TRANSFORMS NUMERCIALS AND TECHNIQUES

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ADAMS-BASHFORTH and ADAMS-MOULTON METHOD

We consider an initial value problem

$$y' = f(x,y)\,, \quad y(x_0) = y_0\,,$$

with ‘f’ such that the problem has a unique solution on some open interval containing x_0. The multistep formula of the **Adams–Bashforth method of fourth order** is given by

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})\text{-------}(5)$$

where

$$\begin{aligned} f_n &= f(x_n, y_n) \\ f_{n-1} &= f(x_{n-1}, y_{n-1}) \\ f_{n-2} &= f(x_{n-2}, y_{n-2}) \\ f_{n-3} &= f(x_{n-3}, y_{n-3}). \end{aligned} \qquad \text{-----------}\ (6)$$

evaluated at equidistant $x - values$,

$$x_0,\ x_1 = x_0 + h,\ x_2 = x_0 + 2h, \ldots, x_n = x_0 + nh.$$

This method expresses the new value y_{n+1} [approximation of the solution y of the initial value problem] in terms of 4 values of ‘f’ computed from the y-values obtained in the preceding 4 steps. In (5) we need f_0, f_1, f_2, f_3. Hence from (6) we see that we must first compute y_1, y_2, y_3 by some other method of comparable accuracy, for instance, by RK4 method.

Similarly, for Adams-Moulton methods, we predict a value y^*_{n+1} by using (5) (i.e.,)

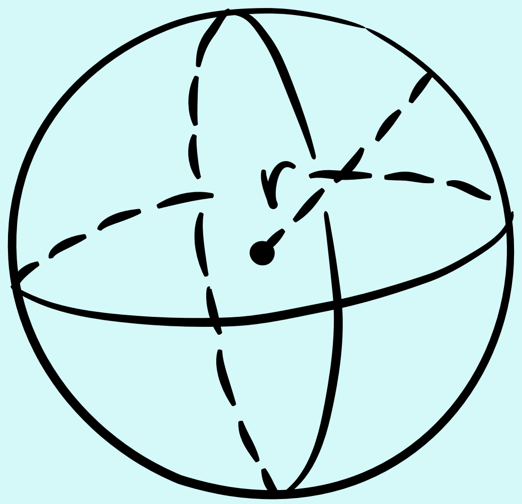
$$y^*_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) \text{ -------- } (7a)$$

and the corrected new value y_{n+1} is then obtained by

$$y_{n+1} = y_n + \frac{h}{24}(9f^*_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}) \text{ --------- } (7b)$$

$$\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$y = mx + b$$



$$V = \frac{4}{3}\pi r^3$$

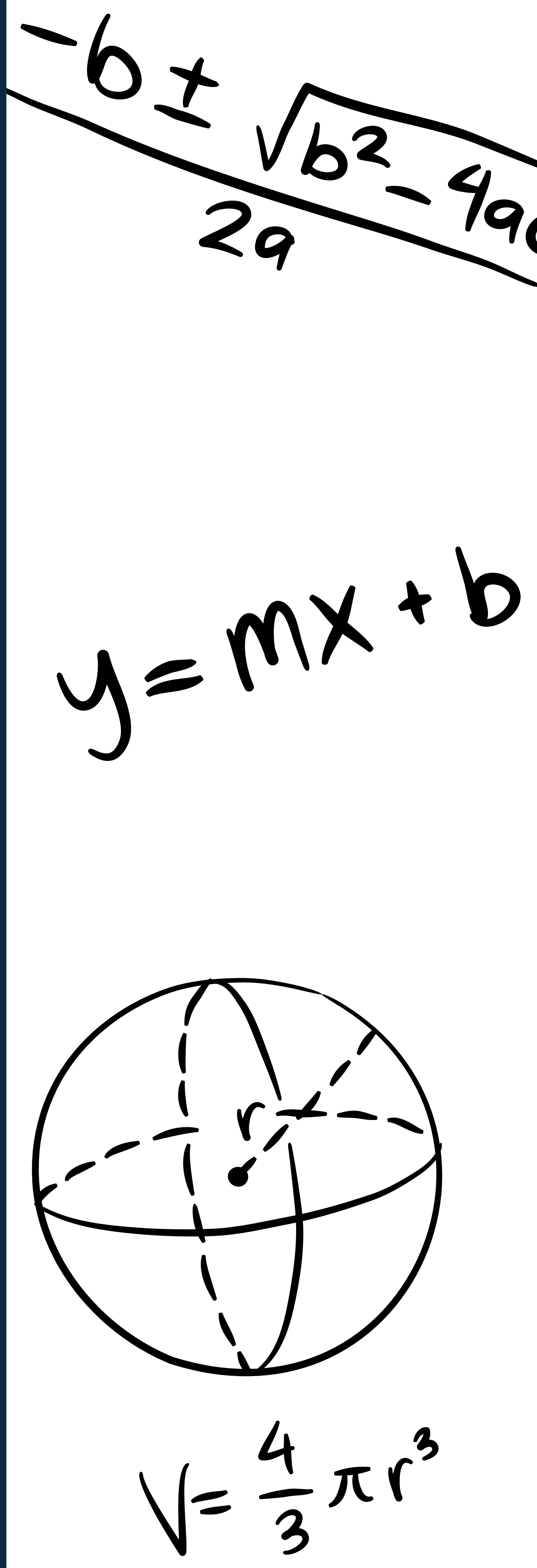
Implicit Nature

The Adams-Moulton method is a family of implicit methods used for solving ordinary differential equations (ODEs), particularly in the context of numerical integration. It is a predictor-corrector method, meaning it first makes an initial prediction using an explicit method (like Adams-Bashforth) and then corrects the prediction using an implicit method.

Key Characteristics:

Implicit Nature: Adams-Moulton methods are implicit because they involve the unknown values at the next time step in the formulation of the method. This requires solving a system of equations at each step, as opposed to explicit methods where the next values are directly calculated.

For example, in the 3rd-order Adams-Moulton method, the next value depends on the current and previous values, but the equation to solve involves the unknown value at the next time step, making it implicit.



Challenges and Limitations of the Adams-Moulton Method

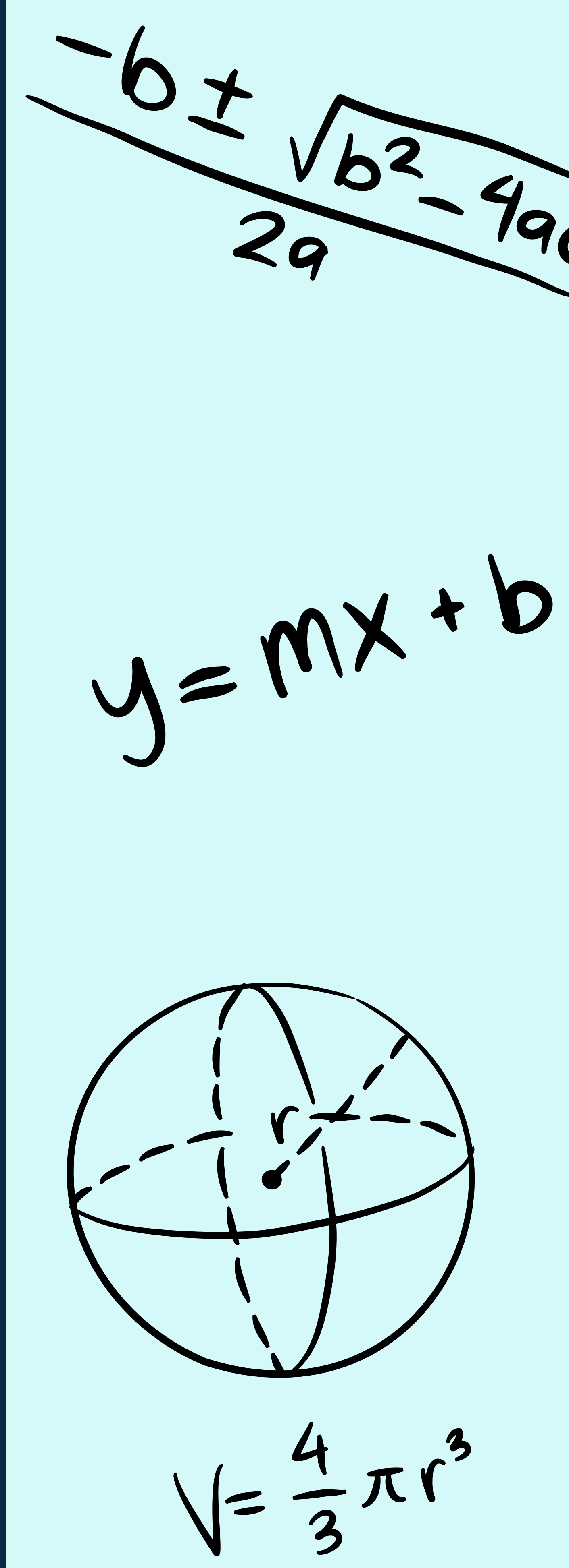
Implicit Nature: The Adams-Moulton method involves solving equations that depend on the unknown future value at each time step, making it more computationally demanding than explicit methods.

Iterative Solvers: Solving the implicit equations often requires iterative techniques like Newton's method, which can be time-consuming and sensitive to initial guesses.

Stiff Problems: While generally stable, the method can still face challenges with stiff ODEs, requiring very small step sizes or special techniques to maintain stability.

Initialization: The method depends on multiple previous values to start. These values are often generated using another method, like Runge-Kutta, adding to the computational overhead.

Error Propagation: Inadequate step-size selection or poor accuracy in earlier steps can lead to the accumulation of errors, reducing the reliability of the solution.



Code and Implementation

```
import numpy as np
import matplotlib.pyplot as plt

# Function for dy/dx = 1 + y^2
def f(x, y):
    return 1 + y * y

def adams_moulton_exact_steps(x0, y0, h, target_steps):
    x = [x0]
    y = [y0]

    # Compute first 3 steps using Runge-Kutta 4th Order
method
    for i in range(3):
        k1 = h * f(x[-1], y[-1])
        k2 = h * f(x[-1] + h / 2, y[-1] + k1 / 2)
        k3 = h * f(x[-1] + h / 2, y[-1] + k2 / 2)
        k4 = h * f(x[-1] + h, y[-1] + k3)
        y_next = y[-1] + (k1 + 2 * k2 + 2 * k3 + k4) / 6
        y.append(y_next)
        x.append(x[-1] + h)

    # Adams-Bashforth Predictor and Adams-Moulton
Corrector
    for i in range(3, target_steps):
        x_next = x[-1] + h
        # Predictor (Adams-Bashforth 4th Order)
        y_pred = y[-1] + h / 24 * (55 * f(x[-1], y[-1]) -
                                   59 * f(x[-2], y[-2]) +
                                   37 * f(x[-3], y[-3]) -
                                   9 * f(x[-4], y[-4]))
        # Corrector (Adams-Moulton 4th Order)
        y_corr = y[-1] + h / 24 * (9 * f(x_next, y_pred) +
                                   19 * f(x[-1], y[-1]) -
                                   5 * f(x[-2], y[-2]) +
                                   f(x[-3], y[-3]))
        y.append(y_corr)
        x.append(x_next)

    return x, y
```

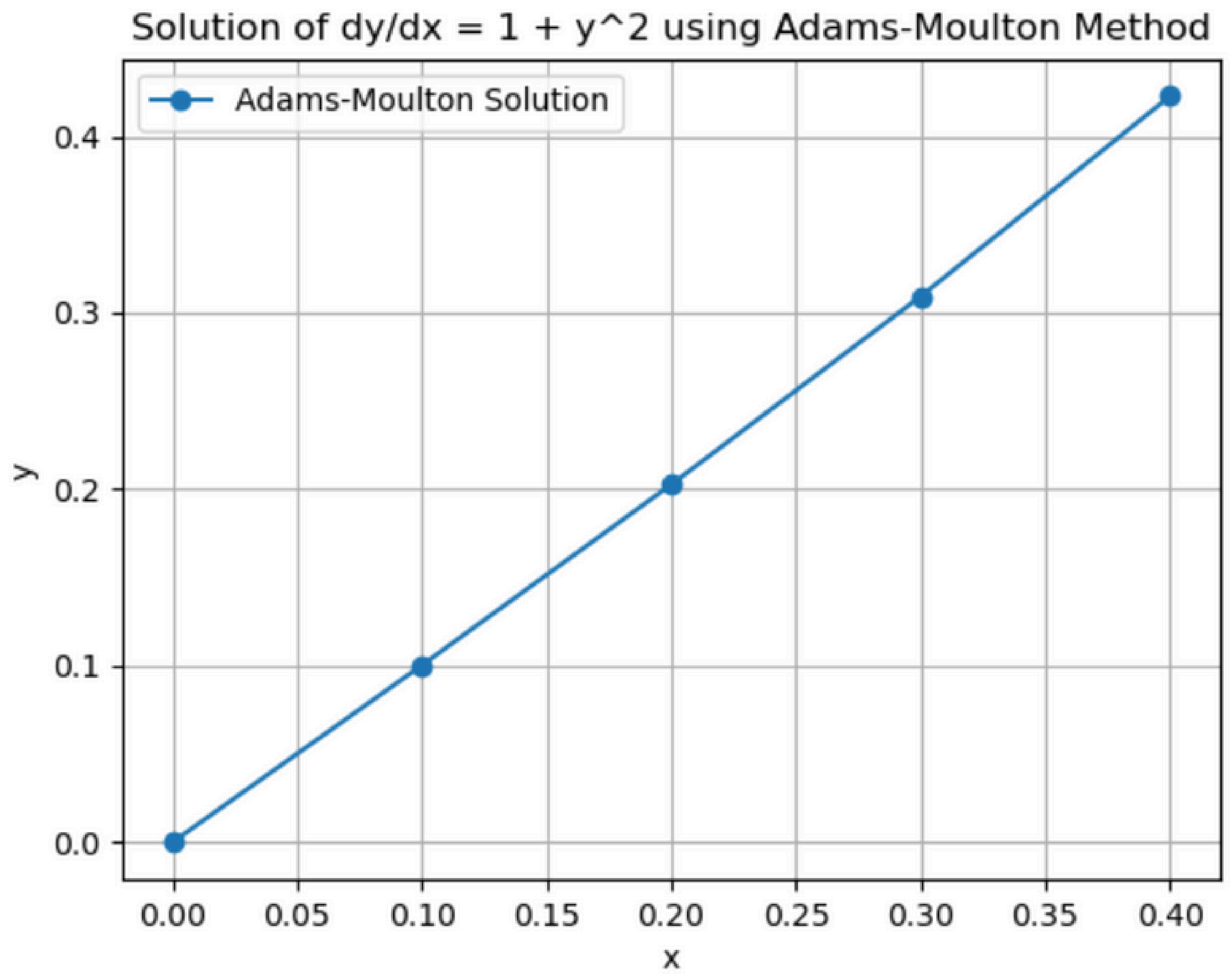
```
# Print results
print("Results:")
for i in range(len(x)):
print(f"x = {x[i]:.3f}, y = {y[i]:.6f}")

# Plot the results
plt.plot(x, y, 'o-', label="Adams-Moulton Solution")
plt.xlabel("x")
plt.ylabel("y")
plt.title("Solution of dy/dx = 1 + y^2 using Adams-Moulton Method")
plt.legend()
plt.grid()
plt.show()
```

Output

```
Results:
x = 0.000, y = 0.000000
x = 0.100, y = 0.100335
x = 0.200, y = 0.202710
x = 0.300, y = 0.309336
x = 0.400, y = 0.422798
```

Graph



Numerical Solution of the Problem

Given, $\frac{dy}{dx} = 1+y^2$, $y(0) = 0$

step size $h = 0.1$

solving y at $x = 0.1, 0.2, 0.3, 0.4$.

By using Runge-Kutta 4th order we will get $y(0.1)$ & $y(0.2)$

By applying Adams-Moulton method we can compute $y(0.3)$

I By using RK method to compute $y(0.1) \longrightarrow$

Formula:- $y_{n+1} = y_n + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$.

where

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left[x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right]$

$k_3 = hf\left[x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right]$

$k_4 = hf[x_n + h, y_n + k_3]$.

$k_1 = 0.1[1 + 0^2] \Rightarrow \boxed{k_1 = 0.1}$

$k_2 = 0.1[1 + 0.05^2] \Rightarrow \boxed{k_2 = 0.10025}$

$k_3 = 0.1[1 + 0.050125] \Rightarrow \boxed{k_3 = 0.100251}$

$k_3 = 0.1[1 + 0.100251^2] \Rightarrow \boxed{k_4 = 0.101005}$

$y(0.1)$:

$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.1 + 2[0.10025] + 2[0.100251] + 0.101005]$

$\boxed{y_1 = 0.1003345}$

$y_2 = y_1 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$k_1 = 0.1[1 + 0.1003345^2]$

$\boxed{k_1 = 0.101007}$

$k_2 = 0.1[1 + 0.150838^2] \Rightarrow \boxed{k_2 = 0.102273.}$

$k_3 = 0.1[1 + 0.151471^2] \Rightarrow \boxed{k_3 = 0.102297.}$

$k_4 = 0.1[1 + 0.202632^2] \Rightarrow \boxed{k_4 = 0.104106}$

$y_2 = 0.1003345 + \frac{1}{6}[0.101007 + 2[0.102273] + 2[0.102297] + 0.104106]$

$\boxed{y_2 = 0.202710}$

Using Adams-Moulton Method for y_3:

$y_{n+1} = y_n + \frac{h}{2}[f[x_{n+1}, y_{n+1}]] + f[x_n, y_n]$

$f(x_2, y_2) = 1 + [0.202710]^2 = 1.041092.$

$y_3 = y_2 + h \cdot f[x_2, y_2]$

$y_3 = 0.202710 + 0.1[1.041092]$

$\boxed{y_3 = 0.3068192} \longrightarrow$ Predicted value.

Correcting using Implicit formula:

$y(0.3) = y_3 = y_2 + \frac{h}{2}[f[0.3, y_p] + f[0.2, y[0.2]]]$

$f[0.3, 0.306819] = [1 + [0.306819]^2] = 1.094136.$

$f[0.2, 0.202710] = 1.041092.$

$y_3 = 0.20271 + \frac{0.1}{2}[1.094136 + 1.041092]$

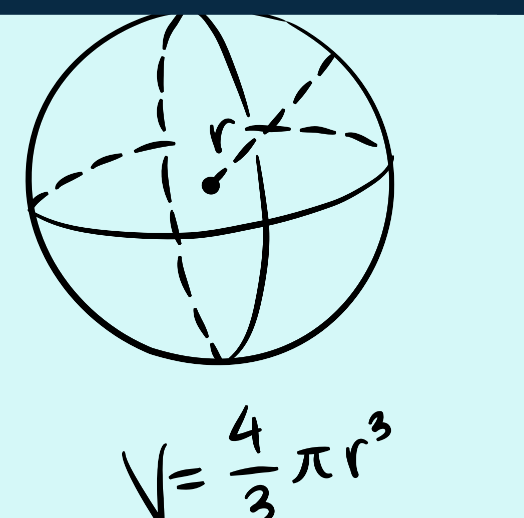
$\boxed{y_3 = 0.309336} \longrightarrow$ corrected value.

$y_4 = y_3 + \frac{0.1}{24}[55[1.094136] - 59.1 + 37 - 9]$

$= 0.309336 + \frac{0.1}{24}[55[1.0909] - 59 + 37 - 9].$

$\boxed{y_4 = 0.422798}.$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Analytical Solution of the Problem

$$\frac{dy}{dx} = 1+y^2.$$

$$\int \frac{dy}{1+y^2} = \int dx.$$

$$\int \frac{1}{1+y^2}\, dy = \int dx.$$

$$\tan^{-1} y = x+c.$$

$$y = \tan(x+c).$$

The solution to the differential equation is y=tan(x+c), Where C is constant. To compute specific value of y, we need the value of C. If no initial condition is provided then we assume C=0.

$y = \tan x.$

Using approximate $\tan(x) \approx x + \frac{x^3}{3}$ for small x

for $x=0.1$

$$y = \tan(0.1) \approx \boxed{0.100330}$$

for $x=0.2$

$$y = \tan(0.2) \approx \boxed{0.202710}$$

for $x=0.3$

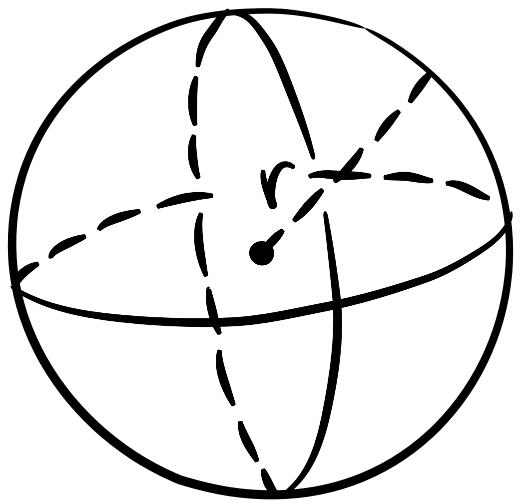
$$y = \tan(0.3) \approx \boxed{0.309340}$$

for $x=0.4$

$$y = \tan(0.4) \approx \boxed{0.422790}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

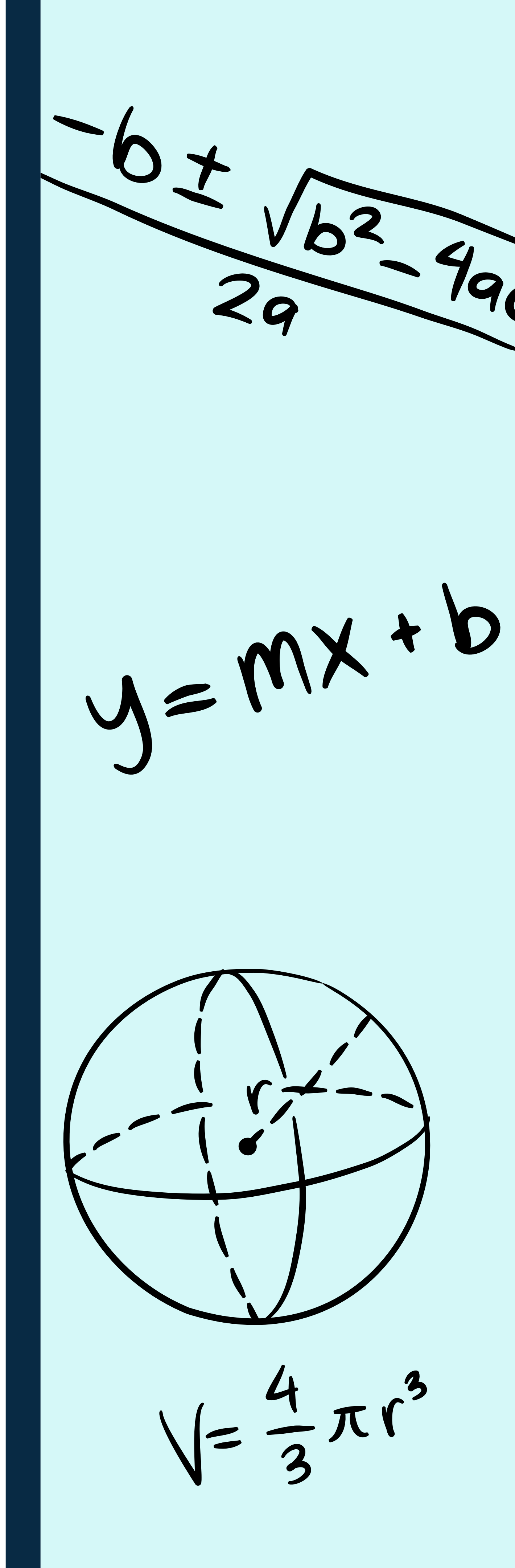
$$y = mx + b$$



$$V = \frac{4}{3}\pi r^3$$

Comparison between Numerical and Analytical solution

X	Numerical Method	Analytical Method	Difference Method
0.0	0.000000	0.000000	0.000000
0.1	0.100335	0.100330	0.000005
0.2	0.202710	0.202710	0.000000
0.3	0.309336	0.309340	0.000004
0.4	0.422798	0.422790	0.000008



Thank you