Laboratory 3

Variant #3

Mariya Zacharneva and Ruslan Melnyk

NOTE: code is available at lab 3.ipynb

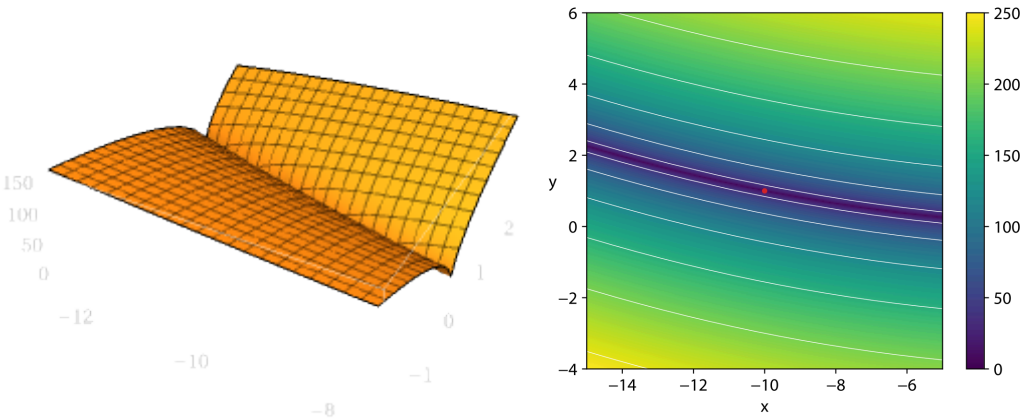
Introduction

For this lab, we have to optimize the Bukin function using Tournament Selection:

$$f(x, y) = 100\sqrt{|y - 0.01x^2|} + 0.01|x + 10|.$$

where x is in the range [-15, 5] and y is in the range [-3, 3].

The Bukin function is a specifically designed test function, also known as artificial landscapes, used to evaluate optimization algorithms. This exact function has its global minimum at the point (-10, 1) with the value of 0.



Tournament selection

Tournament selection is a method that is used in evolutionary algorithms to choose individuals from a population for reproduction. This method randomly selects a small group from the population, and the candidate with the highest fitness is chosen as a parent. This process is repeated until the needed number of parents is reached.

After selecting parents, we run the crossover, or recombination stage. At this stage, the genetic information of the two parents is combined, producing the offspring with mixed features. We are using the Gaussian operator:

$$x_o = \alpha * x_{p1} + (1 - \alpha) * x_{p2}$$
$$y_o = \alpha * y_{p1} + (1 - \alpha) * y_{p2}$$

The next stage of the algorithm is the mutation: at this stage, random mutations are applied to a limited population ratio. In our algorithm, mutation means adding a random value to both X and Y coordinates.

In total, five parameters can be set to obtain the best results:

1. Population size

2. Mutation rate - the ratio of individuals that will be mutated
3. Mutation strength - how big will the changes be when we apply the mutation
4. Crossover rate - the ratio of individuals that will go through the crossover phase
5. The number of generations

Experiments

Best parameters

First of all, we tried a few combinations by hand and found one which gives a fairly good result:

```
population_size=50,
mutation_rate=0.1,
mutation_strength=0.5,
crossover_rate=0.7,
num_generations=100,
```

With such parameters, the best fitness obtained is 0.1958.

Now, let's run a wider experiment with many parameter combinations and see what gives the best results:

1. Population size values: 20, 50, 100, 200
2. Mutation rate values: 0.01, 0.05, 0.1, 0.2, 0.3
3. Mutation strength: 0.1, 0.5, 1, 3
4. Crossover rate: 0.2, 0.5, 0.8, 1
5. Generations: 20, 50, 100, 200

After running a simulation with all of these values, we have found that the 10 best results were obtained with the following parameters:

	Population Size	Mutation Rate	Mutation Strength	Crossover Rate	Generations	Score
887	100	0.2	3.0	0.5	200	0.000982
886	100	0.2	3.0	0.5	100	0.000982
885	100	0.2	3.0	0.5	50	0.000985
871	100	0.2	1.0	0.5	200	0.003310
870	100	0.2	1.0	0.5	100	0.003310
869	100	0.2	1.0	0.5	50	0.003310
868	100	0.2	1.0	0.5	20	0.008065
837	100	0.2	0.1	0.5	50	0.011812
839	100	0.2	0.1	0.5	200	0.011812
838	100	0.2	0.1	0.5	100	0.011812

What we can see from this table:

1. For some parameters, there are clearly winning settings: for example, all top 10 results have a population size of 100, mutation rate of 0.2, and crossover rate of 0.5. However, very good results can be obtained with very different settings for the other two parameters (mutation strength and number of generations).

2. The number of generations seems to be the least important setting: we can see all tested values in the top 10 table. Even in the top 3, the number of generations is the only changing parameter.

3. Even though good results can be obtained with different mutation strengths, the best results were achieved when they were relatively high.

Now, we can tune our parameters even more precisely. We will leave the number of generations and mutation strength as it was before; however, for the other three parameters, we will test results in a close range to our previous winning numbers:

1. **Population size values: 90, 100, 110, 120**
2. **Mutation rate values: 0.18, 0.2, 0.22, 0.25**
3. Mutation strength: 0.1, 0.5, 1, 3
4. **Crossover rate: 0.45, 0.5, 0.55, 0.6**
5. Generations: 20, 50, 100, 200

Results:

	Population Size	Mutation Rate	Mutation Strength	Crossover Rate	Generations	Score
1003	120	0.25	1.0	0.55	200	0.000776
1002	120	0.25	1.0	0.55	100	0.000776
1001	120	0.25	1.0	0.55	50	0.000781
375	100	0.20	3.0	0.50	200	0.000982
374	100	0.20	3.0	0.50	100	0.000982
373	100	0.20	3.0	0.50	50	0.000985
969	120	0.25	0.1	0.55	50	0.001025
971	120	0.25	0.1	0.55	200	0.001025
970	120	0.25	0.1	0.55	100	0.001025
750	110	0.25	1.0	0.60	100	0.001431

Our previous assumption that the number of generations doesn't matter too much is also confirmed here. The same applies to mutation strength; we can see all options in the leaderboard. As for the other three parameters, we can see there definitely are dominant values, but the difference is not dramatic within that small range.

In conclusion, our best-achieved result is:

Parameters	Results

`population_size=120,`
`mutation_rate=0.25,`
`mutation_strength=1,`
`crossover_rate=0.55,`
`num_generations=200`
 	
`seed=42`
	`Best solution: Individual(`
	`x=np.float64(-9.922362064024572),`
	`y=np.float64(0.9845326892959396)`
	`)`
	`Best fitness: 0.0007763793597542801`
	`Average fitness: 20.166185695386353`

Randomness

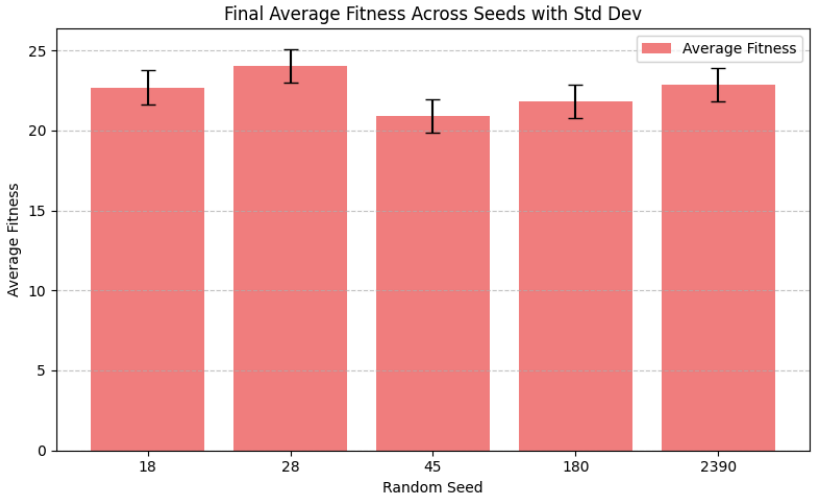
With the obtained best parameters, let's try to use different random seeds to see if the results are stable:

For seed 18,	best solution: [-3.344 0.112]	best fitness: 0.069	average fitness: 22.681
For seed 28,	best solution: [-6.942 0.482]	best fitness: 0.031	average fitness: 24.043
For seed 45,	best solution: [-1.977 0.039]	best fitness: 0.08	average fitness: 20.905
For seed 180,	best solution: [-1.727 0.03]	best fitness: 0.083	average fitness: 21.819
For seed 2390,	best solution: [-11.435 1.308]	best fitness: 0.026	average fitness: 22.852

As we can see, different random seeds can actually change the result a lot, and optimal parameters only work for a given seed. Even though the fitness is very small for all of the results, the found solution is often far from the actual minimum value.

The standard deviation of final best fitness: 0.025

The standard deviation of final average fitness: 1.052



Now we can try to find such a combination of parameters, which would give us good results for all different random seeds, even if it will not be super-precise for any of them in particular. To do that, we will run the result again as in the previous section, but this time each result will be run 5 times for different seeds, and the sum of all fitnesses will be used for evaluation.

	Population Size	Mutation Rate	Mutation Strength	Crossover Rate	Generations	Score
1003	120	0.25	1.0	0.55	200	0.003882
1002	120	0.25	1.0	0.55	100	0.003882
1001	120	0.25	1.0	0.55	50	0.003906
375	100	0.20	3.0	0.50	200	0.004910
374	100	0.20	3.0	0.50	100	0.004910
373	100	0.20	3.0	0.50	50	0.004927
969	120	0.25	0.1	0.55	50	0.005127
971	120	0.25	0.1	0.55	200	0.005127
970	120	0.25	0.1	0.55	100	0.005127
750	110	0.25	1.0	0.60	100	0.007156

The results are not very different from the results with a single seed value.

Let's also rerun the algorithm with decreasing population size and different seed values. In each cell, you can see the best solution, the best fitness, and the average fitness. Results, which are the closest to the actual minimum value, are marked green.

Seed	Population=120	Population=60	Population=30	Population=10
18	[-3.344 0.112] 0.069 22.681	[-9.508 0.904] 0.005 20.966	[-7.498 0.562] 1.522 20.165	[-6.003 0.36] 0.901 19.836
28	[-6.942 0.482] 0.031 24.043	[-1.541 0.024] 0.094 21.376	[-10.258 1.052] 0.815 22.978	[-11.81 1.39] 6.968 15.771
45	[-1.977 0.039] 0.08 20.905	[-12.979 1.684] 0.03 23.582	[-13.927 1.94] 0.04 16.947	[-11.647 1.356] 0.696 13.166
180	[-1.727 0.03] 0.083 21.819	[-2.733 0.075] 0.073 21.754	[-7.922 0.628] 0.063 18.609	[-7.879 0.621] 1.665 16.527
2390	[-11.435 1.308] 0.026 22.852	[-8.163 0.667] 1.354 22.82	[1.287 0.017] 0.274 17.395	[-10.016 1.007] 6.585 13.973

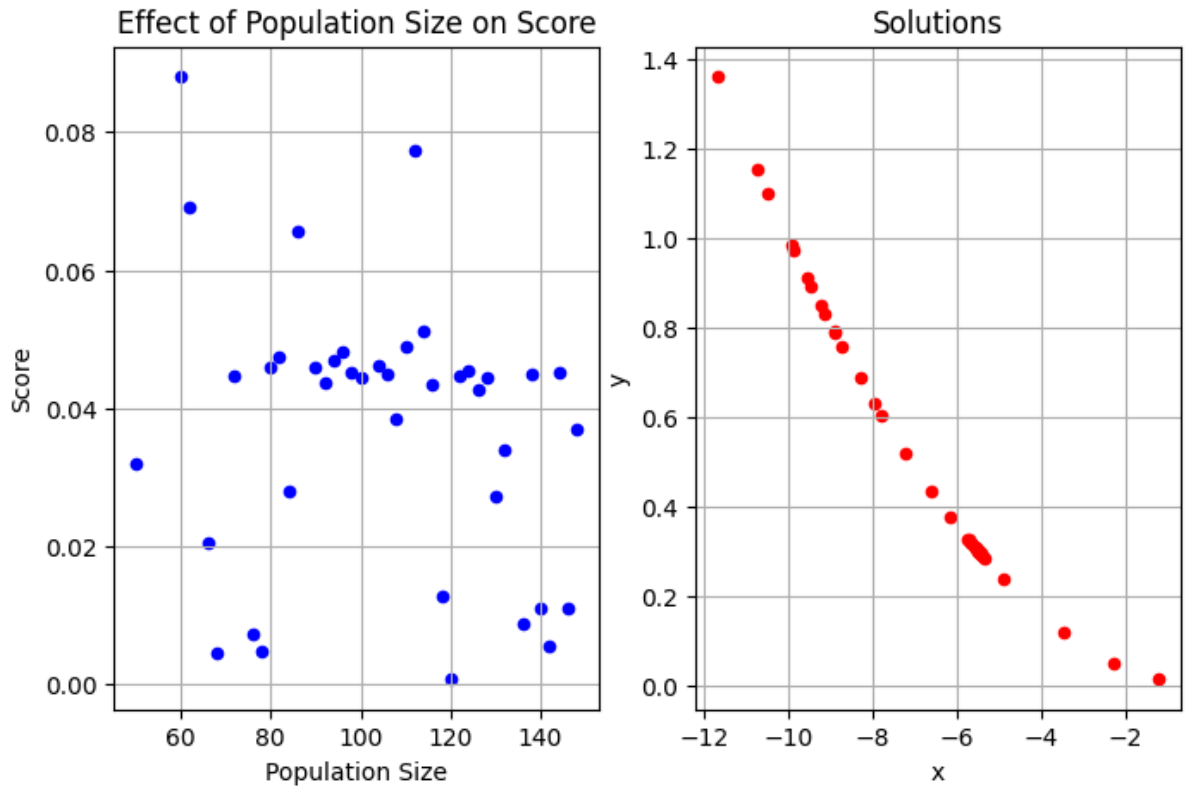
Changing each parameter separately

Let's take our best shot and try to change each parameter individually, to see how the results will change. For the fixed values, we will take:

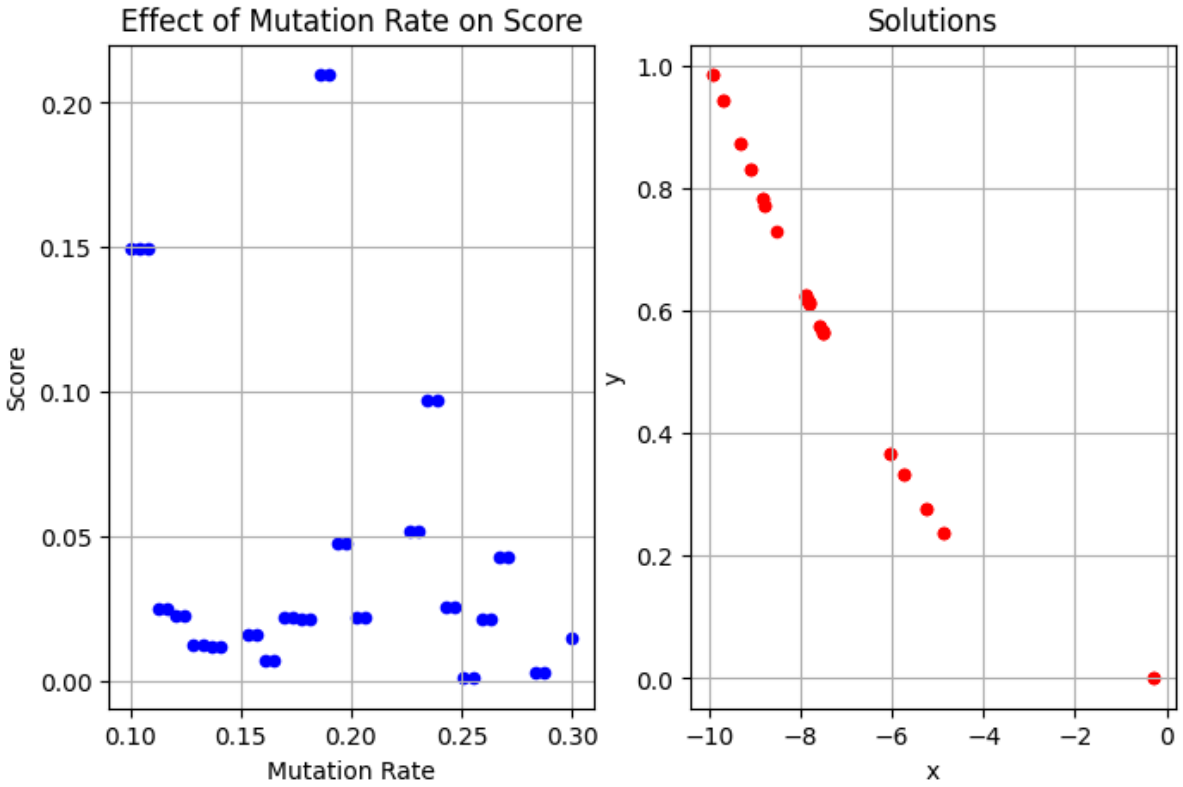
```
population_size=120,
mutation_rate=0.25,
mutation_strength=1,
crossover_rate=0.55,
num_generations=100
```

To get more accurate results, we will drop the worst 10 from each run. This will allow us to see the area near zero more closely and compare close values more precisely.

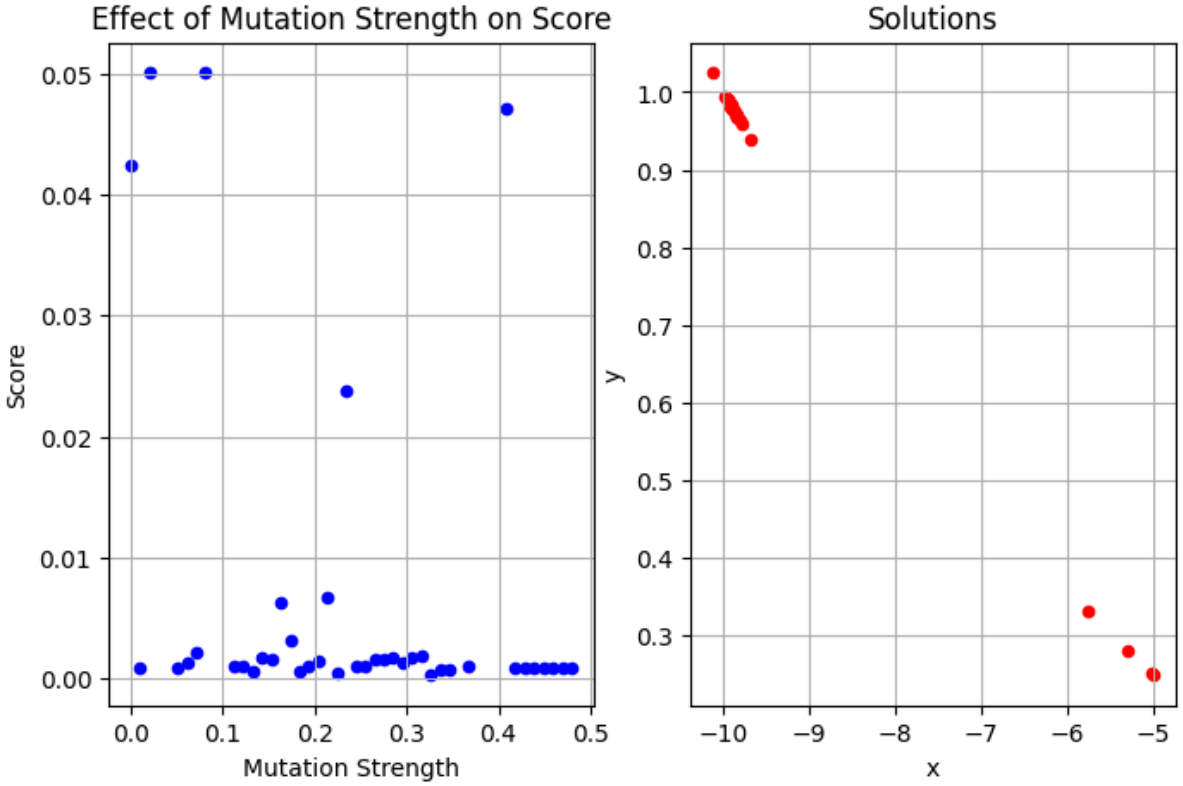
1. Population size: test_values = [x for x in range(50, 150, 2)]



2. Mutation rate: test_values = [x for x in np.linspace(0.1, 0.3, 50)]

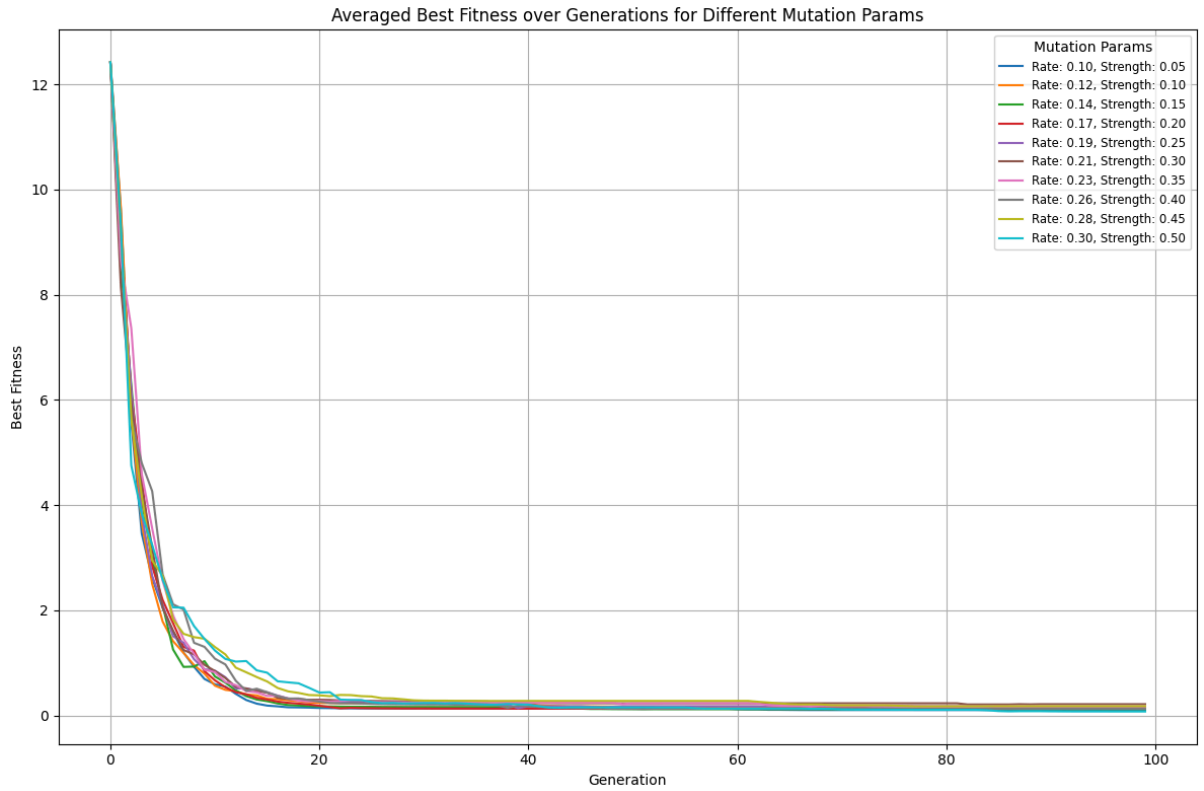


3. Mutation strength: test_values = [x for x in np.linspace(0, 0.5, 50)]

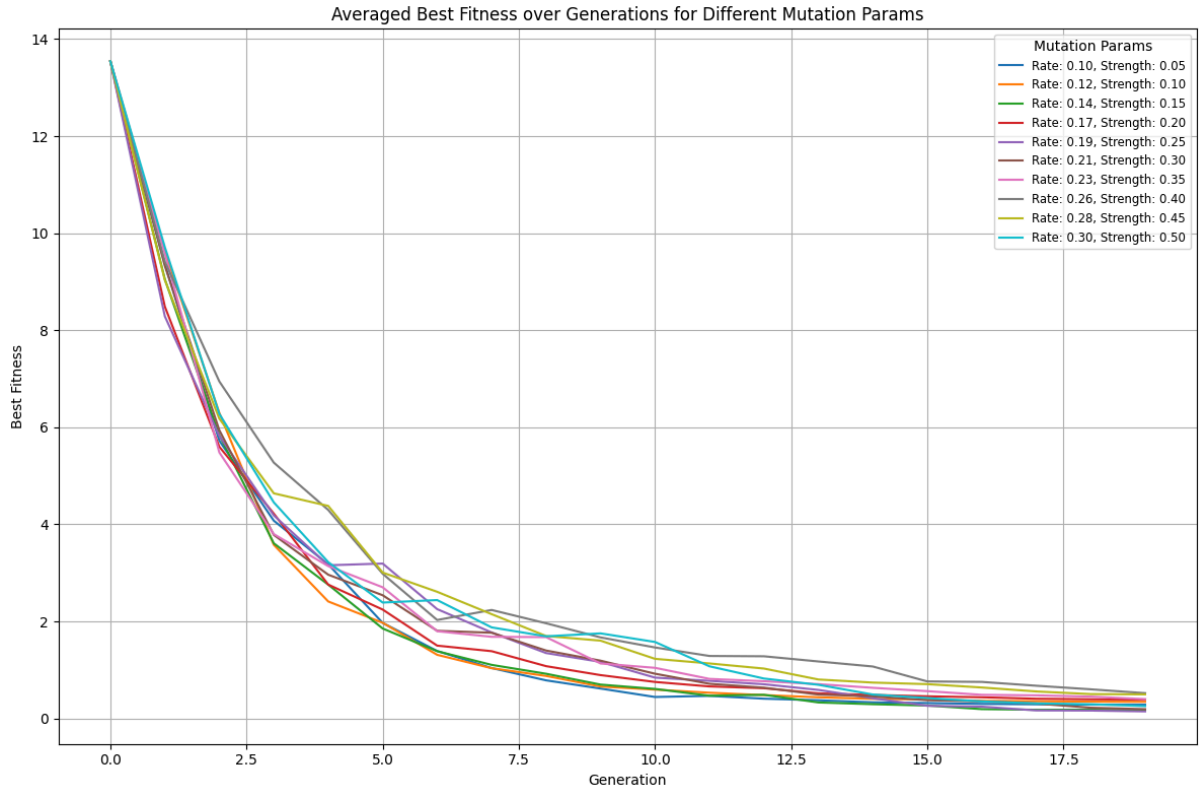


Let's check how the best fitness changes over generations with increasing mutation rates and strength simultaneously.

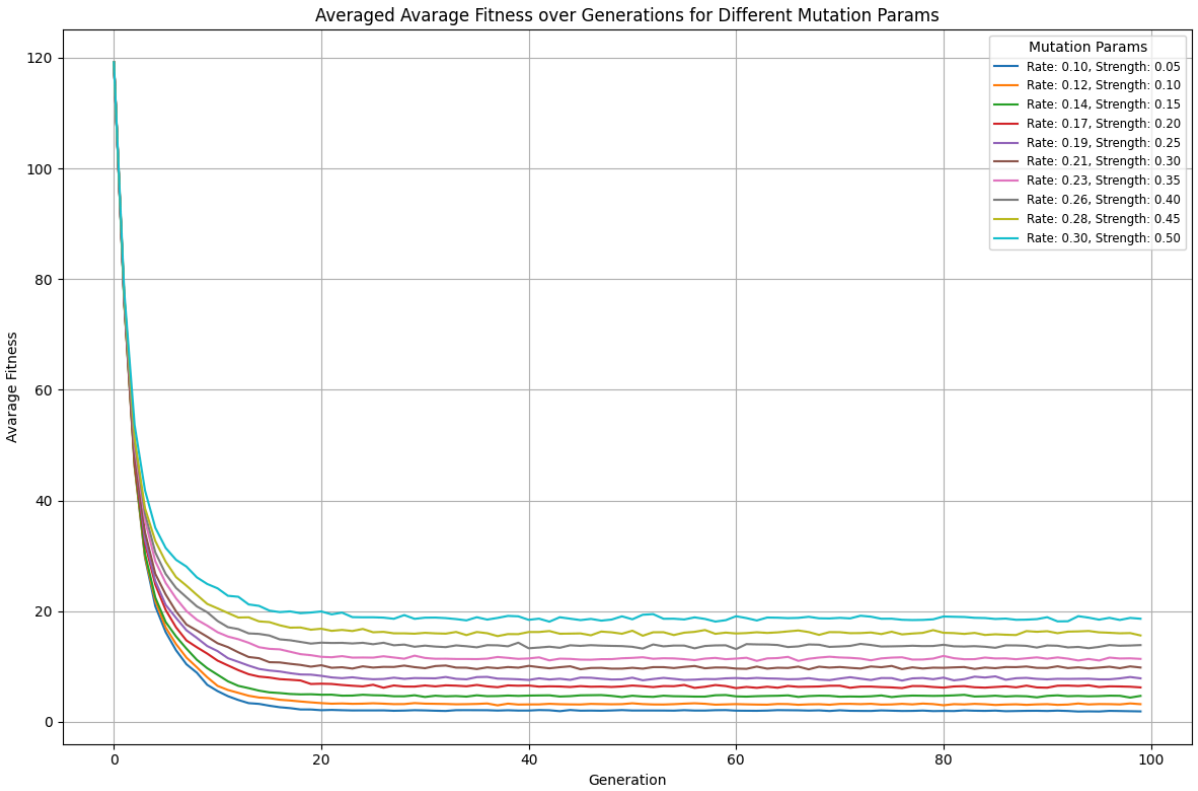
We take the best fitness averaged over 50 seeds, to be sure.



Zooming in to the first 20 generations:

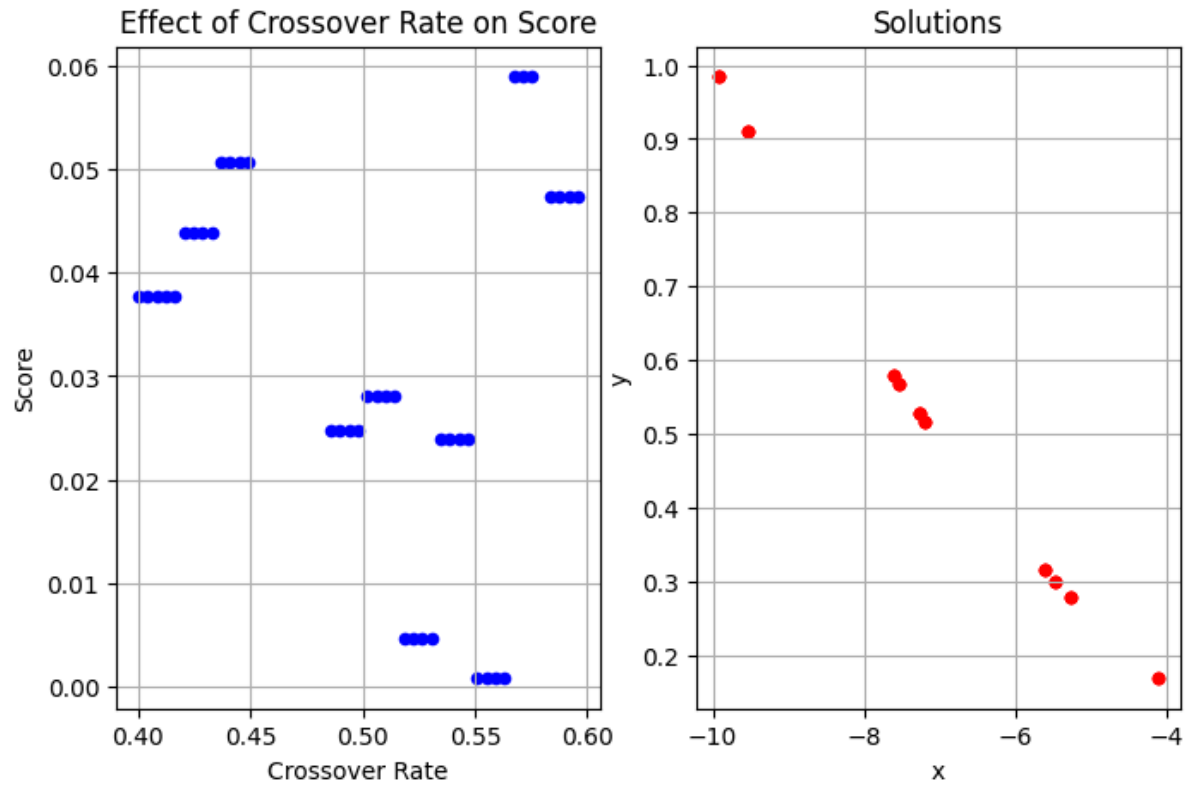


A lower mutation rate and strength seem to result in a faster approach to better fitness. To check the convergence, let's repeat the experiment but now show the average fitness across the generations:



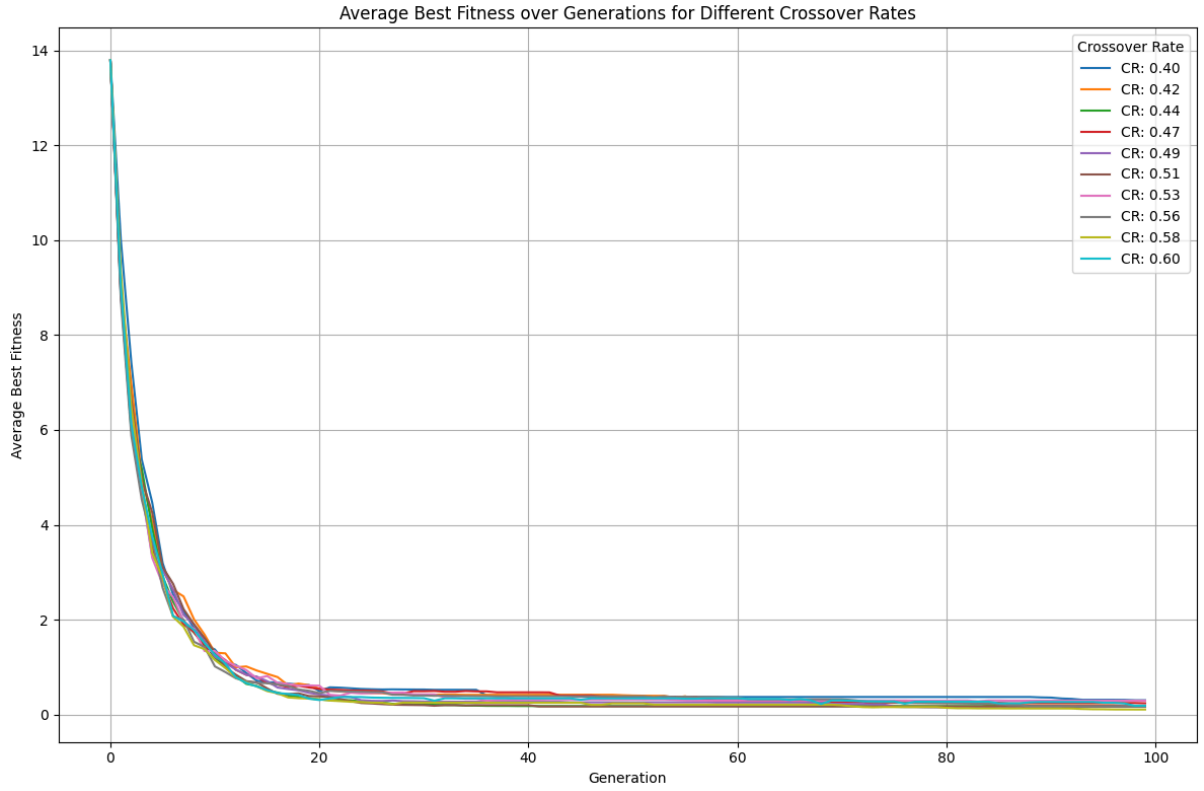
It is clear that a higher mutation rate and strength results in a bigger average fitness. Indeed, it decreases over generations, but a lower mutation rate and strength perform better.

4. Crossover rate: test_values = [x for x in np.linspace(0, 0.5, 50)]

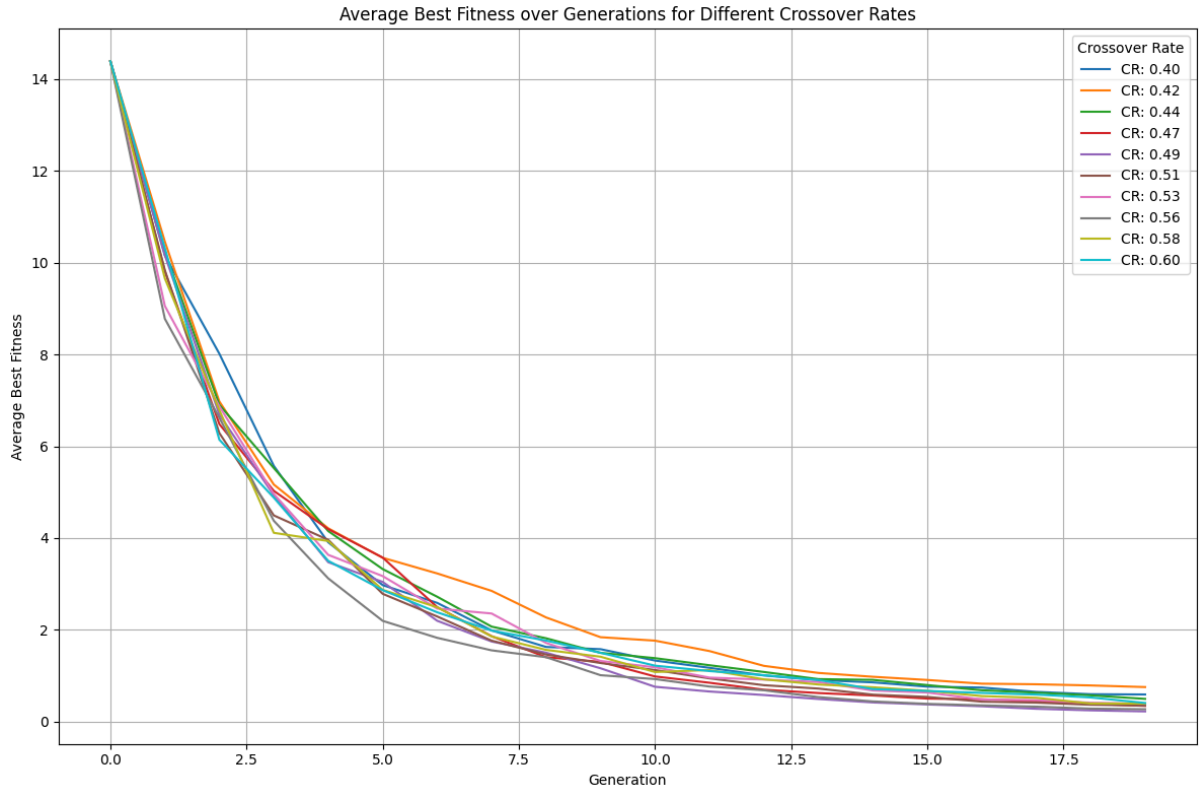


Now, let's stick to 10 crossover rates [x for x in np.linspace(0.4, 0.6, 10)]

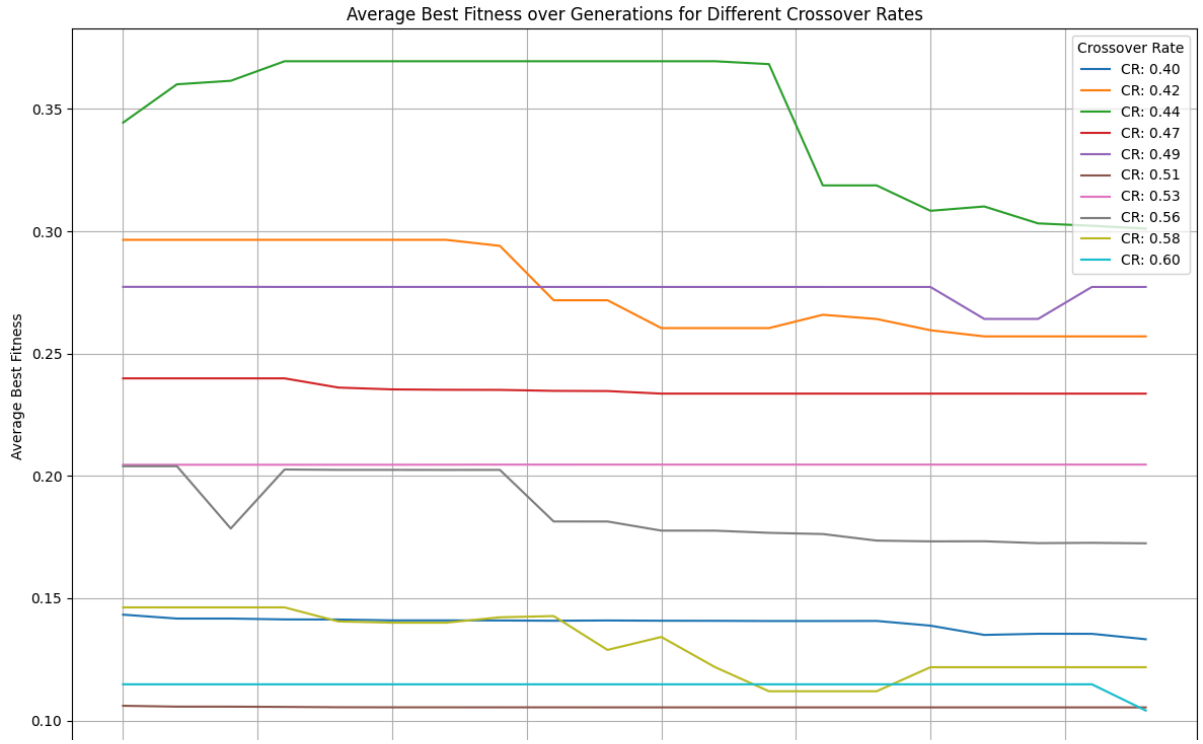
Try 50 different seeds for each crossover rate
And show how the best fitness (averaged over seeds) changes over generations



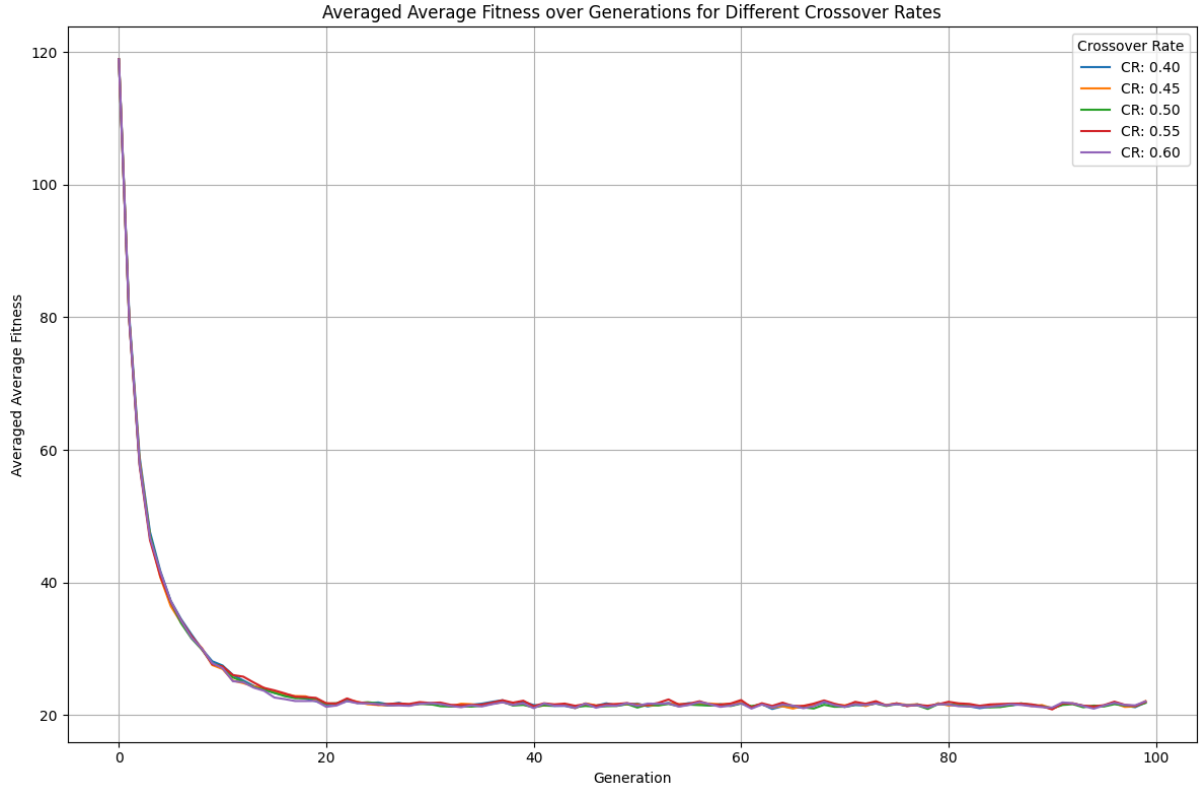
Seems like the effect of the crossover rate is not significant.
Zooming in to the first 20 generations for each crossover rate:



Zooming in to the last 20 generations for each crossover rate:

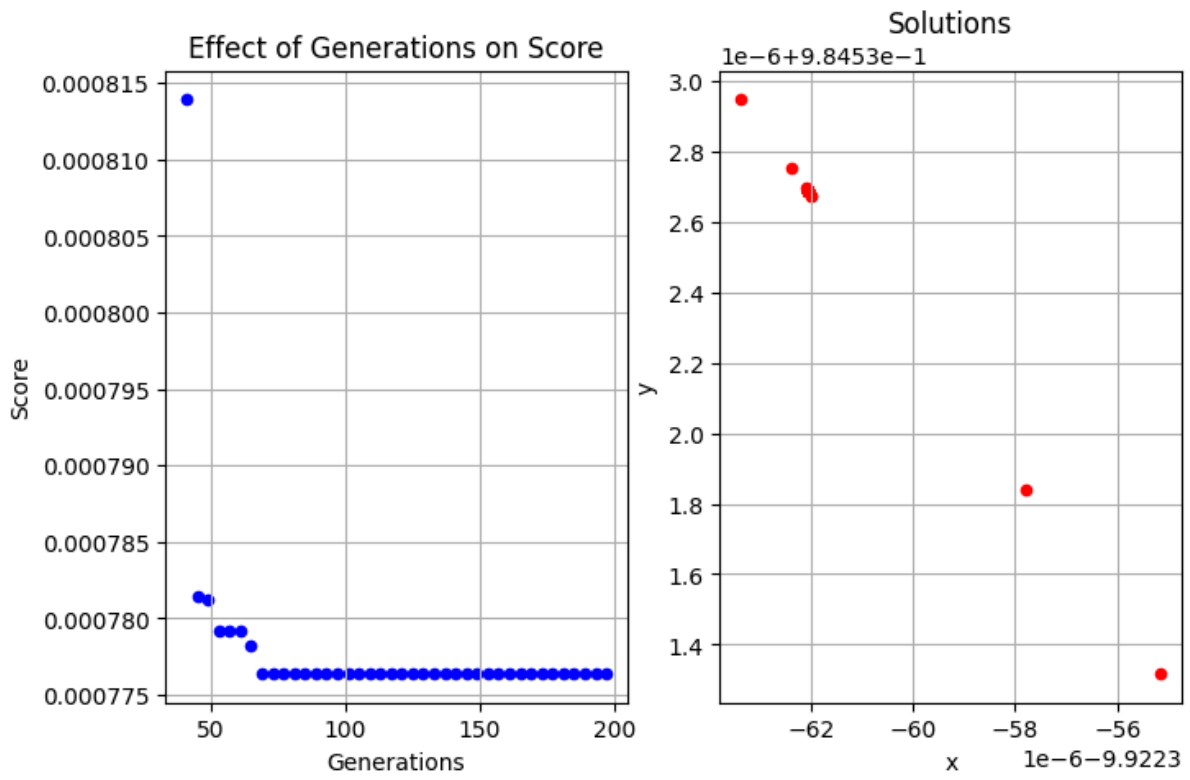


Seems like a crossover rate between 0.5 and 0.6 usually results in better final fitness. Now let's see the average fitness over generations:



We can see that the algorithm converges almost the same way no matter which convergence rate we set.

5. Generations: test_values = [x for x in range(1, 200, 4)]



Conclusion

In this lab, we tested how different parameters affect the performance of an evolutionary algorithm. The main parameters we tested were population size, mutation rate, mutation strength, crossover rate, and the number of generations. The results were shown using tables and graphs to help understand the impact of each setting.

From the experiments, we observed that certain values gave better results than others:

In particular, lower mutation rate and strength perform better (both more stable and give lower fitness quickier over generations).

Crossover rate of 0.5-0.6 usually results in better final fitness, and changing the crossover rate does not affect the algorithm's stability.

The number of generations must be at least 20-50; after 50 generations, the number of generations does not result in better fitness.

Clearly, a population size of approximately 100 is a winning value for this parameter.