$2np - p^2 = 2np$ $\quad$ [when $n >> p$, $n_i, n_f >>$]

$(n-p)^2 = n^2$

$$\boxed{\nu = -\frac{E_1}{h}\left[\frac{2p}{n^3}\right]}$$

when $n = 2$, Error $= 300\%$

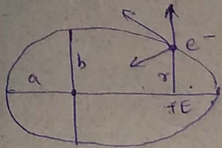
$n = 10000$, Error $= 0.01\%$

20·03·19

Sommerfield model

Problem with Bohr's model – couldn't explain fine structure.

Elliptical orbits



electron moving in elliptical orbits.

r = radius vector, the distance of the e^- from the nucleus $+E$

$\phi \rightarrow$ The angle which the ~~nucleus~~ radius vector makes with the major axis of the ellipse.

Tangential Velocity (v)

One radial $\rightarrow dr/dt$

Transverse at right angle to the radius vector $\rightarrow r\left(\frac{d\phi}{dt}\right)$

$P_r \rightarrow$ radial momentum $\rightarrow m\left(\frac{dr}{dt}\right)$

$P_\phi \rightarrow$ angular or azimuthal $\rightarrow m r^2\left(\frac{d\phi}{dt}\right)$

$m \rightarrow$ mass of e^-.

$$\left.\begin{aligned} \oint P_r \cdot dr &= n_r \cdot h \\ \oint P_\phi \cdot d\phi &= n_\phi \cdot h \end{aligned}\right\} \text{ — (1)}$$

n_r and n_ϕ are the quantum number.

$\therefore n = n_r + n_\phi$ —— (2) $n \rightarrow$ total Quantum number and is identical to Bohr

W = P.E + radial K.E + angular K.E.

$$W = -\frac{Ee}{r} + \frac{1}{2} m \left(\frac{dr}{dt}\right)^2 + \frac{1}{2} m r^2 \left(\frac{d\phi}{dt}\right)^2 \quad \text{—— (3)}$$

$$1 - e^2 = \frac{b^2}{a^2} = \frac{n_\phi^2}{(n_\phi + n_r)^2} \quad \text{—— (4)}$$

$e \rightarrow$ eccentricity of the ellipse whose semi-major and semi-minor axis are a, b respectively.

$$W = -\frac{2\pi^2 m E^2 e^2}{h^2} \left(\frac{1}{n_\phi + n_r}\right)^2$$

$$(1 - e^2)^{1/2} = b/a = \frac{n_\phi}{n_\phi + n_r} = \frac{n_\phi}{n}$$

$n_\phi \rightarrow$ cannot be zero.

$n_\phi \rightarrow$ cannot be greater than n, since b is always less than a.

$n_\phi = n$, The path becomes circular since $b = a$; $e = 0$.

For a given value of 'n', $n_\phi \rightarrow$ different 'n' values

$\downarrow$

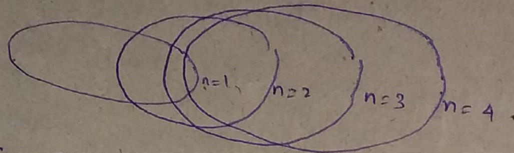
There are possibilities of having 'n' elliptical orbi

$n = 4$

four different ellipses are permitted.

$A_1, A_2, A_3, A_4 \ldots$

n, n_ϕ and n_r.



$$W = \frac{-2\pi^2 m E^2 r^2}{h^2}\left(\frac{1}{n_\phi + n_r}\right)$$

$$= -\frac{2\pi^2 m E^2 e^2}{h^2 n^2}$$

Relativistic relation between mass and velocity.

* mass of e^- from the ~~sat~~ stationary rest frame of the nucleus.

Rosette shape - Processing ellipse.

25/03/19

Sommerfield's Model

$$n = n_r + n_\phi$$

* Splitting of spectral lines in the presence of external fields.
* Some transitions are permanent (selection rules).
* Processing ellipse, in which, e^- have electrostatic force of attraction between e^- and nucleus
* e^- in an orbit is like revolving magnets.

* Magnetic moment which leads to splitting of spectral lines.

* Relativistic theory of sommerfield model help for to known splitting of lines.

* As V changes, mass also changes as it contributes to total energy (extra relativistic energy)

Processing ellipse → doubly period.

* A revolving charge is itself a small magnet
  ↳ non-zero magnetic moment.

* Double periodic besides original period of revolution, there is also the period of procession motion.

'W'

$$W = \frac{-2\pi^2 m E^2 e^2}{h^2}\left[\frac{1}{n^2} + \frac{4\pi^2 E^2 e^2}{c^2 h^2}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)\frac{1}{n^4} + \dots\right].$$

$$= \frac{-2\pi^2 m e^4}{h^2} \cdot \frac{Z^4 \alpha^2}{n^4}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)$$

$E = Ze$

$\alpha = \frac{2\pi e^2}{ch}$

'n' - $n\phi$ on account of $\frac{n}{n_\phi}$.

$n \rightarrow$ corr. to ϕ

α^2, e, c , $\alpha = 7.28 \times 10^{-3}$ [fine structure constant].

Application to the fine structure

$n_1 = 2$ $\qquad$ $n_2 = 3$

$2_2, 2_1$ $\qquad$ $3_3, 3_2, 3_1$

H_α-line of Balmer series.

$3_3 \to 2_2$, $3_3 \to 2_1$,

$3_2 \to 2_2$, $3_2 \to 2_1$

$3_1 \to 2_2$, $3_1 \to 2_1$.

Hydrogen doublet constant Z_2' and Z_1

$$W_{n,n_\phi} = -RZ^2ch\left\{\frac{1}{n^2} + \frac{\alpha^2 Z^2}{n^4}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)\right\}.$$

$W_{n,n_\phi} \to$ Energy corresponding to an orbit of total quantum number 'n' and 'n_ϕ'.

$$R = \frac{2\pi^2 m e^4}{ch^3}.$$

Energy for the 2_2 orbit.

$$W_{2,2} = -RZ^2ch\left\{\frac{1}{Z^2} + \frac{\alpha^2 Z^2}{2^4}\left(\frac{2}{2} - \frac{3}{4}\right)\right\}.$$

For 2_1 orbit.

$$W_{2,1} = -eZ^2ch\left\{\frac{1}{2^2} + \frac{\alpha^2 Z^4}{2^4}\left\{\left(\frac{2}{1} - \frac{3}{4}\right)\right\}\right]$$

$$W_{2,2} - W_{2,1} = RZ^2ch\left\{\frac{\alpha^2 Z^2}{2^4}\left(\frac{2}{1} - \frac{3}{4}\right) - \left(\frac{2}{2} - \frac{3}{4}\right)\right\}.$$

$$= \frac{RZ^4\alpha^2 ch}{2^4}.$$

$$\Delta W = W_{2,2} - W_{2,1} \quad Z = 1$$

$$\Delta \nu = \frac{\Delta W}{h}$$

$$\Delta \bar{\nu} = \frac{\Delta \nu}{c} = \frac{\Delta W}{ch} = \frac{R\alpha^2}{16}$$

$\downarrow$

$\frac{R\alpha^2}{16} \rightarrow$ hydrogen doublet constant d/s

denoted by 'd'.

$R = 109700 \, cm^{-1}$

$\alpha^2 = 5.3 \times 10^{-5}$

$d = 0.364 \, cm^{-1} = \frac{R\alpha^2}{16}$

$$\rightarrow W_{n,n_\phi} = \frac{-RZ^2ch}{n^2} - \frac{RZ^4\alpha^2 ch}{n^4}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)$$

$\Downarrow$ uncorrected energy

$\Downarrow$ Relatively corrections

$$(Z=1) \Rightarrow W_{n,n_\phi} = -W_n = \frac{R\alpha^2}{n^4}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)$$

$\Downarrow$ uncorrected.

26/03/19

Distance between two lines.

$$W_{n,n_\phi} = -\overline{W_n} - \frac{R\alpha^2}{n^4}\left(\frac{n}{n_\phi} - \frac{3}{4}\right)$$

$\overline{W_n} \rightarrow$ uncorrected energy level.

$$n=3 \begin{cases} \overline{W}_{3,3} = -\overline{W}_3 - \frac{16d}{3^4}\left(\frac{3}{3} - \frac{3}{4}\right) = -\overline{W}_3 - \frac{4d}{81} \\ \overline{W}_{3,2} = -\overline{W}_3 - \frac{16d}{3^4}\left(\frac{3}{2} - \frac{3}{4}\right) = -\overline{W}_3 - \frac{12d}{81} \\ \overline{W}_{3,1} = -\overline{W}_3 - \frac{16d}{3^4}\left(\frac{3}{1} - \frac{3}{4}\right) = -\overline{W}_3 - \frac{36d}{81} \end{cases}$$

$$n=2 \begin{cases} \overline{W}_{2,2} = -\overline{W}_2 - d\left(\frac{2}{2} - \frac{3}{4}\right) = -\overline{W}_2 - \frac{d}{4} \\ \overline{W}_{2,1} = -\overline{W}_2 - d\left(\frac{2}{1} - \frac{3}{4}\right) = -\overline{W}_2 - \frac{5d}{4} \end{cases}$$

→ $\overline{W}_3 = \overline{W}_2$

→ $\bar{\nu}_a \quad \bar{\nu}_b \quad \bar{\nu}_c \quad \bar{\nu}_d \quad \bar{\nu}_e \quad \bar{\nu}_f$

relativistic correction levels to splitting. if only single line.

→ $(3_3 \to 2_2) \quad \bar{\nu}_a = \bar{\nu} - \frac{4d}{81} + \frac{d}{4}$

$= \bar{\nu} + \frac{65d}{324}$

→ $(3_3 \to 2_1) \quad \bar{\nu}_b = \bar{\nu} - \frac{4d}{81} + \frac{5d}{4}$

$= \bar{\nu} + \frac{389d}{324}$

$(3_2 \to 2_2) \quad \bar{\nu}_c = \bar{\nu} - \frac{12d}{81} + \frac{d}{4}$

$= \bar{\nu} + \frac{33d}{324}$

$(3_2 \to 2_1) \quad \bar{\nu}_d = \bar{\nu} - \frac{12d}{81} + \frac{5d}{4}$

$= \bar{\nu} + \frac{357}{324}d$

$(3_1 \to 2_2) \quad \bar{\nu}_e = \bar{\nu} - \frac{36d}{81} + \frac{d}{4} = \bar{\nu} - \frac{63d}{324}$

$(3_1 \to 2_1)$ $\bar{\nu}_f = \bar{\nu} - \frac{36d}{81} + \frac{5d}{4} = \bar{\nu} + \frac{261d}{324}$

$\Delta n_\phi = \pm 1$ $(3_2 \to 2_1)$, $(3_2 \to 2_2)$ d

$(3_1 \to 2_1) \to$ forbidden

$$\bar{\nu}_a = \bar{\nu} + \frac{65}{324} d.$$

$$\bar{\nu}_d = \bar{\nu} + \frac{357}{324} d.$$

$$\bar{\nu}_e = \bar{\nu} - \frac{63d}{324}$$

Energy level Diagram

$\bar{\nu}_a$ and $\bar{\nu}_d = \frac{292}{324} d$

$\bar{\nu}_a$ and $\bar{\nu}_e = \frac{128 d}{324}$

$\bar{\nu}_d$ and $\bar{\nu}_e = \frac{420}{324} d.$

Fe $-(26) = Z$

Magnetic Moment $= 2.4$.

→ Partially filled d-shell

↓

Transition metal called

bcz of

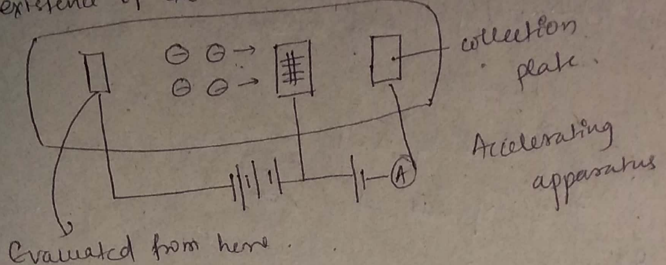
Dirac form (fully relativistic)

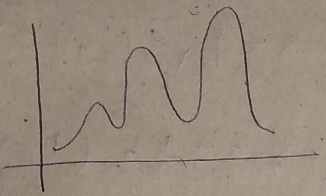
Both L and S.

28·03·19

Franck - Hertz Experiment

existence of excited in Hg atom (1924)





Magnetic moment of Hydrogen atom

Original principle quantum number $n = n_r + n_\phi$ not adequate.

Spin Quantum number:

Irrespective of shape of orbitals, the e^- experiences a spin, e^- → like magnets → so spin contributn also there.

$$||S|| = \sqrt{s(s+1)}\,\hbar$$

S → quantised spin vector.

$||S||$ → norm of spin vector.

s → spin quantum number associated with spin angular momentum.

$\hbar$ → Planck's constant.

Electron Spin

$n \rightarrow$ principle quantum no. (shell no.)

$l \rightarrow$ orbital. $\qquad n=1,\ 1s$

$m_l \rightarrow$ orbital ang. momentum $\qquad n=2,\ 2s,\ 2p$

$1 \rightarrow n^2$

$l \rightarrow 0$ to $n-1$

$s = \frac{1}{2}$, special for e^-

RO_2

$\uparrow$

(Molecular solid)

(p-band or orbital magnetism)

$$s = \hbar\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} = \frac{\sqrt{3}}{2}\hbar$$

$S_z = \pm\frac{1}{2}\hbar$ (thus H-atom 2 lines)

$$\mu_s = -\frac{e}{2m} g S$$

$e \rightarrow$ charge, $m \rightarrow$ mass

$g \rightarrow$ Lande g factor

ESR $\rightarrow$ Electron Spin Resonance.

$\mu_z = \pm\frac{1}{2} g \mu_B$.

$\mu_B \rightarrow$ Bohr Magneton.

Angular Momentum (classical)

$L = r \times p$

$L = I\omega$

$I \rightarrow$ Moment of Inertia

$\omega \rightarrow$ angular velocity.

Angular Momentum (Quantum Mechanics)

$L = r \times p$

$r \rightarrow$ Quantum position operator.

$p \rightarrow$ Quantum momentum operator.

L → Orbital Momentum Operator

J → Total angular momentum

$$\boxed{J = L + S}$$

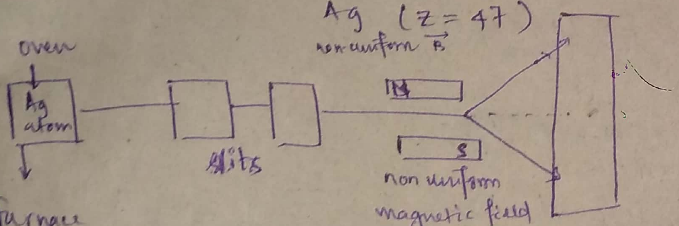
$p_x = -i\hbar \frac{\partial}{\partial x}$

$\vec{p} = i\hbar \vec{\nabla}$

Spatial quantisation of an e^- → spin spin & contributor → also orbital contribution.

02.04.19

STERN - GERLACH EXPERIMENT (1920)



· It is not uniform magnetic field.

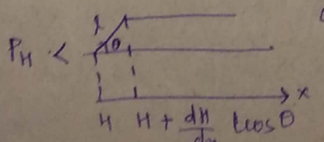
Silver atom has one unpaired e^- in its outermost shell.

for other atoms like H, Cu, Na, K, Ag (e^- rotation corresponding to direction of B).

- $+1/2$ or $-1/2$ e^- spin (spatial quantisation)

If it was uniform B, it will get ~~stee~~ deflected

- Expressions for the amt. of duration w.r.t to distance change in deflectn in x-direction



$\frac{dH}{dx}$ (+ve)

$P \rightarrow$ Pole strength

$l \rightarrow$ length

$M \rightarrow$ Magnetic moment

Field strength $H \rightarrow H + \left(\frac{dH}{dx}\right) l\cos\theta$

$$pH + p\left\{H + \left(\frac{dH}{dx}\right) l\cos\theta\right\}.$$

Extra force is acting on it; the force coming out is not same as the original force pH.

$$p\left(\frac{dH}{dx} l\cos\theta\right) \Rightarrow F_x$$

$$F_x = Pl \cdot \left(\frac{dH}{dx}\right)\cos\theta$$

$$= M\cos\theta \frac{dH}{dx}$$

where $Pl = M$

Each e^- coming out is like tiny magnet, it gets corresponding to strength of N_2S, it gets turned.

velocity $\rightarrow v$, 'm', 'L', time 't'

acceleration, $a_x = \frac{F_x}{m}$

$$D_x = \frac{1}{2} a_x t^2 = \frac{1}{2}\left(\frac{F_x}{m}\right)\left\{\left(\frac{L}{V}\right)^2\right.$$

$\rightarrow t = L/v$

$$D_x = \frac{1}{2} \cdot \frac{\cos\theta}{m} \cdot \frac{dH}{dx}\left(\frac{L}{V}\right)^2$$

$$D_x = \frac{1}{2} \frac{M}{m} \frac{dH}{dx}\left(\frac{L}{V}\right)^2$$

D_x, m, dH/dx

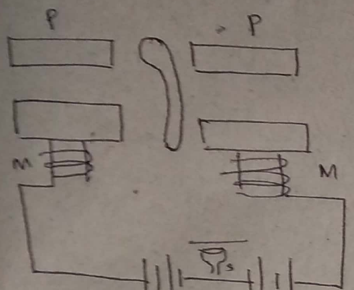
L, V are known

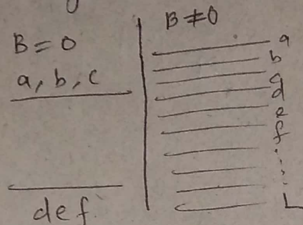
Sanjhwini aayu

If $\frac{H}{dx} = 0$, then $D_x = 0$, $D_x \propto \frac{dH}{dx}$

Zeeman Effect

splitting of spectral lines in several components in the presence of a static magnetic field.

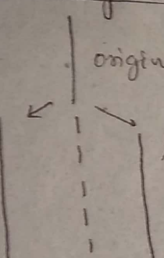




It is reversible
(After ___________,
everything go back to original).

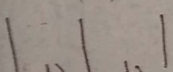
Electromagnetic

original line



Longitudinal view
(Both won't have same ν)

112.

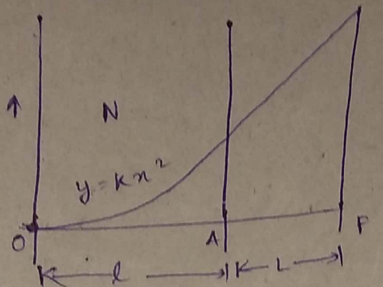


Transverse view
(2 lines adjacent to original lines)

dλ → difference in wavelength.

Zeeman shift → dλ (Splitting of Energy levels.

7/04/19 — Taken by PHD student



$T = 600K$

$l = 0.6m$

$L = 1m$

$\frac{\partial B}{\partial y} = 20Tm^{-1}$

$\frac{1}{2}mv^2 = 2kT$

$\mu_B = 1.$

N.

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- Simple harmonic motion, e^- coming have a vibration towards mean position, if is like SHM.
- The frequency of e^- radiates some energy.
-

- MOKE - Polarise the motion of the e^- like Fe, Co, try to make e^- polarise w.r.t B.
- External B, Additional torque acting on the e^-, energy on the e^- is attained.
- Theoritical Analysis - Spin property; orbital motion - B, additional B dominate this B.

Paschen brackett effect - It will try to clustering original one.

- Total Hamiltonian

$$H = H_0 + V_m.$$

$H_0 \rightarrow$ Unperturbed Hamiltonian.

$V_m \rightarrow$ Perturbation due to B.

(Magnetic Component).

$$V_m = -\vec{\mu} \cdot \vec{B}$$

Total Energy will be altered if we freeze to give ______ perturbation to the system.

-ve sign is due to fact that the perturbation will alter the energy.

$\vec{\mu}$ → Magnetic moment of the atom.

Nucleus also has the spin due to its mass.

$\vec{\mu} = -\mu_B g \vec{J}/\hbar$, μ_B → Bohr Magneton

$\vec{J}$ → Total electronic ang. momentum

$\vec{\mu} = -\mu_B (\underbrace{g_l \vec{L}}_{\text{orbital component}} + \underbrace{g_s \vec{S}}_{\text{spin component}})$

g → g-factor $\vec{L}$ & $\vec{S}$

g_l → normally,

g_s normally.

LS-Coupling

$J = L + S$

$g_J = \langle \sum_i (g_l \vec{l_i} + g_s \vec{s_i}) \rangle$

$= \langle (g_l \vec{L} + g_s \vec{S}) \rangle$

$\vec{L}$ → Total orbital momentum

$\vec{S}$ → Total spin momentum

DEBYE's EXPLANATION

(Normal Zeeman effect)

spin (Neglect) ; $P_l = \frac{l \cdot h}{2\pi}$ —(1)

$\mu_l = \frac{l \cdot eh}{4\pi m_e} = \frac{e}{2m_e} \cdot P_l$ —(2)

e^- have effects of external B, makes its direction towards its direction when e^- was moving $\perp$r to its ~~mot~~ orbital motion.

• E, dE → Field

Larmor Procession

e- moving in a particular orbit.

→ m / e
↓ ↓
mass charge

→ V and $\dot{\omega}$ → Linear and Angular Velocity.

$$F = \frac{mv^2}{r} = mr\omega^2 \quad \text{—— (1)}$$

'H' → Intensity of B,

→ Force acting on e^- is HeV.

Depends upon motion of e^- whether e^- is anticlockwise or clockwise.

H is in incorrect or outward direction.

Let $\delta\omega$,

$$F - HeV = m(\omega + \delta\omega)^2 r \quad \text{—— (3)}$$

Since $F = m\omega^2 r$ and $V = r\omega$

$$m\omega^2 r - He\omega r = m\omega^2 r + 2m\omega\,\delta\omega\, r$$

$$\delta\omega = \frac{-He}{2m}$$ (neglecting terms involving $\delta\omega^2$)

(-ve sign indicates the ~~bae~~ larmor procession to the ~~rotating of~~ motion of the e^-).

n → revolution per sec, $n_0 = \frac{\omega}{2\pi}$

a → area of the circle.

I → current

$$\delta M = ea\,\delta n$$

$$= e \cdot a \left(\frac{\delta\omega}{2\pi} \right)$$

$$= e(\quad)(\quad)$$

$$= \frac{-He^2 n^2}{4\pi}$$

~~...~~ $nia \Rightarrow$ magnetic moment of e^-

$M = Ia$

$P_l \rightarrow$ ang. momentum.

$O = \frac{eH}{2me}$ $\quad O \rightarrow$ laremour proceuional *

$$\Delta E_L = E_H - E_0$$

$$\Delta E = O \cdot P_L$$

$\downarrow$

L.P V.

8/4/19

Schroadinger Equations

Plane wave described by $\psi(\vec{r},t) = Ae^{i(\vec{k}x - \omega t)}$

A = Amplitude ; $\lambda = \frac{2\pi}{k}$, $\vec{k}$ = Phase velocity $\frac{\omega}{k}$.

If we assume the propagation is along x-axis

$\vec{k} = k\hat{x}$

$\psi(x,t) = Ae^{i(kx - \omega t)}$

$p = \hbar k$ and $E = \hbar\omega$.

$\psi(x,t) = A\exp\left[\frac{i}{\hbar}(px - Et)\right]$

$\frac{d\psi}{dt} = A\exp\left[\frac{i}{\hbar}(px - Et)\right]\left(\frac{-iE}{\hbar}\right)$

Multiply by i,

$i\hbar\frac{d\psi}{dt} = E\psi(x,t)$

$\frac{d\psi}{dx} = A\exp\left[\frac{i}{\hbar}(px - Et)\right]\left(\frac{ip}{\hbar}\right)$

Again multiply by i,

$-i\hbar\frac{\partial\psi}{\partial x} = p\psi(x,t)$

$-i\hbar\frac{\partial\psi}{\partial x} = p\psi(x,t) = \frac{\partial\psi}{\partial x} = \frac{-p}{i\hbar}\{\psi(x,t)$

$-i\hbar\frac{\partial^2\psi}{\partial x^2} = p\frac{\partial\psi(x,t)}{\partial x}$.

$$i\hbar \cdot \frac{\partial^2 \psi}{\partial x^2} = p\left(\frac{-p}{i\hbar}\right)\psi(x,t)$$

$$\hbar^2 \frac{\partial^2 \psi}{\partial x^2} = -p^2 \psi(x,t)$$

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{p^2}{2m} \psi(x,t)$$

$$E = \frac{p^2}{2m}$$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}}$$ → One-dimensional time dependent schroading eq^n.

$E \to i\hbar \frac{\partial}{\partial t}$ and $p \to -i\hbar \frac{\partial}{\partial x}$

$$E\psi = \frac{p^2}{2m} \psi(x,t)$$

$$\psi(x,t) = A\,\exp\left[\frac{i}{\hbar}(p_x - Et)\right]$$

$$\psi(\bar{r},t) = A\,\exp\left[\frac{i}{\hbar}(\bar{p}\cdot\bar{r} - Et)\right]$$

$$= A\,\exp\left[\frac{i}{\hbar}(p_x \cdot x + p_y \cdot y + p_z \cdot z) - Et\right]$$

$$i\hbar \frac{d\psi}{dt} = E\psi$$

$$-i\hbar \frac{\partial \psi}{\partial x} = p_x \psi$$

$$\frac{-\hbar^2}{2m} \frac{d^2\psi}{\partial x^2} = \frac{p_x^2 \psi}{2m}$$

$$E = \frac{p^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

$$i\hbar \frac{d\psi}{dt} = E\psi = \frac{p^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right)$$

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \psi.$$ [3D Schroadinger eqn for a free particle]

$$E \to i\hbar \frac{\partial}{\partial t}$$

$$p \to -i\hbar \nabla$$

$$p_x \to i\hbar \frac{\partial}{\partial x}$$

$$p_y \to i\hbar \frac{\partial}{\partial y}$$

$$p_z \to i\hbar \frac{\partial}{\partial z}$$

$$E\psi = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)\psi$$

$$E\psi = \frac{p^2}{2m}\psi$$

$$V(\vec{r}, t) \to P.E$$

$$E = \underbrace{\frac{p^2}{2m}}_{K.E} + \underbrace{V(r,t)}_{P.E}$$

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{p^2}{2m} + V(r,t)\right]\psi.$$

$$= \left[-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi\right]$$

$$H = \frac{p^2}{2m} + V$$

Hamiltonian.

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$-\frac{\hbar}{2m} \nabla^2 + V \longrightarrow \text{Operator}$$

$$i\hbar \frac{d\psi}{dt} = H\psi = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(x)\right]\psi$$

⇓

1-Dimensional time dependent Schroandinger Eq^n.

Experiment → charge density distribution function → Maximum entropy method

$$f(r) = \iiint_N |\psi(r)|^2$$

<u>Physical Interpretation of ψ</u>

$$i\hbar \frac{\partial\psi}{\partial t} = H\psi$$

$$i\hbar \frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi \quad \text{--- (1)}$$

Its complex conjugate,

$$-i\hbar \frac{\partial\psi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^* \quad \text{--- (2)}$$

$V(\vec{r}, t)$ is assumed to be real.

Multiplying (1) by ψ^*;

$$i\hbar\, \psi^* \frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\psi^*\nabla^2\psi + V\psi^*\psi \quad \text{--- (3)}$$

$$-i\hbar\, \psi \frac{\partial\psi^*}{\partial t} = \frac{-\hbar^2}{2m}\psi\nabla^2\psi^* + V\psi\psi^* \quad \text{--- (4)}$$

$$i\hbar\left[\psi^*\frac{\partial\psi}{\partial t}+\psi\frac{\partial\psi^*}{\partial t}\right]=\frac{-\hbar^2}{2m}\left[\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right]$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \quad \text{—— (5)}$$

$$\frac{\partial}{\partial t}(\psi^*\psi)+\left[\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}+\frac{\partial J_z}{\partial z}\right]=0 \quad \text{—— (6)}$$

$$J_x=\frac{i\hbar}{2m}\left[\psi\frac{\partial\psi^*}{\partial x}-\psi^*\frac{\partial\psi}{\partial x}\right]$$

$$J_y=$$

$$J_z=$$

$$J=\frac{i\hbar}{2m}\left(\psi\nabla\psi^*-\psi^*\nabla\psi\right)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\bar{J}=0 \quad \text{—— (7)}$$

$$\rho=\psi^*\psi$$

$$\iiint|\psi^2|\,d\tau=1 \quad \text{(How?)}$$

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Physical Interpretation of wave function

$$\Psi(\bar{r}, t) \; ; \; \psi^*(\bar{r}, t)$$

$$P(\bar{r}, t) = \psi^*(\bar{r}, t)\, \psi(\bar{r}, t) = |\psi(\bar{r}, t)|^2 \quad \text{--- (1)}$$

$|\psi(\bar{r}, t)|^2 \rightarrow$ Probability of finding a system at time 't' in vol $d\tau$.

$$\int_{-\infty}^{+\infty} |\psi(\bar{r}, t)|^2 d\tau = 1 \quad \text{--- (2)}$$

$$|N|^2 \int_{-\infty}^{+\infty} |\psi(\bar{r}, t)|^2 d\tau = 1 \quad \text{--- (3)}$$

$N \rightarrow$ Normalisatn constant.

Probability Curre Density

$$\psi(\bar{r}, t) = V \int_{r} \psi^* \psi \, d\tau$$

$$i\hbar \frac{\partial \psi(\bar{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\bar{r}) \right] \psi \quad \text{--- (I)}$$

$$i\hbar \frac{\partial \psi^*(\bar{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi^* \quad \text{--- (II)}$$

Here $V \rightarrow$ Potential which is real.

Multiplying ① by ψ^* and ⑪ by ψ from left and subtracting from other,

$$i\hbar\left(\psi^*\frac{\partial\psi}{\partial t}+\psi\frac{\partial\psi^*}{\partial t}\right)=\frac{-\hbar^2}{2m}\left[\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right]$$

$$\therefore \frac{\partial}{\partial t}(\psi^*\psi)=-\frac{i\hbar}{2m}\left[\nabla\cdot(\psi^*\nabla\psi-\psi\nabla\psi^*)\right] \quad \text{— (14)}$$

Integrating we get,

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\psi^*\psi\,d\tau=\frac{i\hbar}{2m}\int_{-\infty}^{\infty}\nabla(\psi^*\nabla\psi-\psi\nabla\psi^*)\cdot d\tau$$

$$=\frac{i\hbar}{2m}\left[\psi^*\nabla\psi-\psi\nabla\psi^*\right]_{-\infty}^{\infty} \quad \text{— (3)}$$

$\psi d\psi^* \longrightarrow 0$ [when we have a localised wave packet]

$\gamma \longrightarrow \pm\infty$

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\psi^*\psi\,d\tau=0 \text{ or, } \int_{-\infty}^{\infty}\psi^*\psi\,d\tau=\text{constant in time}$$

Normalisation integral is constant in time

$J(\bar{r},t) \rightarrow$ Probability current density

$$j(\bar{r},t) \Rightarrow \frac{i\hbar}{2m}(\psi\nabla\psi^*-\psi^*\nabla\psi)$$

Subtract the above in (11·1),

$$\frac{\partial \rho(\bar{r},t)}{\partial t} + \nabla \cdot \bar{J}(\bar{r},t) = 0$$

$\Rightarrow$ Equation of continuity for probability.

Expectation Value

Large number of measurement on $\bar{r}$.

$\psi(\bar{r}, t)$

$\langle r \rangle \cdot |\psi(\bar{r},t)|^2$

$$\langle r \rangle = \int r \psi^* \psi \, d\tau = \int \psi^* r \psi \, d\tau$$

$$\langle f(r) \rangle = \int \psi^*(\bar{r},t) \, \gamma(\bar{r}) \, \psi(\bar{r},t) \, d\tau$$

Left multiply by time dependent Schroadinger equation.

$$i\hbar \frac{\partial \psi(\bar{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\bar{r},t)\right] \psi(r,t)$$

Multiplying by ψ^* and integrating from $-\infty$ to ∞

$$\int_{-\infty}^{\infty} \psi^* \left(i\hbar \frac{\partial \psi}{\partial t}\right) d\tau = \int_{-\infty}^{\infty} \psi^* \left(\frac{-\hbar^2}{2m} \nabla^2\right) \psi \, d\tau$$

$$+ \int_{-\infty}^{\infty} \psi^* V(\bar{r},t) \psi \, d\tau.$$

$$\langle i\hbar \frac{\partial}{\partial t} \rangle = \langle \frac{-\hbar^2}{2m} \nabla^2 \rangle + \langle V \rangle$$

$$\langle E \rangle = \langle P^2/2m \rangle + \langle V \rangle$$

$$\langle A \rangle = \int \psi^* A_{op} \psi \, d\tau$$

If the wave function is not normalised

$$\langle A \rangle = \frac{\int \psi^* A_{op} \psi \, d\tau}{\int \psi^* \psi \, d\tau}$$

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Time - Independent Schroedinger Equation

Time dependent S.E -

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r,t) \right] \psi(r,t)$$

$\psi(r,t) = \psi(r)$ and $\phi(t)$

$$\psi(r,t) = \psi(\bar{r}) \, \phi(t)$$

Dividing by $\psi(r) \, \phi(t)$

$$i\hbar \left(\frac{1}{\phi(t)} \right) \frac{\partial \phi(t)}{\partial t} = \frac{1}{\psi(\bar{r})} \left(-\frac{\hbar^2}{2m} \nabla^2 + V(r) \, \psi(r) \right]$$

$$\frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} = \frac{-iE}{\hbar}$$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(r) = E \, \psi(r)$$

$$\frac{\partial \phi(t)}{\phi(t)} = \frac{-iE}{\hbar} \delta t$$

$$\Rightarrow \int \frac{\partial \phi(t)}{\phi(t)} = \frac{iE}{\hbar} \int \delta t$$

$$\phi(t) = c \exp\left(\frac{-iEt}{\hbar}\right)$$ c - constant

$$\Psi(r,t) = \psi(r) \exp\left(-\frac{iEt}{\hbar}\right)$$

$$\Psi(r,t) = \psi(r)\, \phi(t)$$

w.r.t to t and multiply by $i\hbar$,

$$i\hbar \frac{\partial [\Psi(r,t)]}{\partial t} = E \Psi(\bar{r}, t)$$

Multiply both sides by ψ^* from left and integrate over the space coordinates,

$$\int_{-\infty}^{\infty} \psi^* \left(\frac{i\hbar \partial}{\partial t}\right) \Psi(\bar{r}, t) = E \cdot \text{ as } \int_{-\infty}^{+\infty} \psi^* \psi = 1$$

$$\left[\frac{-\hbar^2}{2m} \nabla^2 + V(\bar{r})\right] \psi(r) = E \psi(r)$$

↓

Hamiltonian Operator.

Stationary states:

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r)\right] \psi(r) = E \psi(r)$$

$$\Psi_n(r,t) = \psi_n(r) \exp\left(\frac{-iE_n t}{\hbar}\right)$$

$$\Psi_n(r,t) = \Psi_n(r,0)\exp\left(-\frac{iE_n t}{\hbar}\right)$$

$$P(r,t) = |\Psi_n(r,t)|^2 = |\Psi_n(r,0)|^2$$

$\Downarrow$

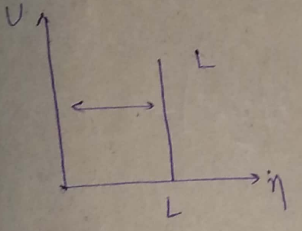
Constant in time

Particle in a box

One dimensional energy eigenvalue problem

$U(\Psi) \rightarrow$ zero outside

$U = 0$ for $0 < x < L$, $U = \infty$ for $x < 0$ or $x > L$



A sq. potential well with infinitely high barriers at each end.

$$\frac{d^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{when } V = 0.$$

$$\psi = A\sin\frac{\sqrt{2mE}}{\hbar}x + B\cos\frac{\sqrt{2mE}}{\hbar}x, \quad A, B \text{ constant}$$

$\psi < 0$ for $x = 0$ and $x = L$.

Since $\cos 0 = 1$, hence $B = 0$.

$$\frac{\sqrt{2mE}}{\hbar} = n\pi, \quad n = 1, 2, 3 \ldots \quad (3)$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}; \quad n = 1, 2, 3 \ldots$$

$$\psi_x = A\sin \frac{\sqrt{2mE_n}}{\hbar} x.$$

$$\psi_x = A\sin \frac{n\pi x}{L}$$

$$\int_{-\infty}^{\infty} |\psi_n|^2 dx = \int_0^L |\psi_n|^2 dx$$

$$= A^2 \cdot \int_0^L \sin^2\left(\frac{n\pi x}{2}\right) dx$$

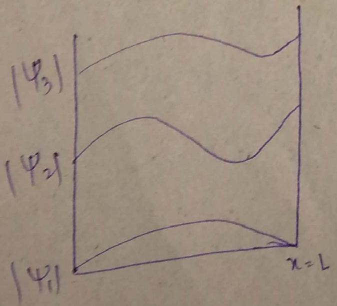
$$= \frac{A^2}{2}\left[\int_0^L dx \int_0^L \cos\left(\frac{2\pi n x}{L}\right)\right]$$

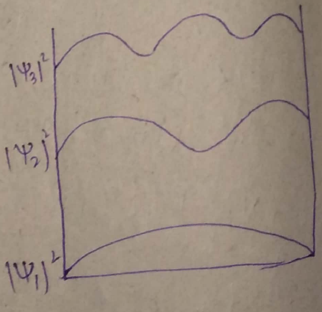
$$= \frac{A^2}{2}\left[x - \frac{L}{2\pi n}\sin\frac{2\pi n x}{L}\right]^2$$

$$= \frac{LA^2}{2}$$

then, $\int |\psi_n|^2 dx = 1$

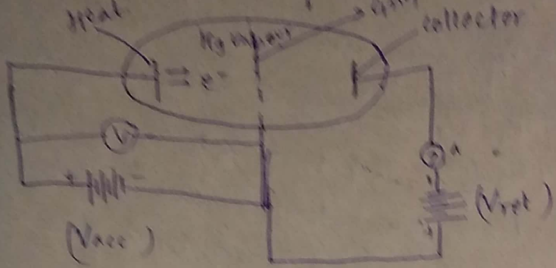
$A = \sqrt{\frac{2}{L}}$, $\psi_x = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$, $n = 1, 2, \cdots$



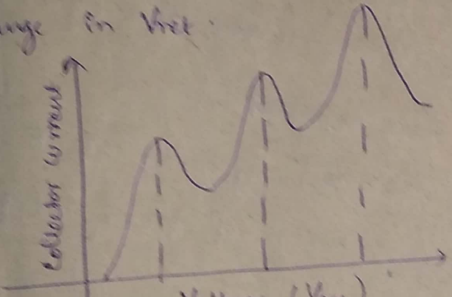


FRANK- HERTZ EXPT

It demonstrates that mechanical energy is absorbed by atoms in discrete quanta.



The energy of e^- passing through Hg vapours and reaching the collector can be determined from variation of collector current with change in V_{ret}.



Voltage (V_{acc})

→ The e^- travel under influence of V_{acc} betwn heater and grid and V_{ret} under betw^n grid and collector.

→ e^- gain energy when they move in electric field existing betw^n heater and grid, on their way the e^- collide with Hg vapors and can lose energy in collision.

- When total energy ≤ 4.9 volts, there was no energy ~~too~~ loss, hence a rise in collector current.

→ When Vacc was increased above 4.9 volts, inelastic collision occured near the grid in which e^- gave up their K.E to Hg vapour atoms, hence they were unable to traverse the Vret and collector current falls.

→ Again Vacc was increased to critical energy of 4.9 eV, after attaining critical energy and loosing it through collisions, e^- picked up new energy and collector current rose again.

→ This process was repeated.

These results show that Hg atoms absorb mechanical energy in quanta of 4.9 eV energy.

→ The expt was repeated in diff. condition of radiation and materials.

It demonstrated the emission or absorption of radiation is not continuous, but takes place in units of quanta of Energy.

Wave-Particle Duality

Wave Nature

→ Interference, diffraction, Polarizatn, reflection, refraction.

Particle Nature

→ BB radiation, photoelectric effect, Compton effect.

Schroandinger Wave Equation

$$\boxed{i\hbar \frac{d\Psi(x,t)}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(x,t) + V\Psi(x,t)}$$

anjiv Puri's ys $-\hbar/i$?? for $i\hbar$

Time dependent one-dimensional eqn.

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\phi = 0}$$

→ 1-D time independent eqn.

where $\phi = A \exp\left[-\frac{i}{\hbar}(Et - px)\right]$

Selection Rules

- The general condition necessary for an atom in an excited state to radiate is that the integral $\int_{-\infty}^{+\infty} x\Psi_{n_1}^* \psi_{n_2}\, dx$ should be non-zero since the intensity of radiation is proportion to it.
- Transitions between any two states n_1 and n_2 for which this integral is finite

are called allowed transitions, while those for which it is zero is called forbidden transitions.

- In case of H-atoms, 3 quantum no. are needed to specify the initial and final states involved in radiative transition.

→ If principle, orbital and magnetic QN of initial state are n_1, l_1, m_{l1} and final states are n_2, l_2, m_{l2}, so conditn for an allowed transition is –

$$\int_{-\infty}^{+\infty} u \, \psi^*_{n_1, l_1, m_{l1}} \cdot \psi_{n_2, l_2, m_{l_2}} \, dv \neq 0.$$

On evaluating the integral, it was found that the transitions for which orbital quantum number l changes by ± 1 and m_l either does not change or change by ± 1, i.e condit^{n}s are the allowed transition, are

$$\boxed{\Delta l = \pm 1 , \quad \Delta m_l = \pm 1}$$

The change in total QN n is not restricted.

Electron Spin

$$S = \sqrt{s(s+1)}\,\hbar$$ $S \rightarrow$ Spin Ang. Momentum

$s \rightarrow$ spin QN.

$$S = \sqrt{\frac{3}{2}}\,\hbar \qquad s = \frac{1}{2}$$

The space quantisation of e^- spin is described by spin magnetic QN m_s. as the orbital ang. momentum vector can have $(2l+1)$ orientations in a magnetic field from $+l$ to $-l$, through zero, the spin ang. momentum vector can have $2s+1 = 2$ orientation specified by $m_s = \pm 1/2$.

- The component S_z of the spin ang. momentum of an e^- along a magnetic field in the z-direction is determined by the spin magnetic QN, so that

$$S_z = m_s \hbar = \pm \frac{1}{2}\hbar$$

ATOMIC STRUCTURE

Thomson's Model

- first model
- Two facts
  - e^- are essential part of atom
  - atom as a whole is electrically neutral.
- +ve charges were uniformly distributed in a sphere of atomic dimensions, while the e^- were arranged inside the positive sphere in a way that the forces of attraction towards the centre of the sphere exactly balance their mutual repulsion.
- He could account for single line in the spectrum of hydrogen.

RUTHERFORD'S MODEL

- Gold-foil Experiment
  → Bombarded α-particles ($_2He^4$ nucleus) on a thin gold foil and observed scattered α-particles on ZnS screen.
  → The scattering of α-particles (+vely charged) is caused by coulomb repulsive forces betwn these particles and +ve charges of atom in scattering target.

→ OBSERVATIONS:

* Atom is composed of tiny nucleus containing all the +ve charge with nearly all the mass of the atom concentrated in it; e^- are arranged some distance away from it.

* α-particles which interacted with the nuclei suffered deflection through large angles.

* Due to large empty spaces present in the atom most of the particles went through foil undeflected or suffered very small deflections.

$$\boxed{N_\theta = \frac{Q \cdot n \cdot t \cdot (Ze)^2 \cdot e^2}{r^2 m^2 v_0^2 \sin^4(\theta/2)}}$$

N_θ - No. of α-particle scattered through θ striking per unit area of the screen at a distance r from the scatterer.

Q - total no. of α-particle that reach the screen.

n - no. of atoms per unit vol in foil.

t - foil thickness.

(Ze) - Nuclear charge

mv_0^2 = K.E of α-particles.

* Limitations of Rutherford's model:

• stability of nuclear atom.

Rutherford assumed that e⁻ should revolve round the nucleus at such a speed that the mechanical centrifugal force would just balance the net excess of electrostatic attraction and as a results stability could be established.

theory was not in accordance to electromagnetic theory, according to which an accelerating electric charge radiate energy in the of EM radiation. This energy can only come from the atomic system and hence it will loose energy continuously. As a result, the e⁻ will approach the nucleus by a spiral path, giving out a radiation of constantly increasing freq. and finally fall into nucleus; thus orbital motion of e⁻ destroys the stability of the atom.

BOHR'S ATOMIC MODEL

Bohr developed the theory of atomic structure on the basis of two postulates:

(1) The e⁻ can revolve only in certain circular orbits satisfying quantum conditions; these orbits were termed as stationary states and hence stable orbits in which e⁻ could revolve without radiating energy as demanded EM theory.

(2) When an e⁻ jumps from one permitted orbit to the other, it emits energy in the form of EM radiation.

For a linear harmonic oscillator, the displacement x and momentum p, are related through the eq^n of ellipse and the phase integral has value equal to integral multiple of h.

$$\oint p_x \, dx = nh$$

If linear mom is replaced by ang. mom,

$$L = I\omega$$

$$\oint L \cdot d\theta = nh$$

If ω = constant, then L becomes constant

$$L \oint d\theta = L \cdot 2\pi$$

$$L \cdot 2\pi = nh$$

$$\boxed{L = \frac{nh}{2\pi}}$$

<u>Important Note</u>: Hence, e^- were allowed to revolve in circular orbits for which the ang. momentum is integral multiple of $h/2\pi$.

(2) <u>Radius of permitted orbits</u>

For H-atom, the electrostatic force of attraction between e^- and proton provides the centripetal force required for revolution in the orbits,

$$\frac{1}{4\pi\epsilon_0} \frac{e\theta}{r^2} = \frac{mv^2}{r}$$

(1) Nature of stationary orbits

For a linear harmonic oscillator, the displacement x and momentum p, are related through the eq^n of ellipse and the phase integral has value equal to integral multiple of h.

$$\oint p_x \, dx = nh$$

If linear mom is replaced by ang. mom,

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(2) Radius of permitted orbits

For H-atom, the electrostatic force of attraction between e^- and proton provides the centripetal force required for revolution in the orbits,

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$$

For H-atom, $Q = e$ $[Z = 1]$

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

eq (1.1).

$$\boxed{v^2 = \frac{\beta e^2}{mr}}$$

$$\frac{e^2}{4\pi\epsilon_0 mr} = v^2 \quad \text{——— (1)}$$

$$\boxed{v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}} = \frac{e}{\sqrt{mr}}\sqrt{\frac{1}{4\pi\epsilon_0}}$$

$$v = \frac{(1.6 \times 10^{-19}) \times (9 \times 10^9)^{1/2}}{(9.11 \times 10^{-31} \times 0.5 \times 10^{-10})^{1/2}}$$

$$\boxed{V \simeq 2.2 \times 10^6 \, m/s}$$

From Bohr's quantisation of ang. momentum,

$$L = n\frac{h}{2\pi}$$

And $L = mvr$.

$$\frac{nh}{2\pi} = mvr.$$

$$v = \frac{nh}{2\pi mr}$$

$$v^2 = \left(\frac{nh}{2\pi mr}\right)^2.$$

$$\boxed{\hbar = \frac{h}{2\pi}}$$

$$v^2 = \left(\frac{n\hbar}{mr}\right)^2 \quad \text{——— (2)}$$

from (1) and (2),

$$\frac{e^2}{4\pi\epsilon_0 m r} = \frac{n^2 \hbar^2}{m^2 r^2}$$

$$\boxed{r = \frac{n^2 \hbar^2 4\pi\epsilon_0}{m e^2}}$$

$$a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m e^2}$$

$$r_n = n^2 a_0$$

$$= 0.53 A^\circ.$$

$$\boxed{r_n = \frac{n^2}{Z} a_0}$$

$$\boxed{r_n = \frac{0.53\, n^2}{Z} A^\circ}$$

(3) Energy of e^- in Permitted Orbits

$$E = P.E + K.E$$

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \cancel{m} \beta \frac{Ze^2}{\cancel{m} r} \quad \text{(from 1.1)}$$

$$\boxed{K.E = \frac{1}{2} \beta \frac{Ze^2}{r}}$$

$$\boxed{P.E = -\frac{\beta Ze^2}{r}}$$

$$E = \frac{1}{2} \beta \frac{Ze^2}{r} - \beta \frac{Ze^2}{r}$$

$$= \beta Ze^2 \left(\frac{1}{2r} - \frac{1}{r} \right)$$

$$E = -\frac{\beta Ze^2}{2r}$$

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$$\boxed{E = -\frac{\beta Z e^2}{2r} \quad ; \quad r = \frac{\beta \cdot m e^2}{n^2 h^2}}$$

$$= -\frac{\beta^2 e^4 Z m \times 4\pi^2}{2 n^2 h^2}$$

$$= -\frac{2\beta^2 e^4 m \pi^2 Z}{n^2 h^2}$$

$$= -\frac{2\pi^2 \beta^2 m e^4 (Z)}{n^2 h^2}$$

$$= -\frac{2\pi^2 m e^4 (Z)}{16\pi^2 \varepsilon_0^2 n^2 h^2}$$

$$\boxed{E = -\frac{Z^2 e^4 m}{8\varepsilon_0^2 n^2 h^2}}$$

-ve energy signifies another imp. concept of binding energy. It means that the e^- is bound to the nucleus by attractive forces so that energy must be supplied to the e^- in order to separate it completely from the nucleus. In this sense, the orbital energy is known as binding energy.

ATOMIC SPECTRA

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{\alpha^2} - \frac{1}{n^2}\right]$$

$\bar{\nu} \rightarrow$ wave no.

$R \rightarrow 109677 \text{ cm}^{-1}$

(Rydberg constant)

Series : $\bar{\nu} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$ $n = 2, 3, \ldots$

series : $\bar{\nu} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

Paschen series : $\bar{\nu} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$

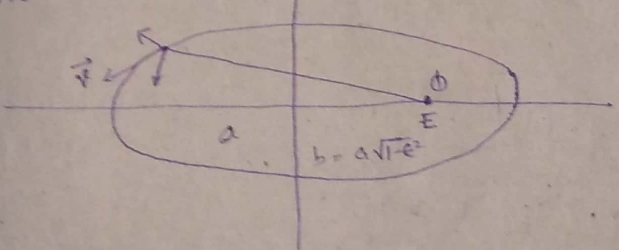
Brackett series, $\bar{\nu} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right]$

Pfund series, $\bar{\nu} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$.

BOHR'S CORRESPONDENCE PRINCIPLE

SOMMERFIELD ATOMIC MODEL

- Bohr's structure couldn't explain the fine structure in the spectral lines of H-atom.
- Sommerfield came up with an argument that since e^- is moving under the influence of a massive nucleus, it might also rotate in the elliptical orbits as planets do.
- Therefore, the positⁿ of e^- at any instant can be fixed in terms of polar coordinates r and ϕ, where r is the radius vector, i.e the distance of the e^- from the nucleus at one of the foci of ellipse and ϕ is the vectorial angle i.e the angle which the radius makes with the major axis of ellipse.



- Tangential Velocity v of e^- at instant can be resolved into two components; one radial i.e along radius vector equal to (dr/dt) and other transverse i.e at the right angles to the radius vector equal to $r\left(\frac{d\phi}{dt}\right)$.

corresponding, radial momentum $p_r = m\left(\frac{dr}{dt}\right)$

ang. or azimuth momentum $P_\phi = mr^2\left(\frac{d\phi}{dt}\right)$

Phase integral of Quantum Theory might be applied to both of these momentum,

$$\oint P_r \cdot dr = n_r h$$

$$\oint P_\phi \cdot d\phi = n_\phi h.$$

- n_r and n_ϕ are related by equation,

$$n = n_r + n_\phi$$

- Total Energy is given by,

$$E = P.E + \text{Radial } K.E + \text{Angular } K.E.$$

$$= -\frac{Ee}{r} + \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}mr^2\left(\frac{d\phi}{dt}\right)^2$$

- Now, $1-\varepsilon^2 = \frac{b^2}{a^2} = \frac{n_\phi^2}{(n_\phi + n_r)^2}$;

$\varepsilon \to$ Eccentricity of ellipse

$a \to$ semi major

$b \to$ semi minor

$$\boxed{\sqrt{(1-\varepsilon)} = \frac{b}{a} = \frac{n_\phi}{n}}$$

$$E = -\frac{2\pi^2 m Z^2 e^4}{h^2}\left(\frac{1}{n_\phi + n_r}\right)^2$$

- n_ϕ cannot be zero, since ellipse would then degenerate into a straight line passing through nucleus.

- n_ϕ cannot be greater than n, since b is always less than a.

- when $n_\phi = n$; path is circular , $b = a$, $\epsilon = 0$

- Given value of n , n_ϕ can assume only n diff. values

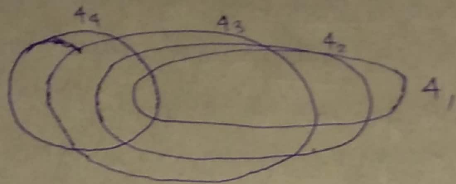
$\Downarrow$

n diff. elliptical orbits with diff. eccentricities

$n = 4,$

$4_1 , 4_2 , 4_3 , 4_4$

$n = 4_4 =$ circular orbital



Relativistic Variation of mass

Zeeman Effect

The splitting up of spectral lines in a strong magnetic field is referred to as Zeeman effect.

- In normal Zeeman effect, a single spectral line is split up into 3 spectral lines, when viewed in transverse direction to magnetic field and into 2 lines when viewed longitudinally.

→ Normal ZE can be explained on the basis of magnetic quantum number (m_l).

- In anomalous Zeeman effect, single line splits up into more than 2 lines.

- To account for fine structure as well as the anomalous Zeeman effect, S. Goudsmith and G. Uhlenbeck proposed that –

" An e^- has an intrinsic ang. momentum commonly known as spin ang. momentum or simply spin. A magnetic field is associated with this ang. momentum.".