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Education and the poverty trap

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Abstract

An overlapping generations model is constructed in which individual wealth is related to educational attainment, and in which liquidity constraints may induce children to invest in a sub-optimal level of education given their ability. Financing for education is obtained from within the family. Abilities differ among children and may be related to parental ability. Stationary state equilibria are found to exist in which children of poorer families are caught in a poverty trap because of an inability to finance their education. The role of redistributive policy is studied in this context.

Keywords: Education; Poverty trap; Overlapping generations model; Stationary equilibria; Redistributive policy

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1. Introduction

There is general agreement that alongside physical capital investment and the generation of new knowledge through research and development, human capital investment is a key determinant of economic progress. Human capital investment

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nonetheless differs from physical capital investment and research and development in that it is largely undertaken by households. [1] Furthermore, households differ with respect to their capacity to accumulate human capital. Some households may be more capable than others, thereby making a given amount of investment more productive; or they may differ in their ability to finance education out of their own resources, and may not be able to use capital markets to make up the difference. Thus, the pattern of human capital investment will not only affect the growth of the economy but also the distribution of welfare among households.

We focus in this paper on the distributional consequences of different households having unequal opportunities to acquire human capital. We construct a simple overlapping generations model of a stationary economy in which households take only two types of decision: an educational investment decision and a saving decision. Education yields a rate of return in the form of higher earnings, whereas savings earn interest. The opportunity cost of education consists of two components – forgone earnings and a financial cost. Additionally, we assume that there are non-pecuniary benefits to education. An important feature of our model is that households cannot borrow in capital markets against future earnings, but only from their parents. [2] Although parents are not altruistic, they are willing to lend money to their children for education since they know whether or not their children will succeed and be able to reimburse their loan.

The two fundamental components of our model are the characterization of household behaviour and the technology of education. In both cases, we work with very simple representations designed to capture more complex underlying structures. We suppose that households live for three distinct periods. During the first period, i.e., their youth, they choose to work or to acquire education at school and, if they attend school, what quality of schooling to acquire. To become educated, they must borrow from their parents to pay for tuition fees and youth consumption. During the second period, i.e., their adulthood, they work a fixed amount of time, receiving a wage which depends on their education. They repay their parents and undertake additional savings for retirement consumption; some of this saving can take the form of lending to their children. They are retired from work in the third period, and finance consumption by decumulating assets. In contrast, agents who choose to work rather than to invest in education during their youth work a fixed amount of time during both the first and the second periods of their lives; during their active lives, they save for retirement consumption. Like educated households, they retire from the labour force during the third period, and finance their consumption out of cumulative savings.

Our analysis concentrates on the distribution of incomes achieved in the stationary state under the above assumptions. In particular, we show how the

[1] Some human capital investment takes place on the job and is part of the activities of the firm.

[2] This assumption is not uncommon in the literature (e.g., see Becker and Tomes, 1976).

existence of a liquidity constraint can condemn some persons to a 'poverty trap'. By poverty trap we mean that capable children remain uneducated, and consequently poor, because their parents' incomes are too small to enable them to finance their education. However, educating these children would be socially efficient: the cost of providing them with a 'good education' is less than the increase in productivity which would result from them becoming educated. We investigate optimal redistributive policy in this setting. Though our model is highly stylized, we think it may be suggestive of one of the reasons for which the public sector is so heavily involved in the provision of education.

The plan of the paper is as follows. In the next section, our model is presented, and the household behaviour is characterised. Section 3 analyses the kinds of equilibrium that can occur if individuals are homogeneous in terms of ability. It is shown that depending on the income of the current generation a family can be caught forever in a poverty trap. In Section 4, we then turn to the case where individuals can be of two ability levels. Over generations the ability level of a family is assumed to evolve according to a Markov process. For the simple subcase presented there we analyse in Section 5 how a Benthamite planner would use lump-sum taxes and transfers. Finally, in the concluding section, we summarize the paper and we present suggestions for further research.

2. The three-period life-cycle model

In this section, we consider the behaviour of a generation-t individual, relating the choices made by this agent to the savings decision of generation $t-1$. All agents are initially assumed to be of equal ability; we relax this assumption in Section 4.

Individuals live for three periods of equal length – childhood (period 0), adulthood (period 1) and retirement (period 2). Throughout the analysis we assume that each adult gives birth to one child (constant population size). The lifetime utility of a generation-t household is assumed to take the simple form

$$U_t = \log(c_{0,t}) + \log(c_{1,t}) + \log(c_{2,t}) + \delta B \tag{1}$$

where the $c_{0,t}$, $c_{1,t}$ and $c_{2,t}$ denote the agent's consumption during childhood, adulthood and retirement respectively while B denotes the utility premium from being educated and δ takes the value one if educated and zero if uneducated. Parameter B is interpreted as the difference in utility between working or becoming educated early in life. Alternatively, it may be viewed as the non-pecuniary benefit of working as an educated person in the next period of life. [3]

[3] For simplicity, we have not considered a functional relationship between the non-pecuniary benefits of schooling and the quality of education acquired. This is consistent with interpreting B as the benefit, in terms of utility, derived from attending school rather than working during childhood.

This utility function is admittedly restrictive, although it is frequently used in this type of dynamic model. [4] It has the advantage of tractability and generates a spectrum of intuitive and interesting qualitative results. We focus on a subset of these in our illustrative analyses below. More general functional forms would increase the spectrum of possibilities even further, but it would also increase the complexity of the model without adding significant insight to the types of results that can occur.

During childhood, the agent must choose either to work for a given wage income, w, and forgo any education or to attend full-time school. A child of generation t who decides to attend school invests in education e_t, the level of which is under his control. During adulthood, an educated person receives wage income of $\omega(e_t)$, where the earnings function $\omega(e_t)$ is increasing in educational expenditure and strictly concave, with $\omega(0)=w$, while an uneducated worker receives wage income w. During retirement, no work is supplied; agents consume the assets they have saved in earlier periods.

Agents can acquire assets which yield a return r, and will do so during their working lives. However, they cannot borrow in capital markets against future earnings to finance their education. Instead, they must borrow from their parents. The absence of a credit market for education can be justified by an argument of adverse selection in a setting of heterogeneity where a child's ability is known only by the child concerned and by his or her parents. The parents are assumed not to make altruistic transfers to their children. [5] However, owing to the shared knowledge between the parent and the child as to the ability of the latter, the parent is willing to loan funds for education to the child for at least the market rate of return r. We assume in this paper that the parents do not attempt to extract more than r from their children, although in principle there is a surplus from the private information that can be shared between the parent and the child. One way to justify this is by supposing a cooperative arrangement within the family according to which parents offer loans to children at the market interest rate, and children agree to repay the loan to their parents fully rather than trying to renegotiate it or even reneging on their debt. [6]

[4] See Glomm and Ravikumar (1992).

[5] This can be because either they are not altruistic or because their altruism is overwhelmed by a free-rider problem associated with leaving bequests for one's offspring. As Bernheim and Bagwell (1988) have pointed out, this is a logical consequence of the Barro model of filial altruism once the biological fact of intermarriage among families is taken into account.

[6] See Cigno (1933) who studies the family as a credit system. However, he emphasises non-cooperative solutions.

In view of the liquidity constraint, we can distinguish between three broad 'categories' of individuals in our model, according to whether or not they are liquidity-constrained and whether or not they choose to have an education at all. The first consists of those persons who choose to remain uneducated. The second category is comprised of those agents who invest in education, but whose parents' savings are too low to allow them to choose their most-preferred level of schooling. The third is made up of those persons who decide to become educated and who can borrow as much as they desire from their parents. We refer to individuals belonging to each category respectively as the uneducated (U), the liquidity-constrained educated (C) and the unconstrained educated (E). All have the same preferences and abilities, but may differ with respect to parental resources. We consider the behaviour of each in turn. Where necessary, we use the superscripts U, C or E to distinguish between them. Where the meaning is clear, the superscripts are deleted. For the remainder of this section, we also delete the generation subscript t as far as possible. In Section 4, we relax the assumption that all individuals are equally able.

2.1. The uneducated (U)

Let σ denote savings during childhood, and s denote savings during the adulthood of an individual. The utility maximisation problem for an uneducated individual can be written as

$$\max_{\sigma,s} \log(w-\sigma)+\log(w+(1+r)\sigma-s)+\log((1+r)s). \tag{2}$$

The solution to this problem is [7]

$$\sigma^U=\frac{w(1+2r)}{3(1+r)} \quad \text{and} \quad s^U=\frac{w(2+r)}{3} \tag{3}$$

with $c_1^U=s^U$, $c_2^U=(1+r)c_1^U$ and $c_0^U=s^U/(1+r)$, so that lifetime utility is

$$V^U=3\log s^U. \tag{4}$$

Note that an uneducated adult divides his disposable income equally between consumption and saving ($c_1^U=s^U$); this feature will also hold for the two other categories of individuals. Observe also that the lifetime utility of uneducated persons can be expressed in terms of adulthood savings only. The above solution characterizes the behaviour and utility of an uneducated individual regardless of the level of parental savings (and so of his generation). It explains why subscript t is deleted from variables related to uneducated persons.

[7] These results are readily obtained by recognizing that differentiating (2) with respect to σ and s yields $c_0=c_1/(1+r)=c_2/(1+r)^2$ and that the sum of consumptions discounted to childhood – also equal to $3c_0$ by virtue of the above equalities – is equal to lifetime wealth i.e. $w+w/(1+r)=w(2+r)/(1+r)$.

2.2. The unconstrained educated (E)

Let b be the amount an unconstrained individual chooses to borrow. The utility maximisation problem for the unconstrained educated person becomes

$$\max_{b,e,s} \log(b-e) + \log(\omega(e) - b(1+r) - s) + \log(s(1+r)) + B. \tag{5}$$

The solution to this problem yields [8]

$$\omega'(e) = 1 + r \tag{6}$$

which implies simply that the marginal rate of return on education must be equal to that on financial assets. We shall denote by e^* the solution to (6) and refer to it as the socially desirable level of education. The solutions for b and s are given by

$$b^E = \frac{\omega(e^*) + 2e^*(1+r)}{3(1+r)} \quad \text{and} \quad s^E = \frac{\omega(e^*) - e^*(1+r)}{3}, \tag{7}$$

with $c_1^E = s^E$, $c_2^E = (1+r)c_1^E$ and $c_0^E = s^E(1+r)^{-1}$ so that lifetime utility is

$$V^E = 3 \log s^E + B. \tag{8}$$

Two observations should be made with respect to the unconstrained solution. First, this outcome can be sustained only if parents who obtained an education of quality level e^* themselves save enough to lend their own children the desired amount b^E, i.e., $s^E \geqslant b^E$. Second, to choose to become educated requires $V^E \geqslant V^U$. In the special case in which $B = 0$, so that there are no non-pecuniary benefits to becoming educated, this is equivalent to $s^E \geqslant s^U$, which can be written as

$$-e^* + \frac{\omega(e^*)}{1+r} \geqslant w + \frac{w}{1+r}. \tag{9}$$

This says that the net present value of education must exceed the present value of the stream of earnings that would have been obtained if uneducated. More generally, if $B > 0$, $V^E \geqslant V^U$ implies $3 \log s^E + B \geqslant 3 \log s^U$. Thus, it is possible for s^E to be less than s^U if $B > 0$.

2.3. The constrained educated (C)

Children of generation t whose parents' savings are less than b^E are constrained in their education decision and solve the utility maximisation problem

$$\max_{e_t, s_t} \log(\bar{b}_t - e_t) + \log(\omega(e_t) - \bar{b}_t(1+r) - s_t) + \log(s_t(1+r)) + B \tag{10}$$

[8] To obtain this solution, we proceed in the same way as sketched in the previous footnote. Here, lifetime wealth net of e is equal to $-e + \omega(e)/(1+r)$. This is first maximized to yield (6).

where $\bar{b}_t$ denotes the amount they can borrow from their parents. The level of education chosen is the solution to [9]

$$\omega'(e_t) = \frac{\omega(e_t) - \bar{b}_t(1+r)}{2(\bar{b}_t - e_t)} \tag{11}$$

and first-period saving is given by

$$s_t^C = \frac{\omega(e_t) - \bar{b}_t(1+r)}{2}. \tag{12}$$

Observe that since $\bar{b}_t = s_{t-1}$, both the levels of education and of savings of a generation-t individual depend upon his or her parent's savings. Observe also that

$$c_{0,t}^C = \bar{b}_t - e_t; \quad c_{1,t}^C = s_t^C \quad \text{and} \quad c_{2,t}^C = (1+r)c_{1,t}^C. \tag{13}$$

Finally, note that by using (12) and (13) we can infer that

$$\frac{\omega'(e_t)}{1+r} = \frac{c_{1,t}^C}{(1+r)c_{0,t}^C}. \tag{14}$$

Recall that in the unconstrained case, $\omega'(e_t) = 1 + r$ and $c_{1,t} = c_{0,t}(1 + r)$; thus, agents' education decisions satisfy (14), regardless of whether or not they are constrained. Here, however, $\omega'(e_t) > 1 + r$ because of the borrowing constraint, and therefore $c_{2,t}^C/(1 + r) = c_{1,t}^C > c_{0,t}^C\ (1 + r)$, whereas in the unconstrained context, $\omega'(e_t) = 1 + r$ and so $c_{2,t}^E/(1 + r) = c_{1,t}^E = c_{0,t}^E(1 + r)$. In other words, constrained households spend a larger proportion of their wealth on c_1 and c_2 than do unconstrained ones.

Lifetime utility – which depends upon s_{t-1} – can now be written

$$V_t^C = 3\log(s_t^C) - \log\left(\frac{\omega'(e_t^C)}{1+r}\right). \tag{15}$$

This expression for V_t^C is in fact directly comparable with V^U and V^E. It is easily shown that as s_{t-1} tends to b^E (see (7)), e_t^C and s_t^C converge to e^* and s^E.

Again, for the constrained educated person to choose to become educated, it must be the case that $V_t^C \geqslant V^U$, or

$$\log(c_{0,t}^C(1+r)) + 2\log(s_t^C) + B \geqslant \log(c_0^U(1+r)) + 2\log(s^U). \tag{16}$$

To interpret (16), note that the utility of a constrained educated person, V_t^C, is increasing in $\bar{b}_t$ and so in s_{t-1}. Denote by $\underline{s}$ the level of parental savings for which

[9] To obtain this solution we first derive the first-order conditions with respect to e_t and s_t: $c_{0,t} = c_{1,t}/\omega'(e_t)$ and $c_{1,t} = c_{2,t}/(1+r)$. Since children are here constrained in their education decision, $\omega'(e_t) > 1 + r$ and so $c_{0,t} < (1+r)^{-1}c_{1,t}$ (which makes the analysis different from that in the previous footnotes). However, the following equality of discounted values still holds: $\omega(e_t) - \bar{b}_t(1+r) = c_{1,t} + (1+r)^{-1}c_{2,t}$. The relations (11) to (14) are obtained from these equalities.

(16) is satisfied as an equality (assuming that $\underline{s}$ exists); this level $\underline{s}$ decreases as B rises. If $\underline{s} \leqslant s^U$, uneducated parents will save enough for their children to acquire education. It means that with $\underline{s} \leqslant s^U$, each family will have at least some of its members being educated. On the contrary, if $\underline{s} > s^U$, any family that starts with an uneducated member will stay uneducated forever.

This completes our characterisation of household behaviour. We now turn to a description of equilibria. We first analyse the sorts of equilibria that can exist in an economy of individuals with identical abilities, but different family histories. Depending on initial family income, which for now we take as historically given, different equilibria can occur.

3. Stationary state equilibria in the homogeneous household case

To identify those stationary states towards which families with different histories will tend, we must characterise the dynamics of intergenerational behaviour. To proceed, note that a person's lifetime utility is weakly monotonic in his or her parents' savings. In particular, for the uneducated, utility is independent of parental saving. As parental saving increases, a first threshold may be attained at which it is just worthwhile for a child to become constrained educated ($s_{t-1} = \underline{s}$). As parental saving increases further, the child's utility increases monotonically until a second threshold is reached at which parental saving is sufficient to eliminate the constraint ($s_{t-1} = b^E$). Beyond this level, the child's utility is independent of the level of parental saving.

The dynamic analysis involves investigating how savings evolve from generation to generation. Consider a constrained educated household of generation t. Let us rewrite Eqs. (11) and (12) as

$$\omega'(e_t) = \frac{\omega(e_t) - s_{t-1}(1+r)}{2(s_{t-1} - e_t)} \tag{11'}$$

and

$$s_t = \frac{\omega(e_t) - s_{t-1}(1+r)}{2}. \tag{12'}$$

From (11′) and (12′) we can infer how family savings evolve as long as successive generations remain constrained. Differentiating (12′) we obtain

$$\frac{\mathrm{d}s_t}{\mathrm{d}s_{t-1}} = \frac{\omega'(e_t)\dfrac{\mathrm{d}e_t}{\mathrm{d}s_{t-1}} - (1+r)}{2} \tag{17}$$

where, from (11′),

$$\frac{\mathrm{d}e_t}{\mathrm{d}s_{t-1}} = \frac{3\omega'(e_t) - (\omega'(e_t) - (1+r))}{3\omega'(e_t) - 2(s_{t-1} - e_t)\omega''(e_t)}. \tag{18}$$

Note first that since $\omega'(e_t) > 1 + r$ for the constrained person, $0 < \mathrm{d}e_t/\mathrm{d}s_{t-1} < 1$ if $\omega(e_t)$ is strictly concave. This implies that additional amounts of borrowing are partly devoted to educational spending and partly to increased childhood consumption. However, we cannot infer the sign of $\mathrm{d}s_t/\mathrm{d}s_{t-1}$ from (17); it may be positive or negative even when $s_{t-1} = \underline{s}$. We can, however, be certain that $\mathrm{d}s_t/\mathrm{d}s_{t-1}$ eventually turns negative for, as $s_{t-1} \to b^E$, $\omega'(e_t) \to 1 + r$.

Given this, a number of different shapes for the curve of s_t versus s_{t-1} are possible, each one giving rise to different types of equilibria. For expositional purposes, we restrict ourselves to the case in which $s_t(s_{t-1})$ is positively sloped at $s_{t-1} = \underline{s}$ and is strictly concave. [10] Below we investigate equilibria that may exist; in each situation considered, a poverty trap may or may not exist. *We shall say that a poverty (or no-education) trap exists if a family stays uneducated forever although it would be socially desirable for a child to become educated. Insufficient parental saving precludes this. It is socially desirable (or efficient) to be educated if the household would become educated had it unlimited access to the capital market.* Given that education is socially profitable, a poverty trap exists if $s^U < \underline{s}$, so that a child with an uneducated parent will not be able to become educated.

Three separate cases are considered – that in which education is socially efficient and no poverty trap exists, that in which a poverty trap exists, and that in which education is socially unprofitable. For each, stable and/or unstable equilibria may exist.

3.1. Case 1: No poverty trap with education socially desirable

This case has five possible subcases, each of which can be obtained by considering successive vertical moves of the $s_t(s_{t-1})$ curve. Fig. 1 illustrates one of these subcases – that of a stable, unconstrained equilibrium.

The curve begins at $s_{t-1} = \underline{s}$, which is the minimum level of saving required to satisfy (16). Recall that, because no poverty trap exists, $\underline{s} \leqslant s^U$. The curve then extends to $s_{t-1} = b^E = e^* + c_0^E$, at which point the household becomes unconstrained educated. As one moves to the right along the curve, the welfare of generation t is strictly increasing. Every family converges to the unconstrained educated equilibrium in finite time. From then on, it will be unconstrained educated, and successive generations will continue to borrow b^E and to save s^E. So, if parental saving is larger than b^E, children do not choose to borrow more than b^E, and would themselves save only s^E. This is represented in Fig. 1 by the

[10] A sufficient (but far stronger than necessary) condition for this to be true with a log-linear utility function is that $\omega''' \leqslant 0$. If we were to work with a more general specification of the utility function, we would not expect $s_t(s_{t-1})$ to be strictly concave, and it is probable that in the discussion below, additional stable and unstable equilibria would exist.

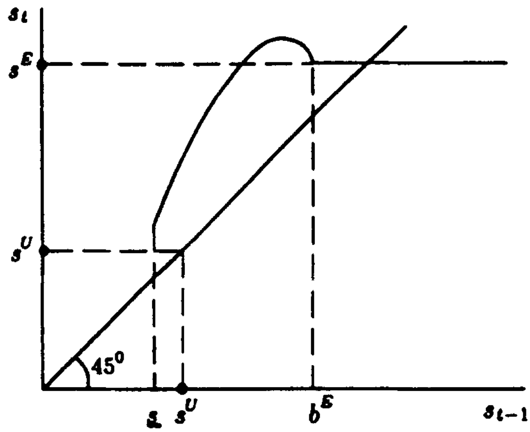

Fig. 1.

horizontal line beyond $s_{t-1} = b^E$. [11] Note that in the steady state, $s^E \geqslant b^E$, this is needed to have a stable unconstrained equilibrium. Observe also that along the $s_t(s_{t-1})$ curve, constrained agents can save *more* than the steady-state level; however, if this occurs, in every subsequent period the family will no longer be constrained.

The four other subcases – (i) one stable, constrained equilibrium, (ii) one unstable constrained, one stable unconstrained equilibrium, (iii) one unstable constrained, one stable constrained equilibrium (see Fig. 2), and (iv) no equilibrium – reflect different positions of the $s_t(s_{t-1})$ curve with respect to the 45° line. When there exist multiple equilibria, as in subcases (ii) and (iii), the income (and education levels) of those families whose incomes are initially greater than the level attained in the unstable equilibrium converges to the stable equilibrium. Thus, in Fig. 2, families with parental savings greater than y converge to the stable constrained equilibrium X after an infinite number of generations; at this equilibrium parental savings are not large enough for their children to spend e^* for education.

In contrast, those families whose parental savings are initially lower than y are caught in a cycle of relative indigence. Eventually, after a finite number of periods, an educated person's child will choose again not to invest in education, but that uneducated individual's own child *will* acquire an education. However, after a finite number of periods an educated person's child will choose again not to invest in education, and the cycle will start anew. In effect, when there exists an unstable constrained equilibrium, no family with an uneducated person in some

[11] A level of parental saving larger than s^E cannot be reached with only one ability type. But it may occur with several ability types (as in the next section).

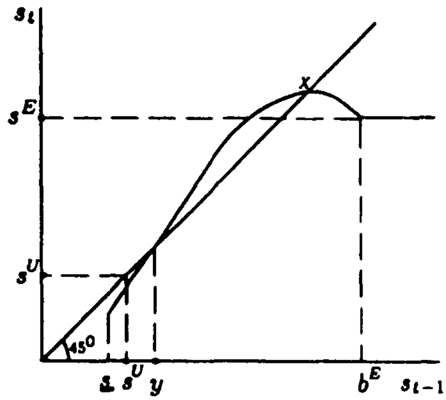

Fig. 2.

generation will ever attain the unconstrained educated equilibrium. Nonetheless, all members of the family would borrow b^E so as to acquire the socially efficient level of education e^* if they had access to the credit market.

Subcase (iv), where there is no equilibrium, occurs when the $s_t(s_{t-1})$ curve lies everywhere below the 45° line with $\underline{s} \leqslant s^U$. In this case all families follow the cycle of indigence described above, and so no family attains the unconstrained solution. By no means this is a poverty trap as families are not stuck in the no-education category.

3.2. Case 2: Poverty trap exists with education socially profitable

The same subcases arise here as in Case 1. The essential difference is that now $s^U < \underline{s}$, and so a poverty trap exists; this implies that once a member of a family decides not to invest in education, all of that individual's descendants will be uneducated also. If, however, parents are educated, different types of equilibria are possible depending upon the initial level of parental saving and the location of the $s_t(s_{t-1})$ curve. For example, if one modifies Fig. 1 by having $s^U < \underline{s}$, it is easy to see that whereas any family with an initial level of savings less than $\underline{s}$ will remain uneducated in perpetuity, those families whose savings exceed $\underline{s}$ will still converge after a finite number of periods to the unconstrained stable equilibrium. More generally, when there exist multiple equilibria, if parental saving exceeds the amount saved in the unstable equilibrium (y in Fig. 2 that one modifies by moving s^U to the left of $\underline{s}$) at some point in time, the family will remain educated in perpetuity. Whether or not it becomes unconstrained depends upon where the $s_t(s_{t-1})$ curve intersects the 45° line. In contrast, families that start out with a level of wealth below that attained in the unstable equilibrium will eventually fall into the poverty trap when parental saving falls below $\underline{s}$. Once the poverty trap is reached (in a finite number of generations), no family members will subsequently invest in education. Thus, unlike Case 1 where at any point in time one can expect to observe a distribution of poor families with savings ranging from $\underline{s}$ to $\bar{s}^U$, when a poverty trap exists and initial saving falls short of its level in the unstable equilibrium (assuming it exists), the society converges to a steady state in which all poor families have savings s^U. It should nonetheless be recalled that, as in Case 1, in the absence of liquidity constraints the household would borrow enough to become educated at the unconstrained level (e^*).

3.3. Case 3: Education not socially profitable

In this case, even in the absence of borrowing constraint on the capital market, individuals do not acquire any education. Therefore, families keep uneducated forever and so stuck in a poverty trap.

This completes our description of household behaviour and of the types of equilibria that may occur when individuals are equally able but their families are initially endowed with different levels of resources. We now wish to study the stationary state equilibria that occur in economies with heterogeneous individuals.

4. Stationary equilibria in a stochastic two-ability model

We consider an economy with individuals of two ability levels, high and low, and with a very simple structure. A person's lifetime utility is determined jointly by ability and parental savings; the latter depends upon the family's history of ability levels (i.e. genetic history). The distribution of wealth and welfare in the stationary state depends upon the locations of the $s_t(s_{t-1})$ curves for the persons of high and low ability. These in turn depend upon the earnings functions of high- and low-ability persons. Let these functions be $\phi_H \omega(e)$ and $\phi_L \omega(e)$ with $\phi_H > \phi_L$ where H and L refer to high and low ability. In this section we suppose that low-ability agents are unable to benefit from education, and so remain uneducated regardless of the level of their parent's savings (i.e., they are characterised by Case 3 above). In contrast, there is no poverty trap for high-ability individuals, and after a sufficiently long string of high-ability members, a family converges to a stable unconstrained equilibrium, i.e., Fig. 1 applies and $\underline{s}_H \leqslant s^U$. Whether or not high-ability agents are constrained in their educational investment decision therefore depends upon the level of their parent's savings.

Ability is determined stochastically at birth. The probability of being born with high ability is assumed to depend only on parents' ability; let π_L and π_H denote this probability conditional on the parents being of low and high ability respectively. Therefore, this probability is assumed to be independent of the previous sequence of abilities within the family (one-generation dependence).[12] Moreover this genetic history is irrelevant both for the behaviour and welfare of low-ability persons since such persons remain uneducated regardless of the income of their parents. In contrast, genetic history is of importance for high-ability persons since it determines parental savings and so the education expenditures that such persons are able to finance. For these, denote by R the number of generations that have passed since a member of the family was last of type L ($R = 1, 2, \ldots$). (In the following, R will also denote the state reached by a high-ability individual.) Since uneducated high-ability workers save $s^U = w(2+r)/3$ during adulthood, it is straightforward to determine the level of education and savings chosen by each successive child of high ability. Consequently, the resources available to a given child of type H are fully characterized by index R. In particular, since Fig. 1 applies, after a finite number of periods denoted by R^*, high-ability children are no longer constrained. For $R < R^*$, household utility and parental savings increase with each successive generation, and thereafter remain constant after the family has been of high ability for at least R^* generations.

In an economy consisting of a very large number of families, a stationary regime will eventually be reached with a given distribution of population among states L and R, where $R = 1, 2 \ldots$. We denote in this regime the proportions of the population in these various states, i.e. the stationary probabilities by the vector

$$P = (P_L, P_1, P_2 \ldots). \tag{19}$$

To determine an explicit expression for P_L and P_R ($R = 1, 2 \ldots$) in terms of probabilities π_L and π_H, note first that the matrix of transition probabilities Π characterizing the underlying Markov chain can be written as

$$\Pi = \begin{bmatrix} 1-\pi_L & \pi_L & 0 & 0 & \cdot \\ 1-\pi_H & 0 & \pi_H & 0 & \cdot \\ 1-\pi_H & 0 & 0 & \pi_H & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{20}$$

where π_{ij} (i.e. the element of this matrix at the intersection of row i and column j) is the probability that a parent in state i will have a child in state j. We know that the vector of stationary probabilities is given by $P\Pi = P$. Therefore, we have

$$\begin{aligned} &P_L(1-\pi_L) + \sum_{R=1}^{\infty} P_R(1-\pi_H) = P_L, \\ &P_L \pi_L = P_1, \\ &P_R \pi_H = P_{R+1} \quad (R = 1, 2, \ldots). \end{aligned} \tag{21}$$

[12] This could be relaxed with little complication.

This gives the relative values of P_L and P_H elements. Note in particular that

$$\frac{P_1}{P_L} = \pi_L \quad \text{and} \quad \frac{P_{R+1}}{P_R} = \pi_H \quad (R = 1, 2, \dots). \tag{22}$$

The absolute values may be obtained by noting that $P_L + \Sigma_{R=1}^{\infty} P_R = 1$. Therefore,

$$P_L = \left[1 + \pi_L (1 - \pi_H)^{-1}\right]^{-1}, \; P_R = P_L \pi_L \pi_H^{R-1} \quad (R = 1, 2, \dots). \tag{23}$$

This characterizes the population distribution across the states.

Observe that, in this simple model of heterogeneous types, genetic history determines lifetime utility. In our simple model, a person of low ability obtains utility V^U, whereas high-ability persons such that $1 \leqslant R < R^*$ obtain V^C, which increases with their parents' saving and so with R. Thus, constrained persons are distributed along the curve of Fig. 1. Finally, persons in state $R \geqslant R^*$ achieve utility V^E. Thus, in the stationary regime utilities are distributed according to the probability distribution P, from the uneducated level, V^U, through the different constrained educated levels, up to the unconstrained level, V^E. At least part of this distribution (that among persons of type $R < R^*$) is accounted for by the liquidity constraint. It is natural to ask how public policy may be used to address this unequal distribution of welfare. We turn to this next.

5. Redistributive tax policy in the simple two-ability model

The distributional problem arises partly because of differences in parental income and partly because of differences in ability. We design tax policies intended to address the former problem. The *informational assumption* is important here. We assume that the planner can only observe the ability of a person when adult. Thus, information available to parents and children during childhood becomes available to the planner once the child is an adult. We return to this informational assumption at the end of the section. Since the planner knows whether or not a given high-ability agent's parents, grandparents, great-grandparents, ... were of high ability, the planner also knows the agent's type R. Therefore, lump-sum taxes and transfers made during adulthood may be contingent on state (L; $R = 1, 2, \dots$). Let T_L and T_R denote the lump-sum transfers to persons in states L and R ($R = 1, 2, \dots$). These may be positive or negative.

The planner's goal is to maximise the sum of the utilities of each of the various types of agents in the community, weighted by their proportion in total population and taking into account the lump-sum transfers. From (3) and (4), the utility of a low-ability person can be written as

$$V_L = 3 \log(s_L) \tag{24}$$

with

$$s_L = \frac{w(2 + r) + T_L}{3}. \tag{25}$$

Among high-ability persons we must distinguish unconstrained educated ($R \geqslant R^*$) from constrained ones ($R < R^*$). The utility of the former is

$$V_R = 3 \log s_R + B, \quad R \geqslant R^*, \tag{26}$$

where s_R is given by

$$s_R = \frac{\phi_H \omega(e^*) - e^*(1+r) + T_R}{3} \tag{27}$$

with $\phi_H \omega'(e^*) = 1 + r$.

For the high-ability persons who are constrained educated we have

$$V_R = \log(s_{R-1} - e_R) + 2 \log s_R + B, \quad R = 1, \cdots, R^* - 1, \tag{28}$$

where $e_R = e(s_{R-1}, T_R)$ and $s_R = s(s_{R-1}, T_R)$ satisfy

$$\phi_H \omega'(e_R) = \frac{\phi_H \omega(e_R) - s_{R-1}(1+r) + T_R}{2(s_{R-1} - e_R)}, \tag{29a}$$

and

$$s_R = \frac{\phi_H \omega(e_R) - s_{R-1}(1+r) + T_R}{2}, \quad R = 1, \ldots, R^* - 1. \tag{29b}$$

Note that $s_0 \equiv s_L$.

Weighting utilities by the stationary probabilities, the planner's problem is

$$\max_{T_L, T_R} P_L \left[3 \log \left(\frac{w(2+r)}{3} + T_L \right) \right] + \sum_{R=1}^{R^*-1} P_R [\log(s_{R-1} - e_R) + 2 \log s_R + B] + \sum_{R=R^*}^{\infty} P_R [3 \log s_R + B], \tag{30}$$

subject to

$$P_L T_L + \sum_{R=1}^{\infty} P_R T_R = 0, \tag{31}$$

where the levels of savings and education in (30) are given by (25), (27) and (29). [13]

With μ being the Lagrange multiplier of budget constraint (31), the first-order conditions of the planner's problem can be expressed as follows.
For $R = R^*, R^* + 1, \ldots$:

$$\frac{1}{s_R} = \mu, \tag{32a}$$

[13] Note that since P_R tends to zero as R tends to infinity, this infinite sum is well defined.

For $R = 1, \dots, R^* - 1$:

$$\frac{1}{s_R} + \pi_H A_{R+1} \frac{\mathrm{d}s_R}{\mathrm{d}T_R} = \mu, \tag{32b}$$

For L:

$$\frac{1}{s_L} + \pi_L A_1 \frac{\mathrm{d}s_L}{\mathrm{d}T_L} = \mu, \tag{32c}$$

where A_R is given by:

$$A_R = \frac{1}{s_R}\left[\phi_H \omega'(e_R) - (1+r)\right] + \pi_H A_{R+1} \frac{\mathrm{d}s_R}{\mathrm{d}s_{R-1}}, \quad R = 1, 2, \dots \tag{33}$$

with $s_0 \equiv s_L$.

The interpretation of these first-order conditions is quite straightforward. For high-ability persons who are unconstrained educated, lump-sum taxes must be chosen such that the marginal benefit of an additional unit of resources to the present generation ($1/s_R$) is equated with the marginal cost of public funds (μ). For the other persons, the first-order conditions are somewhat more complicated because the intergenerational effect must also be taken into account. Thus, as compared with (32a), Eqs. (32b) and (32c) have an additional term which reflects the expected marginal benefit to the current generation's high-ability children, grand-children,... of a rise in the current generation's savings following an increase in T_L or T_R. Those benefits A_R are given by the recurrence formula (33); in this formula, the positive term in brackets reflects the discrepancy between the marginal rate of return on education spending and the interest rate due to the liquidity constraint; an increase in parental savings allows to reduce this discrepancy. Observe that the additional term in (32b) or (32c) is weighted by π_H or π_L because the ability of children is not known by the planner when those lump-sum transfers are given to their parents.

In analyzing the implications of conditions (32) for redistribution, it is useful to single out the case where by using lump-sum taxes and transfers it is possible to eliminate any borrowing constraint for high-ability individuals. In this case, low-ability persons receive transfers large enough to push their savings above b_H^E (as given by (7)); also, the lump-sum taxes imposed on high-ability persons do not induce them to save less than b_H^E.[14] In this case, we have $R^* = 1$, and since $e_R = e^*$ ($R = 1, 2, \dots$), the term within brackets in (33) disappears. This implies that full redistribution obtains ($s_R = s_L$, $R = 1, 2, \dots$).[15]

[14] The occurrence of this case depends upon the parameters of the model, in particular the values of π_L and π_H.

[15] This is equivalent to equating the marginal utilities of disposable income across individuals during their adulthood (i.e. the life period during which lump-sum taxes and transfers are performed).

Full redistribution might also be optimal with the high-ability individuals being constrained educated. By definition of full redistribution ($s_R = s_L$, $R = 1, 2, \dots$), it first requires that all high-ability persons have the same savings, implying that they are in a steady-state equilibrium such as X in Fig. 2. Note that reducing the (possibly negative) transfer to these persons decreases their savings and so moves downwards their $s_t(s_t - 1)$ curve. Therefore, the saving rate in the steady-state equilibrium falls as the transfer received by the high-ability individuals decreases. Let us now find the optimal redistributive policy, i.e. the value of T_L and the common value of the T_R's which satisfy the first-order conditions (32b) and (32c) and balance the government budget; so doing, let us neglect for a moment the requirement that s_L be equal to the common savings of high-ability individuals in the steady-state equilibrium. Note that since the e_R's are all equal in the steady state, and likewise the derivatives $\mathrm{d}s_R/\mathrm{d}s_{R-1}$ and $\mathrm{d}s_R/\mathrm{d}T_R$, the A_R's as well as the second terms on the left-hand side of (32b) are equal. Therefore, identical values for the s_R's satisfy (32b). Solving (32b) and (32c) may result either in full redistribution or in less than full redistribution.

For s_L to be equal to the common value of the s_R's and so for full redistribution to be optimal, first-order conditions (32b) and (32c) impose that at the optimum, $\pi_H \mathrm{d}s_R/\mathrm{d}T_R = \pi_L \mathrm{d}s_L/\mathrm{d}T_L$. This equality is, however, unlikely to be satisfied because we expect $\pi_H \geqslant \pi_L$ and with high-ability persons facing a borrowing constraint we have [16] $\mathrm{d}s_R/\mathrm{d}T_R > \mathrm{d}s_L/\mathrm{d}T_L$. So it is likely that when solving (32b) and (32c) without imposing the equality between s_L and the common value of the s_R's, one ends up with a value of s_L that is too small. This suggests that when it is not possible to redistribute incomes in such a way that high-ability individuals face no liquidity constraint, full redistribution will not obtain at the optimum; on intuitive grounds, we expect that savings s_R will increase with R and tend asymptotically to its steady-state level.

The above analysis assumes that the planner observes the ability of adult persons, which is admittedly restrictive. Alternatively, he could observe whether a given adult invested in education as a child. To infer from this information his ability, we must insure that no high-ability person find worthwhile mimicking a low-ability person by taking no education. This will be the case if the optimal tax-and-transfer policy just discussed provides high-ability individuals with a higher utility than that reached by taking no education and receiving transfer T_L. A large value of B (i.e. the non-pecuniary utility premium from being educated) makes it more likely to occur. [17] If it is not the case, incentive compatibility

[16] Differentiating (29a) and (29b) yields $\mathrm{d}s_R/\mathrm{d}T_R = \frac{1}{2}[(2+\alpha_R)/(3+\alpha_R)]$ with $\alpha_R \equiv -2\omega''(e_R)\omega'(e_R)^{-1}(s_{R-1} - e_R) > 0$. From (25) we obtain $\mathrm{d}s_L/\mathrm{d}T_L = 1/3$.

[17] Note that if a high-ability person in state R is constrained educated, inequality $s_R > s_L$ does not imply that the incentive compatibility constraint is met. To see this, compare (25) and (28) and recall from Section 2 that in (28), $s_{R-1} - e_R < s_R$.

constraints must be included in the optimization problem, which makes it more complex to solve.

6. Conclusion

This paper has studied a simple overlapping generations economy in which individual wealth and welfare are related to educational attainment, and in which liquidity constraints may induce children to invest in a lower quality of education than that which they would choose could they borrow in capital markets against future earnings. One of the principal goals of this analysis has been to examine the circumstances under which children whose parents are unequally wealthy will rationally choose to invest in education, and the different equilibria that may occur in the stationary steady state. More specifically, if individuals can either work or attend school during their youth, but must borrow from their parents to finance both their tuition and their consumption during youth, and repay this loan during their working life, then the children of sufficiently poor parents may not find it worthwhile to acquire an education; they, and their children, will then be caught in a poverty trap. Furthermore, even if the children of uneducated parents find it worthwhile to invest in education, they may themselves save less than their parents, and the family may then be caught in a perpetual cycle of relative indigence. Only when parental saving is sufficiently high, or the education technology is very productive, will subsequent generations attain greater levels of wealth and utility. If an unconstrained optimum exists, it will be attained in a finite number of periods; otherwise, the family will converge to a stable, constrained equilibrium with a positive level of educational attainment.

The second major concern of this paper has been to study optimal redistributive tax policy when individuals differ with respect to their capacity to benefit from a given quality of education. Two cases can be distinguished. In the first, education is productive enough for allowing, with lump-sum taxes and transfers, any high-ability person to reach the optimal level of educational expenditures. In this case, full redistribution obtains (in the sense of equating marginal utility of adulthood income across individuals). In contrast, when resources are not large enough to avoid the liquidity constraint, full redistribution is unlikely to occur at the optimum.

In view of the policy implications of our analysis, it is natural to wonder whether or not the assumed absence of parental altruism plays a critical role in obtaining our results. It is relatively straightforward to appropriately reformulate the model to allow for altruism, and it can then be checked that the descriptive part of our analysis is essentially unaffected. This is true because, for as long as parental resources are limited, then whether the intergenerational transfer takes the form of a gift or a loan is not crucial to the analysis. Instead, what is important is whether or not the level of resources to which the child has access is sufficient to induce him/her to acquire an education, rather than to remain uneducated, and whether or not the returns to education are such that over time the dynasty will attain successively higher levels of welfare until the unconstrained equilibrium is achieved. However, what is less straightforward is to undertake the same analysis of optimal tax policy when agents are altruistic. Such an exercise is complicated by the fact that it is not at all obvious as to what the planner's objective function is. Furthermore, whether or not any role exists for redistributive tax policy will turn on the particular specification of altruism which is adopted. In view of these complications, it is not possible to affirm that the same policies would be optimal were parents altruistic, although one might expect that similar results would be obtained were altruism less than full. A useful discussion of the effect of introducing altruism in models of the family can be found in Cigno (1991).

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