Written Assignment 1

Rushabh Ashok Dharia

September 10, 2018

Q1.

$$f(x) = 3x^2 + 5x + 3$$
$$g(x) = 2x^3 + x - 100$$
$$f(x) = O(g(x))$$
$$To\ show\ that\ \exists\ c, n_0\ such\ that$$
$$0 \leq 3n^2 + 5n + 3 \leq c(2n^3 + n - 100)\ \forall n \geq n_0$$
$$Set\ c = 1\ and\ n_0 = 5$$
$$3(25) + 25 + 3 \leq 1(2(125) + 5 - 100)$$
$$103 \leq 155$$
$$Q.E.D$$
$$Therefore,\ 0 \leq 3n^2 + 5n + 3 \leq 1(2n^3 + n - 100)\ \forall n \geq 5$$

Q2.

$$The\ running\ time\ of\ the\ program\ given\ is\ : (\frac{n}{2})^2$$

Consider n = 10
The outer loop will run 5 times
The inner loop will run 5 times
In total the convoluted loop will run 5x5=25 times
Even for other values of n we can verify that

$$The\ running\ time\ of\ the\ program\ will\ be\ (\frac{n}{2})^2$$
$$Therefore,\ the\ program\ will\ have\ its\ upper\ bound\ as\ O(n^2)$$

Q3.

$$To\ prove\ that\ f(n) = \frac{1}{n} = O(1)$$
$$Therefore,\ f(n) = \frac{1}{n}\ and\ g(n) = O(1)$$

$$To\ show\ that\ \exists\ c, n_0\ such\ that$$
$$0 \leq \frac{1}{n} \leq c(1)\ \forall n \geq n_0$$
$$Set\ c = 1\ and\ n_0 = 1$$
$$\frac{1}{1} \leq 1(1)$$
$$1 \leq 1$$
$$Q.E.D$$
$$Therefore,\ 0 \leq \frac{1}{n} \leq 1(1)\quad \forall n \geq 1$$

Q4.

$$Given\ f(n) = O(g(n))\ and\ f(n) = O(h(n))$$
$$Example\ for\ g(n) = O(h(n))$$
$$Let\ f(n) = x\ and\ g(n) = x^2\ and\ h(n) = x^3$$
$$Here,\ f(n) = O(g(n))\ i.e.\ x = O(x^2)\ and$$
$$f(n) = O(h(n))\ i.e.\ x = O(x^3)$$
$$Also,\ x^2 \leq x^3\ \forall n \geq 0$$
$$Therefore,\ x^2 = 0(x^3)\ by\ definition:\ 0 \leq x^2 \leq c(x^3)\ \forall n \geq 1\ and\ c = 1$$
$$Q.E.D$$

$$Example\ for\ g(n) \neq O(h(n))$$
$$Let\ f(n) = x\ and\ g(n) = x^8\ and\ h(n) = x^3$$
$$Here,\ f(n) = O(g(n))\ i.e.\ x = O(x^8)\ and$$
$$f(n) = O(h(n))\ i.e.\ x = O(x^3)$$
$$But,\ x^8 \geq x^3\ \forall n \geq 0$$
$$Therefore,\ x^8 \neq O(x^3)$$
$$Since,\ x^3 = O(x^8)\ by\ definition:\ 0 \leq x^3 \leq c(x^8)\ \forall n \geq 1\ and\ c = 1$$
$$Therefore,\ g(n) \neq O(h(n))$$
$$Since\ g(n) = \Omega(h(n))$$
$$Q.E.D$$

Q5. i)

$$
\begin{aligned}
f(n) &= 2^n \\
g(n) &= n^2 \\
f(n) &= O(g(n)) \\
&\mathit{To\ show\ that}\ \exists\ c, n_0\ \mathit{such\ that} \\
0 \leq 2^n &\leq c(n^2)\ \forall n \geq n_0 \\
\mathit{Set}\ c = 1\ \mathit{and}\ n_0 &= 2 \\
2^2 &\leq 1(2^2) \\
4 &\leq 4 \\
&Q.E.D \\
\mathit{Therefore},\ 0 \leq 2^n &\leq 1(n^2)\ \forall n \geq 2
\end{aligned}
$$

ii)

$$
\begin{aligned}
f(n) &= 2^n \\
g(n) &= 3^n \\
f(n) &= O(g(n)) \\
&\mathit{To\ show\ that}\ \exists\ c, n_0\ \mathit{such\ that} \\
0 \leq 2^n &\leq c(3^n)\ \forall n \geq n_0 \\
\mathit{Set}\ c = 1\ \mathit{and}\ n_0 &= 1 \\
2^1 &\leq 1(3^1) \\
2 &\leq 3 \\
&Q.E.D \\
\mathit{Therefore},\ 0 \leq 2^n &\leq 1(3^2)\ \forall n \geq 1
\end{aligned}
$$

iii)

$$f(n) = log\ n$$
$$g(n) = log^2 n$$
$$f(n) = O(g(n))$$
$$To\ show\ that\ \exists\ c, n_0\ such\ that$$
$$0 \leq log\ n \leq c(log^2 n)\ \forall n \geq n_0$$
$$Set\ c = 1\ and\ n_0 = 4$$
$$log4 \leq 1(log^2 4)$$
$$2 \leq 2^2$$
$$2 \leq 4$$
$$Q.E.D$$
$$Therefore,\ 0 \leq log\ n \leq 1(log^n)\ \forall n \geq 2$$

iv)

$$f(n) = n^{\sqrt{n}}$$
$$g(n) = n^n$$
$$f(n) = O(g(n))$$
$$To\ show\ that\ \exists\ c, n_0\ such\ that$$
$$0 \leq n^{\sqrt{n}} \leq c(n^n)\ \forall n \geq n_0$$
$$Set\ c = 1\ and\ n_0 = 1$$
$$1^{\sqrt{1}} \leq 1(1^1)$$
$$1 \leq 1$$
$$Q.E.D$$
$$Therefore,\ 0 \leq n^{\sqrt{n}} \leq c(n^n)\ \forall n \geq 1$$