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HW - 01 Probability

Q. 1.1

Given

Let Jerry's Probability going to bank be $P(A) = 0.2$
Susan's Probability going to bank be $P(B) = 0.3$
Together going to bank $P(A \cap B) = 0.08$
$P(A \cup B) = 0.2 + 0.3 - 0.08$
$= 0.42$

a) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.08}{0.3}$

$= 0.26667$

b) $P\left(\frac{A}{B'}\right) = \frac{P(A \cap B')}{P(B')}$

(Here, $P(B')$ is probability of Susan not going bank)

$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$= \frac{0.2 - 0.08}{1 - 0.3}$

$= \frac{0.12}{0.7}$

$= 0.171428$

c) $\dfrac{P[(A\cap B)]}{[(A\cup B)]} = \dfrac{P((A\cap B)\cap(A\cup B))}{P[(A\cup B)]}$ (Atleast one $\Rightarrow P(A\cup B)$, Both $\Rightarrow P(A\cap B)$)

$= \dfrac{P(A\cap B)}{P(A\cup B)}$

$= \dfrac{0.08}{0.42}$

$= 0.1904$

Answers a) 0.26667
b) 0.171428
c) 0.1904

Q 1.2.

Given.

Let Probability of Harold getting 'B' be $P(A) = 0.8$
Probability of Sharon getting 'B' be $P(B) = 0.9$
Probability of atleast getting 'B' is $P(A\cup B) = 0.91$

$\therefore$ Probability of both getting 'B' is $P(A\cap B) = P(A) + P(B) - P(A\cup B)$
$= 0.8 + 0.9 - 0.91$
$= 0.79$

$P(A') = 1 - 0.8$
$= 0.2$
(Probability of Harold not getting 'B')

$P(B') = 1 - 0.9$
$= 0.1$
(Probability of Sharon not getting 'B')

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a) $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.8 - 0.79$

$= 0.01$

b) $P(B \cap A') = P(B) - P(B \cap A)$

$= 0.9 - 0.79 \quad (P(A \cap B) = P(B \cap A))$

$= 0.11$

c) $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.91$

$= 0.09$

Answers a) 0.01
b) 0.11
c) 0.09

Q. 1.3 Given :

$P(A)$: Joey goes to bank = 0.2
$P(B)$: Susan goes to bank = 0.3

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$P(A \cap B)$: Both are at bank $= 0.08$

$P(A)\,P(B) = 0.2 \times 0.3$

$= 0.06$

$P(A \cap B) = 0.08$

$\therefore\ P(A \cap B) \neq P(A).P(B)$

Hence, the events "Jerry is at the bank" & Susan is at the bank" is not independent.

Q 1.4. Given.

Sample space for 2 dice thrown :

$$S = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)$$
$$(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)$$
$$(1,3),(2,3),(3,3),(4,3)(5,3),(6,3)$$
$$(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)$$
$$(1,5),(2,5),(3,5),(4,5)(5,5),(6,5)$$
$$(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\}$$

$\therefore\ n(S) = 36$

a) A is the event where sum is 6

$A = \{(5,1),(4,2),(3,3)(2,4),(1,5)\}$

$n(A) = 5$

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B is the event where second dice shows '5'

$B = \{(1,5)(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$n(B) = 6$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$

~~Ael~~ $A \cap B$ is the event where sum is 6 & second dice is 5

$A \cap B = \{(1,5)\}$

$n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{36}$

$\therefore P(A).P(B) = \frac{5}{36} \times \frac{6}{36} = \frac{5}{36 \times 6}$

$= \frac{5}{216}$

As $P(A \cap B) \neq P(A) \cdot P(B)$, event A & B are not independent.

b) C is the event where sum is 7

$C = \{(6,1),(5,2),(4,3),(3,4),(2,5),(1,6)\}$

$n(C) = 6$

D is the event where first dice shows '5'

$D = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$n(D) = 6$

$\Rightarrow \quad P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

$C \cap D$ is the event where first dice is '5' and sum is 7

$C \cap D = \{(5,2)\}$

$n(C \cap D) = 1$

$P(C \cap D) = \frac{1}{36}$

$P(C) \cdot P(D) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

As $P(C \cap D) = P(C) \cdot P(D)$, the events "the sum is 7" & "first dic shows 5" are independent.

Q. 1.5 Given.

Let probability of being chosen for TX be $P(TX) = 0.6$
probability of NJ being chosen be $P(NJ) = 0.1$
probability of AK being chosen be $P(AK) = 1-0.6-0.1$
$= 0.3$

F be event for finding oil.

Probability of finding oil in TX : $P\left(\frac{F}{TX}\right) = 0.3$

" " oil in AK : $P\left(\frac{F}{AK}\right) = 0.2$

" " oil in NJ : $P\left(\frac{F}{NJ}\right) = 0.1$

1) $P(F) = P\left(\frac{F}{Tx}\right)P(Tx) + P\left(\frac{F}{Ak}\right)P(Ak) + P\left(\frac{F}{NJ}\right) \cdot P(NJ)$

$= 0.3 \times 0.6 + 0.2 \times 0.3 + 0.1 \times 0.1$

$= 0.18 + 0.06 + 0.01$

$P(F) = 0.25$

2) $P\left(\frac{Tx}{F}\right) = \frac{P\left(\frac{F}{Tx}\right) P(Tx)}{P(F)}$ (Bayes Theorem)

$= \frac{0.3 \times 0.6}{0.25} = \frac{0.18}{0.25}$

$= 0.72$

1.6. As mentioned in homework notes, we don't have to include crew members as passenger.

∴ Data would look like for only passenger.

→ Survived

		Cabin			
		1st	2nd	3rd	Sub total
Age	Adult	197	94	151	442
	Child	6	24	27	57
	Sub Total	203	118	178	499

→ Not Survived

		Cabin			
		1st	2nd	3rd	Sub total
Age	Adult	122	167	476	765
	Child			52	52
	Sub total	122	167	528	817

→ Total

		Cabin			
		1st	2nd	3rd	Grand total
Age	Adult	319	261	627	1207
	Child	6	24	79	109
	Grand Total	325	285	706	1316

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a) - Probability that a passengro did not suorive

$$P(A) = \frac{\text{Passengros not suorived}}{\text{Total people}}$$

$$= \frac{817}{2201}$$

$$= 0.3711$$

b) - Probability of a passengro staying in first class

$$P(B) = \frac{\text{Passengros was staying } 1^{st} \text{ class}}{\text{Total people}}$$

$$= \frac{325}{2201}$$

$$= 0.1476$$

c) - Let A : Passengro Staying in first class
B : Passengro survived

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$ (Passengro suorived in first class)

$$= \frac{203/2201}{499/2201}$$

$$= 0.4068$$

d) - Let A : Survival of people ~~ice first class~~
B : Staying in first class

For independent,

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A) = \frac{711}{2201} = 0.323$$

$$P(B) = \frac{325}{2201} = 0.1476$$

$$P(A \cap B) = \frac{203}{2201} = 0.0922$$

$$P(A) \times P(B) = 0.04767 \neq P(A \cap B)$$

As $P(A \cap B) \neq P(A) \cdot P(B)$, events are not independent

e) - Let A : Passenger was staying in first class
B : the passenger was a child
C : passenger survived.

$$\Rightarrow P\left(\frac{A \cap B}{C}\right) = \frac{P(A \cap B \cap C)}{P(C)}$$ (Passenger staying in first class survived & is child)

$$= \frac{6/2201}{499/2201}$$

$$= 0.012$$

f) - Let A : Passenger was an adult
B : Passenger survived

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{442/2201}{499/2201}$$

$$= 0.8857$$

g) - Let A : Adult passenger
B : Child passenger
C : first class passenger
D : passenger survived.

Age & first class would be independent only when first class & child are independent as well as first class & adult passenger are independent.

As it is given, passenger survived, we will take probability where passenger is survived in every case.

For adult,

$$P(A \cap C) = \frac{197}{2201}$$

$$P(A) = \frac{442}{2201}$$

$$P(C) = \frac{203}{2201}$$

For independent, $P(A \cap C) = P(A) \cdot P(C)$

$$P(A) \cdot P(C) = \frac{442}{2201} * \frac{203}{2201}$$

$$= 0.2008 * 0.0922$$

$$= 0.01852$$

$$P(A \cap C) = \frac{197}{2201}$$

$$= 0.0895$$

$\therefore$ $P(A \cap C) \neq P(A)P(C)$, events adult & first class passenger are not independent, age and staying in first class are not independent even if child & first class are.