FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Sunday 06th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

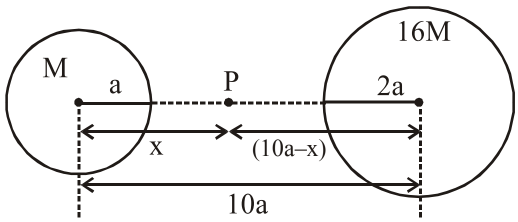
PHYSICS | TEST PAPER WITH ANSWER & SOLUTION

1. Two planets have masses M and 16 M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :

(1) $\sqrt{\frac{GM^2}{ma}}$ (2) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(3) $4\sqrt{\frac{GM}{a}}$ (4) $2\sqrt{\frac{GM}{a}}$

Sol.



$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$

$$\frac{1}{x} = \frac{4}{(10a-x)} \quad \Rightarrow 4x = 10a - x$$

$x = 2a$ ....(i)

COME

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE$$

$$= -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$$

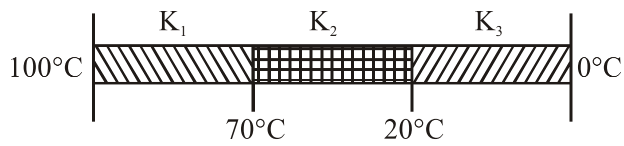
$$KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right]$$

$$V = \sqrt{\frac{90GM}{8a}}$$

$$V = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

2. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2, and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is :



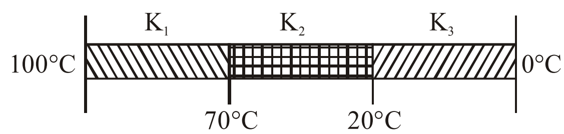
(1) $K_1 : K_3 = 2 : 3$; $K_2 : K_3 = 2 : 5$

(2) $K_1 < K_2 < K_3$

(3) $K_1 : K_2 = 5 : 2$; $K_1 : K_3 = 3 : 5$

(4) $K_1 > K_2 > K_3$

Sol.



Rods are identical have same length (ℓ) and area of cross-section (A)

Combination are in series, so heat current is same for all Rods

$$\left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD} = \text{Heat current}$$

$$\frac{(100-70)K_1A}{\ell} = \frac{(70-20)K_2A}{\ell} = \frac{(20-0)K_3A}{\ell}$$

$30K_1 = 50K_2 = 20K_3$

$3K_1 = 2K_3$

$$\frac{K_1}{K_3} = \frac{2}{3} = 2:3$$

$5K_2 = 2K_3$

$$\frac{K_2}{K_3} = \frac{2}{5} = 2:5$$

3. For a plane electromagnetic wave, the magnetic field at a point x and time t is

$$\vec{B}(x,t) = \left[1.2\times10^{-7}\sin(0.5\times10^{3}x + 1.5\times10^{11}t)\hat{k}\right]T$$

The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is : (speed of light $c = 3 \times 10^8\ ms^{-1}$)

(1) $\vec{E}(x,t) = \left[36\sin(0.5\times10^{3}x + 1.5\times10^{11}t)\hat{k}\right]\frac{V}{m}$

(2) $\vec{E}(x,t) = \left[-36\sin(0.5\times10^{3}x + 1.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

(3) $\vec{E}(x,t) = \left[36\sin(1\times10^{3}x + 0.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

(4) $\vec{E}(x,t) = \left[36\sin(1\times10^{3}x + 1.5\times10^{11}t)\hat{j}\right]\frac{V}{m}$

Sol. $\vec{E}$ and $\vec{B}$ are perpendicular for EM wave

$E_0 = CB_0$

$= 3 \times 10^8 \times 1.2 \times 10^{-7}$

$= 36$

Having same phase

Propagation is along –x-axis, $\vec{B}$ is along z-axis hence $\vec{E}$ must be along y-axis.

So, option (2) is correct

4. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is:

(1) $\frac{R}{2}$ (2) 2R

(3) $\frac{3R}{2}$ (4) $\frac{R}{3}$

Sol.

$R_1 = R_2 = R$

Power (P)

Refractive index is assume (μ_ℓ)

$$P = \frac{1}{f} = (\mu_\ell - 1)\left(\frac{2}{R}\right) \quad ....(i)$$

$$P' = \frac{1}{f'} = (\mu_\ell - 1)\left(\frac{1}{R'}\right) \quad ....(ii)$$

$$P' = \frac{3}{2}P$$

$$(\mu_\ell - 1)\left(\frac{1}{R'}\right) = \mu\frac{3}{2}(\mu_\ell - 1)\left(\frac{2}{R}\right)$$

$$\therefore \quad R' = \frac{R}{3}$$

5. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit :

(1) ammeter is always connected series and voltmeter in parallel.

(2) Both, ammeter and voltmeter mast be connected in series.

(3) Both ammeter and voltmeter must be connected in parallel.

(4) ammeter is always used in parallel and voltmeter is series.

Sol. Conceptual

Option (1) is correct

Ammeter :- In series connection, the same current flows through all the components. It aims at measuring the curent flowing through the circuit and hence, it is connected in series.

Voltmeter :- A voltmeter measures voltage change between two points in a circuit, So we have to place the voltmeter in parallal with the circuit component.

6. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If $\vec{a}$ Ls the acceleration of the particle, then which of the following statements is true for the particle ?

(1) quantity $\vec{v}\cdot\vec{a}$ is constant in time.

(2) kinetic energy of particle is constant in time.

(3) quantity $\vec{v}\times\vec{a}$ is constant in lime.

(4) $\vec{F}$ arises due to a magnetic field.

Sol. $\dfrac{dv_x}{dt} = \dfrac{k}{m}v_y$

$$\frac{dv_y}{dt} = \frac{k}{m}v_x$$

$$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$$

$$v_y^2 = v_x^2 + C$$

$$v_y^2 - v_x^2 = \text{constant}$$

Option (3)

$$\vec{v}\times\vec{a} = (v_x\hat{i} + v_y\hat{j})\times\frac{k}{m}(v_y\hat{i} + v_x\hat{j})$$

$$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m}$$

$$= (v_x^2 - v_y^2)\frac{k}{m}\hat{k}$$

= Constant

7. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell ?

(1) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$ for r > R

(2) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{R^2} > F > 0$ for r < R

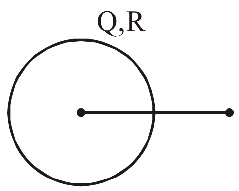
(3) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$ for all r

(4) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{R^2}$ for r < R

Sol. Inside the shell

E = 0

hence F = 0



Oustside the sheell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

hence $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$ for r > R

8. Given the masses of various atomic particles $m_p = 1.0072u$, $m_n = 1.0087u$, $m_e = 0.000548u$, $m_{\bar{v}} = 0$, $m_d = 2.0141u$, where $p \equiv$ proton, $n \equiv$ neutron, $e \equiv$ electron, $\bar{v} \equiv$ antineutrino and $d \equiv$ deuteron. Which of the following process is allowed by momentum and energy conservation ?

(1) $n + p \rightarrow d + \gamma$

(2) $e^+ + e^- \rightarrow \gamma$

(3) $n + n \rightarrow$ deuterium atom

(electron bound to the nucleus)

(4) $p \rightarrow n + e^+ + \bar{v}$

Sol. Only in case-I, $M_{LHS} > M_{RHS}$ i.e.

total mass on reactant side is greater then that on the product side. Hence it will only be allowed.

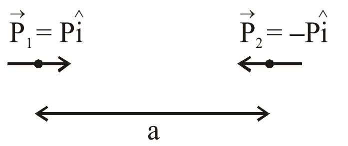
9. Two identical electric point dipoles have dipole moments $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is'm', their speed when they arc infinitely far apart is:

(1) $\frac{p}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (2) $\frac{p}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

(3) $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ (4) $\frac{p}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$

Sol. Using energy conservation:

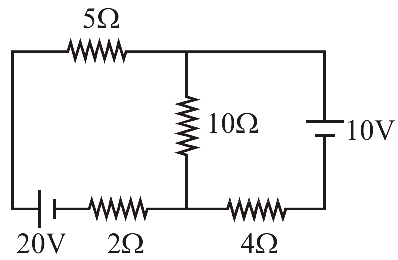
$KE_i + PE_i = KE_f + PE_f$



$$O + \frac{2KP}{a^3} \times P = \frac{1}{2}mv^2 \times 2 + 0$$

$$V = \sqrt{\frac{2P^2}{4\pi\varepsilon_0 a^3 m}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 am}}$$

10. In the figure shown, the current in the 10 V battery is close to :



(1) 0.36 A from negative to positive terminal.

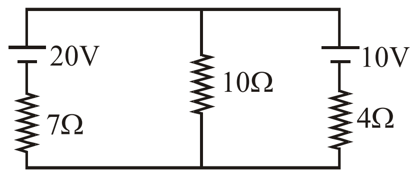
(2) 0.71 A from positive to negative terminal.

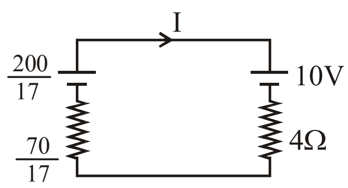
(3) 0.21 A from positive to negative terminal.

(4) 0.42 A from positive to negative terminal.

Sol. $E_{eq} = \frac{20 \times 10}{17} = \frac{200}{17}$

and $R_{eq} = \frac{7 \times 10}{17} = \frac{70}{17}$





$$\therefore \quad I = \frac{\frac{20}{17} - 10}{4 + \frac{70}{17}} = 0.21 \text{ A}$$

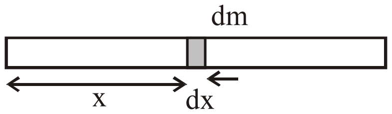
from +ve to –ve terminal

11. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x)=\lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

(1) $\frac{5}{12}ML^2$ (2) $\frac{3}{7}ML^2$

(3) $\frac{2}{5}ML^2$ (4) $\frac{7}{18}ML^2$

Sol.



$$I=\int r^2dm=\int x^2\lambda dx$$

$$I=\int_0^L x^2\lambda_0\left(1+\frac{x}{L}\right)dx$$

$$I=\lambda_0\int_0^L\left(x^2+\frac{x^3}{L}\right)dx$$

$$I=\lambda\left[\frac{L^3}{3}+\frac{L^3}{4}\right]$$

$$I=\frac{7L^3\lambda_0}{12} \quad ....(i)$$

$$M=\int_0^L\lambda dx=\int_0^L\lambda_0\left(1+\frac{x}{L}\right)dx$$

$$M=\lambda_0\left(L+\frac{L}{2}\right)=\lambda_0\frac{3L}{2}$$

$$\frac{2}{3}M=(\lambda_0 L) \quad ....(ii)$$

From (i) & (ii)

$$I=\frac{7}{12}\left(\frac{2}{3}M\right)L^2=\frac{7ML^2}{18}$$

Ans. (4)

12. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as :

(1) (5.5375 ± 0.0739) mm

(2) (5.538 ± 0.074) mm

(3) (5.54 ± 0.07) mm

(4) (5.5375 ± 0.0740) mm

Sol. Use significant figures. Answer must be upto three significant figures.

Ans. (3)

13. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t)=y_0\sin^2\omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is :

(1) $\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{2y_0}}$

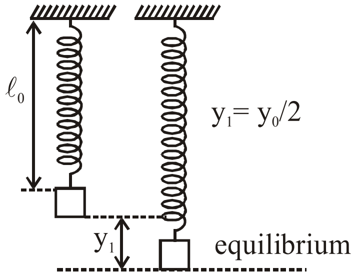
(3) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (4) $\sqrt{\frac{2g}{y_0}}$

Sol. $y=y_0\sin^2\omega t$

$$y=\frac{y_0}{2}(1-\cos 2\omega t)$$

$$y-\frac{y_0}{2}=-\frac{y_0}{2}\cos 2\omega t$$

Amplitude : $\frac{y_0}{2}$



$$\frac{y_0}{2} = \frac{mg}{K}$$

$$2\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2g}{y_0}}$$

$$\omega = \sqrt{\frac{g}{2y_0}}$$

Ans. (2)

14. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as :

(1) $\sqrt{T}$ (2) $\frac{1}{T}$

(3) $\frac{1}{\sqrt{T}}$ (4) T

Sol. $v_{avg} \propto \sqrt{T}$

t_0 : mean time

λ : mean free path

$$t_0 = \frac{\lambda}{v_{avg}} \propto \frac{1}{\sqrt{T}}$$

15. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. P At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to :

(1) $\sqrt{\frac{P}{2\rho} + v^2}$ (2) $\sqrt{\frac{P}{\rho} + v^2}$

(3) $\sqrt{\frac{2P}{\rho} + v^2}$ (4) $\sqrt{\frac{P}{\rho} + v}$

Sol. Applying Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\frac{2P}{2\rho} + \frac{1}{2}\frac{\rho v^2}{\rho} \times 2 = V^2$$

$$\sqrt{\frac{P}{\rho} + v^2} = V$$

Ans. (2)

16. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to : (Given : nitrogen molecule weight : 4.64 × 10^{-26}kg, Boltzman constant : 1.38 × 10^{-23} J/K, Planck constant: 6.63 × 10^{-34} J.s)

(1) 0.34 Å (2) 0.24 Å

(3) 0.20 Å (4) 0.44 Å

Sol. $v_{rms} = \sqrt{\frac{3KT}{m}}$

m $\rightarrow$ mass of one molecule (in kg) = $\frac{\text{molar mass}}{\text{NA}}$

de-Broglie wavelenth,

$$\lambda = \frac{h}{mv}$$

given, v = v_{rms}

$$\lambda = \frac{h}{m\sqrt{\frac{3KT}{m}}}$$

$$\lambda = \frac{h}{\sqrt{3KTm}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times1.38\times10^{-23}\times400\times\left(\frac{28\times10^{-3}}{6.023\times10^{-23}}\right)}}$$

$$\lambda = \frac{6.63\times10^{-11}}{2.77} = 2.39\times10^{-11}\,\text{m}$$

λ = 0.24 Å

17. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i}+\hat{j})ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i}+\sqrt{3}\hat{j})ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is :

(1) 60° (2) 15° (3) –45° (4) 105°

Sol. $\vec{v}_{01} = (\sqrt{3}\hat{i}+\hat{j})$ m / s

$\vec{v}_{02} = \vec{0}$

$m_1 = 2m_2$

After collision, $\vec{v}_1 = (\hat{i}+\sqrt{3}\hat{j})$ m / s

$\vec{v}_2 = ?$

Applying conservation of linear momentum,

$$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$2m_2(\sqrt{3}\hat{i}+\hat{j}) + 0 = 2m_2(\hat{i}+\sqrt{3}\hat{j}) + m_2\vec{v}_2$$

$$\vec{v}_2 = 2(\sqrt{3}\hat{i}+\hat{j}) - 2(\hat{i}+\sqrt{3}\hat{j})$$

$$= 2(\sqrt{3}\hat{i}-\hat{j}) + 2(\hat{i}-\sqrt{3}\hat{j})$$

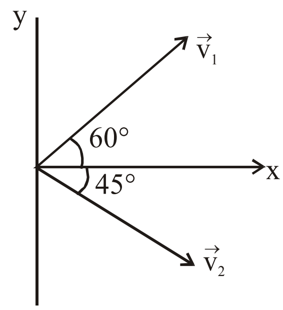
$$\vec{v}_2 = 2(\sqrt{3}-1)(\hat{i}-\hat{j})$$

for angle between $\vec{v}_1$ & $\vec{v}_2$,

$$\cos\theta = \frac{\vec{v}_1\cdot\vec{v}_2}{v_1 v_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2\times2\sqrt{2}(\sqrt{3}-1)}$$

$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 105°$$

or

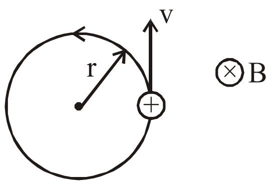


18. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle :

(1) $-\frac{mv^2\vec{B}}{B^2}$ (2) $-\frac{mv^2\vec{B}}{2\pi B^2}$

(3) $\frac{mv^2\vec{B}}{2B^2}$ (4) $-\frac{mv^2\vec{B}}{2B^2}$

Sol.



Magnetic moment

M = iA

$$M = \left(\frac{q}{T}\right) \times \pi r^2 = \frac{q\pi r^2}{\left(\frac{2\pi r}{v}\right)} = \frac{qvr}{2}$$

$$M = \frac{qv}{2} \times \frac{vm}{qB}$$

$$M = \frac{mv^2}{2B}$$

As we can see from the figure, direction of magnetic moment (M) is opposite to magnetic field.

$$\vec{M} = -\frac{mv^2}{2B}\hat{B}$$

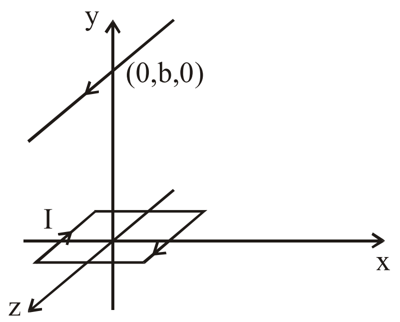
$$= -\frac{mv^2}{2B^2}\vec{B}$$

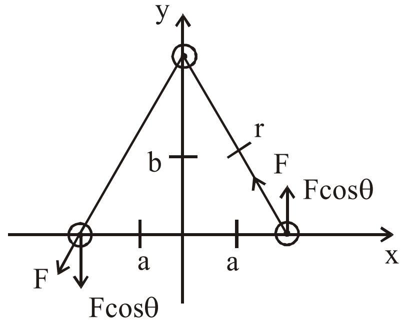
19. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0, b, 0), (b >> a). The magnitude of torque on the loop about z-ax is will be :

(1) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$ (2) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2+b^2)}$

(3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^2}{\pi b}$

Sol.





$r = \sqrt{b^2 + a^2}$

$$F = BI2a = \frac{\mu_0 I}{2\pi r} I \times 2a$$

$$F = \frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}}$$

$$\tau = F\cos\theta \times 2a$$

$$= \frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}} \times \frac{b}{\sqrt{b^2+a^2}} \times 2a$$

$$\tau = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$$

If b >> a then $\tau = \frac{2\mu_0 I^2 a^2}{\pi b}$

But among the given options (1) is most appropriate

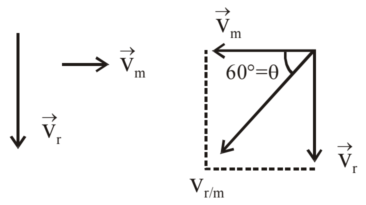
20. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to (1 + β)v, this angle changes to 45°. The value of β is close to:

(1) 0.41 (2) 0.50

(3) 0.37 (4) 0.73

Sol. Rain is falling vertically downwards.

$$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$$

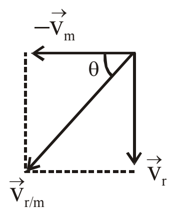


$$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$$

$$v_r = v_m\sqrt{3} = v\sqrt{3}$$

Now, $v_m = (1 + B)v$

and $\theta = 45°$



$$\tan 45 = \frac{v_r}{v_m} = 1$$

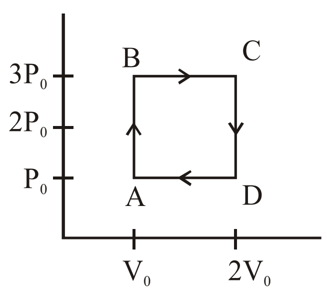
$v_r = v_m$

$$v\sqrt{3} = (1+\beta)v$$

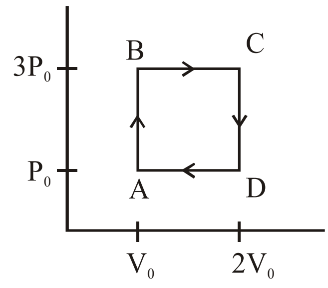
$$\sqrt{3} = 1+\beta$$

$$\Rightarrow \beta = \sqrt{3}-1 = 0.73$$

21. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to ________ .



Sol.



$W_{ABCDA} = 2P_0V_0$

$Q_{in} = Q_{AB} + Q_{BC}$

$Q_{AB} = nC(T_B - T_A)$

$$= \frac{n3R}{2}(T_B - T_A)$$

$$= \frac{3}{2}(P_BV_B - P_AV_A)$$

$$= \frac{3}{2}(3P_BV_0 = P_0V_0) = 3P_0V_0$$

$Q_{BC} = nC_P(T_C - T_B)$

$$= \frac{n5R}{2}(T_C - T_B)$$

$$= \frac{5}{2}(P_CV_C - P_BV_B)$$

$$= \frac{5}{2}(6P_0V_0 - 3P_0V_0) = \frac{15}{2}P_0V_0$$

$$\eta = \frac{W}{Q_{in}} \times 100 = \frac{2P_0V_0}{3P_0V_0 + \frac{15}{2}P_0V_0} \times 100$$

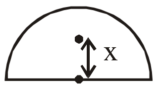
$$\eta = \frac{400}{21} = 19.04 \approx 19$$

$\eta = 19$

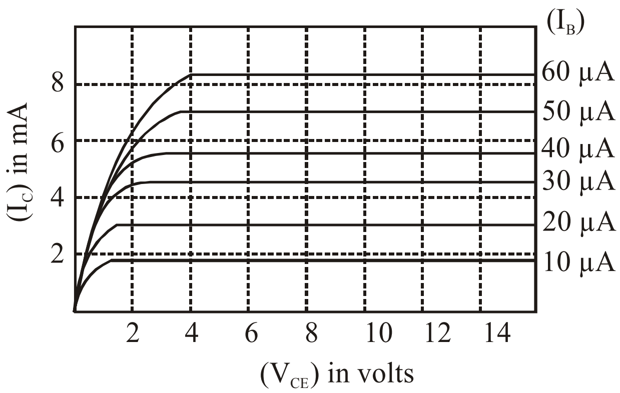
22. The centre of mass of a solid hemisphere of radius 8 cm is X cm from the centre of the flat surface. Then value of x is ________ .

Sol. $x = \frac{3R}{8} = 3\text{cm}$

$x = 3$



23. The output characteristics of a transistor is shown in the figure. When V_{CE} is 10 V and I_C = 4.0 mA, then value of β_{ac} is ________ .



Sol. $\Delta I_B = (30 - 20) = 10\mu A$

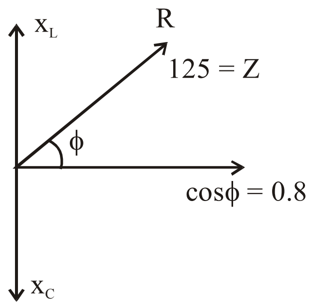
$\Delta I_C = (4.5 - 3)\ \text{mA} = 1.5\text{mA}$

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5\text{mA}}{10\mu A} = 150$$

$\beta_{ac} = 150$

24. In a scries LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is ________ .

Sol.



$$P = \frac{E_{rms}^2}{Z}\cos\phi$$

$$400 = \frac{(250)^2 \times 0.8}{Z}$$

$Z = 25 \times 5 = 125$

$X_L = 125 \sin\phi = 125 \times 0.6 = 75$

25. A Young's doublc-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is A $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is ________ .

Sol. $I_{max} = k$

$I_1 = I_2 = K/4$

$\Delta x = \lambda/6 \quad \Rightarrow \Delta\phi = \pi/3$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$I = \frac{K}{4} + \frac{K}{4} + 2 \times \frac{K}{4}\frac{1}{2}$$

$$= \frac{K}{2} + \frac{K}{4} = \frac{3K}{4} = \frac{9K}{12}$$

$n = 9$

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Sunday 06th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. The value of K_C is 64 at 800 K for the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The value of K_C for the following reaction is :

$$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$$

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) 8 (4) $\frac{1}{64}$

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3 \rightarrow K_C = 64$

$$2NH_3 \rightleftharpoons N_2 + 3H_2 \rightarrow K_C = \frac{1}{64}$$

$$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow K_C = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}$$

2. The element that can be refined by distillation is :

(1) nickel (2) zinc

(3) gallium (4) tin

Sol. Impure zinc is refined by distillation method.

3. The correct match between **Item-I** and **Item-II** :

Item-I		Item-II
(a) Natural rubber	(I)	1, 3-butadiene + styrene
(b) Neoprene	(II)	1, 3-butadiene + acrylonitrile
(c) Buna-N	(III)	Chloroprene
(d) Buna-S	(IV)	Isoprene

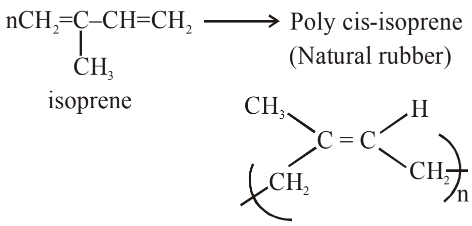
(1) (a) - (III), (b) - (IV), (c) - (I), (d) - (II)

(2) (a) - (IV), (b) - (III), (c) - (II), (d) - (I)

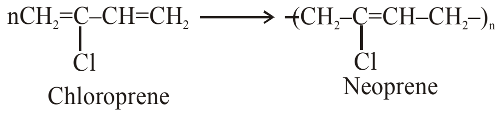
(3) (a) - (IV), (b) - (III), (c) - (I), (d) - (II)

(4) (a) - (III), (b) - (IV), (c) - (II), (d) - (I)

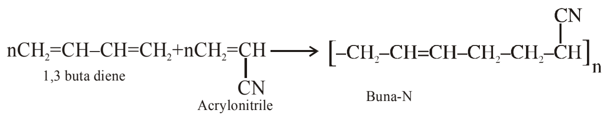
Sol.(a)



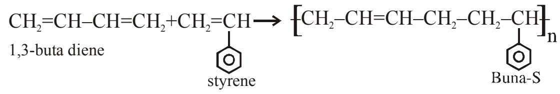
(b)



(c)



(d)



4. Mischmetal is an alloy consisting mainly of:

(1) lanthanoid metals

(2) actinoid metals

(3) actinoid and transition metals

(4) lanthanoid and actinoid metals

Sol. Alloys of lanthanides with Fe are called Misch metal, which consists of a lanthanoid metal (~95%) and iron (~5%) and traces of S, C, Ca and Al.

5. Reaction of an inorganic sulphite X with dilute H_2SO_4 generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and Z, respectively, are:

(1) S and Na_2SO_3

(2) SO_2 and $NaHSO_3$

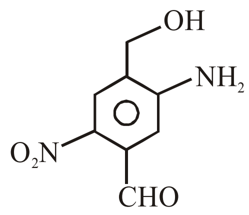
(3) SO_3 and $NaHSO_3$

(4) SO_2 and Na_2SO_3

Sol. $\underset{(X)}{Na_2SO_3} \xrightarrow{\text{dil. } H_2SO_4} \underset{(Y)}{SO_2} \xrightarrow{NaOH} \underset{(X)}{Na_2SO_3}$

$$\underset{(X)}{Na_2SO_3} \xrightarrow{SO_2 + H_2O} \underset{(Z)}{NaHSO_3}$$

6. The IUPAC name of the following compound is :



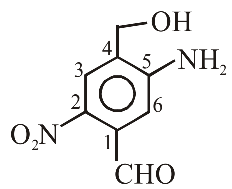
(1) 3-amino-4-hydroxymethyl-5-nitrobenzaldehyde

(2) 2-nitro-4-hydroxymethyl-5-aminobenzaldehyde

(3) 4-amino-2-formyl-5-hydroxymethylnitrobenzene

(4) 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

Sol.



5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

7. Dihydrogen of high purity (> 99.95%) is obtained through:

(1) the electrolysis of warm $Ba(OH)_2$ solution using Ni electrodes.

(2) the reaction of Zn with dilute HCl

(3) the electrolysis of brine solution.

(4) the electrolysis of acidified water using Pt electrodes.

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysing warm aqueous barium hydroxide solution between nickel electrodes.

8. Match the following :

	Test/Method		**Reagent**
(i)	Lucas Test	(a)	$C_6H_5SO_2Cl$/aq. KOH
(ii)	Dumas method	(b)	$HNO_3/AgNO_3$
(iii)	Kjeldahl's method	(c)	CuO/CO_2
(iv)	Hinsberg Test	(d)	Conc. HCl and $ZnCl_2$
		(e)	H_2SO_4

(1) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)

(2) (i)-(b), (ii)-(d), (iii)-(e), (iv)-(a)

(3) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(e)

(4) (i)-(b), (ii)-(a), (iii)-(c), (iv)-(d)

Sol. **Test** — **Correct reagent**

(i) Lucas test $\longrightarrow$ conc. HCl + $ZnCl_2$

(ii) Dumas method $\longrightarrow$ CuO / CO_2

(iii) Kjeldahl's method $\longrightarrow$ H_2SO_4

(iv) Hinsberg Test $\longrightarrow$ $C_6H_5SO_2Cl$ + aq. KOH

9. The reaction of NO with N_2O_4 at 250 K gives :

(1) N_2O_5 (2) NO_2

(3) N_2O (4) N_2O_3

Sol. $2NO + N_2O_4 \xrightarrow{250K} \underset{\text{blue solid}}{2N_2O_3}$

10. For the given cell ;

$Cu(s)|Cu^{2+}(C_1M)||Cu^{2+}(C_2M)|Cu(s)$ change in Gibbs energy (ΔG) is negative, if :

(1) $C_1 = 2C_2$ (2) $C_2 = C1/\sqrt{2}$

(3) $C_1 = C_2$ (4) $C_2 = \sqrt{2}C_1$

Sol. $\Delta G = -n F E_{cell}$

ΔG is negative, if E_{cell} is positive

Anode : $Cu(s) \longrightarrow Cu^{+2}(C_1) + 2e^- : E°$

Cathode : $Cu^{+2}(C_2) + 2e^- \longrightarrow Cu(S) : -E°$

Cell reaction : $Cu^{+2}(C_2) \longrightarrow Cu^{+2}(C_1)\ E^{\circ}_{cell} = 0$

$$E_{cell} = E^{\circ}_{cell} - \frac{2.303RT}{nF}\log Q$$

$$E_{cell} = 0 - \frac{2.303RT}{nF}\log\left(\frac{C_1}{C_2}\right)$$

$$E_{cell} > 0 : \text{ if } \frac{C_1}{C_2} < 1 \Rightarrow C_1 < C_2$$

11. A crystal is made up of metal ions 'M_1' ana 'M_2' and oxide ions. Oxide ions form a ccp lattice structure. The cation 'M_1' occupies 50% of octahedral voids and the cation 'M_2' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of 'M_1' and 'M_2' are, respectively :

(1) +2, +4 (2) +3, +1

(3) +1, +3 (4) +4, +2

Sol. O^{-2} ions form ccp. $\underset{(-8\text{ charge})}{\overset{O_4}{\downarrow}}$

M_1 = 50% of O.V. $\Rightarrow \frac{50}{100}\times 4 = 2:(M_1)_2$

M_2 = 12.5% of T.V. $\Rightarrow \frac{12.5}{100}\times 8 = 1:(M_2)_1$

So formula is : $(M_1)_2\,(M_2)_1O_4$

This must be neutral. Both metals must have +8 charge in total.

From given options : $\left\{\begin{array}{r} \text{O.N. of } M_1 = +2 \\ M_2 = +4 \end{array}\right\}$

12. For a d^4 metal ion in an octahedral field, the correct electronic configuration is :

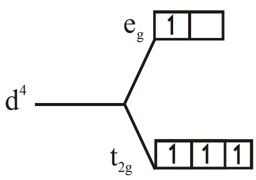
(1) $t_{2g}^4e_g^0$ when $\Delta_O < P$

(2) $e_g^2t_{2g}^2$ when $\Delta_O < P$

(3) $t_{2g}^3e_g^1$ when $\Delta_O < P$

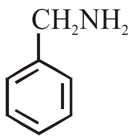
(4) $t_{2g}^3e_g^1$ when $\Delta_O > P$

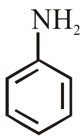
Sol.

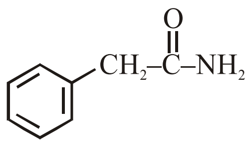


back pairing is not possible because pairing energy $> \Delta_O$.

13. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?

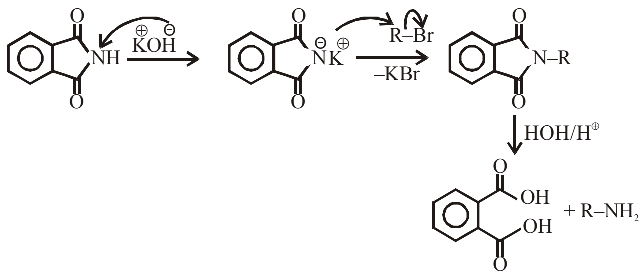
(1) 

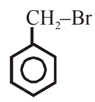
(2) 

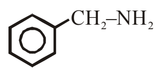
(3) 

(4) $CH_3-CH_2-NHCH_3$

Sol. Gabriel phthalimide synthesis is used for preparation of 1° Aliphatic amine



Here R–Br = 

$R-NH_2$ = 

14. The correct match between **Item-I** (starting material) and **Item-II** (reagent) for the preparation of benzaldehyde is :

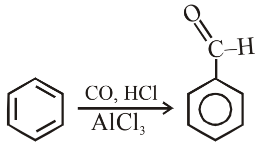
Item-I	**Item-II**
(I) Benzene	(P) HCl and $SnCl_2, H_3O^+$
(II) Benzonitrile	(Q) H_2, Pd-$BaSO_4$, S and quinoline
(III) Benzoyl Chloride	(R) CO, HCl and $AlCl_3$

(1) (I)-(Q), (II)-(R) and (III)-(P)

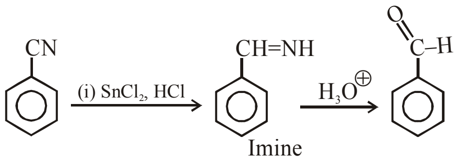
(2) (I)-(R), (II)-(Q) and (III)-(P)

(3) (I)-(R), (II)-(P) and (III)-(Q)

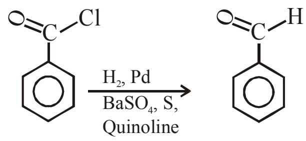
(4) (I)-(P), (II)-(Q) and (III)-(R)

Sol.(i) 

(Gattermann koch reaction)

(ii) 

(Stephen reduction)

(iii) 

(Rosenmund reduction)

15. The average molar mass of chlorine is 35.5 g mol^{-1}. The ratio of ^{35}Cl to ^{37}Cl in naturally occurring chlorine is close to :

(1) 4 : 1

(2) 1 : 1

(3) 2 : 1

(4) 3 : 1

Sol.

	^{35}Cl		^{37}Cl	Av. molar mass = 35.5
let mole ratio	x	:	1	

Av. molar mass = $\dfrac{n_1M_1 + n_2M_2}{(n_1 + n_2)}$

$$35.5 = \frac{x \times 35 + 1 \times 37}{x + 1}$$

x = 3

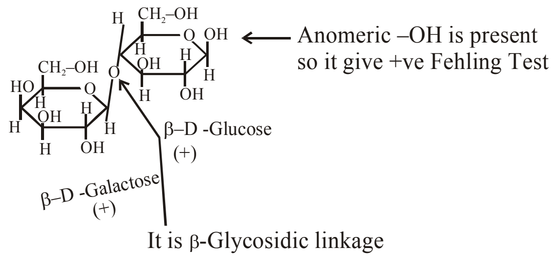
16. Which one of the following statements not true ?

(1) Lactose contains α-glycosidic linkage between C_1 of galactose and C_4 of glucose.

(2) Lactose ($C_{11}H_{22}O_{11}$) is a disaccharide and it contains 8 hydroxyl groups.

(3) On acid hydrolysis, lactose gives one molecule of D(+)-glucose and one molecule of D(+)-galactose.

(4) Lactose is a reducing sugar and it gives Fehling's test.

Sol. 

Structure of Lactose

structure of lactose

17. A set of solutions is prepared using 180 g of water as a solvent and 10 g of different non-volatile solutes A, B and C. The relative lowering of vapour pressure in the presence of these solutes are in the order [Given, molar mass of A = 100 g mol^{-1}; B = 200 g mol^{-1}; C = 10,000 g mol^{-1}]

(1) A > B > C (2) A > C > B

(3) C > B > A (4) B > C > A

Sol. Relative lowering of V.P. = $\dfrac{\Delta P}{P^0} = x_{solute}$

$$\left(\frac{\Delta P}{P^0}\right)_A = \frac{\frac{10}{100}}{\frac{10}{100}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_B = \frac{\frac{10}{200}}{\frac{10}{200}+\frac{180}{18}}$$

$$\left(\frac{\Delta P}{P^0}\right)_C = \frac{\frac{10}{10,000}}{\frac{10}{10,000}+\frac{180}{18}} : \left(\frac{\Delta P}{P^0}\right)_A > \left(\frac{\Delta P}{P^0}\right)_B > \left(\frac{\Delta P}{P^0}\right)_C$$

18. For a reaction,

$4M(s) + nO_2(g) \rightarrow 2M_2O_n(s)$,

the free energy change is plotted as a function of temperature. The temperature below which the oxide is stable could be inferred from the plot as the point at which :

(1) the slope changes from positive to zero

(2) the free energy change shows a change from negative to positive value

(3) the slope changes from negative to positive

(4) the slope changes from positive to negative

19. Match the following compounds (Column-I) with their uses (Column-II) :

S.No.	Column – I	S.No.	Column – II
(I)	$Ca(OH)_2$	(A)	casts of statues
(II)	NaCl	(B)	white wash
(III)	$CaSO_4.\frac{1}{2}H_2O$	(C)	antacid
(IV)	$CaCO_3$	(D)	washing soda preparation

(1) (I)-(D), (II)-(A), (III)-(C), (IV)-(B)

(2) (I)-(B), (II)-(C), (III)-(D), (IV)-(A)

(3) (I)-(C), (II)-(D), (III)-(B), (IV)-(A)

(4) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

Sol. (I) $Ca(OH)_2$ is used in white wash

(II) NaCl is used in preparation of washing soda

$$2NH_3 + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3$$

$$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow 2NH_4HCO_3$$

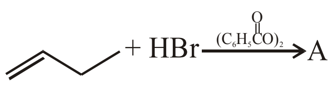
$$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3(s)$$

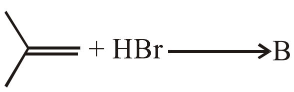
$$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$$

(III) $CaSO_4.\frac{1}{2}H_2O$ (Plaster of Paris) is used for making casts of statues

(IV) $CaCO_3$ is used as an antacid

20. The increasing order of the boiling points of the major products A, B and C of the following reactions will be :

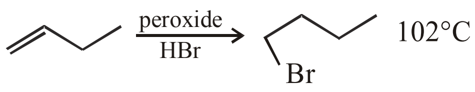
(a) 

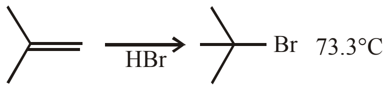
(b) 

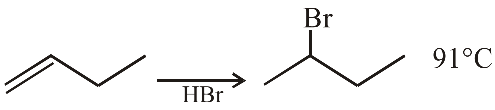
(c) 

(1) C < A < B (2) B < C < A

(3) A < B < C (4) A < C < B

Sol. (a) 

(b) 

(c) 

$$B.P. \propto \frac{1}{\text{Branching}} \qquad \therefore\ a > c > b \text{ (order of B.P.)}$$

21. For Freundlich adsorption isotherm, a plot of log (x/m) (y-axis) and log p (x-axis) gives a straight line. The intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm, is______ $\times 10^{-4}$g.

(log 3 = 0.4771)

Sol. $\frac{x}{m} = KP^{1/n}$

$$\log\left(\frac{x}{m}\right) = \frac{1}{n}\log P + \log K$$

$$\text{slope} = \frac{1}{n} = 2$$

intercept = log K = 0.4771

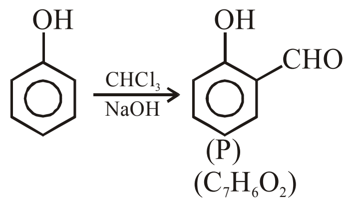
K = 3

mass of gas adsorbed per gm of adsorbent = $\frac{x}{m}$

$$\frac{x}{m} = 3 \times (0.04)^2 = 48 \times 10^{-4}$$

22. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is________. (to the nearest integer)

(Atomic mass : C = 12; H = 1; O = 16)

Sol. 

(Reimer Tiemann reaction)

Molecular weight of $C_7H_6O_2$ = 122

$$\%C = \frac{12 \times 7 \times 100}{122} = 68.85 \approx 69$$

23. If the solubility product of AB_2 is 3.20×10^{-11} M^3, then the solubility of AB_2 in pure water is_ _ _ _ _ $\times 10^{-4}$ mol L^{-1}. [Assuming that neither kind of ion reacts with water]

Sol. $AB_2(s) \rightleftharpoons \underset{S}{A^{+2}_{(aq.)}} + \underset{2S}{2B^{-}_{(aq.)}} : K_{sp}$

$K_{SP} = S^1 \times (2s)^2 = 4s^3$

$3.2 \times 10^{-11} = 4 \times S^3$

$S = 2 \times 10^{-4}$ M/L

24. The rate of a reaction decreased by 3.555 times when the temperature was changed from 40°C to 30°C. The activation energy (in kJ mol^{-1}) of the reaction is_ _ _ _ _ _.

Take; R=8.314 J mol^{-1} K^{-1} In 3.555 = 1.268

Sol. $$\ell n\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$T_1 = 303$ K ; $T_2 = 313$ K

$$\frac{K_{T_2}}{K_{T_1}} = 3.555$$

$$\ell n(3.555) = \frac{E_a}{8.314}\left[\frac{1}{303} - \frac{1}{313}\right]$$

$E_a = 99980.715$

$E_a = 99.98 \dfrac{kJ}{mole}$

25. The atomic number of Unnilunium is _____.

Sol. Unnilunium ⇒ 101

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

1. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x).(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda\sin^2 x + \sin^3 x$

$f'(x) = 2\lambda\sin x\cos x + 3\sin^2 x\cos x$

$f'(x) = \sin x\cos x(2\lambda + 3\sin x)$

$\sin x = 0, \ \frac{-2\lambda}{3}$, $(\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

2. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$

(1) $f''(x) = 0$, for some $x \in (0, 1)$

(2) $f''(0) = 0$

(3) $f''(x) \neq 0$ at every point $x \in (0, 1)$

(4) $f''(x) = 0$ at every point $x \in (0, 1)$

Sol. $f(0) = f(1) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0, 1)$

Now apply Rolles theorem on $y = f'(x)$

in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some , $\beta \in (0, \alpha) \in (0, 1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0, 1)$

3. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line - segement joining the points (1, 0) and (e, e), then c is equal to :

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$

(3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

Sol. $f(x) = x\log_e x$

$$f'(x)\big|_{(c, f(c))} = \frac{e-0}{e-1}$$

$f'(x) = 1 + \log_e x$

$$f'(x)\big|_{(c, f(c))} = 1 + \log_e c = \frac{e}{e-1}$$

$$\log_e c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$$

4. Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is :

(1) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is odd.

(3) For an integer n, if n is odd, then $n^3 - 1$ is even.

(4) For an integer n, if n is even, then $n^3 - 1$ is even.

Sol. Contrapositive of $(p \to q)$ is $\sim q \to \sim p$

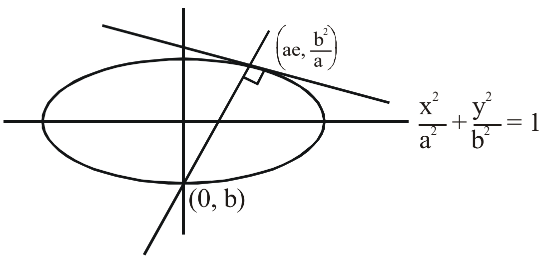
For an integer n, if n is even then $(n^3 - 1)$ is odd

5. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

(1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

Sol.



$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$$

$$\frac{a^2x}{ae} - \frac{b^2y}{b^2}.a = a^2e^2$$

$$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$$

passes through (0, b)

$-b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1 - e^2) = a^2e^4 \Rightarrow e^4 + e^2 = 1$

6. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is :

(1) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(3) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$a = 3$, $b = 3$, $c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

7. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$

(3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

Sol. α and β are the roots of the equation $4x^2 + 2x - 1 = 0$

$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha$ ...(1)

$$\beta = \frac{-1}{2} - \alpha$$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

8. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the :

(1) imaginary axis (2) real axis

(3) line, $y = x$ (4) line, $y = -x$

Sol. $z = x + iy$

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on $y = x$

9. The common difference of the A.P. $b_1, b_2, \ldots, b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \ldots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :

(1) –127 (2) –81

(3) 81 (4) 127

Sol. $a_1, a_2, \ldots, a_n \to (CD = d)$

$b_1, b_2, \ldots, b_m \to (CD = d + 2)$

$a_{40} = a + 39d = -159$ ...(1)

$a_{100} = a + 99d = -399$ ...(2)

Subtract : $60d = -240 \Rightarrow d = -4$

using equation (1)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

$b_{100} = -279$

$b_1 + 99(d + 2) = -279$

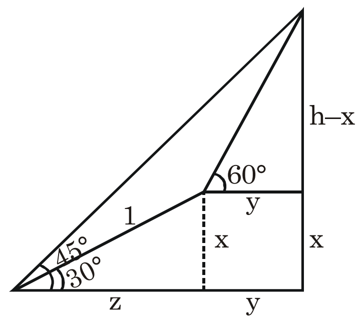
$b_1 - 198 = -279 \Rightarrow b_1 = -81$

10. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climding up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is :

(1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$

(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Sol.



$\sin 30° = x \Rightarrow x = \frac{1}{2}$

$\cos 30° = z \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45° = \frac{h}{y+z} \Rightarrow h = y + z$

$\tan 60^\circ = \frac{h-x}{y} \Rightarrow \tan 60^\circ = \frac{h-x}{h-z}$

$\sqrt{3}(h-z) = h-x$

$(\sqrt{3}-1)h = \sqrt{3}z - x$

$\Rightarrow (\sqrt{3}-1)h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow (\sqrt{3}-1)h = 1$

$h = \frac{1}{\sqrt{3}-1}$

11. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If B = A + A^4, then det(B) :

(1) is one (2) lies in (1, 2)

(3) is zero (4) lies in (2, 3)

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

B = A + A^4

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} (\cos\theta + \cos 4\theta) & (\sin\theta + \sin 4\theta) \\ -(\sin\theta + \sin 4\theta) & (\cos\theta + \cos 4\theta) \end{bmatrix}$

$|B| = (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2$

$|B| = 2 + 2\cos 3\theta$, when $\theta = \frac{\pi}{5}$

$|B| = 2 + 2\cos\frac{3\pi}{5} = 2(1 - \sin 18)$

$|B| = 2\left(1 - \frac{\sqrt{5}-1}{4}\right) = 2\left(\frac{5-\sqrt{5}}{4}\right) = \frac{5-\sqrt{5}}{2}$

12. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \ne -a$ and $f(x) \ne -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) 3

(3) –3 (4) $-\frac{1}{3}$

Sol. $f(x) = \frac{a-x}{a+x}$ $x \in R - \{-a\} \to R$

$f(f(x)) = \frac{a - f(x)}{a + f(x)} = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)}$

$f(f(x)) = \frac{(a^2 - a) + x(a+1)}{(a^2 + a) + x(a-1)} = x$

$\Rightarrow (a^2 - a) + x(a + 1) = (a^2 + a)x + x^2(a - 1)$

$\Rightarrow a(a - 1) + x(1 - a^2) - x^2(a - 1) = 0$

$\Rightarrow a = 1$

$f(x) = \frac{1-x}{1+x}$,

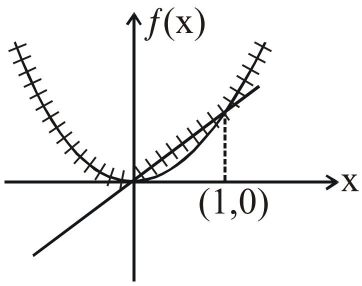
$f\left(\frac{-1}{2}\right) = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 3$

13. Let f : R $\to$ R be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then :

(1) {0, 1} (2) {0}

(3) ϕ(an empty set) (4) {1}

Sol. $f(x) = \max(x, x^2)$



Non-differentiable at x = 0, 1

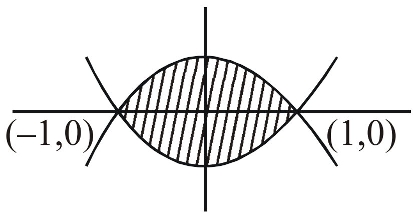
S = {0, 1}

14. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{8}{3}$

(3) $\frac{16}{3}$ (4) $\frac{7}{2}$

Sol. $y = x^2 - 1$ and $y = 1 - x^2$



$$A = \int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$$

$$A = \int_{-1}^{1}(2-2x^2)dx = 4\int_{0}^{1}(1-x^2)dx$$

$$A = 4\left(x-\frac{x^3}{3}\right)_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$$

15. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36]

(3) [0.25, 0.35] (4) [0.20, 0.25]

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$

16. if the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then |k| equals :

(1) 2 (2) 1

(3) 3 (4) 9

Sol. $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r . x^{\frac{10-r}{2}} . (-k)^r . x^{-2r}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r$$

Constant term : $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2 . (-k)^2 = 405$

$$k^2 = \frac{405}{45} = 9$$

$k = \pm 3 \Rightarrow |k| = 3$

17. The integral $\int_1^2 e^x . x^x \left(2+\log_e x\right)dx$ equal :

(1) e(4e + 1) (2) e(2e – 1)

(3) $4e^2 - 1$ (4) e(4e – 1)

Sol. $\int_1^2 e^x . x^x \left(2+\log_e x\right) dx$

$\int_1^2 e^x \left(2x^x + x^x \log_e x\right) dx$

$\int_1^2 e^x \left(\underbrace{x^x}_{f(x)} + \underbrace{x^x\left(1+\log_e x\right)}_{f'(x)}\right) dx$

$\left(e^x . x^x\right)_1^2 = 4e^2 - e$

18. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is :

(1) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$

(3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

Sol. L : $\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x + 3y - 3 = 0$

Image of point (–1, –4)

$$\frac{x+1}{1}=\frac{y+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1}=\frac{y+4}{3}=\frac{16}{5}$$

$$(x,y) \equiv \left(\frac{11}{5}, \frac{28}{5}\right)$$

19. If $y=\left(\frac{2}{\pi}x-1\right)\text{cosec}x$ is the solution of the differential equation,

$\frac{dy}{dx}+p(x)y=\frac{2}{\pi}\text{cosec}x$, $0 < x < \frac{\pi}{2}$, then the function p(x) is equal to

(1) cotx (2) tanx

(3) cosecx (4) secx

Sol. $y=\left(\frac{2x}{\pi}-1\right)\text{cosec}\,x$ ...(1)

$$\frac{dy}{dx}=\frac{2}{\pi}\text{cosec}x-\left(\frac{2x}{\pi}-1\right)\text{cosec}x\cot x$$

$$\frac{dy}{dx}=\frac{2\,\text{cosec}x}{\pi}-y\cot x$$

using equation (1)

$$\frac{dy}{dx}+y\cot x=\frac{2\,\text{cosec}x}{\pi}$$

$$\frac{dy}{dx}+p(x).y=\frac{2\,\text{cosec}x}{\pi} \quad x\in\left(0,\frac{\pi}{2}\right)$$

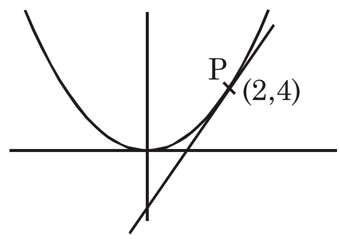
Compare : p(x) = cot x

20. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is :

(1) $\left(\frac{3}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{-16}{5}, \frac{53}{10}\right)$

(3) $\left(\frac{6}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{-53}{10}, \frac{16}{5}\right)$

Sol.



$y = x^2$

$\left.\frac{dy}{dx}\right|_P = 4$

$(y-4)=4(x-2)$

$4x - y - 4 = 0$

Circle : $(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0$

passes through (0, 1)

$4+9+\lambda(-5)=0 \Rightarrow \lambda=\frac{13}{5}$

Circle : $x^2+y^2+x(4\lambda-4)+y(-\lambda-8)+(20-4\lambda)=0$

Centre : $\left(2-2\lambda, \frac{\lambda+8}{2}\right) \equiv \left(\frac{-16}{5}, \frac{53}{10}\right)$

21. The sum of distinct values of λ for which the system of equations

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+3(\lambda-1)z=0,$

has non-zero solutions, is ___________.

Sol. $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+(3\lambda-3)z=0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$R_1 \to R_1-R_2$ & $R_2 \to R_2-R_3$

$$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$$(\lambda-3)^2\begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$(\lambda-3)^2[(3\lambda+1)+(3\lambda-1)]=0$

$6\lambda(\lambda-3)^2=0 \Rightarrow \lambda=0, 3$

Sum = 3

22. Suppose that a function $f: R \to R$ satisfies $f(x+y)=f(x)f(y)$ for all $x, y \in R$ and $f(1)=3$. If $\sum_{i=1}^{n} f(i)=363$, then n is equal to __________.

Sol. $f(x+y)=f(x)\,f(y)$

put $x=y=1$ $\quad f(2)=(f(1))^2=3^2$

put $x=2, y=1$ $\quad f(3)=(f(1))^3=3^3$

$\vdots$

Similarly $f(x)=3^x$

$\sum_{i=1}^{n} f(i)=363 \Rightarrow \sum_{i=1}^{n} 3^i=363$

$(3+3^2+\ldots+3^n)=363$

$\frac{3(3^n-1)}{2}=363$

$3^n-1=242 \Rightarrow 3^n=243$

$\Rightarrow n=5$

23. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ___________.

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$\frac{4!}{2!} \times {}^5C_2 \times \frac{2!}{2!}=12\times 10=120$

24. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ____________.

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r \, {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$\Rightarrow 3^n = 729 \Rightarrow n = 6$

25. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is __________.

Sol. $|\vec{x}+\vec{y}| = |\vec{x}|$

$$\sqrt{|\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}.\vec{y}} = |\vec{x}|$$

$|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0$ .... (1)

Now $(2\vec{x}+\lambda\vec{y}).\vec{y} = 0$

$$2\vec{x}.\vec{y} + \lambda|\vec{y}|^2 = 0$$

from (1)

$$-|\vec{y}|^2 + \lambda|\vec{y}|^2 = 0$$

$$(\lambda - 1)|\vec{y}|^2 = 0$$

given $|\vec{y}| \neq 0$ $\Rightarrow \lambda = 1$