Assignment No. 4

To solve the problem of finding the maximum-length common subsequence of X and Y, where X and Y are two sequences, I constructed two 2-d arrays b and c. Array b is used to print the Longest common subsequence and Array c is used to calculate the length of the longest common subsequence.

As per the Theorem 15.1 on page 392 of the textbook - “Introduction to Algorithms”, we are recursively solving the substructure of the 2 sequences to find common elements which will eventually build up the subsequence. The Theorem states that - Let X and Y be sequences and Z be any LCS of X and Y.
Then
1. If $x_m = y_n$, then $z_k=x_m=y_n$ and Z_{k-1} is an LCS of X_{m-1} and Y_{n-1}
2. If $x_m \neq y_n$, then $z_k \neq x_m$ implies that Z is an LCS of X_{m-1} and Y.
3. If $x_m \neq y_n$, then $z_k \neq y_n$ implies that Z is an LCS of X and Y_{n-1}.

Therefore, array c is basically used to solve these subproblems which are shown by the above theorem

Recursive Formulation

$$
\begin{aligned}
C[i,j] &= 0, && \text{if } i==0 \text{ or } j==0 \\
&= c[i-1,j-1]+1, && \text{if } i,j > 0 \text{ and } x_i = y_j \\
&= \max(c[i,j-1], c[i-1,j]), && \text{if } i,j > 0 \text{ and } x_i \neq y_j
\end{aligned}
$$