

FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS

TEST PAPER WITH ANSWER & SOLUTIONS

SECTION-A

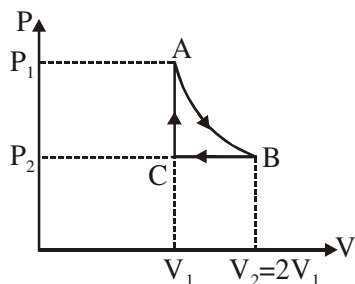
1. n mole a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

A \rightarrow B : Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2 .

B \rightarrow C : Isobaric compression at pressure P_2 to initial volume V_1 .

C \rightarrow A : Isochoric change leading to change of pressure from P_2 to P_1 .

Total workdone in the complete cycle ABCA is :



- (1) 0 (2) $nRT \left(\ln 2 + \frac{1}{2} \right)$
 (3) $nRT \ln 2$ (4) $nRT \left(\ln 2 - \frac{1}{2} \right)$

2. The focal length f is related to the radius of curvature r of the spherical convex mirror by:

- (1) $f = +\frac{1}{2}r$ (2) $f = -r$
 (3) $f = -\frac{1}{2}r$ (4) $f = r$

3. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

- (1) 1 : 4 (2) 3 : 1 (3) 4 : 1 (4) 2 : 1

4. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is :

(1) $\frac{1}{2\pi} \sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi \sqrt{\frac{d^3}{3Gm}}$
 (3) $\frac{1}{2\pi} \sqrt{\frac{3Gm}{d^3}}$ (4) $2\pi \sqrt{\frac{3Gm}{d^3}}$

5. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$ where $\alpha_0 = 20$ A/s and $\beta = 8$ As⁻². Find the charge crossed through a section of the wire in 15 s.

(1) 2250 C (2) 11250 C
 (3) 2100 C (4) 260 C

6. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as ;
 I_1 = M.I. of thin circular ring about its diameter.
 I_2 = M.I. of circular disc about an axis perpendicular to the disc and going through the centre,
 I_3 = M.I. of solid cylinder about its axis and
 I_4 = M.I. of solid sphere about its diameter.
 Then :

(1) $I_1 + I_3 < I_2 + I_4$
 (2) $I_1 + I_2 = I_3 + \frac{5}{2} I_4$
 (3) $I_1 = I_2 = I_3 > I_4$
 (4) $I_1 = I_2 = I_3 < I_4$

7. Given below are two statements :

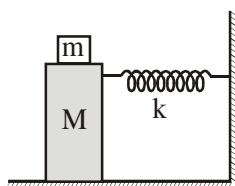
Statement-I : Two photons having equal linear momenta have equal wavelengths.

Statement-II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both Statement I and Statement II are true
 (2) Statement I is false but Statement II is true
 (3) Both Statement I and Statement II are false
 (4) Statement I is true but Statement II is false

8. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k . The mass oscillates on a frictionless surface with time period T and amplitude A . When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :



- (1) $A\sqrt{\frac{M-m}{M}}$ (2) $A\sqrt{\frac{M}{M+m}}$
 (3) $A\sqrt{\frac{M+m}{M}}$ (4) $A\sqrt{\frac{M}{M-m}}$

9. If Y , K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

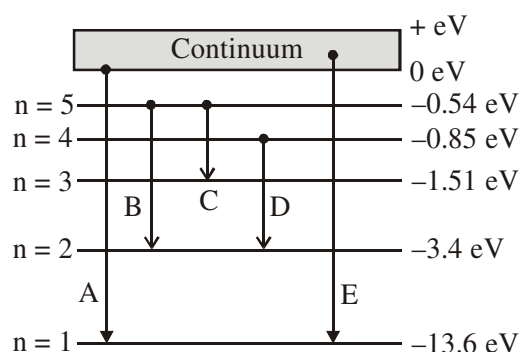
(1) $Y = \frac{9K\eta}{3K - \eta} \text{ N/m}^2$

(2) $\eta = \frac{3YK}{9K + Y} \text{ N/m}^2$

(3) $Y = \frac{9K\eta}{2\eta + 3K} \text{ N/m}^2$

(4) $K = \frac{Y\eta}{9\eta - 3Y} \text{ N/m}^2$

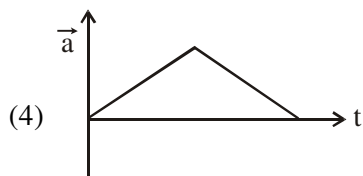
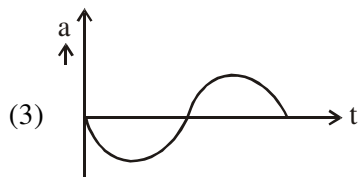
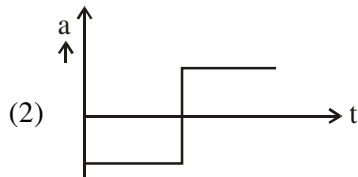
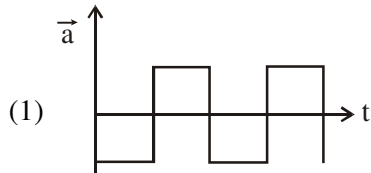
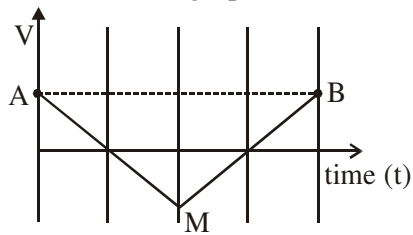
10. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent :



- (1) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.
 - (2) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.
 - (3) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.
 - (4) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.
11. Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

- (1) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$
- (2) $\sqrt{G(1+2\sqrt{2})}$
- (3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$
- (4) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$

12. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



13. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be:

- (1) 4 : 1 (2) 2 : 1
(3) 1 : 4 (4) 1 : 2

14. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :

- (1) 7 (2) 0.5
(3) 0.875 (4) 3.5

15. Match List-I with List-II :

List-I

List-II

- | | |
|----------------|-------------------------------|
| (a) Isothermal | (i) Pressure constant |
| (b) Isochoric | (ii) Temperature constant |
| (c) Adiabatic | (iii) Volume constant |
| (d) Isobaric | (iv) Heat content is constant |

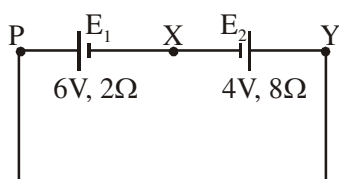
Choose the correct answer from the options given below :

- (1) (a) \rightarrow (i), (b) \rightarrow (iii), (c) \rightarrow (ii), (d) \rightarrow (iv)
 (2) (a) \rightarrow (ii), (b) \rightarrow (iii), (c) \rightarrow (iv), (d) \rightarrow (i)
 (3) (a) \rightarrow (ii), (b) \rightarrow (iv), (c) \rightarrow (iii), (d) \rightarrow (i)
 (4) (a) \rightarrow (iii), (b) \rightarrow (ii), (c) \rightarrow (i), (d) \rightarrow (iv)

16. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is ' α '. The metal sheet is heated uniformly, by a small temperature ΔT , so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

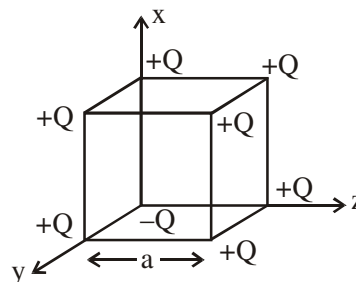
- (1) $3a^3\alpha\Delta T$ (2) $4a^3\alpha\Delta T$
 (3) $4\pi a^3\alpha\Delta T$ (4) $\frac{4}{3}\pi a^3\alpha\Delta T$

17. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is :



- (1) 10.0 V (2) 3.6 V
 (3) 5.6V (4) 2.0 V

18. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is -Q. The electric field at the centre of cube is :



- (1) $\frac{-Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (2) $\frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (3) $\frac{2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (4) $\frac{Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

19. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is :

- (1) 8 : 1 2) 1 : 4
(3) 2 : 1 (4) 1 : 8

20. The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants. Then the dimension of β will be :

- (1) $[M L^2 T^{-2}]$ (2) $[M L T^{-2}]$
(3) $[M^2 L T^2]$ (4) $[M^0 L T^0]$

SECTION-B

1. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be ____N. $[g = 10 \text{ ms}^{-2}]$

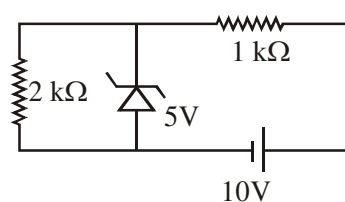
2. A resonance circuit having inductance and resistance $2 \times 10^{-4} \text{ H}$ and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _____. $[\pi = 3.14]$

3. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift ____kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

4. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is _____ cm.

5. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is _____ $\times 10^7$ m/s.

6. In connection with the circuit drawn below, the value of current flowing through $2\text{ k}\Omega$ resistor is _____ $\times 10^{-4}$ A.



7. An audio signal $v_m = 20 \sin 2\pi (1500 t)$ amplitude modulates a carrier $v_C = 80 \sin 2\pi (100,000 t)$. The value of percent modulation is _____.
8. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is _____.
9. A common transistor radio set requires 12V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _____.
10. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be _____ Lumens.

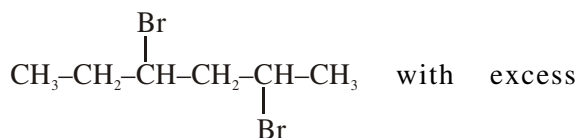
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CHEMISTRY

SECTION-A

1. The product formed in the first step of the reaction of



Mg/Et₂O(Et = C₂H₅) is :

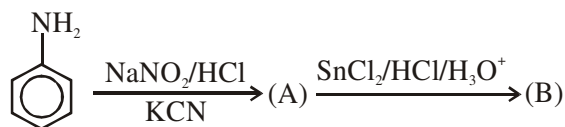
- (1) $\text{CH}_3-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}-\text{CH}_3$
 $\quad \quad \quad \text{CH}_3-\text{CH}-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}_3$
- (2) $\text{CH}_3-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}-\text{CH}_3$
 $\quad \quad \quad \text{CH}_3-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}-\text{CH}_3$
- (3) $\text{CH}_3-\text{CH} \begin{cases} \text{CH}_2 \\ | \\ \text{CH}-\text{CH}_3 \end{cases}$
- (4) $\text{CH}_3\text{CH}_2-\overset{\text{MgBr}}{\text{CH}}-\text{CH}_2-\overset{\text{MgBr}}{\text{CH}}-\text{CH}_3$

2. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is :

- (1) Mg < Al < Si < S < P
 (2) Al < Mg < Si < S < P
 (3) Mg < Al < Si < P < S
 (4) Al < Mg < S < Si < P

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3. 'A' and 'B' in the following reactions are :



- (1) (A) : $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^-$ (B) : $\text{C}_6\text{H}_5\text{CHO}$
- (2) (A) : $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^-$ (B) : $\text{C}_6\text{H}_5\text{Cl}$
- (3) (A) : $\text{C}_6\text{H}_5\text{CN}$ (B) : $\text{C}_6\text{H}_5\text{CHO}$
- (4) (A) : $\text{C}_6\text{H}_5\text{CN}$ (B) : $\text{C}_6\text{H}_5\text{Cl}$

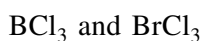
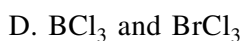
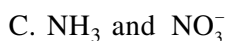
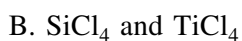
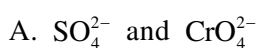
4. Which of the following ore is concentrated using group 1 cyanide salt ?

- (1) Sphalerite (2) Calamine
 (3) Siderite (4) Malachite

5. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

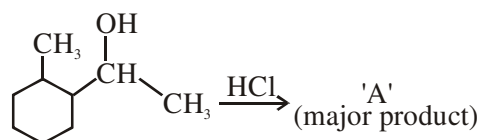
- (1) $\text{X} = \text{Na}[\text{Al}(\text{OH})_4]$, $\text{Y} = \text{SO}_2$, $\text{Z} = \text{Al}_2\text{O}_3$
- (2) $\text{X} = \text{Na}[\text{Al}(\text{OH})_4]$, $\text{Y} = \text{CO}_2$, $\text{Z} = \text{Al}_2\text{O}_3 \cdot x\text{H}_2\text{O}$
- (3) $\text{X} = \text{Al}(\text{OH})_3$, $\text{Y} = \text{CO}_2$, $\text{Z} = \text{Al}_2\text{O}_3$
- (4) $\text{X} = \text{Al}(\text{OH})_3$, $\text{Y} = \text{SO}_2$, $\text{Z} = \text{Al}_2\text{O}_3 \cdot x\text{H}_2\text{O}$

6. Which of the following are isostructural pairs ?



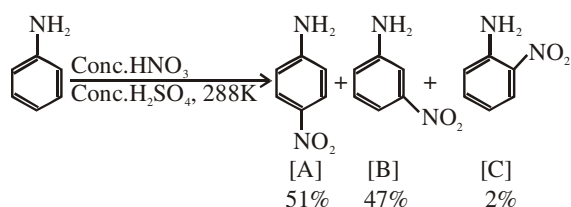
- (1) C and D only
- (2) A and B only
- (3) A and C only
- (4) B and C only

7. What is the final product (major) 'A' in the given reaction ?

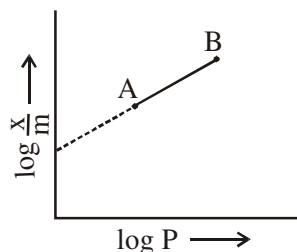


- (1)
- (2)
- (3)
- (4)

8. In the following reaction the reason why meta-nitro product also formed is :

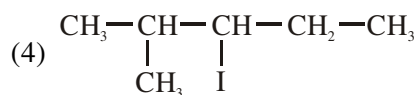
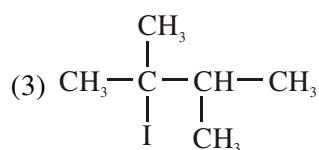
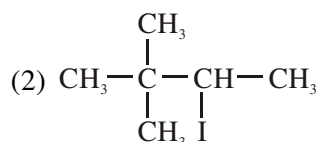
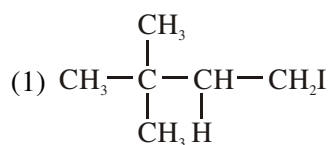
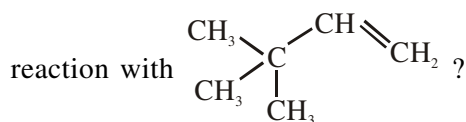


- (1) low temperature
 (2) $-\text{NH}_2$ group is highly meta-directive
 (3) Formation of anilinium ion
 (4) $-\text{NO}_2$ substitution always takes place at meta-position
9. In Freundlich adsorption isotherm, slope of AB line is :
10. (A) $\text{HOCl} + \text{H}_2\text{O}_2 \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^- + \text{O}_2$
 (B) $\text{I}_2 + \text{H}_2\text{O}_2 + 2\text{OH}^- \rightarrow 2\text{I}^- + 2\text{H}_2\text{O} + \text{O}_2$
 Choose the correct option.
- (1) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B)
 (2) H_2O_2 acts as oxidising agent in equation (A) and (B)
 (3) H_2O_2 acts as reducing agent in equation (A) and (B)
 (4) H_2O_2 act as oxidizing and reducing agent respectively in equation (A) and (B)

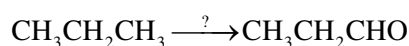


- (1) $\log n$ with ($n > 1$)
 (2) n with ($n, 0.1$ to 0.5)
 (3) $\log \frac{1}{n}$ with ($n < 1$)
 (4) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

11. What is the major product formed by HI on



12. Which of the following reagent is used for the following reaction ?



- (1) Manganese acetate
- (2) Copper at high temperature and pressure
- (3) Molybdenum oxide
- (4) Potassium permanganate

13. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Statement I is true but Statement II is false
- (2) Both Statement I and Statement II are false
- (3) Statement I is false but Statement II is true
- (4) Both Statement I and Statement II are true

14. Out of the following, which type of interaction is responsible for the stabilisation of α -helix structure of proteins ?

- (1) Ionic bonding
- (2) Hydrogen bonding
- (3) Covalent bonding
- (4) vander Waals forces

15. Match List I with List II.

List I (Monomer Unit)	List II (Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1,3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

- (1) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)
 (2) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)
 (3) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)
 (4) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

16. The gas released during anaerobic degradation of vegetation may lead to :

- (1) Ozone hole
 (2) Acid rain
 (3) Corrosion of metals
 (4) Global warming and cancer

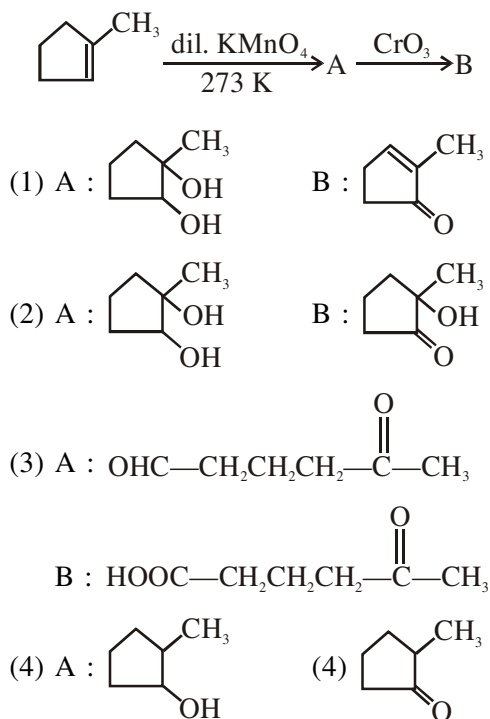
17. The major components in "Gun Metal" are :

- (1) Cu, Zn and Ni (2) Cu, Sn and Zn
 (3) Al, Cu, Mg and Mn (4) Cu, Ni and Fe

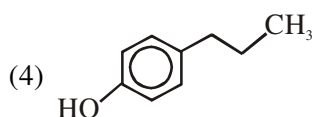
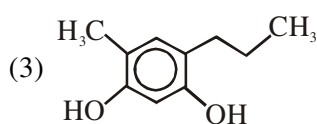
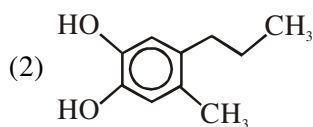
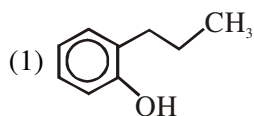
18. The electrode potential of M^{2+} / M of 3d-series elements shows positive value of :

- (1) Zn (2) Fe (3) Co (4) Cu

19. Identify products A and B :



20. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ?



SECTION-B

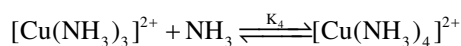
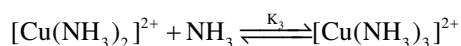
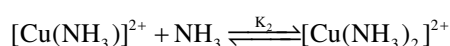
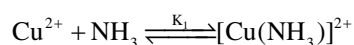
1. When 9.45 g of ClCH_2COOH is added to 500 mL of water, its freezing point drops by 0.5°C . The dissociation constant of ClCH_2COOH is $x \times 10^{-3}$. The value of x is _____. (Rounded off to the nearest integer)

$$[K_{\text{f}(\text{H}_2\text{O})} = 1.86 \text{ K kg mol}^{-1}]$$

2. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer)

3. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value K_p for the reaction $\text{Cl}_{2(g)} \rightleftharpoons 2\text{Cl}_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is _____. (Rounded of to the nearest integer)
4. Number of amphoteric compound among the following is _____
- (A) BeO (B) BaO
(C) Be(OH)₂ (D) Sr(OH)₂
5. The reaction of sulphur in alkaline medium is the below:
- $$\text{S}_{8(s)} + a \text{OH}^-_{(aq)} \rightarrow b \text{S}^{2-}_{(aq)} + c \text{S}_2\text{O}_3^{2-}_{(aq)} + d \text{H}_2\text{O}_{(l)}$$
- The values of 'a' is _____. (Integer answer)
6. For the reaction $\text{A}_{(g)} \rightarrow (\text{B})_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is _____. (Rounded of to the nearest integer) ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ and $\ln 10 = 2.3$)
7. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{\text{Li}}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is _____. (Rounded off to the nearest integer) (Mass of $\text{Li}^{3+} = 8.3$ mass of proton)

8. The stepwise formation of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is given below



The value of stability constants K_1 , K_2 , K_3 and K_4 are 10^4 , 1.58×10^3 , 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is _____. (Rounded off to the nearest integer)

9. The coordination number of an atom in a body-centered cubic structure is _____.

[Assume that the lattice is made up of atoms.]

10. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4} \text{s}^{-1}$ at 153°C . The time in minutes it takes for the isomerization to proceed 40 % to completion at this temperature is _____. (Rounded off to the nearest integer)

FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The statement among the following that is a tautology is :

- (1) $A \vee (A \wedge B)$
- (2) $A \wedge (A \vee B)$
- (3) $B \rightarrow [A \wedge (A \rightarrow B)]$
- (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

2. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes

is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1,1), (2, 2) and (4, 4) respectively. Then which of these stones is / are on the path of the man ?

- (1) A only
- (2) C only
- (3) All the three
- (4) B only

3. The equation of the plane passing through the point (1, 2, -3) and perpendicular to the planes

$3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

- (1) $3x - 10y - 2z + 11 = 0$
- (2) $6x - 5y - 2z - 2 = 0$
- (3) $11x + y + 17z + 38 = 0$
- (4) $6x - 5y + 2z + 10 = 0$

4. The population $P = P(t)$ at time 't' of a certain species follows the differential equation

$\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time

at which population becomes zero is :

- (1) $\log_e 18$
- (2) $\log_e 9$
- (3) $\frac{1}{2} \log_e 18$
- (4) $2 \log_e 18$

5. The system of linear equations

$$3x - 2y - kz = 10$$

$$2x - 4y - 2z = 6$$

$$x + 2y - z = 5m$$

is inconsistent if :

- (1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in \mathbb{R}$
(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

6. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[.]$ denotes the greatest integer function, then f is :

- (1) discontinuous at all integral values of x except at $x = 1$
(2) continuous only at $x = 1$
(3) continuous for every real x
(4) discontinuous only at $x = 1$

7. The distance of the point $(1, 1, 9)$ from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is :

- (1) $2\sqrt{19}$ (2) $19\sqrt{2}$
(3) 38 (4) $\sqrt{38}$

8. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q , then the ordinate of the point which divides PQ internally in the ratio $1 : 2$ is :

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

9. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :
- (1) $(-1, 3)$ (2) $(3, 1)$
 (3) $(1, 3)$ (4) $(1, -3)$

10. The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \dots - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \dots + {}^{14}C_{11}$ is :
- (1) $2^{16} - 1$ (2) $2^{13} - 14$
 (3) 2^{14} (4) $2^{13} - 13$

11. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x :$$

- (1) increases in $\left[\frac{1}{2}, \infty \right)$
 (2) increases in $\left(-\infty, \frac{1}{2} \right]$
 (3) decreases in $\left[\frac{1}{2}, \infty \right)$
 (4) decreases in $\left(-\infty, \frac{1}{2} \right]$

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 1$ and
 $g : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

- (1) onto but not one-one
 (2) both one-one and onto
 (3) one-one but not onto
 (4) neither one-one nor onto

13. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

- (1) $\frac{1}{32}$ (2) $\frac{5}{16}$ (3) $\frac{3}{16}$ (4) $\frac{1}{2}$

14. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

- (1) 1625 (2) 575
 (3) 560 (4) 1050

15. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

- (1) $24\pi + 3\sqrt{3}$ (2) $12\pi - 3\sqrt{3}$
 (3) $24\pi - 3\sqrt{3}$ (4) $12\pi + 3\sqrt{3}$

16. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

- (1) $x^2 - 2x + 2 = 0$
 (2) $x^2 - 2x + 8 = 0$
 (3) $x^2 - 2x + 136 = 0$
 (4) $x^2 - 2x + 16 = 0$

17. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

- (1) $20\sqrt{3}$ (2) $25\sqrt{3}$
(3) 30 (4) 25

18. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} (\sin \sqrt{t}) dt}{x^3}$ is equal to :

- (1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 0 (4) $\frac{1}{15}$

19. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \dots \infty) \log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of

$$\frac{2 \sin x}{\sin x + \sqrt{3} \cos x} \left(0 < x < \frac{\pi}{2} \right) \text{ is}$$

- (1) $2\sqrt{3}$ (2) $\frac{3}{2}$
(3) $\sqrt{3}$ (4) $\frac{1}{2}$

20. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

- (1) $x = -\frac{a}{2}$ (2) $x = \frac{a}{2}$
(3) $x = 0$ (4) $x = a$

SECTION-B

1. If the least and the largest real values of α , for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in \mathbb{C}$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to _____

2. If $\int_{-a}^a (|x| + |x - 2|) dx = 22$, ($a > 2$) and $[x]$ denotes the greatest integer $\leq x$, then $\int_a^{-a} (x + [x]) dx$ is equal to _____.

3. Let $A = \{n \in \mathbb{N} : n \text{ is a 3-digit number}\}$
 $B = \{9k + 2 : k \in \mathbb{N}\}$
 and $C = \{9k + l : k \in \mathbb{N}\}$ for some l ($0 < l < 9$)
 If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400 , then l is equal to _____.

4. Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is _____.

5. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _____.
6. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____.
7. $\lim_{n \rightarrow \infty} \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{1+r+r^2} \right) \right\}$ is equal to _____.
8. Let three vectors \vec{a} , \vec{b} and \vec{c} be such that \vec{c} is coplanar with \vec{a} and \vec{b} , $\vec{a} \cdot \vec{c} = 7$ and \vec{b} is perpendicular to \vec{c} , where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is _____.

9. Let B_i ($i = 1, 2, 3$) be three independent events in a sample space. The probability that only B_1 occur is α , only B_2 occurs is β and only B_3 occurs is γ . Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval $(0,1)$). Then

$\frac{P(B_1)}{P(B_3)}$ is equal to _____.

10. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose

$Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some

non-zero $k \in \mathbb{R}$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$,

then $\alpha^2 + k^2$ is equal to _____.