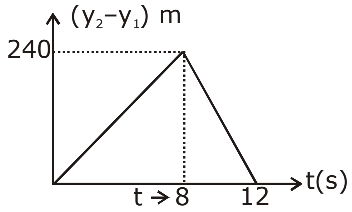
JEE MAIN EXAMINATION - 2015

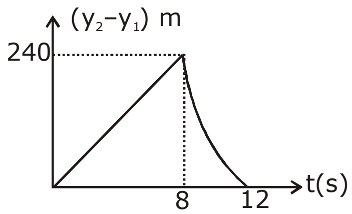
QUESTION WITH SOLUTION

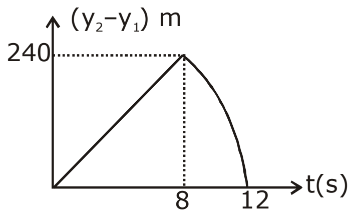
PAPER CODE - A

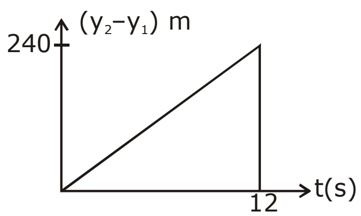
[PHYSICS]

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do no rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$)
(The figures are schematic and not drawn to scale)

(1)


(2)


(3)


(4)


Sol. **3**

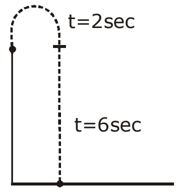
Ist stone

$0 \le t \le 8\,\text{sec}$

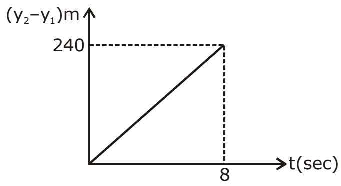
$v_r = 40 - 10$

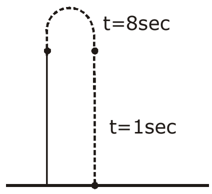
$= 30$ m/s

$a_r = 0$



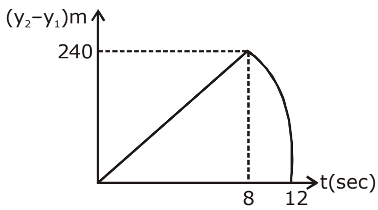
$s_r = v_r \times t = 30 \times 8 = 240$ m





8 sec < t $\le$ 12 sec

v_r increases in magnitude and relative acceleration is g downwards



2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is :

(1) 1% (2) 3%

(3) 2% (4) 5%

Sol. **2**

$$\frac{dT}{T} = \frac{1}{2}\frac{dL}{L} - \frac{1}{2}\frac{dg}{g}$$

$$\frac{90}{100} + \frac{1}{100}$$

$$\frac{1}{2}\frac{dg}{g} = \frac{1}{2}\frac{dL}{L} - \frac{dT}{T}$$

$$\frac{1}{2} \times \frac{0.1}{20} = \frac{1/100}{90/100} = \frac{1}{400} + \frac{1}{90}$$

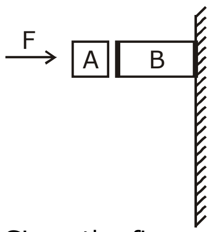
$$\frac{1}{2}\frac{dg}{g} = \frac{1}{400} + \frac{1}{90}$$

$$\frac{dg}{g} = \left(\frac{490}{400 \times 90}\right) \times 2$$

$$= \left(\frac{490}{200 \times 90}\right) = 0.20272$$

$= dg/g \times 100 \approx 2.72\% \approx 3\%$

3.

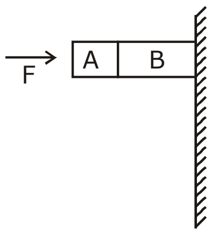


Given the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :

(1) 100 N (2) 150 N

(3) 120 N (4) 80 N

Sol. **3**



Assume the system is in equilibrium. Net gravitational force must be balanced by friction force from the wall.

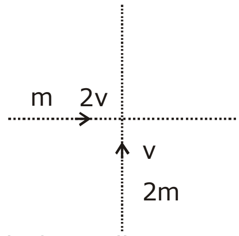
Force of friction = 120 N

4. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. if the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to :

(1) 44% (2) 62%

(3) 56% (4) 50%

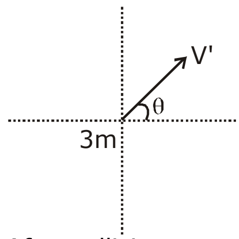
Sol. **3**



before collision

$\vec{P}_x = 2mv\,\hat{i}$

$\vec{P}_y = 2mv\,\hat{j}$



After collision

$\vec{P}_x = 3mv'\cos\theta$

$\vec{P}_y = 3\,v'\sin\theta$

By momentum conservation ;

in horizontal $\rightarrow 2mv = 3mv'\cos\theta$ ...(i)

in vertical $\rightarrow 2mv = 3mv'\sin\theta$ ...(ii)

from (i) and (ii) $\tan\theta = 1$; $\theta = 45°$

final speed $v' = \dfrac{2\sqrt{2}v}{3}$

initial K.E. ; $\rightarrow 1/2\,(m)\,(2v)^2 + 1/2\,(2m)\,(v)^2 = 3mv^2$

final K.E. ; $\rightarrow 1/2\,(3m)\left(\dfrac{2\sqrt{2}v}{3}\right)^2 = 4/3\ mv^2$

% loss $\rightarrow \dfrac{(KE)_i - (KE)_f}{(KE)_i} \times 100\%$

$= 55.55 \simeq 56\%$

5. Distance of the centre of mass of a solid uniform cone from its vertex if z_0. If the radius of its base is R and its height is h then z_0 is equal to :

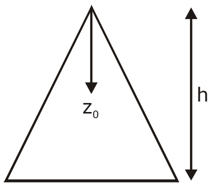
(1) $\dfrac{5h}{8}$ (2) $\dfrac{3h}{4}$

(3) $\dfrac{5h}{8}$ (4) $\dfrac{3h^2}{8R}$

Sol. **2**

$z_0 = \frac{3h}{4}$

(From class theory)



6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$

(3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

Sol. **3**

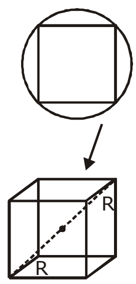
$I = \frac{Mx^2}{6}$

edge length : (x)

$2R = \sqrt{3}\,x$

$x = \frac{2R}{\sqrt{3}}$

Now,
mass of cube :



$$m = \frac{M}{\left(\frac{4}{3}\pi R^3\right)}\left(\frac{2R}{\sqrt{3}}\right)^3$$

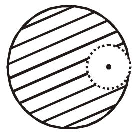
$$\left(\frac{3M}{4\pi R^3}\right)\left(\frac{8R^3}{3\sqrt{3}}\right)$$

$$m = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{1}{3}\left(\frac{2M}{\sqrt{3}\pi}\right)\left[\frac{4R^2}{3}\right]$$

$$= \frac{4MR^2}{9\sqrt{3}\pi}$$

7. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavityh thus formed is :
(G = gravitational constant)



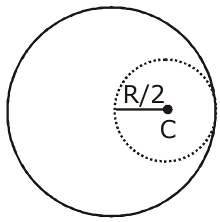
(1) $\frac{-2GM}{3R}$ (2) $\frac{-GM}{R}$

(3) $\frac{-GM}{2R}$ (4) $\frac{-2GM}{R}$

Sol. **2**

Solid sphere is of mass M, radius R.
Spherical portion removed have radius R/2, therefore its mass is M/8.

Potential at the centre of cavity
$= V_{\text{solid sphere}} + V_{\text{removed part}}$



$$= \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right] + \frac{3G(M/8)}{2(R/2)} = \frac{-GM}{R}$$

8. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then 1/Y is euqal to : (g = gravitational acceleration)

(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (2) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(3) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

Sol. **1**

$T = 2\pi\sqrt{\frac{\ell}{g}}$; $T_M = 2\pi\sqrt{\frac{\ell'}{g}}$

$\gamma = \frac{Mg/A}{\Delta\ell/\ell} \Rightarrow \frac{\ell'-\ell}{\ell} = \frac{Mg}{\gamma A} = \frac{\ell'}{\ell} = 1 + \frac{Mg}{\gamma A}$

Also:

$\frac{T_M}{T} = \sqrt{\frac{\ell'}{\ell}}$ $\quad \therefore T_M = T\left[1+\frac{Mg}{\gamma A}\right]^{1/2}$

$\Rightarrow \frac{T_M^2}{T^2} = 1 + \frac{Mg}{\gamma A} \Rightarrow \left[\frac{T_M^2}{T^2}-1\right] = \frac{Mg}{\gamma A}$

$\Rightarrow \frac{1}{\gamma} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2-1\right]$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is :

(1) $T \propto e^{-R}$ (2) $T \propto \frac{1}{R^3}$

(3) $T \propto \frac{1}{R}$ (4) $T \propto e^{-3R}$

Sol. **3**

$u = \frac{U}{V} \propto T^4$

$P = \frac{1}{3}\left(\frac{U}{V}\right)$

Adiabatic expansion

$TV^{\gamma-1} = K$

$TV^{\frac{\gamma}{4}} = C$

$\gamma - 1 = \frac{\gamma}{4}$

$\frac{3\gamma}{4} = 1$

$\gamma = \frac{4}{3}$

$TV^{\frac{\gamma}{4}} = C$

$TV^{\frac{1}{3}} = C$

$T\left(\frac{4}{3}\pi R^3\right)^{\frac{1}{3}} = C$

$T \propto \frac{1}{R}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways :

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoirs supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(1) ln2, 2ln2 (2) ln2, ln2

(3) ln2, 4ln2 (4) 2ln2, 8ln2

Sol. **2**

(i) $\Delta s_1 = \int \frac{dQ}{T} = ms\int_{100}^{150}\frac{dT}{T} + ms\int_{150}^{200}\frac{dT}{T}$

$= \ln\left(\frac{150}{100}\right) + \ln\left(\frac{200}{150}\right)$

$= \ln\left(\frac{3}{2}\right) + \ln\frac{4}{3}$

$\Delta s_1 = \ln 2$

(ii) $\Delta s_2 = \int \frac{dQ}{T} = \int_{100}^{112.5}\frac{dQ}{T} + \int_{112.5}^{125}\frac{dQ}{T} + ...$

$= \ln\left(\frac{112.5}{100}\right) + \ln\left(\frac{125}{112.5}\right) + ...$

$= \ln\left(\frac{9}{8}\right) + \ln\left(\frac{10}{9}\right) + \ln\left(\frac{16}{15}\right)$

$= \ln\left(\frac{16}{8}\right) = \ln 2$

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, when V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_P}{C_V}\right)$

(1) $\frac{3\gamma+5}{6}$ (2) $\frac{\gamma-1}{2}$

(3) $\frac{\gamma+1}{2}$ (4) $\frac{3\gamma-5}{6}$

Sol. **3**

mean free path

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$n = \frac{\text{no. of molecules}}{\text{volume}}$

$v_{avg.} \propto \sqrt{T}$ $\quad T.V^{\gamma-1} = C$

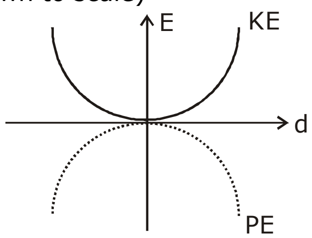
$t = \frac{\lambda}{v_{avg.}} \propto \frac{V}{\sqrt{T}}$ $\quad v \to$ is volume

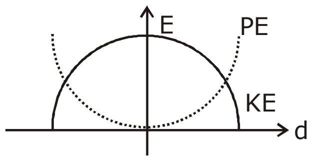
$\frac{V}{\sqrt{\frac{C}{V^{r-1}}}} \propto V^{\frac{\gamma+1}{2}}$

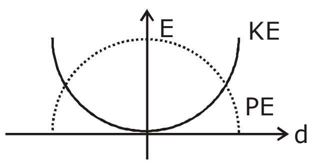
$V^q \propto V^{\frac{\gamma+1}{2}}$

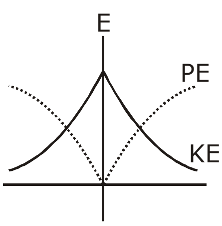
$q = \frac{\gamma+1}{2}$

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale)

(1) 

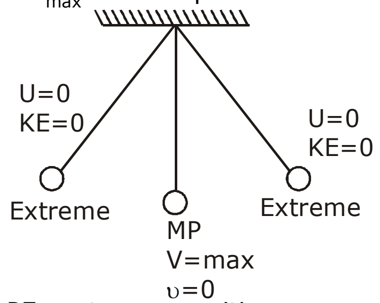
(2) 

(3) 

(4) 

Sol. **2**

KE_{max} at mean position.



PE_{min} at mean position

13. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :

(1) 18% (2) 12%
(3) 6% (4) 24%

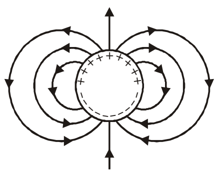
Sol. **2**

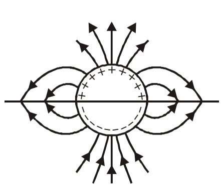
$f_1 = 1000\left(\frac{320}{300-20}\right) = 1066$ Hz

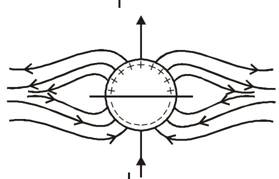
$f_2 = 1000\left(\frac{320}{300+20}\right) = 941$ Hz

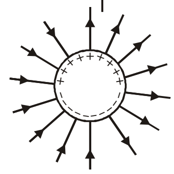
$\therefore$ Change is $\simeq$ 12%

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:
(figures are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

Sol. **1**

Tangent to the electrical field lines will give us the direction at a given point.

15. A uniformly charged solid sphere of radius R has potential V_0 9measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

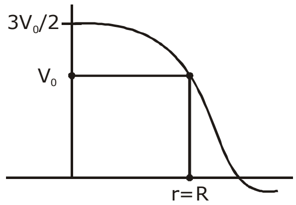
(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(4) $2R < R_4$

Sol. **3 & 4**



$$R_1 = \frac{3V_0}{2} \;;\; R_2 = \frac{5V_0}{4} \;;\; R_3 \frac{3V_0}{4} \;;\; R_4 = \frac{V_0}{4}$$

$$\therefore\ r < R \quad V = \frac{KQ}{2R^3}\left(3R^2 - r^2\right)$$

$$v = \frac{3V_0}{2},\ R_1 = 0$$

$$\frac{5V_0}{4} = \frac{KQ}{2R^3}\left(3R^2 - R_2^2\right)$$

$$\therefore\ R_2 = \frac{R}{\sqrt{2}}$$

$r > R$

$$\frac{3V_0}{4} = \frac{KQ}{R_3}$$

$$R_3 = \frac{4KQ}{3V_0} = \frac{KQ \times R}{3 \times KQ} = \frac{R}{3}$$

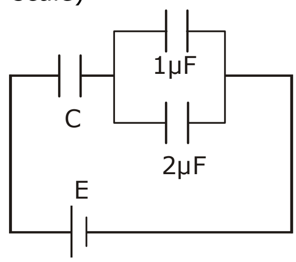
$$\frac{V_0}{4} = \frac{KQ}{R_4}$$

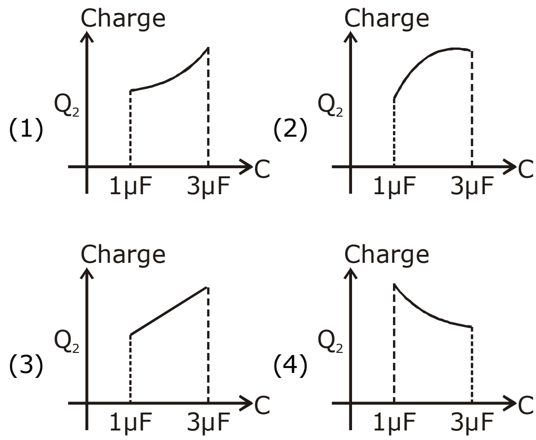
$$\therefore\quad R_4 = \frac{4KQ}{V_0} = \frac{4KQ}{KQ} \times R = 4R$$

On comparing we get

(1) & (2)

16. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figure are drawn shematically and are not to scale)





Sol. **2**

$$q = \left(\frac{3C}{C+3}\right)E \qquad q = CV$$

$$q \propto C$$

$$q_2 = \left(\frac{3C}{C+3}\right)E \ \left(\frac{2}{3}\right)$$

$$q_2 = \left(\frac{2C}{C+3}\right)E$$

$$q_2 = \left(\frac{2C}{1+\frac{3}{C}}\right)E \qquad q = CV$$

$C\uparrow q_2\uparrow$

If $C \rightarrow \infty$, q = constant value.

17. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to :

(1) $1.6 \times 10^{-6}\Omega m$ (2) $1.6 \times 10^{-7}\Omega m$

(3) $1.6 \times 10^{-8}\Omega m$ (4) $1.6 \times 10^{-5}\Omega m$

Sol. **4**

$i = neAV_d$

$\Rightarrow \frac{V}{R} = neAV_d$ $\quad \{R = \frac{\rho l}{A}\}$

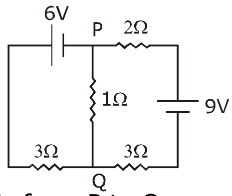
$\Rightarrow \frac{V \times A}{\rho \ell} = neAV_d$

$\Rightarrow \frac{5}{\rho \times 0.1} = 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}$

$\Rightarrow \rho = 1.56 \times 10^{-5}\ \Omega m$

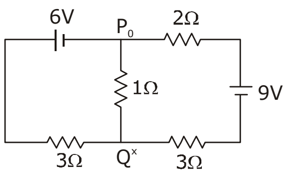
$\Rightarrow \rho \simeq 1.6 \times 10^{-5}\ \Omega m$

18. In the circuit shown, the current in the 1Ω resistor is :



(1) 1.3 A, from P to Q
(2) 0.13 A, from P to Q
(3) 0.13 A, from Q to P
(4) 0A

Sol. **3**



$\frac{x+9}{5} + \frac{x-6}{3} + \frac{x}{1} = 0$

$\Rightarrow \frac{3x+27+5x-30+15x}{15} = 0$

$\Rightarrow x = \frac{3}{23}$ A

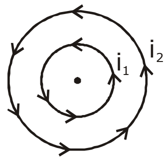
from Q to P

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then:

(1) $\vec{F}_1 = \vec{F}_2 = 0$
(2) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$
(3) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
(4) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards

Sol. **1**

Cross-sectional view



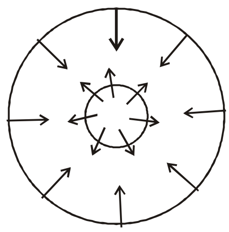
(Both solenoids are taken to be ideal in nature.)

Both wires will attract each other, but net force on each wire will be zero.

Concept:

Two current carrying elements attract each other if direction of current is same.

F.B.D

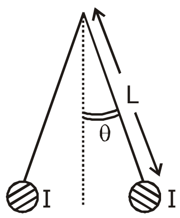


$\vec{F}_1 = 0$

$\vec{F}_2 = 0$

20. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :

(g = gravitational acceleration)



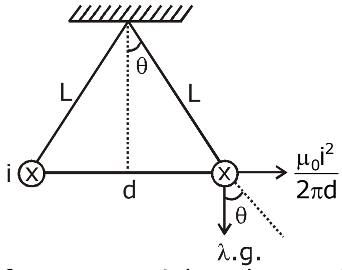
(1) $2\sqrt{\frac{\pi g L}{\mu_0}}\tan\theta$

(2) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$

(3) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$

(4) $\sqrt{\frac{\pi\lambda g L}{\mu_0}\tan\theta}$

Sol. 2



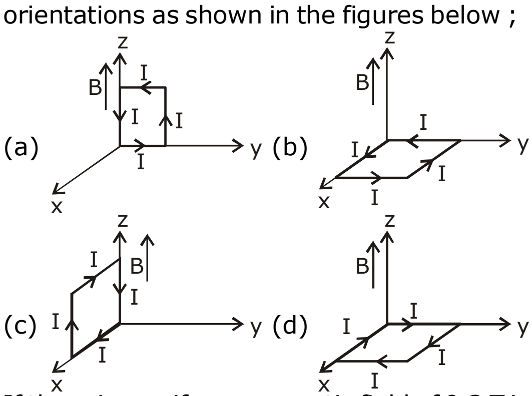
forces per unit length are taken

$$\tan\theta = \frac{\frac{\mu_0 i^2}{2\pi d}}{\lambda . g.}$$

$$i^2 = \frac{\lambda . g . \sin\theta}{\mu_0 . \cos\theta}(2\pi)\, d \qquad [d = 2L \sin\theta]$$

$$i = 2\sin\theta\sqrt{\frac{\lambda g \pi L}{\mu_0 \cos\theta}}$$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below ;



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

(1) (a) and (b), respectively
(2) (b) and (c), respectively
(3) (b) and (d), respectively
(4) (a) and (c), respectively

Sol. 3

For equilibrium $\vec{\tau} = 0$

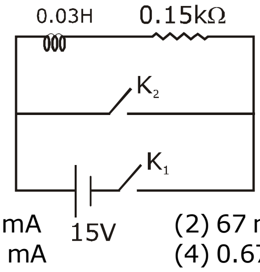
$\vec{\tau} = MB \sin\theta\, \hat{n}$

If, $\sin\theta = 0$; $\vec{\tau} = 0$

If angle between $\vec{M}$ and $\vec{B}$ is zero, then stable equilibrium

If angle between $\vec{M}$ and $\vec{B}$ is π, then unstable equilibrium

22. An inductor (L = 0.03H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be ($e^5 \cong 150$)



(1) 6.7 mA (2) 67 mA
(3) 100 mA (4) 0.67 mA

Sol. 4

According to given conditions:

$$i_0 = \frac{V}{R}$$

$$= \frac{15}{0.15 \times 10^3}$$

$= 0.1\text{A}$

$$i = i_0\, e^{-\frac{Rt}{L}}$$

$$= 0.1 \times e^{-\frac{0.15\times10^3\times10^{-3}}{0.03}}$$

$$= 0.1 \times e^{-5} = \frac{0.1}{150} = 0.67 \text{ mA}$$

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is :

(1) 5.48 V/m (2) 2.45 V/m
(3) 1.73 V/m (4) 7.75 V/m

Sol. 2

$$\text{Intensity} = \frac{P}{A} = \frac{1}{2}\varepsilon_0 E^2 C$$

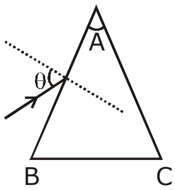
$$\Rightarrow \frac{P}{4\pi R^2} = \frac{1}{2}\times\varepsilon_0 E^2 C \qquad \Rightarrow E = \sqrt{\frac{2P}{4\pi\varepsilon_0 R^2 C}}$$

$$\Rightarrow E = \sqrt{\frac{2\times0.1\times9\times10^9}{1\times1\times3\times10^8}}$$

$$\Rightarrow E = \sqrt{\frac{1.8\times10^9}{3\times10^8}} = \sqrt{\frac{18}{3}}$$

$$\Rightarrow E = \sqrt{6} = 2.45 \text{ V/m}$$

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a rav, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided :



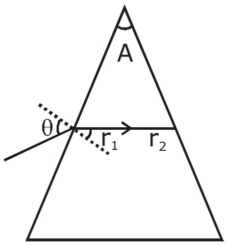
(1) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(2) $\theta < \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(4) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Sol. **1**



$r_2 < \theta_c$

$r_2 < \sin^{-1}(1/\mu)$

$\sin r_2 < 1/\mu$

$\sin\theta = \mu\sin r_1$

$r_1 = \sin^{-1}(\sin\theta/\mu)$

$\sin(A - r_1) < 1/\mu$

$$\sin\left(A-\left(\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)\right)\right) < \frac{1}{\mu}$$

$$A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$\left[\sin\left(A-\sin^{-1}\left(\frac{1}{m}\right)\right)\right] < \frac{\sin\theta}{\mu}$$

$$\sin^{-1}\left[\mu\left[\sin\left(A-\sin^{-1}\left(\frac{1}{m}\right)\right)\right]\right] < \theta$$

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam :

(1) bends downwards

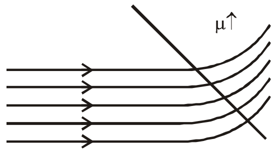
(2) goes horizontally without any deflection

(3) becomes narrower

(4) bends upwards

Sol. **4**

Bends upwards



26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :

(1) 100 μm (2) 30 μm

(3) 1 μm (4) 300 μm

Sol. **2**

$$y = 1.22\frac{\lambda D}{d} = \frac{500\times10^{-9}\times25\times10^{-2}}{2\times0.25\times10^{-2}}$$

$\Rightarrow y = 30\mu m$

27. An an electron makes a transition from a excited state to the ground state of a hydrogen–like stom/ion :

(1) Its kinetic energy increases but potential energy and total energy decrease.

(2) Kinetic energy and total energy decrease but potential energy increase

(3) Kinetic energy decreases, potential energy increases but total energy remains same

(4) Kinetic energy potential energy and total energy decrease

Sol. **1**

$E_{Total} = -13.6\ eV\ \frac{Z^2}{n^2}$

$KE = |E_{Total}|$

$PE = 2\ E_{total}$

As n decreases, Total energy decreases, potential energy decreases and kinetic energy increases.

28. Match List-I (fundamental Experiment) with List –II (its conclusion) and select the correct option from the choice given below the list:

	List–I	**List–II**
(A)	Franck–Hertz Experiment	(i) Particle nature of Experiment light
(B)	Photo-electric experiment	(ii) Discrete energy levels of atom
(C)	Davison-Germer Experiment.	(iii) Wave nature of electron
		(iv) Structure of atom

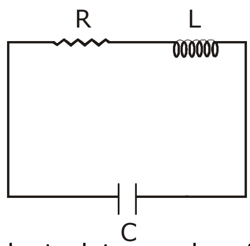
(1) A $\rightarrow$ i ; B $\rightarrow$ iv ; C $\rightarrow$ iii
(2) A $\rightarrow$ iv ; B $\rightarrow$ iii ; C $\rightarrow$ ii
(3) A $\rightarrow$ ii ; B $\rightarrow$ i ; C $\rightarrow$ iii
(4) A $\rightarrow$ ii ; B $\rightarrow$ iv ; C $\rightarrow$ iii

Sol. **3**
Photoelectric experiment is linked with particle nature of light

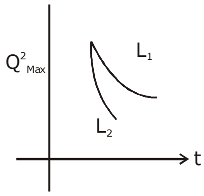
29. A signal of 5 kHZ frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are :
(1) 2 MHz only
(2) 2000 kHz and 1995 kHz
(3) 2005 kHz, 2000 kHz and 1995 kHz
(4) 2005 kHz and 1995 kHz

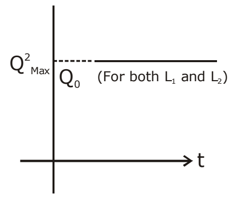
Sol. **3**
Frequencies are
F_C, $F_C \pm F_S$

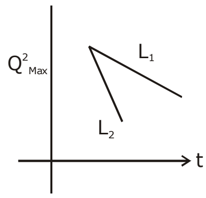
30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

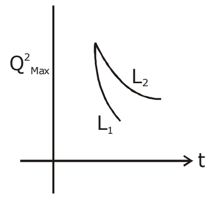


If a student plots graphs of the square of maximum charge (Q^2_{max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (Plots are shematic and not drawn to scale)

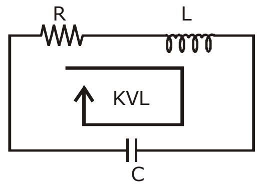
(1)


(2)


(3)


(4)


Sol. **1**



$$IR + L\frac{dI}{dt} - \frac{q}{C} = 0$$

$$L\frac{d^2q}{dt^2} = -R\frac{dq}{dt} + \frac{q}{C}$$

comparing with equation of damped oscillation

$$d\frac{d^2y}{dt^2} = -\gamma\frac{dy}{dt} - ky$$

The eqution of amplitude is $y = Ae^{-bt}$

where $b = \frac{\gamma}{2m} = \frac{R}{2L}$

$\therefore \quad q_{max} = q_0 e^{-\frac{Rt}{2L}}$

$\therefore \quad q^2_{max} = q_0^2 e^{-\frac{Rt}{L}}$

$\therefore$ time constant $\tau = \frac{R}{L}$

since $L_1 > L_2$
$\tau_1 < \tau_2$
Hence correct graph is 3.

Alternative solution

The value of Q_{max} reduces because of energy dissipation in resistor. As the value of inductor increases the time taken for capacity to discharge or charge increases therefore heat dissipation time decreases. Hence corrcect graph is 3.

[CHEMISTRY]

31. The moleclar formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin ?

(1) $\frac{1}{103}$ (2) $\frac{1}{206}$

(3) $\frac{2}{309}$ (4) $\frac{1}{412}$

Sol. **(4)**

$2C_8H_7SO_3Na + Ca^{7+} \longrightarrow (C_8H_7SO_3)_2Ca$

2 mole 1 mole

2×206 gm take 1 mole of Ca^{2+}

$\therefore$ 1gm takes $\frac{1}{412}$ mole of Ca^{2+}.

32. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately:

(1) 1.86 Å (2) 3.22 Å

(3) 5.72 Å (4) 0.93 Å

Sol. **(1)**

$\sqrt{3}a = 4r$

$r = \frac{1.732 \times 4.29}{4} = 1.86Å$

33. Which of the following is the energy of a possible excited state of hydrogen?

(1) +13.6 eV (2) −6.8 eV

(3) −3.4 eV (4) +6.8 eV

Sol. **(3)**

$-\frac{13.6z^2}{n^2} \Rightarrow$ for hydrogen ; z = 1

$-\frac{13.6}{n^2}$

Possible is −13.6, −3.4, −1.5 etc.

34. The intermolecular interation that is dependent on the inverse cube of distance between the molecules is :

(1) ion-ion interaction

(2) ion-dipole interaction

(3) London force

(4) hydrogen bond

Sol. **(4)**

hydrogen bond is dipole-dipole intraction

35. The following reaction is performed at 298 K?

$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K ? ($K_p = 1.6 \times 10^{12}$)

(1) R(298 ln (1.6×10^{12}) − 86600

(2) 86600 + R(298) ln (1.6×10^{12})

(3) $86600 - \frac{\ln(1.6 \times 10^{12})}{R(298)}$

(4) $0.5[2\times86,600 - R(298 \ln (1.6 \times 10^{12})]$

Sol. **(4)**

$-\frac{R \times 298\ \ell n 1.6 \times 10^{12}}{2}$

$= \Delta G^o{}_r = 2\Delta G^0_{NO_2} - 2\Delta G^0_{NO}$

$\Delta G^0_{NO_2} = 86.6 \times 10^3 - \frac{298K\ \ell n\ 1.6 \times 10^{12}}{2}$

36. The vapour pressure of acetone at 20ºC is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20ºC, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is:

(1) 32 (2) 64

(3) 128 (4) 488

Sol. **(2)**

$P^o = 185$

$\frac{P^0 - P}{P} = \frac{n}{N}$

$\frac{185 - 183}{183} = \frac{1.2/M}{100/58}$

M = 64

37. The standard Gibbs energy change at 300K for the reaction $2A \rightleftharpoons B + C$ is 2494.2J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, [B] = 2 and [C] $= \frac{1}{2}$. The reaction proceeds in the : [R = 8.314 J/K/mol, e = 2.718]

(1) forward direction because $Q > K_C$

(2) reverse direction because $Q > K_C$

(3) forward direction because $Q < K_C$

(4) reverse direction because $Q < K_C$

Sol. **(2)**

ΔG^o at 300K = 2494.2 J

2A ------> B + C

$\Delta G^o = -RT\ \ell n\ K$

$-2494.2 = -8.314 \times 300\ \ell n\ K$

K = 10

$$Q = \frac{[B][C]}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4.$$

$Q > K_C \Rightarrow$ reverse direction.

38. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is :
(at. mass of Cu = 63.5 amu)
(1) 0 g (2) 63.5 g
(3) 2g (4) 127g

Sol. **(2)**

$Cu^{2+} + Ze \rightarrow Cu$

2 mole deposit 1 mole of Cu

2F $\Rightarrow$ 2 mole $\rightarrow$ 1 mole of Cu $\Rightarrow$ 63.5 gm.

39. Higher order (>3) reactions are rare due to:
(1) low probability of simultaneous collision of all the reacting species
(2)increase in entropy and activation energy as more molecules are involved
(3) shifting of equilibrium towards reactant due to elastic collisions
(4)loss of active species on collision

Sol. **(1)**

molecularity and order > 3 is not possible because of low probability of simultaneous collision of all the reacting species.

40. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042N. The amount of acetic acid adsorbed (per gram of charcoal) is :
(1) 18 mg (2) 36 mg
(3) 42 mg (4) 54 mg

Sol. **(1)**

CH_3COOH (0.06M)

50 ml

m. moles = 50 × 0.06 = 3

m. moles left = 50 × 0.042 = 2.1

m. moles absorbed = 0.9

$$\text{mass absorbed} = \frac{0.9 \times 10^{-3} \times 60}{3} \times 10^3$$

$$= \frac{54}{3} = 18 \text{ mg}$$

41. The ionic radii (in Å) of N^{3-}, O^{2-} and F^- are respectively:
(1) 1.36, 1.40 and 1.71
(2) 1.36, 1.71 and 1.40
(3) 1.71, 1.40 and 1.36
(4) 1.71, 1.36 and 1.40

Sol. **(3)**

Isoelectronic species. If number of protons are more size will be less.

42. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is **false**?
(1) CO and CO_2 are produced in this process
(2) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
(3) Al^{3+} is reduced at the cathode to form Al
(4) Na_3A/F_6 serves as the electrolyte

Sol. **(4)**

43. From the following statements regarding H_2O_2, choose the incorrect statement?
(1) It can act only as an oxidizing agent
(2) It decomposes on exposure to light
(3) It has to be stored in plastic or wax lined glass bottles in dark
(4) It has to be kept away form dust

Sol. **(1)**

It acts as oxidizing as well as reducing agent.

44. Which one fo the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(1) $CaSO_4$ (2) $BeSO_4$
(3) $BaSO_4$ (4) $SrSO_4$

Sol. **(2)**

$BeSO_4$ is only the soluble sulphate because its hydration energy more than its lattice energy. rest of all are ppt.

45. Which among the following is the most reactive ?
(1) Cl_2 (2) Br_2
(3) I_2 (4) ICl

Sol. **(4)**

It has some dipole moment value and it is polar. rest of all are nonpolar and $\mu = 0$.

46. Match the catalysts to the correct processes:

	Catalyst	Process
(A)	$TiCl_3$	(i) Wacker process
(B)	$PdCl_2$	(ii) Ziegler-Natta polymerization
(C)	$CuCl_2$	(iii) Contact process
(D)	V_2O_5	(iv) Deacon's process

(1) (A) - (iii), (B) - (ii), (C) - (iv), (D) (i)
(2) (A) - (ii), (B) - (i), (C) - (iv), (D) (iii)
(3) (A) - (ii), (B) - (iii), (C) - (iv), (D) (i)
(4) (A) - (iii), (B) - (i), (C) - (ii), (D) (iv)

Sol. **(2)**

47. Which one has the highest boiling point?
(1) He (2) Ne
(3) Kr (4) Xe

Sol. **(4)**
More is the atomic weight more will be boiling point.

48. The number of geometric isomers that can exist for square planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine) :
(1) 2 (2) 3
(3) 4 (4) 6

Sol **(2)**
dsp^2 Mabcd and
hence
Its number of geometrical isomers = 3

49. The color of $KMnO_4$ is due to :
(1) M $\rightarrow$ L charge transfer transitions
(2) d - d transition
(3) L $\rightarrow$ M charge transfer transition
(4) $\sigma \rightarrow \sigma^*$ transition

Sol. **(3)**
Charge transfer from ligand to metal thatswhy $KMnO_4$ is purple colour.

50. **Assertion :** Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen
Reason: The reaction between nitrogen and oxygen requires high temperature.
(1) Both assertion and reason are correct, but the reason is the correct expanation for the assertion
(2) Both assertion and reason are correct, and the reason is not the correct explanation for the assertion
(3) Both the assertion and reason are incorrect
(4) The assertion in incorrect, but the reason in correct

Sol. **(1)**

51. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is : (at. mass Ag = 108, Br = 80)
(1) 24 (2) 36
(3) 48 (4) 60

Sol. **(1)**
moles of Br = 1 × moles of AgBr

$$= 1 \times \frac{141 \times 10^{-3}}{188}$$

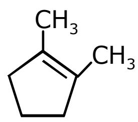
$$\text{mass of Br} = \frac{141 \times 10^{-3}}{188} \times 80$$

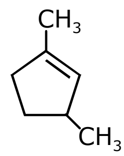
$$\therefore \text{ \% of Br} = \frac{141 \times 10^{-3}}{188} \times \frac{80}{250 \times 10^{-3}} \times 100$$
= 24%

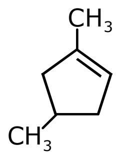
52. Which of the following compounds will exhibit geometrical isomerism ?
(1) 1-Phenyl-2-butene
(2) 3-Phenyl -1-butene
(3) 2-Phenyl-1-butene
(4) 1, 1-Diphenyl-1-propane

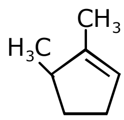
Sol. **(1)**
$H_3C-HC=CH-CH_2-Ph$
Both double bonded carbon are differently disubstituted.

53. Which compound would give 5 - keto - 2 - methyl hexanal upon ozonolysis?

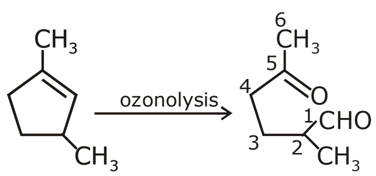
(1) 

(2) 

(3) 

(4) 

Sol. **(2)**



54. The synthesis of alkyl fluorides is best accomplished by :
(1) Free radical fluorination
(2) Sandmeyer's reaction
(3) Finkelstein reaction
(4) Swarts reaction

Sol. **(4)**
R–Cl or R – Br $\xrightarrow{\text{AgF/dmF}}$ R–F + AgCl/AgBr
Swart reaction

55. In the following sequence of reactions:

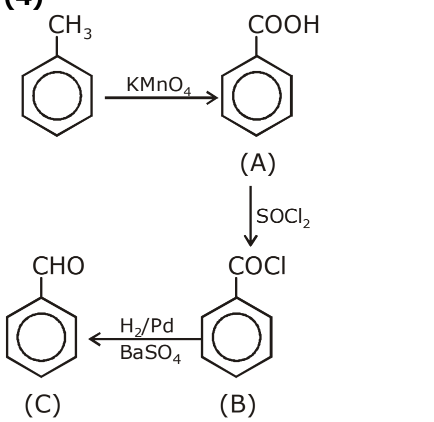
Toluene $\xrightarrow{KMnO_4}$ A $\xrightarrow{SOCl_2}$ B $\xrightarrow{H_2/Pd\ BaSO_4}$ C

the product C is:

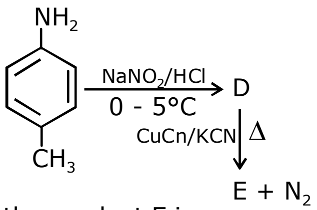
(1) C_6H_5COOH (2) $C_6H_5CH_3$

(3) $C_6H_5CH_2OH$ (4) C_6H_5CHO

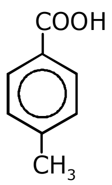
Sol. **(4)**

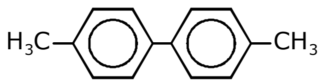


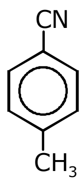
56. In the reaction

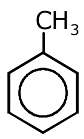


the product E is:

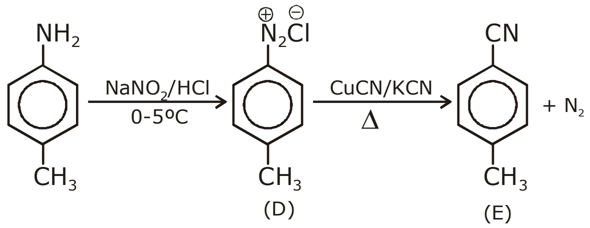
(1) 

(2) 

(3) 

(4) 

Sol. **(3)**



57. Which poloymer is used in the manufacture of paints and lacquers ?

(1) Bakelite

(2) Glyptal

(3) Polypropene

(4) Poly vinyl chloride

Sol. **(2)**

Glyptal polymer is used in the manufacture of paints and lacquers.

58. Which of the vitamins given below is water soluble?

(1) Vitamin C (2) Vitamin D

(3) Vitamin E (4) Vitamin K

Sol. **(1)**

Only vitamine B & C are water soluble while rest of fat soluble.

59. Which of the following compounds is **not** an antacid?

(1) Aluminium hydroxide

(2) Cimetidine

(3) Phenelzine

(4) Ranitidine

Sol. **(3)**

Phenelzine is not an antacid.

60. Which of the following compounds is **not** colored yellow?

(1) $Zn_2[Fe(CN)_6]$

(2) $K_3[Co(NO_2)_6]$

(3) $(NH_4)_3[As(Mo_3O_{10})_4]$

(4) $BaCrO_4$

Sol. **(1)**

(1) $(NH_4)_3[As(Mo_3O_{10})_4]$ = Yellow

(2) $BaCrO_4$ = Yellow

(3) $Zn_2[Fe(CN)_6]$ = White

(4) $K_3[Co(NO_2)_6]$ = Yellow

[MATHEMATICS]

61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :

(1) 219 (2) 256
(3) 275 (4) 510

Sol. **1**

$n(A \times B) = 8$

Total subsets $= 2^8$

${}^8C_0 + 8C_1 + {}^8C_2$

$= 37$

No. of Req. Subsets = 256 – 37 = 219.

62. A complex number z is the said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex number such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular.

Then the point z_1 lies on a :

(1) straight line parallel to x-axis
(2) straight line parallel to y-axis
(3) circle of radius 2.
(4) circle of radius $\sqrt{2}$.

Sol. **3**

$$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$$

$$(z_1 - 2z_2)\ (\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$$

$$|z_1|^2 - 2z_1\bar{z}_2 - 2z_2\bar{z}_1 + 4|z_2|^2 = 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2$$

$$+ |z_1|^2\,|z_2|^2$$

$$|z_1|^2\,|z_2|^2 - |z_1|^2 - 4|z_2|^2 + 4 = 0$$

$$\left(|z_1|^2 - 4\right)\left(|z_2|^2 - 1\right) = 0$$

$\Rightarrow\ |z_1| = 2$

63. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \ge 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to :

(1) 6 (2) –6
(3) 3 (4) –3

Sol. **3**

$x^2 - 6x - 2 = 0 <^{\alpha}_{\beta} \Rightarrow \alpha^2 - 6\alpha - 2 = 0$

$\beta^2 - 6\beta - 2 = 0$ .....(1)

$a_n = \alpha_n - \beta_n\ n \ge 1$

$a_{40} - 29_8 = \alpha^{10} - \beta^{10} - 2\alpha^8 + 2\beta^8$

$= \alpha^8 (a^2 - 2) - b^8 (b^2 - 2)$

$= \alpha^8(6\alpha) - \beta^8(6\beta)$ (using (1)

$= 6\alpha^9 - 6\beta^9$

$= 6a_9$

now

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{6a_9}{2a_9} = 3$$

64. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to

(1) (2, – 1) (2) (– 2, 1)
(3) (2, 1) (4) (– 2, – 1)

Sol. **4**

AAT = 9 I

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ ....(i)

$2a + 2 - 2b = 0 \Rightarrow a - b = -1$ ....(ii)

From i and ii

$3b = -3 \Rightarrow b = -1$

$a = -2$

65. The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(1) is an empty set.
(2) is a singleton.
(3) contains two elements.
(4) contains more than two elements..

Sol. **3**

$\Delta = (2 - \lambda)(\lambda^2 + 3\lambda - 4) + 2(-2\lambda + 2) + 1(4 - 3 - \lambda) = 0$

$-\lambda^3 - \lambda^2 + 6\lambda + 8 - 3 - \lambda - 8 = 0$

$-\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$

$\lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$(\lambda - 1)(\lambda + 3)(\lambda - 1) = 0$

$\lambda = 1, 1, -3$

66. The number of integers greater than 6,000that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(1) 216 (2) 192
(3) 120 (4) 72

Sol. **2**

6/7/8

– – – –

↓

$3 \times {}^4C_3 \times 3! = 72$

– – – – – $= 120$

Total = 192

67. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is

(1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$

(3) $\frac{1}{2}\left(3^{50}-1\right)$ (4) $\frac{1}{2}\left(2^{50}+1\right)$

Sol. **1**

for sum of integral power of x
put x = 1 in

$$\frac{\left(1-2\sqrt{x}\right)^{50}+\left(1+2\sqrt{x}\right)^{50}}{2}$$

$$\Rightarrow \frac{3^{50}+1}{2}.$$

68. If m is the A.M. of two distinct real numbers l and n(l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals.

(1) $4\, l^2mn$ (2) $4\, lm^2n$
(3) $4\, lmn^2$ (4) $4\, l^2m^2n^2$

Sol. **2**

$m = \dfrac{\ell + n}{2}$ $\quad 2m = \ell + \ell r^4$

ℓ	G_1	G_2	G_3	n
ℓ	ℓr	ℓr^2	ℓr^3	$\ell r^4 = n$

$\ell^4r^4 + 2\ell^4r^8 + \ell^4r^{12}$

$\Rightarrow \ell^4r^4(1 + 2r^4 + r^8)$

$\Rightarrow \ell^4r^4 (1 + r^4)^2$

$$\Rightarrow \ell^4r^4\left(\frac{2m}{\ell}\right)^2$$

$$\Rightarrow n \cdot \ell^3\ \frac{4m^2}{\ell^2}$$

$\Rightarrow 4\ell m^2n$

69. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ is :

(1) 71 (2) 96
(3) 142 (4) 192

Sol. **2**

$$T_n = \frac{\Sigma\, n^3}{\Sigma\,(2n-1)} = \frac{n^2(n+1)^2}{4\times n^2}$$

$$\Sigma T_n = \frac{1}{4}\left(\Sigma n^2+2\Sigma n+\Sigma 1\right)$$

$$= \frac{1}{4}\left\{\frac{n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}+n\right\}$$

$$= \frac{1}{4}\left\{\frac{9\times 10\times 19}{6}+90+9\right\}$$

$$= \frac{1}{4}\{285 + 99\} = 96$$

70. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) 4 (2) 3

(3) 2 (4) $\frac{1}{2}$

Sol. **3**

$$\lim_{x\to 0}\frac{(1-\cos 2x)\,(3+\cos x)}{x\tan 4x}$$

$$\lim_{x\to 0}\frac{(1-\cos 2x)\cdot(3+\cos x)}{x\ \dfrac{\tan 4x}{4x}.4x}$$

$$\lim_{x\to 0}\left[\frac{(1-\cos 2x)}{(2x)^2}\right]\cdot\frac{(3+\cos x)}{\dfrac{\tan 4x}{4x}}$$

$$\Rightarrow \frac{1}{2}\cdot(3 + 1) = 2$$

71. If the function $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is :

(1) 2 (2) $\frac{16}{5}$

(3) $\frac{10}{3}$ (4) 4

Sol. **1**

$$g(x) = \begin{cases} k\sqrt{x+1} & x \in [0,3] \\ mx+2 & x \in (3,5] \end{cases}$$

g(x) diff $\Rightarrow$ g(x) continuous

$\therefore$ $g(3^-) = g(3^+)$

$\Rightarrow$ $k\sqrt{4} = 3m + 2$

$\Rightarrow$ 2k = 3m + 2 ......(1)

Again

$g'(3^+) = g'(3^-)$

$\Rightarrow$ $m = \left(\frac{k}{2\sqrt{x+1}}\right)_{x=3} = \frac{k}{4}$

$\Rightarrow$ 4m = k.....(2)

from (1) & (2)

$2k = 3m + 2 \Rightarrow 8m = 3m + 2$

5 m = 2

$m = \frac{2}{5}$

& $k = 4m = \frac{8}{5}$

$\Rightarrow$ $k + m = \frac{10}{5} = 2$

72. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) does not meet the curve again.

(2) meets the curve again in the second quadrant.

(3) meets the curve again in the third quadrant.

(4) meets the curve again in the fourth quadrant.

Sol. **4**

$x^2 + 2xy - 3y^2 = 0$

diff. w.r.t. x

$2x + 2x(y') + 2y - 6yy' = 0$

$2 + 2y' + 2 - 6y' = 0$

$4y' = 4$

$y' = 1$

$\Rightarrow$ slope of normal = – 1

So equation becomes

$y - 1 = -1(x - 1)$

$x + y = 2$

solving it with curve

$x^2 + 2xy - 3y^2 = 0$

$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$

$x^2 + 4x - 2x^2 - 3(x^2 - 4x + 4) = 0$

$-4x^2 + 16x - 12 = 0$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

x = 1, 3

$\Rightarrow$ y = 1, –1

thus second point of intersection is (3, –1) is in 4th qud.

73. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to :

(1) –8 (2) –4

(3) 0 (4) 4

Sol. **3**

f(x) =

$$\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$$

$\Rightarrow$ f(x) must not contain degree 0 & degree 1 term

$\Rightarrow$ $f(x) = ax^4 + bx^3 + cx^2$

now $f'(x) = 4ax^3 + 3bx^2 + 2cx$

$f'(1) = 4a + 3b + 2c = 0$ ......(1)

$f'(2) = 32a + 12b + 4c = 0$ ......(2)

and $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 1 + c = 3$ ......(3)

$\Rightarrow$ c = 2

(1) $\Rightarrow$ 4a + 3b = – 4

(2) $\Rightarrow$ 32a + 12b = – 8

(1) $\Rightarrow$ 32a + 24b = – 32

$\Rightarrow$ $-12b = 24$, $b = -2$, $\Rightarrow a = \frac{1}{2}$

74. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals :

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4} + c$ (2) $(x^4+1)^{1/4} + c$

(3) $-(x^4+1)^{1/4} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4} + c$

Sol. **4**

$$\int \frac{dx}{x^2(x^4+1)^{3/4}}$$

$$= \int \frac{dx}{x^5(1+x^{-4})^{3/4}} = \int \frac{x^{-5}}{(1+x^{-4})^{3/4}} dx \quad ...(1)$$

put $1 + x^{-4} = T^4$

$-4x^{-5}\, dx = 4T^3\, dT$

$\Rightarrow$ (1) become

$$-\int \frac{T^3 dT}{T^3} = -T + C$$

$$= -(1 + x^{-4})^{1/4} + C$$

$$= -\left(\frac{1+x^4}{x^4}\right)^{1/4} + C$$

75. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$ is equal to :

(1) 2 (2) 4

(3) 1 (4) 6

Sol. **3**

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(x-6)^2} dx \quad .....(1)$$

using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_2^4 \frac{\log (6-x)^2}{\log(6-x)^2 + \log x^2} dx \quad ....(2)$$

(1) + (2) gives

$$2I = \int_2^4 1dx = 2$$

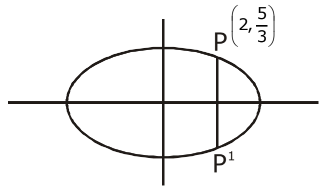
$$I = 1$$

76. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is :

(1) $\frac{27}{4}$ (2) 18

(3) $\frac{27}{2}$ (4) 27

Sol. **4**



$\frac{x^2}{9} + \frac{y^2}{5} = 1$ $a = 3$ $b = \sqrt{5}$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$= 1 - \frac{5}{9} = \frac{4}{9}$$

$$e = \frac{2}{3}$$

now the quadrilateral formed will be a rhombus

with area $= \frac{2a^2}{e}$

$$= \frac{\cancel{2}.9}{\cancel{2}} \times 3$$

$$= 27$$

77. Let y(x) be the solution of the differential equation $(x \log x) \frac{dy}{dx} + y = 2x \log x$, $(x \geq 1)$. Then y(e) is equal to :

(1) e (2) 0

(3) 2 (4) 2e

Sol. **3**

$$\frac{dy}{dx} + \frac{1}{x \log x} . y = 2$$

$$\text{I.F.} = e^{\int \frac{1}{x \log x}} = e^{\log(\log x)} = \log x$$

$$y \cdot \log x = \int 2 \cdot \log x\, dx$$

$y \log x = 2 (x \log x - x) + c$

$x = 1 \Rightarrow c = 2$

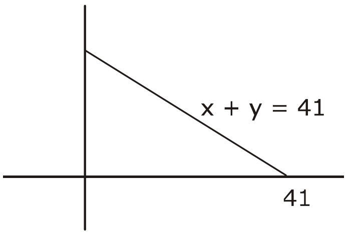
$x = e \Rightarrow y = 2(e - e) + 2 = 2$

78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is

(1) 901 (2) 861

(3) 820 (4) 780

Sol. **4**



$x + y < 41$

1 to each

$x + y < 39 \Rightarrow x + y \leq 38 \Rightarrow x + y + z = 38$

$^{38+3-1}C_{3-1} = {}^{40}C_2 = \dfrac{40 \times 39}{2} = 780$.

79. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k\,(x - 2y + 3) = 0$, $k \in R$, is a :

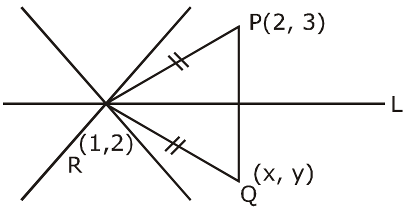
(1) straight line parallel to x-axis

(2) straight line parallel to y-axis

(3) circle of radius $\sqrt{2}$

(4) circle of radius $\sqrt{3}$

Sol. **3**



PR = RQ

$(x - 1)^2 + (y - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

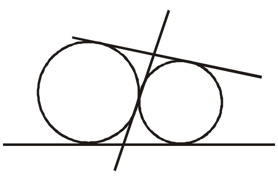
$(x - 1)^2 + (y - 2)^2 = 2$

80. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 1 (2) 2

(3) 3 (4) 4

Sol. **3**



$x^2 + y^2 - 4x - 6y - 12 = 0$

$C_1\,(2, 3)$ $r_1 = \sqrt{2^2 + 3^2 + 12} = 5$

$x^2 + y^2 + 6x + 18y + 26 = 0$

$C_2\,(-3, -9)$, $r_2 = \sqrt{3^2 + 9^2 - 26} = 8$

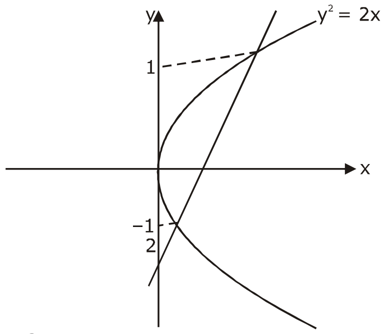
$C_1C_2 = \sqrt{(52 + 122)} = 13$

81. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x$ and $y \geq 4x - 1\}$ is

(1) $\dfrac{7}{32}$ (2) $\dfrac{5}{64}$

(3) $\dfrac{15}{64}$ (4) $\dfrac{9}{32}$

Sol. **4**



$y^2 = 2x$

$\dfrac{y^2}{2} = \dfrac{y+1}{4}$

$2y^2 - y - 1 = 0$

$2y^2 - 2y + y - 1 = 0$

$(2y + 1)\,(y - 1)$

$$A = \int_{-\frac{1}{2}}^{1} \left(\frac{y+1}{4} - \left(\frac{y^2}{2} \right) \right) d$$

$$A = \left(\frac{\frac{y^2}{2} + y}{4} \right)_{-\frac{1}{2}}^{1} - \left(\frac{y^3}{6} \right)_{-\frac{1}{2}}^{1}$$

$$A = \left(\frac{y^2 + 2y}{8} \right)_{-\frac{1}{2}}^{1} - \left(\frac{y^3}{6} \right)_{-\frac{1}{2}}^{1}$$

$$A = \left(\frac{3}{8} - \frac{\left\{ \frac{1}{4} - 1 \right\}}{8} \right) - \left(\frac{1}{6} + \frac{1}{48} \right)$$

$$A = \left(\frac{3}{8} + \frac{3}{32} \right) - \left(\frac{8+1}{48} \right) = \frac{12+3}{32} - \frac{9}{48}$$

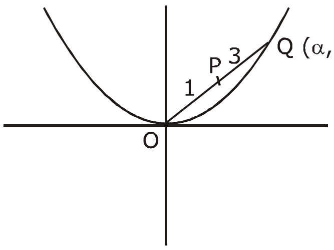
$$A = \frac{15}{32} - \frac{9}{48} = \frac{3}{16}\left(\frac{5}{2} - \frac{3}{3} \right)$$

$$= \frac{3}{16} \times \left(\frac{15-6}{6} \right) = \frac{3 \times 9}{16 \times 6} = \frac{9}{32}$$

82. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is

(1) $x^2 = y$ (2) $y^2 = x$
(3) $y^2 = 2x$ (4) $x^2 = 2y$

Sol. **4**



Let P : (h, k)

$h = \frac{1.\alpha + \beta.0}{4} \Rightarrow \alpha = 4h$

$k = \frac{1.\beta + 3.0}{4} \Rightarrow \beta = 4k$

$\because (\alpha,\beta)$ on Parabola

$\Rightarrow \alpha^2 = 8\beta \Rightarrow (4h^2) = 8.4\,k$

$16h^2 = 32k$

$x^2 = 2y$

83. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is

(1) $2\sqrt{14}$ (2) 8
(3) $3\sqrt{21}$ (4) 13

Sol. **4**

$P(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$

$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 16$

$11\lambda = 11$

$\lambda = 1$

Point of intersection (5, 3, 14)

Distance = $\sqrt{4^2 + 3^2 + 12^2}$

$= \sqrt{169} = 13$

84. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is

(1) $2x + 6y + 12z = 13$
(2) $x + 3y + 6z = -7$
(3) $x + 3y + 6z = 7$
(4) $2x + 6y + 12z = -13$

Sol. **3**

$2x - 5y + z - 3 + \lambda(x + y + 4z - 5) = 0$

$x(\lambda + 2) + y(\lambda - 5) + z(4\lambda + 1) - 3 - 5\lambda = 0$

$\frac{\lambda+2}{1} = \frac{\lambda-5}{3} = \frac{4\lambda+1}{6}$

$3\lambda + 6 = \lambda - 5$ $6\lambda - 30 = 12\lambda + 3$

$2\lambda = -11$ $6\lambda = -33$

$\lambda = -11/2$ $\lambda = -11/2$

$4x - 10y + 2z - 6 - 11x - 11y - 44z + 55 = 0$

$-7x - 21y - 42z + 49 = 0$

$x + 3y + 6z - 7 = 0$

85. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non- zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$
(3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

Sol. **1**

$(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}bc\,\bar{a}$

$(\bar{a}.\bar{c})\,\bar{b} - (\bar{b}.\bar{c})\,\bar{a} = \frac{1}{3}bc\,\bar{a} + 0.\bar{b}$

$-\bar{b}.\bar{c} = \frac{1}{3}bc$, $\bar{a}.\bar{c} = 0$

$-bc\cos\theta = \frac{1}{3}bc$

$\cos\theta = -\frac{1}{3} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$
(3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

Sol. **1**

Success (p) = $\frac{1}{3}$

Failure (q) = $\frac{2}{3}$

Acc. to binomial distribution, we have to find,

$$P(x = 3) = {}^{12}C_3 \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^9$$

$$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}.$$

87. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

(1) 16.8 (2) 16.0
(3) 15.8 (4) 14.0

Sol. **4**

$$\frac{a_1 + a_2 + a_3 + \ldots\ldots + a_{15} + 16}{16} = 16 \quad \ldots\ldots(1)$$

$$\frac{a_1 + a_2 + a_3 + \ldots\ldots + a_{15} + (3+4+5)}{18} = ?? \quad \ldots\ldots(2)$$

(1) $a_1 + a_2 + a_3 + \ldots\ldots + a_{15} = (16)^2 - 16$
now

(2) $\Rightarrow \frac{(16)^2 - 16 + 12}{18}$

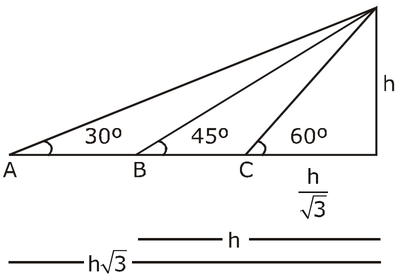
$= \frac{256-4}{18} = \frac{252}{18} = 14$

$\Rightarrow$ mean = 14

88. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC, is :

(1) $\sqrt{3} : 1$ (2) $\sqrt{3} : \sqrt{2}$
(3) $1 : \sqrt{3}$ (4) 2 : 3

Sol. **1**



$$\frac{AB}{BC} = \frac{h(\sqrt{3}-1)}{h\left(1-\frac{1}{\sqrt{3}}\right)} = \sqrt{3} : 1$$

89. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x-x^3}{1-3x^2}$ (2) $\frac{3x+x^3}{1-3x^2}$

(3) $\frac{3x-x^3}{1+3x^2}$ (4) $\frac{3x+x^3}{1+3x^2}$

Sol. **1**

$\tan^{-1} y = \tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2}$

$|x| < \frac{1}{\sqrt{3}}$

$\Rightarrow \tan^{-1} \frac{2x}{1-x^2} = 2 \tan^{-1} x$

$\Rightarrow \tan^{-1} y = \tan^{-1} x + 2\tan^{-1} x$

$= 3 \tan^{-1} x$

$= \tan^{-1} \frac{3x-x^3}{1-3x^2}$

$\Rightarrow y = \frac{3x-x^3}{1-3x^2}$

90. The negation of ~ s v(~ r ^ s) is equivalent to :

(1) s ^ ~ r (2) s ^ (r ^ ~ s)
(3) s v (r v ~ s) (4) s ^ r

Sol. **4**

~ S V (~ r ^ S)

S	r	~ r	~ r ^ S	~ S	~ S V (~ r ^ S)	~ (~ SV(~ r ^ S))
T	T	F	F	F	F	T
T	F	T	T	F	T	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F