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The Behavior of Other as a Reference Point

El comportamiento de los otros como punto de referencia

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The Behavior of Other as a Reference Point¹

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Abstract

We use prospect theory to model reference dependent consumers, where the reference point is the average behavior of the society in the current period. We show that after a finite number of steps under any equilibrium, the distribution of wealth will become and remain equal, or admit a missing class (a particular form of polarization). Under equilibria that admit the highest growth rates, the initial wealth distribution that maximizes this growth rate is one of perfect equality. Conversely, under equilibria that admit the lowest growth rates, perfect equality minimizes this growth rate and societies with a small level of initial inequality grow the fastest. In addition growth rates in corresponding economics without reference dependent consumers admit lower growth rates.

Keywords: Prospect Theory, Reference-dependence, Aspirations

JEL Classification: D11; D91

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El Comportamiento de los Otros Como Punto de Referencia

Resumen

Hacemos uso de la teoría de prospectos para modelar a consumidores cuyo comportamiento sea dependiente de un punto de referencia, donde el punto de referencia es el comportamiento promedio de la sociedad en un periodo presente. Mostramos que luego de un número finito de pasos, en cualquier equilibrio, o la distribución de la riqueza será y permanecerá igual, o mostrará ausencia de una clase (una forma particular de polarización). En los equilibrios que admiten la tasa más alta de crecimiento, la distribución de la riqueza que maximiza esta tasa de crecimiento es de perfecta igualdad. Contrariamente, en los equilibrios que admiten las más bajas tasas de crecimiento, la perfecta igualdad minimiza esta tasa de crecimiento y las sociedades con un pequeño nivel de desigualdad inicial crecerán de manera más rápida. Adicionalmente, las tasas de crecimiento en economías correspondientes pero sin dependencia del punto de referencia son más bajas.

Palabras clave: Teoría de Prospectos, Dependencia del punto de referencia, Aspiraciones

Clasificación JEL: D11; D91

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1. Introduction

It is now an established finding in behavioral economics and psychology that the utility that subjects associate to the different available alternatives often depends on a *reference point* –some amount, or object, that subjects compare the available options with in order to make a choice. That is, the preferences of the individual do not appear to be a fixed ordering irrespective of the environment in which she operates, as suggested by traditional economic modeling, but rather dependent on some ‘point of comparison’ used to guide choice.⁴

These findings have spurred the development of a large literature aimed at identifying the correct functional form to represent such reference-dependence, originating a variety of models –most prominently Prospect Theory of Kahneman and Tversky (1979), Tversky and Kahneman (1991).⁵

This model has seen a large number of applications in various branches of economics, generating a large literature. With few exceptions, however, the entire such literature has focused only on two very specific types of reference-points: the case in which the reference point is the agent’s status quo or endowment, as in almost all the applications of Prospect Theory, and the case in which it is the agent’s expectations, mostly following Köszegi and Rabin (2006) and Köszegi (2010).

While both the status quo and the expectations are important components for the formation of a reference point, there is a third additional element that seems to play an important role: *the behavior of others*. For example, one could argue that a consumer who receives 10 is ‘happier’ when everybody else also receives 10, than she would be if everybody else received 1000. This seems to be due to some form of reference-dependence –but where the reference point is the behavior of *others*. The goal of this paper is to study exactly this case: we study the consequences of reference-dependence when the reference point is what others have or choose. We consider a standard overlapping generations model with a continuum of reference dependent agents in a Prospect Theoretic sense. Each agent has a

⁴ In this his vast literature see, amongst many, Tversky and Kahneman (1974), Kahneman and Tversky (1979), Tversky and Kahneman (1981), Samuelson and Zeckhauser (1988), Camerer (1995).

⁵ Other models, some of which are much related to Prospect Theory, appear in Köszegi and Rabin (2006), Köszegi (2010), as well as Chateauneuf and Wakker (1999), Masatlioglu and Ok (2005, 2008), Diecidue and Van De Ven (2008), Ortoleva (2010), Wakker (2010), and Ok et al. (2011)

trade off between a consumption decision and an investment decision (the bequest left to the offspring in the second period). As opposed to other applications of prospect theory, however, we assume that the agent's reference-point is the average choice of the rest of the society, i.e., the average endowment left to the offspring. We then analyze the consequences that such reference-dependence has on the dynamics of wealth distribution and on growth.

We study a model in which the utility that households derive in the second and last period of their lives depends on the average wealth of the other members of the society during that period, denoted \bar{x}_{t+1} .⁶ We model such reference dependence following prospect theory: we assume that the utility function is *convex* in an interval that ends with \bar{x}_{t+1} , and that it has a kink (it is not differentiable) at \bar{x}_{t+1} , where the left-derivative is greater than the right-derivative. Everything else in the model is entirely standard.

We obtain the following results. After noticing that this economy admits multiple equilibria, we show that all equilibria have a common feature: there is a finite period T after which the wealth distribution of the economy is either of perfect equality, or it admits a *missing class*, i.e., it assigns zero mass to an interval right before the average of the distribution. This can be seen as a particular form of polarization between the rich and the poor. We show that the wealth distribution will maintain one of these forms for all subsequent periods; that no other distribution is possible in the long run; and that one of these distributions must be reached in *finite* time (as opposed to asymptotically, or in steady state). Furthermore, we show that whether the economy converges to a distribution of perfect equality or one with a missing class depends both on the equilibrium under consideration and on the initial allocation. In particular, we show that every society with a non-degenerate initial distribution has an equilibrium in which the wealth distribution will admit a missing class in the long run. On the other hand, a society that has an initial distribution that is 'not too disperse' (the difference between the average and the poorest

⁶ As we discuss below, our analysis could be trivially generalized to the case in which the society is divided into m groups stable over time, e.g., ethnic groups, and each member of the society uses, as a reference point, the average wealth of the group she belongs to.

household is ‘small’) will *also* admit an equilibrium in which the wealth distribution converges to perfect equality in finite time.

We then study how the initial distribution of endowments affects the relative growth of societies with reference-dependence. First, we show that under the equilibrium with the highest growth, a perfectly equal initial distribution attains the highest growth rate. However, things change considerably under the equilibrium with the *lowest* growth: in this case the society with *some* initial inequality, but that still admits an equilibrium of perfect equality will perform the best. In particular, this economy will grow *strictly more* than an economy that starts from perfect equality. This means that if we focus on the equilibria with the lowest growth, then ‘a little bit’ of initial inequality is good for growth.

Finally, we show that if the utility exhibits Constant Relative Risk Aversion, then any society with reference-dependence grows (weakly) more than a corresponding (standard) economy with the same initial endowment, but without reference-dependence. Put differently, we show that the presence of reference-dependence, in the form that we study, cannot be bad for growth. Intuitively, this happens because reference-dependence allows agents to ‘push’ each other, generating growth.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model, the notion of equilibrium that we use, and describes the optimal choice of the households in this environment. Section 3 discusses the consequences of this reference-dependence on the wealth distribution in the long run, while Section 4 analyses its effects on growth. Section 5 concludes. The proofs appear in the appendix.

Our model is directly related to papers that apply Prospect Theory to standard economic models. In this vast literature, Koszegi and Rabin (2009) and Bowman et al. (1999) analyze the consequence of reference-dependent behavior in a consumption-saving model. As opposed to our work, however, their focus is on the case in which the reference point is the agent’s expectations of future consumption. Foellmi et al. (2011) also use prospect theory in a standard Ramsey growth model, but with past consumption as the reference point. Loewenstein and Prelec (1992) apply reference dependence to intertemporal choice. For many other examples of this vast literature see also Spiegel (2011) for applications to Industrial Organization, Sunstein (2003) for applications to Law and Economics, Barberis

and Thaler (2003), Thaler (1993), and Rabin (2002) for applications to finance, Kyle et al. (2006) for a model of liquidation decision by an investor.

Our model is also related to the literature in development economics that studies how the behavior of others might affect an agents' 'aspirations'. Amongst them, Appadurai (2005) and Ray (2006) suggest how aspirations could play both a positive and a negative role in the development of a country: they might induce some subjects to 'work hard' to reach their aspirations; at the same time, those who are too far from reaching them might simply 'give up'. The latter paper in particular was an important source of inspiration for our work. Formal models that study the role of aspirations appear in Banerjee (1990), Genicot and Ray (2009), Dalton et al. (2010), and Mookherjee et al. (2010). The latter studies the case in which agents look at their 'neighbors' to form their aspirations –i.e., they focus on a 'local' origin of aspirations, instead of a society-wide origin like we do. (Furthermore, they use a functional form very different from prospect theory.) Genicot and Ray (2009) similarly study an OLG model in which reference dependent subjects live one period, and admit a utility function that is differentiable everywhere and convex in an interval right before the aspirations level. They first show that there exists an inverse U effect of aspirations on accumulation decision, proving the conjecture in Ray (2006). Then, focusing on the case in which aspirations are the average income of the society, they show that the support of the income distribution in its steady state is generically finite, and that when aspirations are, loosely speaking, 'important enough', a distribution of perfect equality cannot exist. In addition, they also consider the case of 'upward looking' aspirations, i.e., the case in which each individual's aspirations are the average income of those *above* her in the income distribution, and show that in this case continuous income distributions can exist in a steady state if and only if they are Pareto distributions. By contrast, in our paper we focus only on the case in which the reference-point is the average endowment of the society, but consider a utility function which not only has an area of convexity, but also admits a *kink* at a reference point –directly following prospect theory and the notion of loss aversion. This feature allows us obtain our different results on the long run distribution of endowment: that the distribution becomes, and remains, either of perfect equality or has a missing class, therefore also admitting the case of continuous support. (In our model these

features are reached in finite time, as opposed to in the steady state.) Furthermore, it also allows us to characterize the effects of the initial distribution on growth.

Finally, our work is also more broadly connected with the vast literature in macroeconomics usually referred to as *keeping up with the Joneses*, that models agents whose utility for consumption depends also on the relative position of the agent in the wealth distribution (their status). (An analysis similar to that in our work appears in Konrad (1992); Fershtman et al. (1996); Rauscher (1997); Corneo and Jeanne (2001); Cooper et al. (2001); Stark (2006); Hopkins and Kornienko (2006); García-Peñalosa and Turnovsky (2008).) Our approach is different for two reasons. First, these models assume that individuals care about their status using some additive or multiplicative functional form, which renders the model very different from ours in which we assume that subjects care about how their income relates to the average one using a prospect-theoretic form. From this point of view, one can loosely see our work as exploring a similar idea, albeit using a functional form which has been derived from behavioral economics and psychology. Second, most of these works focus more on the properties of the accumulation path than on the implications for the wealth distribution in the long run –as evidenced by the fact that most use a representative agent. In contrast, the development of the wealth distribution, and its effects on growth, are the focus of our study.⁷

2. The Model

2.1 Formal Setup

We study an economy with overlapping generations and warm glow preferences (see Andreoni (1989)). The economy is populated by a size L continuum of agents who live for two periods, make choices in the first period, generate an offspring exogenously in the

⁷ In general, the relation between saving decisions and long run distribution has been studied extensively in macroeconomics: see, amongst many, Stiglitz (1969), Bourguignon (1981), Chatterjee (1994) and Bertola et al. (2005). These works, however, follow a very different approach, as they either consider a saving function which assumed exogenously, in reduced form, or study the distribution under individual optimization, but do not treat the case of reference-dependent preferences.

second period, and have a bequest motive directly in the utility function.⁸ Every agent is endowed with h_{it} of human capital and k_{it} of physical capital, which are determined by the decision taken by the previous generation. Given equilibrium prices of human and physical capital, we denote by x_t the market value of the endowment, and we will refer to it as the endowment, wealth or bequest. We assume constant return to scale in the accumulation of human capital, i.e., by investing h_{it+1} households obtain Ah_{it+1} , and without loss of generality we set $A = 1$. By $\beta \leq 1$ we denote the discount factor; u_t stands for the distribution of endowments in each period, and $\text{supp}(u_t)$ denotes its support; r_t and w_t represent the interest rate and the wage. Finally, in the economy we have a representative firm with a production function $F(K, H)$.

The individual household maximizes:

$$\begin{aligned} & \max_{c_{it}, h_{i,t+1}, k_{i,t+1}} u(c_{it}) + \beta v(x_{i,t+1}, x_{i,t+1}) \\ & \quad \text{such that} \\ & c_{it} + h_{i,t+1} + k_{i,t+1} \geq w_t h_{i,t} + (1 + r_t) k_{i,t} = x_{it} \\ & c_{it}, \quad h_{i,t+1} \geq 0 \\ & x_{i,t+1} = x_{t+1} h_{i,t+1} + (1 + r_{t+1}) k_{i,t+1} \\ & \bar{x}_{i,t+1} = z(\mu_{t+1}) \end{aligned} \tag{1}$$

As opposed to standard OLG problems, there is no consumption in the second period, and the agent's utility for the second period, v depends only on the bequest and can be different from utility function in the first period, u . In particular, the former also depends on an additional term, $\bar{x}_{i,t+1}$, which we will interpret as the agent's *reference-point* for period $t + 1$. The last line states that the reference point is a general function of the distribution of bequest. The non negativity constraints are straightforward, but do not apply to physical capital because there is not limited access to credit.

We start by imposing standard restrictions.

⁸ The presence of an uncountable number of agents is inessential for our results. It is routine to show that everything we prove would hold true qualitatively with finitely many agents.

Assumption 1. $u(\cdot)$ is increasing, concave and satisfies the Inada conditions.

Assumption 2. The production function $F(K, L)$ is increasing, concave, homogeneous of degree one, and satisfies the Inada conditions.

Both requirements above are standard. In particular, agents have a standard utility function in the first period of their lives. (In Section 4 we will also consider the more specific case in which u exhibits Constant Relative Risk Aversion, CRRA.) The main feature of our model is the shape of v .

Assumption 3. There exists a $H > 0$ such that for every $\bar{x} \in \mathbb{R}_+$, the following holds:

1. [*v is continuous and monotone*] $v(\cdot, \bar{x})$ is strictly increasing and continuous; $v(\cdot, \bar{x})$ is continuous for every $x \in \mathbb{R}_+$;
2. [*far away from the reference-point $v(\cdot, \bar{x})$ behaves like u*] for all $x \notin [x - H, x]$, $v(\cdot, \bar{x})$ is twice differentiable and $\frac{dv(x, \bar{x})}{dx} = \frac{du(x)}{dx}$;⁹
3. [*v has a prospect-theory form*]:
 - (a) $v(\cdot, \bar{x})$ is strictly convex on $(\bar{x} - H, \bar{x})$;
 - (b) $\lim_{x \nearrow \bar{x}} \frac{dv(x, \bar{x})}{dx} > \lim_{x \searrow \bar{x}} \frac{dv(x, \bar{x})}{dx}$.

The idea behind Assumption 3 is that our agents have a reference-point which affects their preferences. This reference point affects the second period utility v in two ways, both of which are the fundamental features of prospect theory (part (3) of Assumption 3):

1. *v is steeper for losses than for gains, and in particular it is not differentiable at $v(x, \bar{x})$* : this generates the well-known effect called ‘loss-aversion’, and is motivated by the different approach that subjects have to losses as opposed to gains with respect to the reference point.

2. *v is convex below the reference-point, and concave above it*: this feature, which leads to the so-called ‘diminishing sensitivity’, is motivated by the fact that the marginal

⁹ By $\lim_{x \nearrow \bar{x}} f(x)$ we denote the limit of $f(x)$ as x approaches \bar{x} from below. (The limit from above is defined analogously.)

change in gain-loss sensations is greater the closer we get to the reference point. (Notice that v is strictly concave after \bar{x} , since there it coincides with u .)

(See, among others, Kahneman and Tversky (1979), Tversky and Kahneman (1991, 1992) for more discussion on these properties.) Despite reference-dependence, we assume that v remains ‘well-behaved’: the presence of a reference point does not render the function discontinuous or not-monotone (part (1)). Moreover, as opposed to what usually assumed in prospect theory, we posit that the reference point affects the agents’ utility in the area ‘close to’ the aspiration level, but not ‘far below.’ we posit that there exists a positive H such that v is not subject to reference-effects, i.e., it coincides with u , for levels of x below $\bar{x} - H$ (part (2)). The rationale of this restriction is that reference effects are the strongest in the area immediately preceding the reference point, but then they tend to fade, all the way to disappearance, as we get further below. At the same time, we posit that when the reference point is reached, subjects should go back to their standard behavior: v becomes concave and behaves like u (part (2)).

Because v depends on the reference-point, but u does not, then with Assumption 1 and 3 we are imposing that households are reference dependent *only in the second period of their life*— the reference point affects how they value their bequest. Conversely, they are not reference dependent when they are young. This approach is motivated by the observation that individuals tend to set objectives for themselves to be accomplished when they have reached a certain age, or for their offsprings to reach, as opposed to for their young age. That is, that the accomplishment of life goals tends to be evaluated only in the later part of life. (A similar approach appears in Mookherjee et al. (2010) and Genicot and Ray (2009).) Thus far in our analysis we have not imposed any restriction on the origin of agents’ reference-point. In this paper we choose to focus on a very specific source: we study the case in which the reference-point in period $t + 1$ is the *average* endowment of the society as a whole in period $t + 1$. (Recall that μ_{t+1} denotes the density of endowments at time $t + 1$.)

Assumption 4. For $i \in L$, $t \in T$, $\bar{x}_{i,t+1} = \int_L x_{i,t+1} \mu_{t+1}(x) di$.

While our analysis below focuses on this case, it is straightforward to generalize it to the case in which the population is divided into m groups of positive mass, and the reference point of each agent is the average bequest of the other members *of the group she belongs to*. (Our treatment above would be a special case in which there is only one large group.) This would be the case if, for example, the society is divided into social or ethnic groups, and agents only looked at their relative behavior within their group. If the composition of the groups is stable over time, as it is the case for ethnic groups, then it is immediate to translate all of our results below to apply to the dynamics *within* each group. (The results for the society as a whole would have a more complicated analytical expression, as groups might overlap in the wealth distribution, but would maintain the same basic intuition.) See Section 3.2 for more.

2.2 Discussion

In our model the reference point pertains to the agent's second-period choice, which is the bequest that subjects leave to her offspring. The rationale of having aspirations on this dimension is that it includes elements such as investments in education for the children, housing, or other forms of human capital, which are typical dimensions in which aspirations play a relevant role.

At the same time, it is worth noting that identical results would hold for different specifications as well. First, we obtain identical results if we assume that second period choice –and aspirations– involve both the agent's second period consumption and her bequest, with the additional assumption that the agent chooses a constant fraction in the two dimensions (as would be the case, for example, if preferences in the consumption-bequest space at time $t + 1$ were Cobb Douglas). Second, again similar results would be obtained if the production possibility set of the economy were constructed in a more standard OLG framework– as long as reference-dependence is maintained. For example, one can think of a more standard setup in which labour supply is inelastic (one unit per individual), and households have access to a standard well-behaving production function for which the

technical progress, A , depends on the accumulated capital inside each family.¹⁰ In what follows we won't come back on these alternative formulations, but it is a standard exercise to show that identical results would hold in these cases as well –the mathematical results would be virtually identical, and statements could easily be rephrased accordingly.

2.3 Notion and multiplicity of equilibria

One of the features of the economy described above is that each agent's utility depends on the behavior of the other members of the economy. Since agents decide *simultaneously*, the expectations of the behavior of others will play an essential role: agents are best responding to what they expect others to do. In line with this, we focus our attention on the following (standard) kind of equilibria.

Definition 1. An equilibrium of the economy is a sequence of consumption decisions, human and physical capital investment, factor prices and a distribution $e = \{\{c_{it}\}_{i \in L}, \{k_{it+1}\}_{i \in L}, w_t, r_t, \mu_{t+1}\}_{t=0}^{\infty}$ such that factor markets clear, $(c_{it}, h_{it+1}, k_{i,t+1})$ solves (1) for all $i \in L$ and for all t and the reference point $\bar{x}_{i,t+1} = \int_L x_{i,t+1} \mu_{t+1}(x) di$.

Definition 1 is a standard notion of equilibrium. Notice that by using this definition together with Assumption 4, we are implicitly assuming that subjects can correctly forecast the behavior of others: in fact, reference-points are defined as the average of the true wealth distribution in the second period of the agents' life, but affect their behavior already in the first period – which implies that households can correctly forecast this distribution.

¹⁰ In this case, the household's problem would be

$$\max_{c_{it}, h_{i,t+1}, k_{i,t+1}} u(c_{it}) + \beta v(x_{i,t+1}, \bar{x}_{i,t+1})$$

such that

$$c_{it} + k_{i,t+1} \geq w_{it}$$

$$c_{it} \geq 0$$

$$c_{i,t+1} \leq (1 + r_{t+1})k_{i,t+1}$$

$$\bar{x}_{i,t+1} = z(\mu_{t+1})$$

where $w_{i,t} + (1 + r_{it})k_{it} = A_{it}F(k_{it}, L_{it})$, with $A_{it}(k_{it})$.

It is standard practice to show that one such equilibrium exists for any economy that satisfies Assumptions 1-4. As opposed to the case without reference-points, the economy described in Section 2.1 will not have a unique equilibrium. While indeed the presence of multiple equilibria is not surprising in general, in this economy this is mostly due to the presence of reference-dependence: by rendering the agent's utility dependent on the behavior of others we introduce obvious coordination issues. To see why this leads to multiple equilibria, consider the simplest possible case: an economy in which all agents start with the same endowment in the first period. In this simple case, all agents have the same reference-point, and face the same problem: they will then choose the same consumption-bequest plan.¹¹ And since all households have the same consumption, they will all reach their reference-point—they will all invest exactly what is needed to reach the average. The problem is, however, that this average will depend on the future behavior of other households. Consider for example some equilibrium in which all households choose some $a > 0$ in the second period (and each of them knows that this will happen). Then, consider some alternative equilibrium in which, instead, each household knows that all other agents will choose $a + \epsilon$ (where ϵ is small and positive). In this latter case, if one household were to choose only a , it would fall short of its reference-point. Then, as long as ϵ is small, it might choose instead to save a little more in the first period so that it could reach the rest of the society—and thereby its reference-point. And since this can be true for all agents, then everybody will in fact choose $a + \epsilon$, guaranteeing that this is an equilibrium. (Naturally this is true only as long as ϵ is 'small': there will be some $\bar{\epsilon}$ large enough such that, even if every agent knew that everyone else will consume $a + \bar{\epsilon}$, they'd rather fail to reach their reference-point and choose only a , which means that $a + \bar{\epsilon}$ cannot be an equilibrium.)

In the analysis that follows we will therefore analyze the features of the set of possible equilibria of the economy.

Before we proceed, the following remark will be useful.

¹¹ This requires that we cannot have two subjects who break indifferences in a different way. Lemma 1 in the Appendix proves that this must be the case.

Remark 1. Since there exist two assets (human and physical capital), they should bear equal returns. Given the assumption of competitive markets and the assumption on the differentiability of the production function, at every t we must have $w_t = 1 + r_t$. Then the household problem simplifies to the choice of I_{t+1} to maximize:

$$\max_{I_{t+1}} u(x_{it} - I_{t+1}) + \beta v(w_{t+1}, I_{t+1}, \bar{x}_{i,t+1}) \quad (2)$$

As customary define $K_t = \int_L k_{it} di$, $H_t = \int_L h_{it} di$, $\lambda_t = \frac{K_t}{L_t}$ and $F(K_t, H_t)/H_t$. (Notice that by Assumption 2 there are constant returns to scale.) By the arbitrage condition above, and since factor rental prices are equal to the marginal productivity at every t , we must have

$$f'(\lambda_t) = f(\lambda_t) - f'(\lambda_t)\lambda_t. \quad (3)$$

By Assumption 2 there exists a unique which satisfies (3), since the LHS is always decreasing and the RHS is always increasing.

As a result, the economy is on a balanced growth path since period zero and the growth rate of the economy is equal to the growth rate of investment. From now on, we use $w^* = f(\lambda^*) - f'(\lambda^*)\lambda^*$ to indicate the wage rate at every t .

The remark above also shows why the credit market cannot solve the problem of reaching the aspiration level: if the agent borrows one more unit of physical capital to increase by one unit the educational attainment of the child (increase in human capital), then in the following period she will have an additional cost of $(1 + r_{t+1})$ which, by the no arbitrage condition, must coincide with the additional gain of w_{t+1} .

2.4 Properties of the optimal behavior

Because of the convexity and non-differentiability of the second-period utility function, the behavior of households in this economy will be different from that of the standard model. To better express its features, let us define the optimal choice of a household in the second period as a function of the initial endowment and aspiration level: define $\phi: \mathbb{R}_{++} \times \mathbb{R}_{++} \rightarrow$

\mathbb{R}_{++} such that $x_{i,t+1} = \phi(x_{it}, \bar{x}_{i,t+1})$ is an optimal solution of (1) for a household i with initial endowment x_{it} and reference-point $\bar{x}_{i,t+1}$. Then, the following must hold.

Proposition1. *The following holds for all \bar{x} :*

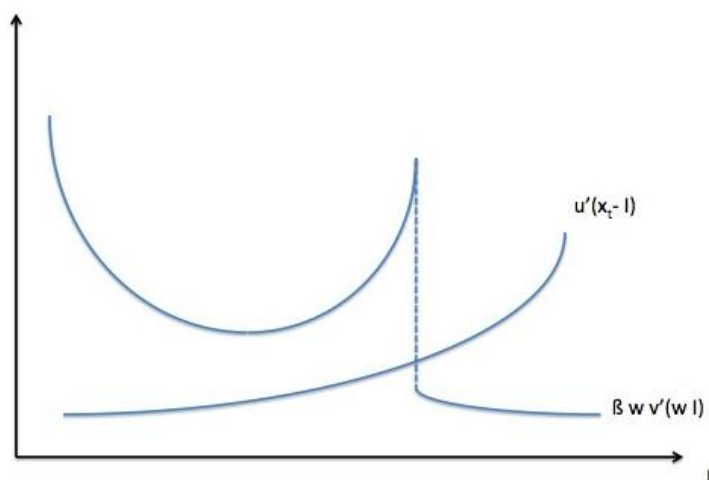
1. *there exist $\gamma \in \mathbb{R}, \gamma > 0$ such that for all $x' \in \mathbb{R}_{++}$, $\phi(x', \bar{x}) \notin (\bar{x} - \gamma, \bar{x})$;*
2. *there exist some $x'_i, x''_i \in \mathbb{R}_{++}$, $x'_i < x''_i$ such that $\phi(x, \bar{x}) = \bar{x}$ for all $x \in (x'_i, x''_i)$.*

Proposition 1 shows two features of the optimal solution of the household problem. Part (1) shows that there is an interval (of positive measure) of second period wealth, right before the reference-point \bar{x} , that will *not* be chosen by any household with reference-point \bar{x} , irrespectively of their initial endowment. Put differently, no household will choose a second-period wealth too close below the reference-point: either they reach their reference-point, or they fall short of it of a non-trivial amount. This is naturally due to the fact that v is convex in an interval that ends in \bar{x} . As we shall see, this has important consequences on the aggregate behavior of the economy.

Part (2) of Proposition 1 shows another feature of the optimal behavior: there is a non-zero interval of initial endowments such that the optimal choice for households with those endowments is the same and equal to the reference-point. That is, a subset of households, equipped with different initial endowments, will choose to consume the same amount in the second period: the reference-point.

An intuition of the result above can be captured by Figure 2.4, which plots the FOCs of the household problem as a function of the investment (directly using the result of Remark 1).

The increasing curve is the derivative of the first period utility, $u'(x_{it} - I)$, which is increasing in the bequest (which reduces consumption) by concavity of the first period utility. The second curve describes the derivative of the utility in the second period, which, following Assumption 3, is increasing in some interval (v is convex), and

Figure 1. An illustration of the first order conditions.

Source: elaboration by the authors.

is not continuous at the reference point. Notice that modifying the initial endowment x_{it} shifts the first curve, which implies that there must exist a full set of endowments for which the household chooses exactly the reference point. At the same time, there must exist a threshold of the x_{it} that makes the household's choice 'jump' from a low level of investment to a high one, due to the area of convexity.

3. Distribution in the Long Run

3.1 Distribution in the long run

We now turn to study the distribution of our economy in the long run. To this end, we will define two kinds of wealth distribution. The first kind is standard: we say that a wealth distribution is of *perfect equality* if it is degenerate.

Definition 2. A distribution μ on \mathbb{R}_+ is of perfect equality if the support of μ is equal to $\{x\}$ for some $x \in \mathbb{R}_+$.

Next, we introduce the notion of *missing class*. We say that an interval (x_1, x_2) is a missing class for a wealth distribution μ if this distribution has zero density on (x_1, x_2) , but it admits a strictly positive density both above and below this interval. The idea is that μ includes households both below x_1 and above x_2 , but has ‘a gap’ between them: the missing class.

Definition 3. For any $x_1, x_2 \in \mathbb{R}_+$ with $x_1 < x_2$, we say that (x_1, x_2) is a *missing class* for a wealth distribution μ if:

1. $\mu([0, x_1])$ and $\mu([x_2, +\infty)) > 0$;
2. $\mu(x) = 0$ for all $x \in (x_1, x_2)$.

A special case of a wealth distribution that admits a missing class is the one in which the average of this distribution, $\mathbb{E}[\mu_t]$, is the *upper* limit of the missing class. In this case, we say that the *wealth distribution admits a missing class below the mean*.

Definition 4. A wealth distribution μ *admits a missing class below the mean* if there exists $x \in \mathbb{R}_{++}$ with $x < \mathbb{E}[\mu_t]$ such that $(x, \mathbb{E}[\mu_t])$ is a missing class for μ .

Any such distribution has a ‘gap’, which lays right below the average that divides the agents with a wealth below the average, from those above or at the average. This means that any distribution of this kind cannot be degenerate. Rather, it must be, in some sense, polarized, because it divides the rich (above or at the mean) from the poor (below the mean).

It turns out that, in the presence of reference-dependence, these two types of distributions are enough to describe the long run behavior of any economy that satisfies our assumptions. (By μ_t^e we denote the distribution of endowment in equilibrium e and period t .)

Theorem 1. Consider an economy as described above that satisfies Assumptions 1-4, and some equilibrium e of this economy. Then there exists some $T \in \mathbb{N}$ such that one of the following must hold for all $t > T$:

- (a) μ_t^e is of perfect equality;
- (b) μ_t^e admits a missing class below the mean.

Theorem 1 shows that the long run distribution of any equilibrium of any economy that satisfies our assumptions can assume only two forms: either it is of perfect equality, or it admit a missing class below the mean. No other distribution is possible –in particular, no continuous and non-degenerate distributions are possible in the long run. Notice, moreover, that this is not an asymptotic result, or a property of the steady state. Theorem 1 shows that one of these two types of distribution will be reached in *finite time* (at time T), and will be stable: after that time the economy will forever remain either of perfect equality or with a missing class.

To analyze the implications of the Theorem 1, recall that in the standard case in which there are no reference effects and the second period utility is equal to the first period utility (i.e., $v(\cdot, \bar{x}) = u(\cdot)$ for all \bar{x}), an economy that started with a continuous and non-degenerate distribution of initial endowments could easily have an equilibrium in which the distribution is continuous and non-degenerate in every period. For example, if u exhibits constant relative risk aversion (CRRA), then any *distribution is possible in the long run* if utilities are strictly concave.¹² In contrast, Theorem 1 shows that the presence of reference-dependence has a strong impact on the type of paths that are attainable in equilibrium: no matter what the initial distribution is, and no matter how small the impact of the reference-point on utilities is, a continuous non-degenerate distribution is no longer possible in the long run. Either the poor get ‘left behind’, failing to attain their reference-point: in this case we have a society with a missing class below the mean, which is polarized between those that attain their reference-point, and those that fail to. Or, everybody catches up with their reference-point – but this can happen only if the distribution of endowments is of perfect equality.

¹² To see why, consider an economy like ours but in which: $v(\cdot, \bar{x}) = u(\cdot)$ for all \bar{x} ; both are strictly concave; both exhibit CRRA. Then, notice that for such economy the distribution of endowments is constant over time. (This is proved in Lemma 2 in the Appendix.) But this implies that the wealth initial distribution is also the long run distribution – which means that any long run distribution is possible.

3.2 Origin of the long run distribution

Theorem 1 shows that in any equilibrium the long run distribution of endowments can be of only forms, but it does not provide conditions that determine which of these two cases takes place. This is the content of the following proposition.

Proposition 2. *Consider an economy as described above that satisfies Assumptions 1-4 with an initial distribution of endowment μ_0 . Then each of the following must hold:*

1. *if μ_0 is not degenerate, then there exists some equilibrium e and some $T \in \mathbb{N}$, such that for all $t > T$, admits a missing class below the mean;*

2. *there exists some $\delta \in \mathbb{R}$, $\delta > 0$, such that, if $\frac{\inf x}{\mathbb{E}(\mu_0)} \geq \delta$, then there exists an equilibrium e and some $T \in \mathbb{N}$ such that for all $t > T$, μ_t^e is of perfect equality.*

Proposition 2 shows that the distribution that the economy will assume in the long run depends both on the initial conditions (μ_0), and also on the equilibrium that we are looking at. In particular, part (1) shows that any economy admits a long run equilibrium that evolves into a distribution with a missing class below the mean, no matter what the initial distribution is (as long as it is not degenerate). To get an intuition of why this is the case, notice that we can always construct an equilibrium in which every member of the society with an endowment equal or above the mean is expected to increase her consumption by at least some b , where b is defined as the maximum amount that the household with an endowment exactly equal to the mean is willing to increase her consumption to if she knew that the mean will increase by b . In this equilibrium everyone with an endowment above or equal to the mean will ‘keep up’, while the rest will not, generating polarization, and a missing class below the mean. If this is true in every period, then there will always be a missing class, leading to part (1) of the proposition.

On the other hand, part (2) of Proposition 2 shows that, if the initial distribution is not ‘too disperse’, i.e., the ratio between the minimum and the average income is high

$\left(\frac{\inf x}{\mathbb{E}(\mu_0)} \geq \delta \right)$, then this economy will *also* admit an equilibrium which evolves into

perfect equality in the long run. The reason is, again, very simple: if the poor are not too much behind the average, as time goes by they will catch up – leading to perfect equality.

We conclude this discussion by recalling that instead of assuming that the reference point of each citizen is the average endowment of the society, we could have instead assumed that the society is divided into m groups of positive mass, and that the reference point of each agent is the average endowment of the other members of the group she belongs to (Section 2.1). We will now argue how our results thus far would immediately apply to this more general case, although they would describe the dynamic *within* each group as opposed to the dynamic of the society as a whole. In fact, the equilibrium prices would be determined according to the arbitrage condition defined in (3), the optimal behavior would still be determined by Proposition 1, and Theorem 1 and Proposition (2) would hold for the distribution within each group, with essentially identical arguments. In turn, the analytic expressions for the society as a whole would become more complicated (as groups can overlap), but the main insight would remain unchanged.

4. Growth

4.1 Comparative Growth

We now turn to investigate the implications of society-dependent reference points on growth. In general, it is well-known that the relation between inequality and growth can be complicated even with standard preferences (see Aghion et al. (1999)). In our simple setup, however, if we removed any reference-effect and focused on CRRA utility (i.e., if we had $v = u$, both CRRA and concave), then we would get a unique growth rate, regardless of the initial distribution.¹³ As we shall see in this Section, it is the presence of reference-effects that will complicate this relationship: with reference effects, the growth rate of the economy will change depending on the initial wealth distribution and on the equilibrium that we consider.

¹³ See Lemma 2 in the Appendix. With a different utility, however, this might not hold (see Lemma 2 again).

In particular, we shall compare the growth rate of different *economies*. An economy is a generic problem as the one expressed in (1), with given preferences (with or without reference dependence), initial distribution of endowments, and technology set. To make comparisons meaningful, we shall compare economies in which the initial distributions have the same mean.

Because, in our model, the wealth distribution affects the utility of the agents, agents in different economies will have different preferences over available actions. This raises the issue of how to make welfare comparisons between these economies, since we cannot use standard notions to rank them (e.g., Pareto). For this reason, we instead focus on a measure that does not depend on the preferences of the agents: we compare the growth rate of the economies. Indeed, we should emphasize that these comparisons do not easily translate to welfare comparisons: precisely because preferences are endogenous, a higher growth rate need not correspond to a higher average utility of the agents (however one may interpret it). For simplicity of exposition, let us introduce the following notions. We begin by saying that an equilibrium e , dominates another equilibrium e' , if the average endowment of the first is above the average endowment of the second for *every* period starting from some period T : that is, if there exists some $T \in \mathbb{N}$, such that $u, \int x \mu_t^e(x) dx \geq \int x \mu_t^{e'}(x) dx$ for all $t \in T$. We say that e *strictly dominates* e' if the former dominates the latter and there exists some $T \in \mathbb{N}$ such that the inequality above is strict for all $t \geq T$. This notion of dominance could be applied either to two equilibria of the same economy, or to two equilibria of different economies that start from the same average endowment.

While the notion above compares two specific equilibria, potentially of two different economies, we now introduce a comparative notion for all the equilibria of two economies. We define three notions: \triangleright , \triangleright_{\max} , \triangleright_{\min} , to represent, respectively, full dominance, higher ‘best’ equilibrium, and higher ‘worst’ equilibrium. We start with ‘full dominance’.

Definition 5. For any two economies E and E' , we say that $E \triangleright E'$ if for all equilibria e of E and e' of E' , e dominates e' . We say $E \triangleright_{\max} E'$ if for all equilibria e of E and e' of E' , e strictly dominates e' .

The idea behind Definition 5 is that if one economy E ‘dominates’ another economy E' ($E \succ E'$), then it means that every equilibrium of E dominates every equilibrium of E' . This is a very demanding notion, since it requires that even the worst of the equilibria of E strictly dominates the best of the equilibria of E' . Two weaker notions would compare the ‘best’ equilibrium of each economy, or the ‘worst’ equilibrium of each economy. This is what our two next definition of \succ_{\max} and \succ_{\min} do.

Definition 6. For any economies E and E' we say that $E \succ_{\max} E'$ if there exists some e of E such that e strictly dominates e' for all e' of E' . We say $E \succeq_{\max} E'$ if there exists some e of E such that e dominates e' for all e' of E' .

Definition 7. For any economies E and E' we say that $E \succ_{\min} E'$ if there exists some e' of E' such that e strictly dominates e' for all e of E . We say that $E \succeq_{\min} E'$ if there exists some e' of E' such that dominates e' for all e of E .

Intuitively, we say that an economy \succ_{\max} -dominates another if its best equilibrium dominates any equilibrium of the other economy. Conversely, we say that an economy \succeq_{\min} -dominates another if even its worst equilibrium dominates *some* equilibrium of the other. It is easy to see that \succeq_{\min} and \succ_{\max} are strictly weaker than \succ : if $E \succeq E'$, then $E \succ_{\min} E'$ and $E \succeq_{\max} E'$, but not *vice versa*.

Finally, to better express our results on growth, we compare the behavior of the following four different types of economies, each of which is assumed to have the same average initial endowment. (Recall that μ_0 denotes the initial distribution.)

Definition 8. Denote by \mathcal{E}_0 the set of economies that satisfy Assumptions 1, 2, $u = v$ and such that μ_0 is of perfect equality. \mathcal{E}_1 is the set of economies that satisfy Assumption 1-4 and such that μ_0 is of perfect equality. \mathcal{E}_2 is the set of economies that satisfy Assumptions 1-4, in which μ_0 is not of perfect equality, but that admit an equilibrium e such that there exists some $T \in \mathbb{N}$ such that for all $t \geq T$, μ_t^e is of perfect equality. \mathcal{E}_3 is the set of economies that satisfy Assumptions 1-4, in which initial distribution is not of perfect

equality, and that admits no equilibrium e' such that there exists some $T \in \mathbb{N}$ such that for all $t \geq T$, μ_t^e is of perfect equality.

\mathcal{E}_0 represents the set of standard economies with a representative agent who has a standard concave utility function that does not admit reference- effects. Economies in \mathcal{E}_1 , \mathcal{E}_2 , and \mathcal{E}_3 , instead, have reference-dependent households as modeled in Assumptions 1-4. They differ from each other because of the initial distribution: economies in \mathcal{E}_1 have an initial distribution with perfect equality; those in \mathcal{E}_2 do not, but admit at least one equilibrium in which the wealth distribution converges to perfect equality (in finite time); those in \mathcal{E}_3 , also start with a distribution characterized by some inequality, but admit no equilibria in which the inequality disappears. From Proposition 2 we know that we can think of economies in \mathcal{E}_2 as those, in which we have some initial inequality, but in which the wealth distribution is not ‘too disperse’ – no household has an initial endowment too far below the mean endowment. By contrast, an economy in \mathcal{E}_3 would be an economy in which the initial distribution has a more acute inequality: there are households with initial endowments far from the mean.

4.2 Growth in the ‘best’ equilibria

We are now ready for our first result on growth. We start by looking at the ‘best’ equilibria – those in which growth is the highest.

Theorem 2. *Consider four economies $E_0 \in \mathcal{E}_0$, $E_1 \in \mathcal{E}_1$, $E_{2,3} \in \mathcal{E}_2 \cup \mathcal{E}_3$ that have the same initial average endowment. Then: $E_1 \succ_{\max} E_{2,3}$ and $E_1 \succ_{\max} E_0$.*

If u exhibit CRRA, then: $E_1 \succ_{\max} E_{2,3} \succeq_{\max} E_0$.

Theorem 2 shows that the presence of reference-dependence could have a strong impact on growth depending on the initial distribution. First of all, it shows that there exists an equilibrium of E_1 that dominates *all* other equilibria of *all* other economies: the equilibrium with the (strictly) highest growth is found in an economy with reference-dependence and with an initial distribution of perfect equality. The intuition is that with reference-dependence and perfect equality, agents could ‘push’ each other into consuming more, and since the distribution is of perfect equality, then by doing this there is no subject that is ‘left

behind’ –leading to the highest growth. Without reference-dependence we lose the mechanism of ‘pushing each other’, which is why the growth of the best equilibrium of E_1 is higher than that of any equilibrium of E_0 ($E_1 \succeq_{\max} E_0$). At the same time, this growth is also higher than that of any equilibrium of a society with reference-dependence and initial in- equality ($E_1 \succeq_{\max} E_2, 3$): while in both economies we have the mechanism of ‘pushing each other’, when there is some initial inequality the wealthy cannot expect the average income of the society to increase too much, since they know that the poor would not follow, and therefore they won’t increase their own income as much as they do in E_1 .

If we further assume that u exhibits constant relative risk aversion, then we obtain the full, transitive rank: $E_1 \succ_{\max} E_{2,3} \succeq_{\max} E_0$. That is, if we look at the best equilibria of each economy, we have that the highest growth is found with reference-dependence and perfect equality, then with reference- dependence and initial inequality, and finally the economy with no reference- dependence. This should be compared with what happens when there is no reference-dependence and both u and v are identical, concave, and CRRA: it is well known (we show it again in Lemma 2 in the Appendix) that in this case the growth rate is constant and independent of the initial distribution. (Notice that this implies that in the case of CRRA utility the ranking above is true also for some E'_0 which, like E_0 , has no reference-dependence, but that has an initial distribution which is not of perfect equality.) This means that: when we focus on ‘best’ equilibria, the presence of reference-dependence is always positive for growth, and it renders growth dependent of the initial distribution, where the presence of initial inequality is actually harmful for growth.

This discussion shows a feature of society-wise reference-dependence. On the one hand, it induces households to ‘push each other’, generating growth. On the other hand, this mechanism works better when households are not ‘too far’ from each other. As we shall see below, however, while this is true for the equilibria with the highest growth, this is not necessarily true in other equilibria.

4.3 Comparative growth in all equilibria

Theorem 2 analyzes the ranking only for the ‘best’ equilibria. It turns out that things can be quite different in other equilibria.

Theorem 3. Consider four economies $E_0 \in \mathcal{E}_0$, $E_1 \in \mathcal{E}_1$, $E_2 \in \mathcal{E}_2$ and $E_3 \in \mathcal{E}_3$ that have the same average initial endowment. Then the following holds:

1. $E_0 \succeq E_1$;
2. for any equilibrium e of E_0 , there exists some equilibrium e' of E_1 such that $\mu_t^e = \mu_t^{e'} \forall t$.

Moreover, if u exhibits CRRA, then:

4. $E_2 \succ E_0$;
5. $E_2 \succ_{\min} E_1$;
6. $E_3 \succeq E_0$;
7. $E_3 \succeq_{\min} E_1$.

Theorem 3 considers other equilibria besides those with the highest growth. First, it shows that the presence of reference-dependence never reduces growth: every equilibrium of E_1 must grow at least as much as any equilibrium of E_0 ($E_0 \succeq E_1$); and if u is CRRA, then also any equilibrium of E_2 and E_3 grows more than one of E_0 ($E_2 \succeq E_0$ and $E_3 \succeq E_0$). In fact, equilibria in E_2 *strictly dominate* all equilibria in E_0 . At the same time, however, the presence of reference-dependence is not sufficient to have a higher growth: for example, there are equilibria of E_1 in which the wealth distribution coincides in every period with that of the (unique) equilibrium of E_0 .

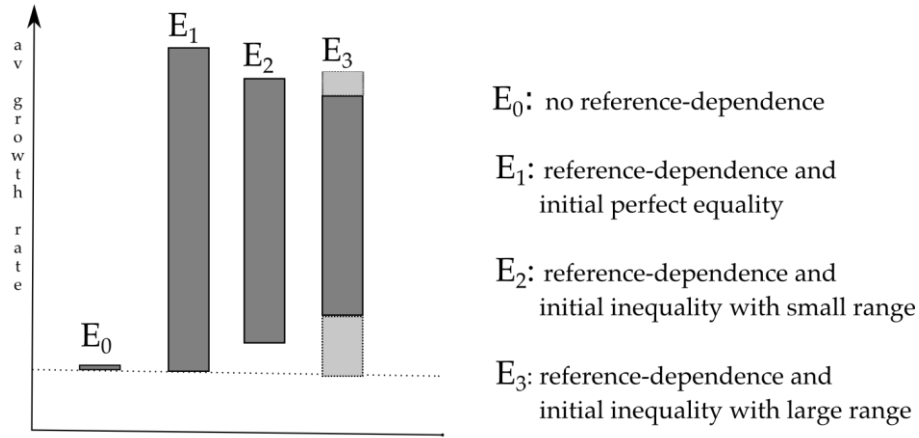
Also in the comparison between E_1 , E_2 , and E_3 , the ranking in Theorem 3 is quite different from the one in Theorem 2. While in the latter we have seen that the economy with the highest growth in the best equilibrium is E_1 , Theorem 3 shows that, if u is CRRA, then a society with a small amount of initial inequality (E_2) has a lowest-growth equilibrium where it grows *strictly* more than the lowest-growth equilibrium of a society with initial perfect equality (E_1): we have $E_2 \succeq_{\min} E_0$. Moreover, we also have that $E_2 \succ E_0$: the presence of a little bit of inequality renders minimal growth *strictly higher* than the case

with no reference-effects. The intuition is that, with reference-dependence, the households that are right below the average income might choose to ‘push up’, and reach their reference-point – generating growth. And since this cannot happen with perfect equality or with no reference-dependence, then minimal growth of an economy of type E_2 must lie strictly above that of an economy of type E_1 or E_0 . A similar argument suggests why we have $E_0 \supseteq_{\min} E_0$, and $E_3 \supseteq_{\min} E_1$. (Here the inequality is weak, \supseteq , since there could be no subjects who ‘push up’ of the kind described above.¹⁴)

The results of Theorems 2 and 3 can then be summarized as follows: the presence of reference-dependence increases the growth rate with respect to the case of no reference-dependence. Depending on the initial distribution of wealth in the economy, the growth rate will be strictly higher in every equilibrium (as is the case if the initial range is small enough), or identical at least for some equilibria (as is the case for initial perfect equality, E_1). If we compare the growth rates between economies with reference-dependence but different initial distributions, the results depend on the equilibrium that we are looking at: economies that have an initial distribution with perfect equality have equilibria with a growth rate strictly higher than any other economy; at the same time, they also have minimal equilibria that are worse than the minimal equilibria of the other economies – strictly worse than the minimal equilibria of E_2 . A graphical intuition of the results appears in Figure 4.3, which represents the set of average growth rates for all equilibria of each type of economy. (Notice that both the highest and the lowest growth rate for E_3 could be either above or below those of E_2 albeit always (weakly) above that of E_0 . To represent this possibility, Figure 4.3 contains the lighter shade area for E_3).

¹⁴ That is, there could be no household with an endowment below but close to the average one.

Figure 2. Average growth rates for different equilibria of economies E_1, E_2, E_3 when u is CRRA



Source: elaboration by the authors.

5. Conclusion

In this paper we study a warm glow OLG model in which agents' utility depends on the average behavior of the rest of society, which acts as a reference-point. In line with this interpretation, we model this reference-dependence using the standard prospect theory functional form. We notice that this leads to multiple equilibria. We then show that in any of these equilibria the wealth distribution will converge (in finite time) to be either of perfect equality or to have a missing class. We then turn to study the growth rates of different economies, and notice that comparisons strongly depend on the equilibrium of focus. If we look at the equilibrium with the highest growth for each society, then a society that starts from perfect equality is the one attaining the highest growth rate. Conversely, if we look at equilibria with the lowest growth, then the society with a small amount of initial inequality grows the fastest. Finally, we show that any society with reference-dependence, no matter what the initial distribution is, grows (weakly) more than any society without reference-dependence.

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Appendix

A. Preliminary Results

Lemma 1. *For all equilibria w , if μ_T^e is of perfect equality, then μ_T^e is of perfect equality $\forall t \geq T$.*

Proof. As in Remark 1, define $w^* = f(\lambda^*) - f'(\lambda^*)\lambda^*$, with $\lambda_t = \frac{K_t}{H_t}$. Assume that μ_T is perfect equality. We claim that $x_{i,T+1} = \mathbb{E}[\mu_{T+1}]$ for all $i \in L$. By means of contradiction, assume instead that exist $i \in L$ such that $x_{i,T+1} \neq \mathbb{E}[\mu_{T+1}]$. If we have $x_{i,T+1} > \mathbb{E}[\mu_{T+1}]$, then must also exist some $j \in I$ such that $\mathbb{E}[\mu_{T+1}] > x_{j,T+1}$, by definition of $\mathbb{E}[\mu_{T+1}]$. We can assume without loss generality that we have $x_{i,T+1} > \mathbb{E}[\mu_{T+1}] > x_{j,T+1}$ for some $i, j \in L$. Notice that for both i and j we must have that the optimal solution meets the FOCS, i.e., $u'(x_T - x_{i,T+1}/w^*) = \beta w^* \frac{dv(x_{i,T+1}, \bar{x})}{dx_{i,T+1}}$ and $u'(x_T - x_{j,T+1}/w^*) = \beta w^* \frac{dv(x_{j,T+1}, \bar{x})}{dx_{j,T+1}}$. By assumption 1 and 3, we know that outside the interval $(\bar{x} - H, \bar{x})$, $v(\cdot, \bar{x})$ behaves like then the u , which implies that, since the u' is everywhere decreasing, $\frac{\delta v(\bar{x} - H, \bar{x})}{\delta(\bar{x} - H)} > \lim_{x \searrow \bar{x}} \frac{dv(x, \bar{x})}{dx}$. For $x_{i,T+1} < x_{j,T+1}$ we have that $\beta \frac{dv(x_{j,T+1}, \bar{x})}{dx_{j,T+1}} < \beta \frac{dv(x_{i,T+1}, \bar{x})}{dx_{i,T+1}}$ which implies that u' is increasing between $x_T - x_{j,T+1}$ and $x_T - x_{i,T+1}$. But this violate Assumption 1, a contradiction.

Lemma 2. *If $u = v$ and u is CRRA, the growth rate of the economy is unique and invariant. At the same time, there exists a strictly concave u such that this is not true.*

Proof. Consider the first order condition for (2) imposing $u(\cdot) = v(\cdot)$ (internal solution is guarantee by assumption 1) and substitute $x_{it+1} = \psi_{it}x_{it}$, we get:

$$-u' \left(x_{it+1} \left(1 - \frac{\psi_{it}}{w^*} \right) \right) + \beta w^* u'(x_{it} \psi_{it}) = 0 \quad (4)$$

applying the implicit function theorem we get:

$$\frac{d\psi_{it}}{dx_{it}} = - \frac{-(1 - \frac{\psi_{it}}{w^*})u''(x_{it}(1 - \frac{\psi_{it}}{w^*})) + \beta w^* u''(x_{it}\psi_{it})\psi_{it}}{(\frac{x_{it}}{w^*})u''(x_{it}(1 - \frac{\psi_{it}}{w^*})) + \beta w^* u''(x_{it}\psi_{it})x_{it}} \quad (5)$$

where the sign is not *a priori* guaranteed and is clearly dependent on the expression at numerator.

Substituting for $u(x) = \frac{c^{1-\theta}}{1-\theta}$ it can be easily checked that the growth rate is unique and invariant to the distribution.

B. Proofs

Proof of Proposition 1

To prove both points consider the households's problem in (1), and notice that the FOCs are

$$\lim_{x' \nearrow \bar{x}_{t+1}} \beta w^* u'(x') \geq u'(x_{it} - \frac{x_{it+1}}{w^*}) \geq \lim_{x' \searrow \bar{x}_{t+1}} \beta w^* u'(x')$$

Notice that if no $x \in (0, x_{it+1})$ satisfy them, the claim is trivially true. Otherwise, define x^α the highest $x \in (0, x_{it+1})$ that satisfied the FOC. (The existence of x^α is guaranteed by standard arguments.) By assumption 3, for all $\epsilon > 0$ we must have $u'(x - x^\alpha/w^* - \epsilon w^* + \beta w^* u' x^\alpha + \epsilon) > 0$ and $-u'(x - x^\alpha/w^* - \epsilon w^* + \beta w^* u' x^\alpha - \epsilon) < 0$, which implies that x^α cannot be a maximum since it fails the second order conditions (recall that since $x < \bar{x}_{it+1}$, the function is differentiable at x^α). In turns, this implies that \bar{x}_{it+1} is an optimal solution for all initial endowments x such that $u'(x - \bar{x}_{t+1}/w^*) \in [\lim_{x' \searrow \bar{x}_{t+1}} \beta w^* u'(x'), \lim_{x' \nearrow \bar{x}_{t+1}} \beta w^* u'(x')]$. *Q.E.D.*

Proof of theorem 1

We start by noticing that, from period 1 onwards, the distribution of endowments must be either of perfect, or admit a missing class below the mean. To see why, consider any initial

distribution μ_0 , and notice that by Proposition 1 we know that there exist some $\gamma > 0$ such that $\mu(\mathbb{E}[\mu_1]_1 - \gamma, \mathbb{E}[\mu_1]_1) = 0$. This implies that, if $\mu_1([0, \mathbb{E}[\mu_1])) > 0$, then $\mu_1([0, \mathbb{E}[\mu_1] - \gamma)) > 0$. Notice also that, by construction, we have that $\mu_1([\mathbb{E}[\mu_1], +\infty)) > 0$. Therefore, if $\mu_1([0, \mathbb{E}[\mu_1])) > 0$, then the distribution admits a missing class below the mean. Conversely, if $\mu_1([0, \mathbb{E}[\mu_1])) = 0$, then we must have that the support of μ is equal to $\{\mathbb{E}[\mu_1]\}$ (every distribution with a support above its average must be degenerate). We have therefore proved that μ_1 can be either of perfect equality, or admit a missing class below the mean. An identical arguments shows that same would hold true for all μ_1 for all t .

We are only left to show that exist some \bar{T} from which the distribution is either of perfect equality, or with a missing class below the mean. Notice that if μ_1 is perfect equality, then we can set $\bar{T} = 1$, since we have proved that μ_1 has a missing class below the mean if it is not perfect equality. *Q.E.D.*

Proof of proposition 2

1) Consider some non-degenerate μ_t , and notice that if μ_1 is not continuous at the mean, then it must have a missing class below the mean. Otherwise, consider some μ_0 which is continuous around its mean, and consider the equilibrium in which each i at the time t choose $\max_{I'_{t+1}} \{I'_{t+1} \in \arg \max_{I_{t+1}} u(x_0 - I_{t+1}) + \beta v(w_t I_{t+1} / w_t I_{t+1}^*)\}$. Then there exist some x_{ti} such that $\lim_{I' \nearrow I} \beta w^* v'(w^* I', w^* I') = u'(x_{it} - \bar{I})$. Notice that we must have that x_{it} is in the interior of the support of u_t , otherwise this would violate Assumption 4 (The common reference point is the average wealth).

This x_{ti} should not stay on the lower bound of the support because it violates Assumption 4. But then $\forall x_{jt} < x_{it}$ household j will not reach her reference-point, and by Proposition 1 the distribution will have a missing class below the mean. Since t has been chosen arbitrarily, this proves the first part of the proposition.

2) By Proposition 1, we know that there exist an interval $S \subseteq \mathbb{R}_+$ such that $\forall x' \in S, \bar{x} = \phi(x', \bar{x})$. This clearly implies that $\text{supp}(\mu_0) \subseteq S$, then μ_1 is of perfect equality. By Lemma

1 we also know that it will remain in perfect equality for all t . We can define $\delta > 0$ implicitly as any $\delta > 0$ such that

$$\frac{\min_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}[\mu_0]} \geq \delta \Rightarrow \text{Supp}(\mu_0) \subseteq S$$

Q.E.D.

Proof of Theorem 2 and 3

Consider four economies $E_0 \in \mathcal{E}_0, E_1 \in \mathcal{E}_1, E_2 \in \mathcal{E}_2$, and $E_3 \in \mathcal{E}_3$ that have the same average initial endowment. Call e_0 the unique equilibrium of E_0 , and $x_t^{e_0}, \dots$, the average endowments in e_0 at period t . Then, the following holds.

Claim 1. For all $t > 0$, there exist a compact, convex, and positive-measure set S_t such that for all equilibria e of E_1 , $\mathbb{E}[\mu_t^e] \in S_t$. Moreover, for all $t > 0$ we have $x_t^{e_0} \in S_t$ and $x_t^{e_0} > x'_t, \forall x' \in S_t$.

Proof. Notice first that any equilibrium of E_1 the distribution must remain of perfect equality in all periods by Lemma 1. Then, in every period every household must choose a bequest exactly equal to her reference-point, which implies that in all equilibria we must have $x_{i,t} = \phi(x_{i,t-1}, x_{i,t})$ for all $i \in L$, for all t . In turns, this means that we have \bar{x}_{t+1} such that $\lim_{x' \nearrow x_{t+1}} \beta w^* v'(x') \geq u'(x - \bar{x}_{t+1}/w^*) \geq \lim_{x' \searrow x_{t+1}} \beta w^* v'(x')$. Since the two limits are finite, by Assumption 1 E_1 has a set of equilibrium solutions such that at every $t > 0$ the optimal x_t^* belongs to a compact convex set S_t . In turn, this implies that S_t is a set of positive measure for all t .

Notice that $x_t^{e_0}$ satisfies first order conditions as an interior point by Assumption 1. Consider now $x'_t \in S_t$. Assume that $x'_t < x_t^{e_0}$. By Assumption 4, reference-point should be equal to x'_t . Since by Assumption 3 the representative agents in E^0 and E^1 have the same utility outside an interval the highest point of which is the reference-point, the between x'_t and $x_t^{e_0}$ the $u'(\cdot)$ must be either constant or increasing and decreasing. But this contradicts Assumption 1.

We are left to show that $x_t^{e_0} \in S_t$. Assume by contradiction that the x_t^{\min} defined as $u'(x_{t-1} - x_t^{\min}/w^*) = \lim_{x \nearrow x_t^{\min}} \beta w^* v'(x)$ is such that $x_t^{\min} > x_t^{e_0}$. This implies $u'(x_{t-1} - x_t^{e_0}/w^*) < \beta w^* v'(x_t^{e_0})$ but since $u' = v'$ above the aspiration level, x_t^{\min} is a stationary point of the problem without reference dependence, which implies a violation of Assumption 1.

Notice that Claim 1 implies $E_1 \succeq_{\max} E_0$. This, together with the observation above that S_t has a positive measure for all t , implies that $E_1 \succ_{\max} E_0$. (The reason is, E_1 has multiple equilibria each inducing different growth rates, and all of them have a growth weakly above E_0 ; but then, there must exist an equilibrium with a growth strictly above E_0 .) In turn, Claim 1 also proves points (1) and (2) of Theorem 3.

We will now show that $E_1 \succ_{\max} E$ for all $E \in \mathcal{E}_2 \cup \mathcal{E}_3$. Denote by e_1 and e the equilibria of maximum expansion of E_1 and E , respectively. Also, denote e_t the equilibrium of maximum expansion of an economy which in period t has the same average endowment as e , but has perfect equality at time t , i.e., and economy of type \mathcal{E}_1 such that $\mu_t^{e_t}(\mathbb{E}[\mu_t^{e_t}]) = 1$. We will first show that e_t has an average endowment that grows strictly more than e between time t and time $t+1$ for all t such that μ_t^e is not imperfect equality. (If μ_t^e is of perfect equality then e and e_t coincide.). To see why, consider first the case in which μ_{t+1}^e is of perfect equality. By definition of maximum rate of expansion, $\mathbb{E}[\mu_{t+1}^{e_t}] = \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_t^{e_t}] - y/w^*) + \beta v(y, y)\}$. We claim that $\mathbb{E}[\mu_{t+1}^e] \leq \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_t^e] + y/w^*) + \beta v(y, y)\}$ where the inequality is strict if μ_t^e is not of perfect equality. If this were not the case, there would exist $i \in L$ such that $x_{i,t+1}^e = \max_{x'} \{x' \in \arg \max_y u(x_{i,t} - y/w^*) + \beta v(y, y)\}$. This however, would mean that then the distribution is not perfect equality, which is a contradiction. The $x_{t+1}^e < \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_t^e] + y/w^*) + \beta v(y, y)\}$, providing that if μ_{t+1}^e is of perfect equality then e_t has an average consumption that grows strictly more than e .

Consider now the case in which μ_{t+1}^e is not of perfect equality.

Fix some $t \geq 0$, and consider $\bar{x} \in \mathbb{R}_{++}$ such that $\mathbb{E}[\mu_{t+1}^e] = \max_{x'} \{x' \in \arg \max_y u(\bar{x} - y/w^* + \beta v(y, y))\}$. This x should be strictly greater than the lower bound of the support and strictly less than the upper bound in any distribution with a missing class below the mean.

Claim 2. $\bar{x} < E[\mu_t^e]$

Proof. By contradiction, assume $\bar{x} \geq \mathbb{E}[\mu_t^e]$. Then by definition of \bar{x} , all those with endowments strictly lower should fail to reach their reference-point, i.e., $\forall j \in L$ such that $x_{jt} < \bar{x}$ we have $x_{j,t+1} < E[\mu_{t+1}^e]$, or $x_{k,t+1} = \gamma(x_{kt})x_{kt} > E[\mu_{t+1}^e]$, where the growth rate $\gamma(x_{kt})$ is defined by $u' \left(x - \frac{x\gamma(x)}{w^*} \right) + \beta w^* v'(x\gamma(x)|x\gamma') = 0$. (Notice that this last expression is twice differentiable.) Then, notice that we must have

$$\frac{d\gamma(x)}{dx} = \frac{\beta w^* \frac{dv'(x\gamma(x)|x\gamma')}{dx\gamma'} \gamma'}{- \left[u'' \left(x - \frac{x\gamma(x)}{w^*} \right) \frac{x}{w^*} + \beta w^* v'(x\gamma(x)|x\gamma') x \right]} \leq 0. \quad (6)$$

Call $\gamma^* = \frac{\mathbb{E}[\mu_{t+1}^e]}{\bar{x}}$, we have:

$$\begin{aligned} \mathbb{E}[\mu_{t+1}^e] &= \int_{j \in L \mid x_{jt} < \bar{x}} x_{j,t+1} d\mu_{t+1} + \int_{k \in L \mid x_{kt} < \bar{x}} x_{k,t+1} d\mu_{t+1} \\ &< \gamma^* \int_{j \in L \mid x_{jt} < \bar{x}} x_{jt} d\mu_t + \gamma^* \int_{k \in L \mid x_{kt} < \bar{x}} x_{kt} d\mu_t = \gamma^* \mathbb{E}[\mu_t^e] \end{aligned}$$

where the inequality is determined by Proposition 1 and Equation (6). But then $\mathbb{E}[\mu_{t+1}^e] < \gamma^* \mathbb{E}[\mu_t^e]$ and $\mathbb{E}[\mu_{t+1}^e] = \gamma^* \bar{x}$ which is a contradiction since $\bar{x} \geq \mathbb{E}[\mu_t^e]$. This completes the proof.

Since $\bar{x} < E[\mu_t^e]$, then we have that $\mathbb{E}[\mu_{t+1}^e] < \max_{x'} \{x' \in \arg \max_y u(\bar{x} - y/w^*) + \beta v(y, y)\} < \max_{x'} \{x' \in \arg \max_y u(\mathbb{E}[\mu_t^e] - y/w^*) + \beta v(y, y)\} = \mathbb{E}[\mu_t^e]$, proving that

also if μ_{t+1}^e is not of perfect equality, then e_t has an average endowment that grows strictly more than e .

We have therefore showed that e_t has an average endowment that grows strictly more than e between time t and time $t + 1$ for all t such that μ_t^e is not of perfect equality. Observe also that if we take two economies $E'_1, E''_1 \in \mathcal{E}_1$ such that the initial endowment of E'_1 is strictly higher than that of E''_1 , then we have $E'_1 \succ_{max} E''_1$. These two observations jointly imply $E_1 \succ_{max} E$ for all $E \in \mathcal{E}_2 \cup \mathcal{E}_3$.

We are left to analyze the case in which u is a CRRA.

Claim 3. If u satisfies CRRA, then $E_3 \supseteq E_0$ and $E_2 \supset E_0$.

Proof. Notice first of all that E_0 has a unique equilibrium E_0 , and that if $u(\cdot)$ exhibits CRRA, then the growth rate of E_0 is constant, and use λ to define it. We need to prove that e never grows less than λ . Call λ^* the growth rate of E_0 the first period, and divide the population L into four groups:

A: Subjects whose initial endowment is above or equal to the average endowment;

B: Subjects whose initial endowment is below the average endowment, but who meet their reference-point in the second period, i.e., $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) = x_0(1 + \hat{\lambda})$;

C: Subjects whose initial endowment is below the average endowment, and who do not meet their reference-point in the second period but have a second period choice in a where v is convex, i.e., $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) \in [x_0(1 + \hat{\lambda}) - \gamma, x_0(1 + \hat{\lambda})]$;

D: Subjects whose initial endowment is below the average endowment, and who do not meet their reference-point in the second period but have a second period choice in a point where v is concave $\phi(x_{i,0}, x_0(1 + \hat{\lambda})) < x_0(1 + \hat{\lambda}) - \gamma$;

(Some of the groups above might be empty.) Define by λ_i^* the growth rate of the average endowment of each of the groups above, for $i = A, B, C, D$. Notice first of all that we must have $\lambda_A^* \geq \lambda$. Notice that every subject in any group would increase her choice of exactly λ if she had $v = u$ instead of being reference dependent. Now, if $\lambda^* \leq \lambda$, the if subjects of group (a) increased their choice by λ , they would remain strictly above their reference-

point: but since above the reference-point $u' = v'$, increasing the choice of λ must remain optimal for them. Therefore, $\lambda_A^* \geq \lambda$. Consider now subjects of group (a) increased their choice by λ must remain optimal for them. Therefore, $\lambda_A^* \leq \lambda$. Consider now subjects in B, and notice that they have initial endowment below the average endowment, but a second period bequest exactly equal to the mean. This means that subjects of group B must have $\lambda_B^* > \lambda$. Consider now subjects in group C: were they not reference-dependent, they would increase their bequest by λ . And, their second-period bequest must lie in an area in which the second period utility v is convex. Therefore, we must have $\lambda_C^* \leq \lambda$. Finally, consider the subjects in group D. Notice that, among them, there cannot exist subjects such that, if they increased their bequest by λ , they would have a second period bequest in a point where v is convex, i.e., $x_{i,0}(1 + \hat{\lambda}) \in [x_0(1 + \hat{\lambda}) - \gamma, x_0(1 + \hat{\lambda})]$. The reason, similar to that presented for subjects in group C, is that if increasing the bequest by λ were optimal with no reference-dependence, it even more so now, with reference-dependence, since v is strictly above u from $x_0(1 + \hat{\lambda}) - \gamma$ on. Therefore the only subjects in group D must be those for whom $x_{i,0}(1 + \lambda) < x_0(1 + \hat{\lambda}) - \gamma$. But since v coincides with u before $x_0(1 + \hat{\lambda}) - \gamma$, then these subjects must increase their choice of at least λ .

We have just proved that: if $\lambda^* \leq \lambda$ then $\lambda_A^* \geq \lambda$; $\lambda_B^* > \lambda$; $\lambda_C^* \leq \lambda$; $\lambda_D^* \geq \lambda$. Clearly this implies that we cannot have $\lambda^* < \lambda$, hence $\lambda^* \geq \lambda$. Notice, moreover, that if group B were not empty, this would imply $\lambda^* > \lambda$.

The argument above must hold true for all periods, i.e., the growth rate of $\lambda^* \leq \lambda$ of e must be above λ for all periods. This means $E_3 \supseteq E_0$ and $E_2 \supseteq E_0$. We are left to show that $E_3 \supseteq E_0$. To see why, notice that E_2 is characterized by the fact that, at some period t , group B above must be non-empty, the distribution must become of perfect equality, which implies that there is a period in which some subjects ‘jump’ from being at the reference point. We have already argued that this implies that the growth rate must be strictly above λ , proving the claim.

Finally, notice that Claim 3 together with (2) imply (4) and (6) of Theorem 3. *Q.E.D.*