The Wright – Guild Experiments and the Development of the CIE 1931 RGB and XYZ Color Spaces

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Summary

The derivation of the CIE 1931 RGB color space from the Wright – Guild color matching data is described. Emphasis is placed on explaining the underlying logic of the process. The transformation of the RGB space to the CIE 1931 XYZ space is briefly described. An argument is made that the principal intent of the color matching experiments was to develop a rigorous, quantitative framework for describing all visible colors. For that purpose, negative chromaticity coefficients or imaginary primary colors are not problematic. Neither of the CIE 1931 color spaces can be displayed on a physical device; and it seems possible that little, if any, consideration was given in 1931 to practical applications of that sort. In general, digital cameras are sensitive to all visible wavelengths, and in principal can encode all humanly visible colors in raw image files. Color spaces with gamuts that exceed the AdobeRGB gamut — currently about the widest that can be reproduced on specialized displays — may be useful for processing raw images. A wide-gamut space such as ProPhotoRGB will, in theory, minimize compression of image color information during editing; thus maximizing head-room for color adjustment. Archived raw image files may also be useful in the future if very-wide gamut displays — possibly using 4, 5, or 6 primary colors — become available.

Key words: RGB color, chromaticity, W. D. Wright, J. Guild, color matching experiment, color matching function, tristimulus value, colorimetry, CIE 1931 XYZ color space, CIE 1931 RGB color space, spectral locus, imaginary color, N.P.L. Standard White Light

Section 1. Introduction

We all "know" about RGB color. RGB is ubiquitous because it is both a framework for defining colors, and a means for producing colors. Displays (CRTs, LEDs) make colors by mixing lights of three primaries: RGB encoding of color data, or its translation to RGB format, is necessary in order to make color images appear on our computer screens. My goal in this paper is to explain the development of the first RGB color standard. The motivation comes from my interest in photography. The approach is historical; because, for me, historical reconstruction facilitates understanding. I assume that it does for at least a few other people. The color-matching experiments conducted by W. D. Wright and J. Guild during the 1920's provided the data necessary for development of the CIE 1931 RGB and XYZ color spaces.[1] Those experiments are the historical source material. I will not dwell on the details of their colorimeters or the experimental procedures, beyond the absolute minimum necessary.

Misconceptions are impediments to understanding. My misconceptions were on at least two levels. The first was to think of the color matching experiments as an end in themselves, rather than as a means to an end. In reality, the goal was not simply to show how to make colors by mixing three primaries: people had been doing that for many years. Furthermore, there could have been little practical application of color mixing in 1930. Color televisions, computers, and digital cameras were far in the future. *The ultimate purpose of the color-matching experiments was to generate the data necessary for constructing a formal, quantitative, rigorous system for characterizing colors: the CIE color spaces.* If that needs reinforcing, it may help to note that neither of the CIE 1931 color spaces is practical for generating color on a physical device: either we need to be able to produce negative color (CIE 1931 RGB), or we need to use physically impossible primary colors (CIE 1931 XYZ).

The second impediment was more subtle. It is in fact difficult to understand the Wright – Guild experiments and the calculus of their data — why they did what they did — without focusing on the more proximal goal. That proximal goal, the immediate purpose of the experiments, was to decompose white light into its constituents; to show, with quantitative precision, how those individual constituents could be generated by mixing three primary colors, and finally to show how the color-mixed constituents could be put back together in a *mathematical* reconstruction of white light. The process required the development of empirical color matching functions. Within that system — defined by a particular set of primaries, and a particular white illuminant — the complex spectral composition of white light could be reduced to three numbers. And, having once achieved that with white light, the same functions could be used to reduce any color, however complex its spectrum, to just three numbers — the three RGB components.

Section 2. Colorimetry

In the 1920's, W. D. Wright and J. Guild, both in England but working independently, carried out a series of experiments.[2] These are usually referred to as the "color matching experiments" although the Guild paper, in particular, contains much more. We should, perhaps, take a clue from the title of his paper: *The Colorimetric Properties of the Spectrum.* Color matching was just one aspect. Although Guild did not publish his results until 1932, he had started his experiments much earlier. And when he did publish, he had the advantage of knowing about Wright's data. For numerical examples, I will rely on data from Guild (1932) Tables 1 and 4. Guild incorporated Wright's (1929) data into his own Table 4 by including averages of his own and Wright's results.

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[1] CIE stands for Commission Internationale de l'Eclairage. In English: International Commission on Illumination

[2] Wright (1929, 1930) and Guild (1932). Unfortunately, neither of these papers appears to be freely available on-line. However, much of the data in Guild (1932) is available here as an Excel spreadsheet.

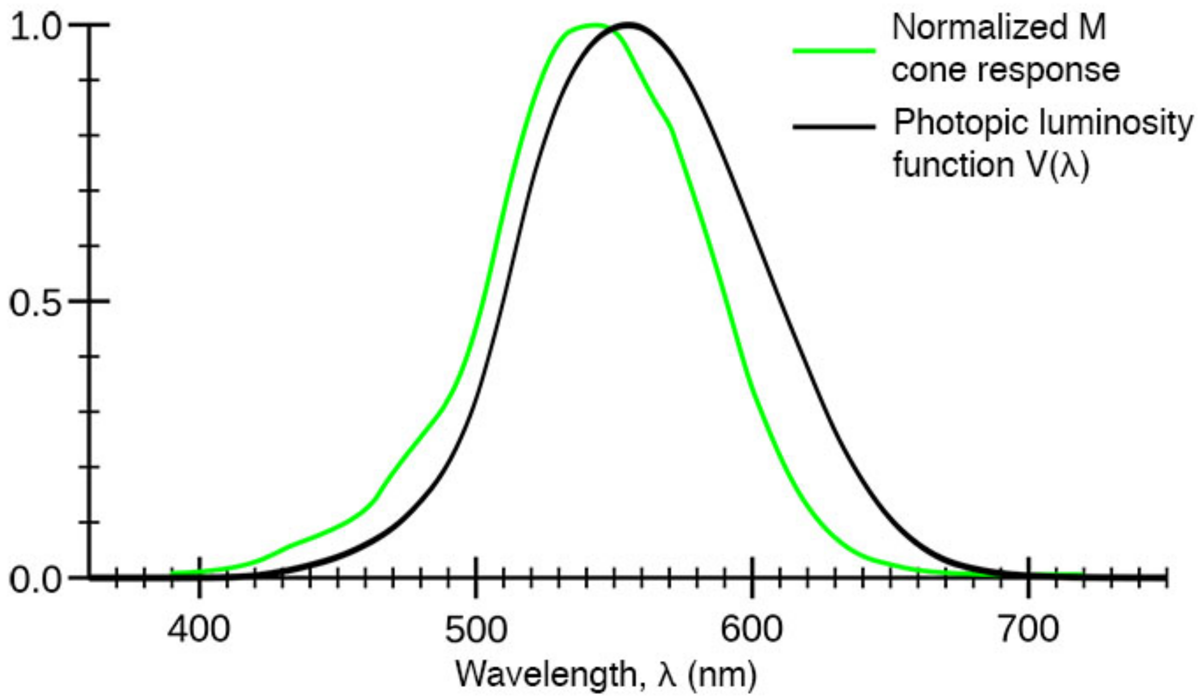

Fig. 1. Human photopic luminous efficiency function (luminosity function) and response function of M cones. Adapted from: https://en.wikipedia.org/wiki/File:Comparison_between_CIE_luminosity_function_and_M_cone_response.svg

2.1. The Reference Illuminant.

Both Wright and Guild used the N.P.L. Standard White Light as their reference white illuminant.[3] Guild (1932, Table 1) gives its spectral characteristics — that is, its radiant power at wavelengths from 380 to 780 nm. I have reproduced the data in the Appendix (my Table 1). In addition to the spectral composition, Guild also needed to determine the brightness of the reference white to human observers. Humans are most sensitive to light with a wavelength of about 555 nm. Sensitivity falls off at shorter and longer wavelengths, eventually declining to zero. This sensitivity as a function of wavelength is described by the photopic luminous efficiency function (or photopic luminosity function), $V(\lambda)$ (Fig. 1).[4] If we multiply the radiant power of the illuminant, at each wavelength, by the luminous efficiency function, at each wavelength, we get the luminance (perceived brightness) of the light source at each wavelength

[3] N.P.L. stands for National Physical Laboratory.

[4] Fig. 1 also shows the spectral sensitivity of M cones (sometimes referred to as the green-sensitive color receptors of the eye). The fact that the two curves are close together suggests that perceived brightness is mostly due to our sensation of green. That will become evident later when we derive an RGB color space, and it plays an important role in the development of the XYZ color space.

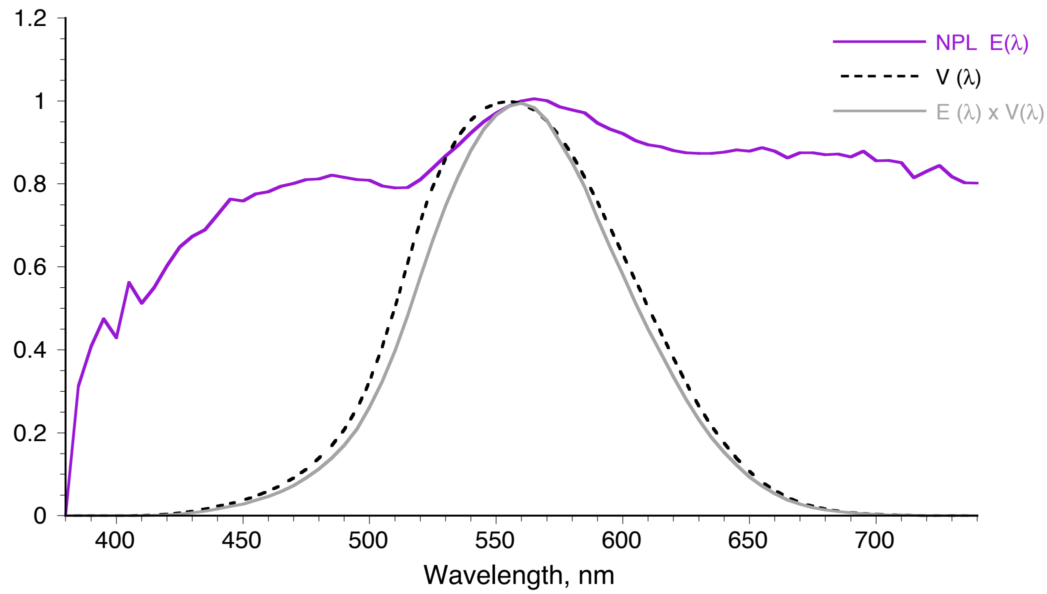

Fig. 2. N.P.L Standard White Light. NPL E(λ) is the "energy distribution curve" of the N.P.L. Standard White Light; *V*(λ) is the human photopic luminous efficiency function; and *E*(λ) x *V*(λ) is the luminance (brightness) of the N.P.L. Standard White as is appears to us.

(Fig. 2).[5] As simple as it looks, the luminance curve of the N.P.L. Standard White Light occupies a central place in this story. After we have derived the color matching functions, we will be able to use them, together with the spectral power distribution of the N.P.L. white, and the relative luminances of the primaries, to exactly re-create that curve.

2.1. Trichromatic Coefficients.

As can be seen (Fig. 2), the reference illuminant is composed of light of many wavelengths. The goal is to recreate that complex mixture with just three primary lights. It will be done piece-by-piece, with the "pieces" being made by color matching light of different wavelengths. In short, observers were asked to match spectral test colors by mixing three primary-color lights. Spectral colors are monochromatic; *i.e.,* consist of a single wavelength. Matches were obtained by adjusting the relative amounts of the three primaries. Wright's observers matched spectral colors between 400 and 700 nm in 10 nm steps (Wright 1930).

[5] Guild never attached units to luminosity. In fact, luminosity does not appear to be a formally defined *photometric* quantity at this time. The units of the luminous efficiency function (Fig. 1) are lumens/watt. Guild (1932, Table 1) calculated the "luminosity" of his white illuminant by multiplying its "energy distribution curve" by the luminous efficiency function. No units were given for the energy distribution curve, but radiant energy is expressed in watts or watts/unit area. Thus, luminosity is either lumens or lumens/unit area (at least in Guild's Table 1). Lumens/unit area is either *luminous intensity* (lumens/steradian) or *luminance* (lumens/steradian/m^2). More recent papers, such as Fairman et al. (1997) appear to use *luminance* in most instances in which Guild (1932) or Wright (1929) use luminosity. I will try to follow Fairman et al. and avoid *luminosity* except when using Guild and Wright's terminology intentionally.

Guild's observers covered the range from 380 to 700 nm in 5 nm steps. Wright and Guild expressed color matching relationships in terms of *trichromatic coefficients* that summed to one. For example, the match to a test color, C, was expressed as

$$C \equiv \alpha R + \beta G + \gamma B \qquad \text{Eq. 1}$$

where α, β, and γ are the trichromatic coefficients. Wright explains how the trichromatic coefficients were determined with a simple example. Suppose that "white" for a particular observer is achieved by a mixture of 30 units of red, 15 units of green, and 60 units of blue; the units being the apertures in the colorimeter through which light from the primaries passes. Since it is desired that "white" be an equal mixture of all three primaries, the instrument readings for the three primaries need to be adjusted by factors that will make them equal. Arbitrarily, we can decide to multiply the red reading by a factor of two and the green by a factor of four so that they both equal 60, which was the reading for blue. These factors are then applied to the instrument readings for color matching. Suppose the same observer obtained a match to a particular test color with instrument readings of 40 red, 10 green, and 20 blue. Then, multiplying red by two and green by four, the "corrected" numbers become 80 red, 40 green and 20 blue. These add to 140, so the trichromatic coefficients become 0.571R, 0.286G, and 0.143B. This procedure compensates both for differences in radiant power of the three primaries, and for the different sensitivity of the human eye to light of different wavelengths. In effect, the trichromatic coefficients are the mixing proportions under the assumption that the primary colors have equal luminance. These "corrected" coefficients are on a scale of dimensionless "trichromatic units" that are the same for each of the three primaries. If you are familiar with *chromaticity,* the similarity of trichromatic coefficients and chromaticity coefficients will be apparent: they are the same. Although Wright did not use the term "chromaticity" he noted that scaling the coefficients of the primaries so that they add up to one "in effect, separates the two variables, luminosity and colour, so that each can be dealt with individually." (Wright 1929, p. 142).

A crucial aspect of these and all similar experiments is that when the test color is a spectral color, it is not possible to obtain a match by adding the three primaries together. Instead, a match can only be obtained by "adding" one of the primaries to the test color, and using the remaining two primaries to make the match.[6] For example, to match a spectral color, C, of wavelength 500 nm, Guild and Wright found that it was necessary to add some red to the spectral color. From Guild's Table 4, we obtain the following relationship:

$$C_{500} + 0.9494R \equiv 1.1727G + 0.7767B \qquad \text{Eq. 2}$$

Wright and Guild treated the primary that was added to the test color as *negative* color, and it is customary to re-write Eq. 2 as:

$$C_{500} \equiv -\,0.9494R + 1.1727G + 0.7767B \qquad \text{Eq. 3}$$

[6] Adding a primary to the test color is known as *de-saturating* the test color. Because both Wright and Guild reported matches only to spectral colors, every color has one negative trichromatic coefficient.

Fig. 3 is a reproduction of Wright's (1929) Fig. 6. The axes are labeled "Red" and "Green" and there is a *color triangle* formed by the locations of the three primaries. In Wright's own words, "The abscissae represent the red [trichromatic] coefficients and the ordinates the green, whilst the values for the blue are given as the differences from unity of the sums of the red

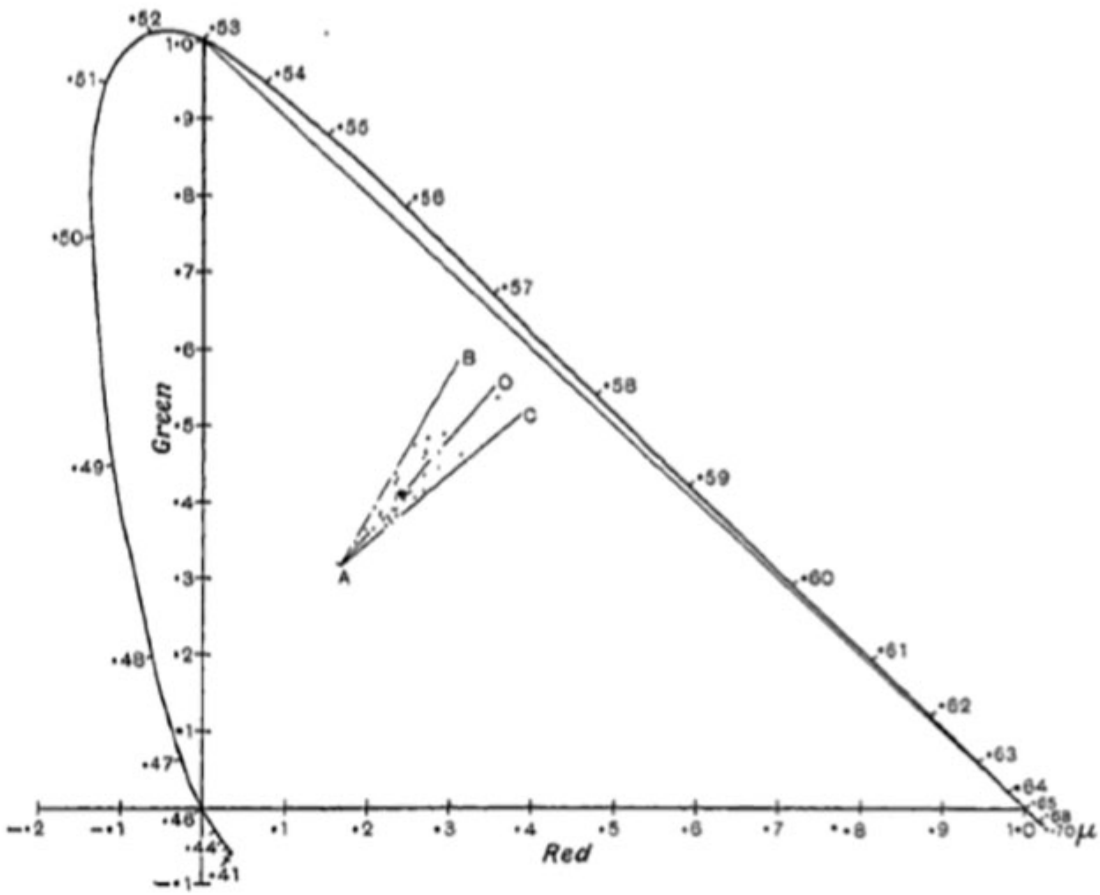

Fig. 6. Colour triangle showing mean spectral locus of 10 observers and white-points of 36 observers

Fig. 3. Wright's Fig. 6, showing the mean spectral locus of 10 observers plotted on axes showing red and green color-matching coefficients (*r-g* trichromatic coefficients). Wright, W. D. 1929. A re-determination of the trichromatic coefficients of the spectral colours. Transactions of the Optical Society. 30:141-164.

and green coefficients. The blue primary is thus represented by the origin." (Wright 1929, p. 152). The curved line in Fig. 3 represents the *spectral locus.* It is the location of the *r-g* trichromatic coefficients of the spectral colors. The reasons for its characteristic horseshoe shape should now be apparent. The bulge to the left of the "Green" axis is due to spectral colors whose match required a negative amount of the red primary (*e.g.*, the spectral color C_{500}, Eq. 3). The slight bulge along the hypotenuse of the triangle is due to spectral color matches that required a negative amount of blue; and the ends of the spectral locus that reach below the "Red" axis represent colors that required a negative amount of green for their matches. The spectral locus is commonly depicted in chromaticity diagrams of the CIE xyY color model. Implicit in this diagram is the fact that it is not possible to synthesize all visible colors by mixing three primaries with only non-negative components. We will return to that point later in the discussion of the CIE 1931 XYZ color space.

2.2. "Luminosity Factors".

As already mentioned, the procedure for calculating trichromatic coefficients separated color information (chromaticity) from brightness (luminance). For the complete description of a color, we need both. Guild proceeded to quantify the "luminosity factors" (relative luminances) of the lights used for the primary colors in his colorimeter.[7] Eventually, after "correcting" his data to standard reference primaries, he settled on the ratio $L_R : L_G : L_B = 1 : 4.390 : 0.048$.[8] It's the ratio that's important — not the actual numbers. It would make no difference if they were all multiplied or divided by ten. This is a scaling factor that will ensure that the color matching functions reproduce the luminance of the standard illuminant at each wavelength. These numbers mean that the green primary was 4.39 times as luminous as the red primary, and 91.46 times as luminous as the blue primary. Guild then used these to estimate the luminance factors of the primary mixtures for each spectral color match. For example, the trichromatic coefficients for the spectral color with wavelength 500 nm were $-0.9494R + 1.1727G + 0.7767B$ (Guild 1932, Table 4). The coefficients were then multiplied by the relative luminances of their respective primaries and summed, to give the luminance factor for $C_{\lambda=500\text{nm}}$:

$$L_{\lambda=500\text{nm}} = -0.9494 + (4.390 \times 1.1727) + (0.048 \times 0.7767) = 4.236 \qquad \text{Eq. 4}$$

Luminance factors for all the spectral color matches are given in Guild (1932) Table 4, and I have reproduced them along with the trichromatic coefficients. (Appendix, Table 2).

Section 3. From Wright – Guild to CIE 1931 RGB

In a second paper, Wright (1930) re-calculated his trichromatic matching coefficients on the assumption that the primaries had been the same as those to which Guild had standardized his own data (Guild 1932, Table 3). Agreement between the standardized Wright and Guild data sets was very close. The fact that independent experiments, conducted in different laboratories with different apparatus and using different primaries, had yielded very similar results increased confidence that the data were good enough for constructing an international standard of color

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[7] For the rest of this paper, I will follow what appears to be current practice, and refer to these as "luminance factors".

[8] The reference primaries are 700 nm, 546.1 nm, and 435.8 nm. These were not the actual working primaries in Guild's colorimeter.

definition — despite the fact that only 17 observers had been used. Both data sets thus became the basis of the CIE 1931 RGB and XYZ color spaces.

3.1. The Color Matching Functions

At this point, I will deviate from the way that Wright and Guild presented their results, in order to frame them in terms that were adopted somewhat later, and with which we are familiar. To summarize so far: we have described the white illuminant (N.P.L. Standard White) in terms of its spectral components and luminance; found empirical color matches for the spectral components; and characterized the relative luminances of the primaries. With these "pieces" in hand, we can begin the process of reconstructing the N.P.L. white light.

The color matching functions are symbolized as:

$$\bar{r}(\lambda) \quad \bar{g}(\lambda) \quad \bar{b}(\lambda)$$

It will be easier to refer to them in the text as CMF_R, CMF_G, and CMF_B. They are calculated by multiplying the trichromatic matching coefficients at each wavelength by the ratio of the photopic luminous efficiency function, $V(\lambda)$, to the luminance factor $L(\lambda)$ of the color match mixture (Appendix, Table 2). In English (I hope): the color matching function encodes the chromaticity of one primary at each wavelength, together with the scaled luminance of the entire

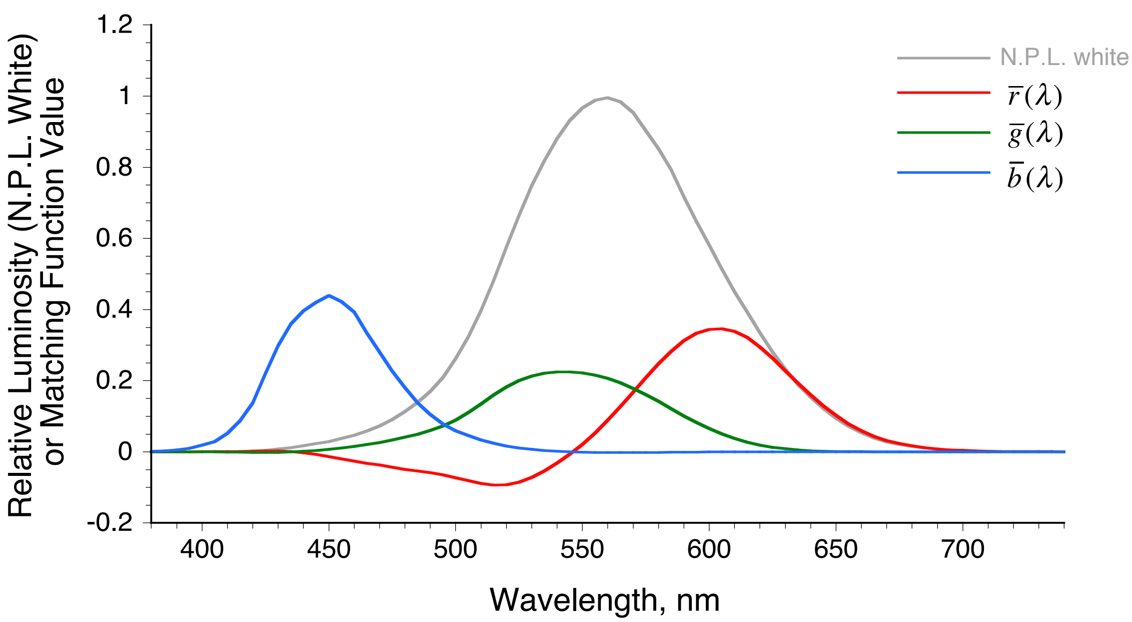

Fig. 4. Color matching functions and luminance curve of the N.P.L. Standard White.

color match at that wavelength. It is worth noting that the color matching functions are specific to the primaries used for color matching, and to the reference white illuminant, because the trichromatic coefficients depend on those parameters. If other primaries are used, or if the reference white is changed, or both, the color matching functions will also change. The color matching functions for the Wright – Guild data (Guild 1932) are shown in Fig. 4 (the data can be seen in the Appendix, Table 2).

3.2. Tristimulus Values.

Now it is time to condense our wavelength-by-wavelength matching function description of the reference illuminant to just three numbers: the tristimulus values *R, G,* and *B*. In words, we multiply the value of the color matching function for a given primary by the energy of the illuminant at that wavelength and sum the results over all wavelengths. Symbolically:

$$R = \sum_{\lambda=380}^{\lambda=780} \bar{r}(\lambda)E(\lambda) \qquad G = \sum_{\lambda=380}^{\lambda=780} \bar{g}(\lambda)E(\lambda) \qquad B = \sum_{\lambda=380}^{\lambda=780} \bar{b}(\lambda)E(\lambda)$$

This calculation is shown in Table 3 (Appendix). The tristimulus values summed over the spectral energy distribution of the N.P.L. reference white are shown at the bottom of columns 6 – 8 of Table 3. The important point is that they are equal (within the limits of the data), which is precisely the point given that these calculations are a description of the reference white illuminant. That is, white is composed of an equal amount of red, green, and blue. Assuming that the N.P.L. light was being used at its full power, it was the brightest white we can make, so we might choose to scale the tristimulus values so that they are each 1.0 (or 255 if we want 8-bit scaling). If we halve the power of light at each wavelength, it will be less bright, and the tristimulus values would also be halved (to 0.5, for example).[9]

Guild (1932) did the tristimulus value calculations for both the N.P.L. illuminant and a hypothetical equal-energy-spectrum (EES) illuminant. These are shown on the right side of his Table 4, and are labeled "Ordinates of the Spectral Distribution Curves". He did not present their sums over wavelengths — the quantities that we are calling tristimulus values. However, we know that he actually did calculate the sums. The geometrical representation of the sums is an area: in this case the area under each matching function curve scaled by the spectral energy distribution of the illuminant. In fact, Guild slightly adjusted the relative luminances of the primaries (L_R, L_G, L_B) so that the areas more closely matched. "The areas under the three distribution curves represent the relative contributions of the primaries in matching the standard white and should be equal" (Guild 1932, p. 166).

3.3. Re-creating the Luminosity Curve of the N.P.L. Standard White Light.

If we multiply the tristimulus values for each wavelength by the relative luminances of the three primaries, we obtain three curves whose areas *sum* to the area under the luminance curve of the N.P.L. white light. The calculations are in Table 4 (Appendix), and the curves are in

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[9] Whether the light would seem half as bright is another issue, which I will not explore.

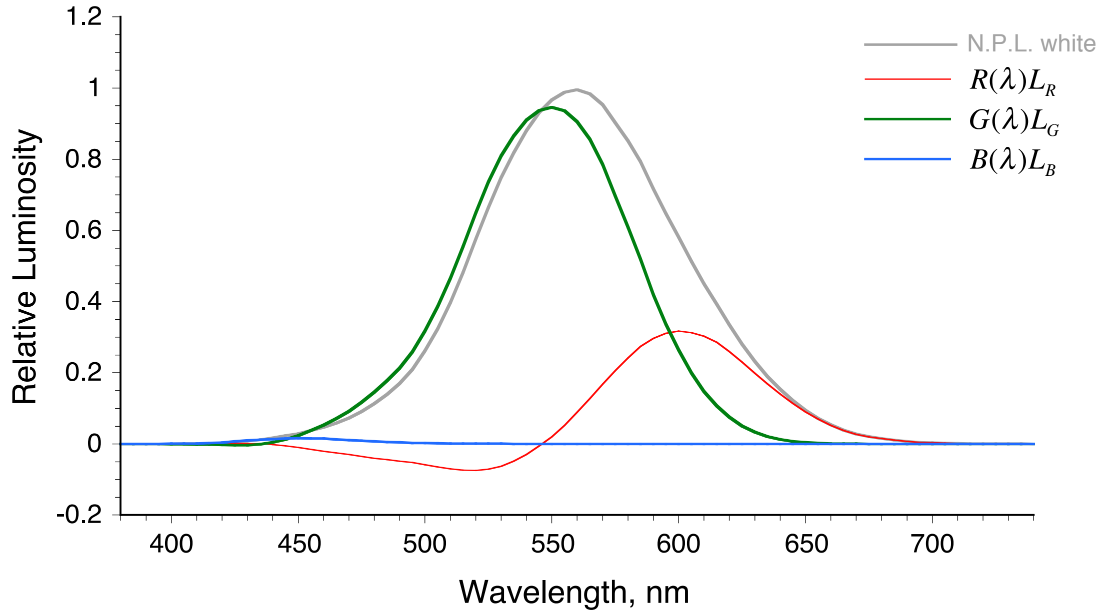

Fig. 5. Luminance of the trichromatic match to the N.P.L. Standard White Light.

Fig. 5. The areas under the three curves are 3.5839, 15.7662; and 0.1724; and sum to 19.5224, which is the area under the N.P.L. white luminance curve, as shown at the bottom of the right-hand column of Table 1. A striking feature of this result is that almost all of the brightness of the standard white has been captured by the green tristimulus value and the luminance of the green primary. That is perhaps not too surprising if we recall that the photopic luminous efficiency function reaches a maximum at 555 nm (Fig. 1). In fact, from the standpoint of luminance of the reference white, blue makes essentially no contribution — about 0.9%.

At this point, I have achieved my stated goal: to decompose the reference white illuminant into its constituent wavelengths; to synthesize those constituent wavelengths with three primary colors; and lastly to reassemble the components into a re-creation of the reference illuminant. The process is summarized in Fig. 6. This would not be a very useful exercise if that were all there was to it. *The value of the color matching functions is that they can be used to specify any color. We have created a universal system for color definition.*[10] The next section gives two examples.

[10] I will call the color space that we have created the Wright – Guild RGB space. It is very similar to the CIE 1931 RGB color space (see Section 4, below).

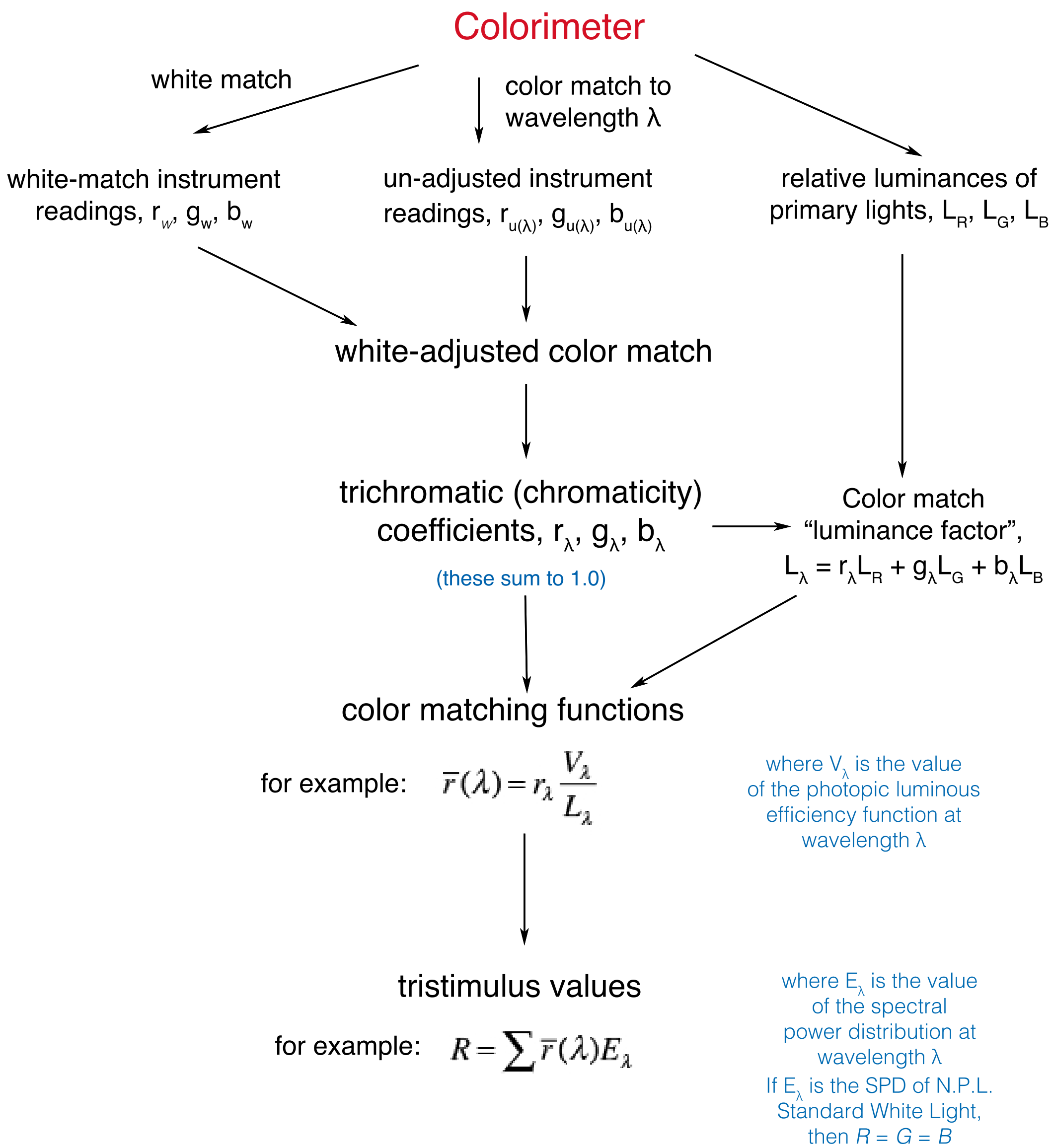

Fig. 6. Flow chart: from color matching to tristimulus values. For brevity, only the red color matching function and tristimulus value are shown.

3.4. Specifying a Color in Wright – Guild RGB Space.

Imagine that we covered the N.P.L. white bulb with a filter that completely blocked all wavelengths from 525 to 625 nm, and allowed the other wavelengths to pass through unaffected. What color would the filter have in the transmitted light? Given the spreadsheet used to generate Tables 1 – 4, it is easy enough to figure out: simply set the spectral power distribution of the illuminant to zero for those wavelengths. After scaling to 8-bits, the RGB tristimulus coefficients are [22, 59, 255]. That seems mostly blue, or perhaps somewhere between blue and

Fig. 7. The Wright – Guild RGB color [22, 59, 255] as simulated in ProPhoto RGB.

cyan. Perhaps the best I can do is show you what those coefficients look like in ProPhotoRGB translated to sRGB (Fig. 7).[11] To my eyes, that is a rather bright, saturated medium blue, although it might well look a bit different in the Wright – Guild RGB space that we have constructed here. If N.P.L. white has a luminance of 1.0, the luminance of this blue is 0.21, about 88% of which is attributable to the green component.

What happens if we use a different filter that allows only wavelenghts from 500 to 550 nm to pass. Well, that will be green. The RGB tristimulus coefficients are [-38, 123, 12]. That is not a typographical error. Negative tristimulus values are perfectly valid in this Wright – Guild RGB space and in the closely related CIE 1931 RGB space. For *defining* a color in numerical terms, negative values are not problematic. But they are nonsensical for *generating* colors with a physical device such as an LED display. The trade-off is that while the Wright – Guild RGB model can define any humanly visible color for which we can obtain a spectral power distribution, it is unsuited for color reproduction. On the other hand, sRGB can be used for color reproduction, but is able to reproduce only a fraction of all visible colors: it has a limited gamut.

Section 4. The CIE 1931 XYZ Color Space

At a meeting in England in 1931,the Colorimetry Committee of the International Commission on Illumination established the CIE 1931 RGB color space. With relatively minor modifications, this is the Wright – Guild RGB space that was defined in Guild (1932, Table 4), and which I have reproduced here. When the CIE standard was established, the relative luminances of the primaries were adjusted slightly (to 1 : 4.5907 : 0.601), and the reference white illuminant became a theoretical equal-energy-spectrum (EES) white, rather than the N.P.L. white.[12] Like its progenitor, the CIE 1931 RGB space includes *all* visible colors. Having

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[11] After translation to sRGB, the HSB coordinates are 221, 100%, 100%: and the Lab coordinates are 41, 37, -93. In fact, this color is *outside* the ProPhotoRGB gamut, so the color of Fig. 6 can only be approximate, at best.

[12] A complete explanation of the transformation of Wright's data and Guild's data to the standard reference primaries (700, 546.1, and 435.8 nm); and the transformation of that Wright-Guild RGB space to the standard CIE 1931 RGB is given by Broadbent, A. D., Calculation from the original experimental data of the CIE 1931 RGB standard observer spectral chromaticity co-ordinates and color matching functions.

established the RGB standard, the committee immediately took up the problem of creating a transformation that would be called the CIE 1931 XYZ space.

The motivation for the XYZ space seems to have been two-fold. First, it was thought desirable to have a system in which all tristimulus values would be positive. This apparently had nothing to do with avoiding the conceptual difficulties of negative color or the need to have a model that could be used for color reproduction. Rather, in an age of hand calculation, it was felt that errors would be minimized if all values were positive. Second, as we have seen (Figs. 1 and 5), almost all the brightness information about a color is in the green "channel". It would be extremely convenient, particularly on computational grounds, if *all* the luminance of a color could be conveyed by just one tristimulus value.[13] The CIE 1931 XYZ color space is a theoretical construct, but it is rooted in the CIE 1931 RGB space, and thus the experimental results of Wright and Guild (and others). On a purely mechanical level, the XYZ space is just a mathematical transformation of the RGB space to a new set of primary "colors". We will not be concerned with the mathematical details.[14] It should be noted that no additional color matching data was needed, or was used.

Chromaticity diagrams are useful geometrical representations of color spaces in two-dimensions (Fig. 3). By convention, chromaticity coordinates are indicated with lower-case letters. For RGB spaces:

Note that $r + g + b = 1$; and, therefore, any two are sufficient to describe the chromaticity of a color. The convention is to show the red and green coordinates, with the understanding that $b =$

$$r = \frac{R}{(R+G+B)} \qquad g = \frac{G}{(R+G+B)} \qquad b = \frac{B}{(R+G+B)}$$

$1 - r - g$. The chromaticity diagram of the CIE 1931 RGB space is shown in Fig. 7.[15] The positions of r (1, 0); g (0, 1); and b (0, 0) on the diagram define a color triangle; and the three vertices represent the primary RGB colors. The only colors that can be made by mixing the primaries, and without resorting to negative coefficients, are colors whose *r-g* chromaticity coordinates lie within the triangle, or along the lines connecting the vertices. The curved line is the spectral locus, which encloses all visible colors. Note that it is everywhere *outside* of the

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13 The idea that one matching function might be able to encode all the luminance information was apparently first suggested by Erwin Schrödinger, winner of the 1933 Nobel Prize for Physics, and best known as the father of quantum mechanics, and for his cat.

14 A detailed explanation of the process of deriving the CIE 1931 XYZ space the CIE 1931 RGB space can be found in Fairman, et al. (1997). Given computers, computation is now routine. It happens almost instantaneously for millions of pixels every time you change your working color space in Photoshop from ProPhotoRGB to sRGB to Lab, and vice versa, for example. The mathematical details of such transformations can be found at brucelindbloom.com.

15 This figure is based on the data for the CIE 1931 RGB color space (i.e., the transformed Wright-Guild data, rather than the Wright-Guild data as it has been used to this point). The data are given in Table 5 (Appendix) and can be downloaded as a spreadsheet here.

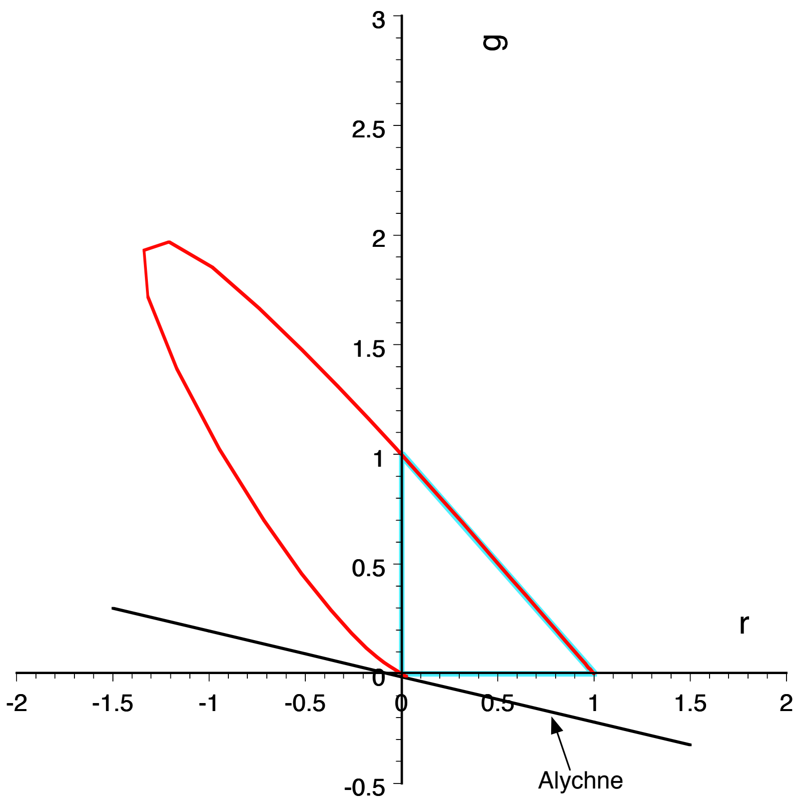

Fig. 7. Chromaticity diagram for the CIE 1931 RGB color space. Spectral locus in red. The color triangle formed by the primaries is defined by the three points (0, 0); (1, 0); and (0, 1).

color triangle (although it lies very close along the hypotenuse). The only way to use *rgb* chromaticity coordinates to specify visible colors outside of the triangle is to use negative values. Given that a stated goal of the XYZ space was to have only positive coefficients, Fig. 7 suggests a geometrical solution: "design" the primaries so that they define a color triangle that completely encloses the spectral locus. Then, all points on the spectral locus will have positive coordinates in the new *x-y* chromaticity space.

The second goal was to design the green color matching function so that it conveyed all the luminance information. The corollary of that is that the red and blue "primaries" should have zero luminance. That can be achieved by placing the new *X* and *Z* primaries on a line known as the alychne, the line of zero luminance (also in Fig. 7).[16] Given that, we can put the new *XYZ* primaries on the *r-g* chromaticity diagram in such as way that the color triangle formed by them completely encloses the spectral locus and alychne (Fig. 8).[17] It remains only to transform Fig. 8 to the new *x-y* chromaticity coordinates (Fig. 9). Just as with *rgb* coordinates, the *xyz*

[16] Alychne was apparently coined by Schrödinger from ancient Greek meaning "light-less" or no light.

[17] The method for determining the exact locations of the *XYZ* primaries is described in Fairman, et al.

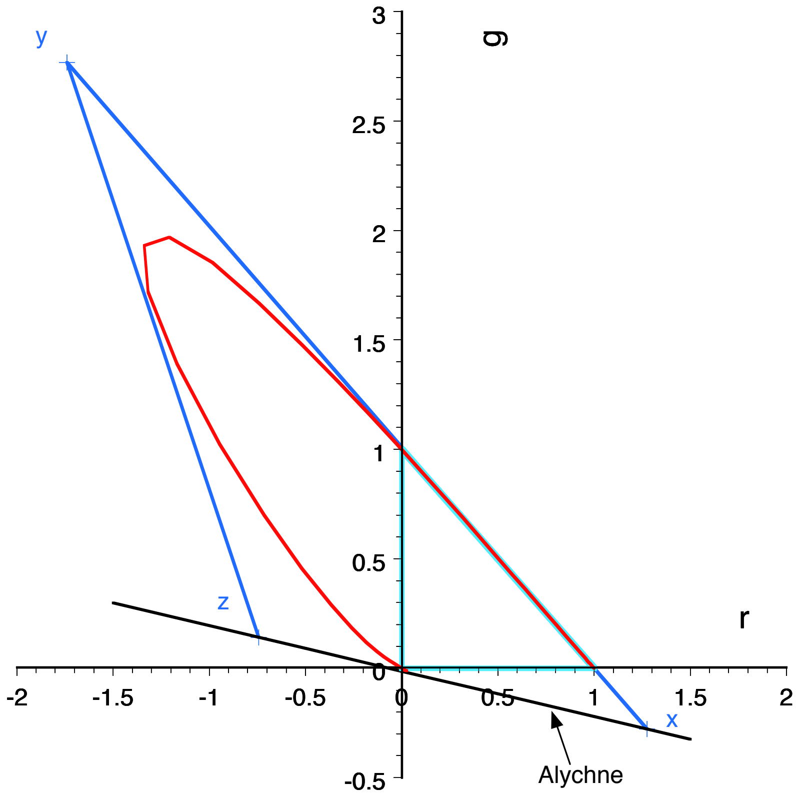

Fig. 8. Chromaticity diagram for CIE 1931 RGB color space with CIE 1931 XYZ color triangle superimposed. Spectral locus in red.

chromaticity coordinates sum to one and only two are needed to completely specify the chromaticity of a color — by convention, these are x and y. Because all the luminance information is in Y, any color in the XYZ space can be described by x, y, and Y.

In the new x-y chromaticity space (Fig. 9), the color triangle (*xyz*) and the spectral locus both lie in the positive (upper right) quadrant, thus achieving the goal of eliminating negative chromaticity coefficients for visible colors. In order to enclose the spectral locus with the *xyz* color triangle, it was necessary to put the XYZ primes in the region outside the locus. Because the spectral locus and the alychne define the region of all the visible colors, we see that the XYZ

$$x = \frac{X}{(X+Y+Z)} \qquad y = \frac{Y}{(X+Y+Z)} \qquad z = \frac{Z}{(X+Y+Z)}$$

primaries are not actual colors. They are often referred to as imaginary colors. Really they are simply mathematical constructs whose sole function is to ensure that the chromaticity coefficients of all visible colors are greater than or equal to zero. Not only are the XYZ primaries not real, two of them have zero luminance — they have chromaticity only.

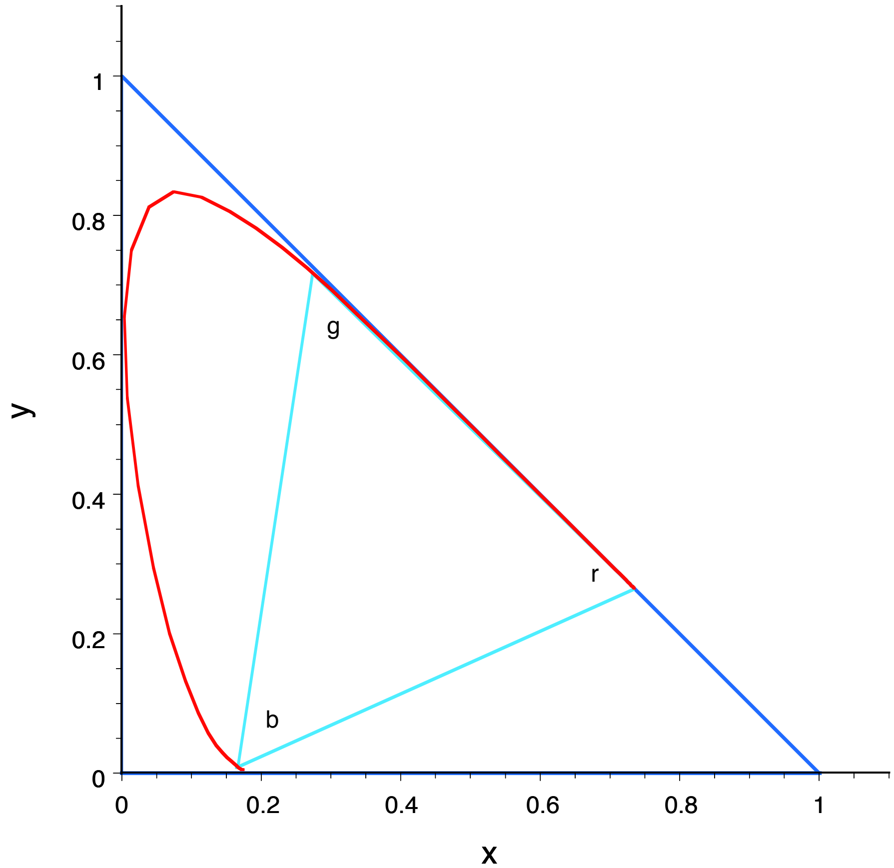

Fig. 9. CIE 1931 XYZ chromaticity diagram. Blue: XYZ color triangle. Red: spectral locus. Cyan: CIE 1931 RGB color triangle. The *x*-axis (*i.e.,* the line where *y*=0) is the alychne.

Section 5: Discussion

The importance of the reference illuminant in defining any RGB model is, I think, mostly overlooked. This re-construction of the development of the Wright-Guild model highlights the role of the white light source. First, color matches to that white were used to adjust the matches to the spectral colors, which determined their chromaticities and ultimately the color matching functions. Second, if those color matching functions are summed over the spectral power distributions of *different* "white" light sources, the relationship $R = G = B$ will not generally hold, and the "white" will be tinted. In short, the reference white is an integral part of the definition of an RGB model. White in an sRGB encoded image (D65 illuminant) will not be white when converted to ProPhotoRGB (D50 illuminant), unless an appropriate chromatic adaptation step is included in the conversion.

Because the XYZ primaries are not real colors, they cannot be used to generate colors on a physical device, such as an LED display. The sole function of the CIE 1931 XYZ color space is to provide a standard for defining all visible colors with non-negative coefficients; and in such a way as to encode all the luminance information in a single parameter, *Y.* If this seems overly abstract, it may help to remember this: physical color spaces, such as sRGB and AdobeRGB are defined by reference to the XYZ standard. Once a color space is defined by reference to XYZ, it is interconvertible with all other spaces that are also referred to XYZ. In effect, the CIE 1931 XYZ space provides a "common language" that allows different color spaces to "speak" with one another. This translation happens whenever you switch between RGB and Lab modes in Photoshop, or whenever you switch between various RGB spaces. Lastly, CIE XYZ is used as a bridge to translate raw images in native camera RGB space to working RGB spaces such as ProPhoto, AdobeRGB, or sRGB. The evidence for this can be seen in the metadata of DNG files made by Adobe DNG Converter, where the camera-to-XYZ matrix is the "Forward Matrix".

It should be pointed out that the frequently used ProPhotoRGB space is also defined by imaginary primaries; but is nevertheless frequently recommended as a working color space for raw image workflow. Digital cameras are input devices: sensors can, in general, "see" all visible wavelengths.[18] Raw data is recorded in red, green, and blue channels, but cameras are not devices for making color, and there is no requirement that the camera RGB space be physically reproducible. An important function of the raw converter is deciding how the camera red, green, and blue channels should be translated into a standard color space, and which space that should be. As already mentioned, Adobe appears to be use XYZ in its raw conversion software. That makes sense because the XYZ gamut is relatively large, it includes all visible colors, and because of the central position of XYZ as a bridge between all RGB spaces.

If you want to preserve the maximum amount of color information captured by a camera, shoot raw and archive the raw files; also convert them to DNG if you want some insurance against loss of propriety raw converters in the future. If you shoot JPEG, the most color information you will ever have is whatever can be contained in the embedded JPEG color space — usually sRGB or AdobeRGB. But, you say, given that we can't see much more than sRGB on a typical display, and given that printer gamuts are also limited, there's no real reason to preserve all that color information. There are at least two arguments against that position. First, at least in theory, more color information means more latitude for image adjustment in post processing. That is why ProPhotoRGB is recommended as a working color space — it's the largest RGB space in common use and will, presumably, entail the least compression of information in the raw file. Second, output device gamuts are likely to improve in the future. For example, displays with more than three "primary" colors can have gamuts that are much larger than either sRGB or AdobeRGB.[19] If such displays eventually make it to market, and if converters can process legacy raw files to the new encoding; then color information, previously hidden, will be revealed.

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[18] This is not the place to discuss camera "gamut", but see, for example, Holm, Jack. Capture color analysis gamuts.

[19] See, for example, Chan, C-C., et al. Development of multi-primary color LCD; and Brill, M.H., and J. Larimer. Metamerism and multi-primary displays.

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Guild, J. 1932. The colorimetric properties of the spectrum. Philosophical Transactions of the Royal Society of London, Series A. 230:149-187.

Wright, W. D. 1929. A re-determination of the trichromatic coefficients of the spectral colours. Transactions of the Optical Society. 30:141-164.

Wright, W. D. 1930. A re-determination of the mixture curves of the spectrum. Transactions of the Optical Society. 31:201-218.

RESOURCES

Broadbent, A. D. Calculation from the original experimental data of the CIE 1931 RGB standard observer spectral chromaticity co-ordinates and color matching functions. This web page also includes a downloadable Excel file with the CIE 1931 RGB data.

Munsell Color Science Laboratory, Rochester Institute of Technology. Excellent resource for downloadable data in Excel files.

Lindbloom, Bruce. www.brucelindbloom.com. An excellent on-line color calculator. Also an excellent source for the many mathematical formulas used for transforming color data among color spaces, calculating color differences, etc.

Appendix

Table 1. Spectral Power Distribution and Luminance of the N.P.L. Standard White Light

Wavelength, λ (nm)	*E*(λ)*	*V*(λ)**	Luminance *E*(λ) x *V*(λ)
380	0.00000	0.00004	0.0000
385	0.31250	0.00006	0.0000
390	0.40833	0.00012	0.0000
395	0.47465	0.00022	0.0001
400	0.42929	0.00040	0.0002
405	0.56250	0.00064	0.0004
410	0.51240	0.00121	0.0006
415	0.55046	0.00218	0.0012
420	0.60250	0.00400	0.0024
425	0.64795	0.00730	0.0047
430	0.67328	0.01160	0.0078
435	0.68943	0.01684	0.0116
440	0.72609	0.02300	0.0167
445	0.76342	0.02980	0.0227
450	0.75921	0.03800	0.0288
455	0.77604	0.04800	0.0372
460	0.78167	0.06000	0.0469
465	0.79432	0.07390	0.0587
470	0.80128	0.09098	0.0729
475	0.81083	0.11260	0.0913
480	0.81211	0.13902	0.1129
485	0.82162	0.16930	0.1391
490	0.81627	0.20802	0.1698
495	0.81052	0.25860	0.2096
500	0.80867	0.32300	0.2612
505	0.79548	0.40730	0.3240
510	0.79066	0.50300	0.3977
515	0.79168	0.60820	0.4815
520	0.81085	0.71000	0.5757
525	0.83800	0.79320	0.6647
530	0.86740	0.86200	0.7477
535	0.89348	0.91485	0.8174
540	0.92338	0.95400	0.8809
545	0.95022	0.98030	0.9315

Table 1. Spectral Power Distribution and Luminance of the N.P.L. Standard White Light

Wavelength, λ (nm)	***E*(λ)***	***V*(λ)****	**Luminance *E*(λ) x *V*(λ)**
550	0.97110	0.99495	0.9662
555	0.98810	1.00000	0.9881
560	1.00000	0.99500	0.9950
565	1.00542	0.97860	0.9839
570	1.00105	0.95200	0.9530
575	0.98656	0.91540	0.9031
580	0.97874	0.87000	0.8515
585	0.97133	0.81630	0.7929
590	0.94663	0.75700	0.7166
595	0.93251	0.69490	0.6480
600	0.92155	0.63100	0.5815
605	0.90438	0.56680	0.5126
610	0.89463	0.50300	0.4500
615	0.89030	0.44120	0.3928
620	0.88110	0.38100	0.3357
625	0.87508	0.32100	0.2809
630	0.87396	0.26500	0.2316
635	0.87327	0.21700	0.1895
640	0.87657	0.17500	0.1534
645	0.88278	0.13820	0.1220
650	0.87944	0.10700	0.0941
655	0.88725	0.08160	0.0724
660	0.87869	0.06100	0.0536
665	0.86362	0.04458	0.0385
670	0.87500	0.03200	0.0280
675	0.87500	0.02320	0.0203
680	0.87059	0.01700	0.0148
685	0.87248	0.01192	0.0104
690	0.86480	0.00821	0.0071
695	0.87891	0.00572	0.0050
700	0.85568	0.00410	0.0035
705	0.85695	0.00293	0.0025
710	0.85127	0.00209	0.0018
715	0.81536	0.00148	0.0012
720	0.83095	0.00105	0.0009
725	0.84459	0.00074	0.0006

Table 1. Spectral Power Distribution and Luminance of the N.P.L. Standard White Light

Wavelength, λ (nm)	***E*(λ)***	***V*(λ)****	**Luminance** ***E*(λ) x *V*(λ)**
730	0.81731	0.00052	0.0004
735	0.80310	0.00036	0.0003
740	0.80257	0.00025	0.0002
745	0.75625	0.00017	0.0001
750	0.75000	0.00012	0.0001
755	0.76651	0.00008	0.0001
760	0.75000	0.00006	0.0000
765	0.00000	0.00004	0.0000
770	0.00000	0.00003	0.0000
775	0.00000	0.00002	0.0000
780	0.00000	0.00001	0.0000
Sum			**19.5224**

* *E*(λ) is the energy distribution curve of the N.P.L. Standard White Light. These numbers, obtained from Broadbent do not precisely match Guild (1932, Table 1).

** *V*(λ) is the CIE (1924) photopic luminous efficiency function. These values are scaled so that V_{555} = 1.0.

Table 2. Wright – Guild Trichromatic Coefficients and RGB Color Matching Functions (Guild, 1932)

		Trichromatic Coefficients (Guild 1932, Table 4)				Color Matching Functions		
Wave-length, λ (nm)	***L*(λ)**	***r***	***g***	***b***	**V(λ)/*L*(λ)***	**CMF_R(λ)**	**CMF_G(λ)**	**CMF_B(λ)**
380	0.021	0.0204	-0.0106	0.9902	0.002	0.0000	-0.0000	0.0019
385	0.021	0.0204	-0.0106	0.9902	0.003	0.0001	-0.0000	0.0028
390	0.021	0.0204	-0.0106	0.9902	0.006	0.0001	-0.0001	0.0056
395	0.021	0.0204	-0.0106	0.9902	0.010	0.0002	-0.0001	0.0102
400	0.021	0.0204	-0.0106	0.9902	0.019	0.0004	-0.0002	0.0185
405	0.022	0.0191	-0.0101	0.9910	0.029	0.0005	-0.0003	0.0284
410	0.023	0.0177	-0.0096	0.9919	0.052	0.0009	-0.0005	0.0518
415	0.025	0.0162	-0.0089	0.9927	0.088	0.0014	-0.0008	0.0873
420	0.029	0.0137	-0.0074	0.9937	0.138	0.0019	-0.0010	0.1375
425	0.033	0.0114	-0.0060	0.9946	0.223	0.0025	-0.0013	0.2214
430	0.039	0.0072	-0.0037	0.9965	0.299	0.0022	-0.0011	0.2980
435	0.047	0.0011	-0.0005	0.9994	0.359	0.0004	-0.0002	0.3590
440	0.058	-0.0070	0.0039	1.0031	0.395	-0.0028	0.0015	0.3959
445	0.071	-0.0178	0.0093	1.0085	0.417	-0.0074	0.0039	0.4207
450	0.088	-0.0315	0.0161	1.0154	0.432	-0.0136	0.0070	0.4389
455	0.116	-0.0487	0.0264	1.0223	0.413	-0.0201	0.0109	0.4221
460	0.157	-0.0697	0.0405	1.0292	0.381	-0.0266	0.0154	0.3921
465	0.229	-0.0983	0.0632	1.0351	0.323	-0.0317	0.0204	0.3343
470	0.338	-0.1376	0.0970	1.0406	0.269	-0.0370	0.0261	0.2800
475	0.519	-0.1990	0.1520	1.0470	0.217	-0.0432	0.0330	0.2274
480	0.805	-0.2846	0.2367	1.0479	0.173	-0.0492	0.0409	0.1810
485	1.267	-0.4019	0.3689	1.0330	0.134	-0.0537	0.0493	0.1380
490	1.959	-0.5527	0.5613	0.9914	0.106	-0.0587	0.0596	0.1053
495	3.001	-0.7480	0.8440	0.9040	0.086	-0.0645	0.0727	0.0779
500	4.236	-0.9494	1.1727	0.7767	0.076	-0.0724	0.0894	0.0592
505	5.424	-1.0800	1.4750	0.6050	0.075	-0.0811	0.1108	0.0454
510	6.347	-1.1203	1.6964	0.4239	0.079	-0.0888	0.1344	0.0336
515	6.729	-1.0311	1.7647	0.2664	0.090	-0.0932	0.1595	0.0241
520	6.657	-0.8637	1.7114	0.1523	0.107	-0.0921	0.1825	0.0162
525	6.288	-0.6739	1.5849	0.0890	0.126	-0.0850	0.1999	0.0112
530	5.839	-0.4879	1.4406	0.0473	0.148	-0.0720	0.2127	0.0070
535	5.367	-0.3165	1.2944	0.0221	0.170	-0.0540	0.2206	0.0038
540	4.900	-0.1617	1.1530	0.0087	0.195	-0.0315	0.2245	0.0017

Table 2. Wright – Guild Trichromatic Coefficients and RGB Color Matching Functions (Guild, 1932)

		Trichromatic Coefficients (Guild 1932, Table 4)				Color Matching Functions		
Wave-length, λ (nm)	***L*(λ)**	***r***	***g***	***b***	**V(λ)/*L*(λ)***	**CMF_R(λ)**	**CMF_G(λ)**	**CMF_B(λ)**
545	4.479	-0.0279	1.0267	0.0012	0.219	-0.0061	0.2247	0.0003
550	4.110	0.0859	0.9166	-0.0025	0.242	0.0208	0.2219	-0.0006
555	3.739	0.1990	0.8064	-0.0054	0.267	0.0532	0.2157	-0.0014
560	3.389	0.3032	0.7029	-0.0061	0.294	0.0890	0.2064	-0.0018
565	3.062	0.3988	0.6068	-0.0056	0.320	0.1274	0.1939	-0.0018
570	2.762	0.4864	0.5185	-0.0049	0.345	0.1676	0.1787	-0.0017
575	2.487	0.5664	0.4376	-0.0040	0.368	0.2085	0.1611	-0.0015
580	2.242	0.6376	0.3655	-0.0031	0.388	0.2474	0.1418	-0.0012
585	2.026	0.7003	0.3020	-0.0023	0.403	0.2822	0.1217	-0.0009
590	1.829	0.7572	0.2442	-0.0014	0.414	0.3134	0.1011	-0.0006
595	1.672	0.8031	0.1979	-0.0010	0.416	0.3338	0.0823	-0.0004
600	1.543	0.8406	0.1600	-0.0006	0.409	0.3438	0.0654	-0.0002
605	1.430	0.8738	0.1267	-0.0005	0.396	0.3463	0.0502	-0.0002
610	1.338	0.9010	0.0995	-0.0005	0.376	0.3388	0.0374	-0.0002
615	1.267	0.9219	0.0786	-0.0005	0.348	0.3210	0.0274	-0.0002
620	1.211	0.9385	0.0620	-0.0005	0.315	0.2954	0.0195	-0.0002
625	1.163	0.9526	0.0479	-0.0005	0.276	0.2630	0.0132	-0.0001
630	1.125	0.9639	0.0366	-0.0005	0.236	0.2271	0.0086	-0.0001
635	1.093	0.9732	0.0273	-0.0005	0.199	0.1932	0.0054	-0.0001
640	1.070	0.9799	0.0206	-0.0005	0.164	0.1602	0.0034	-0.0001
645	1.052	0.9853	0.0152	-0.0005	0.131	0.1294	0.0020	-0.0001
650	1.038	0.9889	0.0111	0.0000	0.103	0.1020	0.0011	0.0000
655	1.028	0.9917	0.0083	0.0000	0.079	0.0787	0.0007	0.0000
660	1.021	0.9937	0.0063	0.0000	0.060	0.0593	0.0004	0.0000
665	1.017	0.9950	0.0050	0.0000	0.044	0.0436	0.0002	0.0000
670	1.013	0.9961	0.0039	0.0000	0.032	0.0315	0.0001	0.0000
675	1.010	0.9971	0.0029	0.0000	0.023	0.0229	0.0001	0.0000
680	1.008	0.9976	0.0024	0.0000	0.017	0.0168	0.0000	0.0000
685	1.005	0.9985	0.0015	0.0000	0.012	0.0118	0.0000	0.0000
690	1.002	0.9993	0.0007	0.0000	0.008	0.0082	0.0000	0.0000
695	1.001	0.9997	0.0003	0.0000	0.006	0.0057	0.0000	0.0000
700	1.000	1.0000	0.0000	0.0000	0.004	0.0041	0.0000	0.0000
705	1.000	1.0000	0.0000	0.0000	0.003	0.0029	0.0000	0.0000
710	1.000	1.0000	0.0000	0.0000	0.002	0.0021	0.0000	0.0000

Table 2. Wright – Guild Trichromatic Coefficients and RGB Color Matching Functions (Guild, 1932)

		Trichromatic Coefficients (Guild 1932, Table 4)				Color Matching Functions		
Wave-length, λ (nm)	***L*(λ)**	***r***	***g***	***b***	**V(λ)/*L*(λ)***	**CMF_R(λ)**	**CMF_G(λ)**	**CMF_B(λ)**
715	1.000	1.0000	0.0000	0.0000	0.001	0.0015	0.0000	0.0000
720	1.000	1.0000	0.0000	0.0000	0.001	0.0011	0.0000	0.0000
725	1.000	1.0000	0.0000	0.0000	0.001	0.0007	0.0000	0.0000
730	1.000	1.0000	0.0000	0.0000	0.001	0.0005	0.0000	0.0000
735	1.000	1.0000	0.0000	0.0000	0.000	0.0004	0.0000	0.0000
740	1.000	1.0000	0.0000	0.0000	0.000	0.0003	0.0000	0.0000
745	1.000	1.0000	0.0000	0.0000	0.000	0.0002	0.0000	0.0000
750	1.000	1.0000	0.0000	0.0000	0.000	0.0001	0.0000	0.0000
755	1.000	1.0000	0.0000	0.0000	0.000	0.0001	0.0000	0.0000
760	1.000	1.0000	0.0000	0.0000	0.000	0.0001	0.0000	0.0000
765	1.000	1.0000	0.0000	0.0000	0.000	0.0000	0.0000	0.0000
770	1.000	1.0000	0.0000	0.0000	0.000	0.0000	0.0000	0.0000
775	1.000	1.0000	0.0000	0.0000	0.000	0.0000	0.0000	0.0000
780	1.000	1.0000	0.0000	0.0000	0.000	0.0000	0.0000	0.0000

* Values for *V*(λ) can be found in Table 1.

Table 3. Color-matching Functions and RGB Tristimulus Values for Wright – Guild Data (Guild, 1932)							
		Color Matching Functions			**(6)**	**(7)**	**(8)**
Wave-length, λ nm	***E*(λ)***	**CMF_R(λ)**	**CMF_G(λ)**	**CMF_B(λ)**	***R*(λ)**	***G*(λ)**	***B*(λ)**
380	0.00000	0.0000	-0.0000	0.0019	0.0000	0.0000	0.0000
385	0.31250	0.0001	-0.0000	0.0028	0.0000	-0.0000	0.0009
390	0.40833	0.0001	-0.0001	0.0056	0.0000	-0.0000	0.0023
395	0.47465	0.0002	-0.0001	0.0102	0.0001	-0.0001	0.0048
400	0.42929	0.0004	-0.0002	0.0185	0.0002	-0.0001	0.0079
405	0.56250	0.0005	-0.0003	0.0284	0.0003	-0.0002	0.0160
410	0.51240	0.0009	-0.0005	0.0518	0.0005	-0.0003	0.0265
415	0.55046	0.0014	-0.0008	0.0873	0.0008	-0.0004	0.0481
420	0.60250	0.0019	-0.0010	0.1375	0.0011	-0.0006	0.0828
425	0.64795	0.0025	-0.0013	0.2214	0.0016	-0.0009	0.1434
430	0.67328	0.0022	-0.0011	0.2980	0.0014	-0.0007	0.2006
435	0.68943	0.0004	-0.0002	0.3590	0.0003	-0.0001	0.2475
440	0.72609	-0.0028	0.0015	0.3959	-0.0020	0.0011	0.2875
445	0.76342	-0.0074	0.0039	0.4207	-0.0057	0.0030	0.3212
450	0.75921	-0.0136	0.0070	0.4389	-0.0103	0.0053	0.3332
455	0.77604	-0.0201	0.0109	0.4221	-0.0156	0.0085	0.3275
460	0.78167	-0.0266	0.0154	0.3921	-0.0208	0.0121	0.3065
465	0.79432	-0.0317	0.0204	0.3343	-0.0252	0.0162	0.2655
470	0.80128	-0.0370	0.0261	0.2800	-0.0297	0.0209	0.2243
475	0.81083	-0.0432	0.0330	0.2274	-0.0350	0.0268	0.1843
480	0.81211	-0.0492	0.0409	0.1810	-0.0399	0.0332	0.1470
485	0.82162	-0.0537	0.0493	0.1380	-0.0441	0.0405	0.1134
490	0.81627	-0.0587	0.0596	0.1053	-0.0479	0.0487	0.0859
495	0.81052	-0.0645	0.0727	0.0779	-0.0523	0.0590	0.0631
500	0.80867	-0.0724	0.0894	0.0592	-0.0585	0.0723	0.0479
505	0.79548	-0.0811	0.1108	0.0454	-0.0645	0.0881	0.0361
510	0.79066	-0.0888	0.1344	0.0336	-0.0702	0.1063	0.0266
515	0.79168	-0.0932	0.1595	0.0241	-0.0738	0.1263	0.0191
520	0.81085	-0.0921	0.1825	0.0162	-0.0747	0.1480	0.0132
525	0.83800	-0.0850	0.1999	0.0112	-0.0712	0.1675	0.0094
530	0.86740	-0.0720	0.2127	0.0070	-0.0625	0.1845	0.0061
535	0.89348	-0.0540	0.2206	0.0038	-0.0482	0.1971	0.0034
540	0.92338	-0.0315	0.2245	0.0017	-0.0291	0.2073	0.0016
545	0.95022	-0.0061	0.2247	0.0003	-0.0058	0.2135	0.0002
550	0.97110	0.0208	0.2219	-0.0006	0.0202	0.2155	-0.0006

Table 3. Color-matching Functions and RGB Tristimulus Values for Wright – Guild Data (Guild, 1932)

		Color Matching Functions			(6)	(7)	(8)
Wave-length, λ nm	*E*(λ)*	$CMF_R(\lambda)$	$CMF_G(\lambda)$	$CMF_B(\lambda)$	*R*(λ)	*G*(λ)	*B*(λ)
555	0.98810	0.0532	0.2157	-0.0014	0.0526	0.2131	-0.0014
560	1.00000	0.0890	0.2064	-0.0018	0.0890	0.2064	-0.0018
565	1.00542	0.1274	0.1939	-0.0018	0.1281	0.1950	-0.0018
570	1.00105	0.1676	0.1787	-0.0017	0.1678	0.1789	-0.0017
575	0.98656	0.2085	0.1611	-0.0015	0.2057	0.1589	-0.0015
580	0.97874	0.2474	0.1418	-0.0012	0.2422	0.1388	-0.0012
585	0.97133	0.2822	0.1217	-0.0009	0.2741	0.1182	-0.0009
590	0.94663	0.3134	0.1011	-0.0006	0.2966	0.0957	-0.0005
595	0.93251	0.3338	0.0823	-0.0004	0.3113	0.0767	-0.0004
600	0.92155	0.3438	0.0654	-0.0002	0.3168	0.0603	-0.0002
605	0.90438	0.3463	0.0502	-0.0002	0.3132	0.0454	-0.0002
610	0.89463	0.3388	0.0374	-0.0002	0.3031	0.0335	-0.0002
615	0.89030	0.3210	0.0274	-0.0002	0.2858	0.0244	-0.0002
620	0.88110	0.2954	0.0195	-0.0002	0.2602	0.0172	-0.0001
625	0.87508	0.2630	0.0132	-0.0001	0.2301	0.0116	-0.0001
630	0.87396	0.2271	0.0086	-0.0001	0.1985	0.0075	-0.0001
635	0.87327	0.1932	0.0054	-0.0001	0.1687	0.0047	-0.0001
640	0.87657	0.1602	0.0034	-0.0001	0.1404	0.0030	-0.0001
645	0.88278	0.1294	0.0020	-0.0001	0.1143	0.0018	-0.0001
650	0.87944	0.1020	0.0011	0.0000	0.0897	0.0010	0.0000
655	0.88725	0.0787	0.0007	0.0000	0.0698	0.0006	0.0000
660	0.87869	0.0593	0.0004	0.0000	0.0521	0.0003	0.0000
665	0.86362	0.0436	0.0002	0.0000	0.0377	0.0002	0.0000
670	0.87500	0.0315	0.0001	0.0000	0.0275	0.0001	0.0000
675	0.87500	0.0229	0.0001	0.0000	0.0200	0.0001	0.0000
680	0.87059	0.0168	0.0000	0.0000	0.0146	0.0000	0.0000
685	0.87248	0.0118	0.0000	0.0000	0.0103	0.0000	0.0000
690	0.86480	0.0082	0.0000	0.0000	0.0071	0.0000	0.0000
695	0.87891	0.0057	0.0000	0.0000	0.0050	0.0000	0.0000
700	0.85568	0.0041	0.0000	0.0000	0.0035	0.0000	0.0000
705	0.85695	0.0029	0.0000	0.0000	0.0025	0.0000	0.0000
710	0.85127	0.0021	0.0000	0.0000	0.0018	0.0000	0.0000
715	0.81536	0.0015	0.0000	0.0000	0.0012	0.0000	0.0000
720	0.83095	0.0011	0.0000	0.0000	0.0009	0.0000	0.0000
725	0.84459	0.0007	0.0000	0.0000	0.0006	0.0000	0.0000

Table 3. Color-matching Functions and RGB Tristimulus Values for Wright – Guild Data (Guild, 1932)

		Color Matching Functions			(6)	(7)	(8)
Wave-length, λ nm	***E*(λ)***	**CMF_R(λ)**	**CMF_G(λ)**	**CMF_B(λ)**	***R*(λ)**	***G*(λ)**	***B*(λ)**
730	0.81731	0.0005	0.0000	0.0000	0.0004	0.0000	0.0000
735	0.80310	0.0004	0.0000	0.0000	0.0003	0.0000	0.0000
740	0.80257	0.0003	0.0000	0.0000	0.0002	0.0000	0.0000
745	0.75625	0.0002	0.0000	0.0000	0.0001	0.0000	0.0000
750	0.75000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
755	0.76651	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
760	0.75000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
765	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
770	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
775	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
780	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
					R	**G**	**B**
	Tristimulus values for the reference N.P.L. Standard White				**3.5839**	**3.5914**	**3.5909**

* *E*(λ) is the energy distribution curve of the N.P.L. Standard White Light. These numbers, obtained from Broadbent do not precisely match Guild (1932, Table 1).

Table 4. Luminance of RGB Tristimulus Values: Wright – Guild Data (Guild, 1932)

Wave-length, λ (nm)	$R(\lambda) \times L_R$	$G(\lambda) \times L_G$	$B(\lambda) \times L_B$	Luminance
380	0.0000	0.0000	0.0000	0.0000
385	0.0000	-0.0000	0.0000	0.0000
390	0.0000	-0.0001	0.0001	0.0000
395	0.0001	-0.0002	0.0002	0.0001
400	0.0002	-0.0004	0.0004	0.0002
405	0.0003	-0.0007	0.0008	0.0004
410	0.0005	-0.0011	0.0013	0.0006
415	0.0008	-0.0019	0.0023	0.0012
420	0.0011	-0.0027	0.0040	0.0024
425	0.0016	-0.0038	0.0069	0.0047
430	0.0014	-0.0033	0.0096	0.0078
435	0.0003	-0.0005	0.0119	0.0116
440	-0.0020	0.0049	0.0138	0.0167
445	-0.0057	0.0130	0.0154	0.0227
450	-0.0103	0.0232	0.0160	0.0288
455	-0.0156	0.0371	0.0157	0.0372
460	-0.0208	0.0529	0.0147	0.0469
465	-0.0252	0.0712	0.0127	0.0587
470	-0.0297	0.0918	0.0108	0.0729
475	-0.0350	0.1175	0.0088	0.0913
480	-0.0399	0.1458	0.0071	0.1129
485	-0.0441	0.1778	0.0054	0.1391
490	-0.0479	0.2136	0.0041	0.1698
495	-0.0523	0.2588	0.0030	0.2096
500	-0.0585	0.3174	0.0023	0.2612
505	-0.0645	0.3868	0.0017	0.3240
510	-0.0702	0.4666	0.0013	0.3977
515	-0.0738	0.5544	0.0009	0.4815

Table 4. Luminance of RGB Tristimulus Values: Wright – Guild Data (Guild, 1932)

Wave-length, λ (nm)	R(λ) x L_R	G(λ) x L_G	B(λ) x L_B	Luminance
520	-0.0747	0.6498	0.0006	0.5757
525	-0.0712	0.7355	0.0005	0.6647
530	-0.0625	0.8099	0.0003	0.7477
535	-0.0482	0.8654	0.0002	0.8174
540	-0.0291	0.9099	0.0001	0.8809
545	-0.0058	0.9373	0.0000	0.9315
550	0.0202	0.9460	-0.0000	0.9662
555	0.0526	0.9356	-0.0001	0.9881
560	0.0890	0.9061	-0.0001	0.9950
565	0.1281	0.8559	-0.0001	0.9839
570	0.1678	0.7853	-0.0001	0.9530
575	0.2057	0.6975	-0.0001	0.9031
580	0.2422	0.6094	-0.0001	0.8515
585	0.2741	0.5189	-0.0000	0.7929
590	0.2966	0.4200	-0.0000	0.7166
595	0.3113	0.3367	-0.0000	0.6480
600	0.3168	0.2647	-0.0000	0.5815
605	0.3132	0.1994	-0.0000	0.5126
610	0.3031	0.1469	-0.0000	0.4500
615	0.2858	0.1070	-0.0000	0.3928
620	0.2602	0.0755	-0.0000	0.3357
625	0.2301	0.0508	-0.0000	0.2809
630	0.1985	0.0331	-0.0000	0.2316
635	0.1687	0.0208	-0.0000	0.1895
640	0.1404	0.0130	-0.0000	0.1534
645	0.1143	0.0077	-0.0000	0.1220
650	0.0897	0.0044	0.0000	0.0941

Table 4. Luminance of RGB Tristimulus Values: Wright – Guild Data (Guild, 1932)

Wave-length, λ (nm)	$R(\lambda) \times L_R$	$G(\lambda) \times L_G$	$B(\lambda) \times L_B$	Luminance
655	0.0698	0.0026	0.0000	0.0724
660	0.0521	0.0015	0.0000	0.0536
665	0.0377	0.0008	0.0000	0.0385
670	0.0275	0.0005	0.0000	0.0280
675	0.0200	0.0003	0.0000	0.0203
680	0.0146	0.0002	0.0000	0.0148
685	0.0103	0.0001	0.0000	0.0104
690	0.0071	0.0000	0.0000	0.0071
695	0.0050	0.0000	0.0000	0.0050
700	0.0035	0.0000	0.0000	0.0035
705	0.0025	0.0000	0.0000	0.0025
710	0.0018	0.0000	0.0000	0.0018
715	0.0012	0.0000	0.0000	0.0012
720	0.0009	0.0000	0.0000	0.0009
725	0.0006	0.0000	0.0000	0.0006
730	0.0004	0.0000	0.0000	0.0004
735	0.0003	0.0000	0.0000	0.0003
740	0.0002	0.0000	0.0000	0.0002
745	0.0001	0.0000	0.0000	0.0001
750	0.0001	0.0000	0.0000	0.0001
755	0.0001	0.0000	0.0000	0.0001
760	0.0000	0.0000	0.0000	0.0000
765	0.0000	0.0000	0.0000	0.0000
770	0.0000	0.0000	0.0000	0.0000
775	0.0000	0.0000	0.0000	0.0000
780	0.0000	0.0000	0.0000	0.0000
Sum	**3.5839**	**15.7662**	**0.1724**	**19.5224**

Table 5. The CIE 1931 RGB Chromaticity Coordinates

Wavelength, λ (nm)	*r*	*g*	*b*
380	0.0272	-0.0115	0.9843
385	0.0268	-0.0114	0.9846
390	0.0263	-0.0114	0.9851
395	0.0256	-0.0113	0.9857
400	0.0247	-0.0112	0.9865
405	0.0237	-0.0111	0.9874
410	0.0225	-0.0109	0.9884
415	0.0207	-0.0104	0.9897
420	0.0181	-0.0094	0.9913
425	0.0142	-0.0076	0.9934
430	0.0088	-0.0048	0.9960
435	0.0012	-0.0007	0.9995
440	-0.0084	0.0048	1.0036
445	-0.0213	0.0120	1.0093
450	-0.0390	0.0218	1.0172
455	-0.0618	0.0345	1.0273
460	-0.0909	0.0517	1.0392
465	-0.1281	0.0762	1.0519
470	-0.1821	0.1175	1.0646
475	-0.2584	0.1840	1.0744
480	-0.3667	0.2906	1.0761
485	-0.5200	0.4568	1.0632
490	-0.7150	0.6996	1.0154
495	-0.9459	1.0247	0.9212
500	-1.1685	1.3905	0.7780
505	-1.3182	1.7195	0.5987
510	-1.3371	1.9318	0.4053
515	-1.2076	1.9699	0.2377
520	-0.9830	1.8534	0.1296
525	-0.7386	1.6662	0.0724

Table 5. The CIE 1931 RGB Chromaticity Coordinates

Wavelength, λ (nm)	*r*	*g*	*b*
530	-0.5159	1.4761	0.0398
535	-0.3304	1.3105	0.0199
540	-0.1707	1.1628	0.0079
545	-0.0293	1.0282	0.0011
550	0.0974	0.9051	-0.0025
555	0.2121	0.7919	-0.0040
560	0.3164	0.6881	-0.0045
565	0.4112	0.5932	-0.0044
570	0.4973	0.5067	-0.0040
575	0.5751	0.4283	-0.0034
580	0.6449	0.3579	-0.0028
585	0.7071	0.2952	-0.0023
590	0.7617	0.2402	-0.0019
595	0.8087	0.1928	-0.0015
600	0.8475	0.1537	-0.0012
605	0.8800	0.1209	-0.0009
610	0.9059	0.0949	-0.0008
615	0.9265	0.0741	-0.0006
620	0.9425	0.0580	-0.0005
625	0.9550	0.0454	-0.0004
630	0.9649	0.0353	-0.0003
635	0.9730	0.0272	-0.0002
640	0.9797	0.0205	-0.0002
645	0.9850	0.0152	-0.0002
650	0.9888	0.0113	-0.0001
655	0.9918	0.0083	-0.0001
660	0.9940	0.0061	-0.0001
665	0.9954	0.0047	-0.0001
670	0.9966	0.0035	-0.0001
675	0.9975	0.0025	0.0000
680	0.9984	0.0016	0.0000
685	0.9991	0.0009	0.0000

Table 5. The CIE 1931 RGB Chromaticity Coordinates

Wavelength, λ (nm)	*r*	*g*	*b*
690	0.9996	0.0004	0.0000
695	0.9999	0.0001	0.0000
700	1.0000	0.0000	0.0000
705	1.0000	0.0000	0.0000
710	1.0000	0.0000	0.0000
715	1.0000	0.0000	0.0000
720	1.0000	0.0000	0.0000
725	1.0000	0.0000	0.0000
730	1.0000	0.0000	0.0000
735	1.0000	0.0000	0.0000
740	1.0000	0.0000	0.0000
745	1.0000	0.0000	0.0000
750	1.0000	0.0000	0.0000
755	1.0000	0.0000	0.0000
760	1.0000	0.0000	0.0000
765	1.0000	0.0000	0.0000
770	1.0000	0.0000	0.0000
775	1.0000	0.0000	0.0000
780	1.0000	0.0000	0.0000