HOMEWORK-1

USC ID: **4382673678**

USC ID: 4382673678
Name: Rupesh Kumar

HW - 1

Problem 1:

Given the line l with parameters (a, b, c), any point $(x, y, 1)$ in 2D image coordinate system is given by,

$$[a \;\; b \;\; c] \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0 \qquad \text{—— (1)}$$

Now,

Suppose $[p \;\; q \;\; r \;\; s]$ represents the plane in 3D camera coordinate system. Any point on this plane is given by,

$$[p \;\; q \;\; r \;\; s] \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = 0 \qquad \text{—— (2)}$$

where $(\hat{x} \;\; \hat{y} \;\; \hat{z} \;\; 1)$ is the point on the ~~xxx~~ above plane.

As we know the projection of a point in camera coordinate system to image coordinate system,

$$p = \frac{1}{\hat{z}} K [Id \;\; 0] P \qquad \text{— (3)}$$

~~From (1) and~~:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \frac{1}{\hat{z}} \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} \quad \text{—(4)}$$

From (1) and (4), we get

$$\begin{bmatrix} a & b & c \end{bmatrix} \frac{1}{\hat{z}} \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = 0.$$

$$\Rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = 0 \quad \text{—(5)}$$

From (2) and (5), we get;

$$\begin{bmatrix} p & q & r & s \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} K & 0 \end{bmatrix}$$

where K is the intrinsic matrix.

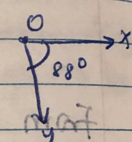
Problem 2.

Sol. Given, $f = 0.05\ m$

$K = l = \dfrac{1 \text{ pixel}}{0.04\ mm} = 25{,}000 \text{ pixel } m^{-1}$

So, $\alpha = \beta = lf = kf = 25{,}000 \times 0.05 = 1250$ pixel

Also, imaging ~~plane~~ surface is 1200 × 1200 pixels



So, $x_0 = 600$

$y_0 = \dfrac{600}{\cos 2^\circ} = 600.37$

$\theta = -88^\circ$

So, $K = \begin{bmatrix} \alpha & -\alpha \cot\theta & x_0 \\ 0 & \beta/\sin\theta & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1250 & -1250 \cot(-88) & 600 \\ 0 & 1250/\sin(-88) & 600.37 \\ 0 & 0 & 1 \end{bmatrix}$

$$K = \begin{bmatrix} 1250 & 43.65 & 600 \\ 0 & -1250.76 & 600.37 \\ 0 & 0 & 1 \end{bmatrix}$$

Q2.2 $O_c = (4, -3, 2)$.

X_c is parallel to X_w & the rotation is 15° about X_c in clockwise direction

$$R = \begin{bmatrix} i_c \cdot i_w & j_c \cdot i_w & k_c \cdot i_w \\ i_c \cdot j_w & j_c \cdot j_w & k_c \cdot j_w \\ i_c \cdot k_w & j_c \cdot k_w & k_c \cdot k_w \end{bmatrix}^T \quad \text{—(1)}$$

Assuming we rotate the matrix about X_c axis in counter-clockwise direction by an angle θ, we have,

$$i_c = i_w$$

$$j_c = j_w \cos\theta - k_w \sin\theta$$

$$k_c = k_w \cos\theta + j_w \sin\theta$$

using above equations in (1), we get

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$t = \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}$$

$$M = K\,[R \; t] \quad \text{— (2)}$$

$$[R \; t] = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & \cos(-15) & -\sin(-15) & -3 \\ 0 & \sin(-15) & \cos(-15) & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0.96 & 0.25 & -3 \\ 0 & -0.25 & 0.96 & 2 \end{bmatrix} \quad \text{— (3)}$$

putting eq (1) in (2), we get

$$M = \begin{bmatrix} 1250 & 43.65 & 600 \\ 0 & -1250.76 & 600.37 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0.96 & 0.25 & -3 \\ 0 & -0.25 & 0.96 & 2 \end{bmatrix}$$

so,

$$M = \begin{bmatrix} 1250 & -113.6 & 590.8 & 6069 \\ 0 & -1363.5 & 256.2 & 4953 \\ 0 & -0.25 & 0.96 & 2 \end{bmatrix}$$

Q3 Vanishing point will lie in the X_w-Z_w plane and homogenous world coordinates $= (a, b, c, 0)$.
Also, y-coordinate will be 0 as it lies in X_w - Z_w plane,
so,
$${}^{w}P = (a, 0, c, 0).$$

Also, $${}^{c}P = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} {}^{w}P$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0.96 & 0.25 & -3 \\ 0 & -0.25 & 0.96 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ c \\ 0 \end{bmatrix}$$

$${}^{c}P = \begin{bmatrix} a \\ 0.258c \\ 0.96c \\ 0 \end{bmatrix}$$

Now, convert camera coordinates to image coordinates,

$$p = \frac{1}{Z} K [\,Id \quad 0\,] \, {}^{c}P$$

$$= \frac{1}{0.96c} \begin{bmatrix} 1250 & 43.65 & 600 & 0 \\ 0 & -1250.76 & 600.37 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0.258c \\ 0.96c \\ 0 \end{bmatrix}$$

$$p = \begin{bmatrix} 1294.1a/c + 611.7 \\ 265.26 \\ 1 \end{bmatrix}$$

where p is the vanishing point in the image coordinate system.