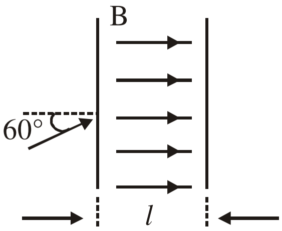
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton $= 1.67 \times 10^{-27}$ kg, charge of the proton $= 1.6 \times 10^{-19}$ C)

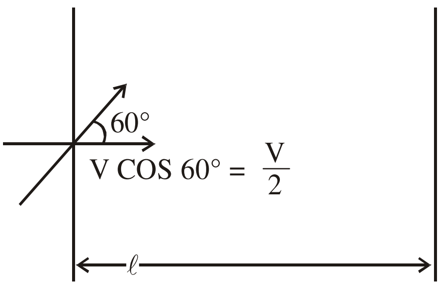


(1) 0.11 m (2) 0.22 m

(3) 0.44 m (4) 0.88 m

Sol. $T = \frac{2\pi m}{qB}$

total time t = 10 T



Kinematics

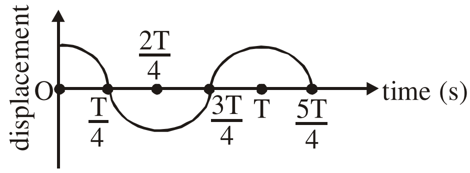
$$\ell = \frac{V}{2}t$$

$$\ell = \frac{V}{2}10 \times \frac{2\pi m}{qB}$$

$$= 4 \times 10^5 \times 10 \times \frac{3.14 \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$$

= 0.439

2. The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion ?

(A) The force is zero $t = \frac{3T}{4}$

(B) The acceleration is maximum at t = T

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (A), (B) and (D)

(2) (B), (C) and (D)

(3) (A) and (D)

(4) (A), (B) and (C)

Sol. (A) F = ma $\qquad a = -\omega^2 x$

at $\frac{3T}{4}$ displacement zero (x = 0), so a = 0

F = 0

(B) at t = T $\qquad$ displacement (x) = A

x maximum, So acceleration is maximum.

(C) $V = \omega\sqrt{A^2 - x^2}$

V_{max} at x = 0

$V_{max} = A\omega$

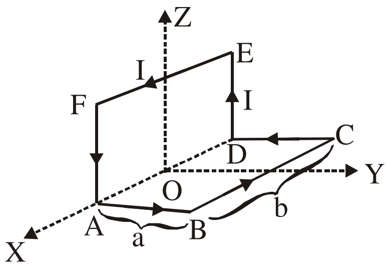
at $t = \frac{T}{4}$, x = 0, So $V_{max.}$

(D) KE = PE

$\therefore$ at $x = \frac{A}{\sqrt{2}}$.

at $t = \frac{T}{2}$ $\quad x = -A$ $\quad$ (So not possible)

3. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is :



(1) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(2) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

(3) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(4) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

Sol. M = NIA

N = 1

For ABCD

$$\vec{M}_1 = abI\ \hat{K}.$$

For DEFA

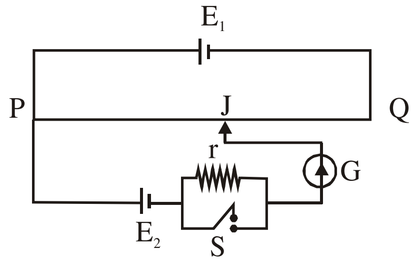
$$\vec{M}_2 = abI\ \hat{j}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2$$

$$= ab\ I\ (\hat{k}+\hat{j})$$

$$= ab\ I\ \sqrt{2}\ \left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$$

4. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is :



(1) 0.02 V/cm (2) 0.04 V/cm

(3) 0.01 V/cm (4) 0.03 V/cm

Sol. Balancing length is measured from P.

So 100 – 49 = 51 cm

$E_2 = \phi \times 51$

Where ϕ = Potential gradient

$1.02 = \phi \times 51$

$\phi = 0.02$ V/cm

5. A heat engine is involved with exchange of heat of 1915 J, –40 J, +125 J and QJ, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(1) 640 J (2) 400 J

(3) 980 J (4) 40 J

Sol. $\eta = \frac{\text{Work done}}{\text{Heat supplied}}$

$$\frac{1}{2} = \eta = \frac{1915-40+125-Q}{1915+125}$$

$$\frac{1}{2} = \frac{2000-Q}{2040}$$

2040 = 4000 – 2Q

2Q = 1960

Q = 980 J

6. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be :

(1) 28 (2) 24 (3) 18 (4) 30

Sol. Let the length of segment is "ℓ"
Let N is the no. of fringes in "ℓ"
and w is fringe width.
→ We can write

$$N\,w = \ell$$

$$N\left(\frac{\lambda D}{d}\right) = \ell$$

$$\frac{N_1\lambda_1 D}{d} = \ell$$

$$\frac{N_2\lambda_2 D}{d} = \ell$$

$N_1\lambda_1 = N_2\lambda_2$
$16 \times 700 = N_2 \times 400$
$N_2 = 28$

7. In a hydrogen atom the electron makes a transition from $(n + 1)^{th}$ level to the n^{th} level. If n>>l, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n^4}$ (2) $\frac{1}{n^3}$ (3) $\frac{1}{n^2}$ (4) $\frac{1}{n}$

Sol. In hydrogen atom,

$$E_n = \frac{-E_0}{n^2}$$

Where E_0 is Ionisation Energy of H.
→ For transition from (n + 1) to n, the energy of emitted radiation is equal to the difference in energies of levels.

$$\Delta E = E_{n+1} - E_n$$

$$\Delta E = E_0\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\Delta E = h\nu = E_0\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right)$$

$$h\nu = E_0\left[\frac{2n+1}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

$$h\nu = E_0\left[\frac{n\left(2+\frac{1}{n}\right)}{n^4\left(1+\frac{1}{n}\right)^2}\right]$$

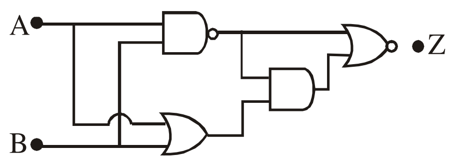
Since n >>> 1

Hence, $\frac{1}{n} \simeq 0$

$$h\nu = E_0\left[\frac{2}{n^3}\right]$$

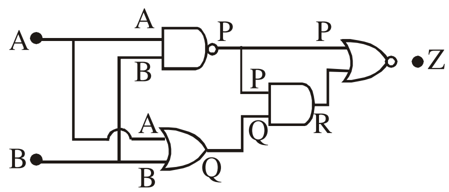
$$\nu \,\alpha\, \frac{1}{n^3}$$

8. In the following digital circuit, what will be the output at 'Z', when the input (A, B) are (1,0), (0,0), (1,1), (0,1):



(1) 1, 0, 1, 1 (2) 0, 1, 0, 0
(3) 0, 0, 1, 0 (4) 1, 1, 0, 1

Sol.



$Z = \overline{(P + R)}$

$Z = \overline{(P + PQ)}$

$Z = \overline{(P(1+Q))}$

$Z = (\overline{P})$ [Using Identity $(1 + A) = 1$]

$Z = \overline{(\overline{AB})}$

Z = AB

Truth table for Z = AB

A	B	Z
1	0	0
0	0	0
1	1	1
0	1	0

9. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is :

(1) $[PA^{-1}T^{-2}]$ (2) $[PA^{1/2}T^{-1}]$

(3) $[P^2AT^{-2}]$ (4) $[P^{1/2}AT^{-1}]$

Sol. Let $[E] = [P]^x [A]^y [T]^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$ML^2T^{-2} = M^x L^{x+2y} T^{-x+z}$

$\to x = 1$

$\to x + 2y = 2$

$1 + 2y = 2$

$y = \dfrac{1}{2}$

$\to -x + z = -2$

$-1 + z = -2$

$z = -1$

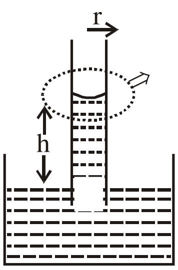
$[E] = [PA^{1/2} T^{-1}]$

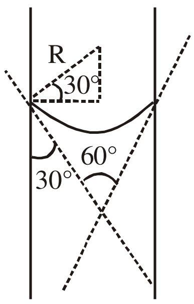
10. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g = 10 ms^{-2}).

(1) 0.137 m (2) 0.172 m

(3) 0.087 m (4) 0.049 m

Sol.





$r \to$ radius of capillary

$R \to$ Radius of meniscus.

From figure, $\dfrac{r}{R} = \cos 30°$

$$R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$= \frac{0.3}{\sqrt{3}} \times 10^{-3} m$$

Height of capillary

$$h = \frac{2T}{\rho g R} = 2\sqrt{3}\,T$$

$$h = \frac{2 \times 0.05}{667 \times 10 \times \left(\frac{0.3 \times 10^{-3}}{\sqrt{3}}\right)}$$

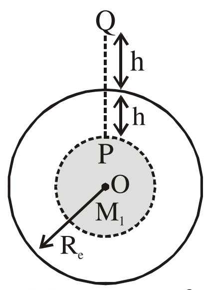
h = 0.087 m

11. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) :

(1) $\dfrac{\sqrt{5}R - R}{2}$ (2) $\dfrac{\sqrt{5}}{2}R - R$

(3) $\dfrac{R}{2}$ (4) $\dfrac{\sqrt{3}R - R}{2}$

Sol.



- M = mass of earth

M_1 = mass of shaded portion

R = Radius of earth

- $M_1 = \dfrac{M}{\frac{4}{3}\pi R^3} \cdot \dfrac{4}{3}\pi (R-h)^3$

$$= \frac{M(R-h)^3}{R}$$

- Weight of body is same at P and Q

i.e. $mg_P = mg_Q$

$g_P = g_Q$

$$\frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$(R - h)(R + h)^2 = R^3$

$R^3 - hR^2 - h^2R - h^3 + 2R^2 h - 2Rh^2 = R^3$

$R^2 - Rh^2 - h^3 = 0$

$R^2 - Rh - h^2 = 0$

$$h^2 + Rh - R^2 = 0 \Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\text{ie } h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

12. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true ?

(A) the mean free path of the molecules decreases.

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time remains unchanged.

(1) (C) and (D) (2) (A) and (B)

(3) (A) and (D) (4) (B) and (C)

Sol. The mean free path of molecules of an ideal gas is given as:

$$\lambda = \frac{V}{\sqrt{2}\pi d^2 N}$$

where : V = Volume of container
N = No of molecules

Hence with increasing temp since volume of container does not change (closed container), so mean free path is unchanged.

Average collision time

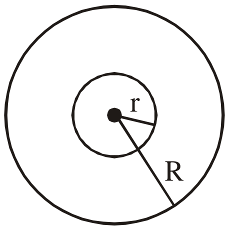
$$= \frac{\text{mean free path}}{V_{av}} = \frac{\lambda}{\text{(avg speed of molecules)}}$$

$\because$ avg speed $\alpha \sqrt{T}$

$\therefore$ Avg coll. time $\alpha \dfrac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

13. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is :



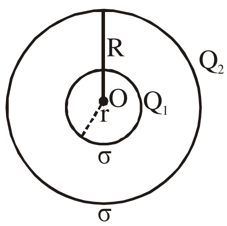
(1) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(R+2r)Q}{2(R^2+r^2)}$

(2) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(R+r)}{2(R^2+r^2)}Q$

(3) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(R+r)}{(R^2+r^2)}Q$

(4) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(2R+r)}{(R^2+r^2)}Q$

Sol. Let the charges on inner and outer spheres are Q_1 and Q_2.



Since charge density 'σ' is same for both spheres, so

$$\sigma = \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r^2}{R^2}$$

$$Q_1 + Q_2 = Q \Rightarrow \frac{Q_2 r^2}{R^2} + Q_2 = Q$$

$$\Rightarrow Q_2 = \frac{QR^2}{(r^2 + R^2)}$$

$$Q_1 = \frac{r^2}{R^2} \cdot \frac{QR^2}{(R^2 + r^2)} = \frac{Qr^2}{(R^2 + r^2)}$$

Potential at centre 'O' = $\frac{kQ_1}{r} + \frac{kQ_2}{R}$

$$= k\left[\frac{Qr^2}{r(R^2 + r^2)} + \frac{QR^2}{R(R^2 + r^2)}\right]$$

$$= \frac{kQ(r + R)}{(R^2 + r^2)} = \frac{1}{4\pi \epsilon_0} \frac{(R + r)}{(R^2 + r^2)} Q$$

14. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is :

(1) 1.1×10^{-2} H (2) 1.1×10^{-1} H

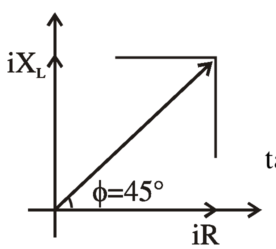
(3) 5.5×10^{-5} H (4) 6.7×10^{-7} H

Sol. 

- Reactance of inductance coil

$$= \sqrt{R^2 + x_L^2} = 100 \quad .....(i)$$

- f = 1000 Hz of applied AC signal
- Voltage leads current by 45°

ie 

$\tan 45° = \frac{iX_L}{iR} = \frac{\omega L}{R}$

ie R = X_L = ωL

Putting in eqn (i) : $\sqrt{X_L^2 + X_L^2} = 100$

$$\sqrt{2} X_L = 100 \Rightarrow X_L = 50\sqrt{2}$$

ie ωL = $50\sqrt{2}$

$$L = \frac{50\sqrt{2}}{\omega} = \frac{50\sqrt{2}}{2\pi f} = \frac{25\sqrt{2}}{\pi \times 1000} \text{ H}$$

= 1.125×10^{-2} H

15. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m² and 10 rad s⁻¹ respectively while those for the second one are 0.2 kg-m² and 5 rad s⁻¹ respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is :

(1) $\frac{10}{3}$J (2) $\frac{2}{3}$J (3) $\frac{5}{3}$J (4) $\frac{20}{3}$J

Sol.
- Both discs are rotating in same sense
- Angular momentum conserved for the system

i.e. $L_1 + L_2 = L_{final}$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$0.1 \times 10 + 0.2 \times 5 = (0.1+0.2) \times \omega_f$

$$\omega_f = \frac{20}{3}$$

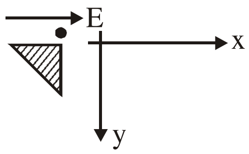
- Kinetic energy of combined disc system

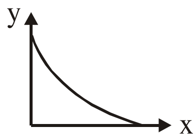
$$\Rightarrow \frac{1}{2}(I_1 + I_2)\omega_f^2$$

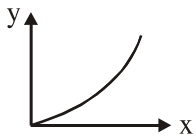
$$= \frac{1}{2}(0.1 + 0.2) \cdot \left(\frac{20}{3}\right)^2$$

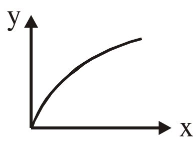
$$= \frac{0.3}{2} \times \frac{400}{9} = \frac{120}{18} = \frac{20}{3} \text{J}$$

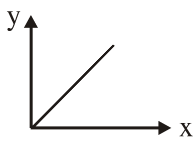
16. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale).

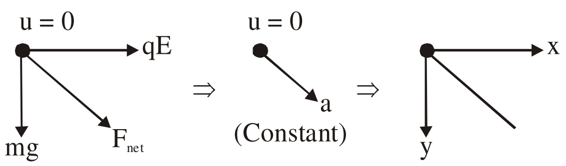


(1)


(2)


(3)


(4)


Sol.


Since initial velocity is zero and acceleration of particle will be constant, so particle will travel on a straight line path.

17. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

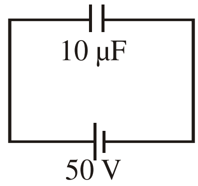
(1) 10 μF

(2) 15 μF

(3) 20 μF

(4) 30 μF

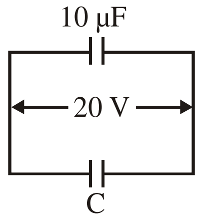
Sol. Initially



- Charge on capacitor 10 μF

Q = CV = (10 μF) (50V)

Q = 500 μC



- Final Charge on 10 μF capacitor

Q = CV = (10 μF) (20V)

Q = 200 μC

- From charge conservation,

Charge on unknown capacitor

C = 500 μC – 200 μC = 300 μC

$$\Rightarrow \text{Capacitance (C)} = \frac{Q}{V} = \frac{300\ \mu C}{20\ V} = 15\ \mu F$$

18. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to:

(1) 0.008 (2) 0.06

(3) 0.8 (4) 2.3

Sol. Given $\frac{\Delta L}{L} = 0.02\%$

$\therefore \Delta L = L\alpha\Delta T \Rightarrow \frac{\Delta L}{L} = \alpha\Delta T = 0.02\%$

$\therefore$ β = 2α (Areal coefficient of expansion)

$\Rightarrow$ βΔT = 2αΔT = 0.04%

Volume = Area × Length

$$\text{Density}(\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Area} \times \text{Length}} = \frac{M}{AL}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \cancelto{0}{\frac{\Delta M}{M}} - \frac{\Delta A}{A} - \frac{\Delta L}{L} \text{ (Mass remains constant)}$$

$$\Rightarrow \left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$$

= 0.04% + 0.02%

= 0.06%

19. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i}-2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

(1) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (2) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$

(3) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$ (4) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

Sol. $\hat{E}=\hat{k}$

$$\vec{B}=2\hat{i}-2\hat{j} \Rightarrow \hat{B}=\frac{\vec{B}}{|B|}=\frac{2\hat{i}-2\hat{j}}{2\sqrt{2}}$$

$$\Rightarrow \hat{B}=\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$$

Direction of wave propagation $=\hat{C}=\hat{E}\times\hat{B}$

$$\hat{C}=\hat{k}\times\left[\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})\right]$$

$$\hat{C}=\frac{1}{\sqrt{2}}(\hat{k}\times\hat{i}-\hat{k}\times\hat{j})$$

$$\hat{C}=\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

20. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to :

(1) 4.8×10^{-27} kg

(2) 1.2×10^{-28} kg

(3) 9.1×10^{-31} kg

(4) 9.7×10^{-28} kg

Sol. Let mass of particle = m

Let speed of e^- = V

$\Rightarrow$ speed of particle = 5V

Debroglie wavelength $\lambda_d=\frac{h}{P}=\frac{h}{mv}$

$$\Rightarrow (\lambda_d)_P=\frac{h}{m(5V)} \quad .....(1)$$

$$\Rightarrow (\lambda_d)_e=\frac{h}{m_e.V} \quad .....(2)$$

According to question

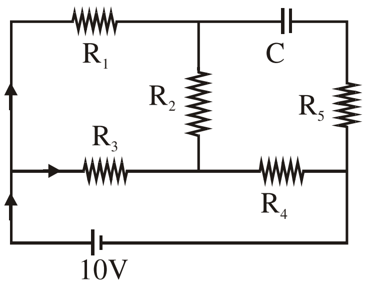
$$\frac{(1)}{(2)}=\frac{m_e}{5m}=1.878\times10^{-4}$$

$$\Rightarrow m=\frac{m_e}{5\times1.878\times10^{-4}}$$

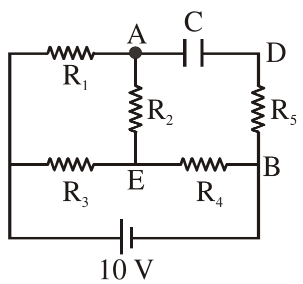
$$\Rightarrow m=\frac{9.1\times10^{-31}}{5\times1.878\times10^{-4}}$$

$\Rightarrow m = 9.7 \times 10^{-28}$ kg

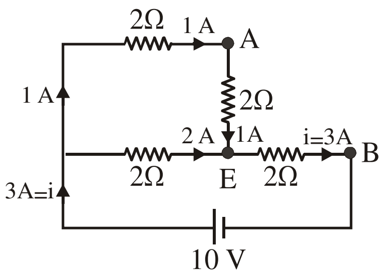
21. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is ________.



Sol.



- R_1 to R_5 $\rightarrow$ each 2Ω
- Cap. is fully charged
- So no current is there in branch ADB
- Effective circuit of current flow :



$$R_{eq} = \left(\frac{4\times 2}{4+2}\right) + 2$$

$$R_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\Omega$$

$$i = \frac{10}{10/3} = 3A$$

So potential different across AEB

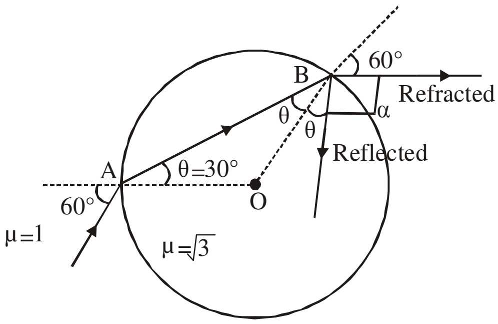
$\Rightarrow 2 \times 1 + 2 \times 3 = 8V$

Hence potential difference across

Capacitor = $\Delta V = V_{AEB} = 8V$

22. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is______.

Sol.



By Snell's law at A :

$1 \times \sin 60° = \sqrt{3} \times \sin\theta$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

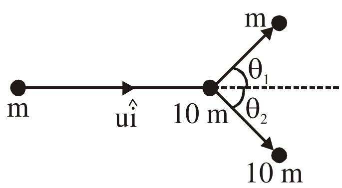
So at B :

$\theta + 60° + \alpha = 180°$

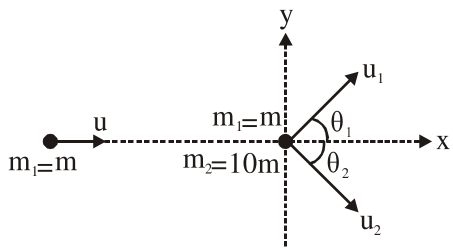
$30° + 60° + \alpha = 180°$

$\alpha = 90°$

23. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is _______.



Sol.



By momentum conservation along y :

$m_1u_1\sin\theta_1 = m_2u_2\sin\theta_2$

i.e. $mu_1\sin\theta_1 = 10mu_2\sin\theta_2$

$$\Rightarrow \boxed{u_1\sin\theta_1 = 10u_2\sin\theta_2} \quad .....(i)$$

$$kf_{m_1} = \frac{1}{2}ki_{m_1} \quad \text{i.e. } \frac{1}{2}mu_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$$

$$\text{i.e. } \boxed{u_1 = \frac{u}{\sqrt{2}}} \quad .....(ii)$$

Also collision is elastic : $k_i = k_f$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}.10m.u_2^2$$

$$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}\times 10m.u_2^2$$

$$\frac{1}{4}mu^2 = \frac{1}{2}\times 10\times mu_2^2$$

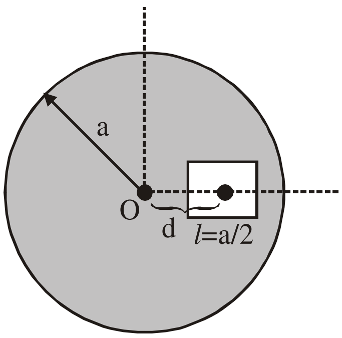
$$\boxed{u_2 = \frac{u}{\sqrt{20}}} \quad ....(iii)$$

Putting (ii) & (iii) in (i)

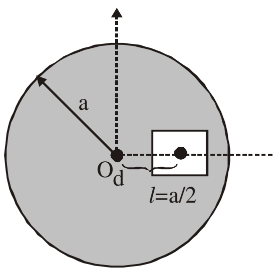
$$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$$

$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2}$ → Hence n = 10

24. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is _____.



Sol.



$$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

where :

- m_1 = mass of complete disc
- m_2 = removed mass
- Let σ = surface mass density of disc material

$$\text{wrt 'O'} : X_{com} = \frac{\sigma\pi a^2(O) - \sigma.\frac{a^2}{4}.d}{\sigma\pi a^2 - \sigma\frac{a^2}{4}} = \frac{-\frac{a^2}{4}d}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

So, X = $2(4\pi-1) = (8\pi-2) = 23.12$

So, nearest integer value of X = 23

25. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire Y = 9×10^{10} Nm^{-2}), (to the nearest integer),_________.

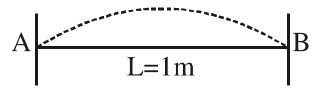
Sol. $\rho_{wire} = 9 \times 10^{-3}\frac{kg}{cm^3} = \frac{9 \times 10^{-3}}{10^{-6}} kg/m^3$

$= 9000$ kg/m^2

(A = CSA of wire)

(Y = 9×10^{10} Nm^2)

(Strain = 4.9×10^{-4})



$$\Rightarrow L = 1m = \frac{\lambda}{2} \Rightarrow \lambda = 2m$$

$$\Rightarrow v = f\lambda \Rightarrow \sqrt{\frac{T}{\mu}} = f\,\lambda$$

$$\text{Where } Y = \frac{T/A}{\text{strain}} \Rightarrow T = Y.A.\text{strain}$$

$$\Rightarrow \sqrt{\frac{Y.A.\text{strain}}{m/L}} = f \times 2 \Rightarrow \sqrt{\frac{Y.A.L.\text{strain}}{M}} = f \times 2$$

$$\Rightarrow \sqrt{\frac{Y \times V \times \text{strain}}{M}} = f \times 2 \Rightarrow \sqrt{\frac{Y \times \text{strain}}{\rho}} = f \times 2$$

$$f = \frac{1}{2}.\sqrt{\frac{Y \times \text{strain}}{\rho}} = \frac{1}{2}\sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9000}}$$

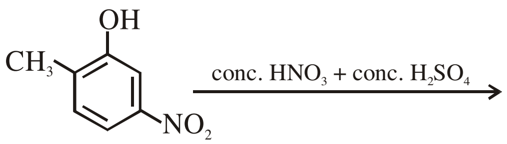
$$f = \frac{1}{2}.\sqrt{\frac{9 \times 10^3}{9} \times 4.9} = \frac{1}{2}\sqrt{4900} = \frac{70}{2} = 35\text{ Hz}$$

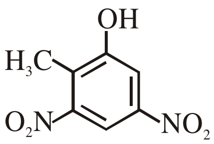
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

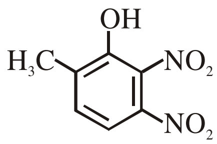
(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

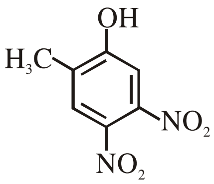
CHEMISTRY | TEST PAPER WITH ANSWER & SOLUTIONS

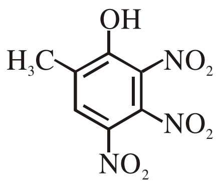
1. The major product of the following reaction is:



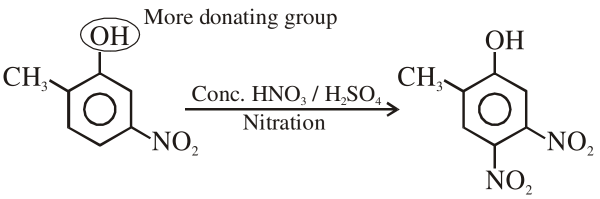
(1) 

(2) 

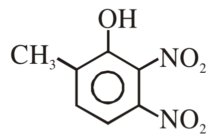
(3) 

(4) 

Sol.



(Major)

+ 

[Minor due to crowding]

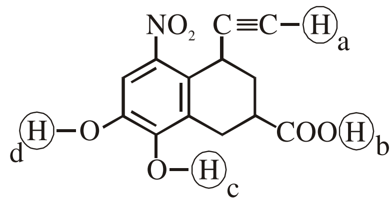
2. If you spill a chemical toilet cleaning liquid on your hand, your first aid would be :

(1) aqueous NH_3 (2) vinegar

(3) aqueous $NaHCO_3$ (4) aqueous NaOH

Sol. Toilet cleaning liquid has about 10.5% w/v HCl; to neutralise its affect aqueous $NaHCO_3$ is used while NaOH is avoid for this purpose because its highly corosive in nature and can burn body.

3. Arrange the following labelled hydrogens in decreasing order of acidity :



(1) b > c > d > a

(2) c > b > a > d

(3) b > a > c > d

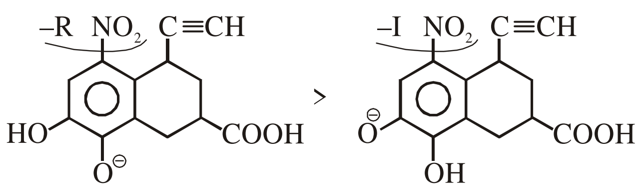
(4) c > b > d > a

Sol. Acidic strength order :

$$R-\overset{\overset{O}{\|}}{C}-OH > R-OH > R-C\equiv CH$$

Reason : $R-\overset{\overset{O}{\|}}{C}-O^{\ominus}$ stable by equivalent resonance.

Stable :



So answer is b > c > d > a.

4. Cast iron is used for the manufacture of :

(1) wrought iron and pig iron

(2) wrought iron and steel

(3) wrought iron, pig iron and steel

(4) pig iron, scrap iron and steel

Sol. Cast iron is used for manufacturing of wrought iron and steel.

5. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. products C and D show following chemical tests.

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are :

(1) $C = H_3C-C(CH_3)_2-OH$;
$D = H_3C-CH_2-CH(OH)-CH_3$

(2) $C = H_3C-CH_2-CH_2-CH_2-OH$;
$D = H_3C-C(CH_3)_2-OH$

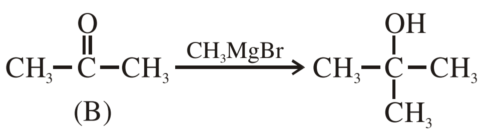
(3) $C = H_3C-CH_2-CH(OH)-CH_3$;
$D = H_3C-C(CH_3)_2-OH$

(4) $C = H_3C-CH_2-CH_2-CH_2-OH$;
$D = H_3C-CH_2-CH(OH)-CH_3$

Sol.

(A) (C) 2° Alcohol

CAN test for alcohol : ✓
Iodoform test : ✓



(B) 3° Alcohol

CAN test for alcohol : ✓
Lucas test : Immediately
Iodoform test : ✗

6. The shape/structure of $[XeF_5]^-$ and XeO_3F_2, respectively, are :

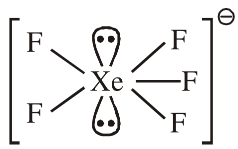
(1) pentagonal planar and trigonal bipyramidal

(2) trigonal bipyramidal and pentagonal planar

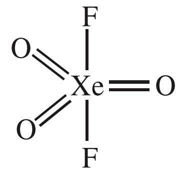
(3) octahedral and square pyramidal

(4) trigonal bipyramidal and trigonal bipyramidal

Sol.



XeF_5^-
sp^3d^3
Pentagonal planar



XeO_3F_2
sp^3d
Trigonal bipyramidal

7. The major product obtained from E_2-elimination of 3-bromo-2-fluoropentane is:

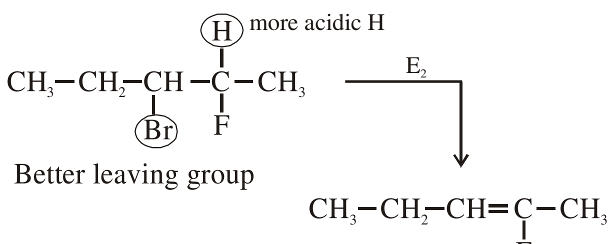
(1) $CH_3CH_2-CH(Br)-CH=CH_2$

(2) $CH_3-CH_2-C(Br)=CH-CH_3$

(3) $CH_3-CH=CH-CH(F)-CH_3$

(4) $CH_3CH_2CH=C(CH_3)-F$

Sol.



Stable alkene having 5 αH

8. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is :

(1) Z < Y < X (2) X < Z < Y

(3) X < Y < Z (4) Y < X < Z

Sol. When we are moving from left to right in a periodic table acidic character of oxides increases (as well as atomic number of atom increases)

$\therefore$ X < Y < Z (acidic character)

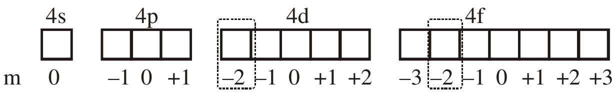
X < Y < Z (atomic number)

9. The number of subshells associated with n = 4 and m = –2 quantum numbers is :

(1) 4 (2) 8 (3) 16 (4) 2

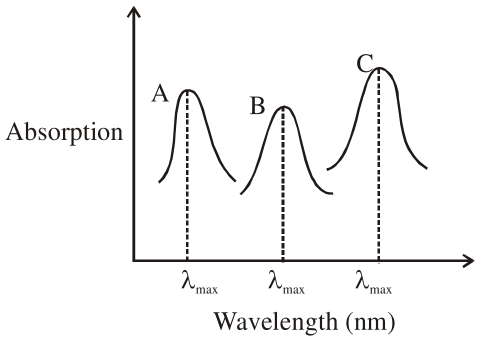
Sol. For n = 4

ℓ = 0, 1, 2, 3



$\therefore$ 4d & 4f subshell associated with n = 4, m = –2

10. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of M^{n+} ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is :



(i) $[M(NCS)_6]^{(-6+n)}$ (ii) $[MF_6]^{(-6+n)}$

(iii) $[M(NH_3)_6]^{n+}$

(1) A-(ii), B-(i), C-(iii) (2) A-(iii), B-(i), C-(ii)

(3) A-(ii), B-(iii), C-(i) (4) A-(i), B-(ii), C-(iii)

Sol. Strength of ligand $F^- < NCS^- < NH_3$

$[MF_6]^{(-6+n)}$ (ii) $[M(NCS)]^{(-6+n)}$ (i) $[M(NH_3)_6]^{n+}$ (iii)

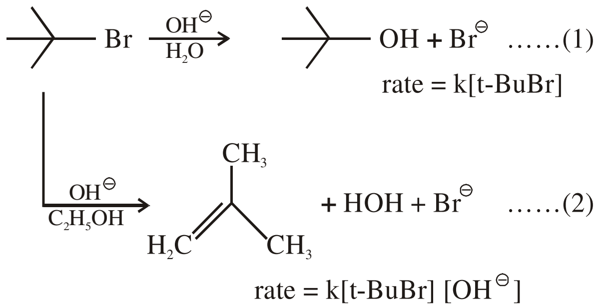
$\longrightarrow \Delta_{Oh} \uparrow$

$\longrightarrow \lambda_{max} \downarrow$

As given in graph : A < B < C (λ_{max})

$\therefore$ Correct matching is A–(iii), B-(i), C-(ii)

11. Consider the reaction sequence given below :



Which of the following statements is true :

(1) Changing the concentration of base will have no effect on reaction (1)

(2) Changing the concentration of base will have no effect on reaction (2)

(3) Changing the base from $OH^{\ominus}$ to $^{\ominus}OR$ will have no effect on reaction (2)

(4) Doubling the concentration of base will double the rate of both the reactions.

Sol. Reaction 1 : SN_1

Reaction 2 : E_2

SN_1 is independent of concentration of nucleophile/base

12. The results given in the below table were obtained during kinetic studies of the following reaction:

$$2A + B \longrightarrow C + D$$

Experiment	$[A]/molL^{-1}$	$[B]/molL^{-1}$	Initial rate/$molL^{-1}$ min^{-1}
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}
V	0.3	Y	2.88×10^{-1}

X and Y in the given table are respectively :

(1) 0.3, 0.4 (2) 0.4, 0.3

(3) 0.4, 0.4 (4) 0.3, 0.3

Sol. From rate law

$$r = -\frac{1}{2}\frac{d[A]}{dt} = \frac{-d[B]}{dt}$$

$$= K[A]^x [B]^y$$

$6 \times 10^{-3} = K(0.1)^x (0.1)^y$ ......(1)

$2.4 \times 10^{-2} = K(0.1)^x (0.2)^y$ ......(2)

$1.2 \times 10^{-2} = K(0.2)^x (0.1)^y$ ......(3)

(3) ÷ (1) $\Rightarrow$ x = 1

(2) ÷ (3) $\Rightarrow$ x = 2

So, other with respect to A = 1

Order with respect to B = 2

(4) ÷ (3)

$$\left(\frac{x}{0.2}\right)\times\left(\frac{0.2}{0.1}\right)^2 = \frac{7.2\times10^{-2}}{1.2\times10^{-2}}$$

$$x = \frac{6\times0.2}{4}$$

x = 0.3 M

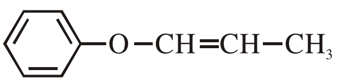
(5) ÷ (4)

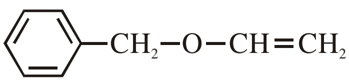
$$\left(\frac{y}{0.2}\right)^2 = \frac{2.88\times10^{-1}}{7.2\times10^{-2}}$$

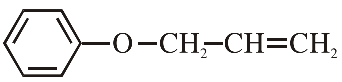
$y^2 = 4 \times 0.2^2$

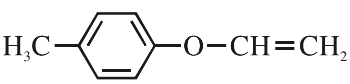
y = 0.4 M

13. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with $AgNO_3$ where as 'C' tautomerizes to 'D'. 'D' gives positive idoform test. 'A' could be :

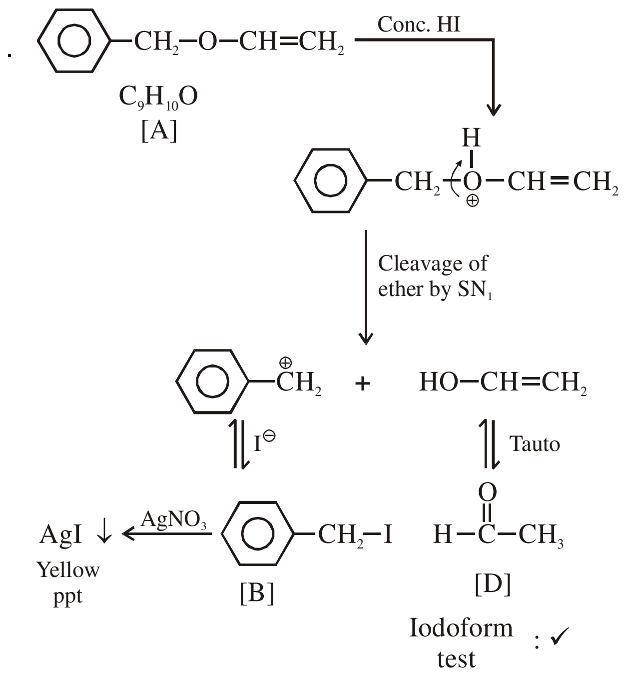
(1) 

(2) 

(3) 

(4) 

Sol



14. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following processes can explain this ?

(1) Diffusion (2) Dialysis

(3) Osmosis (4) Reverse osmosis

Sol. Raw mango shrink in salt solution due to net transfer of water molecules from mango to salt solution due to phenomenon of osmosis.

15. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :

(1) Na and C (2) Li and Mg

(3) Cs and Ba (4) Na and Rb

Sol. Both Li and Mg form nitride when reacts directly with nitrogen.

The hydrogen carbonate of both Li and Mg does not exist in solid state.

All alkali metal hydrogen carbonate exist in solid state except $LiHCO_3$.

16. The one that is not expected to show isomerism is :

(1) $[Ni(NH_3)_4(H_2O)_2]^{2+}$ (2) $[Ni(NH_3)_2Cl_2]$

(3) $[Pt(NH_3)_2Cl_2]$ (4) $[Ni(en)_3]^{2+}$

Sol. $[Ni(NH_3)_2Cl_2]$ is tetrahedral complex, therefore does not show geometrical and optical isomerism.

$[Ni(NH_3)_2Cl_2]$ does not show structural isomerism

$[Ni(NH_3)_4(H_2O)_2]^{2+}$ & $[Pt(NH_3)_2Cl_2]$ show geometrical isomerism

$[Ni(en)_3]^{2+}$ show optical isomerism

17. Amongst the following statements regarding adsorption, those that are valid are :

(a) ΔH becomes less negative as adsorption proceeds.

(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.

(c) On adsorption, the residual force acting along the surface of the adsorbent increases.

(d) With increase in temperature, the equilibrium concentration of adsorbate increases.

(1) (b) and (c) (2) (a) and (b)

(3) (d) and (a) (4) (c) and (d)

Sol.(a) Since adsorption is exothermic process, as adsorption proceeds number of active sites present over adsorbent decreases, so less heat is evolved.

(b) Since NH_3 has higher force of attraction on adsorbent due to its polar nature (high value of 'a').

(c) As the adsorption increases, residual forces over surface decreases.

(d) Since process is exothermic, on increasing temperature it shift to backward direction, so concentration of adsorbate particle decreases.

18. Match the type of interaction in Column A with the distance dependence of their interaction energy in Column B :

A	B
(I) iron - ion	(a) $\frac{1}{r}$
(II) dipole - dipole	(b) $\frac{1}{r^2}$
(III) London dispersion	(c) $\frac{1}{r^3}$
	(d) $\frac{1}{r^6}$

(1) (I)-(a), (II)-(b), (III)-(c)

(2) (I)-(a), (II)-(c), (III)-(d)

(3) (I)-(a), (II)-(b), (III)-(d)

(4) (I)-(b), (II)-(d), (III)-(c)

Sol.

Type of interaction	Interaction Energy(E)
ion - ion	$E \propto \frac{1}{r}$
dipole - dipole	$E \propto \frac{1}{r^3}$
London dispersion	$E \propto \frac{1}{r^6}$

19. The correct observation in the following reactions is :

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Glycosidic bond}} A + B \xrightarrow[\text{reagent}]{\text{Seliwanoff 's}} ?$$

(1) Formation of blue colour

(2) Formation of violet colour

(3) Formation of red colour

(4) Gives no colour

Sol. Seliwanoff's test is used to distinguish between aldose and ketone sugars; when added to a solution containing ketose, red colour is formed rapidly.

$$\text{Sucrose} \xrightarrow{\text{Hydrolysis}} \begin{matrix}\text{Glucose}\\ +\\ \text{Fructose}\end{matrix} \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} \begin{matrix}\text{Red}\\ \text{colour}\end{matrix}$$

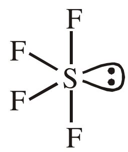
20. The molecular geometry of SF_6 is octahedral. What is the geometry of SF_4 (including lone pair(s) of electrons, if any) ?

(1) Trigonal bipyramidal

(2) Square planar

(3) Tetrahedral

(4) Pyramidal

Sol. 

4σ bonds +1 lone pair

$\therefore$ Shape (including lone pair of electrons) is Trigonal bipyramidal

21. The heat of combustion of ethanol into carbon dioxides and water is –327 kcal at constant pressure. The heat evolved (in cal) at constant volume and 27°C (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1})

Sol. $C_2H_5OH_{(\ell)} + 3O_{2(g)} \longrightarrow 2CO_{2(g)} + 3H_2O_{(\ell)}$

$\Delta n_g = 2 - 3 = -1$

$\Delta_cH = \Delta_cU + (\Delta n_g)\,RT$

$\Delta_cH = \Delta_cU - RT$

$\Delta_cU = \Delta_cH + RT$

$= -327 \times 10^3 + 2 \times 300$

$= -326400$ cal.

$\therefore$ Heat evolved

$= 326400$ cal.

22. For the disproportionation reaction $2Cu^+ (aq) \rightleftharpoons Cu(s) + Cu^{2+}(aq)$ at 298 K, ln K (where K is the equilibrium constant) is _____ $\times 10^{-1}$.

Given

($E^0_{Cu^{2+}/Cu^+} = 0.16V$

$E^0_{Cu^+/Cu} = 0.52V$

$\frac{RT}{F} = 0.025$)

Sol.

$$\begin{array}{rcl} Cu^+ & \longrightarrow & Cu + e^- \\ Cu^+ + e^- & \longrightarrow & Cu(s) \\ \hline 2Cu^+ & \longrightarrow & Cu^{2+} + Cu \end{array}$$

$E^o_{cell} = E^o_{Cu^+/Cu} - E^o_{Cu^{2+}/Cu^+}$

$= 0.52 - 0.16$

$= 0.36$ V

At equilibrium $\rightarrow E_{cell} = 0$

$E^o_{cell} = \frac{RT}{nF}\, \ell n\ K$

$\ell n\ K = \frac{E^o_{cell} \times nF}{RT}$

$\ell n\ K = \frac{0.36 \times 1}{0.025}$

$= 14.4 = 144 \times 10^{-1}$

23. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is ____.

Sol. $K_2Cr_2O_7$

$2\,(+1) + 2x + 7(-2) = 0$

$x = +6$

In $K_2Cr_2O_7$, Transition metal (Cr) present in +6 oxidation state.

$KMnO_4$

$(+1) + y + 4(-2) = 0$

$x = +7$

In $KMnO_4$, transition metal (Mn) present in +7 oxidation state

K_2FeO_4

$2(+1) + z + 4(-2) = 0$

$x = +6$

In K_2FeO_4 , transition metal (Fe) present in +6 oxidation state

So, $x = +6$

$y = +7$

$z = +6$

$x + y + z = 19$

24. The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is _______ .

Sol. C : H = 4 : 1

C : O = 3 : 4

Mass ratio

C : H : O = 12 : 3 : 16

Mole ratio

C : H : O = 1 : 3 : 1

Empirical formula = CH_3O

Molecular formula = $C_2H_6O_2$

(saturated acyclic organic compound)

$$C_2H_6O_2 + \frac{5}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$$

2 mole 5 mol

Moles of O_2 required = 5 moles

25. The work function of sodium metal is 4.41×10^{-19} J. If the photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be ($h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^{8}$ m/s) ________ $\times 10^{-21}$ J.

Sol. $E = W + K{\cdot}E_{max}$

$K{\cdot}E_{max} = E - W$

$$= \frac{hc}{\lambda} - 4.41 \times 10^{-19}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{300\times10^{-9}} - 4.41 \times 10^{-19}$$

$= 2.22 \times 10^{-19}$ J

$= 222 \times 10^{-21}$ J

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

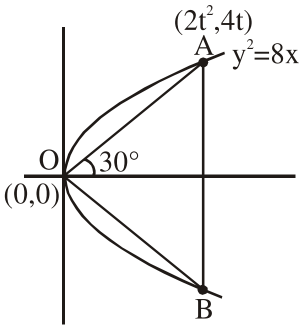
MATHEMATICS | TEST PAPER WITH SOLUTION

1. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

Sol.



$$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$$

$$AB = 8t = 16\sqrt{3}$$

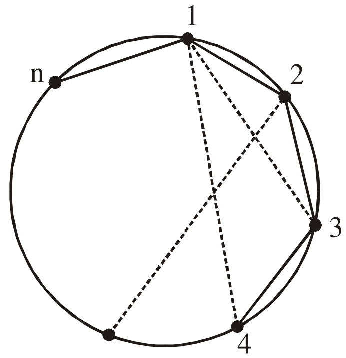
$$\text{Area} = 256.3 \cdot \frac{\sqrt{3}}{4} = 192\sqrt{3}$$

2. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-

(1) 199 (2) 101

(3) 201 (4) 200

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$$= {}^nC_2 - n$$

$${}^nC_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} - n = 99n$$

$$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$$

3. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4}, -1\right)$ (4) $\left[-1, -\frac{1}{2}\right]$

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

4. Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :

(1) $(-3, -1)$ (2) $(1, 3)$

(3) $(-1, 0)$ (4) $(0, 1)$

Sol. $f(x) = a(x - 3)(x - \alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

5. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x, y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is :

(1) 5 (2) 9

(3) 20 (4) 4

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n - 1)$

$g(n) = 20 \Rightarrow n(n - 1) = 20$

$n = 5$

6. Let $a, b, c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be :

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) 3 (4) $\frac{1}{3}$

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 (1 - 0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

7. Let $f : (-1, \infty) \to R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1 + x), x \neq 0$. Then the function f:

(1) decreases in $(-1, \infty)$

(2) decreases in $(-1, 0)$ and increases in $(0, \infty)$

(3) increases in $(-1, \infty)$

(4) increases in $(-1, 0)$ and decreases in $(0, \infty)$

Sol. $f'(x) = \dfrac{\dfrac{x}{1+x} - \ell n(1+x)}{x^2}$

$= \dfrac{x-(1+x)\,\ell n(1+x)}{x^2(1+x)}$

Suppose h(x) = x – (1 + x) $\ell n(1+x)$

$\Rightarrow$ h'(x) = 1 – $\ell n(1+x)$ – 1 = –$\ell n(1+x)$

h'(x) > 0, $\forall$ x $\in$ (–1, 0)

h'(x) < 0, $\forall$ x $\in$ (0, ∞)

h(0) = 0 $\Rightarrow$ h'(x) < 0 $\forall$ x $\in$ (–1, ∞)

$\Rightarrow$ f'(x) < 0 $\forall$ x$\in$(–1, ∞)

$\Rightarrow$ f(x) is a decreasing function for all x$\in$(–1, ∞)

8. If the sum of first 11 terms of an A.P., a_1 a_2, a_3,... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5,...,a_{23} is ka_1, where k is equal to :

(1) $\dfrac{121}{10}$ (2) $-\dfrac{72}{5}$

(3) $\dfrac{72}{5}$ (4) $-\dfrac{121}{10}$

Sol. $a_1 + a_2 + a_3 + \ldots + a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \dfrac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1 = -5d}$

$a_1 + a_3 + a_5 + \ldots + a_{23}$

$= (a_1 + a_{23}) \times \dfrac{12}{2} = (a_1 + a_1 + 22d) \times 6$

$= \left(2a_1 + 22\left(\dfrac{-a_1}{5}\right)\right) \times 6$

$= -\dfrac{72}{5}a_1 \Rightarrow K = \dfrac{-72}{5}$

9. The imaginary part of

$\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be :

(1) $-2\sqrt{6}$ (2) 6

(3) $\sqrt{6}$ (4) $-\sqrt{6}$

Sol. $\left(3+2\sqrt{-54}\right) = 3 + 2\times 3\times\sqrt{6}\,i$

$= \left(3+\sqrt{6}\,i\right)^2$

$\left(3-2\sqrt{54}\right) = \left(3-\sqrt{6}\,i\right)^2$

$\left(3+2\sqrt{-54}\right)^{1/2} + \left(3-2\sqrt{-54}\right)^{1/2}$

$= \pm\left(3+\sqrt{6}\,i\right) \pm \left(3-\sqrt{6}\,i\right)$

$= 6, -6, 2\sqrt{6}i, -2\sqrt{6}i,$

10. $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

Sol. $\lim\limits_{x\to 0}\left\{\tan\left(\dfrac{\pi}{4}+x\right)\right\}^{1/x}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{1+\tan x - 1 + \tan x}{x(1-\tan x)}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

11. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y = 4x + 2$ (2) $x + 4y = 8$

(3) $y + 4x = 2$ (4) $2y + x = 4$

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x=0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y=2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$$

Put $x = 0$ & $y = 2$

$$y' = e^{2\times 2 \ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve $= -\frac{1}{4}$ $\{m_1m_2=-1\}$

Hence equation of normal at (0, 2) is

$$y - 2 = -\frac{1}{4}(x-0)$$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x+4y=8}$

12. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

Sol. Given $\theta \in \left(0, \frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$$\Rightarrow \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$$

Hence eccentricity of hyperbola

$$(e_H) = \sqrt{1 + \frac{10\cos^2\theta}{10}} \qquad \text{...(1)}$$

$$\left\{e = \sqrt{1 + \frac{b^2}{a^2}}\right\}$$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$$\Rightarrow \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \qquad \left\{e = \sqrt{1 - \frac{a^2}{b^2}}\right\}$$

Hence eccenticity of ellipse

$$(e_E) = \sqrt{1 - \frac{5\cos^2\theta}{5}}$$

$$(e_E) = \sqrt{1-\cos^2\theta} = |\sin\theta| = \sin\theta \qquad \text{...(2)}$$

$$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$ ...(3)

Now length of latus rectum of ellipse

$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

13. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$ (2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$

(3) $(p \rightarrow q) \wedge (q \rightarrow p)$ (4) $(\sim q) \vee (p \wedge q) \rightarrow q$

Sol. Option (1) is

$\sim p \wedge (p \vee q) \rightarrow q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$\equiv C \vee (\sim p \wedge q) \rightarrow q$

$\equiv (\sim p \wedge q) \rightarrow q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee t$

so $\sim p \wedge (p \vee q) \rightarrow q$ is a tautology

14. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to:

(1) –10 (2) 5

(3) 10 (4) –5

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through (α, –3, 5) so this point satisfies the equation of plane so put in equation (1)

$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

15. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

Sol. Given E_1, E_2, E_3 are pairwise indepedent events

so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$

$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$

= 1 – $P(E_2)$ – $P(E_3)$

= [1 – $P(E_3)$] – $P(E_2)$

= $P(E_3^C) - P(E_2)$

16. Let A = {X = $(x, y, z)^T$: PX = 0 and

$x^2 + y^2 + z^2 = 1$} where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$,

then the set A :

(1) is a singleton

(2) contains exactly two elements

(3) contains more than two elements

(4) is an empty set

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \left.\begin{matrix} x + 2y + z = 0 \\ -2x + 3y - 4z = 0 \\ x + 9y - z = 0 \end{matrix}\right\}$ D = 0, so system have

infinite many solutions,

By solving these equation

we get $x = \frac{-11\lambda}{2}; y = \lambda; z = \frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$

$\Rightarrow \lambda = \pm\frac{1}{\sqrt{\frac{121}{4} + 1 + \frac{49}{4}}}$

so, there are 2 values of λ.

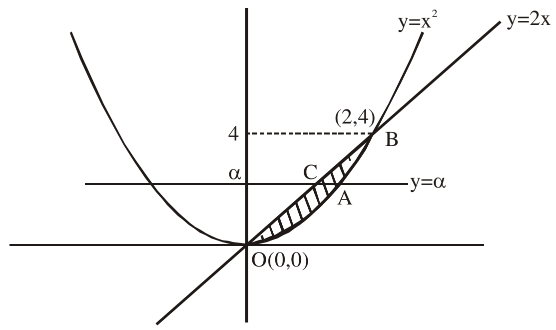
$\therefore$ so, there are 2 solution set of (x,y,z).

17. Consider a region R = {$(x, y) \in R^2 : x^2 \le y \le 2x$}. If a line y = α divides the area of region R into two equal parts, then which of the following is true?

(1) $\alpha^3 - 6\alpha^2 + 16 = 0$ (2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$ (4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

Sol.



* $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

$\Rightarrow \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)dy = 2\int_0^{\alpha}\left(\sqrt{y} - \frac{y}{2}\right).dy$

$$\Rightarrow \frac{4}{3} = 2\left[\frac{2}{3}\alpha^{3/2} - \frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2 - 8\alpha^{3/2} + 8 = 0}$$

18. If a curve y = f(x), passing through the point (1,2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

(3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$

Sol. $2x^2dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{2x^2} \quad \{\text{Homogeneous D.E.}\}$$

$$\left\{\begin{array}{l}\text{let } y = xt \\ \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}\end{array}\right\}$$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$$

$$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$$

$$\Rightarrow 2\int \frac{dt}{t^2} = \int \frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C \quad \left\{\text{Put } t = \frac{y}{x}\right\}$$

$$\Rightarrow -\frac{2x}{y} = \ell n\, x + C \quad \left\{\begin{array}{l}\text{Put } x = 1 \,\&\, y = 2 \\ \text{then we get } C = -1\end{array}\right\}$$

$$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1$$

$$\Rightarrow y = \frac{2x}{1-\ell n\, x}$$

$$\Rightarrow f(x) = \frac{2x}{1-\log_e x}$$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1+\log_e 2}}$

19. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3+(k+4)a\}+\{x^4+(k + 6)a\}+.....$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :

(1) –5 (2) 1

(3) –3 (4) 3

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] +......9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4+.....9 \text{ terms}) + (ka + ka + ka + ka +.......9 \text{ terms}) + (0 + 2a + 4a + 6a + ...9 \text{ terms})$

$$\Rightarrow S = x\left[\frac{x^9-1}{x-1}\right] + 9ka + 72a$$

$$\Rightarrow S = \frac{(x^{10}-x) + (9k+72)a(x-1)}{(x-1)}$$

Compare with given sum, then we get, (9k + 72) = 45

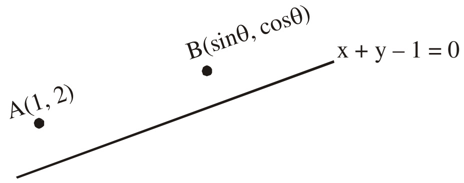
$$\Rightarrow \boxed{k = -3}$$

20. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is :

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

Sol. Given that both points $(1, 2)$ & $(\sin\theta, \cos\theta)$ lie on same side of the line $x + y - 1 = 0$



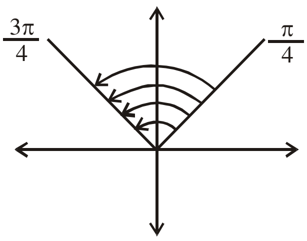
So, $\begin{pmatrix}\text{Put } (1,2) \text{ in}\\ \text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta, \cos\theta \text{ in}\\ \text{given line}\end{pmatrix} > 0$

$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \ \left\{\div \text{by} \sqrt{2}\right\}$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$



$$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

$$\Rightarrow \boxed{0 < \theta < \frac{\pi}{2}}$$

21. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \ldots. b_{11}$ is 90, then the common difference of this A.P. is________.

Sol. Let a be the first term and d be the common difference of the given A.P. Where $d > 0$

$$\bar{X} = a + \frac{0 + d + 2d + \ldots + 10d}{11}$$

$= a + 5d$

$$\Rightarrow \text{varience} = \frac{\Sigma(\bar{X} - x_i)^2}{11}$$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

22. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$,

then $\frac{dy}{dx}$ at $x = 0$ is________.

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5} \quad 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

$= \cos\alpha \,.\, \cos kx - \sin\alpha \,.\, \sin kx$

$= \cos(\alpha + kx)$

As we have to find derivate at $x = 0$

We have $\cos^{-1}(\cos(\alpha + kx))$

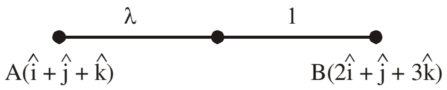
$= (\alpha + kx)$

$$\Rightarrow y = \sum_{k=1}^{6}(\alpha + kx)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{\text{at } x=0} = \sum_{k=x}^{6} k = \frac{6 \times 7 \times 13}{6} = 91$$

23. Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ $(\lambda > 0)$. If O is the origin and $\overrightarrow{OB} \cdot \overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6$, then λ is equal to______.

Sol.



Using section formula we get

$$\overline{OP} = \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{\lambda+1}{\lambda+1}\hat{j} + \frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB}\cdot\overline{OP} = \dfrac{4\lambda+2+\lambda+1+9\lambda+3}{\lambda+1}$

$$= \frac{14\lambda+6}{\lambda+1}$$

$$\overline{OA}\times\overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \dfrac{2\lambda+1}{\lambda+1} & 1 & \dfrac{3\lambda+1}{\lambda+1} \end{vmatrix}$$

$$= \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA}\times\overline{OP}|^2 = \frac{(2\lambda+1)^2+\lambda^2+\lambda^2}{(\lambda+1)^2}$$

$$= \frac{6\lambda^2+1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1} - 3\times\frac{(6\lambda^2+1)}{(\lambda+1)^2} = 6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0, \dfrac{8}{10} = 0.8$

$\Rightarrow \lambda = 0.8$

24. For a positive integer n, $\left(1+\dfrac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{2}{5}$

$\Rightarrow 7r = 2n + 2$ ...(1)

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$\Rightarrow 17r = 5n - 12$ ...(2)

On solving (1) & (2)

$\Rightarrow n = 118$

25. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is_______.

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\displaystyle\int_1^2 |2x-[3x]|\,dx$

$$= \int_1^{4/3}(3-2x)dx + \int_{4/3}^{5/3}(4-2x)\,dx + \int_{5/3}^{2}(5-2x)dx$$

$$= \frac{2}{9}+\frac{3}{9}+\frac{4}{9} = 1$$