A simple life annuity

LIFE INSURANCE PRODUCTS VALUATION IN R

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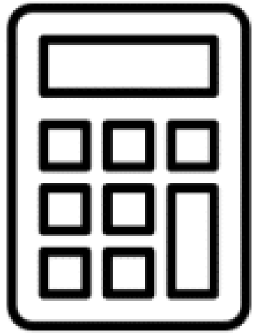
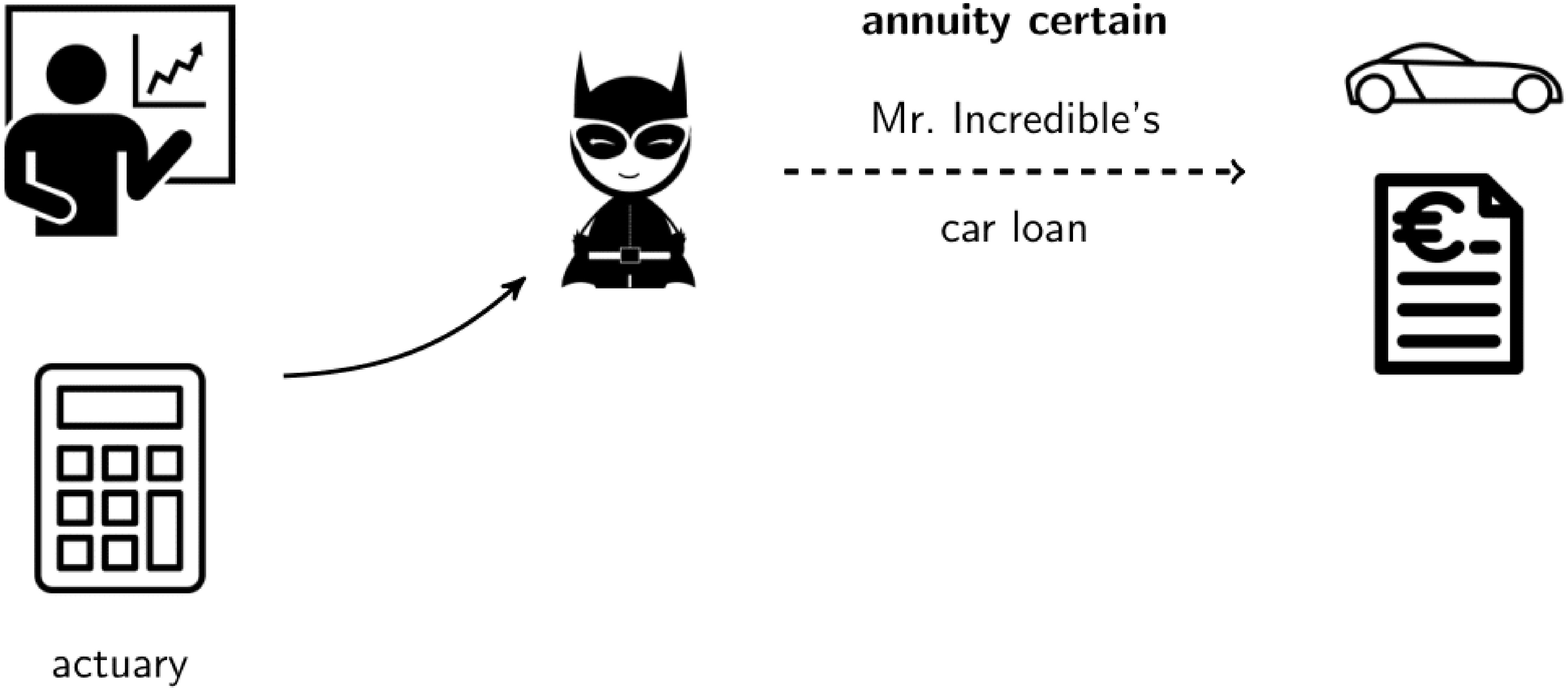
The life annuity

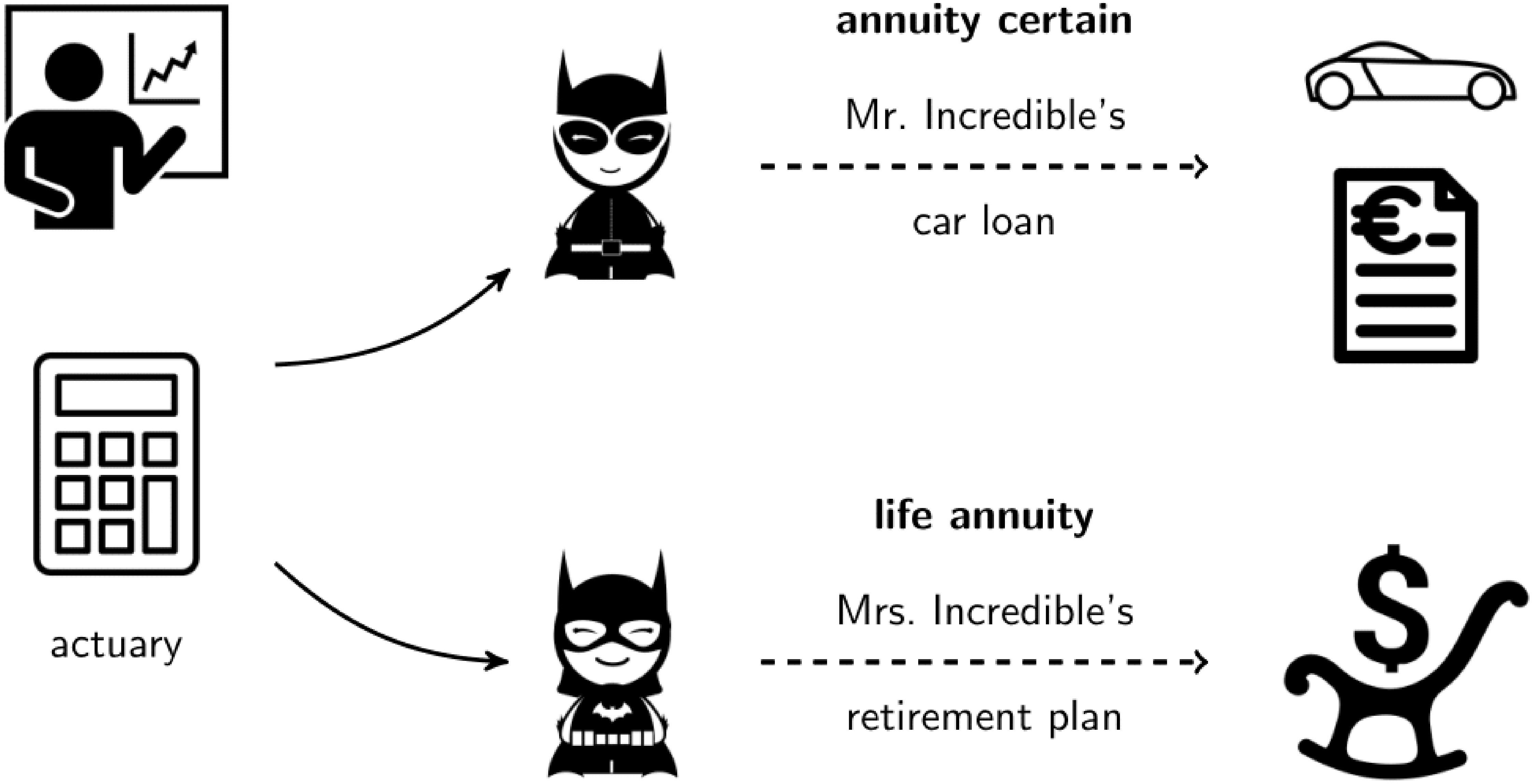


actuary

The life annuity

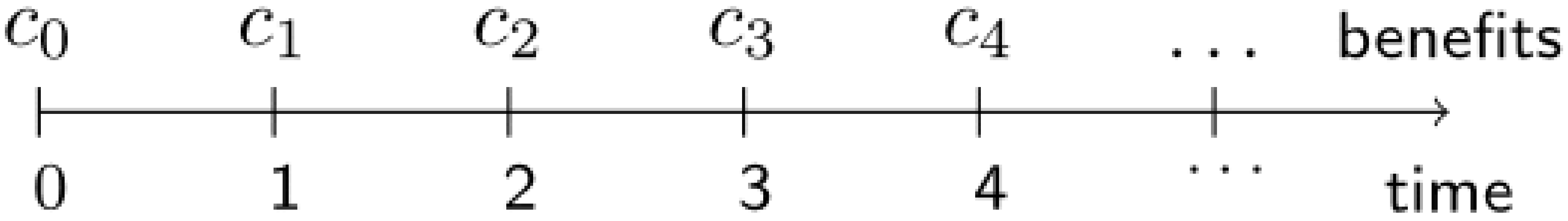


The life annuity

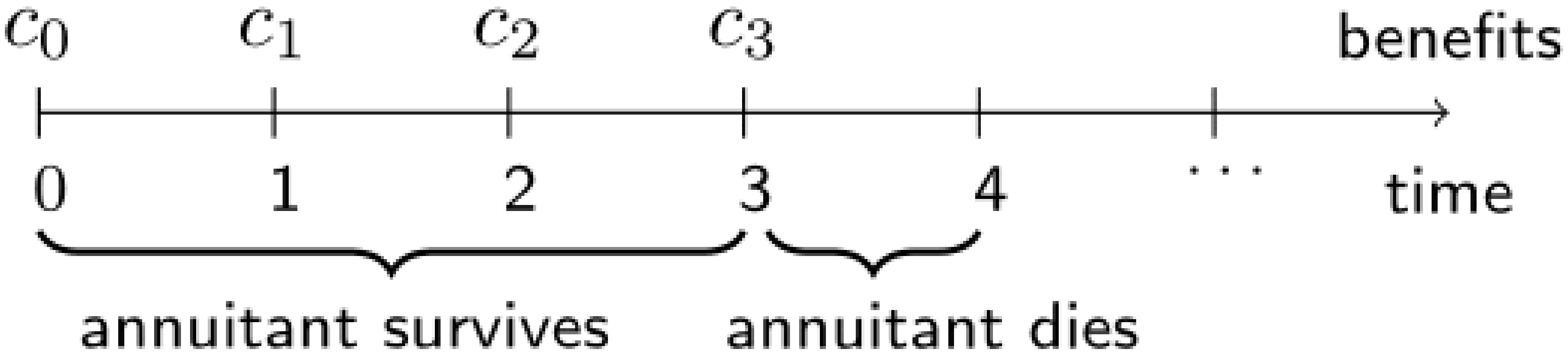


Annuity vs. life annuity: mind the difference!

- **Annuity (certain)** offers a guaranteed series of payments.

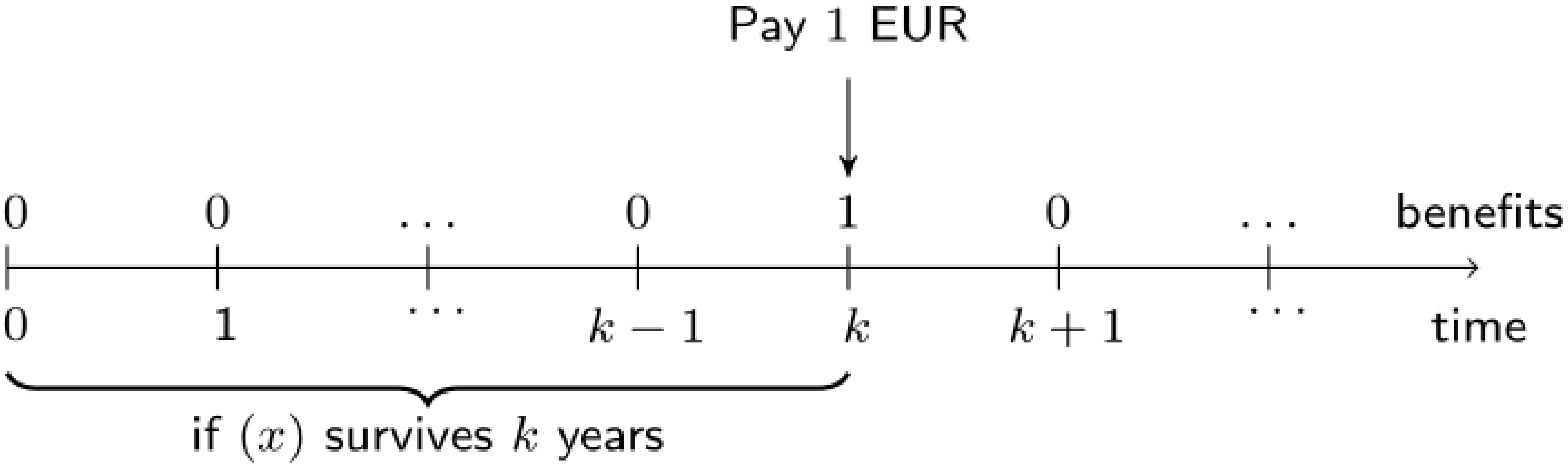


- **Life annuity** depends on the **survival** of the recipient.



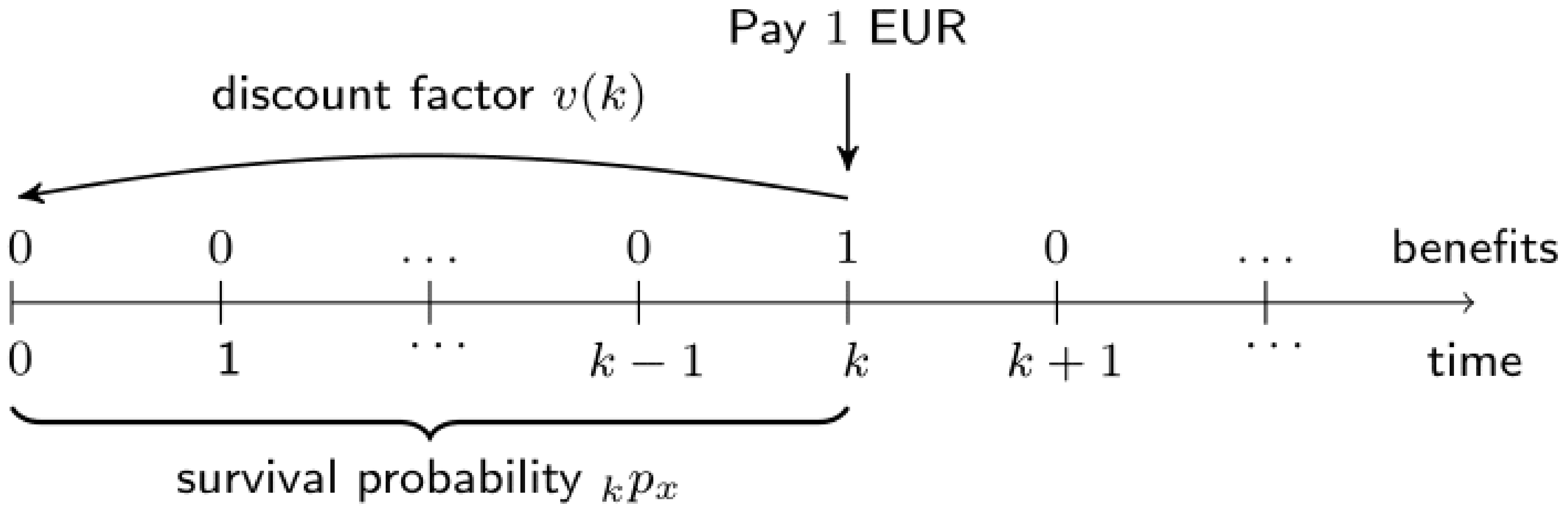
Pure endowment

- The product is sold to (x) at time 0.



EPV of pure endowment

- **Expected Present Value:**



The EPV is

$$ {}_kE_x = 1 \cdot \ v(k) \cdot \ {}_kp_x \, . $$

Annuity vs. life annuity: mind the difference!

- With an **annuity certain**, the benefit of 1 euro at time k is **guaranteed**.

  PV is $v(k)$.

```
i <- 0.03
discount_factor <- (1 + i) ^ - 5
1 * discount_factor
```

```
0.8626088
```

Annuity vs. life annuity: mind the difference!

- With a **pure endowment**, the benefit of 1 euro at time k is **not guaranteed**.

  Expected PV is $v(k) \cdot {}_kp_x$.

```
qx <- life_table$qx; px <- 1 - qx
kpx <- prod(px[(65 + 1):(69 + 1)])
kpx
```

```
0.9144015
```

```
1 * discount_factor * kpx
```

```
0.7887708
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

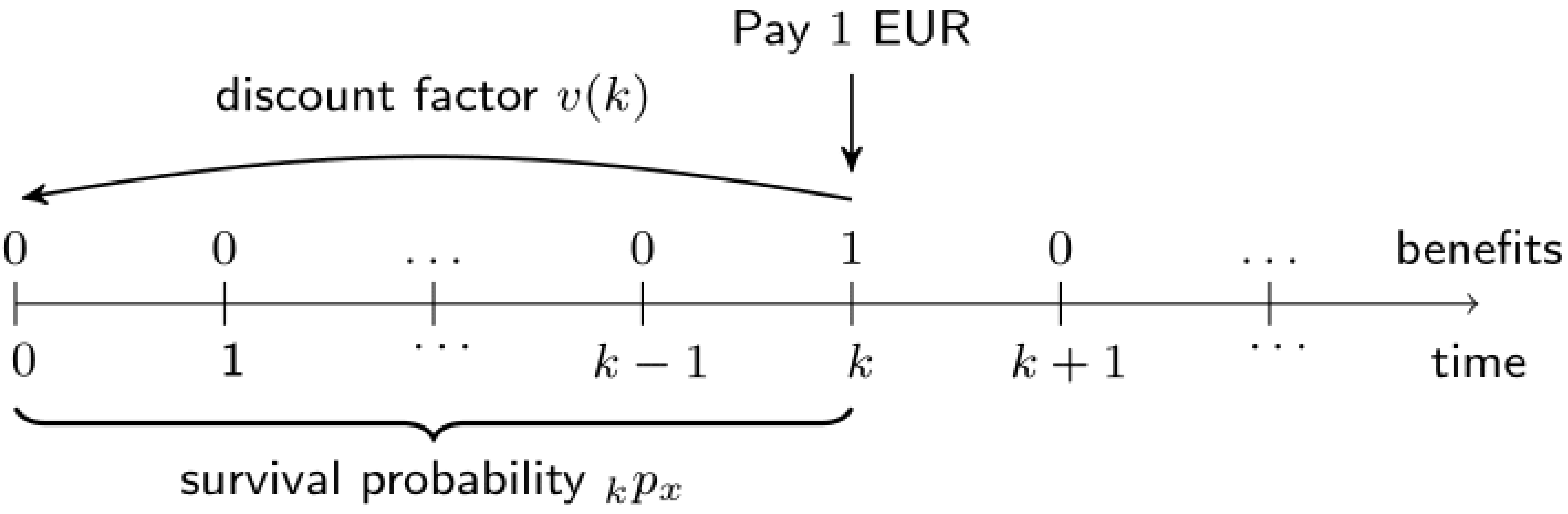
The whole, temporary and deferred life annuity

LIFE INSURANCE PRODUCTS VALUATION IN R

Katrien Antonio, Ph.D.

Professor, KU Leuven and University of Amsterdam

A series of benefits



- What if?
  - The benefit is c_k EUR instead of 1 EUR?
  - A series of such pure endowments instead of just one?

General setting

- A life annuity on (x) with **benefit** vector

$$(c_0, c_1, \ldots, c_k, \ldots)$$

- Sequence of **pure endowments:**
  - each with $c_k \cdot v(k) \cdot {}_k p_x$ as Expected Present Value (EPV)
  - together:

$$\sum_{k=0}^{+\infty} c_k \cdot v(k) \cdot {}_k p_x$$

    the EPV.

Life annuities in R

500	400	300	200	200	200	200	200	benefits
0	1	2	3	4	5	6	7	time
65	66	67	68	69	70	71	72	age
1	1.03^{-1}	1.03^{-2}	1.03^{-3}	1.03^{-4}	1.03^{-5}	1.03^{-6}	1.03^{-7}	discount factors
1	${}_1p_{65}$	${}_2p_{65}$	${}_3p_{65}$	${}_4p_{65}$	${}_5p_{65}$	${}_6p_{65}$	${}_7p_{65}$	survival probabilities

```r
benefits <- c(500, 400, 300, rep(200, 5))
discount_factors <- (1 + 0.03) ^ - (0:7)
kpx <- c(1, cumprod(px[(65 + 1):(71 + 1)]))
sum(benefits * discount_factors * kpx)
```

```
1945.545
```

Whole life annuity due

Whole life annuity **due:** pay c_k at *beginning* of year $(k+1)$.

1	1	1	1	1	1	...	benefits
0	1	2	3	4	5	...	time
1	$v(1)$	$v(2)$	$v(3)$	$v(4)$	$v(5)$	...	discount factors
1	${}_1p_x$	${}_2p_x$	${}_3p_x$	${}_4p_x$	${}_5p_x$	...	survival probabilities

$\ddot{a}_x$ for constant benefit of 1 EUR and constant discount factor v

Whole life immediate annuity

Whole life **immediate** annuity: pay c_k at *end* of year $(k+1)$.

0	1	1	1	1	1	$\dots$	benefits
0	1	2	3	4	5	$\cdots$	time
	$v(1)$	$v(2)$	$v(3)$	$v(4)$	$v(5)$	$\cdots$	discount factors
	${}_1p_x$	${}_2p_x$	${}_3p_x$	${}_4p_x$	${}_5p_x$	$\cdots$	survival probabilities

$$a_x$$

for constant benefit of 1 EUR

and constant discount factor v

Whole life annuities in R

Compute $\ddot{a}_{35}$ (due) for constant interest rate $i = 3\%$

```
# whole-life annuity due of (35)
kpx <-
  c(1, cumprod(px[(35 + 1):length(px)]))
discount_factors <-
  (1 + 0.03) ^ - (0:(length(kpx) - 1))
benefits <- rep(1, length(kpx))
sum(benefits * discount_factors * kpx)
```

```
24.44234
```

and a_{35} (immediate)

```
# whole-life immediate annuity of (35)
kpx <- cumprod(px[(35 + 1):length(px)])
discount_factors <-
   (1 + 0.03) ^ - (1:length(kpx))
benefits <- rep(1, length(kpx))
sum(benefits * discount_factors * kpx)
```

```
23.44234
```

Temporary life annuity due

Temporary annuity due: maximum of n years, at time 0 until $n-1$.

1	1	1	$\dots$	1	0	0	benefits
0	1	2	$\dots$	$n-1$	n	$n+1$	time
1	$v(1)$	$v(2)$	$\cdots$	$v(n-1)$			discount factors
1	${}_1p_x$	${}_2p_x$	$\cdots$	${}_{n-1}p_x$			survival probabilities

$$\ddot{a}_{x:\overline{n}|}$$

for constant benefit of 1 EUR

and constant discount factor v

Deferred whole life annuity due

Deferred whole life annuity due: no payments in first u years.

0	0	0	$\ldots$	0	1	1	$\ldots$	benefits
0	1	2	$\ldots$	$u-1$	u	$u+1$	$\ldots$	time
					$v(u)$	$v(u+1)$	$\cdots$	discount factors
					${}_{u}p_x$	${}_{u+1}p_x$	$\cdots$	survival probabilities

$${}_{u}\ddot{a}_x$$

for constant benefit of 1 EUR

and constant discount factor v

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

Guaranteed payments

LIFE INSURANCE PRODUCTS VALUATION IN R

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Statistician, Finity Consulting

Guaranteed payments

- Additional flexibility: life annuities with a **guaranteed period**.
- A typical contract:
  - Initially:

    benefits are paid **regardless** of whether the annuitant is **alive or not**.
  - Afterwards:

    benefits are paid **conditional on survival**.

Mr. Incredible's prize!

Mr. Incredible is 35 years old.

He won a special prize: a life annuity of 10,000 EUR each year for life!

The first payment starts at the end of the first year. Moreover, the first 10 payments are **guaranteed**.

Can you calculate the **value** of his prize?

Mr. Incredible's prize in R

- He is 35-years-old, living in Belgium, year 2013.
- Interest rate is 3%.
- Survival probabilities of (35)

```
# Survival probabilities of (35)
kpx <- c(1, cumprod(px[(35 + 1):length(px)]))
```

- Discount factors

```
# Discount factors
discount_factors <- (1 + 0.03) ^ - (0:(length(kpx) - 1))
```

Mr. Incredible's prize pictured

35	36	37	...	45	46	47	...	age
0	1	2	...	10	11	12	...	time
0	10^4	10^4	...	10^4	0	0	...	guaranteed
0	0	0	...	0	10^4	10^4	...	upon survival

```
# Benefits guaranteed
benefits_guaranteed <- c(0, rep(10^4, 10), rep(0, length(kpx) - 11))

# Benefits nonguaranteed
benefits_nonguaranteed <- c(rep(0, 11), rep(10^4, length(kpx) - 11))
```

```r
# PV of the guaranteed annuity
sum(benefits_guaranteed * discount_factors)
```

```
85302.03
```

```r
# EPV of the nonguaranteed life annuity
sum(benefits_nonguaranteed * discount_factors * kpx)
```

```
149675.3
```

```r
# PV of the guaranteed annuity + EPV of the nonguaranteed annuity
sum(benefits_guaranteed * discount_factors) + sum(benefits_nonguaranteed * discount_factors * kpx)
```

```
234977.3
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

On premium payments and retirement plans

LIFE INSURANCE PRODUCTS VALUATION IN R

Katrien Antonio, Ph.D.
Professor, KU Leuven and University of Amsterdam

Paying premiums

- **Goal** of premium calculation:
  - Premiums + interest earnings should **match** benefits.
- **Solution**:
  - Set up **actuarial equivalence** between premium vector and benefit vector.
  - Treat premium payments as a life annuity on (x).

Mrs. Incredible's retirement plan

Mrs. Incredible is 35 years old.

She wants to buy a **life annuity** that provides 12,000 EUR annually for life, **beginning at age 65.**

She will finance this product with **annual premiums**, payable for 30 years beginning at age 35. Premiums reduce by one-half after 15 years.

What is her initial premium?

Mrs. Incredible's retirement plan pictured

35	$\dots$	49	50	$\dots$	64	65	66	$\dots$	age
0	$\dots$	14	15	$\dots$	29	30	31	$\dots$	time
0	$\cdots$	0	0	$\cdots$	0	$12 \cdot 10^3$	$12 \cdot 10^3$	$\cdots$	benefits
P	$\cdots$	P	$0.5P$	$\cdots$	$0.5P$	0	0	$\cdots$	premiums

Mrs. Incredible's retirement plan in R

- She is 35-years-old, living in Belgium, year 2013.
- Interest rate is 3%.
- Survival probabilities

```
# Survival probabilities of (35)
kpx <- c(1, cumprod(px[(35 + 1):length(px)]))
```

- Discount factors

```
# Discount factors
discount_factors <- (1 + 0.03) ^ - (0:(length(kpx) - 1))
```

- Benefits

```r
# Benefits
benefits <- c(rep(0, 30), rep(12000, length(kpx) - 30))
# EPV of the life annuity benefits
sum(benefits * discount_factors * kpx)
```

```
70928.84
```

- Premium pattern rho

```r
# Premium pattern rho
rho <- c(rep(1, 15), rep(0.5, 15), rep(0, length(kpx) - 30))
# EPV of the premium pattern
sum(rho * discount_factors * kpx)
```

```
16.01978
```

Mrs. Incredible's retirement plan in R

- Actuarial equivalence

$$P = \frac{\text{EPV}(\text{benefits})}{\text{EPV}(\text{rho})}.$$

```
# The ratio of the EPV of the life annuity benefits
# and the EPV of the premium pattern
sum(benefits * discount_factors * kpx) / sum(rho * discount_factors * kpx)
```

```
4427.578
```

Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R