FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

(1) 2.123 cm (2) 2.125 cm

(3) 2.121 cm (4) 2.124 cm

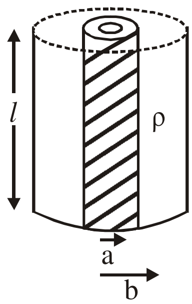
Sol. $LC = \frac{\text{pitch}}{\text{CSD}} = \frac{0.1\text{ cm}}{50} = 0.002\text{ cm}$

So any measurement will be integral

Multiple of LC.

So ans. will be 2.124 cm

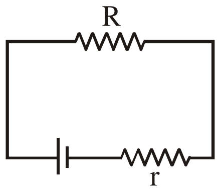
2. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for:-



(1) $R = \frac{2\rho}{\pi l} ln\left(\frac{b}{a}\right)$ (2) $R = \frac{\rho}{\pi l} ln\left(\frac{b}{a}\right)$

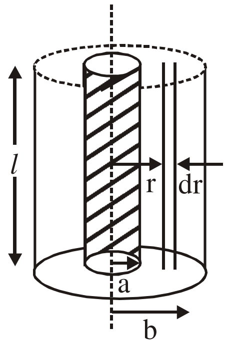
(3) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$ (4) $R = \frac{\rho}{2\pi l} ln\left(\frac{b}{a}\right)$

Sol. Maximum power in external resistance is generated when it is equal to internal resistance of battery.



$P_R = \left(\frac{\in}{r+R}\right)^2 R$

P_R is max. when r = R



$\int dr = \int_a^b \frac{\rho dr}{2\pi r l} \Rightarrow r = \frac{\rho}{2\pi l} ln\frac{b}{a}$

3. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 0.61 eV (2) 0.52 eV

(3) 0.81 eV (4) 1.02 eV

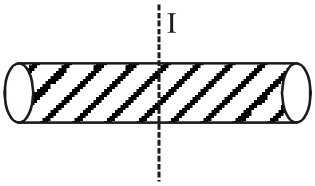
Sol. $\frac{3}{1} = \frac{\frac{hc}{200\text{ nm}} - \phi}{\frac{hc}{500\text{ nm}} - \phi}$, hc = 1240 eV-nm

On solving ϕ = 0.61 eV

4. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of material, the ratio L/R for it to have minimum possible I is :-

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{2}{3}$

Sol.



$$I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right) \quad .....(1)$$

as mass is constant $\Rightarrow$ m = ρV = constant

V = constant

$\pi^2 Rl$ = constant $\Rightarrow$ R^2L = constant

$$2RL + R^2\frac{dL}{dR} = 0 \quad .....(2)$$

From equation (1)

$$\frac{dI}{dR} = M\left(\frac{2R}{4}+\frac{2L}{12}\times\frac{dL}{dr}\right) = 0$$

$$\frac{R}{2}+\frac{L}{6}\frac{dL}{dR} = 0$$

Substituting value of $\frac{dL}{dR}$ from eqution (2)

$$\frac{R}{2}+\frac{L}{6}\left(\frac{-2L}{R}\right) = 0$$

$$\frac{R}{2} = \frac{L^2}{3R} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

5. The magnetic field of a plane electromagnetic wave is

$\vec{B} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{i}$ T

Where c = 3 × 10^8 ms^{-1} is the speed of light.

The corresponding electric field is :

(1) $\vec{E} = -10^{-6}\sin[200\pi(y+ct)]\hat{k}$ V/m

(2) $\vec{E} = -9\sin[200\pi(y+ct)]\hat{k}$ V/m

(3) $\vec{E} = 9\sin[200\pi(y+ct)]\hat{k}$ V/m

(4) $\vec{E} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{k}$ V/m

Sol. $\vec{B} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{i}$ T

$E_0 = CB_0 \Rightarrow E_0 = 3\times10^8\times3\times10^{-8}$

= 9 V/m

and direction of wave propagation is given as

$\left(\vec{E}\times\vec{B}\right)\parallel\vec{C}$

$\hat{B} = \hat{i}$ & $\hat{C} = -\hat{j}$

so $\hat{E} = -\hat{k}$

$\therefore \vec{E} = E_0\sin[200\pi(y+ct)](-\hat{k})$ V/m

6. A charged particle carrying charge 1 μC is moving with velocity $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ ms^{-1}. If an external magnetic field of $\left(5\hat{i}+3\hat{j}-6\hat{k}\right)\times10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times10^{-9}$ N. The vector $\vec{F}$ is :

(1) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$

(2) $-300\hat{i}+320\hat{j}-90\hat{k}$

(3) $-30\hat{i}+32\hat{j}-9\hat{k}$

(4) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

Sol. $\vec{F} = 9\left(\vec{V}\times\vec{B}\right)$ (Force on charge particle moving in magnetic field)

$\vec{V}\times\vec{B} = (2\hat{i}+3\hat{j}+4\hat{k})\times(5\hat{i}+3\hat{j}-6\hat{k})\times10^{-3}$

$$= \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{pmatrix} \times 10^{-3}$$

$$= \left[\hat{i}[-18-12]-\hat{j}[-12-20]+\hat{k}[6-15]\right]\times 10^{-3}$$

$$= \left[\hat{i}[-30]+\hat{j}[32]+\hat{k}[-9]\right]\times 10^{-3}$$

$$\text{Force} = 10^{-6}[-30\hat{i}+32\hat{j}-9\hat{k}]\times 10^{-3}$$

$$= 10^{-9}[-30\hat{i}+32\hat{j}-9\hat{k}]$$

7. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω , an inductance of 0.1803 H and a capacitance of 10 μ F all in series. The time in which the resistance (heat capacity 2J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to :

(1) 418 s (2) 245 s (3) 348 s (4) 365 s

Sol. f = 750 Hz, V_{rms} = 20V,

R = 100 Ω , L = 0.1803 H,

C = 10 μ F, S = 2 J/°C

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$= \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

Putting values

|Z| = 834Ω

In AC power P = $V_{rms}\, i_{rms} \cos\phi$

$$\cos\phi = \frac{R}{|Z|} \qquad i_{rms} = \frac{V_{rms}}{|Z|}$$

$$= \frac{V_{rms}^2 R}{(|Z|)^2}$$

$$= \left(\frac{20}{834}\right)^2 \times 100 = 0.0575\ \text{J/s}$$

H = Pt = S$\Delta\theta$

$$t = \frac{2(10)}{0.0575} = 348\ \text{sec}$$

8. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is :

(1) $\frac{3}{4}$ (2) $\frac{7}{8}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

Sol. First order decay

$N(t) = N_0 e^{-\lambda t}$

Given $N(t) / N_0 = 9/16 = e^{-\lambda t}$

Now, $N(t/2) = N_0\, e^{-\lambda t/2}$

$$\frac{N(t/2)}{N_0} = \sqrt{e^{-\lambda t}} = \sqrt{9/16}$$

$N(t/2) = 3/4\ N_0$

9. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

(1) Irreversible isothermal

(2) Irreversible adiabatic

(3) Reversible adiabatic

(4) Reversible isothermal

Sol. Bursting of helium balloon is irreversible & adiabatic.

10. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 8 : 1 (2) 0.8 : 1

(3) 2 : 1 (4) 4 : 1

Sol. $\Delta P_1 = 0.01 = 4T/R_1$ .....(1)

$\Delta P_2 = 0.02 = 4T/R_2$ .....(2)

Equation (1) ÷ (2)

$$\frac{1}{2} = \frac{R_2}{R_1}$$

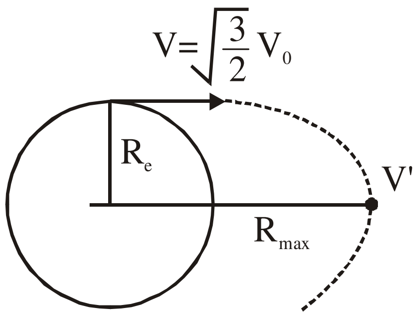
$R_1 = 2R_2$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = 8$$

11. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that is become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R, value of R is :

(1) $4R_e$ (2) $3R_e$ (3) $2R_e$ (4) $2.5R_e$

Sol.



$$V_0=\sqrt{\frac{GM}{R_e}}$$

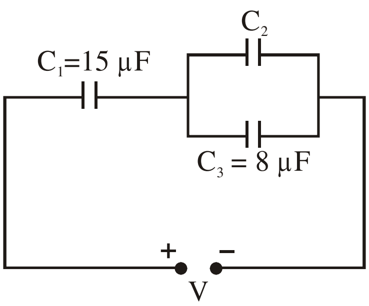
$$\frac{-GMm}{R_e}+\frac{1}{2}mv^2=\frac{-GMm}{R_{max}}+\frac{1}{2}mv'^2 \quad \text{....(i)}$$

$VR_e = V'R_{max}$ ....(ii)

Solving (i) & (ii)

$\boxed{R_{max}=3R_e}$

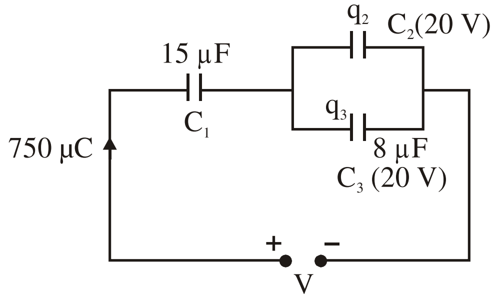
12. In the circuit shown in the figure, the total charge in 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 590 μC (2) 450 μC

(3) 650 μC (4) 160 μC

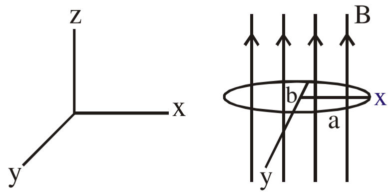
Sol.



$q_3 = 20 \times 8 = 160$ μC

$\therefore$ $q_2 = 750–160 = 590$ μC

13. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is :



(1) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$ (2) Zero

(3) $\frac{\pi^2a^2b^2B^2\omega^2}{R}$ (4) $\frac{\pi abB\omega}{R}$

Sol. $\in$ = NABωcosωt $\boxed{N=1}$

$$P_{avg}=<\frac{\in^2}{R}>=<\frac{(AB\omega\cos\omega t)^2}{R}>$$

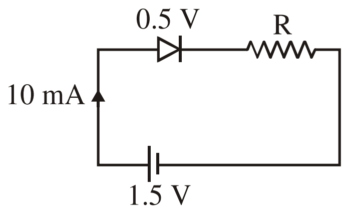
$$=\frac{A^2B^2\omega^2}{R}\frac{1}{2}=\frac{\pi^2a^2b^2B^2\omega^2}{2R}$$

14. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

(1) 100 Ω (2) 50 Ω

(3) 300 Ω (4) 200 Ω

Sol.



$1.5 - 0.5 - R \times 10 \times 10^{-3} = 0$

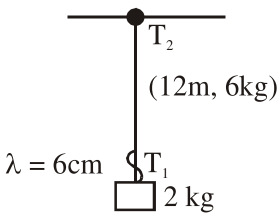
$\therefore\ R = 100\ \Omega$

15. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 9 (2) 12 (3) 6 (4) 3

Sol. $V \propto \lambda$ $\quad T_2 = 8g$

$T_1 = 2g$

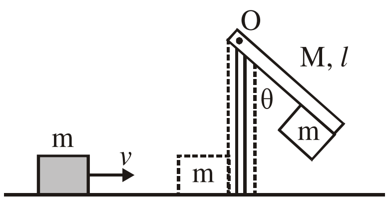


$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{V_2}{V_1}\lambda_1 = \sqrt{\frac{T_2}{T_1}} \times \lambda_1$$

$$= \sqrt{\frac{8g}{2g}}\,\lambda_1 = 2 \times 6 = 12\ \text{cm}$$

16. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1 m, the value of θ is approximately : (Take g = 10 m/s²)



(1) 69° (2) 63° (3) 55° (4) 49°

Sol. Angular momentum conservation

$$mvl = \frac{Ml^2}{3}\omega + ml^2\omega$$

$$\Rightarrow \omega = \frac{1 \times 6 \times 1}{\frac{2}{3} + 1} = \frac{18}{5}$$

Now using energy consevation

$$\frac{1}{2}\left(M\frac{l^2}{3}\right)\omega^2 + \frac{1}{2}(ml^2)\omega^2$$

$$= (m + M)r_{cm}(1 - \cos\theta)$$

$$= (m + M)\left(\frac{ml + \frac{Ml}{2}}{m + M}\right)g(1 - \cos\theta)$$

$$\frac{5}{6} \times \left(\frac{18}{5}\right)^2 = 20(1 - \cos\theta)$$

$$\Rightarrow 1 - \cos\theta = \frac{18}{5} \times \frac{3}{20}$$

$$\cos\theta = 1 - \frac{27}{50}$$

$$\cos\theta = \frac{23}{50} \Rightarrow \theta \simeq 63°$$

17. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.07° (2) 0.17° (3) 1.7° (4) 0.57°

Sol. $\Delta\theta_0 = \left(\frac{\lambda}{d} \times \frac{180}{\pi}\right)^0$

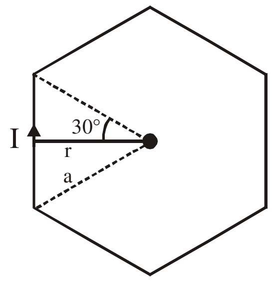
$= 0.57°$

18. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is :

(1) $250\sqrt{3}$ (2) $5\sqrt{3}$

(3) $500\sqrt{3}$ (4) $50\sqrt{3}$

Sol.



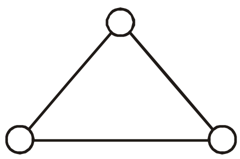
r = acos30°

$$B = \frac{6\mu_0 I}{4\pi a\cos 30°} \times 2\sin 30° \times 50$$

$$= \frac{\mu_0 I}{\pi}\frac{150}{\sqrt{3}a} = \frac{50\sqrt{3}}{0.1}\frac{\mu_0 I}{\pi}$$

$$= 500\sqrt{3}\frac{\mu_0 I}{\pi}$$

19.



Consider a gas of triatomic molecules. The molecules are assumed to the triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

(1) $\frac{9}{2}RT$ (2) $\frac{3}{2}RT$

(3) $\frac{5}{2}RT$ (4) 3RT

Sol. DOF = 3 + 3 = 6

$$U = \frac{f}{2}nRT = 3RT$$

20. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

(1) 6 μC and 3 μC

(2) +4.5 μC and –4.5 μC

(3) 3 μC and 6 μC

(4) 4.5 μC on both

Sol. Now

$Q_1 + Q_2 = Q'_1 + Q'_2 = 12\ \mu C - 3\ \mu C = 9\ \mu C$

$$\&\ \ V_1 = V_2 \Rightarrow \frac{KQ'_1}{\frac{2R}{3}} = \frac{KQ'_2}{\frac{R}{3}}$$

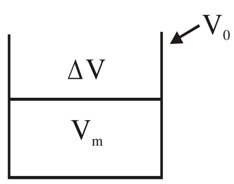
$$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$$

$$\Rightarrow Q'_2 = 3\ \mu C$$

& $Q'_1 = 6\ \mu C$

21. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}\ °C^{-1}$ and $\gamma_{(mercury)} = 1.5 \times 10^{-4}\ °C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to___________.

Sol.



$\boxed{\Delta V = (V_0 - V_m)}$

After increasing temperature

$\Delta V' = (V'_0 - V'_m)$

$\Delta V' = \Delta V$

$V_0 - V_m = V_0(1+\gamma_b\Delta T) - V_m(1+\gamma_M\Delta T)$

$V_0\gamma_b = V_m\gamma_m$

$$V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6\times10^{-6})}{(1.5\times10^{-4})}$$

$= 20$ CC

22. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F(in N) is ($g = 10$ ms^{-2})_______.

Sol. $W_F = \frac{1}{2}mv^2 = mgh$

F(S) = mgh

F(0.2) = (0.15) (10) (20)

$\boxed{F = 150N}$

23. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is [$\rho_{(liquid)} = 900$ kgm^{-3}, $g = 10$ ms^{-2}] (Give answer in closest integer)________.

Sol. Capillary rise

$$h = \frac{2S\cos\theta}{\rho gr}$$

$$S = \frac{\rho grh}{2\cos\theta}$$

$$= \frac{(900)(10)(15\times10^{-5})(15\times10^{-2})}{2}$$

$S = 1012.5 \times 10^{-4}$

$S = 101.25 \times 10^{-3} = 101.25$ mN/m

In question closest integer is asked

so closest integer = 101.00 Ans.

24. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis as 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre__________.

Sol. $L_i = L_f$

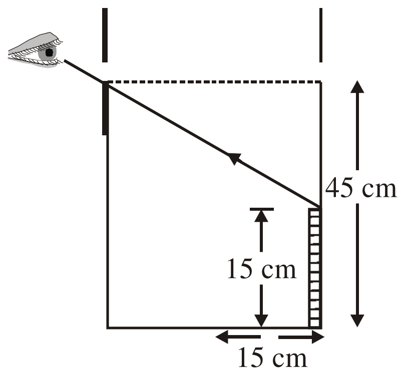
$$\left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$$

$180\omega_0 = 100\omega_1$

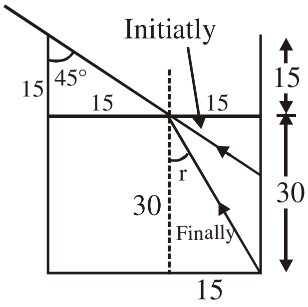
$\omega_1 = 1.8\omega_0 = 1.8 \times 5$

$= 9$ rpm

25. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid N/100, where N is an integer, the value of N is___________.



25. $\tan r = \frac{15}{30} = \frac{1}{2}$



$\sin r = \frac{1}{\sqrt{5}}$

$1 \sin 45° = \mu \sin r$

$\frac{1}{\sqrt{2}} = \mu\left(\frac{1}{\sqrt{5}}\right)$

$\boxed{\mu = \sqrt{\frac{5}{2}} = 1.581}$

$\frac{N}{100} = \mu$

$N = 100\ \mu$

$N = 158.11$

So integer value of $N = 15800$

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CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

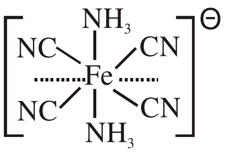
1. The complex that can show optical activity is:

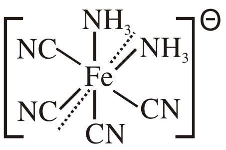
(1) trans-$[Fe(NH_3)_2(CN)_4]^-$

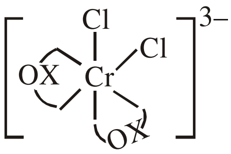
(2) cis-$[Fe(NH_3)_2(CN)_4]^-$

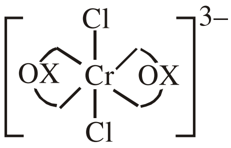
(3) cis-$[CrCl_2(ox)_2]^{3-}$ (ox = oxalate)

(4) trans-$[Cr(Cl_2)(ox)_2]^{3-}$

Sol. (1)  optically inactive

(2)  optically inactive

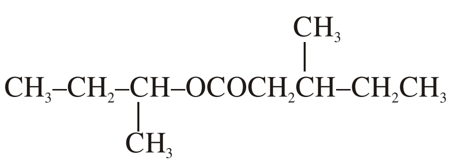
(3)  optically active

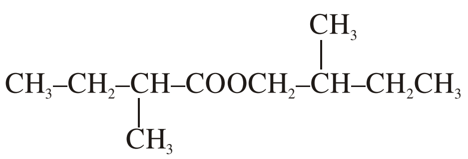
(4)  optically inactive

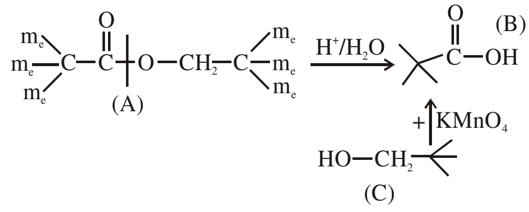
2. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and alcohol [C]. Oxidation of [C] with $CrO_3 - H_2SO_4$ produced [B]. Which of the following structures are not possible for [A] ?

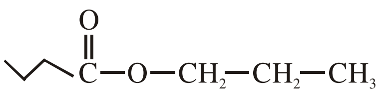
(1) $(CH_3)_3$–C–$COOCH_2C(CH_3)_3$

(2) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

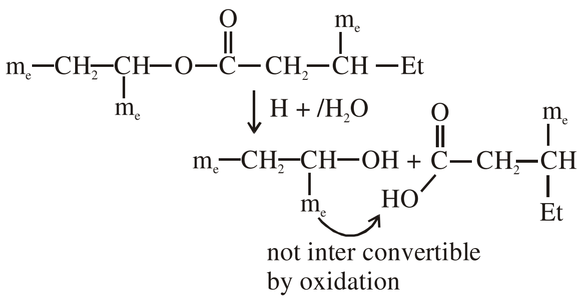
(3) 

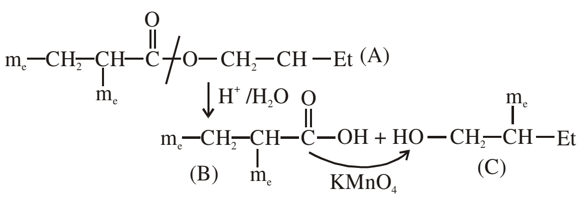
(4) 

Sol. (1) 

(2) 

Total 8 'C' → so molecular formula not matched.

(3) 

(4) 

3. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be :

(1) Greater than 300 K but less than 373 K

(2) Less than 300 K

(3) Equal to 373 K

(4) More than 373 K

Sol. Boiling point of H_2S < Boiling point of H_2O

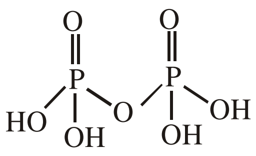
(213 K) (373 K)

4. In a molecule of pyrophosphoric acid, the number of P–OH, P=O and P–O–P bonds/ moiety(ies) respectivey are :

(1) 3, 3 and 3 (2) 2, 4 and 1

(3) 4, 2 and 0 (4) 4, 2 and 1

Sol. Pyrophosphoric acid.



P – OH linkages = 4

P = O linkages = 2

P–O–P linkages = 1

5. It is true that :

(1) A zero order reaction is a single step reaction

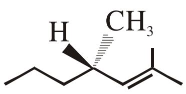
(2) A second order reaction is always a multistep reaction

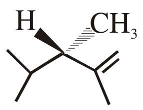
(3) A first order reaction is always a single step reaction

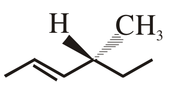
(4) A zero order reaction is a multistep reaction

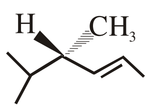
Sol. Zero order reaction is multiple step reaction.

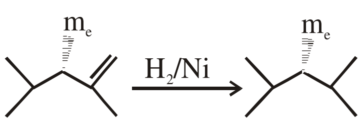
6. Which of the following compounds produces an optically inactive compound on hydrogenation ?

(1) 

(2) 

(3) 

(4) 

Sol. 

Optically active Optically inactive

7. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that :

(1) The pressure of a 55.5 molal solution of γ is 1 bar

(2) The pressure of a 55.5 molal solution of δ is 250 bar

(3) Solubility of γ at 308 K is lower than at 298 K

(4) α has the highest solubility in water at a given pressure

Sol. (1) $P_\gamma = K_H X_Y$

$$P_\gamma = 2\times10^{-15}\times\frac{55.5}{55.5+\frac{1000}{18}} = 2\times10^{-5}\,\text{K bar}$$

= 2×10^{-2} bar

(2) $P_\delta = K_H\, X_\delta$

$$P_\delta = 0.5\times\frac{55.5}{55.5+\frac{1000}{18}} = .249\,\text{K bar} = 249\,\text{bar}$$

(3) On increasing temperature solubility of gases decreases

(4) $K_H \downarrow$ solubility $\uparrow$ and lowest K_H is for γ.

8. Tyndall effect of observed when :

(1) The diameter of dispersed particles is much smaller than the wavelength of light used

(2) The diameter of dispersed particles is much larger than the wavelength of light used

(3) The diameter of dispersed particles is similar to the wavelength of light used

(4) The refractive index of dispersed phase is greater than that of the dispersion medium

Sol. The diameter of disperseed particles is similar to wavelength of light used.

9. Thermal power plants can lead to :

(1) Ozone layer depletion

(2) Eutrophication

(3) Acid rain

(4) Blue baby syndrome

Sol. Thermal power plants lead to acid rain.

10. The electronic spectrum of $[Ti(H_2O)_6]^{3+}$ shows a single broad peak with a maximum at 20,300 cm^{-1}. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1}, is :

(1) 242.5

(2) 83.7

(3) 145.5

(4) 97

Sol. CFSE = 0.4 Δ_0

$$= 0.4 \times \frac{20300}{83.7}$$

$$= 97 \text{ kJ/mol}$$

11. Aqua regia is used for dissolving noble metals (Au, Pt, etc). The gas evolved in this process is :

(1) N_2

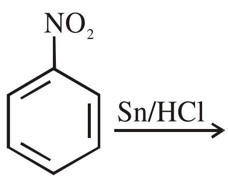
(2) N_2O_3

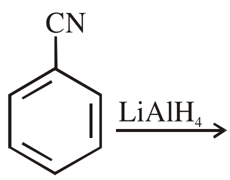
(3) NO

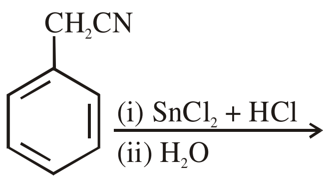
(4) N_2O_5

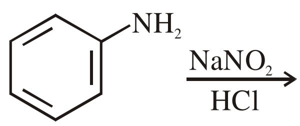
Sol. $Au + HNO_3 + 4HCl \rightarrow HAuCl_4 + NO + 2H_2O$

12. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products ?

(a) 

(b) 

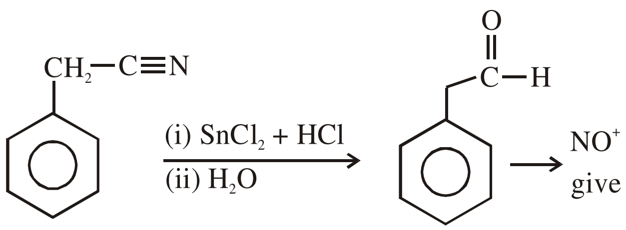
(c) 

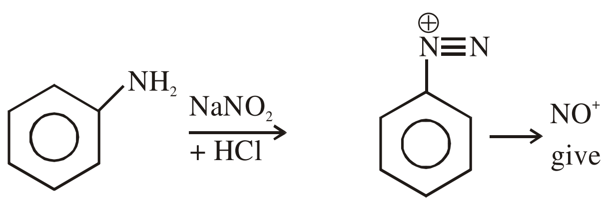
(d) 

(1) a and d (2) c and d

(3) a, c and d (4) b and c

Sol. Kjeldahl method is used for N estimation But not given by 'Diazo' compounds





13. The mechanism of S_N1 reaction is given as :

$$R-X \rightarrow \underset{\text{Ion pair}}{R^{\oplus}\,X^{\ominus}} \rightarrow \underset{\text{Solvent separated ion pair}}{R^{\oplus}||X^{\ominus}} \xrightarrow{Y^{\ominus}} R-Y+X^{\ominus}$$

A student writes general characteristics based on the given mechanism as :

(a) The reaction is favoured by weak nucleophiles

(b) $R^{\oplus}$ would be easily formed if the substituents are bulky

(c) The reaction is accompained by recemization

(d) The reaction is favoured by non-polar solvents.

Which observations are correct ?

(1) b and d (2) a and c

(3) a, b and c (4) a and b

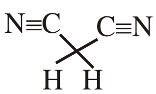
Sol. S_N1 favours

(a) The reaction is favoured by weak nucleophiles

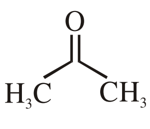
(b) $R^{\oplus}$ would be easily formed if the substituents are bulky

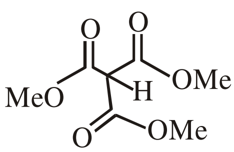
(c) The reaction is accompained by recemization

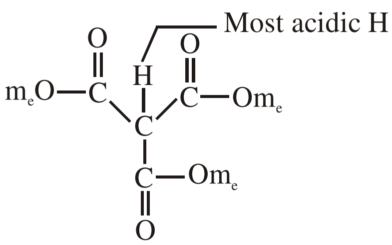
14. Which one of the following compounds possesses the most acidic hydrogen ?

(1) 

(2) $H_3C-C\equiv C-H$

(3) 

(4) 

Sol. 

Due to presence of 3 (–R) groups

15. Glycerol is separated in soap industries by :

(1) Steam distillation

(2) Differential extraction

(3) Distillation under reduced pressure

(4) Fractional distillation

Sol. Glycerol is separated by reduced pressure distillation in soap industries.

16. Of the species, NO, NO^+, NO^{2+}, NO^-, the one with minimum bond strength is :

(1) NO^{2+} (2) NO^+ (3) NO (4) NO^-

Sol. Bond order of NO^{2+} = 2.5

Bond order of NO^+ = 3

Bond order of NO = 2.5

Bond order of NO^- = 2

Bond order α bond strength.

17. The atomic number of the element unnilennium is :

(1) 119 (2) 108 (3) 102 (4) 109

Sol. 1 0 9

un nil enn

Hence correct name $\rightarrow$ unnilennium

18. An acidic buffer is obtained on mixing :

(1) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

(2) 100 mL of 0.1 M CH_3COOH and 100 mL of 0.1 M NaOH

(3) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

(4) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl

Sol.

$$\underset{\substack{10\text{ mili mol}\\ —}}{HCl} + \underset{\substack{20\text{ mili mol}\\ 10\text{ mili mol}}}{CH_3COONa} \rightarrow \underset{\substack{—\\ 10\text{ mili mol}}}{CH_3COOH} + \underset{\substack{—\\ 10\text{ mili mol}}}{NaCl}$$

So finaly we get mixture of $CH_3COOH + CH_3COONa$ that will work like acidic buffer solution.

19. Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solutions of NaCl and $BaSO_4$, respectively, at a temperature T. Which of the following is false ?

(1) Ionic mobilities of ions from both salts increase with T

(2) $C_{NaCl} >> C_{BaSO_4}$ at a given T

(3) $C_{NaCl}(T_2) > C_{NaCl}(T_1)$ for $T_2 > T_1$

(4) $C_{BaSO_4}(T_2) > C_{BaSO_4}(T_1)$ for $T_2 > T_1$

Sol. Dissolution of $BaSO_4$ is an endothermic reaction 50 on increasing temperature number of ions of $BaSO_4$ decrease so it's conduction also decrease.

20. The antifertility drug 'Novestrol" can react with :

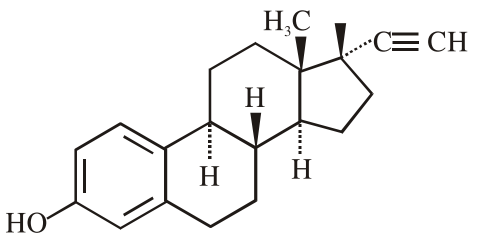
(1) Br_2/water; $ZnCl_2$/HCl; $FeCl_3$

(2) Alcoholic HCN; NaOCl; $ZnCl_2$/HCl

(3) Br_2/water; $ZnCl_2$/HCl; NaOCl

(4) $ZnCl_2$/HCl; $FeCl_3$; Alcoholic HCN

Sol.



Ethynylestradiol (novestrol)

gives (1) $Br_2 + H_2O$ test

(2) Lucas test with $ZnCl_2$ + HCl

(3) $FeCl_3$ test of phenolic group.

21. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. (R=0.0821 L atm K^{-1} mol^{-1}) (rounded off to the nearest integer)

Sol. Volume strength of H_2O_2 at 1 atm

273 kelvin = M × 11.2 = 8.9 × 11.2 = 99.68

Ans : 100

22. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is ______.

Sol. $X_{C_6H_{12}O_6} = 0.1$

Let total mole is 1 mol then mole of glucose will be 0.1 and mole of water will be 0.9

$$\text{so mass \% of water} = \frac{0.9\times18}{0.1\times180+0.9\times18}\times100$$

$$= 47.36$$

Ans : 47

23. The photoelectric current from Na (work function, w_0 = 2.3 eV) is stopped by the output voltage of the cell

$Pt(s)|H_2(g, 1bar)|HCl(aq., pH = 1)|AgCl(s)|Ag(s)$

The pH of aq. HCl required to stop the photoelectric current from K(w_0 = 2.25eV), all other conditions remaining the same, is_____ × 10^{-2} (to the nearest integer).

Given, $2.303\frac{RT}{F}=0.06V; E^0_{AgCl|Ag|Cl^-}=0.22V$

Sol.
$$\frac{1}{2}H_2 \rightarrow H^+ + e^{\ominus}$$
$$e^{\ominus} + AgCl_{(s)} \rightarrow Ag_{(s)} + Cl^{\ominus}$$
$$\frac{1}{2}H_2 + AgCl_{(s)} \rightarrow H^+_{(aq)} + Ag_{(s)} + Cl^{\ominus}_{(aq)}$$

$$E = \varepsilon^0 - \frac{.06}{1}\log\frac{[H^+][Cl^{\ominus}]}{P_{H_2}^{\frac{1}{2}}}$$

$$E = 0.22 - .06\log\frac{(10^{-1})(10^{-1})}{1^{\frac{1}{2}}}$$

E = 0.22 + .12 = .34 volt

⇒ total energy of photon will be (for Na)

= 2.3 + 0.34 = 2.64 eV

$\Rightarrow$ stopping potential required for K

$= 2.64 - 2.25 = 0.39$ volt

$$E = \varepsilon^0 - \frac{.06}{1}\log\frac{[H^+][Cl^-]}{P_{H_2}^{\frac{1}{2}}}$$

as $[H^+] = [Cl^{\ominus}]$ so

$$0.39 = 0.22 - .06\log\frac{[H^+]^2}{1^{\frac{1}{2}}}$$

$0.17 = +\ .12$ pH

pH $= 1.4166 \Rightarrow 1.42$

24. An element with molar mass 2.7×10^{-2} kgmol^{-1} forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^3 kgm^{-3}, the radius of the element is approximately ____ $\times 10^{-12}$ m (to the nearest integer).

Sol. $$d = \frac{z\left(\frac{M}{N_A}\right)}{a^3}$$

$$2.7\times10^3 = z\frac{\left(\frac{2.7\times10^{-2}}{6\times10^{23}}\right)}{\left(405\times10^{-12}\right)^3}$$

$$2.7\times10^3 = z\frac{\left(2.7\times10^{-2}\right)}{6\times10^{23}\left(4.05\times10^{-10}\right)^3}$$

$$2.7\times10^3 = z\frac{\left(2.7\times10^{-2}\right)}{6\times10^{23}\times66.43\times10^{-30}}$$

$3.98 = z$

$z \approx 4$ structure is fcc

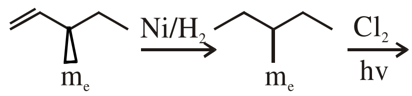
$$\frac{a}{\sqrt{2}} = 2r$$

$$r = \frac{a}{2\sqrt{2}} = \frac{\sqrt{2}a}{4} = \frac{1.414\times405\times10^{-12}}{4}$$

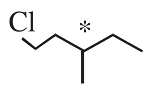
$r = 143.16 \times 10^{-12}$

25. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is ______.

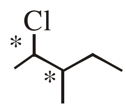
$$\underset{\substack{\text{(simplest optically}\\ \text{active alkene)}}}{A} \xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i)}H_2/Ni/\Delta}$$

Sol. 

Simplest
O.A. Alkene

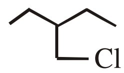


2



4

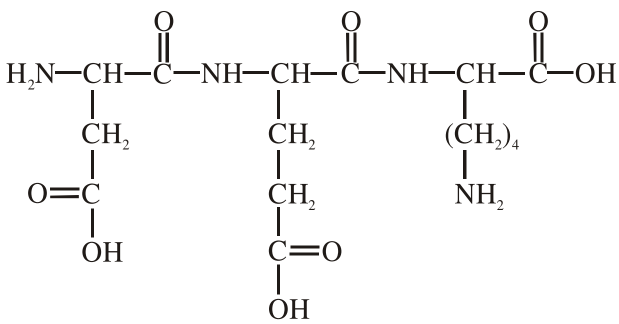
1



1

Alter

Str. of Tri peptide



FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

(1) $\frac{1}{8}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability = $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

2. The lines

$\vec{r} = (\hat{i} - \hat{j}) + \ell(2\hat{i} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) Intersect when ℓ = 1 and m = 2

(2) Intersect when ℓ = 2 and m = $\frac{1}{2}$

(3) Do not intersect for any values of ℓ and m

(4) Intersect for all values of ℓ and m

Sol. $\vec{r} = \hat{i}(1 + 2\ell) + \hat{j}(-1) + \hat{k}(\ell)$

$\vec{r} = \hat{i}(2 + m) + \hat{j}(m - 1) + \hat{k}(-m)$

For intersection

$1 + 2\ell = 2 + m$ ...... (i)

$-1 = m - 1$ ...... (ii)

$\ell = -m$ ...... (iii)

from (ii) m = 0

from (iii) ℓ = 0

These values of m and ℓ do not satisfy equation (1).

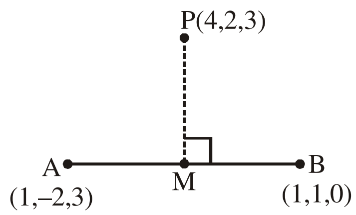
Hence the two lines do not intersect for any values of ℓ and m.

3. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane :

(1) x + 2y – z = 1 (2) x – 2y + z = 1

(3) x – y – 2z = 1 (4) 2x + y – z = 1

Sol.



Equation of AB = $\vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$

Let coordinates of M = (1, (1 + 3λ), –3λ).

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} . \overrightarrow{AB} = 0$

$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$

$\Rightarrow \lambda = -\frac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

4. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{3} = 1$

eccentricity $= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore$ foci $= (\pm 1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$$\frac{x^2}{1/2} - \frac{y^2}{b^2} = 1$$

eccentricity $= \sqrt{1 + 2b^2}$

$\therefore$ foci $= \left(\pm\sqrt{\frac{1+2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$$\Rightarrow \sqrt{\frac{1+2b^2}{2}} = 1$$

$$\Rightarrow b^2 = \frac{1}{2}$$

$\therefore$ Equation of hyperbola : $\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$$\Rightarrow x^2 - y^2 = \frac{1}{2}$$

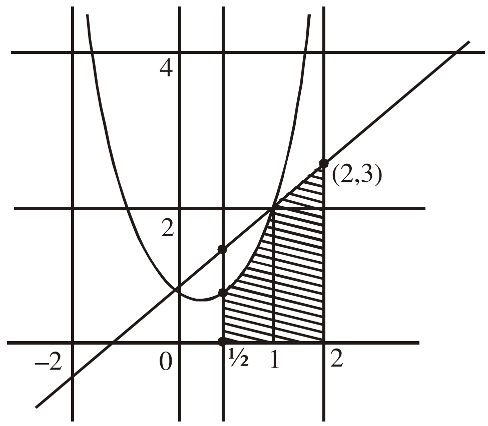
Clearly $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

5. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\}$ is :

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$

(3) $\frac{79}{24}$ (4) $\frac{23}{16}$

Sol. $0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2$



Required area $= \int_{1/2}^{1} (x^2 + 1)dx + \frac{1}{2}(2+3)\times 1$

$$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$$

6. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

(1) $\frac{1}{4}$ (2) $\frac{1}{5}$

(3) $\frac{1}{7}$ (4) $\frac{1}{6}$

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow$ $(T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

$\Rightarrow$ $(T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$\Rightarrow$ $\frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$

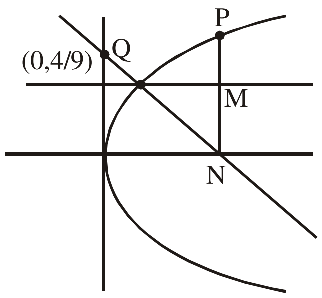
$\Rightarrow$ $d = \frac{1}{6}$

7. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

(1) $MQ = \frac{1}{3}$ (2) $PN = 3$

(3) $MQ = \frac{1}{4}$ (4) $PN = 4$

Sol.



Let $P = (3t^2, 6t)$; $N = (3t^2, 0)$

$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$\therefore \quad Q = \left(\frac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

y-intercept of NQ $= 4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$\therefore \quad MQ = \frac{9}{4}t^2 = \frac{1}{4}$

$PN = 6t = 2$

8. For the frequency distribution :

Variate (x) :	x_1	x_2	x_3 .....x_{15}
Frequency (f) :	f_1	f_2	f_3 ....f_{15}

where $0 < x_1 < x_2 < x_3 < .... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

(1) 2 (2) 1

(3) 4 (4) 6

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore \quad \sigma^2 < \frac{1}{4}(10 - 0)^2$

$\Rightarrow \quad 0 < \sigma < 5$

$\therefore \quad \sigma \ne 6.$

9. $\int_{-\pi}^{\pi} \left| \pi - |x| \right| dx$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

Sol. $\int_{-\pi}^{\pi} \left| \pi - |x| \right| dx = 2\int_{0}^{\pi} |\pi - x| dx$

$$= 2\int_{0}^{\pi} (\pi - x) dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$$

10. Consider the two sets :

A = {m ∈ R : both the roots of

$x^2 - (m + 1)x + m + 4 = 0$ are real} and

B = [–3, 5).

Which of the following is not true ?

(1) A – B = (–∞, –3) ∪ (5, ∞)

(2) A ∩ B = {–3}

(3) B – A = (–3, 5)

(4) A ∪ B = R

Sol. A : D ≥ 0

⇒ $(m + 1)^2 - 4(m + 4) \geq 0$

⇒ $m^2 + 2m + 1 - 4m - 16 \geq 0$

⇒ $m^2 - 2m - 15 \geq 0$

⇒ $(m - 5)(m + 3) \geq 0$

⇒ $m \in (-\infty, -3] \cup [5, \infty)$

∴ $A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

11. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then :

(1) $|y''(0)| = 2$ (2) $|y'(0)| + |y''(0)| = 3$

(3) $|y'(0)| + |y''(0)| = 1$ (4) $y''(0) = 0$

Sol. $y^2 + \ln(\cos^2 x) = y$ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for x = 0 y = 0 or 1

Differentiating wrt x

⇒ $2yy' - 2\tan x = y'$

At (0, 0) y' = 0

At (0, 1) y' = 0

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At (0, 0) y'' = –2

At (0, 1) y'' = 2

∴ $|y''(0)| = 2$

12. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in R$, is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$

(4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$$\Rightarrow f'(x) = 5x^{2/3} - \frac{14}{3x^{1/3}}$$

$$= \frac{15x-14}{3x^{1/3}} > 0$$

+   0   −   14/15   +

$$\therefore \quad f'(x) > 0 \ \forall \ x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

13. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - ....$ up to 51^{th} term) + $(1! - 2! + 3! - .....$ up to 51^{th} term) is equal to :

(1) $1 + (51)!$ (2) $1 - 51(51)!$

(3) $1 + (52)!$ (4) 1

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2$ .... upto 51 terms)

$+ (1! + 2! + 3!$ .... upto 51 terms)

$[\because {}^np_{n-1} = n!]$

$\therefore \quad S = (2 \times 1! - 3 \times 2! + 4 \times 3! .... + 52.51!)$

$+ (1! - 2! + 3! .... (51)!)$

$= (2! - 3! + 4! .... + 52!)$

$+ (1! - 2! + 3! - 4! + ... + (51)!)$

$= 1! + 52!.$

14. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to :

(1) –1   (2) 1

(3) –3   (4) 9

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$

$= Ax^3 + Bx^2 + Cx + D.$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$$

$$= (x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore \quad B + C = 12 - 15 = -3$

15. The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$, which passes through the point (0, 1), is :

(1) $y^2 = 1 + y \log_e\left(\frac{1+e^x}{2}\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^x}{2}\right) + 2\right)$

(3) $y^2 = 1 + y \log_e\left(\frac{1+e^{-x}}{2}\right)$

(4) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$

Sol. $(1 + e^{-x})(1 + y^2)\dfrac{dy}{dx} = y^2$

$\Rightarrow (1 + y^{-2})\,dy = \left(\dfrac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \dfrac{1}{y}\right) = \ell n(1+e^x) + c$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\,\ell n\left(\dfrac{1+e^x}{2}\right)$

16. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

(1) 264 (2) 256

(3) 128 (4) 248

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}}$ $(n \geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ... , 32 × 8

$\therefore$ least value of n = 256.

17. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \dfrac{1}{\alpha}\right)\left(\beta - \dfrac{1}{\beta}\right)\left(\alpha + \dfrac{1}{\beta}\right)\left(\beta + \dfrac{1}{\alpha}\right)$ is equal to:

(1) $\dfrac{9}{4}(9 + p^2)$ (2) $\dfrac{9}{4}(9 - q^2)$

(3) $\dfrac{9}{4}(9 - p^2)$ (4) $\dfrac{9}{4}(9 + q^2)$

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ & $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p$ $\quad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

18. Let [t] denote the greatest integer $\leq t$. If for some $\lambda \in R - \{0, 1\}$, $\lim\limits_{x \to 0}\left|\dfrac{1 - x + |x|}{\lambda - x + [x]}\right| = L$, then L is equal to :

(1) 1 (2) 2

(3) $\dfrac{1}{2}$ (4) 0

Sol. LHL : $\lim\limits_{x \to 0^-}\left|\dfrac{1 - x - x}{\lambda - x - 1}\right| = \left|\dfrac{1}{\lambda - 1}\right|$

RHL : $\lim\limits_{x \to 0^+}\left|\dfrac{1 - x + x}{\lambda - x + 1}\right| = \left|\dfrac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$$\Rightarrow \frac{1}{|\lambda-1|}=\frac{1}{|\lambda|}\Rightarrow \lambda=\frac{1}{2}$$

$$\therefore \quad L=\frac{1}{|\lambda|}=2$$

19. $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to:

(1) $\frac{7\pi}{4}$ (2) $\frac{5\pi}{4}$

(3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

Sol. $2\pi-\left(\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)\right)$

$$=2\pi-\left(\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$=2\pi-\left(\tan^{-1}\left(\frac{63}{16}\right)+\tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$$

20. The proposition $p \to \sim(p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$ (2) q

(3) $(\sim p) \wedge q$ (4) $(\sim p) \vee (\sim q)$

Sol. $p \to \sim(p\wedge \sim q)$

$$=\sim p \vee \sim(p\wedge \sim q)$$

$$=\sim p \vee \sim p \vee q$$

$$=\sim(p\wedge q)\vee q$$

$$=\sim p \vee q$$

21. Let $A=\begin{bmatrix} x & 1\\ 1 & 0\end{bmatrix}$, $x \in R$ and $A^4=[a_{ij}]$. If $a_{11}=109$, then a_{22} is equal to _______ .

Sol. $A=\begin{bmatrix} x & 1\\ 1 & 0\end{bmatrix}$

$$A^2=\begin{bmatrix} x & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix} x & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix} x^2+1 & x\\ x & 1\end{bmatrix}$$

$$A^4=\begin{bmatrix} x^2+1 & x\\ x & 1\end{bmatrix}\begin{bmatrix} x^2+1 & x\\ x & 1\end{bmatrix}$$

$$=\begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x\\ x(x^2+1)+x & x^2+1\end{bmatrix}$$

$a_{11}=(x^2+1)^2+x^2=109$

$\Rightarrow x=\pm 3$

$a_{22}=x^2+1=10$

22. If $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is _______ .

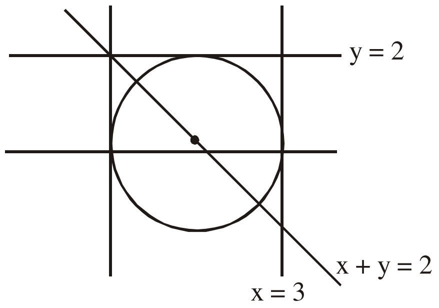
Sol. $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$

$$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2}=\frac{1}{8}\times\frac{1}{32}=2^{-k}$$

$\Rightarrow 2^{-8}=2^{-k} \Rightarrow k=8.$

23. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is _______ .

Sol.



$\because$ center lies on x + y = 2 and in 1st quadrant

center = $(\alpha, 2 - \alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches x = 3 and y = 2

$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)| =$ radius

$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \frac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = $2\alpha = 3$.

24. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\text{to }\infty\right)}$ is equal to _______ .

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots\ldots\text{to }\infty\right)}$

$$=\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$$

$$=\left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)}=\left(\frac{1}{2}\right)^{-2}=4$$

25. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{i-1}\right)^{\frac{n}{3}}=1$, (m, n $\in$ N) then the greatest common divisor of the least values of m and n is _______ .

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2}=\left(\frac{1+i}{i-1}\right)^{n/3}=1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2}=\left(\frac{(1+i)^2}{-2}\right)^{n/3}=1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of m = 8 and n = 12.

$\therefore$ GCD = 4