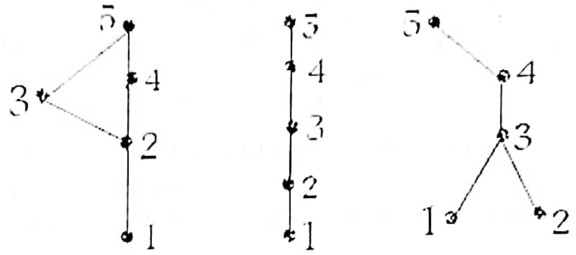
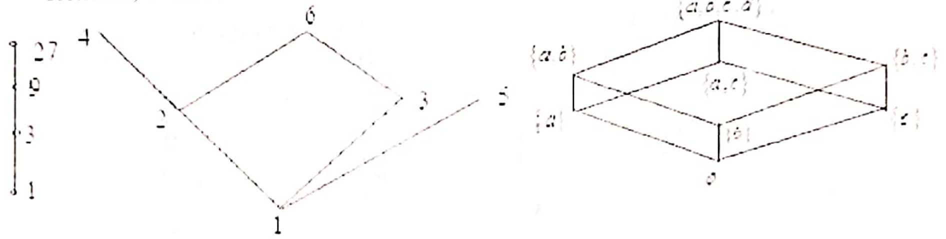
Relation II

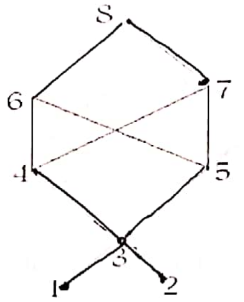
1. Define partial ordering on a set and let $A = \{a, b\}$, describe all partial order relation on A.
2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 15\}$ then (A, /) is a poset. Draw the Hasse diagram of (A, /).
3. Let $A = \{1, 2, 3, 4, 5\}$. Determine the relation represented by the following Hasse Diagram:



4. Determine whether the poset represented by Hasse diagram have a greatest element, least element, minimal element and maximal elements:



5. Find the least and greatest element in the poset $(Z^{+}, /)$, if they esist.
6. Prove that a finite partial ordered set has
   i. At most one greatest element.
   ii. At most one least element.
7. Consider the poset $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ under the partial order whose Hasse diagram is as shown below. Consider the subsets $B = \{1, 2\}$ and $C = \{3, 4, 5\}$ of A. Find i) All the lower and upper bounds of B and C.
   ii) glb(B), lub(B), glb(C) and lub(C).



8. Is the poset shown in above Hasse diagram is lattices?
9. Draw Hasse diagrams of all lattices with upto five elements.
10. For any positive integer m, let D_m denotes the set of divisors of m ordered by divisibility. Draw the Hasse diagrams of D_m for m=30, and show that *glb(a, b)=gcd(a, b)* and *lub(a ,b)=lcm(a ,b)* exist for any pair a, b in D_m.

11. If R & S are equivalence relations on a non-empty set A, then S.T. (i) $R \cap S$ (ii) $R \cup S$ are also equivalence relation on set A. (ii) R^{-1} (iii) $\bar{R}$

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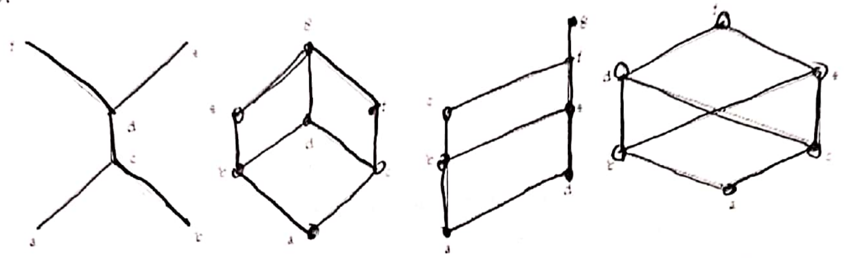
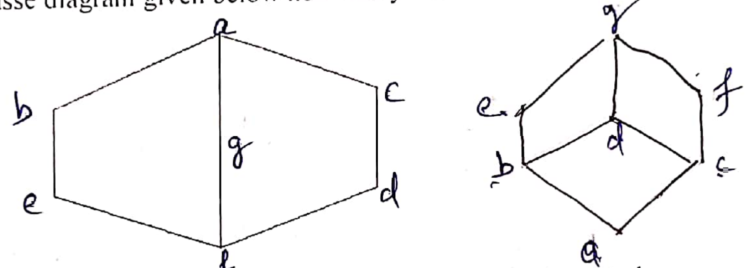
Lattice I

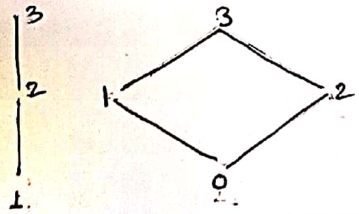
1. Determine whether the poset represented by each of the Hass diagram in the fig. are lattice:



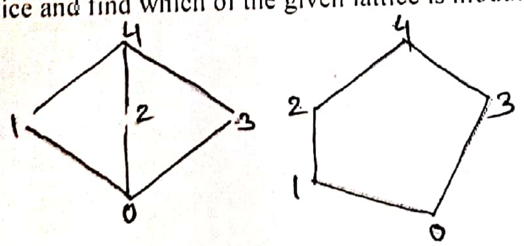
2. Show that, in any lattice the distribution inequalities holds for any a, b, c.
$(i)\,(a \wedge b) \vee c \geq (a \vee c) \wedge (b \vee c)$ $\quad (ii)\,(a \vee b) \wedge c \leq (a \wedge c) \vee (b \wedge c)$
3. In any bounded distributive lattice, the elements having complements, form a sublattice, prove it.
4. Let L be a bounded distributive lattices, then show that complements are unique if they exist.
5. In the Hasse diagram given below how many comlements does the element e have?



6. Show that the lattice $(L^3, \leq)$ of 3 tuples of 0 and 1 is complimented.
7. Show that every chain is a distributive lattice.
8. Write the dual of each statement:
$(i)\,(a \wedge b) \vee c = (b \vee c) \wedge (c \vee a)$. $(ii)\,(a \wedge b) \vee c = a \wedge (b \vee a)$
9. Show with an example that the union of two sublattice may not be a sub lattice.
10. Let L be a lattice and let a and b be elements of L such that $a \leq b$. The interval [a, b] is defined as set of all $x \in L$ such that $a \leq x \leq b$. Prove that [a, b] is a sublattice.
11. Define product of two lattice and draw the Hasse diagram for $L_1 \times L_2$ for given posets



12. Define modular lattice and find which of the given lattice is modular:



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13. Let A be a given finite set & S(A) is it powe set. Draw Hasse diagrams for (i) A = {a, b, c}, (S(A), ⊆) (ii) A = {a, b, c, d}, with partial ordering "⊆".
14. Draw Hasse diagram for (i) m=12 (ii) m=210 (iii) m=12 for Poset $(D_m, /)$.

Boolean algebra I

1. Define Boolean algebra axiomatically and find the values of the Boolean function represented by

   $i)\ F(x, y, z) = xy + \overline{z}$

   $ii)\ F(x, y, z) = x\overline{y} + \overline{(xyz)}$

   $iii)\ F(x, y, z) = \overline{y}(xz + \overline{xz})$

2. Show that $x\overline{y} + y\overline{z} + \overline{x}z = \overline{x}y + \overline{y}z + x\overline{z}$.
3. State and prove De Morgen's Law for Boolean Algebra.
4. Prove the Distribution law and commutative law fro Boolean Algebra using truth table.
5. Show that in a Boolean algebra, the Idempotent law, Law of the double compliment hold for every x.
6. Find the sum of products expansion and product of sum expansion for the function $F(x, y, z) = (x + y)\overline{z}$.
7. Use K-maps to minimize the sum of products expansions:

   $i)\ xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}\,\overline{z}$

   $ii)\ x\overline{y}z + x\,\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$