## PART–A : MATHEMATICS

1. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of $b$ is

(1) $\frac{9}{2}$ (2) 6

(3) $\frac{7}{2}$ (4) 4

**Answer (1)**

**Sol.** $y^2 = 6x$ ; slope of tangent at $(x_1, y_1)$ is $m_1 = \frac{3}{y_1}$

also $9x^2 + by^2 = 16$; slope of tangent at $(x_1, y_1)$ is

$m_2 = \frac{-9x_1}{by_1}$

As $m_1m_2 = -1$

$\Rightarrow \frac{-27x_1}{by_1^2} = -1$

$\Rightarrow b = \frac{9}{2}\left(\text{as } y_1^2 = 6x_1\right)$

2. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + k$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b} = 24$, then $|\vec{u}|^2$ is equal to

(1) 84 (2) 336

(3) 315 (4) 256

**Answer (2)**

**Sol.** Clearly, $\vec{u} = \lambda(\vec{a}\times(\vec{a}\times\vec{b}))$

$\Rightarrow \vec{u} = \lambda((\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2\,\vec{b})$

$\Rightarrow \vec{u} = \lambda(2\vec{a} - 14\vec{b}) = 2\lambda\left\{(2\hat{i} + 3\hat{j} - \hat{k}) - 7(\hat{j} + \hat{k})\right\}$

$\Rightarrow \vec{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$

as, $\vec{u}\cdot\vec{b} = 24$

$\Rightarrow 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k})\cdot(\hat{j} + \hat{k}) = 24$

$\Rightarrow \lambda = -1$

So, $\vec{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$

$\Rightarrow |\vec{u}|^2 = 336$

3. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to $t$. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right)$$

(1) Does not exist (in $R$) (2) Is equal to 0

(3) Is equal to 15 (4) Is equal to 120

**Answer (4)**

**Sol.** As $\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \le \frac{1}{x}$

$\frac{2}{x} - 1 < \left[\frac{2}{x}\right] \le \frac{2}{x}$

$\sum_{r=1}^{15}\left(\frac{r}{x} - 1\right) < \sum_{r=1}^{15}\left(\frac{r}{x}\right) \le \sum_{r=1}^{15}\frac{r}{x}$

$120 < \lim_{x\to 0^+} x\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right) \le 120$

$\Rightarrow \lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right) = 120$

4. If $L_1$ is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and $L_2$ is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines $L_1$ and $L_2$, is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$

(3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

**Answer (3)**

**Sol.** $L_1$ is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j}$

$L_2$ is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$

Also, $L_2$ passes through $\left(\frac{5}{7}, \frac{8}{7}, 0\right)$

So, required plane is $\begin{vmatrix} x-\frac{5}{7} & y-\frac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$

$\Rightarrow$ $7x - 7y + 8z + 3 = 0$

Now, perpendicular distance $= \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

5. Then value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$

(3) $\frac{\pi}{2}$ (4) $4\pi$

**Answer (1)**

**Sol.** $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x dx}{1+2^x}$ ... (i)

Also, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x dx}{1+2^x}$

Adding (i) and (ii)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

$$2I = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \text{... (iii)}$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad \text{... (iv)}$$

Adding (iii) & (iv)

$$2I = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

6. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha$, $\beta$ ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is

(1) $\frac{1}{2}(\sqrt{2}-1)$ (2) $\frac{1}{2}(\sqrt{3}-1)$

(3) $\frac{1}{2}(\sqrt{3}+1)$ (4) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

**Answer (2)**

**Sol.** $18x^2 - 9\pi x + \pi^2 = 0$

$(6x-\pi)(3x-\pi) = 0$

$\therefore \ x = \frac{\pi}{6}, \ \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}, \ \beta = \frac{\pi}{3}$

$y = (gof)(x) = \cos x$

Area $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx = (\sin x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}$

$= \frac{1}{2}(\sqrt{3}-1)$ sq. units

7. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right) \cdot \cos\left(\frac{\pi}{6}-x\right) - \frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then $k$ is equal to :

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$

(3) $\frac{13}{9}$ (4) $\frac{8}{9}$

**Answer (3)**

**Sol.** $8\cos x \cdot \left(\cos^2 \frac{\pi}{6} - \sin^2 x - \frac{1}{2}\right) = 1$

$\Rightarrow$ $8\cos x\left(\frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x\right) = 1$

$\Rightarrow$ $8\cos x\left(\frac{-3+4\cos^2 x}{4}\right) = 1$

$\Rightarrow$ $\cos 3x = 1$

$\Rightarrow$ $\cos 3x = \frac{1}{2}$

$\Rightarrow \quad 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\Rightarrow \quad x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$\Rightarrow \quad \text{Sum} = \frac{13\pi}{9}$

$\Rightarrow \quad k = \frac{13}{9}$

8. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$.

If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) $2\sqrt{2}$ (2) 3

(3) –3 (4) $-2\sqrt{2}$

**Answer (1)**

**Sol.** $h(x) = \dfrac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}}$

$= \left(x - \frac{1}{x}\right) + \dfrac{2}{\left(x - \frac{1}{x}\right)}$

$x - \frac{1}{x} > 0, \ \left(x - \frac{1}{x}\right) + \dfrac{2}{\left(x - \frac{1}{x}\right)} \in (2\sqrt{2}, \infty]$

$x - \frac{1}{x} < 0, \ \left(x - \frac{1}{x}\right) + \dfrac{2}{\left(x - \frac{1}{x}\right)} \in (-\infty, -2\sqrt{2}]$

Local minimum is $2\sqrt{2}$

9. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to

(1) $\dfrac{-1}{1+\cot^3 x} + C$

(2) $\dfrac{1}{3(1+\tan^3 x)} + C$

(3) $\dfrac{-1}{3(1+\tan^3 x)} + C$

(4) $\dfrac{1}{1+\cot^3 x} + C$

(where $C$ is a constant of integration)

**Answer (3)**

**Sol.** $I = \displaystyle\int \frac{\sin^2 x.\cos^2 x \, dx}{\left\{(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)\right\}^2}$

Dividing the numberator and denominator by $\cos^6 x$

$\Rightarrow \quad I = \displaystyle\int \frac{\tan^2 x \sec^2 x \, dx}{(1+\tan^3 x)^2}$

Let, $\tan^3 x = z$

$\Rightarrow \quad 3\tan^2 x.\sec^2 x dx = dz$

$I = \frac{1}{3} \displaystyle\int \frac{dz}{z^2} = \frac{-1}{3z} + C$

$= \dfrac{-1}{3(1+\tan^3 x)} + C$

10. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{3}{4}$ (2) $\frac{3}{10}$

(3) $\frac{2}{5}$ (4) $\frac{1}{5}$

**Answer (3)**

**Sol.** $E_1$ : Event that first ball drawn is red.

$E_2$ : Event that first ball drawn is black.

$E$ : Event that second ball drawn is red.

$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)$

$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{2}{5}$

11. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If $C$ is the circumcentre of this triangle, then the radius of the circle having line segment $AC$ as diameter, is

(1) $\frac{3\sqrt{5}}{2}$

(2) $\sqrt{10}$

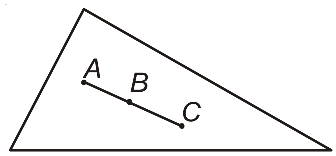
(3) $2\sqrt{10}$

(4) $3\sqrt{\frac{5}{2}}$

**Answer (4)**

**Sol.** $A$ (–3, 5)

$B$ (3, 3)




So, $AB = 2\sqrt{10}$

Now, as, $AC = \frac{3}{2}AB$

So, radius = $\frac{3}{4}AB = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

12. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of $c$ is

(1) 95 (2) 195

(3) 185 (4) 85

**Answer (1)**

**Sol.** Equation of tangent at (1, 7) to curve $x^2 = y - 6$ is

$$x - 1 = \frac{1}{2}(y + 7) - 6$$

$2x - y + 5 = 0$ …(i)

Centre of circle = (–8, –6)

Radius of circle $= \sqrt{64 + 36 - c} = \sqrt{100 - c}$

$\because$ Line (i) touches the circle

$$\therefore \left|\frac{2(-8) - (-6) + 5}{\sqrt{4 + 1}}\right| = \sqrt{100 - c}$$

$$\sqrt{5} = \sqrt{100 - c}$$

$\Rightarrow$ $c = 95$

13. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(1) 2 (2) –1

(3) 0 (4) 1

**Answer (4)**

**Sol.** $x^2 - x + 1 = 0$

Roots are $-\omega, -\omega^2$

Let $\alpha = -\omega, \beta = -\omega^2$

$$\alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107}$$
$$= -(\omega^{101} + \omega^{214})$$
$$= -(\omega^2 + \omega)$$
$$= 1$$

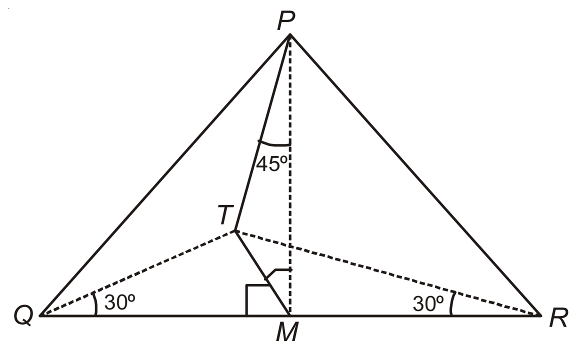
14. $PQR$ is a triangular park with $PQ = PR = 200$ m. A T.V. tower stands at the mid-point of $QR$. If the angles of elevation of the top of the tower at $P$, $Q$ and $R$ are respectively 45º, 30º and 30º, then the height of the tower (in m) is

(1) $50\sqrt{2}$ (2) 100

(3) 50 (4) $100\sqrt{3}$

**Answer (2)**

**Sol.**




Let height of tower $TM$ be $h$

$\therefore$ $PM = h$

In $\Delta TQM$, $\tan 30° = \frac{h}{QM}$

$QM = \sqrt{3}h$

In $\Delta PMQ$, $PM^2 + QM^2 = PQ^2$

$h^2 + (\sqrt{3}h)^2 = 200^2$

$\Rightarrow$ $4h^2 = 200^2$

$\Rightarrow$ $h = 100$ m

15. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ...., x_9$ is

(1) 3 (2) 9

(3) 4 (4) 2

**Answer (4)**

**Sol.** Standard deviation of $x_i - 5$ is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{9}(x_i - 5)^2}{9} - \left(\frac{\sum_{i=1}^{9}(x_i - 5)}{9}\right)^2}$$

$\Rightarrow$ $\sigma = \sqrt{5 - 1} = 2$

As, standard deviation remains constant if observations are added/subtracted by a fixed quantity.

So, $\sigma$ of $x_i$ is 2

16. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

(1) 2 (2) –1

(3) 0 (4) 1

**Answer (1)**

**Sol.** $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$$=2\left[{}^5C_0\,x^5+{}^5C_2\,x^3(x^3-1)+{}^5C_4\,x(x^3-1)^2\right]$$

$$=2\left[x^5+10(x^6-x^3)+5x(x^6-2x^3+1)\right]$$

$$=2\left[x^5+10x^6-10x^3+5x^7-10x^4+5x\right]$$

$$=2\left[5x^7+10x^6+x^5-10x^4-10x^3+5x\right]$$

Sum of odd degree terms coefficients

= 2(5 + 1 – 10 + 5)

= 2

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points $P$ and $Q$. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of $\Delta PTQ$ is

(1) $36\sqrt{5}$ (2) $45\sqrt{5}$

(3) $54\sqrt{3}$ (4) $60\sqrt{3}$

**Answer (2)**

**Sol.** Clearly $PQ$ is a chord of contact,

*i.e.*, equation of $PQ$ is $T \equiv 0$

$\Rightarrow$ $y = -12$

Solving with the curve, $4x^2 - y^2 = 36$

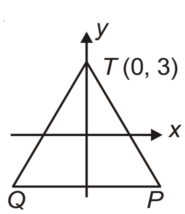
$\Rightarrow$ $x=\pm 3\sqrt{5},\ y=-12$

*i.e.*, $P(3\sqrt{5},-12)$; $Q(-3\sqrt{5},-12)$; $T(0,3)$

Area of $\Delta PQT$ is

$$\Delta=\frac{1}{2}\times 6\sqrt{5}\times 15$$

$= 45\sqrt{5}$




18. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) At least 750 but less than 1000

(2) At least 1000

(3) Less then 500

(4) At least 500 but less than 750

**Answer (2)**

**Sol.** Number of ways of selecting 4 novels from 6 novels $= {}^6C_4$

Number of ways of selecting 1 dictionary from 3 dictionaries $= {}^3C_1$

Required arrangements $= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

$\Rightarrow$ Atleast 1000

19. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution $(x, y, z)$, then $\dfrac{xz}{y^2}$ is equal

(2) –10

(3) 10 (4) –30

**Answer (3)**

**Sol.** $\because$ System of equation has non-zero solution.

$$\therefore\ \begin{vmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{vmatrix}=0$$

$\Rightarrow$ $44 - 4k = 0$

$\therefore$ $k = 11$

Let $z = \lambda$

$\therefore$ $x + 11y = -3\lambda$

and $3x + 11y = 2\lambda$

$$\therefore\ x=\frac{5\lambda}{2},\ y=-\frac{\lambda}{2},\ z=\lambda$$

$$\therefore\ \frac{xz}{y^2}=\frac{\frac{5\lambda}{2}\cdot\lambda}{\left(-\frac{\lambda}{2}\right)^2}=10$$

20. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair $(A, B)$ is equal to

(1) (4, 5)
(2) (–4, –5)
(3) (–4, 3)
(4) (–4, 5)

**Answer (4)**

**Sol.** $\Delta = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

$x = -4$ makes all three row identical

hence $(x + 4)^2$ will be factor

Also, $C_1 \to C_1 + C_2 + C_2$

$$\Delta = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$\Rightarrow$ $5x - 4$ is a factor

$\Delta = \lambda(5x-4)(x+4)^2$

$\therefore$ $B = 5$, $A = -4$

21. Two sets $A$ and $B$ are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$, then

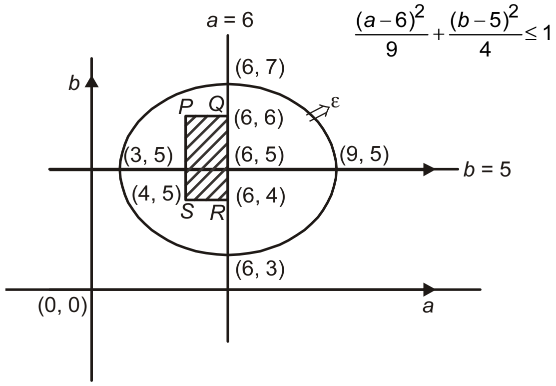
(1) Neither $A \subset B$ nor $B \subset A$
(2) $B \subset A$
(3) $A \subset B$
(4) $A \cap B = \phi$ (an empty set)

**Answer (3)**

**Sol.** As, $|a - 5| < 1$ and $|b - 5| < 1$

$\Rightarrow$ $4 < a, b < 6$ and $\dfrac{(a-6)^2}{9} + \dfrac{(b-5)^2}{4} \le 1$

Taking axes as $a$-axis and $b$-axis




The set $A$ represents square $PQRS$ inside set $B$ representing ellipse and hence $A \subset B$.

22. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at $A$ and $B$, respectively. If $C$ is the centre of the circle through the points $P$, $A$ and $B$ and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(1) $\dfrac{4}{3}$

(2) $\dfrac{1}{2}$

(3) 2

(4) 3

**Answer (3)**

**Sol.** $y^2 = 16x$

Tangent at $P(16, 16)$ is $2y = x + 16$ ... (1)

Normal at $P(16, 16)$ is $y = -2x + 48$ ... (2)

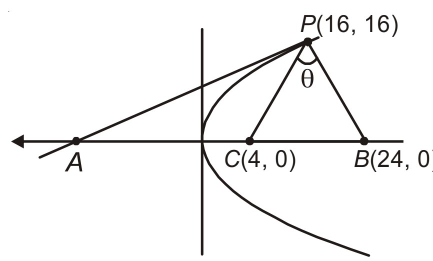
*i.e.*, $A$ is $(-16, 0)$; $B$ is $(24, 0)$

Now, Centre of circle is $(4, 0)$

Now, $m_{PC} = \dfrac{4}{3}$

$m_{PB} = -2$

*i.e.*, $\tan\theta = \left|\dfrac{\frac{4}{3}+2}{1-\frac{8}{3}}\right| = 2$




23. Let $S = \{t \in R : f(x) = |x-\pi| \cdot (e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set $S$ is equal to

(1) $\{0, \pi\}$
(2) $\phi$ (an empty set)
(3) $\{0\}$
(4) $\{\pi\}$

**Answer (2)**

**Sol.** $f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$

$x = \pi$, 0 are repeated roots and also continuous.

Hence, '$f$' is differentiable at all $x$.

24. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q$ is equivalent to

(1) $\sim q$

(2) ~p

(3) $p$

(4) $q$

**Answer (2)**

**Sol.** $(p \vee q) \vee (\sim p \wedge q)$

By property, $(\quad p \wedge \quad q) \vee (\quad p \wedge q)$

$= \sim p$

25. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points $P$ and $Q$. If $O$ is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(1) $3x + 2y = 6xy$

(2) $3x + 2y = 6$

(3) $2x + 3y = xy$

(4) $3x + 2y = xy$

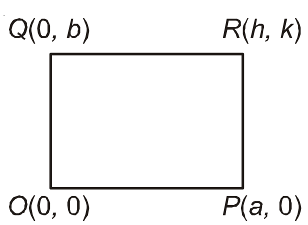
**Answer (4)**

**Sol.** Let the equation of line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ...(i)

(i) passes through the fixed point (2, 3)

$\Rightarrow \dfrac{2}{a} + \dfrac{3}{b} = 1$ ...(ii)

$P(a, 0)$, $Q(0, b)$, $O(0, 0)$, Let $R(h, k)$,




Midpoint of $OR$ is $\left(\dfrac{h}{2}, \dfrac{k}{2}\right)$

Midpoint of $PQ$ is $\left(\dfrac{a}{2}, \dfrac{b}{2}\right) \Rightarrow h = a, \quad k = b$ ... (iii)

From (ii) & (iii),

$\dfrac{2}{h} + \dfrac{3}{k} = 1 \quad \Rightarrow$ locus of $R(h, k)$

$\dfrac{2}{x} + \dfrac{3}{y} = 1 \quad \Rightarrow 3x + 2y = xy$

26. Let $A$ be the sum of the first 20 terms and $B$ be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots..$

If $B - 2A = 100\lambda$, then $\lambda$ is equal to

(1) 496

(2) 232

(3) 248

(4) 464

**Answer (3)**

**Sol.** $A = 1^2 + 2.2^2 + 3^2 + \ldots. + 2.20^2$

$= (1^2 + 2^2 + 3^2 + \ldots. + 20^2) + 4(1^2 + 2^2 + 3^2 + \ldots. + 10^2)$

$= \dfrac{20 \times 21 \times 41}{6} + \dfrac{4 \times 10 \times 11 \times 21}{6}$

$= 2870 + 1540 = 4410$

$B = 1^2 + 2.2^2 + 3^2 + \ldots. + 2.40^2$

$= (1^2 + 2^2 + 3^2 + \ldots. + 40^2) + 4(1^2 + 2^2 + 3^2 + \ldots. + 20^2)$

$= \dfrac{40 \times 41 \times 81}{6} + \dfrac{4 \times 20 \times 21 \times 41}{6}$

$= 22140 + 11480 = 33620$

$\Rightarrow B - 2A = 33620 - 8820 = 24800$

$\Rightarrow 100\lambda = 24800$

$\lambda = 248$

27. Let $y = y(x)$ be the solution of the differential equation $\sin x \dfrac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$. If $y = \left(\dfrac{\pi}{2}\right) = 0$, then $y\left(\dfrac{\pi}{6}\right)$ is equal to :

(1) $-\dfrac{4}{9}\pi^2$

(2) $\dfrac{4}{9\sqrt{3}}\pi^2$

(3) $\dfrac{-8}{9\sqrt{3}}\pi^2$

(4) $-\dfrac{8}{9}\pi^2$

**Answer (4)**

**Sol.** $\sin x \dfrac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$

$\dfrac{dy}{dx} + y \cot x = \dfrac{4x}{\sin x}$

$\therefore \quad \text{I.F.} = e^{\int \cot x \, dx} = \sin x$

$\therefore$ Solution is given by

$$y\sin x = \int \frac{4x}{\sin x}\cdot \sin x\, dx$$

$y\cdot\sin x = 2x^2 + c$

when $x = \frac{\pi}{2}$, $y = 0 \Rightarrow c = -\frac{\pi^2}{2}$

$\therefore$ Equation is : $y\sin x = 2x^2 - \frac{\pi^2}{2}$

when $x = \frac{\pi}{6}$ then $y\cdot\frac{1}{2} = 2\cdot\frac{\pi^2}{36} - \frac{\pi^2}{2}$

$\therefore$ $y = -\frac{8\pi^2}{9}$

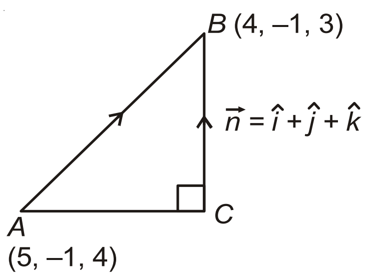
28. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is:

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

**Answer (1)**

**Sol.**




Normal to the plane $x + y + z = 7$ is $\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{AB} = -\hat{i} - \hat{k} \Rightarrow |\overrightarrow{AB}| = AB = \sqrt{2}$

$BC$ = Length of projection of $\overrightarrow{AB}$ on $\vec{n}$ = $|\overrightarrow{AB}\cdot\hat{n}|$

$$= \left|\left(-\hat{i} - \hat{k}\right)\cdot\frac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

Length of projection of the line segment on the plane is $AC$

$$AC^2 = AB^2 - BC^2 = 2 - \frac{4}{3} = \frac{2}{3}$$

$$AC^2 = \sqrt{\frac{2}{3}}$$

29. Let $S = \{x \in R : x \geq 0$ and

$2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$. Then $S$ :

(1) Contains exactly four elements

(2) Is an empty set

(3) Contains exactly one element

(4) Contains exactly two elements

**Answer (4)**

**Sol.** $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$

$2|\sqrt{x} - 3| + (\sqrt{x} - 3 + 3)(\sqrt{x} - 3 - 3) + 6 = 0$

$2|\sqrt{x} - 3| + (\sqrt{x} - 3)^2 - 3 = 0$

$(\sqrt{x} - 3)^2 + 2|\sqrt{x} - 3| - 3 = 0$

$(|\sqrt{x} - 3| + 3)(|\sqrt{x} - 3| - 1) = 0$

$\Rightarrow |\sqrt{x} - 3| = 1, |\sqrt{x} - 3| + 3 \neq 0$

$\Rightarrow \sqrt{x} - 3 = \pm 1$

$\Rightarrow \sqrt{x} = 4, 2$

x = 16, 4

30. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then $m$ is equal to

(1) 33 (2) 66

(3) 68 (4) 34

**Answer (4)**

**Sol.** Let $a_1 = a$ and common difference = $d$

Given, $a_1 + a_5 + a_9 + \ldots + a_{49} = 416$

$\Rightarrow a + 24d = 32$ ...(i)

Also, $a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ ...(ii)

Solving (i) & (ii),

We get $d = 1$, $a = 8$

Now, $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$

$\Rightarrow 8^2 + 9^2 + \ldots + 24^2 = 140m$

$$\Rightarrow \frac{24\times25\times49}{6} - \frac{7\times8\times15}{6} = 140m$$

$\Rightarrow \boxed{m = 34}$

# PART–B : PHYSICS

31. Three concentric metal shells $A$, $B$ and $C$ of respective radii $a$, $b$ and $c$ ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell $B$ is

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

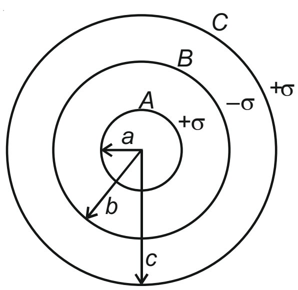
(2) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

(3) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(4) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

**Answer (3)**

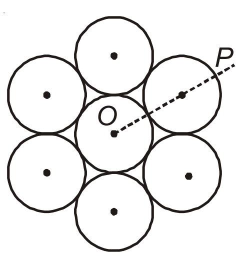
**Sol.**




$$V_B = \left[\frac{\sigma 4\pi a^2}{4\pi\varepsilon_0 b} - \frac{\sigma 4\pi b^2}{4\pi\varepsilon_0 b} + \frac{\sigma 4\pi c^2}{4\pi\varepsilon_0 c}\right]$$

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$$

32. Seven identical circular planar disks, each of mass $M$ and radius $R$ are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point $P$ is




(1) $\frac{181}{2}MR^2$ (2) $\frac{19}{2}MR^2$

(3) $\frac{55}{2}MR^2$ (4) $\frac{73}{2}MR^2$

**Answer (1)**

**Sol.** $I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$

$I_P = I_0 + 7M(3R)^2$

$= \frac{181}{2}MR^2$

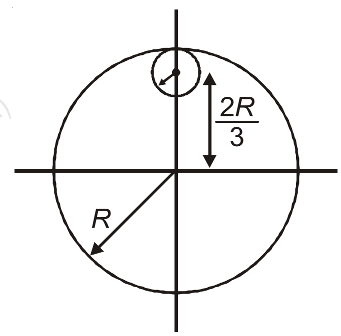
33. From a uniform circular disc of radius $R$ and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

(1) $\frac{37}{9}MR^2$

(2) $4MR^2$

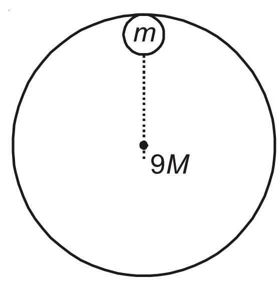
(3) $\frac{40}{9}MR^2$

(4) $10MR^2$




**Answer (2)**

**Sol.**




$m = \frac{(9M)}{9} = M$

$I_1 = \frac{(9M)\times R^2}{2}$

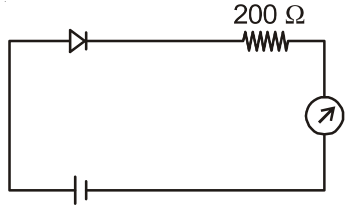
$I_2 = \frac{M\times\left(\frac{R}{3}\right)^2}{2} + M\times\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$

$\therefore\ I_{req} = I_1 - I_2$

$= \frac{9}{2}MR^2 - \frac{MR^2}{2}$

$= 4MR^2$

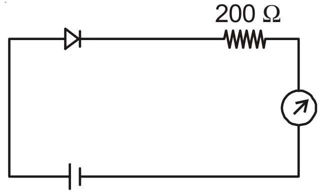
34. The reading of the ammeter for a silicon diode in the given circuit is




(1) 13.5 mA (2) 0

(3) 15 mA (4) 11.5 mA

**Answer (4)**




**Sol.** $I = \dfrac{V - V_{diode}}{R}$

$= \left[\dfrac{3-0.7}{200} \times 1000\right]$ mA

= 11.5 mA

35. Unpolarized light of intensity $I$ passes through an ideal polarizer $A$. Another identical polarizer $B$ is placed behind $A$. The intensity of light beyond $B$ is found to be $\dfrac{I}{2}$. Now another identical polarizer $C$ is placed between $A$ and $B$. The intensity beyond $B$ is now found to be $\dfrac{I}{8}$. The angle between polarizer $A$ and $C$ is

(1) 60° (2) 0°

(3) 30° (4) 45°

**Answer (4)**

**Sol.** Polaroids $A$ and $B$ are oriented with parallel pass axis

Let polaroid $C$ is at angle $\theta$ with $A$ then it makes $\theta$ with $B$ also.

$\because \quad \dfrac{I}{8} = \left(\dfrac{I}{2} \times \cos^2\theta\right) \times \cos^2\theta$

$\Rightarrow \quad \cos^2\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 45°$

36. For an RLC circuit driven with voltage of amplitude $v_m$ and frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$ the current exibits resonance. The quality factor, $Q$ is given by

(1) $\dfrac{CR}{\omega_0}$ (2) $\dfrac{\omega_0 L}{R}$

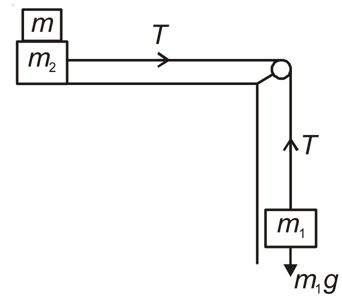
(3) $\dfrac{\omega_0 R}{L}$ (4) $\dfrac{R}{(\omega_0 C)}$

**Answer (2)**

**Sol.** Quality factor, $Q = \dfrac{\omega_0}{(2\Delta\omega)}$

$Q = \dfrac{\omega_0 L}{R}$

37. Two masses $m_1$ = 5 kg and $m_2$ = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight $m$ that should be put on top of $m_2$ to stop the motion is




(1) 10.3 kg (2) 18.3 kg

(3) 27.3 kg (4) 43.3 kg

**Answer (3)**

**Sol.** To stop the moving block $m_2$, acceleration of $m_2$ should be opposite to velocity of $m_2$

$m_1g < \mu(m + m_2)g$

$\Rightarrow \quad 5 < 0.15(10 + m_2)$

$\Rightarrow \quad m_2 > 23.33$ kg

$\therefore$ Minimum mass = 27.3 kg (according to given options)

38. In a collinear collision, a particle with an initial speed $v_0$ strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(1) $\dfrac{v_0}{\sqrt{2}}$ (2) $\dfrac{v_0}{4}$

(3) $\sqrt{2}v_0$ (4) $\dfrac{v_0}{2}$

**Answer (3)**

**Sol.** It is a case of superelastic collision

$mv_0 = mv_1 + mv_2$ ...(i)

$\Rightarrow \quad v_1 + v_2 = v_0$

$\dfrac{1}{2}m\left(v_1^2 + v_2^2\right) = \dfrac{3}{2}\left(\dfrac{1}{2}mv_0^2\right)$

$\Rightarrow \quad \left(v_1^2+v_2^2\right)=\frac{3}{2}v_0^2 \qquad ...(ii)$

$\Rightarrow \quad (v_1+v_2)^2 = v_1^2+v_2^2+2v_1v_2$

$\Rightarrow \quad v_0^2 = \frac{3v_0^2}{2}+2v_1v_2$

$\Rightarrow \quad 2v_1v_2 = -\frac{v_0^2}{2} \qquad ...(iii)$

$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v_0^2+v_0^2$

$\Rightarrow \quad v_1 - v_2 = \sqrt{2}v_0$

39. A particle is moving with a uniform speed in a circular orbit of radius $R$ in a central force inversely proportional to the $n^{th}$ power of $R$. If the period of rotation of the particle is $T$, then

(1) $T \propto R^{n/2}$ (2) $T \propto R^{3/2}$ for any $n$

(3) $T \propto R^{\frac{n}{2}+1}$ (4) $T \propto R^{(n+1)/2}$

**Answer (4)**

**Sol.** $m\omega^2R = k\,R^{-n} = \frac{k}{R^n}$

$\Rightarrow \quad \frac{1}{T^2} \propto \frac{1}{R^{n+1}}$

$\Rightarrow \quad T \propto R^{\left(\frac{n+1}{2}\right)}$

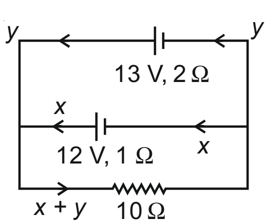
40. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between

(1) 11.7 V and 11.8 V (2) 11.6 V and 11.7 V

(3) 11.5 V and 11.6 V (4) 11.4 V and 11.5 V

**Answer (3)**

**Sol.**




Applying KVL in loops

$12 - x - 10(x + y) = 0$

$\Rightarrow \quad 12 = 11x + 10y \qquad ...(i)$

$13 = 10x + 12y \qquad ...(ii)$

Solving $x = \frac{7}{16}$A, $y = \frac{23}{32}$A

$V = 10(x + y) = 11.56$ V

Aliter : $r_{eq} = \frac{2}{3}\Omega$ , $R = 10\ \Omega$

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1}+\frac{E_2}{r_2} \Rightarrow E_{eq} = \frac{37}{3}\text{ V}$$

$$V = \frac{E_{eq}}{R+r_{eq}}R = 11.56\text{ V}$$

41. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100\sin 30t$

$i = 20\sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) 50, 0 (2) 50, 10

(3) $\frac{1000}{\sqrt{2}}, 10$ (4) $\frac{50}{\sqrt{2}}, 0$

**Answer (3)**

**Sol.** $P_{av} = E_{rms}\,I_{rms}\cos\phi$

$$= \frac{100}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

$i_{wattless} = i_{rms}\sin\phi = \frac{20}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = 10$

42. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number $k$ and frequency $\nu$ refer to their values in air. The medium is non-magnetic. If $\varepsilon_{r_1}$ and $\varepsilon_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (2) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$

(3) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (4) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

**Answer (4)**

**Sol.** $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ air

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z-ct)\right]$ medium

During refraction, frequency remains unchanged, whereas wavelength gets changed.

$\therefore$ $k' = 2k$ (From equations)

$$\Rightarrow \frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda_0}\right)$$

$$\Rightarrow \lambda' = \frac{\lambda_0}{2}$$

$$\Rightarrow v = \frac{c}{2}$$

$$\Rightarrow \frac{1}{\sqrt{\mu_0\varepsilon_2}} = \frac{1}{2}\times\frac{1}{\sqrt{\mu_0\varepsilon_1}}$$

$$\Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{4}$$

43. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) $2 \times 10^6$ (2) $2 \times 10^3$

(3) $2 \times 10^4$ (4) $2 \times 10^5$

**Answer (4)**

**Sol.** Frequency of carrier = $10 \times 10^9$ Hz

Available bandwidth = 10% of $10 \times 10^9$ Hz

= $10^9$ Hz

Bandwidth for each telephonic channel = 5 kHz

$\therefore$ Number of channels $= \frac{10^9}{5\times10^3}$

$= 2 \times 10^5$

44. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3$ kg/m$^3$ and its Young's modulus is $9.27 \times 10^{10}$ Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 7.5 kHz (2) 5 kHz

(3) 2.5 kHz (4) 10 kHz

**Answer (2)**

**Sol.** $f_0 = \frac{V}{2L} = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$

$$= \frac{1}{2\times0.6}\sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}} = 4.88 \text{ kHz} \approx 5 \text{ kHz}$$

45. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is $p_d$; while for its similar collision with carbon nucleus at rest, fractional loss of energy is $p_c$. The values of $p_d$ and $p_c$ are respectively

(1) (0, 1)

(2) (.89, .28)

(3) (.28, .89)

(4) (0, 0)

**Answer (2)**

**Sol.** $mu = mv_1 + 2m \times v_2$ ...(i)

$u = (v_2 - v_1)$ ...(ii)

$$\Rightarrow v_1 = -\frac{u}{3}$$

$$\therefore \frac{\Delta E}{E} = p_d = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{3}\right)^2}{\frac{1}{2}mu^2}$$

$$= \frac{8}{9} = 0.89$$

And $mu = mv_1 + (12m) \times v_2$ ...(iii)

$u = (v_2 - v_1)$ ...(iv)

$$\Rightarrow v_1 = -\frac{11}{13}u$$

$$\therefore \frac{\Delta E}{E} = p_c = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{11}{13}u\right)^2}{\frac{1}{2}mu^2} = \frac{48}{169} = 0.28$$

46. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 6% (2) 2.5%

(3) 3.5% (4) 4.5%

**Answer (4)**

**Sol.** $\rho = \frac{m}{l^3}$

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 3\frac{dl}{l}$$

= (1.5 + 3 × 1)

= 4.5%

47. Two moles of an ideal monoatomic gas occupies a volume $V$ at 27°C. The gas expands adiabatically to a volume 2 $V$. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) 2.7 kJ

(2) (a) 189 K (b) 2.7 kJ

(3) (a) 195 K (b) –2.7 kJ

(4) (a) 189 K (b) –2.7 kJ

**Answer (4)**

**Sol.** $TV^{\gamma-1}$ = Constant

$$T_f = 300\left(\frac{V}{2V}\right)^{\frac{5}{3}-1} = 189\text{ K}$$

$$\Delta U = nC_v\Delta T = 2\times\frac{3R}{2}\times[189-300]$$

= –2.7 kJ

48. A solid sphere of radius $r$ made of a soft material of bulk modulus $K$ is surrounded by a liquid in a cylindrical container. A massless piston of area of $a$ floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass $m$ is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(1) $\frac{mg}{Ka}$

(2) $\frac{Ka}{mg}$

(3) $\frac{Ka}{3mg}$

(4) $\frac{mg}{3Ka}$

**Answer (4)**

**Sol.** $K = -V\frac{dP}{dV}$

$$\Rightarrow \frac{-dV}{V} = \frac{dP}{K} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{-3dr}{r} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{dr}{r} = -\frac{mg}{3Ka}$$

49. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(1) 0.9 nC

(2) 1.2 nC

(3) 0.3 nC

(4) 2.4 nC

**Answer (2)**

**Sol.** $C' = KC_0$

$Q = KC_0V$

$$Q_{\text{induced}} = Q\left(1-\frac{1}{K}\right)$$

$$= \frac{5}{3}\times 90\times 10^{-12}\times 20\left(1-\frac{3}{5}\right)$$

= 1.2 nC

50. The dipole moment of a circular loop carrying a current $I$, is $m$ and the magnetic field at the centre of the loop is $B_1$. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is $B_2$. The ratio $\frac{B_1}{B_2}$ is

(1) $\frac{1}{\sqrt{2}}$ (2) 2

(3) $\sqrt{3}$ (4) $\sqrt{2}$

**Answer (4)**

**Sol.** $m = I(\pi R^2)$, $m' = 2m = I \times \left(\pi\sqrt{2}R\right)^2$

$$\therefore \quad R' = \sqrt{2}R$$

$$B_1 = \frac{\mu_0 I}{2R}$$

$$B_2 = \frac{\mu_0 I}{2\times\left(\sqrt{2}R\right)}$$

$$\therefore \quad \frac{B_1}{B_2} = \sqrt{2}$$

51. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let $\lambda_n$, $\lambda_g$ be the de Broglie wavelength of the electron in the $n^{th}$ state and the ground state respectively. Let $\Lambda_n$ be the wavelength of the emitted photon in the transition from the $n^{th}$ state to the ground state. For large $n$, ($A$, $B$ are constants)

(1) $\Lambda_n^2 \approx \lambda$ (2) $\Lambda_n \approx A + \dfrac{B}{\lambda_n^2}$

(3) $\Lambda_n \approx A + B\lambda_n$ (4) $\Lambda_n^2 \approx A + B\lambda_n^2$

**Answer (2)**

**Sol.** $P_n = \dfrac{h}{\lambda_n}$, $P_g = \dfrac{h}{\lambda_g}$

$$k = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}, \quad E = -k = -\frac{h^2}{2m\lambda^2}$$

$$E_n = -\frac{h^2}{2m\lambda_n^2}, \quad E_g = -\frac{h^2}{2m\lambda_g^2}$$

$$E_n - E_g = \frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right) = \frac{hc}{\Lambda_n}$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right) = \frac{hc}{\Lambda_n}$$

$$\Lambda_n = \frac{2mc}{h}\left(\frac{\lambda_g^2\lambda_n^2}{\lambda_n^2 - \lambda_g^2}\right)$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\frac{\lambda_n^2}{\lambda_n^2\left(1-\dfrac{\lambda_g^2}{\lambda_n^2}\right)}$$

$$= \frac{2mc\lambda_g^2}{h}\left[1-\frac{\lambda_g^2}{\lambda_n^2}\right]^{-1}$$

$$= \frac{2mc\lambda_g^2}{h}\left[1+\frac{\lambda_g^2}{\lambda_n^2}\right]$$

$$= \frac{2mc\lambda_g^2}{h} + \left(\frac{2mc\lambda_g^4}{h}\right)\frac{1}{\lambda_n^2}$$

$$= A + \frac{B}{\lambda_n^2}$$

$$A = \frac{2mc\lambda_g^2}{h}, \quad B = \frac{2mc\lambda_g^4}{h}$$

52. The mass of a hydrogen molecule is $3.32 \times 10^{-27}$ kg. If $10^{23}$ hydrogen molecules strike, per second, a fixed wall of area 2 cm$^2$ at an angle of 45° to the normal, and rebound elastically with a speed of $10^3$ m/s, then the pressure on the wall is nearly

(1) $4.70 \times 10^2$ N/m$^2$ (2) $2.35 \times 10^3$ N/m$^2$

(3) $4.70 \times 10^3$ N/m$^2$ (4) $2.35 \times 10^2$ N/m$^2$

**Answer (2)**

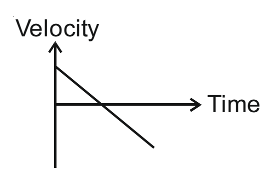
**Sol.** $F = nmv\cos\theta \times 2$

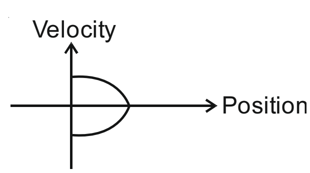
$$P = \frac{F}{A} = \frac{2.nmv\cos\theta}{A}$$

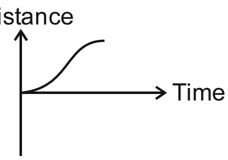
$$= \frac{2\times10^{23}\times3.32\times10^{-27}\times10^3}{\sqrt{2}\times2\times10^{-4}}\text{N/m}^2$$

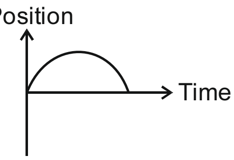
$= 2.35 \times 10^3$ N/m$^2$

53. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1) 


(2) 

(3) 


(4) 


**Answer (3)**

**Sol.** Options (1), (2) and (4) correspond to uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration, whereas option (3) does not correspond to this motion.

54. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii $r_e$, $r_p$, $r_\alpha$ respectively in a uniform magnetic field $B$. The relation between $r_e$, $r_p$, $r_\alpha$ is

(1) $r_e < r_\alpha < r_p$

(2) $r_e > r_p = r_\alpha$

(3) $r_e < r_p = r_\alpha$

(4) $r_e < r_p < r_\alpha$

**Answer (3)**

**Sol.** $r = \dfrac{\sqrt{2mk}}{qB}$

$$\frac{r_\alpha}{r_p} = \frac{\sqrt{2m_\alpha}}{q_\alpha} \times \frac{q_p}{\sqrt{2m_p}} \qquad \begin{bmatrix} m_\alpha = 4m_p \\ q_\alpha = 2q_p \end{bmatrix}$$

$= 1$

Mass of electron is least and charge $q_e = e$

So, $r_e < r_p = r_\alpha$

55. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 910 Ω

(2) 990 Ω

(3) 505 Ω

(4) 550 Ω

**Answer (4)**

**Sol.** $\dfrac{R_1}{R_2} = \dfrac{l}{(100 - l)}$

$\dfrac{R_2}{R_1} = \dfrac{(l - 10)}{(110 - l)}$

$(100 - l)(110 - l) = l(l - 10)$

$11000 + l^2 - 210l = l^2 - 10l$

$\Rightarrow \quad l = 55$ cm

$R_1 = R_2\left(\dfrac{55}{45}\right)$

$R_1 + R_2 = 1000\ \Omega$

$R_1 = 550\ \Omega$

56. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 2.5 Ω

(2) 1 Ω

(3) 1.5 Ω

(4) 2 Ω

**Answer (3)**

**Sol.** $\because \quad E \propto l_1$

and $E - ir \propto l_2$

$\therefore \quad \dfrac{E}{E - ir} = \dfrac{l_1}{l_2}$

$\Rightarrow \quad \dfrac{E}{E - \left(\dfrac{E}{r+5}\right) \times r} = \dfrac{52}{40}$

$\Rightarrow \quad \dfrac{r+5}{5} = \dfrac{13}{10}$

$\Rightarrow \quad r = 1.5\ \Omega$

57. If the series limit frequency of the Lyman series is $\nu_L$, then the series limit frequency of the Pfund series is

(1) $\nu_L/25$

(2) $25\ \nu_L$

(3) $16\ \nu_L$

(4) $\nu_L/16$

**Answer (1)**

**Sol.** $h\nu_L = E\left[\frac{1}{12} - \frac{1}{\infty}\right] = E$

$$h\nu_P = E\left[\frac{1}{5^2} - \frac{1}{\infty}\right] = \frac{E}{25}$$

$$\Rightarrow \quad \nu_P = \frac{\nu_L}{25}$$

58. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?

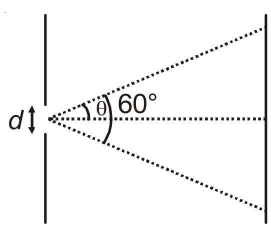
(*i.e.* distance between the centres of each slit.)

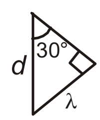
(1) 100 μm (2) 25 μm

(3) 50 μm (4) 75 μm

**Answer (2)**

**Sol.** $d\sin\theta = \lambda$




$\lambda = \frac{d}{2}$ [$d = 1 \times 10^{-6}$ m]

$\Rightarrow \lambda = 5000$ Å

Fringe width, $B = \frac{\lambda D}{d'}$ ($d'$ is slit separation)

$$10^{-2} = \frac{5000 \times 10^{-10} \times 0.5}{d'}$$

$\Rightarrow d' = 25 \times 10^{-6}$ m = 25 μm

59. A particle is moving in a circular path of radius $a$ under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(1) $-\frac{3}{2}\frac{k}{a^2}$ (2) $-\frac{k}{4a^2}$

(3) $\frac{k}{2a^2}$ (4) Zero

**Answer (4)**

**Sol.** $F = \frac{-dU}{dr}$ $\left[U = -\frac{k}{2r^2}\right]$

$\frac{mv^2}{r} = \frac{k}{r^3}$ [This force provides necessary centripetal force]

$$\Rightarrow mv^2 = \frac{k}{r^2}$$

$$\Rightarrow K.E = \frac{k}{2r^2}$$

$$\Rightarrow P.E = -\frac{k}{2r^2}$$

Total energy = Zero

60. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}$/second. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = $6.02 \times 10^{23}$ gm mole$^{-1}$)

(1) 5.5 N/m

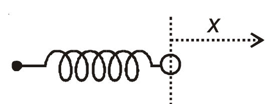
(2) 6.4 N/m

(3) 7.1 N/m

(4) 2.2 N/m

**Answer (3)**

**Sol.**




$Kx = ma \Rightarrow a = (K/m)x$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = 10^{12}$$

$$= \frac{1}{4\pi^2} \times \frac{K}{m} = 10^{24}$$

$$K = 4\pi^2 m \times 10^{24} = \frac{4 \times 10 \times 108 \times 10^{-3}}{6.02 \times 10^{23}} \times 10^{24}$$

= 7.1 N/m

# PART–C : CHEMISTRY

61. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

    (1) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

    (2) $[Co(H_2O)_6]Cl_3$

    (3) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

    (4) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$

**Answer (1)**

**Sol.** The solution which shows maximum freezing point must have minimum number of solute particles.

(1) $[Co(H_2O)_3Cl_3] \cdot 3H_2O \rightarrow [Co(H_2O)_3Cl_3]$, i = 1

(2) $[Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^-$, i = 4

(3) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O \rightarrow [Co(H_2O)_5Cl]^{2+} + 2Cl^-$, i = 3

(4) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \rightarrow [Co(H_2O)_4Cl_2]^+ + Cl^-$, i = 2

So, solution of 1 molal $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ will have minimum number of particles in aqueous state.

Hence, option (1) is correct.

62. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively.

    (1) $H_2O$ and $(H_2O + OH^-)$

    (2) $(H_2O + O_2)$ and $H_2O$

    (3) $(H_2O + O_2)$ and $(H_2O + OH^-)$

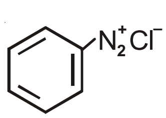
    (4) $H_2O$ and $(H_2O + O_2)$

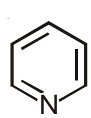
**Answer (4)**

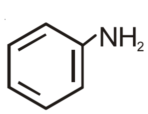
**Sol.** $[Fe(CN)_6]^{4-} + \frac{1}{2}H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

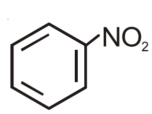
$[Fe(CN)_6]^{3-} + \frac{1}{2}H_2O_2 + OH^\ominus \longrightarrow [Fe(CN)_6]^{4-} + H_2O + \frac{1}{2}O_2$

63. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

    (1) 


    (2) 


    (3) 


    (4) 


**Answer (3)**

**Sol.** Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

64. Glucose on prolonged heating with HI gives

    (1) 6-iodohexanal

    (2) *n*-Hexane

    (3) 1-Hexene

    (4) Hexanoic acid

**Answer (2)**

**Sol.** $CHO{-}(CH{-}OH)_4{-}CH_2{-}OH \xrightarrow{HI,\ \Delta} \underset{\text{n-Hexane}}{CH_3{-}CH_2{-}CH_2{-}CH_2{-}CH_2{-}CH_3}$

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

| | Base | Acid | End point |
|---|---|---|---|
| (1) | Strong | Strong | Pink to colourless |
| (2) | Weak | Strong | Colourless to pink |
| (3) | Strong | Strong | Pinkish red to yellow |
| (4) | Weak | Strong | Yellow to pinkish red |

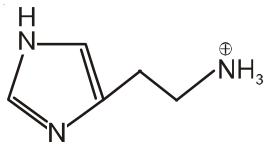
**Answer (4)**

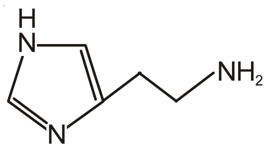
**Sol.** The pH range of methyl orange is

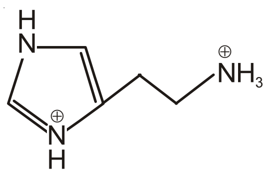
$\xleftarrow{\text{Pinkish red}}$ 3.9—4.5 $\xrightarrow{\text{Yellow}}$

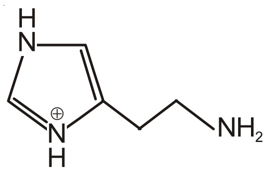
Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

66. The predominant form of histamine present in human blood is ($pK_a$, Histidine = 6.0)

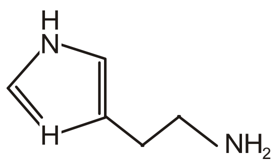
(1) 

(2) 

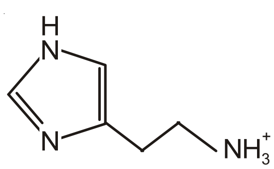
(3) 

(4) 

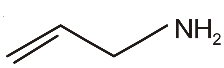
**Answer (1)**

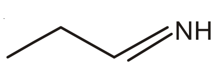
**Sol.** Histamine 

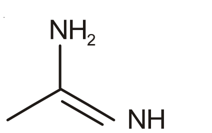
At pH (7.4) major form of histamine is protonated at primary amine.

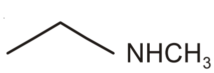


67. The increasing order of basicity of the following compound is

(a) 

(b) 

(c) 

(d) 

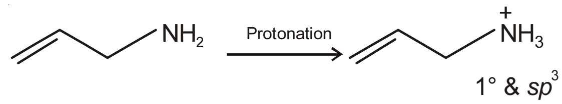
(1) (d) < (b) < (a) < (c)

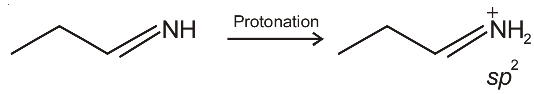
(2) (a) < (b) < (c) < (d)

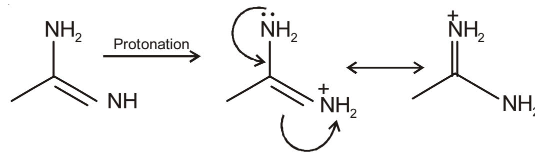
(3) (b) < (a) < (c) < (d)

(4) (b) < (a) < (d) < (c)

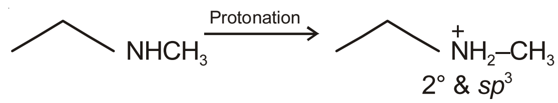
**Answer (4)**

**Sol.** (a) 


(b) 

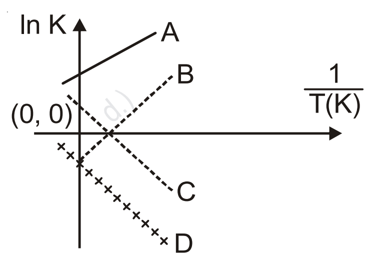

(c) 


[Equivalent resonance]

(d) 


$\therefore$ Correct order of basicity : b < a < d < c.

68. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?




(1) A and D (2) A and B

(3) B and C (4) C and D

**Answer (2)**

**Sol.** Equilibrium constant $K = \left(\frac{A_f}{A_b}\right)e^{-\frac{\Delta H^\circ}{RT}}$

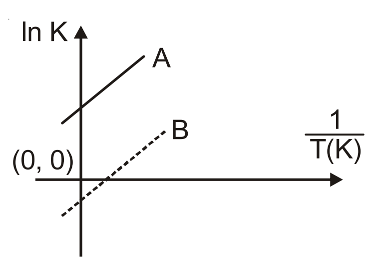
$$\ln K = \ln\left(\frac{A_f}{A_b}\right) - \frac{\Delta H^\circ}{R}\left(\frac{1}{T}\right)$$

$$y = \quad C \quad + \quad m \quad x$$

Comparing with equation of straight line,

$$\text{Slope} = \frac{-\Delta H^\circ}{R}$$

Since, reaction is exothermic, $\Delta H^\circ$ = –ve, therefore, slope = +ve.




Hence, option (2) is correct.

69. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(1) 1.6 hours (2) 6.4 hours

(3) 0.8 hours (4) 3.2 hours

**Answer (4)**

**Sol.** $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

27.66 of $B_2H_6$ = 1 mole of $B_2H_6$ which requires three moles of oxygen ($O_2$) for complete burning

$6H_2O \longrightarrow 6H_2 + 3O_2$ (On electrolysis)

Number of faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

$t = \dfrac{12 \times 96500}{100}$ second

$t = \dfrac{12 \times 96500}{100 \times 3600}$ hour $\Rightarrow$ t = 3.2 hours

70. Consider the following reaction and statements

$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer

(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.

(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.

(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

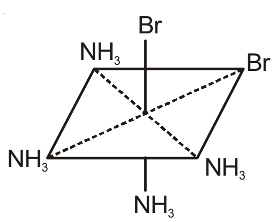
The correct statements are:

(1) (II) and (IV) (2) (I) and (II)

(3) (I) and (III) (4) (III) and (IV)

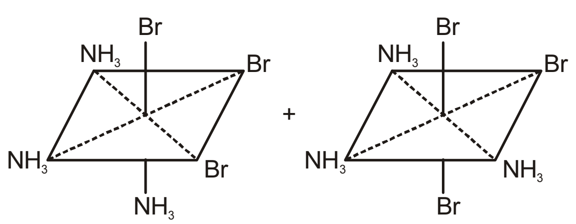
**Answer (3)**

**Sol.**



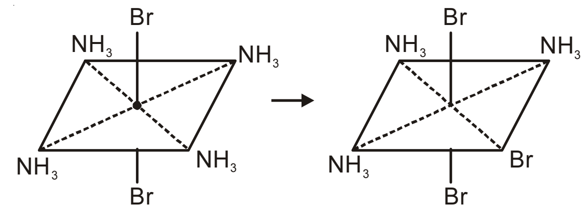
*cis*-isomer

$\downarrow +Br^-$



fac- + mer-

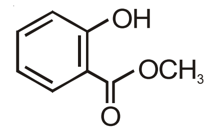
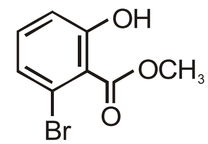
(2 isomer)

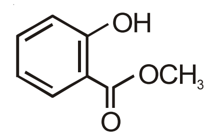
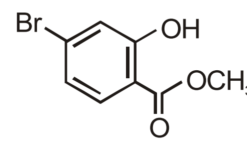


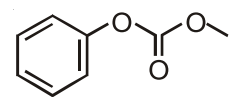
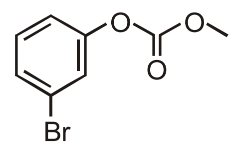
trans → Mer (1 isomer)

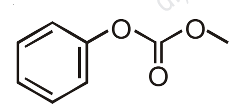
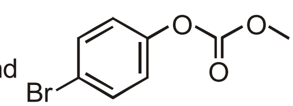
So option (3) is correct.

71. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with $Br_2$ to form product B. A and B are respectively

(1)  and 

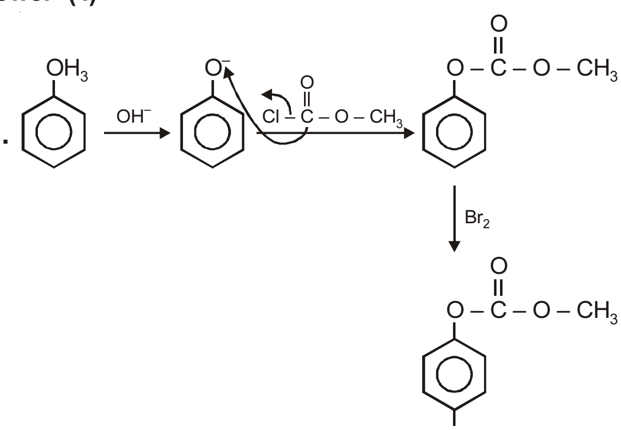
(2)  and 

(3)  and 

(4)  and 

**Answer (4)**

**Sol.**



Hence, option (4) is correct.

72. An aqueous solution contains an unknown concentration of $Ba^{2+}$. When 50 mL of a 1 M solution of $Na_2SO_4$ is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is $1 \times 10^{-10}$. What is original concentration of $Ba^{2+}$?

(1) $1.0 \times 10^{-10}$ M (2) $5 \times 10^{-9}$ M

(3) $2 \times 10^{-9}$ M (4) $1.1 \times 10^{-9}$ M

**Answer (4)**

**Sol.** Final concentration of $[SO_4^{--}] = \dfrac{[50 \times 1]}{[500]} = 0.1$ M

$K_{sp}$ of $BaSO_4$,

$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$

$[Ba^{2+}][0.1] = \dfrac{10^{-10}}{0.1} = 10^{-9}$ M

Concentration of $Ba^{2+}$ in final solution = $10^{-9}$ M

Concentration of $Ba^{2+}$ in the original solution.

$M_1V_1 = M_2V_2$

$M_1$ (500 – 50) = $10^{-9}$ (500)

$M_1 = 1.11 \times 10^{-9}$ M

So, option (4) is correct.

73. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 torr, was 1.00 torr $s^{-1}$ when 5% had reacted and 0.5 torr $s^{-1}$ when 33% had reacted. The order of the reaction is

(1) 0 (2) 2

(3) 3 (4) 1

**Answer (2)**

**Sol.** Assume the order of reaction with respect to acetaldehyde is x.

**Condition-1 :**

Rate = $k[CH_3CHO]^x$

$1 = k[363 \times 0.95]^x$

$1 = k[344.85]^x$ ...(i)

**Condition-2 :**

$0.5 = k[363 \times 0.67]^x$

$0.5 = k[243.21]^x$ ...(ii)

Divide equation (i) by (ii),

$$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$$

$\Rightarrow$ x = 2

74. The combustion of benzene (l) gives $CO_2$(g) and $H_2O$(l). Given that heat of combustion of benzene at constant volume is –3263.9 kJ $mol^{-1}$ at 25° C; heat of combustion (in kJ $mol^{-1}$) of benzene at constant pressure will be

(R = 8.314 $JK^{-1}$ $mol^{-1}$)

(1) –3267.6 (2) 4152.6

(3) –452.46 (4) 3260

**Answer (1)**

**Sol.** $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$

$\Delta H = \Delta U + \Delta n_g RT$

$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$

$= -3263.9 + (-3.71)$

$= -3267.6$ kJ $mol^{-1}$

75. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound $C_XH_Y$ completely to $CO_2$ and $H_2O$. The empirical formula of compound $C_XH_YO_Z$ is

(1) $C_2H_4O_3$ (2) $C_3H_6O_3$

(3) $C_2H_4O$ (4) $C_3H_4O_2$

**Answer (1)**

**Sol.**

| Element | Relative mass | Relative mole | Simplest whole number ratio |
|---|---|---|---|
| C | 6 | $\frac{6}{12} = 0.5$ | 1 |
| H | 1 | $\frac{1}{1} = 1$ | 2 |

So, X = 1, Y = 2

Equation for combustion of $C_XH_Y$

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required = $2\left(X + \frac{Y}{4}\right)$

As per information,

$2\left(X + \frac{Y}{4}\right) = 2Z$

$\Rightarrow \left(1 + \frac{2}{4}\right) = Z$

$\Rightarrow$ Z = 1.5

Molecule can be written

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow C_2H_4O_3$

76. The *trans*-alkenes are formed by the reduction of alkynes with

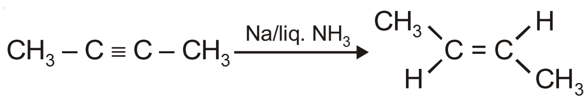
(1) Sn - HCl

(2) $H_2$ - Pd/C, $BaSO_4$

(3) $NaBH_4$

(4) Na/liq. $NH_3$

**Answer (4)**

**Sol.** 

Trans alkene

So, option (4) is correct.

77. Which of the following are Lewis acids?

(1) $BCl_3$ and $AlCl_3$ (2) $PH_3$ and $BCl_3$

(3) $AlCl_3$ and $SiCl_4$ (4) $PH_3$ and $SiCl_4$

**Answer (1)***

**Sol.** $BCl_3$ – electron deficient, incomplete octet

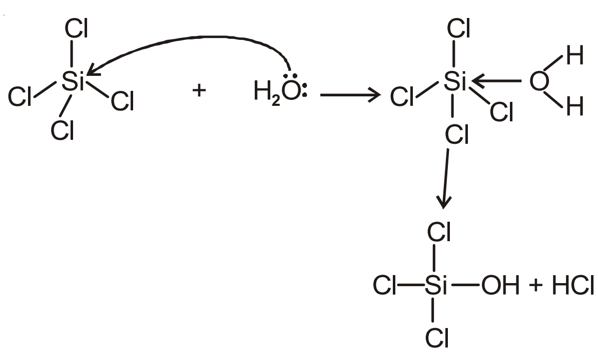
$AlCl_3$ – electron deficient, incomplete octet

Ans-(1) $BCl_3$ and $AlCl_3$

$SiCl_4$ can accept lone pair of electron in *d*-orbital of silicon hence it can act as Lewis acid.

* Although the most suitable answer is (1). However, both option (1) & (3) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



Hence option (3), $AlCl_3$ and $SiCl_4$ is also correct.

78. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is

(1) Fe (2) Zn

(3) Ca (4) Al

**Answer (4)**

**Sol.** $Al^{3+} \xrightarrow{NaOH} \underset{\text{White gelatinous ppt.}}{Al(OH)_3 \downarrow} \xrightarrow[NaOH]{Excess} \underset{\text{Sodium meta aluminate (soluble)}}{NaAlO_2}$

$$2Al(OH)_3 \xrightarrow{\text{Strong heating}} Al_2O_3 + 3H_2O$$

$Al_2O_3$ is used in column chromatography.

79. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) $H_2^{2-}$

(2) $He_2^{2+}$

(3) $He_2^{+}$

(4) $H_2^{-}$

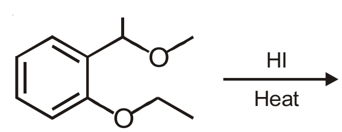
**Answer (1)**

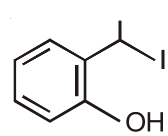
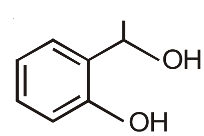
**Sol.**

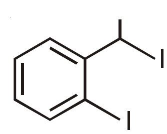
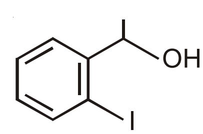
| | Electronic configuration | Bond order |
|---|---|---|
| $He_2^{+}$ | $\sigma_{1s^2}\sigma^*_{1s^1}$ | $\frac{2-1}{2} = 0.5$ |
| $H_2^{-}$ | $\sigma_{1s^2}\sigma^*_{1s^1}$ | $\frac{2-1}{2} = 0.5$ |
| $H_2^{2-}$ | $\sigma_{1s^2}\sigma^*_{1s^2}$ | $\frac{2-2}{2} = 0$ |
| $He_2^{2+}$ | $\sigma_{1s^2}$ | $\frac{2-0}{2} = 1$ |

Molecule having zero bond order will not be a viable molecule.

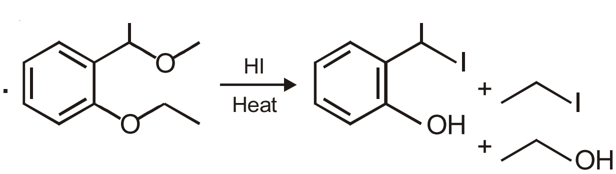
80. The major product formed in the following reaction is



(1)  (2) 

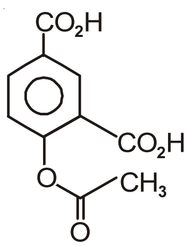
(3)  (4) 

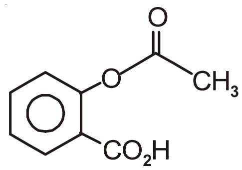
**Answer (1)**

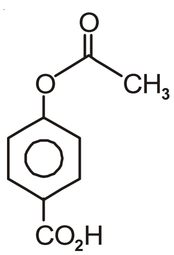
**Sol.** 

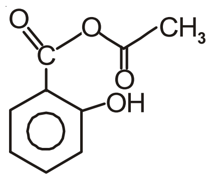
Hence, option (1) is correct.

81. Phenol on treatment with $CO_2$ in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of $H_2SO_4$ produces

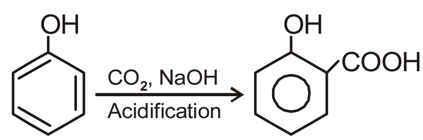
(1) 


(2) 

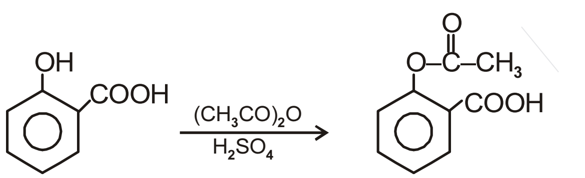

(3) 


(4) 


**Answer (2)**

**Sol.** 


(Major)




Acetyl salicylic acid (Aspirin)

82. Which of the following compounds contain(s) no covalent bond(s)?

$KCl$, $PH_3$, $O_2$, $B_2H_6$, $H_2SO_4$

(1) $KCl$, $B_2H_6$ (2) $KCl$, $B_2H_6$, $PH_3$

(3) $KCl$, $H_2SO_4$ (4) $KCl$

**Answer (4)**

**Sol.** KCl – Ionic bond between $K^+$ and $Cl^-$

$PH_3$ – Covalent bond between P and H

$O_2$ – Covalent bond between O atoms

$B_2H_6$–Covalent bond between B and H atoms

$H_2SO_4$ – Covalent bond between S and O and also between O and H.

$\therefore$ Compound having no covalent bonds is KCl only.

83. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Metal deficiency defect

(2) Schottky defect

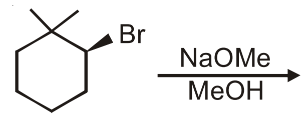
(3) Vacancy defect

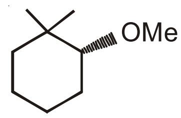
(4) Frenkel defect

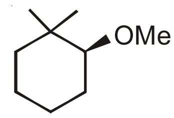
**Answer (4)**

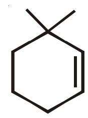
**Sol.** In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

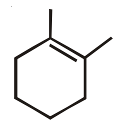
84. The major product of the following reaction is




(1) 


(2) 


(3) 

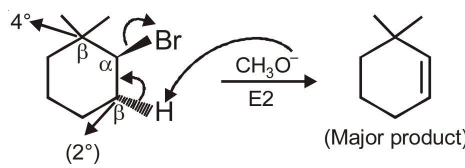
(4) 

**Answer (3)**

**Sol.** $CH_3O^-$ is a strong base and strong nucleophile, so favourable condition is $S_N2$/E2.

Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over $S_N2$.

Also, polarity of $CH_3OH$ (solvent) is not as high as $H_2O$, so E1 is also dominated by E2.




85. The compound that does not produce nitrogen gas by the thermal decomposition is

(1) $(NH_4)_2SO_4$ (2) $Ba(N_3)_2$

(3) $(NH_4)_2Cr_2O_7$ (4) $NH_4NO_2$

**Answer (1)**

**Sol.** $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

Among all the given compounds, only $(NH_4)_2SO_4$ do not form dinitrogen on heating, it produces ammonia gas.

86. An aqueous solution contains 0.10 M $H_2S$ and 0.20 M HCl. If the equilibrium constant for the formation of $HS^-$ from $H_2S$ is $1.0 \times 10^{-7}$ and that of $S^{2-}$ from $HS^-$ ions is $1.2 \times 10^{-13}$ then the concentration of $S^{2-}$ ions in aqueous solution is

(1) $5 \times 10^{-19}$ (2) $5 \times 10^{-8}$

(3) $3 \times 10^{-20}$ (4) $6 \times 10^{-21}$

**Answer (3)**

**Sol.** In presence of external $H^+$,

$$H_2S \rightleftharpoons 2H^+ + S^{2-}, \quad K_{a_1} \cdot K_{a_2} = K_{eq}$$

$$\therefore \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1\times10^{-7}\times1.2\times10^{-13}$$

$$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2\times10^{-20}$$

$[S^{2-}] = 3 \times 10^{-20}$

87. The oxidation states of

Cr in $\left[Cr(H_2O)_6\right]Cl_3$, $\left[Cr(C_6H_6)_2\right]$, and

$K_2\left[Cr(CN)_2(O)_2(O_2)(NH_3)\right]$ respectively are

(1) +3, 0 and +4

(2) +3, +4 and +6

(3) +3, +2 and +4

(4) +3, 0 and +6

**Answer (4)**

**Sol.** $\left[Cr(H_2O)_6\right]Cl_3 \Rightarrow x + 0\times6 - 1\times3 = 0$

$\therefore x = +3$

$\left[Cr(C_6H_6)_2\right] \Rightarrow x + 2\times0 = 0$

$x = 0$

$K_2\left[Cr(CN)_2(O_2)(O_2)NH_3\right]$

$\Rightarrow 1\times2 + x - 1\times2 - 2\times2 - 2\times1 = 0$

$\Rightarrow x - 6 = 0$

$x = +6$

88. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to

(1) $[3\{Ca(OH)_2\}.CaF_2]$ (2) $[CaF_2]$

(3) $[3(CaF_2).Ca(OH)_2]$ (4) $[3Ca_3(PO_4)_2.CaF_2]$

**Answer (4)**

**Sol.** $F^-$ ions make the teeth enamel harder by converting

$$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} \text{ to } \underset{\text{Fluorapatite}}{[3Ca_3(PO_4)_2.CaF_2]}$$

89. Which of the following salts is the most basic in aqueous solution?

(1) $Pb(CH_3COO)_2$ (2) $Al(CN)_3$

(3) $CH_3COOK$ (4) $FeCl_3$

**Answer (3)**

**Sol.** $CH_3COOK + H_2O \longrightarrow CH_3COOH + \underset{\text{Basic}}{KOH}$

$FeCl_3$ – Acidic solution

$Al(CN)_3$ – Salt of weak acid and weak base

$Pb(CH_3COO)_2$ – Salt of weak acid and weak base

$CH_3COOK$ is salt of weak acid and strong base.

Hence solution of $CH_3COOK$ is basic.

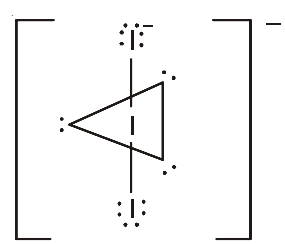
90. Total number of lone pair of electrons in $I_3^-$ ion is

(1) 12 (2) 3

(3) 6 (4) 9

**Answer (4)**

**Sol.** Structure of $I_3^-$



Number of lone pairs in $I_3^{\ominus}$ is 9.

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