# FINAL JEE–MAIN EXAMINATION – APRIL,2019

**(Held On Friday 12th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM**

## PHYSICS

## TEST PAPER WITH ANSWER & SOLUTION

**1.** The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1) 0.24 μm (2) 0.48 μm

(3) 0.12 μm (4) 0.38 μm

**Sol.** Numerical aperature of the microscope is given as

$$NA = \frac{0.61\lambda}{d}$$

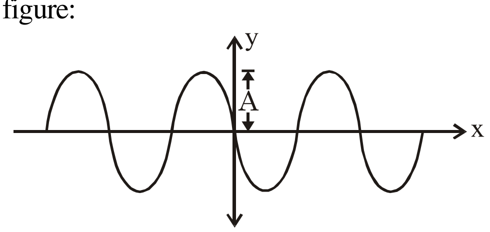
Where d = minimum sparation between two points to be seen as distinct

$$d = \frac{0.61\lambda}{NA} = \frac{(0.61)\times(5000\times 10m^{-10})}{1.25}$$

$= 2.4 \times 10^{-7}$ m

$= 0.24$ μm

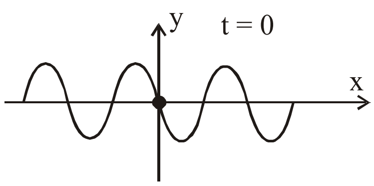
**2.** A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin (kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure:




For this wave, the phase $\phi$ is :

(1) 0 (2) $-\frac{\pi}{2}$ (3) $\pi$ (4) $\frac{\pi}{2}$

**Sol.**

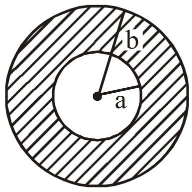



$y = A \sin (kx - wt + \phi)$

at x = 0, t = 0, y = 0 and slope is negative

$\Rightarrow \phi = \pi$

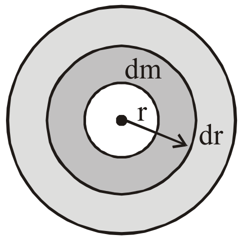
**3.** A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :




(1) $\frac{a+b}{2}$ (2) $\frac{a+b}{3}$

(3) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (4) $\sqrt{\frac{a^2+b^2+ab}{3}}$

**Sol.**




$dI = (dm)r^2$

$= (\sigma\, dA)r^2$

$= \left(\frac{\sigma_0}{r}2\pi r dr\right)r^2$

$= (\sigma_0\, 2\pi)r^2 dr$

$$I = \int dI = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$$

$$m = \int dm = \int \sigma dA$$

$$= \sigma_0 2\pi \int_a^b dr$$

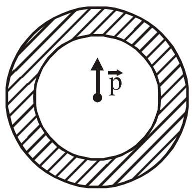
$m = \sigma_0\, 2\pi\, (b\text{-}a)$

Radius of gyration

$$k = \sqrt{\frac{I}{m}} \quad = \sqrt{\frac{(b^3 - a^3)}{3(b-a)}}$$

$$= \sqrt{\left(\frac{a^3 + b^3 + ab}{3}\right)}$$

**4.** Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case :



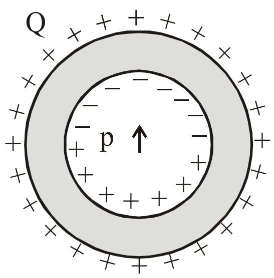
(1) Electric field outside the shell is the same as that of a point charge at the centre of the shell.

(2) Surface charge density on the inner surface of the shell is zero everywhere.

(3) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$.

(4) Surface charge density on the outer surface depends on $|\vec{p}|$

**Sol.**



Total charge of dipole = 0, so charge induced on outside surface = 0.

But due to non uniform electric field of dipole, the charge induced on inner surface is non zero and non uniform.

So, for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.

$$\boxed{E = \frac{KQ}{r^2}}$$

**5.** A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude $\theta_0$. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $\ell$ ($\ell << L$), is close to :

(1) $Mg\ell$

(2) $Mg\ell\,(1 + \theta_0^2)$

(3) $Mg\ell\,(1 - \theta_0^2)$

(4) $Mg\ell\left(1 + \frac{\theta_0^2}{2}\right)$

**Sol.** Angular momentum conservation.

$$\cancel{M}V_0L = \cancel{M}V_1(L-\ell)$$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$w_g + w_p = \Delta KE$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1 - \frac{\ell}{L}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1 + \frac{2\ell}{L}\right) - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_p = mg\ell + mv_0^2\frac{\ell}{L}$$

here, $V_0$ = maximum velocity
$= \omega \times A$

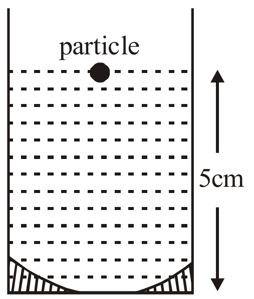
$$= \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$V_0 = \theta_0\sqrt{gL}$$

$$\text{so, } w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$$

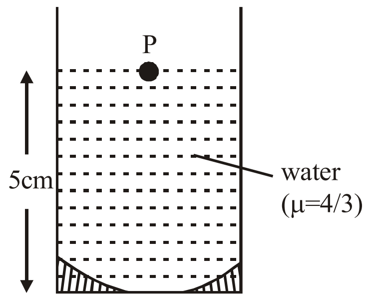
$$= mg\ell\left(1+\theta_0^2\right)$$

**6.** A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to : (Refractive index of water = 1.33)




(1) 8.8 cm (2) 11.7 cm
(3) 6.7 cm (4) 13.4 cm

**Sol.** Light incident from particle P will be reflected at mirror




$$u = -5\text{cm}, \ f = -\frac{R}{2} = -20\text{cm}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\boxed{v_1 = +\frac{20}{3}\text{cm}}$$

This image will act as object for light getting refracted at water surface

$$\text{So, object distance } d = 5+\frac{20}{3} = \frac{35}{3}\text{cm}$$

below water surface.
After refraction, final image is at

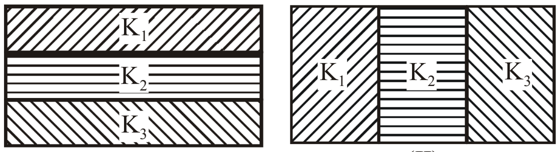
$$d' = d\left(\frac{\mu_2}{\mu_1}\right)$$

$$= \left(\frac{35}{3}\right)\left(\frac{1}{4/3}\right)$$

$$= \frac{35}{4} = 8.75\text{cm}$$

$$\approx 8.8\text{cm}$$

**7.** Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants $K_1$, $K_2$ and $K_3$. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II.
If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be ($E_1$ refers to capacitor (I) and $E_2$ to capacitor (II)) :




(I) (II)

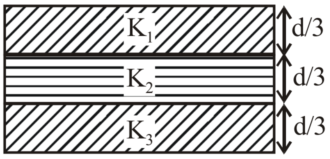
(1) $\dfrac{E_1}{E_2} = \dfrac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(2) $\dfrac{E_1}{E_2} = \dfrac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(3) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(4) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

**Sol.** I.




$$C_1 = \frac{3\varepsilon_0 AK_1}{d}$$

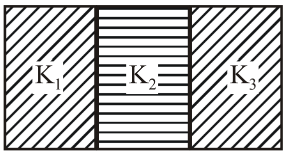
$$C_2 = \frac{3\varepsilon_0 AK_2}{d}$$

$$C_3 = \frac{3\varepsilon_0 AK_3}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)} \quad \text{........(1)}$$

II.




$$C_1 = \frac{\varepsilon_0 K_1 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 K_2 A}{3d}$$

$$C_3 = \frac{\varepsilon_0 K_3 A}{3d}$$

$C'_{eq} = C_1 + C_2 + C_3$

$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \quad \text{........(2)}$$

Now,

$$\boxed{\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq}.V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}}$$

**8.** A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V = 0 at infinity) :

(1) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(2) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

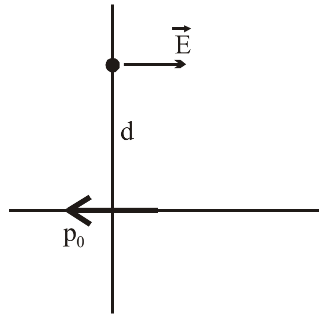
(3) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(4) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

**Sol.** V = 0

$$E = -\frac{K\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\varepsilon_0 d^3}$$




**9.** When $M_1$ gram of ice at –10ºC (specific heat = 0.5 cal $g^{-1}$ºC$^{-1}$) is added to $M_2$ gram of water at 50ºC, finally no ice is left and the water is at 0ºC. The value of latent heat of ice, in cal $g^{-1}$ is:

(1) $\frac{5M_1}{M_2} - 50$

(2) $\frac{50M_2}{M_1}$

(3) $\frac{50M_2}{M_1} - 5$

(4) $\frac{5M_2}{M_1} - 5$

**Sol.** Heat lost = Heat gain

$\Rightarrow M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1.L_f$

$$\Rightarrow L_f = \frac{50M_2 - 5M_1}{M_1}$$

$$= \frac{50M_2}{M_1} - 5$$

**10.** The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle $\theta_0$ with speed $v_0$ then ($g = 10\ ms^{-2}$) :

(1) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$

(2) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$

(4) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$

**Sol.** Equation of trajectory is given as

$y = 2x - 9x^2$ ........ (1)

Comparing with equation :

$$y = x \tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2 \qquad \text{........ (2)}$$

We get;

$\tan\theta = 2$

$$\therefore \boxed{\cos\theta = \frac{1}{\sqrt{5}}}$$

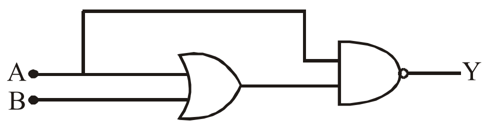
$$\text{Also, } \frac{g}{2u^2\cos^2\theta} = 9$$

$$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2} = u^2$$

$$\Rightarrow u^2 = \frac{25}{9}$$

$$\Rightarrow \boxed{u = \frac{5}{3}\text{m/s}}$$

**11.** The truth table for the circuit given in the fig. is:




(1)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

(2)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

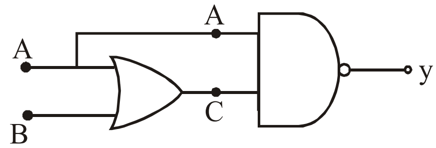
(3)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

(4)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**Sol.**




$C = A + B$

and $y = \overline{A.C}$

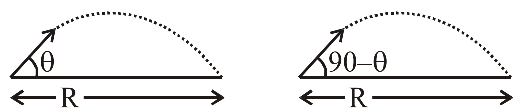
| A | B | $C = (A+B)$ | A.C. | $y = \overline{A.C}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |

**12.** A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If $t_1$ and $t_2$ are the values of the time taken by it to hit the target in two possible ways, the product $t_1t_2$ is:

(1) R/g (2) R/4g

(3) 2R/g (4) R/2g

**Sol.** Range will be same for time $t_1$ & $t_2$, so angles of projection will be 'θ' & '90° – θ'




$$t_1 = \frac{2u\sin\theta}{g} \qquad t_2 = \frac{2u\sin(90°-\theta)}{g}$$

$$\text{and } R = \frac{u^2\sin 2\theta}{g}$$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$$

$$= \frac{2R}{g}$$

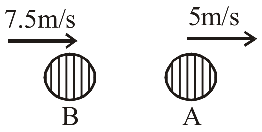
**13.** A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency $\nu$. The value of $\nu$ is close to :

(Speed of sound in water = 1500 $ms^{-1}$)

(1) 499 Hz (2) 502 Hz

(3) 507 Hz (4) 504 Hz

**Sol.**




$f_0 = 500$ Hz

frequency recieved by A

$$\Rightarrow \left(\frac{1500-5}{1500-7.5}\right) f_0 = f_1$$

and frequency recieved By B again =

$\longleftarrow 1500$

(B) (A) & $\Rightarrow$

7.5 m/s $\longrightarrow$ $\longrightarrow$ 5 m/sec

$$f_2 = \left(\frac{1500+7.5}{1500+5}\right) \times \left(\frac{1500-5}{1500-7.5}\right) f_0 = 502 \text{ Hz.}$$

**14.** Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume ? (R = 8.3 J/mol K)

(1) 21.6 J/mol K

(2) 19.7 J/mol K

(3) 17.4 J/mol K

(4) 15.7 J/mol K

**Sol.** $$f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{2\times3+3\times5}{5} = \frac{21}{5}$$

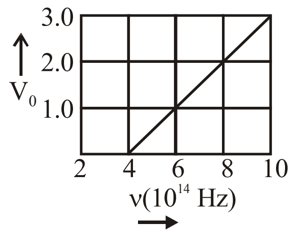
$$C_V = \frac{fR}{2} = \frac{21}{5} \times \frac{R}{2} = 17.4\,\text{J/mol K}$$

**15.** The stopping potential $V_0$ (in volt) as a function of frequency ($\nu$) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be :

(Given : Planck's constant

(h) = 6.63 × $10^{-34}$ Js, electron

charge e = 1.6 × $10^{-19}$ C)




(1) 1.95 eV (2) 1.82 eV

(3) 1.66 eV (4) 2.12 eV

**Sol.** $h\nu = \phi + ev_0$

$$v_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

$v_0$ is zero for $\nu = 4 \times 10^{14}$ Hz

$$0 = \frac{h\nu}{e} - \frac{\phi}{e} \Rightarrow \phi = h\nu$$

$$= \frac{6.63\times10^{-34}\times4\times10^{14}}{1.6\times10^{-19}} = 1.66 \text{ ev.}$$

**16.** At 40ºC, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40ºC to 20ºC it regains its original length of 0.2 m. The value of M is close to :

(Coefficient of linear expansion and Young's modulus of brass are $10^{-5}$/ºC and $10^{11}$ N/$m^2$, respectively; g = 10 $ms^{-2}$)

(1) 1.5 kg (2) 9 kg

(3) 0.9 kg (4) 0.5 kg

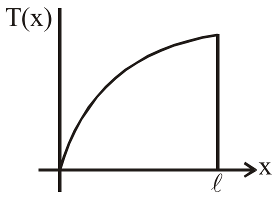
**Sol.** $$Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$$

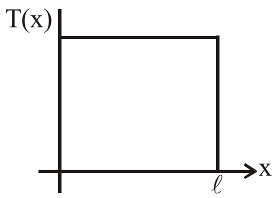
$$\frac{\Delta\ell}{\ell} = \alpha\Delta T$$

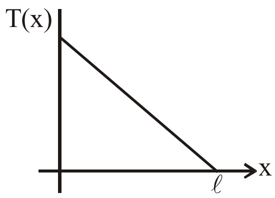
Mg = (Ay)$\alpha\Delta$T = 2$\pi$

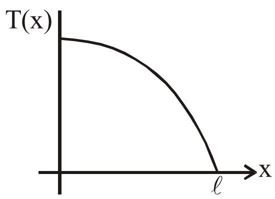
It is closest to 9.

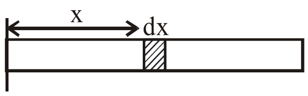
**17.** A uniform rod of length $\ell$ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

(1)



(2)



(3)



(4)



**Sol.**



$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\, \omega^2 x$$

$$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

$$\boxed{T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)}$$

**18.** Which of the following combinations has the dimension of electrical resistance ($\varepsilon_0$ is the permittivity of vacuum and $\mu_0$ is the permeability of vacuum) ?

(1) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

(2) $\frac{\mu_0}{\varepsilon_0}$

(3) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$

(4) $\frac{\varepsilon_0}{\mu_0}$

**Sol.** $[\varepsilon_0] = M^{-1} L^{-3} T^4 A^2$

$[\mu_0] = M L T^{-2} A^{-2}$

$[R] = M L^2 T^{-3} A^{-2}$

$$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

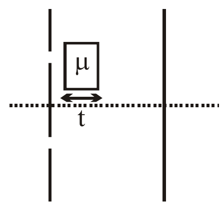
**19.** In a double slit experiment, when a thin film of thickness t having refractive index $\mu$ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is ($\lambda$ is the wavelength of the light used) :

(1) $\frac{\lambda}{2(\mu-1)}$

(2) $\frac{\lambda}{(2\mu-1)}$

(3) $\frac{2\lambda}{(\mu-1)}$

(4) $\frac{\lambda}{(\mu-1)}$

**Sol.**



$\Delta X = (\mu - 1)t = 1\lambda$

for one maximum shift

$$\boxed{t = \frac{\lambda}{\mu - 1}}$$

**20.** An electromagnetic wave is represented by the electric field

$\vec{E} = E_0 \hat{n} \sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propogation $\hat{s}$, is :

(1) $\hat{s} = \frac{4\hat{j} - 3\hat{k}}{5}$ (2) $\hat{s} = \frac{3\hat{i} - 4\hat{j}}{5}$

(3) $\hat{s} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$ (4) $\hat{s} = \frac{-4\hat{k} + 3\hat{j}}{5}$

**Sol.** $\vec{E} = E_0 \hat{n} \sin(\omega t + (6y - 8z))$

$= E_0 \hat{n} \sin(\omega t + \vec{k}.\vec{r})$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and $\vec{k}.\vec{r} = 6y - 8z$

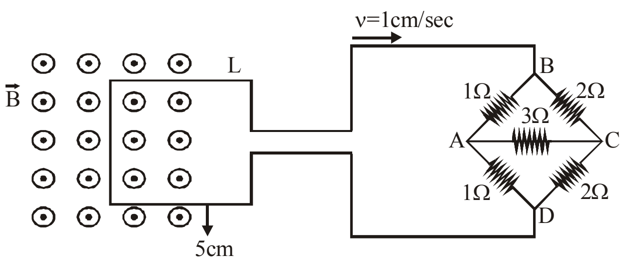
$\Rightarrow \vec{k} = 6\hat{j} - 8\hat{k}$

direction of propagation

$\hat{s} = -\hat{k}$

$= \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$

**21.** The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cms$^{-1}$. At some instant, a part of L is in a uniform magnetic field of 1T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to :




(1) 115 μA (2) 170 μA
(3) 60 μA (4) 150 μA

**Sol.** Since it is a balanced wheatstone bridge, its equivalent resistance = $\frac{4}{3}\Omega$

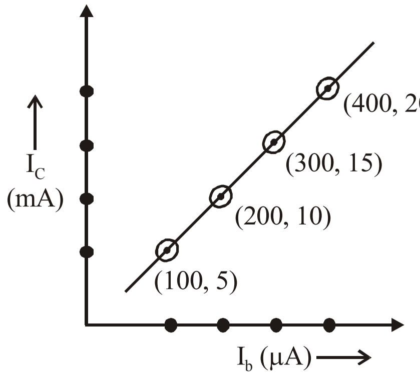
$\varepsilon = B\ell v = 5 \times 10^{-4}$ V

So total resistance

$R = \frac{4}{3} + 1.7 \approx 3\Omega$

$\therefore i = \frac{\varepsilon}{R} \approx 166\mu A \approx 170\mu A$

**22.** The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The Voltage and Power gain, are respectively :




(1) $5 \times 10^4$, $5 \times 10^5$
(2) $5 \times 10^4$, $5 \times 10^6$
(3) $5 \times 10^4$, $2.5 \times 10^6$
(4) $2.5 \times 10^4$, $2.5 \times 10^6$

**Sol.** $V_{gain} = \left(\frac{\Delta I_C}{\Delta I_B}\right)\frac{R_{out}}{R_{in}}$

$= \left(\frac{5 \times 10^{-3}}{100 \times 10^{-6}}\right) \times 10^3$

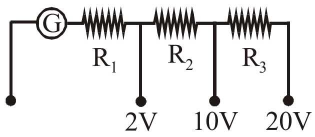
$= \frac{1}{20} \times 10^6 = 5 \times 10^4$

$P_{gain} = \left(\frac{\Delta I_C}{\Delta I_b}\right)(V_{gain})$

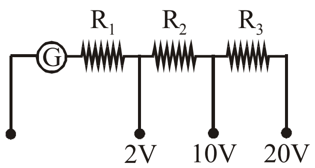
$= \left(\frac{5 \times 10^{-3}}{100 \times 10^{-6}}\right)(5 \times 10^4)$

$= 2.5 \times 10^6$

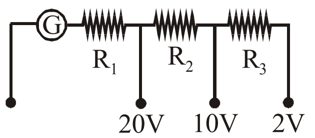
**23.** A galvanometer of resistance 100Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0–2 V, 0–10 V and 0–20 V. The appropriate circuit to do so is :

(1) 

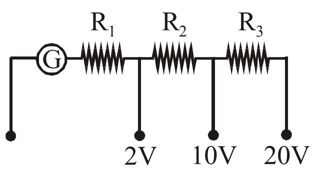

$R_1 = 1900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 19900\ \Omega$

(2) 


$R_1 = 2000\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

(3) 


$R_1 = 19900\ \Omega$
$R_2 = 9900\ \Omega$
$R_3 = 1900\ \Omega$

(4) 


$R_1 = 1900\ \Omega$
$R_2 = 8000\ \Omega$
$R_3 = 10000\ \Omega$

**Sol.** $20 \times 50 \times 10^{-6} = 10^{-3}$ Amp.

$$V_1 = \frac{2}{10^{-3}} = 100 + R_1$$

$1900 = R_1$

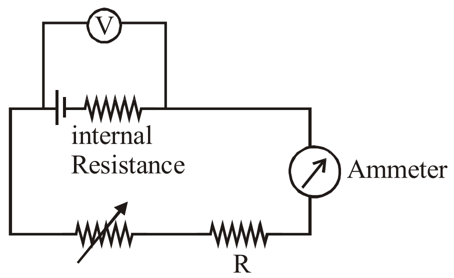
$$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$$

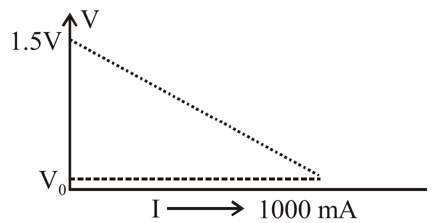
$R_2 = 8000$

$$V_3 = \frac{20}{10^{-3}} = 10 \times 10^3 + R_3$$

$10 \times 10^3 = R_3$

**24.** To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained:



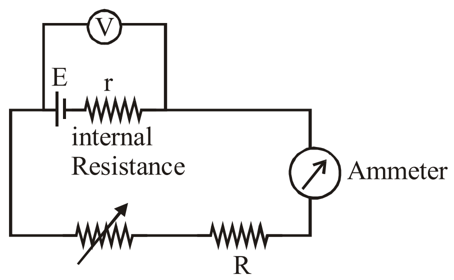

If $V_0$ is almost zero, identify the correct statement:

(1) The value of the resistance R is 1.5 Ω

(2) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

(3) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.

**Sol.** 


$V = E - Ir$

when $V = V_0 = 0 \Rightarrow 0 = E - Ir$

$\therefore\ E = r$

when $I = 0,\ V = E = 1.5V$

$\therefore\ r = 1.5\Omega.$

**25.** An excited $He^{+}$ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength $\lambda$, energy $E = \frac{1240\,eV}{\lambda(\text{in nm})}$) :

(1) n = 5 (2) n = 4

(3) n = 6 (4) n = 7

**Sol.** $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)z^2$

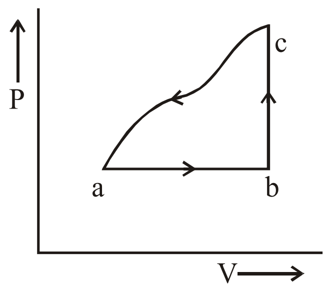
$$\frac{1}{1085} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)2^2$$

$$\frac{1}{304} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)2^2$$

$\therefore$ m = 2

$\therefore$ n = 5

**26.** A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :




(1) 100 J (2) 120 J

(3) 140 J (4) 130 J

**Sol.**

| | ΔE | ΔW | ΔQ |
|---|---|---|---|
| ab | | | 250 |
| bc | | 0 | 60 |
| ca | –180 | | |

| | ΔE | ΔW | ΔQ |
|---|---|---|---|
| ab | 120 | 130 | 250 |
| bc | 60 | 0 | 60 |
| ca | –180 | | |

**27.** A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 $\pi$ rad $s^{-1}$ about its axis, perpendicular to its plane. If the magnetic field at its centre is $3.8 \times 10^{-9}$ T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A$^2$) :

(1) $2 \times 10^{-6}$ C (2) $3 \times 10^{-5}$ C

(3) $4 \times 10^{-5}$ C (4) $7 \times 10^{-6}$ C

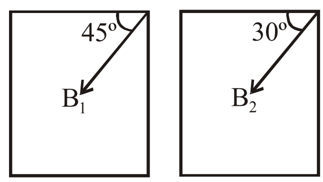
**Sol.** $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q\omega}{2R\,2\pi}$

$\Rightarrow$ q = $3 \times 10^{-5}$ C

**28.** A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If $B_1$ and $B_2$ are respectively the total magnetic field due to the earth at the two places, then the ratio $B_1/B_2$ is best given by :

(1) 2.2 (2) 1.8 (3) 0.7 (4) 3.6

**Sol.**




$$f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1 \cos 45^\circ}{I}} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2 \cos 30^\circ}{I}}$$

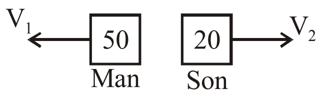
$$\frac{f_1}{f_2} = \sqrt{\frac{B_1 \cos 45^\circ}{B_2 \cos 30^\circ}} \qquad \therefore \frac{B_1}{B_2} \approx 0.7$$

**29.** A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 $ms^{-1}$ with respect to the man. The speed of the man with respect to the surface is :

(1) 0.20 $ms^{-1}$ (2) 0.14 $ms^{-1}$

(3) 0.47 $ms^{-1}$ (4) 0.28 $ms^{-1}$

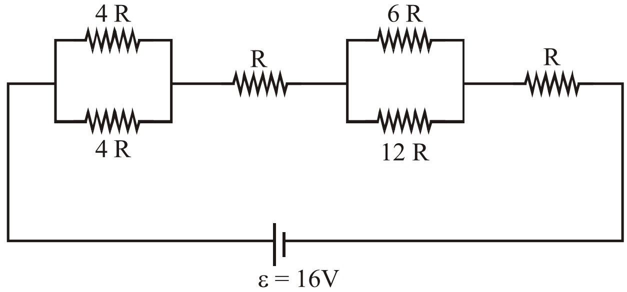
**Sol.**




$\Rightarrow 0 = 50V_1 - 20V_2$ and $V_1 + V_2 = 0.7$

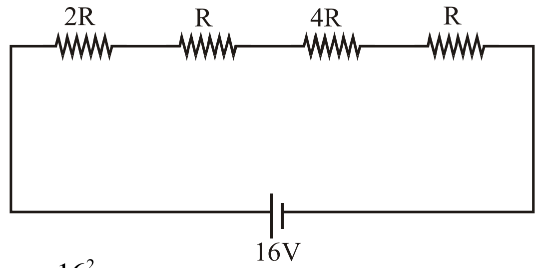
$\Rightarrow V_1 = 0.2$

**30.** The resistive network shown below is connected to a D.C. source of 16V. The power consumed by the network is 4 Watt. The value of R is :




(1) 8Ω (2) 6Ω (3) 1Ω (4) 16Ω

**Sol.**




$$P = \frac{16^2}{8R} = 4 \quad \therefore R = 8\Omega$$

# FINAL JEE–MAIN EXAMINATION – APRIL,2019

**(Held On Friday 12th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM**

## CHEMISTRY

## TEST PAPER WITH ANSWER & SOLUTION

**1.** 5 moles of $AB_2$ weigh $125 \times 10^{-3}$ kg and 10 moles of $A_2B_2$ weigh $300 \times 10^{-3}$ kg. The molar mass of A($M_A$) and molar mass of B($M_B$) in kg mol$^{-1}$ are :

(1) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$

(2) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

(3) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

(4) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$

**Sol.** $5[M_A + 2M_B] = 125$

$M_A + 2M_B = 25$ ....(1)

$2M_A + 2M_B = 30$ ....(2)

from eq. (1) & (2)

$M_A = 5$

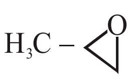
$M_B = 10$

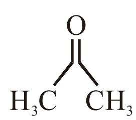
**2.** The major product of the following addition reaction is :

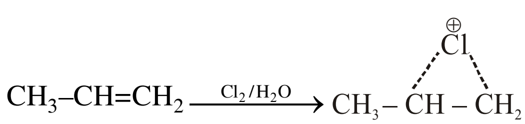
$$H_3C - CH = CH_2 \xrightarrow{Cl_2/H_2O}$$

(1) $CH_3 - \underset{Cl}{CH} - \underset{OH}{CH_2}$

(2) $H_3C - \underset{OH}{CH} - \underset{Cl}{CH_2}$

(3) 

(4) 

**Sol.** 

$$\xrightarrow{H_2\ddot{O}} CH_3-\underset{OH}{CH}-\overset{Cl}{CH_2}$$

**3.** What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution ? Given that, solubility product of $Al(OH)_3 = 2.4 \times 10^{-24}$ :

(1) $12 \times 10^{-23}$ (2) $12 \times 10^{-21}$

(3) $3 \times 10^{-19}$ (4) $3 \times 10^{-22}$

**Sol.** $Al(OH)_3 \rightleftharpoons Al^{+3} + 3OH^-$

$S'$ $\quad 0.2 + 3(S') \simeq 0.2$

$S' \times (0.2)^3 = k_{sp} = 2.4 \times 10^{-24}$

$(S') = 3 \times 10^{-22}$ M

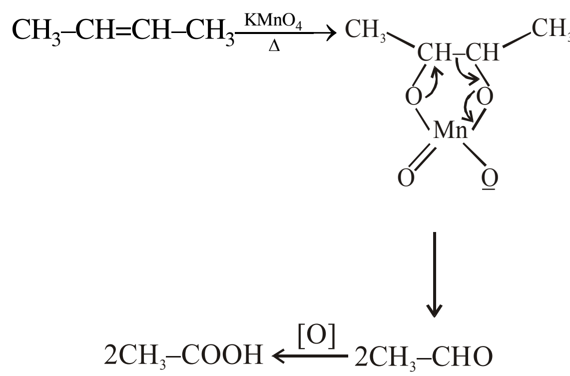
**4.** But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give :

(1) one molecule of $CH_3CHO$ and one molecule of $CH_3COOH$

(2) $CH_3-\underset{OH}{CH}-\underset{OH}{CH}-CH_3$

(3) 2 molecules of $CH_3COOH$

(4) 2 molecules of $CH_3CHO$

**Sol.** 

**5.** The correct sequence of thermal stability of the following carbonates is

(1) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

(2) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(3) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

(4) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

**Sol.** Thermal stability of Alkaline earth metals carbonates increases down the group.

because down the group polarizing power of cation decreases. So thermal stability increases.

Hence, Thermal stability order :

$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

**6.** The correct statement among the following is

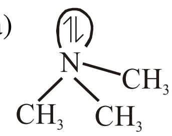
(1) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

(2) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

(3) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

(4) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

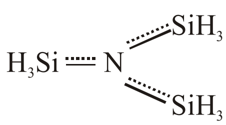
**Sol.** (a)



nitrogen is $sp^3$ hybrid and pyramidal

no back-bonding i.e.more basic

(b)



Nitrogen $sp^2$ hybrid and planar due to back bonding and less basic because lone pair is not available for donation.

**7.** Peptization is a :

(1) process of converting a colloidal solution into precipitate

(2) process of converting precipitate into colloidal solution

(3) process of converting soluble particles to form colloidal solution

(4) process of bringing colloidal molecule into solution

**8.** Given :

$Co^{3+} + e^- \rightarrow Co^{2+}$ ; $E^o = +1.81$ V

$Pb^{4+} + 2e^- \rightarrow Pb^{2+}$ ; $E^o = +1.67$ V

$Ce^{4+} + e^- \rightarrow Ce^{3+}$ ; $E^o = +1.61$ V

$Bi^{3+} + 3e^- \rightarrow Bi$ ; $E^o = +0.20$ V

Oxidizing power of the species will increase in the order :

(1) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$

(2) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$

(3) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$

(4) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

**Sol.** $E^{\circ}_{Red} \uparrow \Rightarrow$ oxidizing power $\uparrow$

**9.** The metal that gives hydrogen gas upon treatment with both acid as well as base is :

(1) zinc (2) iron

(3) magnesium (4) mercury

**Sol.** $Zn + 2HCl \rightarrow ZnCl_2 + H_2 \uparrow$

$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2 \uparrow$

**10.** The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are

(1) 16, 5 and 2 (2) 16, 6 and 3

(3) 15, 5 and 3 (4) 15, 6 and 2

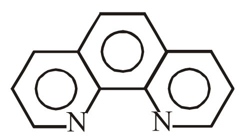
**Sol.** Atomic number (Z) = 15 $\Rightarrow$ P $\rightarrow$ [Ne] $3s^2\ 3p^3$

Phosphorus belongs to $15^{th}$ group

number of valence electrons = 5

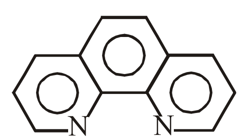
and valency = 3 in ground state.

**11.** The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is

(Phen =  and ignore pairing energy)

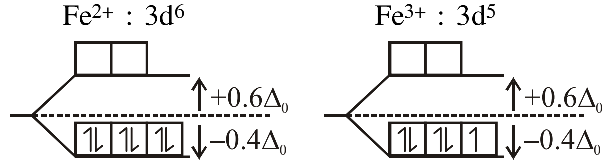
(1) $[Fe(phen)_3]^{2+}$ (2) $[Zn(phen)_3]^{2+}$

(3) $[Ni(phen)_3]^{2+}$ (4) $[Co(phen)_3]^{2+}$

**Sol.** Phen =  is a strong field symmetrical bidentate ligand.

(1) $[Fe(phen)_3]^{2+} \xrightarrow{-e^-} [Fe(phen)_3]^{3+}$

$Fe^{2+}$ : $3d^6$ $Fe^{3+}$ : $3d^5$




$\boxed{C.F.S.E = -2.4\,\Delta_0}$ $\boxed{C.F.S.E = -2.0\Delta_0}$

By oxidation of $Fe^{2+}$ into $Fe^{3+}$, the CFSE value decrease.

(2) $[Zn(phen)_3]^{2+} \xrightarrow{-e^-} [Zn(phen)_3]^{3+}$

$Zn^{2+}$ : $3d^{10}$ $\quad$ $Zn^{3+}$ : $3d^9$

$\boxed{C.F.S.E = 0}$ $\quad$ $\boxed{C.F.S.E = -0.6\Delta_0}$

By oxidation of $Zn^{2+}$ into $Zn^{3+}$, the CFSE value increase.

(3) $[Ni(phen)_3]^{2+} \xrightarrow{-e^-} [Ni(phen)_3]^{3+}$

$Ni^{2+}$ : $3d^8$ $\quad$ $Ni^{3+}$ : $3d^7$

$\boxed{C.F.S.E = -1.2\,\Delta_0}$ $\quad$ $\boxed{C.F.S.E = -1.8\Delta_0}$

by oxidation of $Ni^{2+}$ into $Ni^{3+}$, the CFSE value increase.

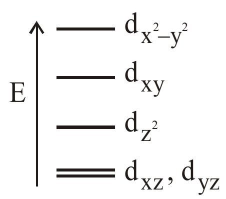
(4) $[Co(phen)_3]^{2+} \xrightarrow{-e^-} [Co(phen)_3]^{3+}$

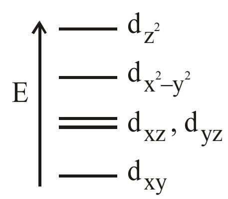
$Co^{2+}$ : $3d^7$ $\quad$ $Co^{3+}$ : $3d^6$

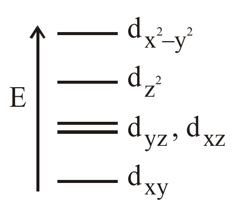
$\boxed{C.F.S.E = -1.8\,\Delta_0}$ $\quad$ $\boxed{C.F.S.E = -2.4\Delta_0}$

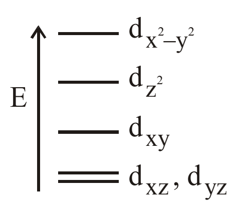
by oxidation of $Co^{2+}$ into $Co^{3+}$, the CFSE value increase.

**12.** Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

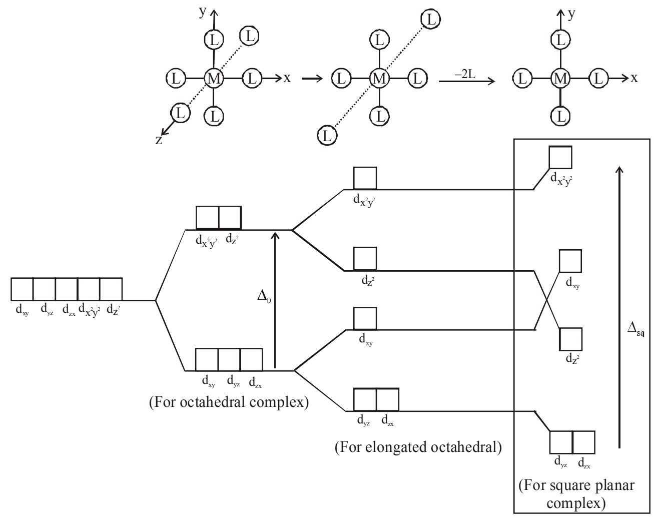
(1) 


(2) 

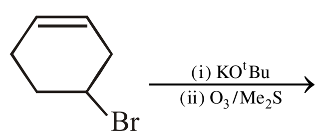

(3) 

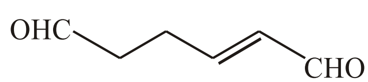

(4) 


**Sol.** If both ligands present along z-axis removed from octachedral field and converted into square planar field, then

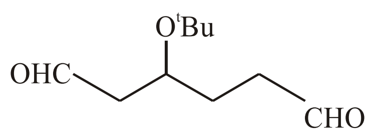


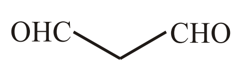

**13.** The major product(s) obtained in the following reaction is/are :

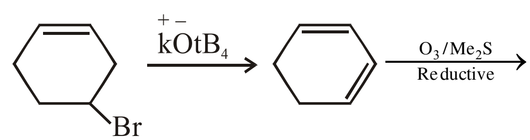



(1) 

(2) 


(3) 

(4) 

**Sol.** 


$CHO{-}CH_2{-}CH_2{-}CHO + OHC{-}CHO$

**14.** An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is :

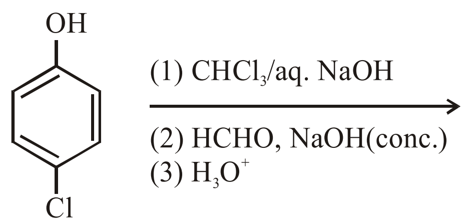
(1) $\frac{a}{2}$ (2) a

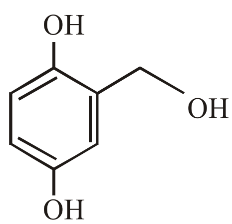
(3) $\frac{3}{2}$ a (4) $\sqrt{2}$ a

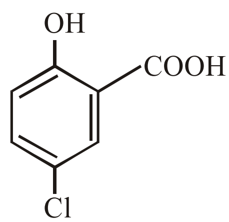
**Sol.** Distance between two nearest tetrahedral void

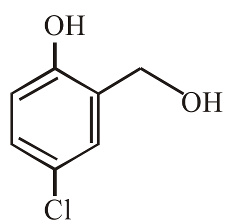
$= \left(\frac{a}{2}\right)$

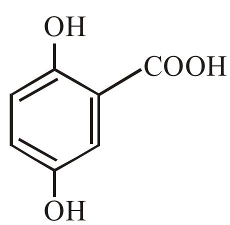
**15.** The major products of the following reaction are :



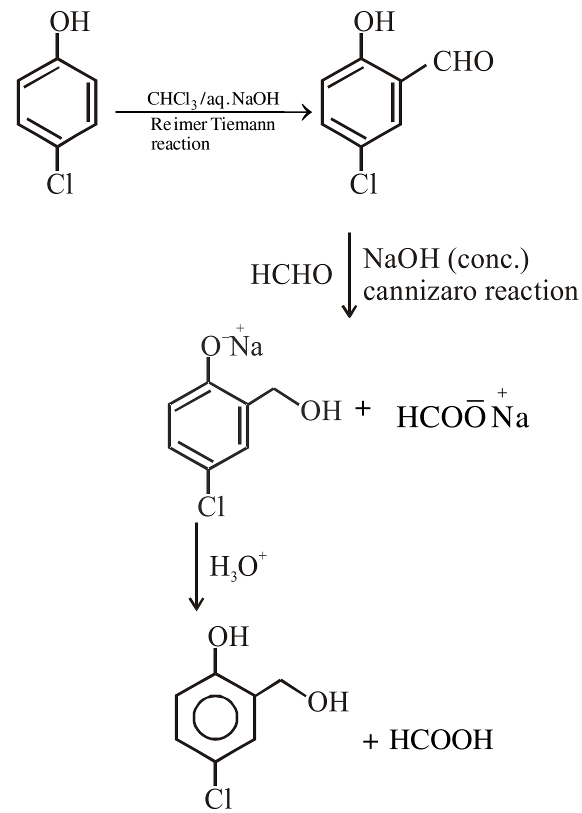

(1)  and Formic acid

(2)  and Methanol

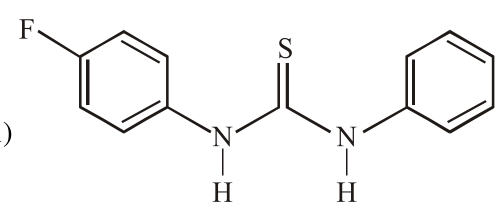
(3)  and Formic acid

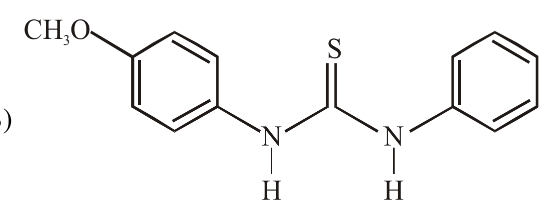
(4)  and Methanol

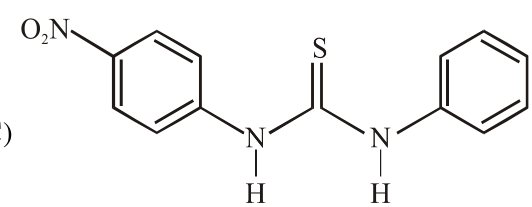
**Sol.**

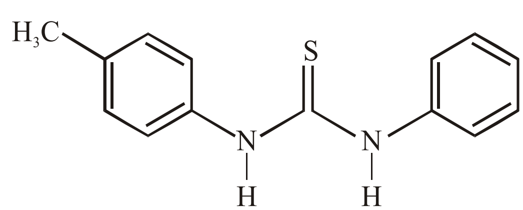



**16.** The increasing order of the $pK_b$ of the following compound is :

(A) 

(B) 

(C) 

(D) 

**Options :**

(1) (A) < (C) < (D) < (B)

(2) (B) < (D) < (A) < (C)

(3) (C) < (A) < (D) < (B)

(4) (B) < (D) < (C) < (A)

**Sol.** B < D < A < C

Basicity $\propto +R \propto \frac{1}{-R}$

$\propto +H \propto \frac{1}{-H}$

**17.** Which of the following statements is not true about RNA ?

(1) It has always double stranded $\alpha$-helix structure

(2) It usually does not replicate

(3) It is present in the nucleus of the cell

(4) It controls the synthesis of protein

**Sol.** RNA is a single stranded structure.

**18.** In the following reaction; $xA \rightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(1) n-Butane and Iso-butane

(2) $C_2H_4$ and $C_4H_8$

(3) $N_2O_4$ and $NO_2$

(4) $C_2H_2$ and $C_6H_6$

**Sol.** $\log \frac{-d[A]}{dt} = \log \frac{d[B]}{dt} + 0.3010$

$$\frac{-d[A]}{dt} = 2 \times \frac{d[B]}{dt}$$

$$\frac{1}{2} \times \frac{-d[A]}{dt} = \frac{d[B]}{dt}$$

$2A \longrightarrow B$

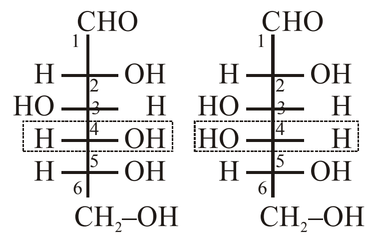
$2C_2H_4 \longrightarrow C_4H_8$

**19.** Glucose and Galactose are having identical configuration in all the positions except position.

(1) C–3 (2) C–2

(3) C–4 (4) C–5

**Sol.** Glucose and galactose are C-4 Epimer's



glucose galactose

**20.** An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1bar. The work done in kJ is :

(1) –9.0 (2) +10.0

(3) –0.9 (4) –2.0

**Sol.** $W = -P_{ext}(V_2 - V_1)$

$= -1\text{bar} \times (10-1)\text{lit}$

$= -9$ bar–lit

$= -900$ J

$= -0.9$ kJ

**21.** Which of the following is a thermosetting polymer?

(1) Buna–N (2) PVC

(3) Bakelite (4) Nylon 6

**Sol.** Bakelite is thermoselting polymer

**22.** The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are:

(1) fisher woman and concentration

(2) washer man and reduction

(3) washer woman and concentration

(4) fisher man and reduction

**Sol.** The idea of froth floatation method came from washerwoman and this process is related to concentration of sulphide ores.

**23.** An example of a disproportionation reaction is :

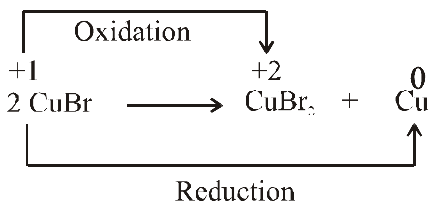
(1) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(2) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

(3) $2CuBr \rightarrow CuBr_2 + Cu$

(4) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

**Sol.**




**24.** The correct set of species responsible for the photochemical smog is :

(1) $NO$, $NO_2$, $O_3$ and hydrocarbons

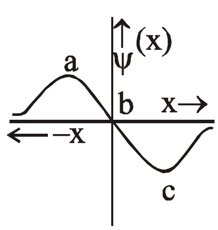
(2) $N_2$,$O_2$, $O_3$ and hydrocarbons

(3) $N_2$, $NO_2$ and hydrocarbons

(4) $CO_2$, $NO_2$, $SO_2$ and hydrocarbons

**Sol.** The common component of photochemical smog are ozone,nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate (PAN).

**25.** The electrons are more likely to be found :




(1) in the region a and b

(2) in the region a and c

(3) only in the region c

(4) only in the region a

**Sol.** $P(x) = 4\pi x^2 \times [\Psi(x)]^2$

Probability will be maximum at a and c

**26.** The basic structural unit of feldspar, zeolites, mica, and asbestos is :

(1) $(SiO_3)^{2-}$ (2) $SiO_2$

(3) $(SiO_4)^{4-}$ (4) $\left(\begin{matrix} R \\ | \\ -Si-O- \\ | \\ R \end{matrix}\right)_n$ (R=Me)

**Sol.** Fledespar - $KAlSi_3O_8$ – $NaAlSi_3O_8$ - $CaAl_2Si_2O_8$

Zeolites - $NaAlSi_2O_6.H_2O$

mica - $KAl_3Si_3O_{10}(OH)_2$

asbestos - $Mg_3Si_2O_5(OH)_4$

These all are silicates having basic unit $(SiO_4)^{4-}$

**27.** An organic compound 'A' is oxidized with $Na_2O_2$ followed by boiling with $HNO_3$.The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate.

Based on above observation, the element present in the given compound is :

(1) Sulphur (2) Nitrogen

(3) Fluorine (4) Phosphorus

**Sol.** The phosphorus containing organic compound are detected by 'Lassaigne's test' by heated with an oxidizing agent (sodium peroxide)

The phosphorus present in the compound in oxidised to phosphate.

The solution is boiled with nitric acid and then treated with ammonium molybdate to produced canary yellow precipitate.

$Na_3PO_4 + 3HNO_3 \rightarrow H_3PO_4 + 3NaNO_3$

$H_3PO_4 + 12\ \underset{\text{(Ammonium molybdate)}}{(NH_4)_2MoO_4} + 21HNO_3 \rightarrow$

$(NH_4)_3PO_4.12MoO_3\downarrow + 21\ NH_4NO_3 + 12\ H_2O$

(Ammonium phosphomolybdate)

(canary yellow precipitate)

**28.** Enthalpy of sublimation of iodine is 24 cal $g^{-1}$ at 200°C. If specific heat of $I_2(s)$ and $I_2(vap)$ are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal $g^{-1}$ is :

(1) 2.85 (2) 11.4 (3) 5.7 (4) 22.8

**Sol.** $I_{2(s)} \rightarrow I_{2(g)}$: $\Delta H_1 = 24$ cal/g at 200°C

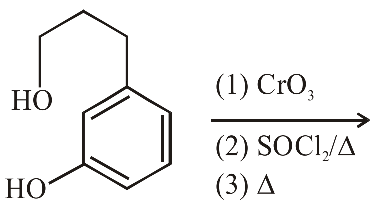
$\Delta H_2 = \Delta H_1 + \Delta C_{P_{rxn}} (T_2 - T_1)$

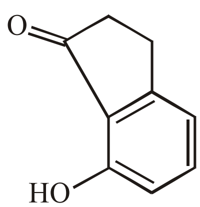
$= 24 + (0.031 - 0.055) \times 50$

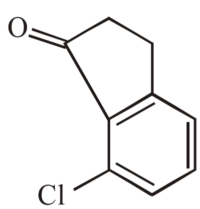
$= 24 - 1.2$

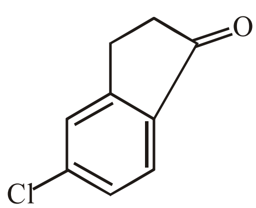
$= 22.8$ Cal/g

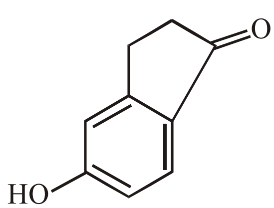
**29.** The major product of the following reaction



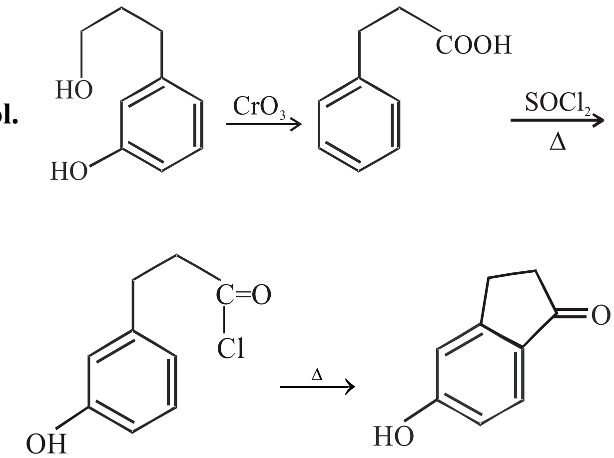

(1) 


(2) 


(3) 


(4) 


**Sol.**




**30.** The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol $kg^{-1}$) of the aqueous solution is

(1) $13.88 \times 10^{-1}$ (2) $13.88 \times 10^{-2}$

(3) 13.88 (4) $13.88 \times 10^{-3}$

**Sol.** $X_{solvent} = 0.8$

If $n_T = 1$

$n_{Solvent} = 0.8$

$n_{Solute} = 0.2$

$$\text{molality} = \frac{0.2}{\frac{0.8 \times 18}{1000}} = 13.88$$

# FINAL JEE–MAIN EXAMINATION – APRIL,2019

**(Held On Friday 12th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM**

## MATHEMATICS | TEST PAPER WITH ANSWER & SOLUTION

**1.** If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4,\ 3\sqrt{2})$ (2) $(4,\ 3\sqrt{3})$

(3) $(3,\ 3\sqrt{3})$ (4) $(5,\ 3\sqrt{6})$

**Sol.** $f(x) = x\sqrt{kx-x^2}$

$f'(x) = \dfrac{3kx-4x^2}{2\sqrt{kx-x^2}}$

For ↑ $f'(x) \geq 0$ | $3kx-4x^2 \geq 0$

$kx - x^2 \geq 0$ | $4x^2-3kx \leq 0$

$x^2 - kx \leq 0$ | $4x(x-\dfrac{3k}{4}) \leq 0$

$x(x-k) \leq 0$ so $x\in[0,\ 3]$ | $3-\dfrac{3k}{4}\leq 0$

+ve $\boxed{x\geq 3}$ | $\boxed{k\geq 4}$

minimum value of k is $\boxed{m=4}$

$f(x) = x\sqrt{kx-x^2}$

$=3\sqrt{4\times 3-3^2}=3\sqrt{3},\ M=3\sqrt{3}$

**2.** The equation $|z-i| = |z-1|$, $i = \sqrt{-1}$, represents:

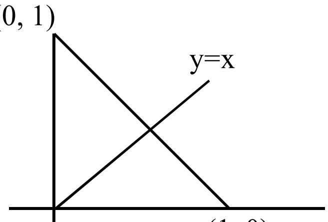
(1) the line through the origin with slope –1.

(2) a circle of radius 1.

(3) a circle of radius $\dfrac{1}{2}$.

(4) the line through the origin with slope 1.

**Sol.**




$|z - i| = |z - 1|$

$y = x$

**3.** For $x\in\left(0,\ \dfrac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x)=\dfrac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi=\left(\dfrac{\pi}{3}\right)$ is equal to :

(1) $\tan\dfrac{\pi}{12}$ (2) $\tan\dfrac{7\pi}{12}$

(3) $\tan\dfrac{11\pi}{12}$ (4) $\tan\dfrac{5\pi}{12}$

**Sol.** $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$fog\ (x) = \sqrt{\tan x}$

$hofog\ (x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\dfrac{\pi}{4}-x\right)$

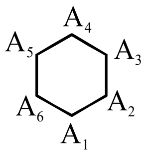
$\phi(x) = \tan\left(\dfrac{\pi}{4}-x\right)$

$\phi\left(\dfrac{\pi}{3}\right)=\tan\left(\dfrac{\pi}{4}-\dfrac{\pi}{3}\right)=\tan\left(-\dfrac{\pi}{12}\right)=-\tan\dfrac{\pi}{12}$

$=\tan\left(\pi-\dfrac{\pi}{12}\right)=\tan\dfrac{11\pi}{12}$

**4.** If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\dfrac{3}{10}$ (2) $\dfrac{1}{10}$ (3) $\dfrac{3}{20}$ (4) $\dfrac{1}{5}$

**Sol.** 

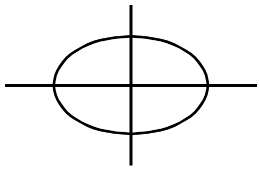

Only two equilateral tringles are possible $A_1 A_3 A_5$ and $A_2A_5A_6$

$$\frac{2}{6_{C_3}} = \frac{2}{20} = \frac{1}{10}$$

**5.** If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{221}}{2}$ (2) $\frac{\sqrt{157}}{2}$ (3) $\frac{\sqrt{61}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

**Sol.** $3x^2 + 4y^2 = 12$



$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$x = 2\cos\theta,\ y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3}\sin\theta)$

Equation of normal is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$$2x\sin\theta - \sqrt{3}\cos\theta y = \sin\theta\cos\theta$$

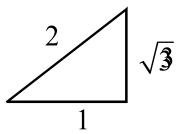
Slope $\frac{2}{\sqrt{3}}\tan\theta = -2 \quad \therefore\ \tan\theta = -\sqrt{3}$

Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + 2\sqrt{3}\sin\theta\ y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$




$$\cos\theta = -\frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2} \quad \therefore\ \theta = 120°$$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$$P\left(-1, \frac{3}{2}\right),\ Q\ (4, 4)$$

$$PQ = \frac{5\sqrt{5}}{2}$$

**6.** If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

(1) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

(3) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

**Sol.** $e^y = xy = e$

differentiate w.r.t. x

$$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(x + e^y) = -y,\ \left.\frac{dy}{dx}\right|_{(0,\,1)} = -\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot e^y \cdot \frac{dy}{dx} + x\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$(x + e^y)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot e^y + 2\frac{dy}{dx} = 0$$

$$e\frac{d^2y}{dx^2} + \frac{1}{e^2}e + 2\left(-\frac{1}{e}\right) = 0$$

$$\therefore\ \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

**7.** If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the palne $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to :

(1) $2\sqrt{14}$ (2) $\sqrt{14}$ (3) $2\sqrt{7}$ (4) 14

**Sol.** $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

$x = 3\lambda + 2,\ y = 2\lambda - 1,\ z = -\lambda + 1$

Intersection with plane $2x + 3y - z + 13 = 0$

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0 \quad \boxed{\lambda = -1}$

$\therefore$ P(–1, –3, 2)

Intersection with plane

$3x + y + 4z = 16$

$3(3\lambda+2) + (2\lambda-1) + 4(-\lambda+1) = 16$

$\lambda = 1$

Q(5, 1, 0)

$PQ = \sqrt{6^2+4^2+2^2} = \sqrt{56} = 2\sqrt{14}$

**8.** The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to:

(1) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$  (2) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

(3) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$  (4) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$

**Sol.** $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$

**9.** If $\alpha$ and $\beta$ are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r+\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$  (2) $\frac{29}{358}$  (3) $\frac{1}{12}$  (4) $\frac{7}{116}$

**Sol.** $375x^2 - 25x - 2 = 0$

$\alpha+\beta=\frac{25}{375}$, $\alpha\beta=\frac{-2}{375}$

$\Rightarrow$ ($\alpha + \alpha^2$ + ... upto infinite terms) + ($\beta + \beta^2$ + ... upto infinite terms) = $=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}=\frac{1}{12}$

**10.** If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx = m(\pi + n)$, then m . n is equal to :

(1) –1  (2) 1  (3) $\frac{1}{2}$  (4) $-\frac{1}{2}$

**Sol.** $\int\limits_0^{\pi/2}\frac{\cot x dx}{\cot x+\operatorname{cosec} x}$

$\int\limits_0^{\pi/2}\frac{\cos x}{\cos x+1}=\int\frac{2\cos^2\frac{x}{2}-1}{2\cos^2\frac{x}{2}}$

$\int\limits_0^{\pi/2}\left(1-\frac{1}{2}\sec^2\frac{x}{2}\right)dx$

$\left[x-\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi-2]$  $m=\frac{1}{2},\ n=-2$

$mn = -1$

**11.** The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) $2^{20}$  (2) $2^{20} - 1$

(3) $2^{20} + 1$  (4) $2^{21}$

**Sol.**

| 10 Identical Object | 21Distinct | 1 | 0 |
|---|---|---|---|
| 0 | 10 | $^{21}C_{10} \times 1$ | |
| 1 | 9 | $^{21}C_9 \times 1$ | |
| ⋮ | ⋮ | ⋮ | |
| 10 | 0 | $^{21}C_0 \times 1$ | |

$^{21}C_0 + .... + {}^{21}C_{10} + {}^{21}C_1 + ...... + {}^{21}C_0 = 2^{21}$

$({}^{21}C_0 + .... + {}^{21}C_{10}) = 2^{20}$

**12.** If the data $x_1, x_2, ...., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) 4  (2) 2  (3) $\sqrt{2}$  (4) $2\sqrt{2}$

**Sol.** $x_1 + ... + x_4 = 44$

$x_5 + ... + x_{10} = 96$

$\bar{x} = 14,\ \Sigma x_i = 140$

Variance = $\frac{\Sigma x_i^2}{n}-\bar{x}^2 = 4$

Standard deviation = 2

**13.** The number of solutions of the equation

$1+\sin^4 x=\cos^2 3x,\ x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$ is :

(1) 5 (2) 4 (3) 7 (4) 3

**Sol.** $1 + \sin^4 x = \cos^2 3x$

$\sin x = 0$ & $\cos 3x = 1$

$0,\ 2\pi,\ -2\pi,\ -\pi,\ \pi$

**14.** Let $S_n$ denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then $S_{10}$ is equal to :

(1) –320 (2) –260 (3) –380 (4) –410

**Sol.** $2\{2a+3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$\boxed{d=-12}$

$S_{10} = 5\ \{44 - 9 \times 12\}$

$= -320$

**15.** If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of $\alpha$ for which det (A) + 1 = 0, is :

(1) 0 (2) 2 (3) 1 (4) –1

**Sol.** $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha^2 - 2\alpha - 25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

**16.** Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(x\le 2)=\frac{k}{2^{16}}$, then k is equal to :

(1) 17 (2) 1 (3) 121 (4) 137

**Sol.** $np = 8$

$npq = 4$

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

$n = 16$

$p\ (x = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p\ (x \le 2) = \frac{{}^{16}C_0+{}^{16}C_1+{}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

**17.** If the truth value of the statement $P \to (\sim p \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T (2) T, F, F

(3) T, T, F (4) T, F, T

**Sol.** $P \to (\sim q \vee r)$

$\sim p \vee (\sim q \vee r)$

$\left.\begin{matrix}\sim p \to F\\ \sim q \to F\\ r \to F\end{matrix}\right\} \Rightarrow \left.\begin{matrix}p \to T\\ q \to T\\ r \to F\end{matrix}\right\}$

**18.** Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, the the value of x for which y = 2, is :

(1) $\frac{1}{2}+\frac{1}{\sqrt{e}}$ (2) $\frac{3}{2}-\sqrt{e}$

(3) $\frac{5}{2}+\frac{1}{\sqrt{e}}$ (4) $\frac{3}{2}-\frac{1}{\sqrt{e}}$

**Sol.** $y^2dx + xdy = \frac{dy}{y}$

$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$

$IF = e^{\int\frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$e^{-\frac{1}{y}}\cdot x=\int e^{-\frac{1}{y}}\cdot\frac{1}{y^3}dy+C$

$xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}+\frac{e^{-\frac{1}{y}}}{y}+C$

$C = -\frac{1}{e}$

$x = \frac{3}{2}-\frac{1}{\sqrt{e}}$ when y = 2

**19.** Let $f : R \to R$ be a continuously differentiable function wuch that $f(2) = 6$ and $f(2) = \frac{1}{48}$.

If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :

(1) 24 (2) 36 (3) 12 (4) 18

**Sol.** $\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x-2}$

$= \lim_{x\to 2} \frac{4.f^3(x)\cdot f'(x)}{1}$

$= 4f^3(2)\ f'(2) = 18$

**20.** The coefficient of $x^{18}$ in the product $(1+x)(1-x)^{10}(1 + x + x^2)^9$ is :

(1) –84 (2) 84 (3) 126 (4) –126

**Sol.** $(1 + x)\ (1 - x)^{10}\ (1 + x + x^2)^9$

$(1 - x^2)\ (1 - x^3)^9$

${}^9C_6 = 84$

**21.** For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is

(1) –153 (2) –133 (3) –131 (4) –135

**Sol.** $$\underbrace{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+...+\left[-\frac{1}{3}-\frac{66}{100}\right]}_{(-1)67}$$

$$\underbrace{+\left[-\frac{1}{3}-\frac{67}{100}\right]+...+\left[-\frac{1}{3}-\frac{99}{100}\right]}_{-2(33)}=-133$$

**22.** The equation $y = \sin x \sin(x + 2) - \sin^2(x+1)$ represents a straight line lying in :

(1) second and third quadrants only

(2) third and fourth quadrants only

(3) first, third and fourth quadrants

(4) first, second and fourth quadrants

**Sol.** $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1-\cos(2x+2))$

$= \cos 2 - 1$

$2y = -2\sin^2\frac{1}{2}$

$y = -\sin^2\frac{1}{2} \le 0$

**23.** Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:

(1) 5:4 (2) 14:13 (3) 2:1 (4) 13:11

**Sol.** Equation of tangents

$y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$\frac{x^2}{1}-\frac{y^2}{8}=1 \Rightarrow y = mx \pm \sqrt{m^2-8}$

Since they are common tangent

$\therefore \frac{3}{m} = \pm\sqrt{m^2-8}$ | $\frac{x^2}{1}-\frac{y^2}{8}=1$

$m^4 - 8m^2 - 9 = 0$ | $e = 3$

$m = \pm 3$ | $ae = 3$

$\therefore$ $y = 3x + 1$, $y = -3x - 1$ $>$ $P\left(-\frac{1}{3}, 0\right)$ , $S = (3, 0)$, $S' = (-3, 0)$

$\longleftarrow 8/3 \longrightarrow\longleftarrow 10/3 \longrightarrow$

$S'(-3,0)$ $\left(-\frac{1}{3}, 0\right)$ $S(3,0)$

**24.** If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then $\lambda$ is equal to :

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

**Sol.** Volume of parallelopiped $= \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

$\because$ Question is asking minimum value of volume of parallelopiped & corresponding value of $\lambda$; the minimum value is zero, $\because$ cubic always has atleast one real root.

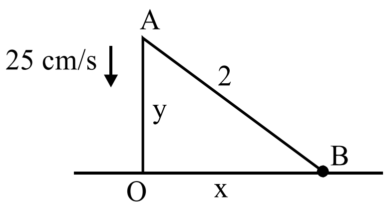
Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

So the problem setter is interested in local minimum value of volume therefore we can chose option '3' as most appropriate option.

**25.** A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/ sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

(1) $25\sqrt{3}$ (2) 25 (3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{3}$

**Sol.**




$$x^2 + y^2 = 4 \left(\frac{dy}{dt} = -25\right)$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$\sqrt{3}\frac{dx}{dt} - 1(25) = 0$$

$$\frac{dx}{dt} = \frac{25}{\sqrt{3}} \text{ cm / sec}$$

**26.** If a is A symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

**Sol.** A = A', B = –B'

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \quad ...(1)$$

$$A' + B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \quad ...(2)$$

After adding Eq. (1) & (2)

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

**27.** Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i} + 2\hat{j} - \hat{k})$ (2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$

(3) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (4) $4(2\hat{i} + 2\hat{j} + \hat{k})$

**Sol.** $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$= 2(\vec{b} \times \vec{a})$

$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$

$= 2(8\hat{i} - 8\hat{j} + 4\hat{k})$

Required vector $= \pm 12\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$

$= \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

**28.** The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\frac{x^3+1}{x}\right|+C$

(2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$

(4) $\log_e\frac{|x^3+1|}{x^2}+C$

**Sol.** $\int \frac{2x^3-1}{x^4+x}dx$

$\int \frac{2x-\frac{1}{x^2}}{x^2+\frac{1}{x}}dx$

$x^2+\frac{1}{x}=t$

$\left(2x-\frac{1}{x^2}\right)dx=dt$

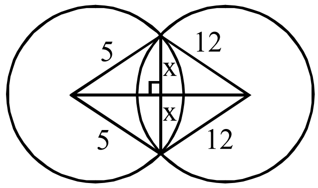
$\int \frac{dt}{t}=\ell n(t)+C$

$=\ell n\left(x^2+\frac{1}{x}\right)+C$

**29.** If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{60}{13}$ (2) $\frac{120}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

**Sol.**




Let length of common chord = 2x

$\sqrt{25-x^2}+\sqrt{144-x^2}=13$

after solving

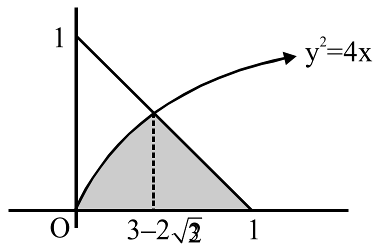
$x=\frac{12\times 5}{13}$

$2x=\frac{120}{13}$

**30.** If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge O\}$ is $a\sqrt{2}+b$, then a – b is equal to :

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) 6 (4) $-\frac{2}{3}$

**Sol.** $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$




$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx+\frac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-(3-2\sqrt{2})\right)$

$=\frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2}+\frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$

$=\frac{8\sqrt{2}}{3}+\left(-\frac{10}{3}\right)$

$a = \frac{8}{3}$, $b = -\frac{10}{3}$

a – b = 6