# A simple life insurance

LIFE INSURANCE PRODUCTS VALUATION IN R

**Roel Verbelen, Ph.D.**
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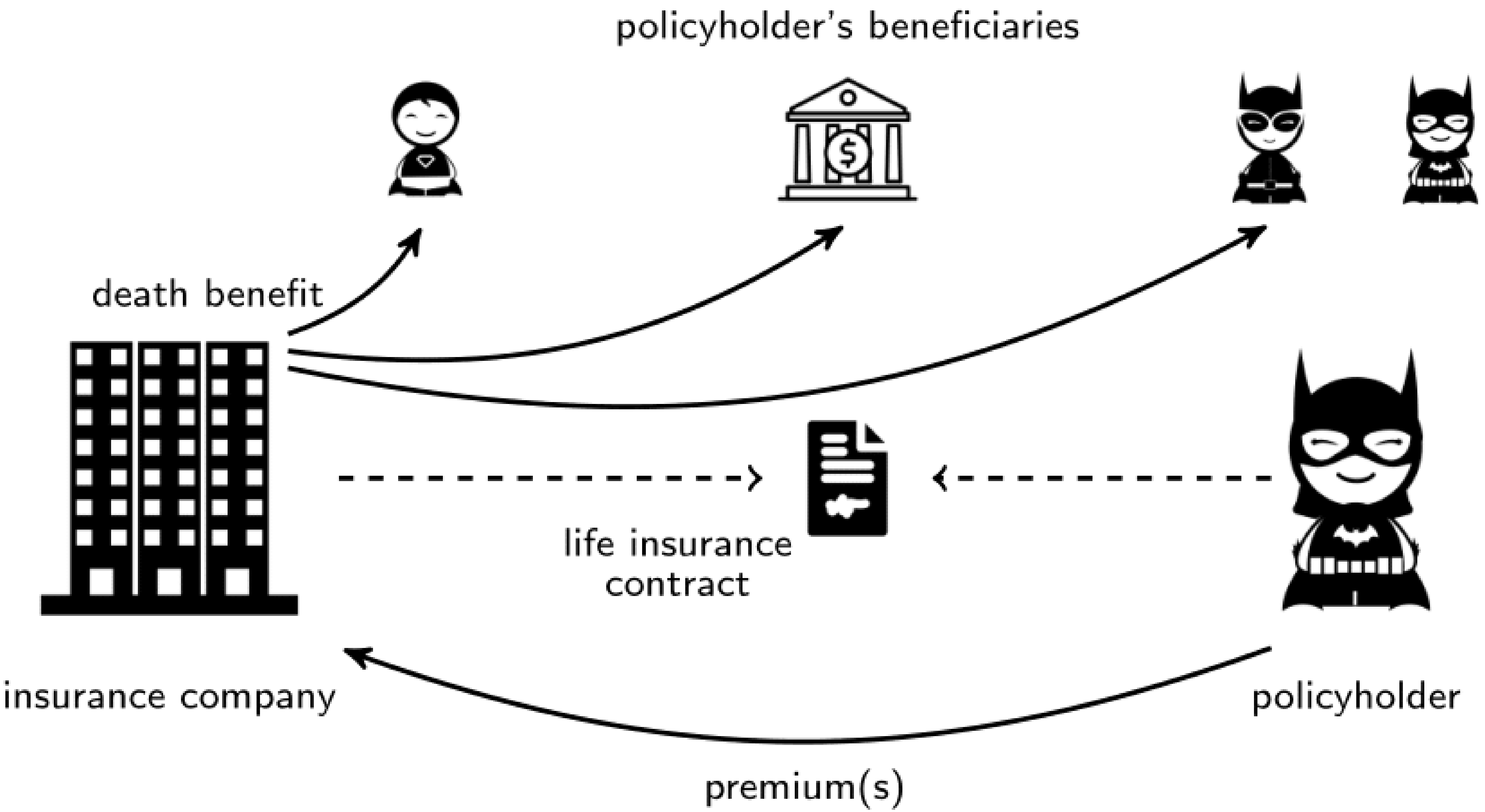
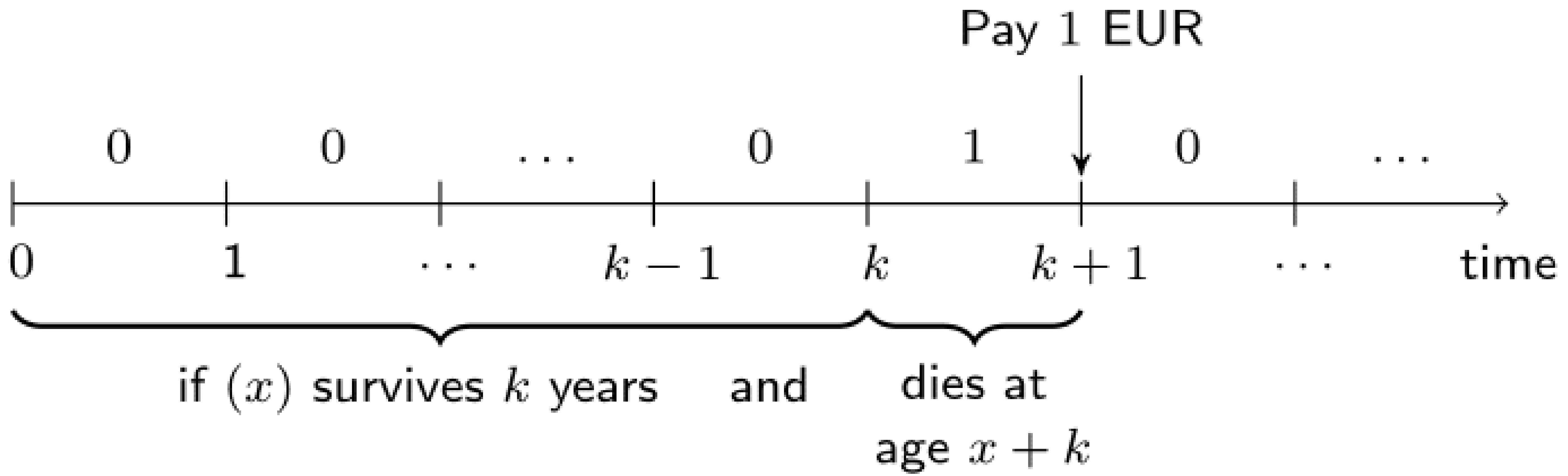
# The life insurance



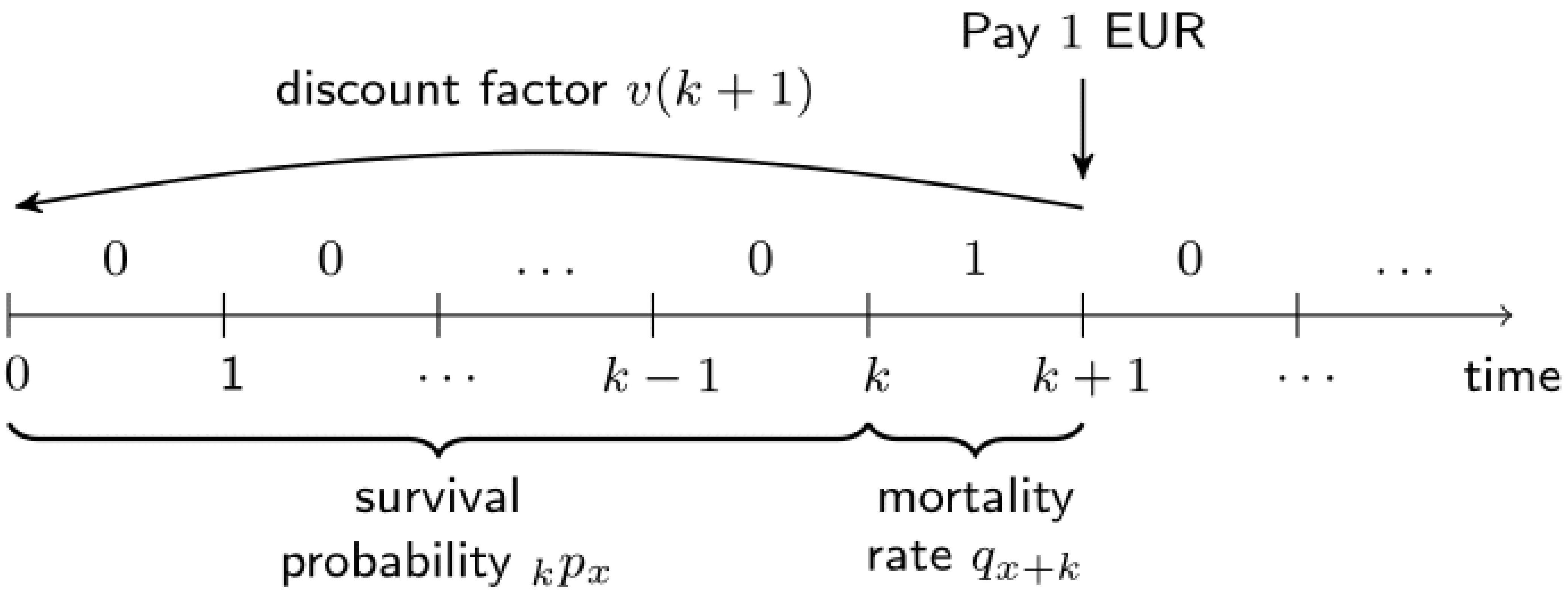
# A simple life insurance

- The product is sold to $(x)$ at time 0.



# A simple life insurance

- **Expected Present Value:**




The EPV is

$${}_{k|1}A_x = 1 \cdot \ v(k+1) \cdot \ {}_kp_x \cdot \ q_{x+k} = 1 \cdot \ v(k+1) \cdot \ {}_{k|}q_x \, .$$

# A simple life insurance in R

Compute ${}_{5|1}A_{65} = 1 \cdot\ v(6) \cdot\ {}_{5|}q_{65} = 1 \cdot\ v(6) \cdot\ {}_{5}p_{65} \cdot\ q_{70}$ for constant $i = 3\%$.

```
# Mortality rates and one-year survival probabilities
qx <- life_table$qx
px <- 1 - qx
```

```
# 5-year deferred mortality probability of (65)
kpx <- prod(px[(65 + 1):(69 + 1)])
kqx <- kpx * qx[70 + 1]
kqx
```

```
0.02086664
```

# A simple life insurance in R (cont.)

```
# Discount factor
discount_factor <- (1 + 0.03) ^ - 6
discount_factor
```

```
0.8374843
```

```
# EPV of the simple life insurance
1 * discount_factor * kqx
```

```
0.01747548
```

# Let's practice!

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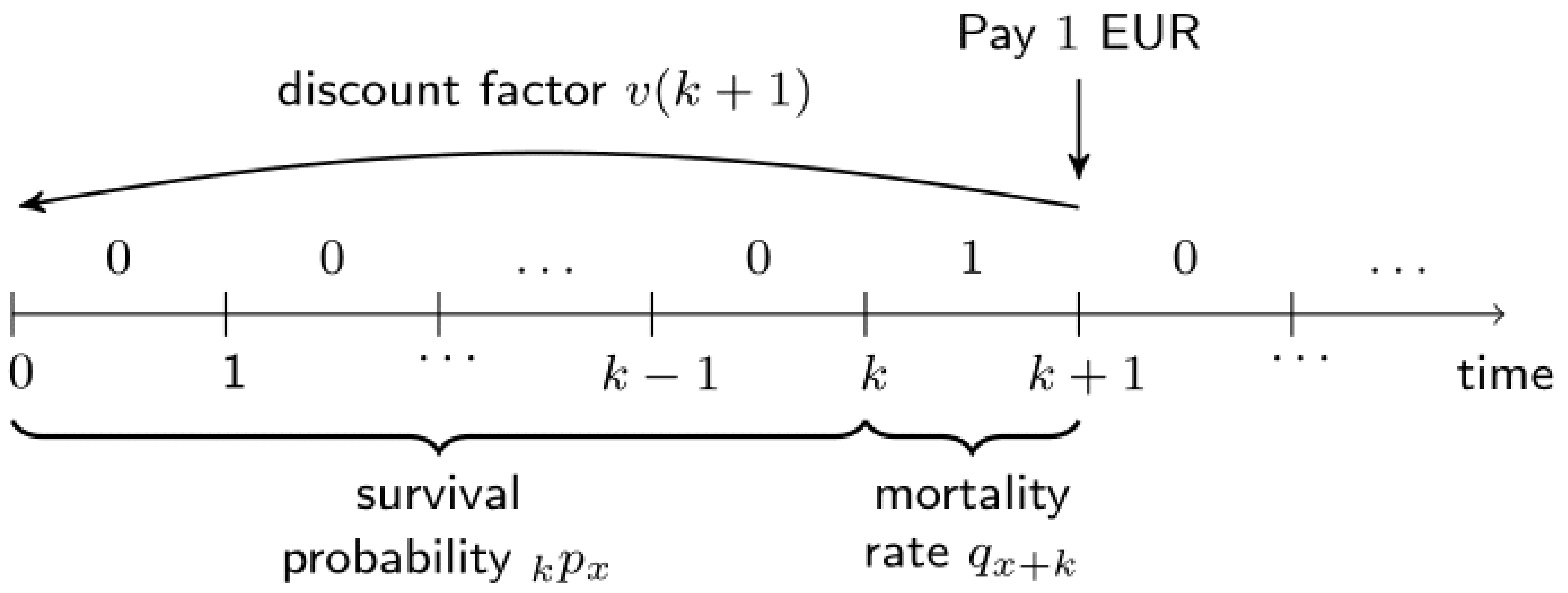
# The whole, temporary and deferred life insurance

LIFE INSURANCE PRODUCTS VALUATION IN R

**Katrien Antonio, Ph.D.**
Professor, KU Leuven and University of Amsterdam

# A series of one-year contracts




- What if?
  - The benefit is $b_k$ EUR instead of 1 EUR?
  - A series of one-year contracts instead of just one?

# General setting

- A life insurance on $(x)$ with **death benefit** vector

$$(b_0, b_1, \ldots, b_k, \ldots)$$

- Series of **one-year** contracts:
  - Each with $b_k \cdot v(k+1) \cdot {}_kp_x \cdot q_{x+k}$ as Expected Present Value (EPV)
  - Together:

$$\sum_{k=0}^{+\infty} b_k \cdot v(k+1) \cdot {}_kp_x \cdot q_{x+k} = \sum_{k=0}^{+\infty} b_k \cdot v(k+1) \cdot {}_{k|}q_x$$

the EPV.

# Whole life insurance

**Whole** life insurance: *lifelong*.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 1 | 1 | 1 | 1 | $\ldots$ | benefits |
| 0 | 1 | 2 | 3 | 4 | 5 | $\cdots$ | time |
| | $v(1)$ | $v(2)$ | $v(3)$ | $v(4)$ | $v(5)$ | $\cdots$ | discount factors |
| | $q_x$ | ${}_{1\vert}q_x$ | ${}_{2\vert}q_x$ | ${}_{3\vert}q_x$ | ${}_{4\vert}q_x$ | $\cdots$ | deferred mortality probabilities |

$$A_x$$

for constant benefit of 1 EUR

and constant discount factor $v$

# Temporary life insurance

**Temporary** (or: **term**) life insurance: maximum of $n$ years.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| benefits | | 1 | 1 | $\ldots$ | 1 | 1 | 0 |
| time | 0 | 1 | 2 | $\ldots$ | $n-1$ | $n$ | $n+1$ |
| discount factors | | $v(1)$ | $v(2)$ | $\cdots$ | $v(n-1)$ | $v(n)$ | |
| deferred mortality probabilities | | $q_x$ | ${}_{1\|}q_x$ | $\cdots$ | ${}_{n-2\|}q_x$ | ${}_{n-1\|}q_x$ | |

$$A^{\,1}_{x:\overline{n}|}$$

for constant benefit of 1 EUR

and constant discount factor $v$

# Deferred whole life insurance

**Deferred** whole life insurance: no payments in first $u$ years.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $0$ | $\ldots$ | $0$ | $1$ | $1$ | $\ldots$ | | benefits |
| $0$ | $1$ | $\ldots$ | $u$ | $u+1$ | $u+2$ | $\ldots$ | time |
| | | | | $v(u+1)$ | $v(u+2)$ | $\cdots$ | discount factors |
| | | | | ${}_{u\vert}q_x$ | ${}_{u+1\vert}q_x$ | $\cdots$ | deferred mortality probabilities |

$${}_{u|}A_x$$

for constant benefit of 1 EUR

and constant discount factor $v$

# Life insurances in R

Compute $A_{35}$ for constant interest rate $i = 3\%$.

```
# Whole-life insurance of (35)
kpx <- c(1, cumprod(px[(35 + 1):(length(px) - 1)]))
kqx <-  kpx * qx[(35 + 1):length(qx)]
discount_factors <- (1 + 0.03) ^ - (1:length(kqx))
benefits <- rep(1, length(kqx))
sum(benefits * discount_factors * kqx)
```

```
0.2880872
```

Now do ${}_{20|}A_{35}$.

```
# Deferred whole-life insurance of (35)
kpx <- c(1, cumprod(px[(35 + 1):(length(px) - 1)]))
kqx <-  kpx * qx[(35 + 1):length(qx)]
discount_factors <- (1 + 0.03) ^ - (1:length(kqx))
benefits <- c(rep(0, 20), rep(1, length(kqx) - 20))
sum(benefits * discount_factors * kqx)
```

```
0.2552956
```

# Let's practice!

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# Combined benefits

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**Roel Verbelen, Ph.D.**
Statistician, Finity Consulting

# Endowment insurance

| | | | | | | |
|---|---|---|---|---|---|---|
| death benefits | | 1 | 1 | $\cdots$ | 1 | 1 |
| pure endowment | 0 | 0 | 0 | $\ldots$ | 0 | 1 |
| time | 0 | 1 | 2 | $\ldots$ | $n-1$ | $n$ |
| discount factors | | $v(1)$ | $v(2)$ | $\cdots$ | $v(n-1)$ | $v(n)$ |
| deferred mortality probabilities | | $q_x$ | ${}_{1\|}q_x$ | $\cdots$ | ${}_{n-2\|}q_x$ | ${}_{n-1\|}q_x$ |
| survival probability | | | | | | ${}_np_x$ |

$A_{x:\overline{n}|}$ for constant benefit of 1 EUR and constant discount factor $v$

# Sending baby Incredible to college

Mrs. Incredible is 35 years old.

She wants to **save money** to send her baby to college. She needs 75,000 EUR when he gets 18.

Given her dangerous lifestyle as a superhero, at the same time **she wants to cover her life.**

The sum insured is 50,000 euro.

Can you design this type of life insurance policy?

# Sending baby Incredible to college pictured

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 35 | | 36 | | 37 | | ... | | 52 | | 53 | age |
| 0 | | 1 | | 2 | | ... | | 17 | | 18 | time |
| | $50 \cdot 10^3$ | | $50 \cdot 10^3$ | | $\cdots$ | | $50 \cdot 10^3$ | | $50 \cdot 10^3$ | | death benefits |
| 0 | | 0 | | 0 | | $\cdots$ | | 0 | | $75 \cdot 10^3$ | pure endowment |
| $P$ | | $P$ | | $P$ | | $\cdots$ | | $P$ | | 0 | premiums |

# Sending baby Incredible to college in R

- She is 35-years-old, living in Belgium, year 2013.
- Interest rate is 3%.

```
i <- 0.03
```

- Death benefits (using the deferred mortality probabilities $q_{35}$, ${}_{1|}q_{35}$ to ${}_{17|}q_{35}$)

```
kqx <-  c(1, cumprod(px[(35 + 1):(51 + 1)])) * qx[(35 + 1):(52 + 1)]
discount_factors <- (1 + i) ^ - (1:length(kqx))
benefits <- rep(50000, length(kqx))
EPV_death_benefits <- sum(benefits * discount_factors * kqx)
EPV_death_benefits
```

```
870.8815
```

# Sending baby Incredible to college in R

- Pure endowment (using the survival probability ${}_{18}p_{35}$)

```
EPV_pure_endowment <- 75000 * (1 + i) ^ - 18 * prod(px[(35 + 1):(52 + 1)])
EPV_pure_endowment
```

```
42975.86
```

- Premium pattern rho (using the survival probabilities ${}_{0}p_{35}$ to ${}_{17}p_{35}$)

```
# Premium pattern rho
kpx <- c(1, cumprod(px[(35 + 1):(51 + 1)]))
discount_factors <- (1 + i) ^ - (0:(length(kpx) - 1))
rho <- rep(1, length(kpx))
EPV_rho <- sum(rho * discount_factors * kpx)
EPV_rho
```

```
14.06193
```

# Sending baby Incredible to college in R

- Actuarial equivalence

$$P = \frac{\text{EPV(death benefits)} + \text{EPV(pure endowment)}}{\text{EPV(rho)}}.$$

```
# Premium level
(EPV_death_benefits + EPV_pure_endowment) / EPV_rho
```

```
3118.116
```

# Let's practice!

LIFE INSURANCE PRODUCTS VALUATION IN R

# Congratulations!

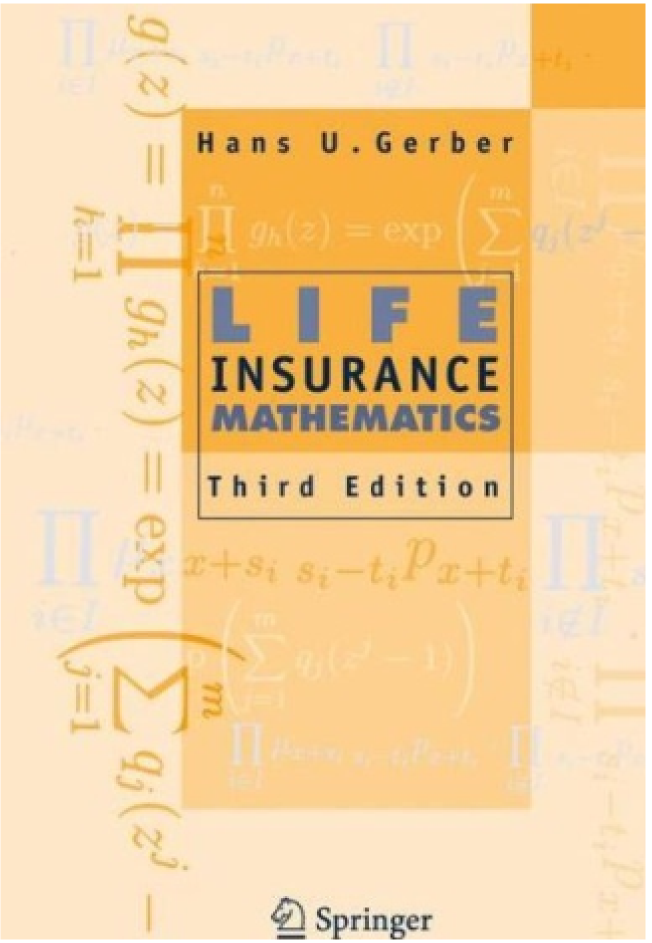
LIFE INSURANCE PRODUCTS VALUATION IN R

**Katrien Antonio and Roel Verbelen**
Professor, KU Leuven and University of Amsterdam Postdoctoral researcher,

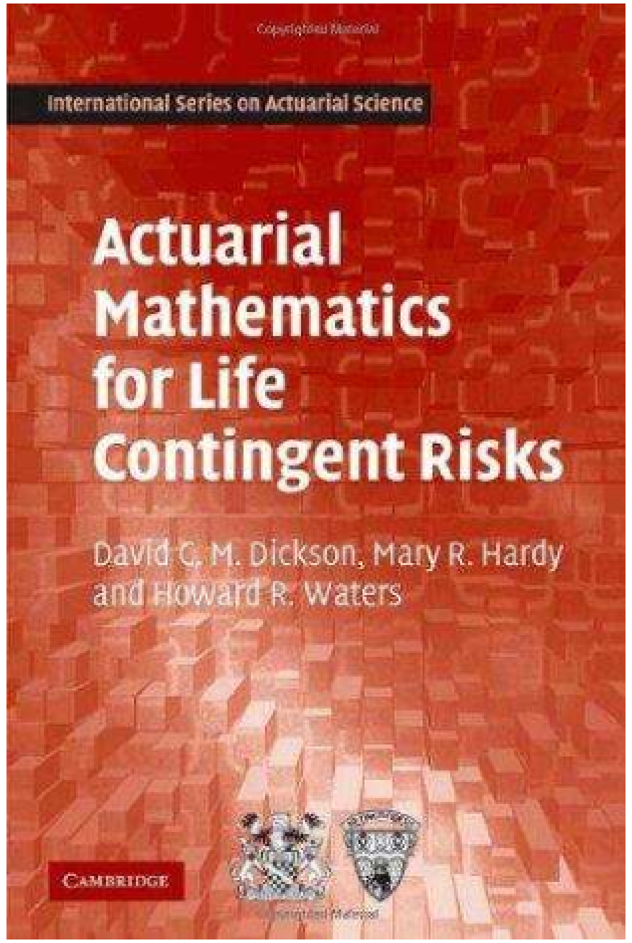
# What you've learned

- Valuation of cash flows
- Life tables
- Life annuities
- Life insurances

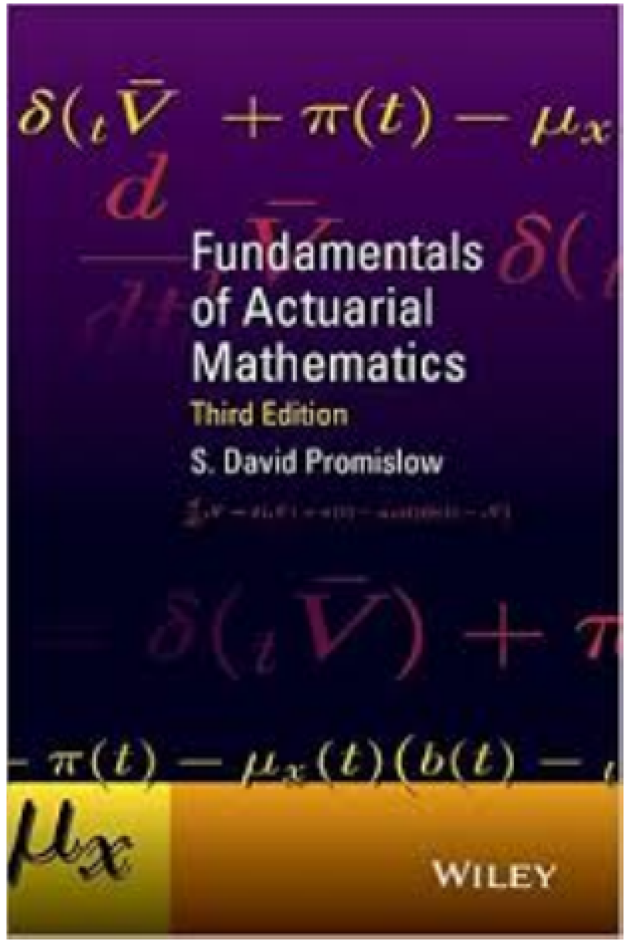
# Want to know more?




Gerber
(1997, Springer)




Dickson, Hardy & Waters
(2013, Cambridge University Press)




Promislow
(2015, Wiley)

# What else is there?

- More advanced life insurance products.
- Loss models for frequencies and severities.
- Data science in insurance.

# Enjoy your journey as an actuary!

LIFE INSURANCE PRODUCTS VALUATION IN R