⇒ logical connectives:-

From statement p and q, we constru the statements: Negation $\sim p$, conjunct $p \wedge q$, disjunction $p \vee q$, implication $p \rightarrow$ biimplication $p \leftrightarrow q$.

The symbol $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$ are calle the logical connectives.

p, q, r, ... called statement Variable

Ex let A be the statement formula:

$$(\sim(p \vee q)) \rightarrow (q \wedge p).$$

construct Truth Table for A

→ Truth table for A.

p	q	$(p \vee q)$	$(\sim(p \vee q))$	$(q \wedge p)$	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

Ex Consider the formula: $A : (\sim p \wedge q) \rightarrow r$

$A : (\sim p \wedge q) \leftrightarrow r$

construct the truth table for A.

now,

p	q	r	~p	~p∧q	(~p∧q) → r
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	T
F	F	F	T	F	T

⇒ <u>Tautology:</u> (It is denoted by F)

- A statement formula A is said to be a tautology if the truth value of A is T for any assignment of the truth values T and F to the statement variables occuring in A.

⇒ <u>contradiction:</u>

- A statement formula A is said to be a contradiction if the truth value of A is F for any assignment of the truth values T & F to the statement variables occuring in A.

Ex for tautology: let A be the statement formula A: $(\sim p \land q) \to (\sim(q \to p))$ Prove that A is a Tautology.

p	~p	q	(~p∧q)	q → p	~(q → p)	A
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	T	F	F	T	F	T

From the truth table it follows that truth value of A is T for any assignm of truth values T and F to p and q.

- Hence, A is a tautology

eg. for contradiction

Let B be the statement formula, then $\sim p \wedge p$. Prove that B is a contradiction

B: $\sim p \wedge p$

p	$\sim p$	q	$\sim p \wedge p$
T	F	F	F
F	T	T	F

From the table, it follows that B is a contradiction

⇒ **logically imply:**

A statement formula A is said to logically imply a statement formula B if the statement formula $A \to B$ is a tautology. If A logically implies B, then symbolically we write $A \to B$.

⇒ **logically equivalent:**

- A statement formula A is said to be logically equivalent to a statement formula B if the statement formula $A \leftrightarrow B$ is a tautology. If A is logically equivalent to B, then symbolically

$A: p \wedge (p \to q)$

$B: q$ $\qquad$ $p \wedge (p \to q) \equiv q$

we write $A \equiv B$ (or) $A \Leftrightarrow B$)

Ex let A denote the statement formula, $p \wedge (p \to q)$ and B be q. then show that A is logically implies B.

logically implies

p	q	$p \to q$	$p \wedge (p \to q)$	$A \to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

From the truth table it follows that $A \to B$ is a tautology and hence A is logically implies B.

Ex show that $p \to q$ is equivalent to $\sim p \vee q$.

p	q	$\sim p$	A $(p \to q)$	B $\sim p \vee q$	$A \equiv B$ $(p \to q) \leftrightarrow \sim p \vee q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

From this table, it follows that $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology and hence, $p \to q$ is equivalent to $\sim p \vee q$,

i.e., $p \to q \equiv \sim p \vee q$.

⇒ let p, q and r be statements. The following logical equivalences hold

① <u>commutative laws:</u>

$p \wedge q \equiv q \wedge p$ and

② <u>Associative laws:</u>

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ and
$(p \vee q) \vee r \equiv p \vee (q \vee r)$

③ <u>Distributive laws:</u>

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ and
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

④ <u>Absorption laws:</u>

$p \wedge (p \vee q) \equiv p$ and
$p \vee (p \wedge q) \equiv p$

⑤ <u>Idempotent laws:</u>

$p \wedge p \equiv p$ & $p \vee p \equiv p$

⑥ <u>Double negation law:</u>

$\sim\sim p \equiv p$

⑦ <u>De Morgan's laws:</u> $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ &
$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

⇒ Principle of substitution:

If a statement formula A is a tautology containing statement letters $P_1, P_2, \dots, P_n$ and statement B is obtained from statement A by substituting statement formulas $A_1, A_2, \dots, A_n$ for $P_1, P_2, \dots, P_n$, respectively, then statement B is also a tautology.

Proof: let ⊨ A. To show that B is a tautology, we consider an arbitrary assignment of truth values to the statement letters of B.

- For this assignment, let the truth values ~~to the statement letters of~~ of $A_1, A_2, \dots, A_n$ be $x_1, x_2, \dots, x_n$, respectively, in A, then the resulting ~~the~~ truth value of A is the same as the truth value of B.

- Now the truth value of A is T for the above assignment since A is a tautology.

- Thus, the truth value of B is T.

- Hence we find that the truth value of B is T for any assignment of the truth values to the statement letters occuring in B.

- Therefore, B is a tautology

⇒ Validity of Arguments:

- A finite sequence $A_1, A_2, \cdots, A_{n-1}, A_n$ Statements is called an argument.

- The final statement A_n is called the con and the statements $A_1, A_2, \cdots, A_{n-1}$ are called the premises of the argument.

→ An argument $A_1, A_2, \cdots, A_{n-1}, A_n$ is cal logically valid if the statement formula

$$(A_1 \wedge A_2 \wedge A_3 \wedge \cdots \wedge A_{n-1}) \rightarrow A_n$$

is a tautology.

→ Sometimes we write an argument in th following form.

$$\begin{array}{l} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{n-1} \\ \therefore A_n \end{array}$$

To test the logical validity of an argumen written in a natural language, we first write each of premises & the conclusion with the help of statement letters and logical connectives

Then we check whether the conjunction

$$A_1 \wedge A_2 \wedge A_3 \wedge \cdots \wedge A_{n-1}$$

logically implies A_n. If it does, ~~not~~ then the argument is logically valid, otherwise not.

Ex consider the Argument:
If Sheila solved seven problems correctly, then sheila obtained the grade A.

→ let.

p: sheila solved seven problems correctly.
q: sheila obtained the grade A.

So the argument takes the form

$$p \to q$$
$$p$$
$$\therefore q.$$

Now consider the truth table for $((p\to q)\wedge p) \to q$

p	q	$p\to q$	$(p\to q)\wedge p$	$((p\to q)\wedge p)\to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$((p\to q)\wedge p)\to q$ is a tautology, so it follows that the given argument is valid

⇒ Application:

Judicial court system:

In Judicial court work system you can't whatever the person say is True (or) not

Doctor says this.... A_1
police Says this.... A_2
Some person's say this.... A_{n-1}
Using these premises, ⇓
you can derive conclusion A_n

So this is how a judge deals with an argument whether doctor's argument, or a lawyer's arguments or other argument are valid and judge take a decision that argument is valid (or) not!

So based on those premises, we are able to derive a conclusion 'c' then we can say the argument is valid otherwise it is not

Note: we don't know whether all premises are T (or) F (or) conclusion is T (or) F but I'm saying using those premises, we can not derive a conclusion

If that is a case then the argument is Invalid otherwise it is valid.

⇒ Some Valid argument forms:

1) Modus ponens:

Consider the following argument form.

$p \to q$ — If today is Tuesday then John will go to work

p — Today is Tuesday

$\therefore q$ — ∴ John will go to work

eg. $((p \to q) \wedge p) \to q \Rightarrow$ tautology

Valid argument form

called Modus Ponens

Latin meaning is method of affirming.

2) Modus tollens:

Consider the following argument form.

$p \to q$ — If A is true then B is true

$\sim q$ — B is not true

$\therefore \sim p$ — ∴ A is not true.

eg. $((p \to q) \wedge (\sim q)) \to (\sim p) \Rightarrow$ tautology

valid argument form

called Modus tollens

Latin meaning is method of denying

3) Disjuctive syllogisms.

Consider the following argument form,

a.) $p \vee q$
$\sim p$
$\therefore q$

b.) $p \vee q$
$\sim q$
$\therefore p$

These two are valid argument forms.

4) hypothetical syllogism.

The following argument form is also valid argument form.

$$\left.\begin{array}{l} p \to q \\ q \to r \\ \therefore p \to r \end{array}\right\}$$ Valid argument form

$[(p \to q) \land (q \to r)] \to (p \to r)$

5) dilemma :-

consider the following argument form.

$$\left.\begin{array}{l} p \lor q \\ p \to r \\ q \to r \\ \therefore r \end{array}\right\}$$ Valid argument form.

6) conjunctive Simplifications!

consider the following two argument forms.

a) $p \land q$
$\therefore p$

b) $p \land q$
$\therefore q$

These two forms are Valid argument form

7) disjunctive additions:
consider the following two argument forms.

a.) p
$\therefore p \lor q$

b.) q
$\therefore p \lor q$

These 2 are valid argument form

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8) conjunctive addition:

The following argument form is also a valid argument form.

$$\begin{array}{l} p \\ q \\ \therefore p \wedge q. \end{array}$$

⇒ logically Valid Argument:

A statement formula 'A' is said to follow logically from the statement formulas $A_1, A_2, \ldots, A_{n-1}, A_n$ written as,

$$A_1, A_2, \cdots, A_{n-1}, A_n \models A$$

if there exists an argument $B_1, B_2, \cdots B_{m-1}, B_m$ satisfying the following conditions:

1.) B_m is A.
2.) For $1 \le i \le m$, either

i) B_i is one of $A_1, A_2, \ldots, A_{n-1}, A_n$ (we say B_i is a hypothesis). or
ii) B_i is a tautology or
iii) for $i \geq 2$, there exists $B_{i_1}, B_{i_2}, \ldots B_{i_t}$, where $\{i_1, i_2, \ldots, i_t\} \subseteq \{1, 2, 3, \ldots, i-1, i\}$ such that $B_{i_1}, B_{i_2}, \ldots, B_{i_t}$ is a logically valid argument form and B_{i_t} is B_i; i.e., $B_{i_1} \wedge B_{i_2} \wedge \ldots \wedge B_{i_{t-1}} \rightarrow B_{i_t}$ is a tautology.

eg: Show that $p, q, p \to r, q \to s \models r \wedge s$.

→ $B_1 : p \to r$ hypothesis

$B_2 : p$ hypothesis

$B_3 : r$ $B_1, B_2; B_3$ is a logically Vali[d] argument, by modus pon[ens] Sometimes we write B_3 follows from B_2 & B_1 b[y] modus ponens.

$B_4 : q \to s$ hypothesis

$B_5 : q$ hypothesis

$B_6 : s$ B_4, B_5, B_6 is a logically Valid argument by modus ponens.

$B_7 : r \wedge s$ B_3, B_6, B_7 is a logically Valid argument by : conjunc[tion] addition.

Thus, we find that there exists an argumen[t] $B_1, B_2, \cdots, B_7$ satisfying the conditions of the Definition.

Hence, $p, q, p \to r, q \to s \models r \wedge s$.