SOLUTION TO AIEEE-2005

PHYSICS

1. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flights in the two cases, then the product of the two time of flights is proportional to

(1) R^2 (2) $1/R^2$
(3) $1/R$ (4) R

1. **(4)**

$$t_1t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2R}{g}$$

2. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1/F_2 is

(1) $\frac{R_2}{R_1}$ (2) $\left(\frac{R_1}{R_2}\right)^2$
(3) 1 (4) $\frac{R_1}{R_2}$

2. **(4)**

$$\frac{F_1}{F_2} = \frac{R_1\omega^2}{R_2\omega^2} = \frac{R_1}{R_2}$$

3. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(1) $\mu_k = 1 - \frac{1}{n^2}$ (2) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$
(3) $\mu_s = 1 - \frac{1}{n^2}$ (4) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

3. **(1)**

$$d = \frac{1}{2}\frac{g}{\sqrt{2}}t_1^2$$

$$d = \frac{1}{2}\frac{g}{\sqrt{2}}\left(1 - \mu_k\right)t_2^2$$

$$\frac{t_2^2}{t_1^2} = n^2 = \frac{1}{1 - \mu_k}$$

4. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

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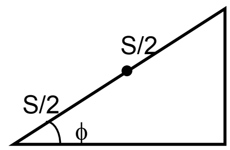
(1) $2\sin\phi$ (2) $2\cos\phi$
(3) $2\tan\phi$ (4) $\tan\phi$

4. **(3)**



mg s sin ϕ **=** $\mu mg\cos\phi.\frac{s}{2}$

5. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?
(1) 3.0 cm (2) 2.0 cm
(3) 1.5 cm (4) 1.0 cm

5. **(4)**
$F.3 = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{4}$
$F(3+x) = \frac{1}{2}mv^2$
x = 1 cm

6. Out of the following pair, which one does NOT have identical dimensions is
(1) angular momentum and Planck's constant
(2) impulse and momentum
(3) moment of inertia and moment of a force
(4) work and torque

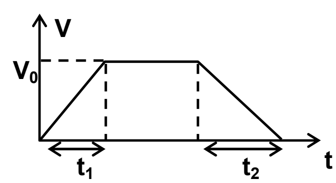
6. **(3)**
Using dimension

7. The relation between time t and distance x is $t=ax^2+bx$ where a and b are constants. The acceleration is
(1) $-2abv^2$ (2) $2bv^3$
(3) $-2av^3$ (4) $2av^2$

7. **(3)**
$t = ax^2 + bx$
by differentiating acceleration = $-2av^3$

8. A car starting from rest accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate f/2 to come to rest. If the total distance traversed is 15 S, then
(1) S=ft (2) S = 1/6 ft^2
(3) S = 1/2 ft^2 (4) S = 1/4 ft^2

8. **(none)**



$S = \frac{ft_1^2}{2}$
$v_0 = \sqrt{2Sf}$
During retardation
$S_2 = 2S$

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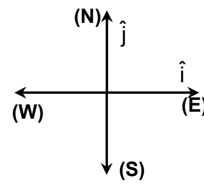
During constant velocity
15S -3S = 12S = $v_o t$

$$\Rightarrow S = \frac{ft^2}{72}$$

9. A particle is moving eastwards with a velocity of 5 m/s in 10 seconds the velocity changes to 5 m/s northwards. The average acceleration in this time is

(1) $\frac{1}{\sqrt{2}}\,m/s^2$ towards north-east
(2) $\frac{1}{2}\,m/s^2$ towards north.
(3) zero
(4) $\frac{1}{\sqrt{2}}\,m/s^2$ towards north-west

9. (4)



$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{t}$$

$$= \frac{5\hat{j} - 5\hat{i}}{10} = \frac{1}{2}\left(\hat{j} - \hat{i}\right)$$

$$\therefore a = \frac{1}{\sqrt{2}} ms^{-2} \text{ towards north west}$$

10. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

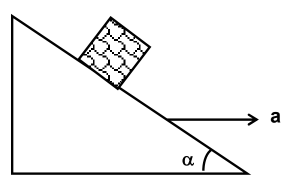
(1) 91 m
(2) 182 m
(3) 293 m
(4) 111 m

10. (3)

$$s = 50 + \left(\frac{3^2 - (2\times 10\times 50)}{2(-2)}\right)$$

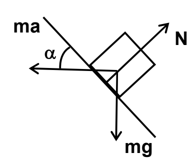
= 293 m.

11. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(1) g/tanα
(2) g cosecα
(3) g
(4) g tanα

11. (4)



mg sinα = ma cos α
$\therefore a = g \tan\alpha$

12. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

(1) 40 m/s
(2) 20 m/s
(3) 10 m/s
(4) $10\sqrt{30}$ m/s

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12. **(1)**

mgh = ½ mv^2

$v = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 80} = 40\ \text{m/s}$

13. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass 1/3 M and a body C of mass 2/3 M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(1) depends on height of breaking
(2) does not shift
(3) body C
(4) body B

13. **(2)**

No horizontal external force is acting

$\therefore a_{cm} = 0$

since $v_{cm} = 0$

$\therefore \Delta x_{cm} = 0$

14. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is

(1) $\frac{1}{4}Mr^2$
(2) $\frac{2}{5}Mr^2$
(3) Mr^2
(4) $\frac{1}{2}Mr^2$

14. **(4)**

$2I = 2M\frac{R^2}{2}$

$\therefore I = \frac{MR^2}{2}$

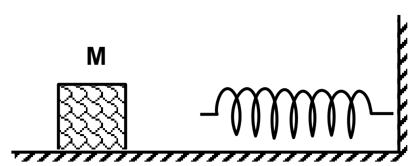
15. A particle of mass 0.3 kg is subjected to a force F=–kx with k=15 N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

(1) 3 m/s^2
(2) 15 m/s^2
(3) 5 m/s^2
(4) 10 m/s^2

15. **(4)**

$a = \frac{kx}{m} = 10\,m/s^2$

16. The block of mass M moving on the frictionless horizontal surface collides with a spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is



(1) $\sqrt{MK}\,L$
(2) $\frac{KL^2}{2M}$
(3) zero
(4) $\frac{ML^2}{K}$

16. **(1)**

$\frac{1}{2}KL^2 = \frac{P^2}{2m} \quad \therefore P = \sqrt{MK}\ L$

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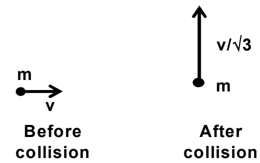
17. A mass 'm' moves with a velocity v and collides inelastically with another identical mass. After collision the 1st mass moves with velocity v/√3 in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



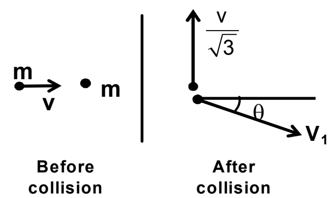
(1) v (2) √3 v
(3) 2v/√3 (4) v/√3

17. (3)

$mv = mv_1 \cos\theta$

$0 = \dfrac{mv}{\sqrt{3}} - mv_1 \sin\theta$

$\therefore v_1 = \dfrac{2}{\sqrt{3}} v$



18. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be

(1) 8 cm (2) 10 cm
(3) 4 cm (4) 20 cm

18. (4)

Water will rise to the full length of capillary tube

19. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(1) $2S^2Y$ (2) $S^2/2Y$
(3) $2Y/S^2$ (4) $S/2Y$

19. (2)

$U = \dfrac{1}{2}\text{stress} \times \text{strain} = \dfrac{S^2}{2Y}$

20. Average density of the earth

(1) does not depend on g (2) is a complex function of g
(3) is directly proportional to g (4) is inversely proportional to g

20. (3)

$g = \dfrac{G4\pi}{3}\rho_{av}R$

21. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function time is given by

(1) $\dfrac{mv^2}{T^2}\cdot t$ (2) $\dfrac{mv^2}{T^2}\cdot t^2$
(3) $\dfrac{1}{2}\dfrac{mv^2}{T^2}\cdot t$ (4) $\dfrac{1}{2}\dfrac{mv^2}{T^2}\cdot t^2$

21. (1)

P= (ma).v

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$= m\, a^2\, t$

$= m\, \frac{V^2}{T^2} t$

22. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is $[\mu_k = 0.5]$

(1) 800 m (2) 1000 m
(3) 100 m (4) 400 m

22. (2)

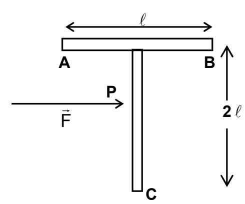
$\mu_k mgs = \frac{1}{2} mu^2$

$s = \frac{u^2}{2\mu_k g} = 1000m$

23. Which of the following is incorrect regarding the first law of thermodynamics?
(1) It is not applicable to any cyclic process
(2) It is a restatement of the principle of conservation of energy
(3) It introduces the concept of the internal energy
(4) It introduces the concept of the entropy

23. (none)
More than one statements are incorrect

24. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force F is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C



(1) $\frac{2}{3}\ell$ (2) $\frac{3}{2}\ell$
(3) $\frac{4}{3}\ell$ (4) ℓ

24. (3)
P will be the centre of mass of system

25. The change in the value of g at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?

(1) $d = \frac{h}{2}$ (2) $d = \frac{3h}{2}$
(3) $d = 2h$ (4) $d = h$

25. (3)

$\frac{GM}{(R+h)^2} = \frac{GM}{R^3}(R-d)$

$\Rightarrow d = 2h$

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26. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere
(you may take G = 6 . 67$\times 10^{-11}$ Nm^2 / kg^2)
(1) 13.34×10^{-10} J
(2) 3.33×10^{-10} J
(3) 6.67×10^{-9} J
(4) 6.67×10^{-10} J

26. (4)
w = GMm / R = 6.67×10^{-10} J

27. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_P}{C_v}$ of the mixture is
(1) 1.59
(2) 1.62
(3) 1.4
(4) 1.54

27. (2)
$$c_v = \frac{n_1c_{v1} + n_2c_{v2}}{n_1 + n_2} = \frac{29R}{18}$$
$$c_P = \frac{47R}{18}, \qquad \frac{c_P}{c_v} = 1.62$$

28. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $\frac{I}{8}$. The thickness of lead which will reduce the intensity to $\frac{I}{2}$ will be
(1) 6 mm
(2) 9 mm
(3) 18 mm
(4) 12 mm

28. (4)
Use $I = I_0 e^{-\mu x}$

29. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is
(1) 1.1 eV
(2) 2.5 eV
(3) 0.5 eV
(4) 0.7 eV

29. (3)
$$E_g = \frac{hc}{\lambda} = 0.5\,\text{eV}$$

30. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed $\frac{1}{2}$ m away, the number of electrons emitted by photo cathode would
(1) decrease by a factor of 4
(2) increase by a factor of 4
(3) decrease by a factor of 2
(4) increase by a factor of 2

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30. (2)

$I \propto \frac{1}{r^2}$

31 Starting with a sample of pure ^{66}Cu, 7/8 of it decays into Zn in 15 minutes. The corresponding half-life is

(1) 10 minutes (2) 15 minutes

(3) 5 minutes (4) $7\frac{1}{2}$ minutes

31. (3)

$$N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8}$$

$3t_{1/2} = 15 \;\therefore t_{1/2} = 5$

32. If radius of $^{27}_{13}Al$ nucleus is estimated to be 3.6 Fermi then the radius $^{125}_{52}Te$ nucleus be nearly

(1) 6 fermi (2) 8 fermi

(3) 4 fermi (4) 5 fermi

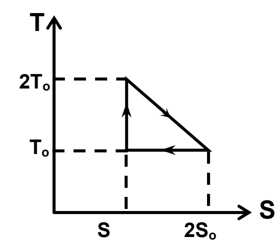
32. (1)

$$\frac{R}{3.6} = \left(\frac{125}{27}\right)^{\frac{1}{3}} \Rightarrow R = 6 \text{ fermi}$$

33. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is

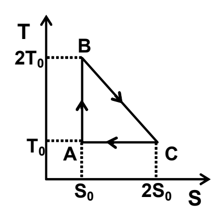
(1) 1/2 (2) 1/4

(3) 1/3 (4) 2/3

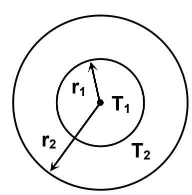


33. (3)

$$\eta = \frac{\Delta W}{Q_{BC}} = \frac{\frac{S_0T_0}{2}}{\frac{3S_0T_0}{2}} = 1/3$$



34. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric sphere is proportional to



(1) $\frac{r_2 - r_1}{r_1r_2}$ (2) $\ln\left(\frac{r_2}{r_1}\right)$

(3) $\frac{r_1r_2}{r_2 - r_1}$ (4) $\ln(r_2 - r_1)$

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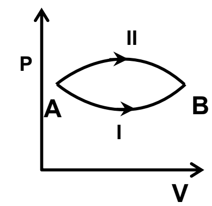
34. **(3)**

$$\left(\frac{dQ}{dt}\right) = (T_1 - T_2)\frac{4\pi r_1 r_2 K}{(r_2 - r_1)}$$

35. A system goes from A to B via two processes I and II as shown in the figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, the



(1) $\Delta U_1 = \Delta U_2$
(2) relation between ΔU_1 and ΔU_2 can not be determined
(3) $\Delta U_2 > \Delta U_1$
(4) $\Delta U_2 < \Delta U_1$

35. **(1)**
Internal energy is state function

36. The function $\sin^2(\omega t)$ represents
(1) a periodic, but not simple harmonic motion with a period $2\pi/\omega$
(2) a periodic, but not simple harmonic motion with a period π/ω
(3) a simple harmonic motion with a period $2\pi/\omega$
(4) a simple harmonic motion with a period π/ω.

36. **(4)**

$$y = \frac{(1-\cos 2\omega t)}{2}$$

37. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is
(1) hyperbola (2) circle
(3) straight line (4) parabola

37. **(3)**
Straight line
Note: If instead of young's double slit experiment, young's double hole experiment was given shape would have been hyperbola.

38. Two simple harmonic motions are represented by the equation $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos\pi t$. The phase difference of the velocity of particle 1 w.r.t. the velocity of the particle 2 is
(1) $-\pi/6$ (2) $\pi/3$
(3) $-\pi/3$ (4) $\pi/6$

38. **(1)**
Phase difference $(\phi) = 99\pi t + \pi/3 - \pi/2$
at $t = 0$ $\phi = -\pi/6$.

39. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle in cm is
(1) $36\sqrt{7}$ (2) $36/\sqrt{7}$
(3) $36\sqrt{5}$ (4) $4\sqrt{5}$

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39. (2)

$$r = \frac{h}{\sqrt{\mu^2 - 1}} = \frac{36}{\sqrt{7}}$$

40. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]

(1) 5 m (2) 1m
(3) 6 m (4) 3m

40. (1)

$$\frac{1.22\lambda}{(3\text{mm})} = \text{Resolution limit} = \frac{(1\text{mm})}{R}$$

$\therefore$ R = 5 m

41. A thin glass (refractive index 1.5) lens has optical power of – 5D in air. Its optical power in a liquid medium with refractive index 1.6 will be

(1) 1 D (2) -1D
(3) 25 D (4) – 25 D

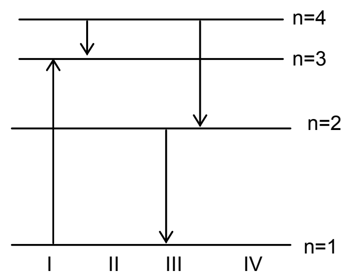
41. (none)

$$\frac{P_m}{P_{air}} = \frac{\left(\frac{\mu_\ell}{\mu_a} - 1\right)}{\left(\frac{\mu_\ell}{\mu_m} - 1\right)}$$

P_m=5/8 D

42. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy ?

(1) III (2) IV
(3) I (4) II



42. (1)

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

43. If the kinetic energy of a free electron doubles. Its deBroglie wavelength changes by the factor

(1) $\frac{1}{2}$ (2) 2
(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

43. (3)

$$\lambda = \frac{h}{\sqrt{2Km}}$$

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44. In a common base amplifier, the phase difference between the input signal voltage and output voltage is

(1) $\frac{\pi}{4}$ (2) π

(3) 0 (4) $\frac{\pi}{2}$

44. (3)

No phase difference between input and output signal.

45. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be

(1) 50 Hz (2) 25 Hz

(3) 100 Hz (4) 70.7 Hz

45. (3)

frequency = 2 (frequency of input signal).

46. A nuclear transformation is denoted by X(n, α) ${}_{3}^{7}Li$. Which of the following is the nucleus of element X ?

(1) ${}^{12}C_6$ (2) ${}_{5}^{10}B$

(3) ${}_{5}^{9}B$ (4) ${}_{4}^{11}Be$

46. (2)

$X + {}_{0}^{1}n \rightarrow {}_{2}^{4}He + {}_{3}^{7}Li$

47. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be

(1) 10^3 (2) 10^5

(3) 99995 (4) 9995

47. (4)

I_g=15mA   Vg = 75mV

$R = \frac{V}{I_g} - \frac{V_g}{I_g}$

48. Two voltameters one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltameters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively the charge which flows through the silver voltameter is

(1) $\frac{q}{1+\frac{z_1}{z_2}}$ (2) $\frac{q}{1+\frac{z_2}{z_1}}$

(3) $q\frac{z_1}{z_2}$ (4) $q\frac{z_2}{z_1}$

48. (2)

$q_1Z_1=q_2Z_2$

$q=q_1+q_2$

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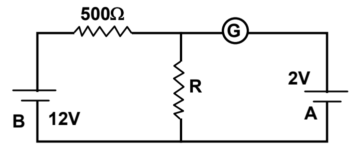
$\therefore\ q_2 = \dfrac{q}{1+\dfrac{Z_2}{Z_1}}$

49. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(1) 200 Ω (2) 100 Ω
(3) 500 Ω (4) 1000 Ω

49. (2)

$\dfrac{12R}{500+R} = 2$

50. Two sources of equal emf are connected to an external resistance R. The internal resistance of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then

(1) $R = R_2 \times (R_1 + R_2)/R_2 - R_1)$ (2) $R = R_2 - R_1$
(3) $R = R_1R_2 / (R_1 + R_2)$ (4) $R = R_1R_2 / (R_2 - R_1)$

50. (2)

$I = \dfrac{2E}{R_1 + R_2 + R}$

$E - R_2 I = 0$

$\Rightarrow R = R_2 - R_1$

51. A fully charged capacitor has a capacitance 'C' it is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT'. The potential difference V across the capacitance is

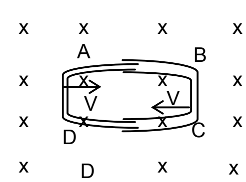
(1) $\sqrt{\dfrac{2mC\Delta T}{s}}$ (2) $\dfrac{mC\Delta T}{s}$

(3) $\dfrac{ms\Delta T}{C}$ (4) $\sqrt{\dfrac{2ms\Delta T}{C}}$

51. (4)

Dimensionally only 4th option is correct.

52. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. if each tube moves towards the other at a constant speed V, then the emf induced in the circuit in terms of B, ℓ and V where ℓ is the width of each tube will be



(1) $B\ell V$ (2) $-B\ell V$
(3) zero (4) $2B\ell V$

52. (4)

$\left|\dfrac{d\phi}{dt}\right| = 2B\ell v$

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53. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be
(1) doubled (2) four times
(3) one fourth (4) halved

53. (1)

$$H = \frac{V^2 \Delta t}{R}$$

$$H' = \frac{V^2}{R'} \Delta t \qquad \text{Given } R' = R/2$$

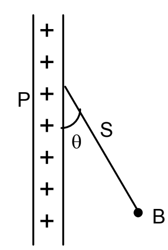
54. Two thin, long parallel wires separated by a distance 'd' carry a current of 'i' A in the same direction. They will
(1) attract each other with a force of $\mu_0 i^2/(2\pi d)$
(2) repel each other with a force of $\mu_0 i^2 / (2\pi d)$
(3) attract each other with a force of $\mu_0 i^2(2\pi d^2)$
(4) repel each other with a force of $\mu_0 i^2/(2\pi d^2)$

54. (1)
Using the definition of force per unit length due to two long parallel wires carrying currents.

55. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is
(1) $\frac{1}{2}I_o$ (2) $\frac{1}{4}I_o$
(3) zero (4) I_0

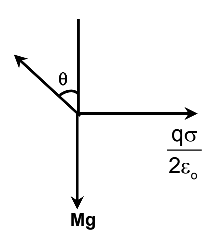
55. (1)
When unpolarised light of intensity I_o is incident on a polarizing sheet, only $I_o/2$ is transmitted.

56. A charged ball B hangs from a silk thread S which makes an angle θ with a large charged conducting sheet P, as show in the figure. The surface charge density σ of the sheet is proportional to
(1) $\cos\theta$ (2) $\cot\theta$
(3) $\sin\theta$ (4) $\tan\theta$



56. (4)

$$\tan\theta = \frac{q\sigma}{(2\varepsilon_o)mg}$$



57. Two point charges + 8q and – 2q are located at x = 0 and x = L respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is
(1) 2L (2) L/4

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(3) 8L (4) 4L

57. (1)

$$-\frac{k2q}{(x-L)^2}+\frac{k8q}{x^2}=0$$

$\Rightarrow$ x = 2L

58. Two thin wires rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are +q and –q. The potential difference between the centres of the two rings is

(1) $QR/4\pi\varepsilon_0 d^2$

(2) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(3) zero

(4) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

58. (2)

$$v_1=\frac{kq}{R}-\frac{kq}{\sqrt{R^2+d^2}}$$

$$v_2=\frac{-kq}{R}+\frac{kq}{\sqrt{R^2+d^2}}$$

59. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is

(1) (n – 1)C (2) (n + 1)C
(3) C (4) nC

59. (1)

C_{eq}=(n–1) C ($\because$ all capacitors are in parallel)

60. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per seconds are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

(1) 200 Hz (2) 202 Hz
(3) 196 Hz (4) 204 Hz

60. (3)

$|f_1-f_2| = 4$

Since mass of second tuning fork increases so f_2 decrease and beats increase so $f_1>f_2$

$\Rightarrow f_2=f_1-4 = 196$

61. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2}$ + αx = 0, its time period is

(1) $\frac{2\pi}{\alpha}$ (2) $\frac{2\pi}{\sqrt{\alpha}}$

(3) $2\pi\alpha$ (4) $2\pi\sqrt{\alpha}$

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61. **(2)**

$\omega^2=\alpha$

$\omega = \sqrt{\alpha}$

$T = \frac{2\pi}{\sqrt{\alpha}}$

62. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillation bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(1) first increase and then decrease to the original value.
(2) first decreased then increase to the original value.
(3) remain unchanged.
(4) increase towards a saturation value.

62. **(1)**

First CM goes down and then comes to its initial position.

63. An observer moves towards a stationary source of sound, with a velocity one fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(1) zero. (2) 0.5%
(3) 5% (4) 20%

63. **(4)**

$f = \frac{v + v/5}{v} f = \frac{6f}{5}$

% increase in frequency = 20%

64. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?
(1) $2I_0$ (2) $4I_0$
(3) I_0 (4) $I_0/2$

64. **(3)**

Maximum intensity is independent of slit width.

65. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 Ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in Weber/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)
(1) 12×10^{-5} (2) 10^{-5}
(3) 5×10^{-5} (4) 7×10^{-5}

65. **(3)**

$B = \frac{\mu_o}{2r}\sqrt{I_1^2 + I_2^2}$

$B = \frac{4\pi \times 10^{-7}}{2 \times 2\pi \times 10^{-2}} \times 5$

$B = 5\times10^{-5}$

66. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in
(1) 0.05 s (2) 0.1 s

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(3) 0.15 s (4) 0.3 s

66. **(2)**

$$I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$$

$$0.693 = \frac{R}{L}t$$

$$t = \frac{.3 \times 0.693}{2} = 0.1\text{sec}$$

67. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

(1) $4\mu F$ (2) $8\mu F$

(3) $1\mu F$ (4) $2\mu F$

67. **(3)**

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{4 \times \pi^2 f^2 \times 10}$$

$C = 1\mu F$

68. An energy source will supply a constant current into the load of its internal resistance is

(1) equal to the resistance of the load.

(2) very large as compared to the load resistance.

(3) zero.

(4) non-zero but less than the resistance of the load.

68. **(2)**

$$I = \frac{E_o}{R + r} \approx \frac{E}{r} \text{ if } R << r$$

69. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be

(1) 0.8 (2) 0.4

(3) 1.25 (4) 0.125

69. **(1)**

$$\cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$$

70. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?

(1) C alone (2) R.L

(3) L. C (4) L alone

70. **(2)**

0<phase difference for R-L circuit < $\pi/2$

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71. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then
(1) its velocity will decrease. (2) its velocity will increase.
(3) it will turn towards right of direction of motion. (4) it will turn towards left of direction of motion.

71. (1)
$\vec{F} = -e\left[\vec{E} + \vec{v} \times \vec{B}\right] = -e\vec{E}$

$\vec{a} = -\frac{e\vec{E}}{m}$

$v(t) = v_o - \frac{eE}{m}t$

72. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is
(1) $\frac{2\pi mq}{B}$ (2) $\frac{2\pi q^2 B}{m}$
(3) $\frac{2\pi qB}{m}$ (4) $\frac{2\pi m}{qB}$

72. (4)
$m\omega^2 r = Bq\omega r$
$\omega = Bq/m$
$T = \frac{2\pi m}{qB}$

73. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2Ω the balancing length becomes 120 cm. The internal resistance of the cell is
(1) 1 Ω (2) 0.5 Ω
(3) 4 Ω (4) 2 Ω

73. (4)
$r = R\left[\frac{\ell_1}{\ell_2} - 1\right] = 2\left[\frac{240}{120} - 1\right] = 2\Omega$

74. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?
(1) 40 Ω (2) 20 Ω
(3) 400 Ω (4) 200 Ω

74. (1)
$R_{hot} = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\Omega$
cold resistance = $R_{hot}/10$ = 400/10 = 40 Ω

75. A magnetic needle is kept in a non-uniform magnetic field. It experiences
(1) a torque but not a force (2) neither a force nor a torque
(3) a force and a torque. (4) a force but not a torque.

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75. **(3)**
In non uniform magnetic field, dipole experiences both force and torque.

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SOLUTION TO AIEEE-2005

CHEMISTRY

76. Which of the following oxides is amphoteric in character?
(1) CaO (2) CO_2
(3) SiO_2 (4) SnO_2

76. (4)
CaO $\longrightarrow$ basic
SiO_2 & CO_2 $\longrightarrow$ acidic
SnO_2 $\longrightarrow$ amphoteric

77. Which one of the following species is diamagnetic in nature?
(1) He_2^+ (2) H_2
(3) H_2^+ (4) H_2^-

77. (2)
H_2 $\sigma 1s^2\ \sigma^* 1s^0$, no unpaired so diamagnetic

78. If α is the degree of dissociation of Na_2SO_4, the vant Hoff's factor (i) used for calculating the molecular mass is
(1) $1+\alpha$ (2) $1-\alpha$
(3) $1+2\alpha$ (4) $1-2\alpha$

78. (3)
Na_2SO_4 $\quad$ $2Na^+ + SO_4^{-2}$
$1-\alpha$ $\quad$ 2α $\quad$ α
Total moles = $1+2\alpha$

79. The oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$ is
(1) +3 (2) +2
(3) +1 (4) 0

79. (1)
$(Cr(NH_3)_4Cl_2)^+$
$X + 4\times0 + 2\times-1 = 1$
$X = +3$

80. Hydrogen bomb is based on the principle of
(1) Nuclear fission (2) Natural radioactivity
(3) Nuclear fusion (4) Artificial radioactivity

80. (3)

81. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be
(1) AB (2) A_2B
(3) AB_3 (4) A_3B

81. (3)
$A = \frac{1}{8}\times 8 = 1$
(Corner)

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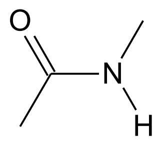
$B = \frac{1}{2} \times 6 = 3$
(Face centre)
$\therefore AB_3$

82. For a spontaneous reaction the ΔG , equilibrium constant (K) and E^{o}_{cell} will be respectively
(1) -ve, >1, +ve
(2) +ve, >1, -ve
(3) -ve, <1, -ve
(4) -ve, >1, -ve

82. (1)

83. Which of the following is a polyamide?
(1) Teflon
(3) Nylon – 66
(3) Terylene
(4) Bakelite

83 (2)



Amide $\longrightarrow$ Nylon 66

84. Which one of the following types of drugs reduces fever?
(1) Analgesic
(2) Antipyretic
(3) Antibiotic
(4) Tranquiliser

84. (2)

85. Due to the presence of an unpaired electron, free radicals are:
(1) Chemically reactive
(2) Chemically inactive
(3) Anions
(4) Cations

85. (1)

86. Lattice energy of an ionic compounds depends upon
(1) Charge on the ion only
(2) Size of the ion only
(3) Packing of ions only
(4) Charge on the ion and size of the ion

86. (4)

87. The highest electrical conductivity of the following aqueous solutions is of
(1) 0.1 M acetic acid
(2) 0.1 M chloroacetic acid
(3) 0.1 M fluoroacetic acid
(4) 0.1 M difluoroacetic acid

87. (4)

88. Aluminium oxide may be electrolysed at $1000^{\circ}C$ to furnish aluminium metal (Atomic mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is
$Al^{3+} + 3e^{-} \longrightarrow Al^{o}$
To prepare 5.12 kg of aluminium metal by this method would require
(1) 5.49×10^{7} C of electricity
(2) 1.83×10^{7} C of electricity
(3) 5.49×10^{4} C of electricity
(4) $\times 10^{1}$ C of electricity

88. (1)

$$Q = \frac{mFZ}{M} = \frac{5.12 \times 10^{5} \times 96500 \times 3}{27}$$
$= 5.49 \times 10^{7}$ C

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89. Consider an endothermic reaction, $X \longrightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general
(1) $E_b < E_f$
(2) $E_b > E_f$
(3) $E_b = E_f$
(4) There is no definite relation between E_b and E_f

89. (1)
$\Delta H = E_f - E_b$
For ΔH = Positive, $E_b < E_f$

90. Consider the reaction: $N_2 + 3H_2 \longrightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true?
(1) $\Delta H = 0$ (2) $\Delta H = \Delta U$
(3) $\Delta H < \Delta U$ (4) $\Delta H > \Delta U$

90. (3)
$\Delta H = \Delta U + \Delta nRT$
$\Delta n = -2$
$\Delta H = \Delta U - 2RT$
$\Delta H < \Delta U$

91. Which one of the following statements is NOT true about the effect of an increase in temperature on the distribution of molecular speeds in a gas?
(1) The most probable speed increases
(2) The fraction of the molecules with the most probable speed increases
(3) The distribution becomes broader
(4) The area under the distribution curve remains the same as under the lower temperature

91. (2)
Most probable velocity increase and fraction of molecule possessing most probable velocity decreases.

92. The volume of a colloidal particle, V_C as compared to the volume of a solute particle in a true solution V_S, could be
(1) $\frac{V_C}{V_S} = 1$ (2) $\frac{V_C}{V_S} = 10^{23}$
(3) $\frac{V_C}{V_S} = 10^{-3}$ (4) $\frac{V_C}{V_S} = 10^{3}$

92. (4)

93. The solubility product of a salt having general formula MX_2, in water is: 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is
(1) 2.0×10^{-6} M (2) 1.0×10^{-4} M
(3) 1.6×10^{-4} M (4) 4.0×10^{-10} M

93. (2)
$MX_2 \quad M^{+2} + 2X^-$
$\quad\quad\quad S \quad\; 2S$

$K_{sp} = 4s^3$, $S = \sqrt[3]{\frac{K_{sp}}{4}} = 1 \times 10^{-4}$

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94. Benzene and toluene form nearly ideal solutions. At $20^\circ C$, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at $20^\circ C$ for a solution containing 78 g of benzene and 46 g of toluene in torr is

(1) 50 (2) 25
(3) 37.5 (4) 53.5

94. (1)

$P_B = P_B^\circ \times B = 75 \times \frac{1}{1.5} = 50$ torr

95. The exothermic formation of ClF_3 is represented by the equation:

$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}; \quad \Delta rH = -329$ kJ

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(1) Increasing the temperature (2) Removing Cl_2
(3) Increasing the volume of the container (4) Adding F_2

95. (4)

$M_3V_3 = M_1V_2 + M_2V_2$

M = $\frac{480(1.5)+520(1.2)}{1000} = 1.344M$

96. Two solutions of a substance (non electrolyte) are mixed in the following manner. 480 ml of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture?

(1) 1.20 M (2) 1.50 M
(3) 1.344 M (4) 2.70 M

96. (3)

97. For the reaction

$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$,

($K_c = 1.8 \times 10^{-6}$ at $184^\circ C$)

(R = 0.0831 kJ / (mol.K)

When K_p and K_c are compared at $184^\circ C$, it is found that

(1) K_p is greater than K_c
(2) K_p is less than K_c
(3) $K_p = K_c$
(4) Whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure

97. (1)

$Kp = Kc\ RT^{\Delta n}$, $\Delta n = 1$

Kp > Kc

98. Hydrogen ion concentration in mol / L in a solution of pH = 5.4 will be

(1) 3.98×10^{8} (2) 3.88×10^{6}
(3) 3.68×10^{-6} (4) 3.98×10^{-6}

98. (4)

$p^H = -\log(H^+)$

99. A reaction involving two different reactants can never be

(1) Unimolecular reaction (2) First order reaction
(3) second order reaction (4) Bimolecular reaction

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99. (1)

100. If we consider that $\frac{1}{6}$, in place of $\frac{1}{12}$; mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of a substance will
(1) Decrease twice
(2) Increase two fold
(3) Remain unchanged
(4) Be a function of the molecular mass of the substance

100. (3)

101. In a multi – electron atom, which of the following orbitals described by the three quantum numbers will have the same energy in the absence of magnetic acid and electric fields?
(a) n = 1, l = 0, m = 0
(b) n = 2, l = 0, m = 0
(c) n = 2, l = 1, m = 1
(d) n = 3, l = 2, m = 1
(e) n = 3, l = 2, m = 0
(1) (a) and (b) (2) (b) and (c)
(3) (c) and (d) (4) (d) and (e)

101. (4)
n = same

102. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud' These are
(1) Sn and Ag (2) Pb and Zn
(3) Ag and Au (4) Fe and Ni

102. (3)

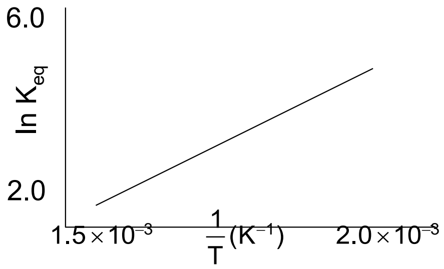
103.

Electrolyte	KCl	KNO_3	HCl	NaOAc	NaCl
$\wedge^{\infty}$(S cm^2mol^{-1})	149.9	145.0	426.2	91.0	126.5

Calculate $\wedge^{\infty}_{HOAc}$ Using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25°C
(1) 517.2 (2) 552.7
(3) 390.7 (4) 217.5

103. (3)
$\wedge^{\infty}_{AcOH} = \wedge^{\infty}_{HCl} + \wedge^{\infty}_{AcONa} - \wedge^{\infty}_{NaCl}$
= 390.7

104. A schematic plot of In K_{eq} versus inverse of temperature for a reaction is shown below



The reaction must be

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(1) exothermic
(2) endothermic
(3) one with negligible enthalpy change
(4) highly spontaneous at ordinary temperature

104. (1)

$K_{eq} = A\ e^{-\frac{\Delta H}{RT}}$

105. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectively, which of the following statements is NOT correct?

(1) magnesium chloride solution coagulates, the gold sol more readily than the iron (III) hydroxide sol.
(2) sodium sulphate solution causes coagulation in both sols
(3) mixing the sols has no effect
(4) coagulation in both sols can be brought about by electrophoresis

105. (3)

106. Based on lattice energy and other considerations which one of the following alkali metal chlorides is expected to have the highest melting point.
(1) LiCl
(2) NaCl
(3) KCl
(4) RbCl

106. (2)

Although lattice energy of LiCl higher than NaCl but LiCl is covalent in nature and NaCl ionic there after , the melting point decreases as we move NaCl because the lattice energy decreases as a size of alkali metal atom increases (lattice energy $\propto$ to melting point of alkali metal halide)

107. Heating mixture of Cu_2O and Cu_2S will give
(1) $Cu + SO_2$
(2) $Cu + SO_3$
(3) $CuO + CuS$
(4) Cu_2SO_3

107. (1)

$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

108. The molecular shapes of SF_4, CF_4 and XeF_4 are
(1) the same with 2,0 and 1 lone pairs of electrons on the central atom, respectively
(2) the same with 1, 1 and 1 lone pair of electrons on the central atoms, respectively
(3) different with 0, 1 and 2 lone pair of electrons on the central atoms, respectively
(4) different with 1, 0 and 2 lone pairs of electron on the central atoms respectively

108 (4)

109. The number and type of bonds between two carbon atoms in calcium carbide are
(1) One sigma, one pi
(2) One sigma, two pi
(3) Two sigma, one pi
(4) Two sigma, two pi

109. (2)

CaC_2 $\quad Ca^{+2} \begin{matrix} C^{-} \\ ||| \\ C^{-} \end{matrix}$

One σ

Two π

110. The oxidation state of chromium in the final product formed by the reaction between KI and acidified potassium dichromate solution is
(1) +4
(2) +6
(3) +2
(4) +3

110. (4)

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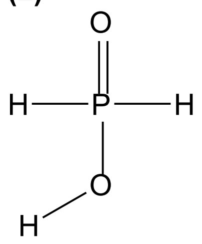
$$\underset{+6}{Cr_2O_7^{-2}} + 14H^+ + 6I^- \longrightarrow \underset{+3}{2Cr^{+3}} + 7H_2O + 3I_2$$

111. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is
(1) zero
(2) two
(3) one
(4) three

111. **(2)**



112. What is the conjugate base of OH^-?
(1) O_2
(2) H_2O
(3) O^-
(4) O^{-2}

112. **(4)**
$OH^- \longrightarrow O^{-2} + H^+$

113. The correct order of the thermal stability of hydrogen halides (H – X) is
(1) HI > HBr > HCl > HF
(2) HF > HCl > HBr > HI
(3) HCl < HF > HBr < HI
(4) HI > HCl < HF < HBr

113. **(2)**

114. Heating an aqueous solution of aluminium chloride to dryness will give
(1) $AlCl_3$
(2) Al_2Cl_6
(3) Al_2O_3
(4) $Al(OH)Cl_2$

114. **(3)**
$Al_2Cl_6\ 6H_2O \longrightarrow Al_2O_3 + + 6HCl + 3H_2O\uparrow$

115. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
(1) $HgNH_2Cl$
(2) $NH_2 – Hg – Hg – Cl$
(3) Hg_2O
(4) HgO

115. **(1)**
$Hg_2Cl_2 + 2NH_4OH \longrightarrow Hg + Hg(NH_2)Cl + NH_4Cl + 2H_2O$

116. In which of the following arrangements the order is NOT according to the property indicated against it?
(1) $Al^{3+} < Mg^{2+} < Na^+ < F^-$
Increasing ionic size
(2) B < C < N < O
Increasing first ionization enthalpy
(3) I < Br < F < Cl
Increasing electron gain enthalpy (with negative sign)
(4) Li < Na < K < Rb
Increasing metallic radius

116. **(2)**
B < C < O < N

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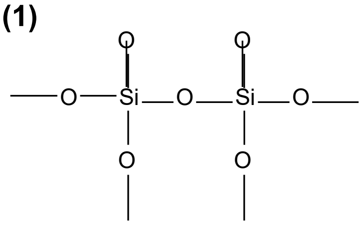
117. In silicon dioxide
(1) Each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms
(2) Each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms
(3) Silicon atoms is bonded to two oxygen atoms
(4) there are double bonds between silicon and oxygen atoms

117. (1)



118. Of the following sets which one does NOT contain isoelectronic species?
(1) $PO_4^{-3}, SO_4^{-2}, ClO_4^-$
(2) CN^-, N_2, C_2^{-2}
(3) $SO_3^{-2}, CO_3^{-2}, NO_3^-$
(4) $BO_3^{-3}, CO_3^{-2}, NO_3^-$

118. (3)

119. The lanthanide contraction is responsible for the fact that
(1) Zr and Y have about the same radius
(2) Zr and Nb have similar oxidation state
(3) Zr and Hf have about the same radius
(4) Zr and Zn have the same oxidation

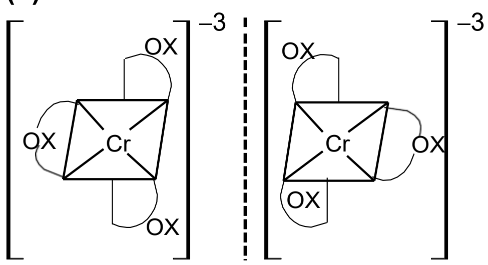
119. (3)
Due to Lanthanide contraction.

120. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is
(1) Potassium hexacyanoferrate (II)
(2) Potassium hexacyanoferrate (III)
(3) Potassium hexacyanoiron (II)
(4) tripotassium hexcyanoiron (II)

120. (2)

121. Which of the following compounds shows optical isomerism?
(1) $[Cu(NH_3)_4]^{+2}$
(2) $[ZnCl_4]^{-2}$
(3) $[Cr(C_2O_4)_3]^{-3}$
(4) $[Co(CN)_6]^{-3}$

121. (3)



122. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour?
(1) $[Cr(CN)_6]^{-3}$
(2) $[Mn(CN)_6]^{-3}$
(3) $[Fe(CN)_6]^{-3}$
(4) $[Co(CN)_6]^{-3}$
(At. No. Cr = 24, Mn = 25, Fe = 26, Co = 27)

122. (4)

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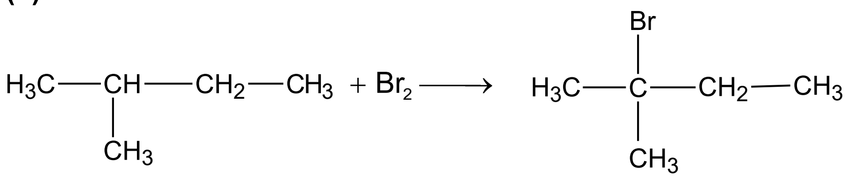
123. 2 methylbutane on reacting with bromine in the presence of sunlight gives mainly
(1) 1 – bromo -2 - methylbutane (2) 2 – bromo -2 - methylbutane
(3) 2 – bromo -3 - methylbutane (4) 1 – bromo -3 – methylbutane

123. (2)



Major

124. The photon of hard gamma radiation knocks a proton out of $^{24}_{12}Mg$ nucleus to form
(1) the isotope of parent nucleus (2) the isobar of parent nucleus
(3) the nuclide $^{23}_{11}Na$ (4) the isobar of $^{23}_{11}Na$

124. (3)

125. The best reagent to convert pent -3- en-2-ol into pent -3-en-2-one is
(1) Acidic permanganate (2) Acidic dichromate
(3) Chromic anhydride in glacial acetic acid (4) Pyridinium chloro – chromate

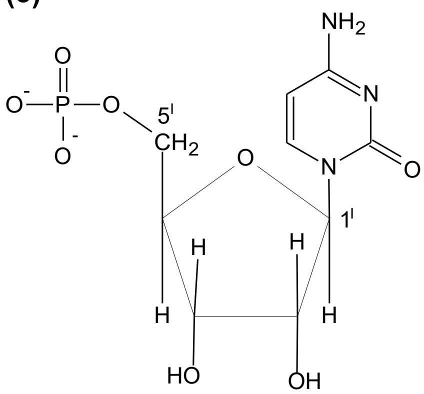
125. (3)

126. Tertiary alkyl halides are practically inert to substitution by S_N2 mechanism because of
(1) insolubility (2) instability
(3) inductive effect (4) steric hindrance

126. (4)

127. In both DNA and RNA, heterocyclic base and phosphate ester linkages are at-
(1) C'_5 and C'_2 respectively of the sugar molecule
(2) C'_2 and C'_5 respectively of the sugar molecule
(3) C'_1 and C'_5 respectively of the sugar molecule
(4) C'_5 and C'_1 respectively of the sugar molecule

127. (3)



128. Reaction of one molecule of HBr with one molecule of 1,3-butadiene at 40^0C gives predominantly
(1) 3-bromobutene under kinetically controlled conditions
(2) 1-bromo-2-butene under thermodymically controlled conditions
(3) 3-bromobutene under thermodynamically controlled conditions
(4) 1-bromo-2-butene under kinetically controlled conditions

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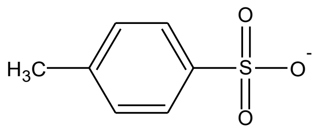
128. (2)

129. Among the following acids which has the lowest pK_a value?
(1) CH_3COOH
(2) $HCOOH$
(3) $(CH_3)_2COOH$
(4) CH_3CH_2COOH

129. (2)

130. The decreasing order of nucleophilicity among the nucleophiles
(a) $CH_3-\underset{\underset{O}{\|}}{C}-O^-$
(b) CH_3O^-
(c) CN^-
(d)


(1) (a), (b), (c), (d) (2) (d), (c), (b), (a)
(3) (b), (c), (a), (d) (4) (c), (b), (a), (d)

130. (4)

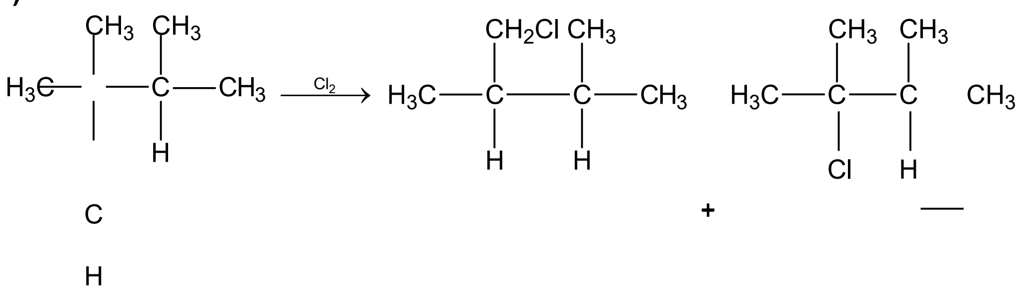
131. Which one of the following methods is neither meant for the synthesis nor for separation of amines?
(1) Hinsberg method
(2) Hofmann method
(3) Wurtz reaction
(4) Curtius reaction

131. (3)

132. Which of the following is fully fluorinated polymer?
(1) Neoprene (2) Teflon
(3) Thiokol (4) PVC

132. (2)

133. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is
(1) n-hexane
(2) 2, 3-dimethylbutane
(3) 2,2-dimethylbutane
(4) 2-methylpentane

133. (2)


134. Alkyl halides react with dialkyl copper reagents to give
(1) alkenes (2) alkyl copper halides
(3) alkanes (3) alkenyl halides

134. (3)
$R_2CuLi + R'X \longrightarrow R-R' + R-Cu + LiX$

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135. Acid catalyzed hydration of alkenes except ethene leads to the formation of
(1) primary alcohol
(2) secondary or tertiary alcohol
(3) mixture of primary and secondary alcohols
(4) mixture of secondary and tertiary alcohols

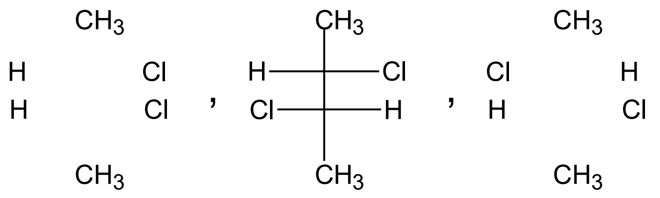
135. (4)

136. Amongst the following the most basic compound is
(1) benzylamine (2) aniline
(3) acetanilide (4) p-nitroaniline

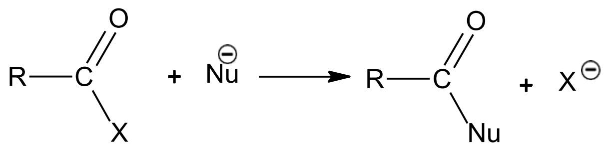
136. (1)
-NH_2 group is not linked with benzene ring.

137. Which types of isomerism is shown by 2,3-dichlorobutane?
(1) Diastereo (2) Optical
(3) Geometric (4) Structural

137. (2)



138. The reaction



is fastest when X is
(1) Cl (2) NH_2
(3) OC_2H_5 (4) OCOR

138. (1)
Conjugated acid of Cl^- is a stronger acid i.e. HCl.

139. Elimination of bromine from 2-bromobutane results in the formation of-
(1) equimolar mixture of 1 and 2-butene (2) predominantly 2-butene
(3) predominantly 1-butene (4) predominantly 2-butyne

139. (2)
Saytzeffs product.

140. Equimolar solutions in the same solvent have
(1) Same boiling point but different freezing point
(2) Same freezing point but different boiling point
(3) Same boiling and same freezing points
(4) Different boiling and different freezing points

140. (3)

141. Which of the following statements in relation to the hydrogen atom is correct?
(1) 3s orbital is lower in energy than 3p orbital
(2) 3p orbital is lower in energy than 3d orbital
(3) 3s and 3p orbitals are of lower energy than 3d orbital
(4) 3s, 3p and 3d orbitals all have the same energy

141. (4)

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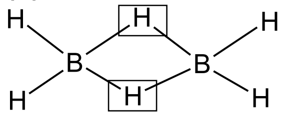
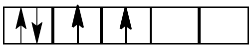
142. The structure of diborane (B_2H_6) contains
(1) four 2c-2e bonds and two 3c-2e bonds
(2) two 2c-2e bonds and four 3c-2e bonds
(3) two 2c-2e bonds and two 3c-3e bonds
(4) four 2c-2e bonds and four 3c-2e bonds

142. (1)



143. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is
(1) d^4 (in strong ligand filed)
(2) d^4 (in weak ligand field)
(3) d^3 (in weak as well as in strong fields)
(4) d^5 (in strong ligand field)

143. (1)



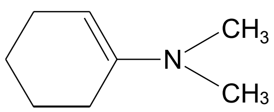
d^4 in strong field, so unpaired electrons = 2.

144. Which of the following factors may be regarded as the main cause of lanthanide contraction?
(1) Poor shielding of one of 4f electron by another in the subshell
(2) Effective shielding of one of 4f electrons by another in the subshell
(3) Poorer shielding of 5d electrons by 4f electrons
(4) Greater shielding of 5d electrons by 4f electrons

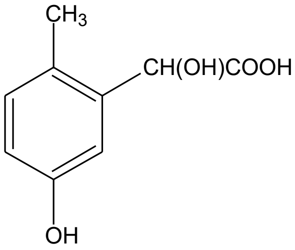
144. (1)

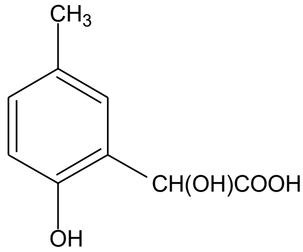
145. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as
(1) a Schiff's base (2) an enamine
(3) an imine (4) an amine

145. (2)



146. p-cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form, the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

(1) 

(2) 

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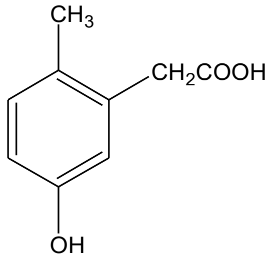
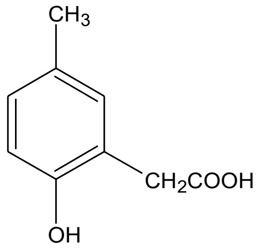
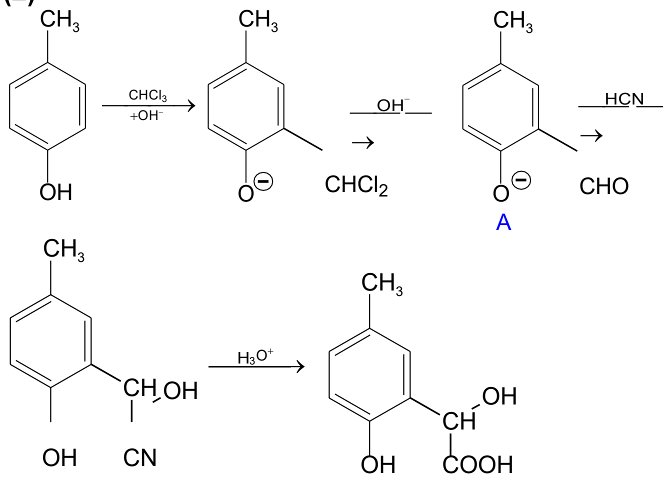
(3) 

(4) 

146. (2)



147. An organic compound having molecular mass 60 is found to contain C = 20%, H = 6.67% and N = 46.67% while rest is oxygen. On heating it gives NH_3 alongwith a solid residue. The solid residue give violet colour with alkaline copper sulphate solution. The compound is

(1) CH_3NCO (2) CH_3CONH_2
(3) $(NH_2)_2CO$ (4) $CH_3CH_2CONH_2$

147. (3)

148. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1:1:0.5 and ΔH for the formation of XY is -200 kJ $mole^{-1}$. The bond dissociation energy of X_2 will be

(1) 100 kJ mol^{-1} (2) 200 kJ mol^{-1}
(3) 300 kJ mol^{-1} (4) 400 kJ mol^{-1}

148. -

(None of the options is correct.)

$$XY \longrightarrow X_{(g)} + Y_{(g)}; \quad \Delta H = +a \text{ kJ / mole} \,.............\, (i)$$

$$X_2 \longrightarrow 2X; \quad \Delta H = +a \text{ kJ/ mole} \,................\, (ii)$$

$$Y_2 \longrightarrow 2Y; \quad \Delta H = +0.5 \text{ a kJ/ mole} \,.............\, (iii)$$

$$\frac{1}{2} \times (ii) + \frac{1}{2} \times (iii) - (i), \text{ Gives}$$

$$\frac{1}{2} X_2 + \frac{1}{2} Y_2 \longrightarrow XY; \quad \Delta H = \left(+\frac{a}{2} + \frac{0.5}{2} a - a \right) \text{kJ / mole}$$

$$+\frac{a}{2} + \frac{0.5 \text{ a}}{2} - a = -200$$

a = 800.

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149. $t_{1/4}$ can be taken as the time taken for the concentration of a reactant to drop to $\frac{3}{4}$ of its initial value. If the rate constant for a first order reaction is K, the $t_{1/4}$ can be written as

(1) 0.10/ K (2) 0.29 /K
(3) 0.69/ K (4) 0.75 /K

149. (2)

$$t_{1/4} = \frac{2.303}{K}\log\frac{1}{1-\frac{1}{4}} = \frac{0.29}{K}.$$

150. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm. Pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm. The equilibrium constant for NH_4HS decomposition at this temperature is

(1) 0.30 (2) 0.18
(3) 0.17 (4) 0.11

150. (4)

$$NH_4HS \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$$

a 0.5 atm

$a - x$ $0.5 + x$ x

Total pressure = 0.5 + 2x = 0.84

i.e., x = 0.17

$K_p = p_{NH_3} \cdot p_{H_2S}$

= (0.67). (0.17)

= 0.1139.

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SOLUTION TO AIEEE-2005

MATHEMATICS

1. If $A^2 - A + I = 0$, then the inverse of A is

(1) $A + I$ (2) A

(3) $A - I$ (4) $I - A$

1. **(4)**

Given $A^2 - A + I = 0$

$A^{-1}A^2 - A^{-1}A + A^{-1} - I = A^{-1}\cdot 0$ (Multiplying A^{-1} on both sides)

$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

2. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are

(1) $-1, -1 + 2\omega, -1 - 2\omega^2$ (2) $-1, -1, -1$

(3) $-1, 1 - 2\omega, 1 - 2\omega^2$ (4) $-1, 1 + 2\omega, 1 + 2\omega^2$

2. **(3)**

$(x-1)^3 + 8 = 0 \Rightarrow (x-1) = (-2)(1)^{1/3}$

$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$

or $n = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.

3. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12} be a relation on the set A = {3, 6, 9, 12}. The relation is

(1) reflexive and transitive only (2) reflexive only

(3) an equivalence relation (4) reflexive and symmetric only

3. **(1)**

Reflexive and transitive only.

e.g. (3, 3), (6, 6), (9, 9), (12, 12) [Reflexive]

(3, 6), (6, 12), (3, 12) [Transitive].

4. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

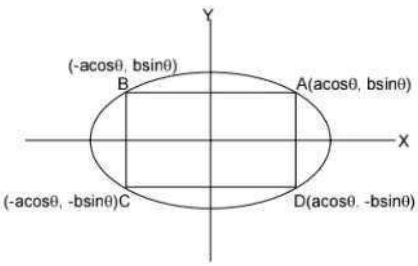
(1) 2ab (2) ab

(3) $\sqrt{ab}$ (4) $\frac{a}{b}$

4. **(1)**

Area of rectangle ABCD = $(2a\cos\theta)(2b\sin\theta) = 2ab\sin 2\theta$

$\Rightarrow$ Area of greatest rectangle is equal to 2ab when $\sin 2\theta = 1$.



5. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where c > 0, is a parameter, is of order and degree as follows:

(1) order 1, degree 2 (2) order 1, degree 1

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(3) order 1, degree 3 (4) order 2, degree 2

5. **(3)**

$y^2 = 2c(x + \sqrt{c})$ ...(i)

$2yy' = 2c\cdot 1$ or $yy' = c$ ...(ii)

$\Rightarrow y^2 = 2yy'\,(x + \sqrt{yy'})$ [on putting value of c from (ii) in (i)]

On simplifying, we get

$(y - 2xy')^2 = 4yy'^3$ ...(iii)

Hence equation (iii) is of order 1 and degree 3.

6. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n^2}\sec^2 1\right]$ equals

(1) $\frac{1}{2}\sec 1$ (2) $\frac{1}{2}\operatorname{cosec}1$

(3) $\tan 1$ (4) $\frac{1}{2}\tan 1$

6. **(4)**

$\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ is equal to

$\lim\limits_{n\to\infty}\frac{r}{n^2}\sec^2\frac{r^2}{n^2} = \lim\limits_{n\to\infty}\frac{1}{n}\cdot\frac{r}{n}\sec^2\frac{r^2}{n^2}$

$\Rightarrow$ Given limit is equal to value of integral $\int\limits_0^1 x\sec^2 x^2 dx$

or $\frac{1}{2}\int\limits_0^1 2x\sec x^2 dx = \frac{1}{2}\int\limits_0^1 \sec^2 t dt$ [put $x^2 = t$]

$= \frac{1}{2}(\tan t)_0^1 = \frac{1}{2}\tan 1.$

7. ABC is a triangle. Forces $\vec{P}, \vec{Q}, \vec{R}$ acting along IA, IB and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then P : Q : R is

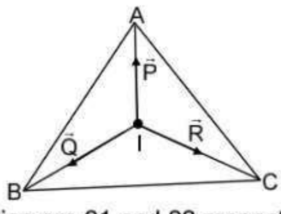
(1) sinA : sin B : sinC (2) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

(3) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$ (4) cosA : cosB : cosC

7. **(3)**

Using Lami's Theorem

$\therefore P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$.



8. If in a frequently distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(1) 22.0 (2) 20.5

(3) 25.5 (4) 24.0

8. **(4)**

Mode + 2Mean = 3 Median

$\Rightarrow$ Mode = $3 \times 22 - 2 \times 21 = 66 - 42 = 24$.

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9. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(1) $y^2 - 4x + 2 = 0$
(2) $y^2 + 4x + 2 = 0$
(3) $x^2 + 4y + 2 = 0$
(4) $x^2 - 4y + 2 = 0$

9. **(1)**

$P = (1, 0)$

$Q = (h, k)$ such that $k^2 = 8h$

Let (α, β) be the midpoint of PQ

$\alpha = \frac{h+1}{2}$, $\beta = \frac{k+0}{2}$

$2\alpha - 1 = h$ $2\beta = k$.

$(2\beta)^2 = 8(2\alpha - 1) \Rightarrow \beta^2 = 4\alpha - 2$

$\Rightarrow y^2 - 4x + 2 = 0$.

10. If C is the mid point of AB and P is any point outside AB, then

(1) $\overline{PA} + \overline{PB} = 2\overline{PC}$
(2) $\overline{PA} + \overline{PB} = \overline{PC}$
(3) $\overline{PA} + \overline{PB} + 2\overline{PC} = 0$
(4) $\overline{PA} + \overline{PB} + \overline{PC} = 0$

10. **(1)**

$\overline{PA} + \overline{AC} + \overline{CP} = 0$

$\overline{PB} + \overline{BC} + \overline{CP} = 0$

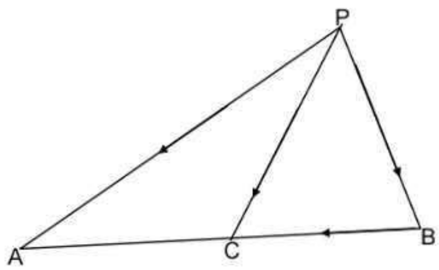
Adding, we get

$\overline{PA} + \overline{PB} + \overline{AC} + \overline{BC} + 2\overline{CP} = 0$

Since $\overline{AC} = -\overline{BC}$

& $\overline{CP} = -\overline{PC}$

$\Rightarrow \overline{PA} + \overline{PB} - 2\overline{PC} = 0$.



11. If the coefficients of rth, (r+ 1)th and (r + 2)th terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation

(1) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$
(2) $m^2 - m(4r+1) + 4r^2 + 2 = 0$
(3) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(4) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

11. **(3)**

Given ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r}$

$= \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0$.

12. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

(1) $a = b + c$
(2) $c = a + b$
(3) $b = c$
(4) $b = a + c$

12. **(2)**

$\tan\left(\frac{P}{2}\right)$, $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$

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$$\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)=\frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right)+\tan\left(\frac{Q}{2}\right)}{1-\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)}=\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=1$$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}}=1 \Rightarrow -\frac{b}{a}=\frac{a}{a}-\frac{c}{a}\Rightarrow -b=a-c$$

$c = a + b$.

13. The system of equations
$\alpha x + y + z = \alpha - 1$,
$x + \alpha y + z = \alpha - 1$,
$x + y + \alpha z = \alpha - 1$
has no solution, if α is
(1) -2
(2) either – 2 or 1
(3) not -2
(4) 1

13. (1)

$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + z\alpha = \alpha - 1$

$$\Delta=\begin{vmatrix}\alpha & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$
$= \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$
$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 1 - 1] = 0$
$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 2] = 0$
$[\alpha^2 + 2\alpha - \alpha - 2] = 0$
$(\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)] = 0$
$(\alpha - 1) = 0, \alpha + 2 = 0 \Rightarrow \alpha = -2, 1$; but $\alpha \neq 1$.

14. The value of α for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is
(1) 1
(2) 0
(3) 3
(4) 2

14. (1)

$x^2 - (a - 2)x - a - 1 = 0$
$\Rightarrow \alpha + \beta = a - 2$
$\alpha\beta = -(a + 1)$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6 = (a - 1)^2 + 5$
$\Rightarrow a = 1$.

15. If roots of the equation $x^2 - bx + c = 0$ be two consectutive integers, then $b^2 - 4c$ equals
(1) – 2
(2) 3

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(3) 2 (4) 1

15. **(4)**

Let $\alpha, \alpha + 1$ be roots

$\alpha + \alpha + 1 = b$

$\alpha(\alpha + 1) = c$

$\therefore b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1.$

16. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(1) 601 (2) 600

(3) 603 (4) 602

16. **(1)**

Alphabetical order is

A, C, H, I, N, S

No. of words starting with A – 5!

No. of words starting with C – 5!

No. of words starting with H – 5!

No. of words starting with I – 5!

No. of words starting with N – 5!

SACHIN – 1

601.

17. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(1) $^{55}C_4$ (2) $^{55}C_3$

(3) $^{56}C_3$ (4) $^{56}C_4$

17. **(4)**

$$^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$$

$$\Rightarrow {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3\right]$$

$$= \left({}^{50}C_4 + {}^{50}C_3\right) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$\Rightarrow \left({}^{51}C_4 + {}^{51}C_3\right) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$\Rightarrow {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4.$$

18. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical indunction

(1) $A^n = nA - (n-1)I$ (2) $A^n = 2^{n-1}A - (n-1)I$

(3) $A^n = nA + (n-1)I$ (4) $A^n = 2^{n-1}A + (n-1)I$

18. **(1)**

By the principle of mathematical induction (1) is true.

19. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax^2 - \left(\frac{1}{bx}\right)\right]^{11}$, then a and b satisfy the relation

(1) $a - b = 1$ (2) $a + b = 1$

(3) $\frac{a}{b} = 1$ (4) $ab = 1$

19. **(4)**

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T_{r+1} in the expansion $\left[ax^2 + \frac{1}{bx}\right]^{11} = {}^{11}C_r\left(ax^2\right)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r\,(a)^{11-r}\,(b)^{-r}\,(x)^{22-2r-r}$

$\Rightarrow 22 - 3r = 7 \quad \Rightarrow r = 5$

$\therefore$ coefficient of $x^7 = {}^{11}C_5(a)^6\,(b)^{-5}$ ......(1)

Again T_{r+1} in the expansion $\left[ax - \frac{1}{bx^2}\right]^{11} = {}^{11}C_r\,(ax)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{11}C_r\,a^{11-r}\,(-1)^r \times (b)^{-r}\,(x)^{-2r}\,(x)^{11-r}$

Now $11 - 3r = -7 \quad \Rightarrow 3r = 18 \quad \Rightarrow \quad r = 6$

$\therefore$ coefficient of $x^{-7} = {}^{11}C_6\,a^5 \times 1 \times (b)^{-6}$

$\Rightarrow {}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6 a^5 \times (b)^{-6}$

$\Rightarrow ab = 1.$

20. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left[0, \frac{\pi}{2}\right)$

(3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

20. (4)

Given $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ for $x \in (-1, 1)$

clearly range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ co-domain of function = $B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

21. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg z_1 - \arg z_2$ is equal to

(1) $\frac{\pi}{2}$ (2) $-\pi$

(3) 0 (4) $-\frac{\pi}{2}$

21. (3)

$|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

22. If $\omega = \dfrac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on

(1) an ellipse (2) a circle

(3) a straight line (4) a parabola.

22. (3)

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As given $w = \dfrac{z}{z - \frac{1}{3}i} \Rightarrow |w| = \dfrac{|z|}{|z - \frac{1}{3}i|} = 1 \Rightarrow$ distance of z from origin and point $\left(0, \dfrac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence z lies on a straight line.

23. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then f(x) is a polynomial of degree

(1) 1 (2) 0

(3) 3 (4) 2

23. (4)

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}, \text{ Applying } C_1 \to C_1 + C_2 + C_3$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} \quad \because a^2+b^2+c^2+2 = 0$$

$$f(x) = \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}; \text{ Applying } R_1 \to R_1 - R_2, \quad R_2 \to R_2 - R_3$$

$f(x) = (x - 1)^2$

Hence degree = 2.

24. The normal to the curve $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

(1) it passes through the origin

(2) it makes angle $\dfrac{\pi}{2} + \theta$ with the x-axis

(3) it passes through $\left(a\dfrac{\pi}{2}, -a\right)$

(4) it is at a constant distance from the origin

24. (4)

Clearly $\dfrac{dy}{dx} = \tan\theta \Rightarrow$ slope of normal $= -\cot\theta$

Equation of normal at 'θ' is

$y - a(\sin\theta - \theta\cos\theta) = -\cot\theta(x - a(\cos\theta + \theta\sin\theta)$

$\Rightarrow y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta = -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$

$\Rightarrow x\cos\theta + y\sin\theta = a$

Clearly this is an equation of straight line which is at a constant distance 'a' from origin.

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25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
(1) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(2) $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(3) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(4) $(-\infty, -4]$	$x^3 + 6x^2 + 6$

25. (3)

Clearly function $f(x) = 3x^2 - 2x + 1$ is increasing when

$f'(x) = 6x - 2 \geq 0 \quad \Rightarrow x \in [1/3, \infty)$

Hence (3) is incorrect.

26. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim\limits_{x\to\alpha} \frac{1-\cos\left(ax^2+bx+c\right)}{\left(x-\alpha\right)^2}$ is equal to

(1) $\frac{a^2}{2}(\alpha-\beta)^2$ (2) 0

(3) $-\frac{a^2}{2}(\alpha-\beta)^2$ (4) $\frac{1}{2}(\alpha-\beta)^2$

26. (1)

Given limit = $\lim\limits_{x\to\alpha} \frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2} = \lim\limits_{x\to\alpha} \frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha} \frac{2}{(x-\alpha)^2} \times \frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}} \times \frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$= \frac{a^2(\alpha-\beta)^2}{2}.$$

27. Suppose f(x) is differentiable x = 1 and $\lim\limits_{h\to0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

(1) 3 (2) 4

(3) 5 (4) 6

27. (3)

$f'(1) = \lim\limits_{h\to0}\frac{f(1+h)-f(1)}{h}$; As function is differentiable so it is continuous as it is given that $\lim\limits_{h\to0}\frac{f(1+h)}{h} = 5$ and hence f(1) = 0

Hence $f'(1) = \lim\limits_{h\to0}\frac{f(1+h)}{h} = 5$

Hence (3) is the correct answer.

28. Let f be differentiable for all x. If f(1) = - 2 and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(1) $f(6) \geq 8$ (2) $f(6) < 8$

(3) $f(6) < 5$ (4) $f(6) = 5$

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28. **(1)**

As f(1) = - 2 & $f'(x) \geq 2 \ \forall \ x \in [1, 6]$

Applying Lagrange's mean value theorem

$$\frac{f(6)-f(1)}{5} = f'(c) \geq 2$$

$\Rightarrow f(6) \geq 10 + f(1)$

$\Rightarrow f(6) \geq 10 - 2$

$\Rightarrow f(6) \geq 8$.

29. If **f** is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, x, y $\in$ R and f(0) = 0, then f(1) equals

(1) -1 (2) 0

(3) 2 (4) 1

29. **(2)**

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$|f'(x)| = \lim_{h\to 0}\left|\frac{f(x+h)-f(x)}{h}\right| \leq \lim_{h\to 0}\left|\frac{(h)^2}{h}\right|$$

$\Rightarrow |f'(x)| \leq 0 \quad \Rightarrow f'(x) = 0 \quad \Rightarrow f(x) = \text{constant}$

As f(0) = 0 $\quad \Rightarrow f(1) = 0$.

30. If x is so small that x^3 and higher powers of x may be neglected, then

$\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

(1) $1-\frac{3}{8}x^2$ (2) $3x+\frac{3}{8}x^2$

(3) $-\frac{3}{8}x^2$ (4) $\frac{x}{2}-\frac{3}{8}x^2$

30. **(3)**

$$(1-x)^{1/2}\left[1+\frac{3}{2}x+\frac{3}{2}\left(\frac{3}{2}-1\right)x^2-1-3\left(\frac{1}{2}x\right)-3(2)\left(\frac{1}{2}x\right)^2\right]$$

$$= (1-x)^{1/2}\left[-\frac{3}{8}x^2\right] = -\frac{3}{8}x^2.$$

31. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and |a| < 1, |b|<1, |c|< 1, then x, y, z are in

(1) G.P. (2) A.P.

(3) Arithmetic – Geometric Progression (4) H.P.

31. **(4)**

$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad a = 1-\frac{1}{x}$

$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \qquad b = 1-\frac{1}{y}$

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$$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \qquad c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$2b = a + c$

$$2\left(1-\frac{1}{y}\right) = 1-\frac{1}{x}+1-\frac{1}{y}$$

$$\frac{2}{y} = \frac{1}{x}+\frac{1}{z}$$

$\Rightarrow$ x, y, z are in H.P.

32. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the the triangle ABC, then 2 (r + R) equals

(1) b + c
(2) a + b
(3) a + b + c
(4) c + a

32. (2)

$$2r + 2R = c + \frac{2ab}{(a+b+c)} = \frac{(a+b)^2 + c(a+b)}{(a+b+c)} = a + b \qquad (\text{ since } c^2 = a^2 + b^2).$$

33. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to

(1) $2 \sin 2\alpha$
(2) 4
(3) $4 \sin^2 \alpha$
(4) $-4 \sin^2 \alpha$

33. (3)

$$\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$$

$$\cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$$

$$\cos^{-1}\left(\frac{xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2}}{2}\right) = \alpha$$

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2 = 4 \cos^2\alpha + x^2y^2 - 4xy \cos\alpha$

$\Rightarrow 4x^2 + y^2 - 4xy \cos\alpha = 4 \sin^2\alpha.$

34. If in a triangle ABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in

(1) G.P.
(2) A.P.
(3) Arithmetic – Geometric Progression
(4) H.P.

34. (2)

$$\Delta = \frac{1}{2}p_1a = \frac{1}{2}p_2b = \frac{1}{2}p_3b$$

p_1, p_2, p_3 are in H.P.

$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P

$\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ sinA, sinB, sinC are in A.P.

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35. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then

(1) $I_2 > I_1$ (2) $I_1 > I_2$

(3) $I_3 = I_4$ (4) $I_3 > I_4$

35. (2)

$I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_0^1 2^{x^2} dx$, $I_4 = \int_0^1 2^{x^3} dx$

$\forall\ 0 < x < 1,\ x^2 > x^3$

$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx$

$\Rightarrow I_1 > I_2$.

36. The area enclosed between the curve $y = \log_e (x + e)$ and the coordinate axes is

(1) 1 (2) 2

(3) 3 (4) 4

36. (1)

Required area (OAB) = $\int_{1-e}^{0} \ln(x+e) dx$

$= \left[x\ln(x+e) - \int \frac{1}{x+e} x\, dx \right]_0^1 = 1.$

37. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(1) 1 : 2 : 1 (2) 1 : 2 : 3

(3) 2 : 1 : 2 (4) 1 : 1 : 1

37. (4)

$y^2 = 4x$ and $x^2 = 4y$ are symmetric about line $y = x$

$\Rightarrow$ area bounded between $y^2 = 4x$ and $y = x$ is $\int_0^4 \left(2\sqrt{x} - x\right) dx = \frac{8}{3}$

$\Rightarrow A_{s_2} = \frac{16}{3}$ and $A_{s_1} = A_{s_3} = \frac{16}{3}$

$\Rightarrow A_{s_1} : A_{s_2} : A_{s_3} :: 1 : 1 : 1.$

38. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(1) $y \log\left(\frac{x}{y}\right) = cx$ (2) $x \log\left(\frac{y}{x}\right) = cy$

(3) $\log\left(\frac{y}{x}\right) = cx$ (4) $\log\left(\frac{x}{y}\right) = cy$

38. (3)

$\frac{x\,dy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$

Put $y = vx$

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$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

$$\Rightarrow \; v + \frac{xdv}{dx} = v(\log v + 1)$$

$$\frac{xdv}{dx} = v\log v$$

$$\Rightarrow \; \frac{dv}{v\log v} = \frac{dx}{x}$$

put log v = z

$$\frac{1}{v}dv = dz$$

$$\Rightarrow \; \frac{dz}{z} = \frac{dx}{x}$$

ln z = ln x + ln c

z = cx

log v = cx

$$\log\left(\frac{y}{x}\right) = cx.$$

39. The line parallel to the x–axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0, where (a, b) $\neq$ (0, 0) is

(1) below the x–axis at a distance of $\frac{3}{2}$ from it

(2) below the x–axis at a distance of $\frac{2}{3}$ from it

(3) above the x–axis at a distance of $\frac{3}{2}$ from it

(4) above the x–axis at a distance of $\frac{2}{3}$ from it

39. (1)

ax + 2by + 3b + λ(bx – 2ay – 3a) = 0

$\Rightarrow$ (a + bλ)x + (2b – 2aλ)y + 3b - 3λa = 0

a + bλ = 0 $\;\Rightarrow\;$ λ = -a/b

$$\Rightarrow ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$$

$$\Rightarrow ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$$

$$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$$

$$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$$

$$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$$

$y = -\frac{3}{2}$ so it is 3/2 units below x-axis.

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40. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness than melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(1) $\frac{1}{36\pi}$ cm/min
(2) $\frac{1}{18\pi}$ cm/min
(3) $\frac{1}{54\pi}$ cm/min
(4) $\frac{5}{6\pi}$ cm/min

40. (2)

$$\frac{dv}{dt} = 50$$

$$4\pi r^2 \frac{dr}{dt} = 50$$

$$\Rightarrow \frac{dr}{dt} = \frac{50}{4\pi(15)^2} \quad \text{where } r = 15$$

$$= \frac{1}{16\pi}.$$

41. $\int \left\{\frac{(\log x - 1)}{(1+(\log x)^2)}\right\}^2 dx$ is equal to

(1) $\frac{\log x}{(\log x)^2 + 1} + C$
(2) $\frac{x}{x^2+1} + C$
(3) $\frac{xe^x}{1+x^2} + C$
(4) $\frac{x}{(\log x)^2 + 1} + C$

41. (4)

$$\int \frac{(\log x - 1)^2}{\left(1+(\log x)^2\right)^2} dx$$

$$= \int \left[\frac{1}{\left(1+(\log x)^2\right)} - \frac{2\log x}{\left(1+(\log x)^2\right)^2}\right] dx$$

$$= \int \left[\frac{e^t}{1+t^2} - \frac{2t\, e^t}{\left(1+t^2\right)^2}\right] dt \quad \text{put } \log x = t \Rightarrow dx = e^t\, dt$$

$$\int e^t \left[\frac{1}{1+t^2} - \frac{2t}{\left(1+t^2\right)^2}\right] dt$$

$$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\log x)^2} + c$$

42. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim\limits_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals

(1) 24
(2) 36
(3) 12
(4) 18

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42. **(4)**

$$\lim_{x\to 2}\int_0^{f(x)}\frac{4t^3}{x-2}dt$$

Applying L Hospital rule

$$\lim_{x\to 2}\left[4f(x)^2\ f'(x)\right] = 4f(2)^3\ f'(2)$$

$$= 4\times 6^3\times\frac{1}{48} = 18.$$

43. Let f (x) be a non–negative continuous function such that the area bounded by the curve y = f (x), x–axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(1) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (2) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(3) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

43. **(4)**

Given that $\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$

Differentiating w. r. t β

$f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

$$f\left(\frac{\pi}{2}\right)=\left(1-\frac{\pi}{4}\right)\sin\frac{\pi}{2}+\sqrt{2}=1-\frac{\pi}{4}+\sqrt{2}.$$

44. The locus of a point P (α, β) moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(1) an ellipse (2) a circle

(3) a parabola (4) a hyperbola

44. **(4)**

Tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$y = mx \pm \sqrt{a^2m^2-b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola

$\Rightarrow m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

$\therefore a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

45. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ the value of λ is

(1) $\frac{5}{3}$ (2) $\frac{-3}{5}$

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(3) $\frac{3}{4}$ (4) $\frac{-4}{3}$

45. **(1)**

Angle between line and normal to plane is

$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$ where θ is angle between line & plane

$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3}$

$\Rightarrow \lambda = \frac{5}{3}$.

46. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(1) 0^0 (2) 90^0

(3) 45^0 (4) 30^0

46. **(2)**

Angle between the lines $2x = 3y = -z$ & $6x = -y = -4z$ is $90°$

Since $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

47. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals

(1) -1 (2) 1

(3) -2 (4) 2

47. **(3)**

Plane

$2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the centre of spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ respectively

centre of spheres are (-3, 4, 1) & (5, - 2, 1)

Mid point of centre is (1, 1, 1)

Satisfying this in the equation of plane, we get

$2a - 3a + 4a + 6 = 0 \Rightarrow a = -2$.

48. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(1) $\frac{10}{9}$ (2) $\frac{10}{3\sqrt{3}}$

(3) $\frac{3}{10}$ (4) $\frac{10}{3}$

48. **(2)**

Distance between the line

$\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$ is

equation of plane is $x + 5y + z = 5$

$\therefore$ Distance of line from this plane

= perpendicular distance of point (2, -2, 3) from the plane

i.e. $\left|\frac{2-10+3-5}{\sqrt{1+5^2+1}}\right| = \frac{10}{3\sqrt{3}}$.

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49. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

(1) $3\vec{a}^2$ (2) $\vec{a}^2$

(3) $2\vec{a}^2$ (4) $4\vec{a}^2$

49. (3)

Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{a}\times\hat{i}=z\hat{j}-y\hat{k}$

$\Rightarrow \left(\vec{a}\times\hat{i}\right)^2=y^2+z^2$

similarly $\left(\vec{a}\times\hat{j}\right)^2=x^2+z^2$

and $\left(\vec{a}\times\hat{k}\right)^2=x^2+y^2 \Rightarrow \left(\vec{a}\times\hat{i}\right)^2=y^2+z^2$

similarly $\left(\vec{a}\times\hat{j}\right)^2=x^2+z^2$

and $\left(\vec{a}\times\hat{k}\right)^2=x^2+y^2$

$\Rightarrow \left(\vec{a}\times\hat{i}\right)^2+\left(\vec{a}\times\hat{j}\right)^2+\left(\vec{a}\times\hat{k}\right)^2=2\left(x^2+y^2+z^2\right)=2\vec{a}^2$.

50. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is

(1) (-1, 2) (2) (-1, -2)

(3) (1, -2) (4) $\left(1, -\frac{1}{2}\right)$

50. (3)

a, b, c are in H.P.

$\Rightarrow \frac{2}{b}-\frac{1}{a}-\frac{1}{c}=0$

$\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$

$\Rightarrow \frac{x}{-1}=\frac{y}{2}=\frac{1}{-1}$ $\therefore$ x = 1, y = -2

51. If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (-1, 2) and (3, 2), then the centroid of the triangle is

(1) $\left(-1, \frac{7}{3}\right)$ (2) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

(3) $\left(1, \frac{7}{3}\right)$ (4) $\left(\frac{1}{3}, \frac{7}{3}\right)$

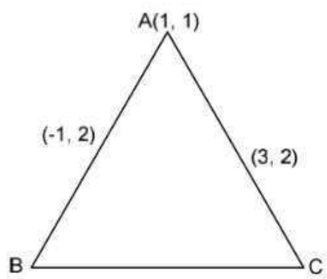
51. (3)

Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (-1, 2) and (3, 2)

$\Rightarrow$ vertex B and C come out to be (-3, 3) and (5, 3)

$\therefore$ centroid is $\frac{1-3+5}{3}, \frac{1+3+3}{3}$

$\Rightarrow$ (1, 7/3)



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52. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for

(1) exactly one value of a (2) no value of a

(3) infinitely many values of a (4) exactly two values of a

52. (2)

$S_1 = x^2 + y^2 + 2ax + cy + a = 0$

$S_2 = x^2 + y^2 - 3ax + dy - 1 = 0$

Equation of radical axis of S_1 and S_2

$S_1 - S_2 = 0$

$\Rightarrow 5ax + (c - d)y + a + 1 = 0$

Given that $5x + by - a = 0$ passes through P and Q

$\Rightarrow \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a}$

$\Rightarrow a + 1 = -a^2$

$a^2 + a + 1 = 0$

No real value of a.

53. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(1) an ellipse (2) a circle

(3) a hyperbola (4) a parabola

53. (4)

Equation of circle with centre (0, 3) and radius 2 is

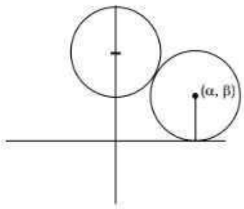
$x^2 + (y - 3)^2 = 4$.

Let locus of the variable circle is (α, β)

$\because$ It touches x-axis.

$\therefore$ It equation $(x - \alpha)^2 + (y - \beta)^2 = \beta^2$

Circles touch externally



$\therefore \sqrt{\alpha^2 + (\beta - 3)^2} = 2 + \beta$

$\alpha^2 + (\beta - 3)^2 = \beta^2 + 4 + 4\beta$

$\alpha^2 = 10(\beta - 1/2)$

$\therefore$ Locus is $x^2 = 10(y - 1/2)$ which is parabola.

54. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

(1) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ (2) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(3) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ (4) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

54. (4)

Let the centre be (α, β)

$\because$ It cut the circle $x^2 + y^2 = p^2$ orthogonally

$2(-\alpha) \times 0 + 2(-\beta) \times 0 = c_1 - p^2$

$c_1 = p^2$

Let equation of circle is $x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

It pass through (a, b) $\Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

Locus $\therefore$ $2ax + 2by - (a^2 + b^2 + p^2) = 0$.

55. An ellipse has OB as semi minor axis, F and F′ its focii and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

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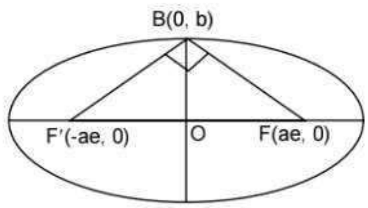
(3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

55. **(1)**



$\because \angle FBF' = 90^{o}$

$\therefore \left(\sqrt{a^2e^2+b^2}\right)^2 + \left(\sqrt{a^2e^2+b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2$

$\Rightarrow e^2 = b^2/a^2$

Also $e^2 = 1 - b^2/a^2 = 1 - e^2$

$\Rightarrow 2e^2 = 1,\ e = \frac{1}{\sqrt{2}}.$

56. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(1) the Geometric Mean of a and b (2) the Arithmetic Mean of a and b

(3) equal to zero (4) the Harmonic Mean of a and b

56. **(1)**

Vector $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab$$

$\therefore$ a, b, c are in G.P.

57. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then

$\left[\lambda\left(\vec{a}+\vec{b}\right) \lambda^2\vec{b}\ \lambda\vec{c}\right] = \left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$ for

(1) exactly one value of λ (2) no value of λ

(3) exactly three values of λ (4) exactly two values of λ

57. **(2)**

$$\left[\lambda\left(\vec{a}+\vec{b}\right)\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right] = \left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$$

$$\begin{vmatrix} \lambda & \lambda & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$\Rightarrow \lambda^4 = -1$

Hence no real value of λ.

58. Let $\vec{a} = \hat{i} - \hat{k},\ \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $\left[\vec{a},\ \vec{b},\ \vec{c}\right]$ depends on

(1) only y (2) only x

(3) both x and y (4) neither x nor y

58. **(4)**

$\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

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$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = \hat{i}\,(1 + x - x - x^2) - \hat{j}\,(x + x^2 - xy - y + xy) + \hat{k}\,(x^2 - y)$$

$\vec{a}.\left(\vec{b}\times\vec{c}\right) = 1$

which does not depend on x and y.

59. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(1) $\frac{2}{9}$ (2) $\frac{1}{9}$

(3) $\frac{8}{9}$ (4) $\frac{7}{9}$

59. **(2)**

For a particular house being selected

Probability = $\frac{1}{3}$

Prob(all the persons apply for the same house) = $\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3 = \frac{1}{9}$.

60. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals

(1) $\frac{2}{e^2}$ (2) 0

(3) $1-\frac{3}{e^2}$ (4) $\frac{3}{e^2}$

60. **(3)**

$P(x = k) = e^{-\lambda}\,\frac{\lambda^k}{k!}$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - e^{-\lambda} - e^{-\lambda}\left(\frac{\lambda}{1!}\right)$

$= 1 - \frac{3}{e^2}$.

61. Let A and B be two events such that $P\left(\overline{A\cup B}\right) = \frac{1}{6}$, $P\left(A\cap B\right) = \frac{1}{4}$ and $P\left(\bar{A}\right) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are

(1) equally likely and mutually exclusive
(2) equally likely but not independent
(3) independent but not equally likely
(4) mutually exclusive and independent

61. **(3)**

$P\left(\overline{A\cup B}\right) = \frac{1}{6}$, $P(A\cap B) = \frac{1}{4}$ and $P\left(\bar{A}\right) = \frac{1}{4}$

$\Rightarrow P(A\cup B) = 5/6 \;\; P(A) = 3/4$

Also $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow P(B) = 5/6 - 3/4 + 1/4 = 1/3$

$P(A)\,P(B) = 3/4 - 1/3 = 1/4 = P(A\cap B)$

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Hence A and B are independent but not equally likely.

62. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

(1) 20 s (2) 1 s

(3) 21 s (4) 24 s

62. (3)

$\frac{1}{2}2t^2 = 21 + 20t$

$\Rightarrow t = 21.$

63. Two points A and B move from rest along a straight line with constant acceleration f and f respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $(f - f')m^2 = ff'n$ (2) $(f + f')m^2 = ff'n$

(3) $\frac{1}{2}(f + f')m = ff'n^2$ (4) $(f' - f)n = \frac{1}{2}ff'm^2$

63. (4)

$v^2 = 2f(d + n) = 2f'd$

$v = f'(t) = (m + t)f$

eliminate d and m we get

$(f' - f)n = \frac{1}{2}ff'm^2$.

64. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance

(1) $\frac{2H}{A-B}$ (2) $\frac{H}{A+B}$

(3) $\frac{H}{2(A+B)}$ (4) $\frac{H}{A-B}$

64. (2)

$(A + B) = d = H$

$d = \left(\frac{H}{A+B}\right)$.

65. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is

(1) 2 : 1 (2) $3 : \sqrt{2}$

(3) 3 : 2 (4) $3 : 2\sqrt{2}$

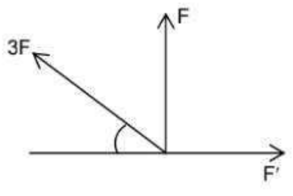
65. (4)

$F' = 3F \cos\theta$

$F = 3F \sin\theta$

$\Rightarrow F' = 2\sqrt{2}\, F$

$F : F' :: 3 : 2\sqrt{2}$.



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66. The sum of the series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+\ldots\ldots\ldots$ ad inf. is

(1) $\frac{e-1}{\sqrt{e}}$ (2) $\frac{e+1}{\sqrt{e}}$

(3) $\frac{e-1}{2\sqrt{e}}$ (4) $\frac{e+1}{2\sqrt{e}}$

66. (4)

$\frac{e^x+e^{-x}}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots\ldots$

putting x = 1/2 we get

$\frac{e+1}{2\sqrt{e}}$.

67. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, a > 0, is

(1) $a\pi$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{a}$ (4) 2π

67. (2)

$\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}\,dx=\int_0^{\pi}\cos^2 x\,dx=\frac{\pi}{2}$.

68. The plane x + 2y – z = 4 cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius

(1) 3 (2) 1

(3) 2 (4) $\sqrt{2}$

68. (2)

Perpendicular distance of centre $\left(\frac{1}{2},0,-\frac{1}{2}\right)$ from x + 2y – 2 = 4

$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}}=\sqrt{\frac{3}{2}}$

radius = $\sqrt{\frac{5}{2}-\frac{3}{2}}=1$.

69. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(1) $3a^2-10ab+3b^2=0$ (2) $3a^2-2ab+3b^2=0$

(3) $3a^2+10ab+3b^2=0$ (4) $3a^2+2ab+3b^2=0$

69. (4)

$\left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|=1$

$\Rightarrow (a+b)^2=4(a^2+b^2+ab)$

$\Rightarrow 3a^2+3b^2+2ab=0$.

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70. Let x_1, x_2, ...,x_n be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(1) 15 (2) 18
(3) 9 (4) 12

70. (2)

$\frac{\sum x_i^2}{n} \geq \left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow n \geq 16.$

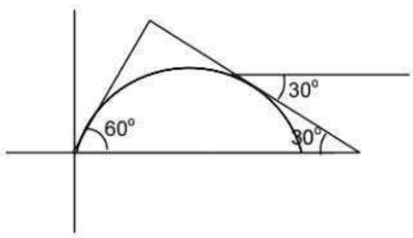
71. A particle is projected from a point O with velocity *u* at an angle of 60^o with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

(1) $\frac{u}{3}$ (2) $\frac{u}{2}$
(3) $\frac{2u}{3}$ (4) $\frac{u}{\sqrt{3}}$

71. (4)

$u \cos 60^o = v \cos 30^o$

$v = \frac{4}{\sqrt{3}}$.



72. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(1) (5, 6] (2) (6, ∞)
(3) (-∞, 4) (4) [4, 5]

72. (3)

$\frac{-b}{2a} < 5$

$f(5) > 0$

$\Rightarrow k \in (-\infty, 4)$.

73. If a_1, a_2, a_3,..., a_n,... are in G.P., then the determinant

$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(1) 1 (2) 0
(3) 4 (4) 2

73. (2)

$C_1 - C_2$, $C_2 - C_3$

two rows becomes identical

Answer: 0.

74. A real valued function f(x) satisfies the functional equation f(x – y) = f(x) f(y) – f(a – x) f(a + y) where a is a given constant and f(0) = 1, f(2a – x) is equal to

(1) –f(x) (2) f(x)
(3) f(a) + f(a – x) (4) f(-x)

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74. **(1)**

f(a – (x – a)) = f(a) f(x – a) – f(0) f(x)

= -f(x) $\left[\because\ x = 0,\ y = 0,\ f(0) = f^2(0) - f^2(a)\ \Rightarrow\ f^2(a) = 0\ \Rightarrow\ f(a) = 0\right]$.

75. If the equation

$a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is

(1) greater than α

(2) smaller than α

(3) greater than or equal to α

(4) equal to α

75. **(2)**

f(0) = 0, f(α) = 0

$\Rightarrow$ f'(k) = 0 for some k$\in$(0, α).

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