MATHEMATICS

1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(1) $\frac{1}{2}(1-\sqrt{5})$ (2) $\frac{1}{2}\sqrt{5}$

(3) $\sqrt{5}$ (4) $\frac{1}{2}(\sqrt{5}-1)$

Ans. (4)

Sol: Given $ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow 1 = r + r^2$

$\therefore r = \frac{\sqrt{5}-1}{2}$.

2. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is

(1) 1 (2) 3

(3) 4 (4) 5

Ans. (2)

Sol: $\sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5} \Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\therefore x = 3$.

3. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then $\frac{a}{b}$ equals

(1) $\frac{5}{n-4}$ (2) $\frac{6}{n-5}$

(3) $\frac{n-5}{6}$ (4) $\frac{n-4}{5}$

Ans. (4)

Sol: ${}^nC_4\, a^{n-4}(-b)^4 + {}^nC_5\, a^{n-5}(-b)^5 = 0$

$\Rightarrow \left(\frac{a}{b}\right) = \frac{n-5+1}{5}$.

4. The set S = {1, 2, 3, …, 12) is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(1) $\frac{12!}{3!\,(4!)^3}$ (2) $\frac{12!}{3!\,(3!)^4}$

(3) $\frac{12!}{(4!)^3}$ (4) $\frac{12!}{(3!)^4}$

Ans. (3)

Sol: Number of ways is ${}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$= \dfrac{12!}{(4!)^3}$.

5. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ is defined, is

(1) $[0, \pi]$

(2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$

(4) $\left[0, \frac{\pi}{2}\right)$

Ans. (4)

Sol: f (x) is defined if $-1 \le \frac{x}{2} - 1 \le 1$ and $\cos x > 0$

or $0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$.

6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are

(1) 12 kg and 13 kg

(2) 5 kg and 5 kg

(3) 5 kg and 12 kg

(4) 5 kg and 13 kg

Ans. (3)

Sol: $T_2 \cos\left(\frac{\pi}{2} - \theta\right) = T_1 \cos\theta \Rightarrow T_1 \cos\theta = T_2 \sin\theta$

$T_1 \sin\theta + T_2 \cos\theta = 13$.

$\because OC = CA = CB$

$\Rightarrow \angle AOC = \angle OAC$ and $\angle COB = \angle OBC$

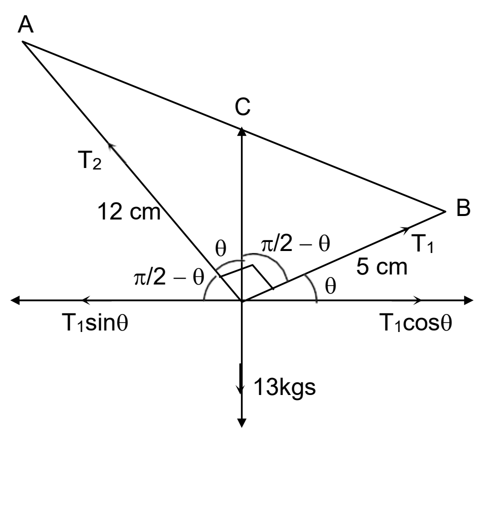
$\therefore \sin\theta = \sin A = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

$\Rightarrow \frac{T_1}{T_2} = \frac{5}{12} \Rightarrow T_1 = \frac{5}{12} T_2$

$T_2\left(\frac{5}{12} \cdot \frac{5}{13} + \frac{12}{13}\right) = 13$

$T_2\left(\frac{169}{12 \cdot 13}\right) = 13$

$T_2 = 12$ kgs. $\Rightarrow T_1 = 5$ kgs.



7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is
(1) 1/729 (2) 8/9
(3) 8/729 (4) 8/243

7. (4)

Sol: Probability of getting score 9 in a single throw = $\frac{4}{36} = \frac{1}{9}$

Probability of getting score 9 exactly twice = ${}^{3}C_{2} \times \left(\frac{1}{9}\right)^{2} \times \frac{8}{9} = \frac{8}{243}$.

8. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval
(1) $0 < k < ½$ (2) $k \geq ½$
(3) $-½ \leq k \leq ½$ (4) $k \leq ½$

Ans. (2)

Sol: Equation of circle $(x - h)^2 + (y - k)^2 = k^2$
It is passing through (– 1, 1) then
$(-1 - h)^2 + (1 - k)^2 = k^2$
$h^2 + 2h - 2k + 2 = 0$
$D \geq 0$
$2k - 1 \geq 0 \Rightarrow k \geq 1/2$.

9. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angles α with the positive x-axis, then $\cos\alpha$ equals
(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{2}$
(3) 1 (4) $\frac{1}{\sqrt{2}}$

Ans. (1)

Sol: If direction cosines of L be l, m, n, then
$2l + 3m + n = 0$
$l + 3m + 2n = 0$
Solving, we get, $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$
$\therefore l : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$.

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is
(1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$
(3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

Ans. (3)

Sol: General equation of all such circles is
$x^2 + y^2 + 2gx = 0$.
Differentiating, we get
$2x + 2y\dfrac{dy}{dx} + 2g = 0$
$\therefore$ Desired equation is
$x^2 + y^2 + \left(-2x - 2y\dfrac{dy}{dx}\right)x = 0$
$\Rightarrow y^2 = x^2 + 2xy\dfrac{dy}{dx}$.

11. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is

(1) 2
(2) 1/2
(3) $\dfrac{1}{\sqrt{2}}$
(4) $\sqrt{2}$

Ans. (4)

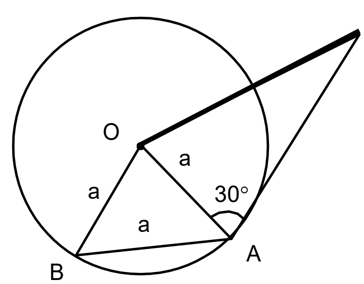
Sol: Using A.M. $\ge$ G.M.
$\dfrac{p^2 + q^2}{2} \ge pq$
$\Rightarrow pq \le \dfrac{1}{2}$
$(p + q)^2 = p^2 + q^2 + 2pq$
$\Rightarrow p + q \le \sqrt{2}$.

12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is

(1) $\dfrac{2a}{\sqrt{3}}$
(2) $2a\sqrt{3}$
(3) $\dfrac{a}{\sqrt{3}}$
(4) $a\sqrt{3}$

Ans. (3)

Sol: ΔOAB is equilateral
$\therefore$ OA = OB = AB = a
Now $\tan 30° = \dfrac{h}{a}$
$\therefore h = \dfrac{a}{\sqrt{3}}$



13. The sum of the series
$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$ is

(1) $-{}^{20}C_{10}$
(2) $\dfrac{1}{2}\,{}^{20}C_{10}$
(3) 0
(4) $^{20}C_{10}$

Ans. (2)

Sol: $(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \ldots + {}^{20}C_{10}x^{10} + \ldots + {}^{20}C_{20}x^{20}$
put x = – 1,
$0 = {}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \ldots + {}^{20}C_{20}$
$0 = 2\,({}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9) + {}^{20}C_{10}$
$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + \ldots + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}.$

14. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

(1) ellipse (2) parabola
(3) circle (4) hyperbola

Ans. (1), (4)

Sol: Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow G \equiv \left(x + y\frac{dy}{dx}, 0\right)$

$\left|x + y\frac{dy}{dx}\right| = |2x|$

$\Rightarrow y\frac{dy}{dx} = x$ or $y\frac{dy}{dx} = -3x$

y dy = x dx or ydy = –3xdx

$\frac{y^2}{2} = \frac{x^2}{2} + c$ or $\frac{y^2}{2} = -\frac{3x^2}{2} + c$

$x^2 - y^2 = -2c$ or $3x^2 + y^2 = 2c$.

15. If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is

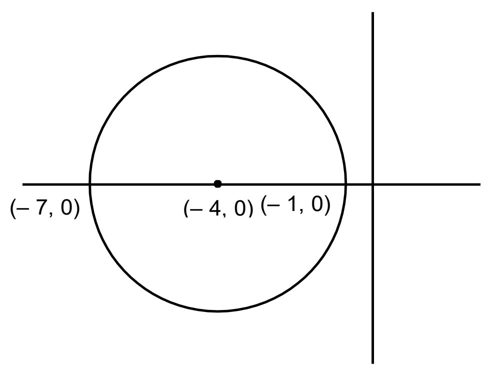
(1) 4 (B) 10
(3) 6 (4) 0

Ans. (3)

Sol: From the Argand diagram maximum value of $|z + 1|$ is 6.

Alternative:

$|z + 1| = |z + 4 - 3|$
$\le |z + 4| + |-3| = 6.$

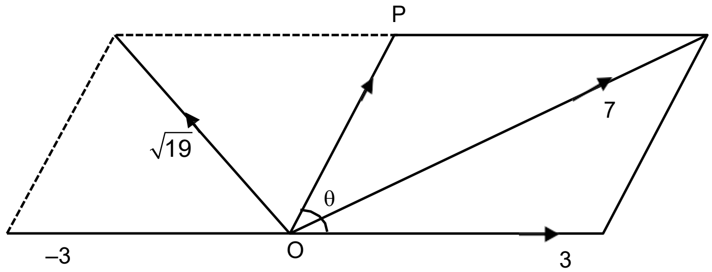


16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is

(1) 5 N (2) 6 N
(3) 3N (4) 4N

Ans. (1)

Sol:



$7^2 = P^2 + 3^2 + 2 \times 3 \times P\cos\theta$ …(1)

$\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2 \times (-3) \times P\cos\theta$ …(2)

adding we get

$68 = 2P^2 + 18 \Rightarrow P = 5.$

17. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(1) 0.06 (2) 0.14
(3) 0.2 (3) 0.7

Ans. ( )

Sol: The desired probability

$= 0.7 \times 0.2 + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \ldots$

$= 0.14\,[1 + (0.56) + (0.56)^2 + \ldots]$

$= 0.14\left[\dfrac{1}{1-0.56}\right] = \dfrac{0.14}{0.44} = \dfrac{7}{22}$

18. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0$, $y \neq 0$ then D is

(1) divisible by neither x nor y (2) divisible by both x and y
(3) divisible by x but not y (4) divisible by y but not x

Ans. (2)

Sol: $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy.$

Hence D is divisible by both x and y.

19. For the hyperbola $\dfrac{x^2}{\cos^2\alpha} - \dfrac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies?

(1) eccentricity (2) directrix
(3) abscissae of vertices (4) abscissae of foci

Ans. (4)

Sol: $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$
coordinates of focii are $(\pm ae, 0)$
$\therefore b^2 = a^2(e^2 - 1) \Rightarrow e = \sec\alpha$.
Hence abscissae of foci remain constant when α varies.

20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol: $l = \cos\frac{\pi}{4}$, $m = \cos\frac{\pi}{4}$
we know $l^2 + m^2 + n^2 = 1$
$\frac{1}{2} + \frac{1}{2} + n^2 = 1$
$\Rightarrow n = 0$
Hence angle with positive direction of z-axis is $\frac{\pi}{2}$.

21. A value of C for which the conclusion of Mean Value Theorem holds for the function f(x) = $\log_e x$ on the interval [1, 3] is

(1) $2\log_3 e$ (2) $\frac{1}{2}\log_e 3$

(3) $\log_3 e$ (4) $\log_e 3$

Ans. (1)

Sol: Using mean value theorem
$f'(c) = \frac{f(3) - f(1)}{3 - 1}$
$\Rightarrow \frac{1}{c} = \frac{\log 3 - \log 1}{2}$
$\Rightarrow c = \frac{2}{\log_e 3} = 2\log_3 e$.

22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ans. (2)

Sol: $f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$

$$= \frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{1+(\sin x+\cos x)^2}$$

f(x) is increasing if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$-\frac{3\pi}{4} < x < \frac{\pi}{4}$

hence f(x) is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

23. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

(1) 5^2 (2) 1
(3) 1/5 (4) 5

Ans. (3)

Sol: $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$625\alpha^2 = 25$

$\Rightarrow |\alpha| = \frac{1}{5}$.

24. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots$ upto infinity is

(1) e^{-2} (2) e^{-1}
(3) $e^{-1/2}$ (4) $e^{1/2}$

Ans. (2)

Sol: $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

put x = 1

$\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \cdots = e^{-1}$.

25. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for

(1) exactly two values of θ (2) more than two values of θ
(3) no value of θ (4) exactly one value of θ

Ans. (4)

Sol: $|2\hat{u} \times 3\hat{v}| = 1$

$6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\sin\theta = \frac{1}{6}$

Hence there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

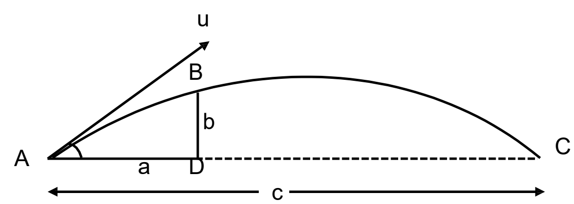
26. A particle just clears a wall of height b at distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

(1) $\tan^{-1}\frac{b}{ac}$ (2) 45°

(3) $\tan^{-1}\frac{bc}{a(c-a)}$ (4) $\tan^{-1}\frac{bc}{a}$

Ans. (3)

Sol: a = (u cosα)t and b = (u sinα)$t - \frac{1}{2}gt^2$



$b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$

also, c = $\frac{u^2\sin 2\alpha}{g}$

$b = a\tan\alpha - \frac{a^2 g}{2}\left(\frac{\sin 2\alpha}{cg}\right)\sec^2\alpha$

$b = a\tan\alpha - \frac{a^2}{2c}2\tan\alpha$

$\Rightarrow \left(a - \frac{a^2}{c}\right)\tan\alpha = b$

$\tan\alpha = \frac{bc}{a(c-a)}$.

27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(1) 40 (2) 20
(3) 80 (4) 60

Ans. (3)

Sol: 52x + 42y = 50 (x + y)
2x = 8y

$\Rightarrow \frac{x}{y} = \frac{4}{1}$ and $\frac{x}{x+y} = \frac{4}{5}$

∴ % of boys = 80.

28. The equation of a tangent to the parabola $y^2 = 8x$ is y = x + 2. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

(1) (–1, 1) (2) (0, 2)
(3) (2, 4) (4) (–2, 0)

Ans. (4)

Sol: Point must be on the directrix of the parabola.
Hence the point is (–2, 0).

29. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

(1) (4, 9, –3) (2) (4, –3, 3)
(3) (4, 3, 5) (4) (4, 3, –3)

Ans. (1)

Sol: Coordinates of centre (3, 6, 1)
Let the coordinates of the other end of diameter are (α, β, γ)
then $\frac{\alpha + 2}{2} = 3, \frac{\beta + 3}{2} = 6, \frac{\gamma + 5}{2} = 1$
Hence $\alpha = 4$, $\beta = 9$ and $\gamma = -3$.

30. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

(1) 0 (2) 1
(3) –4 (4) –2

Ans. (4)

Sol: $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

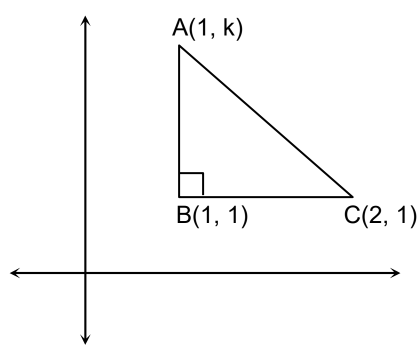
$$\begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$3x + 2 - x + 2 = 0$
$2x = -4$
$x = -2$.

31. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(1) {1, 3} (2) {0, 2}
(3) {–1, 3} (4) {–3, –2}

Ans. (3)



Sol: $\frac{1}{2} \times 1(k - 1) = \pm 1$
$k - 1 = \pm 2$
$k = 3$
$k = -1$

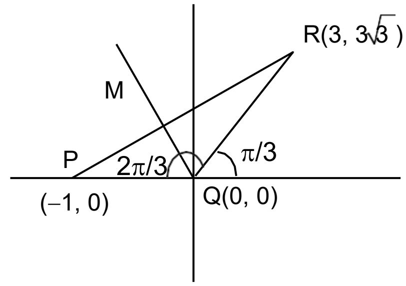
32. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR

(1) $\sqrt{3}x + y = 0$ (2) $x + \frac{\sqrt{3}}{2}y = 0$
(3) $\frac{\sqrt{3}}{2}x + y = 0$ (4) $x + \sqrt{3}y = 0$

Ans. (1)

Sol: Slope of the line QM is $\tan\frac{2\pi}{3} = -\sqrt{3}$

Hence equation is line QM is $y = -\sqrt{3}\,x$.



33. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is
(1) $-1/2$ (2) -2
(3) 1 (4) 2

Ans. (3)

Sol: Equation of bisectors of lines $xy = 0$ are $y = \pm x$
put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get $(1 - m^2)\,x^2 = 0$
$\Rightarrow m = \pm 1$.

34. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then F(e) equals
(1) $\frac{1}{2}$ (2) 0
(3) 1 (4) 2

Ans. (1)

Sol: $f(x) = \int_1^x \frac{\log t}{1+t}dt$

$F(e) = f(e) + f\left(\frac{1}{e}\right)$

$F(e) = \int_1^e \frac{\log t}{1+t}dt + \int_1^{1/e} \frac{\log t}{1+t}dt$

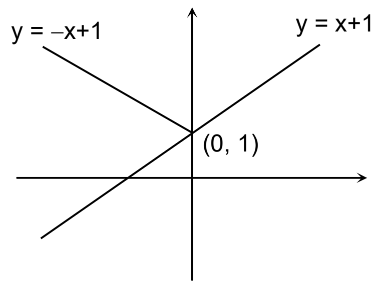
$= \int_1^e \frac{\log t}{1+t} + \int_1^e \frac{\log t}{t(1+t)}\,dt$

$= \int_1^e \frac{\log t}{t}dt = \frac{1}{2}$.

35. Let $f : R \to R$ be a function defined by $f(x) = \text{Min}\,\{x + 1, |x| + 1\}$. Then which of the following is true?
(1) $f(x) \geq 1$ for all $x \in R$ (2) f(x) is not differentiable at x = 1
(3) f(x) is differentiable everywhere (4) f(x) is not differentiable at x = 0

Ans. (3)

Sol: $f(x) = \min\{x + 1, |x| + 1\}$
$f(x) = x + 1 \ \forall\, x \in R.$



36. The function f: R ~ {0} → R given by
$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$
can be made continuous at x = 0 by defining f(0) as
(1) 2 (2) –1
(3) 0 (4) 1

Ans. (4)

Sol: $\lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x} - 1}$
$\lim_{x\to 0} \frac{e^{2x} - 1 - 2x}{x(e^{2x} - 1)}$
$\lim_{x\to 0} \frac{2e^{2x} - 2}{(e^{2x} - 1) + 2xe^{2x}}$
$\lim_{x\to 0} \frac{4e^{2x}}{4e^{2x} + 4xe^{2x}} = 1.$

37. The solution for x of the equation
$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is
(1) 2 (2) π
(3) $\frac{\sqrt{3}}{2}$ (4) $2\sqrt{2}$

Ans. ( )

Sol: $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$
$\left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$
$\sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$
$\sec^{-1} x = \frac{3\pi}{4}$
$x = -\sqrt{2}$.

38. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

(1) $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c$
(2) $\frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + c$
(3) $\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c$
(4) $\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + c$

Ans. (1)

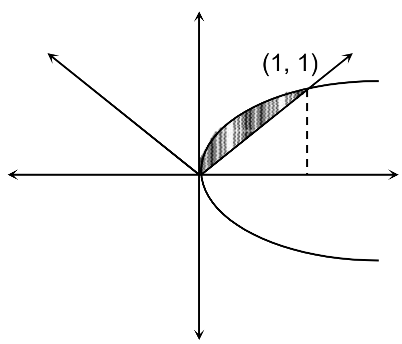
Sol:
$$\int \frac{dx}{\cos x + \sqrt{3}\sin x}$$
$$= \frac{1}{2}\int \sec\left(x - \frac{\pi}{3}\right)dx$$
$$= \frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + c$$
$$= \frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c.$$

39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(1) 2/3
(2) 1
(3) 1/6
(4) 1/3

Ans. (3)

Sol:
$$A = \int_0^1 \left(\sqrt{x} - x\right)dx$$
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$$
$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$



40. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(1) $(-3, 3)$
(2) $(-3, \infty)$
(3) $(3, \infty)$
(4) $(-\infty, -3)$

Ans. (1)

Sol:
$x^2 + ax + 1 = 0$
$\alpha + \beta = a \qquad \alpha\beta = 1$
$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
$|\alpha - \beta| = \sqrt{a^2 - 4}$
$\sqrt{a^2 - 4} < \sqrt{5}$
$a^2 - 4 < 5$
$a^2 - 9 < 0$
$a \in (-3, 3).$

Physics Code-O

41. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metres. The time at which the maximum speed first occurs is
(1) 0.5 s (2) 0.75 s
(3) 0.125 s (4) 0.25 s

Sol. **(1)**
$x = 2 \times 10^{-2} \cos \pi t$
$v = -0.02\pi \sin \pi t$
v is maximum at $t = \frac{1}{2} = 0.5\,\text{sec}$

42. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by
(1) $P = \frac{E_0 I_0}{\sqrt{2}}$ (2) P = zero
(3) $P = \frac{E_0 I_0}{2}$ (4) $P \sqrt{2}\, E_0 I_0$

Sol. **(2)**
$\cos \phi = 0$
So power = 0

43. An electric charge 10^{-3} μC is placed at the origin (0, 0) of X–Y co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be
(1) 9 volt (2) zero
(3) 2 volt (4) 4.5 volt

Sol. **(2)**
Both points are at same distance from the charge

44. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be
(1) 1 (2) 2
(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Sol. **(4)**
$$\frac{\frac{1}{2}qv}{qv} = \frac{1}{2}$$

45. An ideal coil of 10H is connected in series with a resistance of 5 Ω and a battery of 5V. 2 second after the connection is made the current flowing in amperes in the circuit is
(1) (1 – e) (2) e
(3) e^{-1} (4) $(1 - e^{-1})$

Sol. **(4)**

$$i = i_0\left(1 - e^{\frac{Rt}{L}}\right)$$

$$= \left(1 - e^{-1}\right)$$

46. A long straight wire of radius 'a' caries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\frac{a}{2}$ and 2a is

(1) $\frac{1}{4}$ (2) 4

(3) 1 (4) $\frac{1}{2}$

Sol. **(3)**

$$B2\pi\frac{a}{2} = \mu_0 \frac{i}{\pi a^2}\left(\frac{\pi a^2}{4}\right)$$

$$B_1 = \frac{\mu_0 i}{4\pi a} \quad \ldots(i)$$

$$B_2 2\pi(2a) = \mu_0 i$$

$$B_2 = \frac{\mu_0 i}{4\pi a} \quad \ldots(ii)$$

$$\frac{B_1}{B_2} = 1$$

47. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(1) the magnetic field is zero only on the axis of the pipe
(2) the magnetic field is different at different points inside the pipe
(3) the magnetic field at any point inside the pipe is zero
(4) the magnetic field at all points inside the pipe is the same, but not zero

Sol. **(3)**
Use Ampere's law

48. If M_O is the mass of an oxygen isotope ${}_8O^{17}$, M_p and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is

(1) $(M_O - 8M_P)C^2$ (2) $(M_O - 8M_P - 9M_N)C^2$

(3) M_OC^2 (4) $(M_O - 17M_N)C^2$

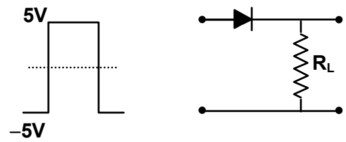
Sol. **(2)**
Binding energy = $(M_O - 8M_P - 9\ M_N)C^2$

49. In gamma ray emission from a nucleus
(1) both the neutron number and the proton number change
(2) there is no change in the proton number and the neutron number.
(3) only the neutron number changes
(4) only the proton number changes

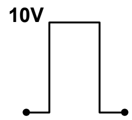
Sol. **(2)**

Solutions to AIEEE - 2007 -PHYSICS

50. If in a p-n junction diode, a square input signal of 10V is applied as shown



Then the output signal across R_L will be

(1)


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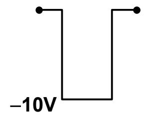

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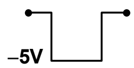

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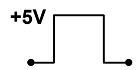


Sol. (4)

51. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is

(1) ν/c (2) $h\nu c$
(3) $h\nu/c^2$ (4) $h\nu/c$

Sol. (4)

$$P = \frac{h}{\lambda} = \frac{h\nu}{c}$$

52. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is x = 0 at t = 0, then its displacement after unit time (t = 1) is

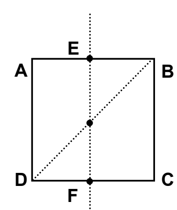
(1) $v_0 + 2g + 3f$ (2) $v_0 + g/2 + f/3$
(3) $v_0 + g + f$ (4) $v_0 + g/2 + f$

Sol. (2)

$$\int_0^x dx = \int_0^1 (V_0 + gt + ft^2)dt$$

$$x = v_0 + g\left(\frac{1}{2}\right) + f\left(\frac{1}{3}\right)$$

53. For the given uniform square lamina ABCD, whose centre is O,



(1) $\sqrt{2}I_{AC} = I_{EF}$
(2) $I_{AD} = 3I_{EF}$
(3) $I_{AC} = I_{EF}$
(4) $I_{AC} = \sqrt{2}I_{EF}$

Sol. (3)

$I_{AC} = I_{EF}$ (from $\perp^{rd}$ axis theorem)

54. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as

$a = A\cos(\omega t + \delta)$

(1) $A = x_0$, $\delta = -\pi/4$ (2) $A = x_0\omega^2$, $\delta = -\pi/4$
(3) $A = x_0\omega^2$, $\delta = -\pi/4$ (4) $A = x_0\omega^2$, $\delta = 3\pi/4$

Sol. **(4)**

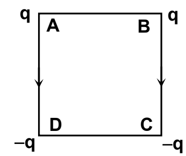
$v = -x_0\omega \sin(\omega t - \pi/4)$

$a = -x_0\omega^2\cos\left(\omega t + \pi - \frac{\pi}{4}\right)$

$a = A\cos(\omega t + \delta)$

$A = x_0\omega^2;\ \ \delta = \frac{3\pi}{4}$

55. Charges are placed on the vertices of a square as shown. Let E be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then



(1) $\vec{E}$ remains unchanged, V changes
(2) Both $\vec{E}$ and V change
(3) $\vec{E}$ and V remains unchanged
(4) $\vec{E}$ changes, V remains unchanged

Sol. **(4)**

As $\vec{E}$ is a vector quantity

56. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then
(1) X will decay faster than Y
(2) Y will decay faster than X
(3) X and Y have same decay rate initially
(4) X and Y decay at same rate always.

Sol. **(2)**

$t_{1/2} = \frac{\ln 2}{\lambda_x}$

$\tau_{mean} = \frac{1}{\lambda_y};\ \frac{dN}{dt} = -\lambda N$

$\frac{\ln 2}{\lambda_x} = \frac{1}{\lambda_y} \Rightarrow \lambda_x = \lambda_y(0.6932) \Rightarrow \lambda_y > \lambda_x$

57. A Carnot engine, having an efficiency of η = 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(1) 99 J (2) 90 J
(3) 1 J (4) 100 J

Sol. **(2)**

$W = Q_2\left(\frac{T_1}{T_2} - 1\right) \qquad \eta = 1 - \frac{T_2}{T_1}$

$10 = Q_2\left(\frac{10}{9} - 1\right) \qquad \frac{1}{10} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{10} = \frac{9}{10}$

$10 = Q_2\left(\frac{1}{9}\right) \qquad \Rightarrow \frac{T_1}{T_2} = \frac{10}{9}$

$Q_2 = 90$ J

58. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate?
(1) The number of free conduction electrons is significant in C but small in Si and Ge.
(2) The number of free conduction electrons is negligible small in all the three.
(3) The number of free electrons for conduction is significant in all the three.
(4) The number of free electrons for conduction is significant only in Si and Ge but small in C.

Sol. **(4)**

59. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(1) $\vec{v} = \vec{E} \times \vec{B} / B^2$
(2) $\vec{v} = \vec{B} \times \vec{E} / B^2$
(3) $\vec{v} = \vec{E} \times \vec{B} / E^2$
(4) $\vec{v} = \vec{B} \times \vec{E} / E^2$

Sol. **(1)**

$\vec{v} \times \vec{B} = -\vec{E}$

60. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by V(x) = 20/(x^2 – 4) Volts. The electric field E at x = 4 μm is given by

(1) 5/3 Volt/μm and in the –ve x direction
(2) 5/3 Volt/μm and in the +ve x direction.
(3) 10/9 Volt /μm and in the –ve x direction
(4) 10/9 Volt/μm and in the +ve x direction.

Sol. **(4)**

$$V_x = \frac{20}{x^2 - 4}$$

$$E = -\frac{dV}{dx} = \frac{20}{(x^2-4)^2}(2x-0) \;\; = \frac{160}{144} = \frac{10}{9}$$

61. Which of the following transitions in hydrogen atoms emit photons of highest frequency?

(1) n = 2 to n = 6
(2) n = 6 to n = 2
(3) n = 2 to n = 1
(4) n = 1 to n = 2

Sol. **(3)**

$$h\nu = Rhcz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

62. A block of mass ‘m’ is connected to another block of mass ‘M’ by a spring (massless) of spring constant ‘k’. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force ‘F’ starts acting on the block of mass ‘M’ to pull it. Find the force on the block of mass ‘m’

(1) $\frac{mF}{M}$
(2) $\frac{(M+m)F}{m}$
(3) $\frac{mF}{(m+M)}$
(4) $\frac{MF}{(m+M)}$

Sol. **(3)**

$$Kx = ma = \frac{mF}{m+M}$$

63. Two lenses of power -15 D and + 5D are in contact with each other. The focal length of the combination is

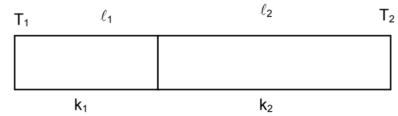
(1) –20 cm
(2) – 10 cm
(3) + 20 cm
(4) + 10 cm

Sol. **(2)**

$P = P_1 + P_2 = -10$

$$f = \frac{1}{P} \Rightarrow -0.1 \text{ m} \Rightarrow -10 \text{ cm}$$

64. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths ℓ_1 and ℓ_2 and thermal conductivities k_1 and k_2 respectively. The temperature at the interface of the two sections is



(1) $(k_2\ell_2T_1 + k_1\ell_1T_2) / (k_1\ell_1 + k_2\ell_2)$ (2) $(k_2\ell_1 T_1 + k_1\ell_1T_2) / (k_2\ell_1 + k_1\ell_2)$

(3) $(k_1\ell_2 T_1 + k_2\ell_1T_2) / (k_1\ell_2 + k_2\ell_1)$ (4) $(k_1\ell_1 T_1 + k_2\ell_2T_2) / (k_1\ell_1 + k_2\ell_2)$

Sol. **(3)**

$$\frac{(T_1 - T)k_1}{\ell_1} = \frac{(T - T_2)k_2}{\ell_2}$$

$$T = \frac{T_1k_1\ell_2 + T_2k_2\ell_1}{k_1\ell_2 + k_2\ell_1}$$

65. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(1) 1000 (2) 10000

(3) 10 (4) 100

Sol. **(4)**

$$B_1 = 10\log\left(\frac{I}{I_0}\right)$$

$$B_2 = \log\left(\frac{I'}{I_0}\right)$$

given $B_2 - B_1 = 20$

$$20 = 10\log\left(\frac{I'}{I}\right)$$

$$I' = 100I$$

66. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(1) $C_p - C_v = R/28$ (2) $C_p - C_v = R/14$

(3) $C_p - C_v = R$ (4) $C_p - C_v = 28R$

Sol. **(1)**

Mayer Formula

67. A charged particle moves through a magnetic field perpendicular to its direction. Then

(1) the momentum changes but the kinetic energy is constant

(2) both momentum and kinetic energy of the particle are not constant

(3) both, momentum and kinetic energy of the particle are constant

(4) kinetic energy changes but the momentum is constant

Sol. **(1)**

68. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance 'd' from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(1) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{1/2}$ (2) $\frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$

(3) $\frac{\mu_0}{2\pi d}\left(I_1 + I_2\right)$ (4) $\frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)$

Sol. **(2)**

$$\frac{\mu_0 I}{2\pi d}\sqrt{(I_1^2 + I_2^2)}$$

69. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0 °C will be

(1) 2 ohm
(2) 1 ohm
(3) 4 ohm
(4) 3 ohm

Sol. **(3)**

$$\frac{5}{6} = \frac{1+50\alpha}{1+100\alpha}$$

$5 = R_0(1 + \alpha \times 50)$

$R_O = 4$

70. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(1) ½ $(K-1)CV^2$
(2) $CV^2(K-1)/K$
(3) $(K-1)CV^2$
(4) zero

Sol. **(4)**

71. If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

(1) 1
(2) 0
(3) g_E/g_m
(4) g_m/g_E

Sol. **(1)**

72. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of α is

(1) 1/3
(2) 1/2
(3) 1/6
(4) 1/4

Sol. **(1)**

In this question distance of centre of mass of new disc is αR not $\frac{\alpha}{R}$.

$$-\frac{3M}{4}\alpha R + \frac{M}{4}R = 0$$

$$\Rightarrow \alpha = \frac{1}{3}$$

73. A round uniform body of radius R, mass M and moment of inertia 'I', rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(1) $\dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$ (2) $\dfrac{g\sin\theta}{1+\dfrac{MR^2}{I}}$

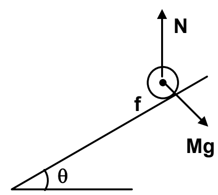
(3) $\dfrac{g\sin\theta}{1-\dfrac{I}{MR^2}}$ (4) $\dfrac{g\sin\theta}{1-\dfrac{MR^2}{I}}$

Sol. **(1)**

$Mg\sin\theta - f = Ma$

$fR = I\,\dfrac{a}{R}$

$\Rightarrow a = \dfrac{g\sin\theta}{\left(1+\dfrac{I}{MR^2}\right)}$



74. Angular momentum of the particle rotating with a central force is constant due to

(1) Constant Force (2) Constant linear momentum.

(3) Zero Torque (4) Constant Torque

Sol. **(3)**

75. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000. N/m. The spring compresses by

(1) 5.5 cm (2) 2.5 cm

(3) 11.0 cm (4) 8.5 cm

Sol. **(1)**

76. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(1) K (2) Zero

(3) K/2 (4) K/4

Sol. **(4)**

77. In a Young's double slit experiment the intensity at a point where the path difference is $\dfrac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, $\dfrac{I}{I_0}$ is equal to

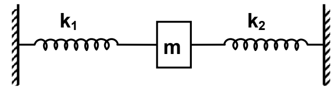
(1) $\dfrac{1}{\sqrt{2}}$ (2) $\dfrac{\sqrt{3}}{2}$

(3) 1/2 (4) 3/4

Sol. **(4)**

$$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$$

78. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(1) f/2 (2) f/4

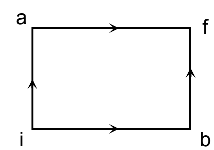
(3) 4f (4) 2f

Sol. **(4)**

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

$$f' = \frac{1}{2\pi}2\sqrt{\frac{k_1 + k_2}{m}} = 2f$$

79. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path' ibf Q = 36 cal. W along the path ibf is



(1) 6 cal (2) 16 cal.

(3) 66 cal. (4) 14 cal.

Sol. **(1)**

80. A particle of mass m executes simple harmonic motion with amplitude 'a' and frequency 'ν'. The average kinetic energy during its motion from the position of equilibrium to the end is

(1) $\pi^2 ma^2\nu^2$ (2) $\frac{1}{4}\pi^2 ma^2\nu^2$

(3) $4\pi^2 ma^2\nu^2$ (4) $2\pi^2 ma^2\nu^2$

Sol. **(1)**

$$\frac{1}{4}ma^2\omega^2 = \pi^2 f^2 ma^2$$

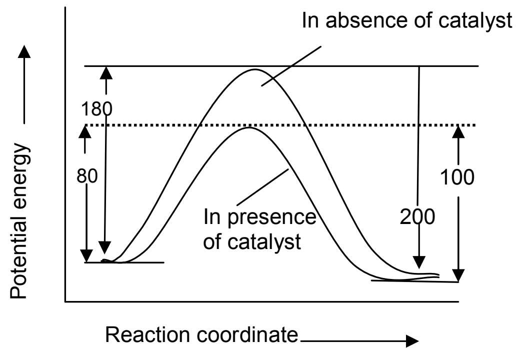
****

81. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction $\left(A_2 + B_2 \longrightarrow 2AB\right)$ in the presence of catalyst will be (in kJ mol^{-1})

(1) 300 (2) 120
(3) 280 (4) 20

Ans. (4)

Sol.



So, $\Delta H_{Reaction} = E_f - E_b$
= 80 – 100 = -20
Hence, (4) is correct.

82. The cell, $Zn\,|\,Zn^{2+}(1M)\,||\,Cu^{2+}(1M)\,|\,Cu\left(E^0_{cell} = 1.10V\right)$, was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left[\frac{[Zn^{2+}]}{[Cu^{2+}]}\right]$ is

(1) antilog (24.08) (2) 37.3
(3) $10^{37.3}$ (4) 9.65×10^4

Ans. (3)

Sol. $E_{cell} = E^o_{cell} - \frac{0.0591}{n}\log Q$

Where Q = $\frac{[Zn^{2+}]}{[Cu^{2+}]}$

For complete discharge E_{cell} = 0

So $E^o_{cell} = \frac{0.591}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$

$\Rightarrow \left|\frac{[Zn^{2+}]}{[Cu^{2+}]}\right| = 10^{37.3}$

Hence, (3) is correct.

83. The pKa of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid is ionized is

(1) 4.5 (2) 2.5
(3) 9.5 (4) 7.0

Ans. (3)

Sol. For buffer solution

pH = pK_a + log $\frac{[Salt]}{[Acid]}$

= 4.5 + log $\frac{[Salt]}{[Acid]}$

as HA is 50% ionized so [Salt] = [Acid]

pH = 4.5
pH + pOH = 14
$\Rightarrow$ pOH = 14 – 4.5 = 9.5
Hence (3) is correct.

84. Consider the reaction,
$2A + B \rightarrow \text{Products}$
When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is
(1) L mol^{-1} s^{-1} (2) no unit
(3) mol L^{-1} s^{-1} (4) s^{-1}

Ans. (1)

Sol. $2A + B \rightarrow \text{Product}$
When conc. of B is doubled, the half life did not change, hence reaction is of first order w.r.t. B.
When concentration of A is doubled, reaction rate is doubled, hence reaction is of first order w.r.t. A.
Hence over all order of reaction is 1 + 1 = 2
So, unit of rate constant mol^{-1} lit s^{-1}.
Hence, (1) is correct.

85. Identify the incorrect statement among the following
(1) d-Block elements show irregular and erratic chemical properties among themselves
(2) La and Lu have partially filled d orbitals and no other partially filled orbitals
(3) The chemistry of various lanthanoids is very similar
(4) 4f and 5f orbitals are equally shielded

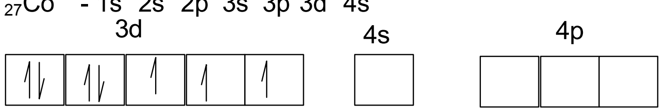
Ans. (4)

Sol. 4f and 5f belongs to different energy levels, hence the shielding effect is on them is not the same.
Shielding of 4f is more than 5f.
Hence (4) is correct.

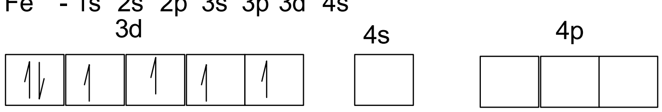
86. Which one of the following has a square planar geometry?
(1) $[CoCl_4]^{2-}$ (2) $[FeCl_4]^{2-}$
(3) $[NiCl_4]^{2-}$ (4) $[PtCl_4]^{2-}$

Ans. (4)

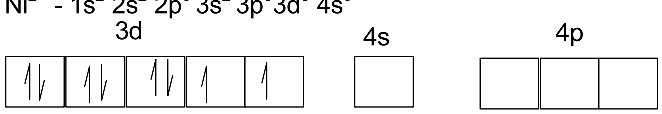
Sol. ${}_{27}Co^{2+}$ - $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6 3d^7\ 4s^0$



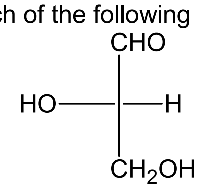
As Cl^- is weak field ligand so no pairing up.
Hence it is sp^3 hybridized giving tetrahedral geometry.
Fe^{2+} - $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6 3d^6\ 4s^0$

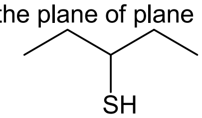


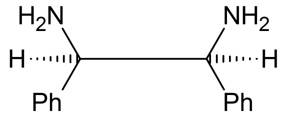
Due to Cl^-, back pairing is not observed so it will be sp^3 hybridized giving tetrahedral geometry.
Ni^{2+} - $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6 3d^8\ 4s^0$

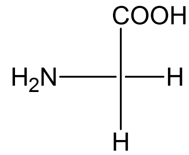


Because weak ligand, back pairing is not observed so it will be sp^3 i.e. tetrahedral geometry.
All the complexes of Pt^{2+} are square planar including those with weak field ligand such as halide ions thus (4) is correct.

87. Which of the following molecules is expected to rotate the plane of plane polarized light?
(1)


(2)


(3) 

(4) 

Ans. (1)

Sol. The plane of polarized light is rotated by optically active compound, i.e. it should be chiral.
So, (1) has, chiral C-atom. So, it is optically active.
In (2), (3) and (4) plane of symmetry is present.
Hence, (1) is correct.

88. The secondary structure of a protein refers to
(1) α-helical backbone
(2) hydrophobic interactions
(3) sequence of α-amino acids
(4) fixed configuration of the polypeptide backbone

Ans. (1)

Sol. Secondary structure of proteins involves $\alpha-$helical back bond and $\beta-$sheet structures. These structures are formed as a result of H-bonding between different peptide groups.
Hence, (1) is correct

89. Which of the following reactions will yield 2, 2-dibromopropane?
(1) $CH_3-C\equiv CH+2HBr\longrightarrow$
(2) $CH_3CH\equiv CHBr+HBr\longrightarrow$
(3) $CH\equiv CH+2HBr\longrightarrow$
(4) $CH_3-CH=CH_2+HBr\longrightarrow$

Ans. (1)

Sol. $CH_3-C\equiv CH+HBr\xrightarrow[\text{(from HBr)}]{\text{electrophilic addition of } H^+}CH_3-\underset{\underset{Br}{|}}{C}=CH_2\xrightarrow{HBr}CH_3-\overset{\overset{Br}{|}}{\underset{\underset{Br}{|}}{C}}-CH_3$

Hence, (1) is correct.

90. In the chemical reaction,
$CH_3CH_2NH_2-CHCl_3+3KOH\longrightarrow(A)+(B)+3H_2O$, the compound (A) and (B) are respectively
(1) C_2H_5CN and 3KCl
(2) $CH_3CH_2CONH_2$ and 3KCl
(3) C_2H_5NC and K_2CO_3
(4) C_2H_5NC and 3KCl

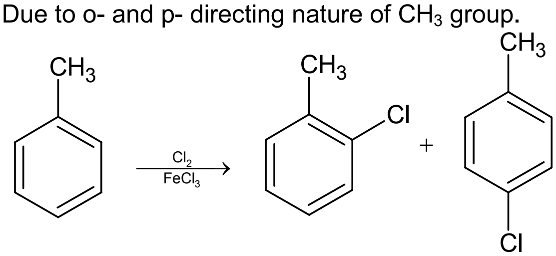
Ans. (4)

Sol. It is example of carbylamine reaction. so, the product will be C_2H_5NC and KCl.
Hence, (4) is the correct answer.

91. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly
(1) benzoyl chloride
(2) benzyl chloride
(3) o-and p-chlorotoluene
(4) m-chlorotoluene

Ans. (3)

Sol. Due to o- and p- directing nature of CH_3 group.



Hence, (3) is correct answer.

92. Presence of a nitro group in a benzene ring
(1) activates the ring towards electrophilic substitution
(2) renders the ring basic
(3) deactivates the ring towards nucleophilic substitution
(4) deactivates the ring towards electrophilic substitution

Ans. (4)

Sol. - NO_2 group shows – M effect, so withdraws the electron density from the ring and hence deactivate the ring towards electrophilic aromatic substitution.
Hence, (4) is correct.

93. In which of the following ionization processes, the bond order has increased and the magnetic behaviour has changed?

(1) $C_2 \longrightarrow C_2^+$ (2) $NO \longrightarrow NO^+$

(3) $O_2 \longrightarrow O_2^+$ (4) $N_2 \longrightarrow N_2^+$

Ans. (2)

Sol. In $C_2 - C_2^+$ electron is removed from bonding molecular orbital so bond order decreases. In NO $\longrightarrow$ NO^+, electron is removed from anti bonding molecular orbital so bond order increases and nature changes from paramagnetic to diamagnetic.

Hence, (2) is correct.

94. The actinoids exhibits more number of oxidation states in general than the lanthanoids. This is because

(1) the 5f orbitals are more buried than the 4f orbitals
(2) there is a similarity between 4f and 5f orbitals in their angular part of the wave function
(3) the actinoids are more reactive than the lanthanoids
(4) the 5f orbitals extend further from the nucleus than the 4f orbitals

Ans. (4)

Sol. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because the 5f orbitals extend further from the nucleus than the 4f orbitals.

Hence, (4) is correct.

95. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is

(1) $\frac{2}{3}$ (2) $\frac{1}{3}\times\frac{273}{298}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. Let the mass of methane and oxygen is w

mole fraction of oxygen = $\dfrac{\frac{w}{32}}{\frac{w}{32}+\frac{w}{16}}$

$= \dfrac{\frac{1}{32}}{\frac{1}{32}+\frac{1}{16}} = \dfrac{\frac{1}{32}}{\frac{3}{32}} = \frac{1}{3}$

Let the total pressure be P

The pressure exerted by oxygen (partial pressure) = $X_{O_2} \times P_{total}$

$\Rightarrow P \times \frac{1}{3}$

Hence, (3) is correct.

96. A 5.25 % solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be

(1) 90.0 g mol^{-1} (2) 115.0 g mol^{-1}
(3) 105.0 g mol^{-1} (4) 210.0 g mol^{-1}

Ans. (4)

Sol. Solutions with the same osmotic pressure are isotonic

Let the molar mass of the substance be M

$\pi_1 = C_1RT = C_2RT = \pi_2$

So, $C_1 = C_2$

As density of the solutions are same

So $\frac{5.25}{M} = \frac{15}{60}$

$M = \frac{5.25 \times 60}{1.5} = 210$

Hence (4) is correct

97. Assuming that water vapour is an ideal gas, the internal energy (ΔU) when 1 mol of water is vapourised at 1 bar pressure and 100°C, (Given: Molar enthalpy of vapourization of water at 1 bar and 373 K = 41 kJ mol^{-1} and R = 8.3 J $mol^{-1}K^{-1}$) will be

(1) 4.100 kJ mol^{-1}
(2) 3.7904 kJ mol^{-1}
(3) 37.904 kJ mol^{-1}
(4) 41. 00 kJ mol^{-1}

Ans. (3)

Sol. $H_2O(\ell) \xrightarrow{\text{vaporisation}} H_2O(g)$

$\Delta n_g = 1 - 0 = 1$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta U = \Delta H - \Delta n_g RT$

$= 41 - 8.3 \times 10^{-3} \times 373$

$= 37.9$ kJ mol^{-1}

Hence, (3) is correct.

98. In a sautrated solution of the sparingly soluble strong electrolyte $AgIO_3$ (Molecular mass = 283) the equilibrium which sets in is

$AgIO_{3(s)} \rightleftharpoons Ag^+_{(aq)} + IO^-_{3(aq)}$

If the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 ml of its saturated solution?

(1) 28.3×10^{-2} g
(2) 2.83×10^{-3} g
(3) 1.0×10^{-7} g
(4) 1.0×10^{-4} g

Ans. (2)

Sol. $AgIO_3(s) \rightleftharpoons Ag^+(aq) + IO_3^-(aq)$

Let the solubility of $AgIO_3$ be s

$K_{sp} = [Ag^+][IO_3^-]$

$1.0 \times 10^{-8} = s^2$

$s = 10^{-4}$ mol/litre

$= \frac{10^{-4} \times 283}{1000} \times 100$

$= 283 \times 10^{-5}$

$= 2.83 \times 10^{-3}$ g/ 100 ml

Hence, (2) is correct.

99. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial activity is ten times the permissible value, after how many days will it be safe to enter the room?

(1) 1000 days
(2) 300 days
(3) 10 days
(4) 100 days

Ans. (4)

Sol. Activity $\left(-\frac{dN}{dt}\right) \propto N$

$N = N_o \left(\frac{1}{2}\right)^n$

$\frac{N}{N_o} = \left(\frac{1}{2}\right)^n$

$\frac{1}{10} = \left(\frac{1}{2}\right)^n \Rightarrow 10 = 2^n$

log10 = nlog2

$\Rightarrow n = \frac{1}{0.301} = 3.32$

$t = n \times t_{112}$

= 3.32 × 30 = 99.6 days
Hence, (4) is correct.

100. Which one of the following conformation of cyclohexane is chiral?
(1) Twist boat (2) Rigid
(3) Chair (4) Boat

Ans. (1)
Sol. Twisted boat is chiral as it does not have plane of symmetry.
Hence, (1) is correct.

101. Which of the following is the correct order of decreasing SN^2 reactivity?
(1) $RCH_2X > R_3CX > R_2CHX$ (2) $RCH_2X > R_2CHX > R_3CX$
(3) $R_3CX > R_2CHX > RCH_2X$ (4) $R_2CHX > R_3CX > RCH_2X$
(X = a halogen)

Ans. (2)
Sol. More is the steric hindrance at the carbon bearing the halogen, lesser is the S_N2 reactivity.
Hence, (2) is correct.

102. In the following sequence of reactions,
$$CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$
the compound 'D' is
(1) butanal (2) n-butyl alcohol
(3) n-propyl alcohol (4) propanal

Ans. (3)
Sol.
$$CH_3CH_2OH \xrightarrow{P+I_2} \underset{(A)}{CH_3CH_2I} \xrightarrow[\text{dry ether}]{Mg} \underset{(B)}{CH_3CH_2MgI}$$
$$\xrightarrow{H-\underset{H}{\overset{|}{C}}=O} \underset{(C)}{CH_3-CH_2-CH_2OMgI} \xrightarrow{H_2O} \underset{(D)}{CH_3CH_2CH_2OH} + Mg(OH)I$$
∴ the compound D is n-propyl alcohol.
Hence, (3) is correct option.

103. Which of the following sets of quantum numbers represents the highest energy of an atom?
(1) $n = 3, l = 2, m = 1, s = +1/2$ (2) $n = 3, l = 2, m = 1, s = +1/2$
(3) $n = 4, l = 0, m = 0, s = +1/2$ (4) $n = 3, l = 0, m = 0, s = +1/2$

Ans. (2)
Sol. (2) is the correct option because it has the maximum value of $n + \ell$
Hence, (2) is correct.

104. Which of the following hydrogen bonds is the strongest?
(1) O–H…….N (2) F–H…….F
(3) O–H…….O (4) O–H…….F

Ans. (2)
Sol. The hydrogen bond in HF is strongest, because fluorine is the most electronegative element.
Thus, (2) is the correct option.

105. In the reaction. $2Al_{(s)} + 6HCl_{(s)} \longrightarrow 2Al^{3+}_{(aq)} + 6Cl^-_{(aq)} + 3H_{2(g)}$,
(1) 6 L $HCl_{(aq)}$ is consumed for every 3L $H_{2(g)}$ produced
(2) 33.6 L $H_{2(g)}$ is produced regardless of temperature and pressure for every mole Al that reacts
(3) 67.2 L $H_{2(g)}$ at STP is produced for every mole Al that reacts
(4) 11.2 $H_{2(g)}$ at STP is produced for every mole $HCl_{(aq)}$ consumed

Ans. (4)
Sol. $2Al(s) + 6HCl(aq) \longrightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$
For each mole of HCl reacted, 0.5 mole of H_2 gas is formed at STP.
1 mole of an ideal gas occupies 22.4 lit at STP.
Volume of H_2 gas formed at STP per mole of HCl reacted is 22.4 × 0.5 litre
Hence, (4) is correct.

106. Regular use of which of the following fertilizer increases the acidity of soil?
(1) Potassium nitrate (2) Urea
(3) Superphosphate of lime (4) Ammonium sulphate

Ans. (4)

Sol. $(NH_4)_2\,SO_4$ is a salt of strong acid and weak base, on hydrolysis it ill produce H^+ ion. This will increase the acidity of soil.

$$(NH_4)_2\,SO_4 \longrightarrow 2NH_4^+ + SO_4^{2-}$$

$$NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$$

Hence, (4) is correct answer.

107. Identify the correct statement regarding a spontaneous process
(1) For a spontaneous process in an isolated system, the change in entropy is positive
(2) Endothermic processes are never spontaneous
(3) Exothermic processes are always spontaneous
(4) Lowering of energy in the reaction process is the only criterion for spontaneity

Ans. (1)

Sol. For a spontaneous process in an isolated system, the change in entropy is positive.
Hence, (1) is correct.

108. Which of the following nuclear reactions will generate an isotope?
(1) neutron particle emission
(2) positron emission
(3) α-particle emission
(4) β-particle emission

Ans. (1)

Sol. $${}_{Z}^{A}X \longrightarrow {}_{Z}^{A-1}X + {}_{0}^{1}n$$

Hence, (1) is correct.

109. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below:

$$\wedge^{\circ}_{CH_3COONa} = 91.0 \text{ S cm}^2/\text{equiv}$$

$$\wedge^{\circ}_{HCl} = 426.2 \text{ S cm}^2/\text{equiv}$$

What additional information/quantity one needs to calculate $\wedge^{\circ}$ of an aqueous solution of acetic acid?
(1) $\wedge^{\circ}$ of NaCl
(2) $\wedge^{\circ}$ of CH_3COOK
(3) The limiting equivalent conductance of $H^+\left(\wedge^{\circ}_{H^+}\right)$
(4) $\wedge^{\circ}$ of chloroacetic acid (C/CH_2COOH)

Ans. (1)

Sol. From Kohlrausch's law

$$\Lambda^{\circ}_{CH_3COOH} = \Lambda^{\circ}_{CH_3COONa} + \Lambda^{\circ}_{HCl} - \Lambda^{\circ}_{NaCl}$$

Hence, (1) is the correct answer.

110. Which one of the following is the strongest base in aqueous solution?
(1) Trimethylamine
(2) Aniline
(3) Dimethylamine
(4) Methylamine

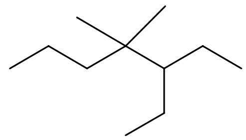
Ans. (3)

Sol. In aqueous solution basicity order of 1^o, 2^o and 3^o amine with methyl group is
$2^o > 1^o > 3^o$
In case of aniline lone pair of nitrogen is involved in resonance, so it is weaker base than aliphatic amines.
Hence, (3) is correct.

111. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(1) benzophenone
(2) acetophenone
(3) benzoic acid
(4) benzyl alcohol

Ans. (3)

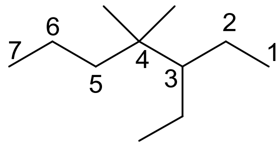
Sol. Any aliphatic carbon with hydrogen attached to it, in combination with benzene ring, will be oxidized to benzoic acid by $KMnO_4/H^+$.
Hence, (3) is correct.

112. The IUPAC name of  is

(1) 1, 1-diethyl-2,2-dimethylpentane
(2) 4, 4-dimethyl-5, 5-diethylpentane
(3) 5, 5-diethyl-4, 4-diemthylpentane
(4) 3-ethyl-4, 4-dimethylheptane

Ans. (4)

Sol.



The correct answer is 3-ethyl-4, 4-dimethylheptane.
Hence, (4) is correct.

113. Which of the following species exhibits the diamagnetic behaviour?

(1) O_2^{2-}
(2) O_2^{+}
(3) O_2
(4) NO

Ans. (1)

Sol. The correct option is O_2^{2-}

This species has 18 e^-, which are filled in such a way that all molecular orbitals are fully filled, so diamagnetic.

$\sigma 1s^2 \sigma^* 1s^2, \sigma 2s^2 \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$

Hence, (1) is correct.

114. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence

(1) $GeX_2 \ll SiX_2 \ll SnX_2 \ll PbX_2$
(2) $SiX_2 \ll GeX_2 \ll PbX_2 \ll SnX_2$
(3) $SiX_2 \ll GeX_2 \ll SnX_2 \ll PbX_2$
(4) $PbX_2 \ll SnX_2 \ll GeX_2 \ll SiX_2$

Ans. (3)

Sol. Due to inert pair effect, the stability of +2 oxidation state increases as we move down this group.

$\therefore SiX_2 \ll GeX_2 \ll SnX_2 \ll PbX_2$

Hence, (3) is correct.

115. Identify the incorrect statement among the following

(1) Ozone reacts with SO_2 to give SO_3
(2) Silicon reacts with $NaOH_{(aq)}$ in the presence of air to give Na_2SiO_3 and H_2O
(3) Cl_2 reacts with excess of NH_3 to give N_2 and HCl
(4) Br_2 reacts with hot and strong NaOH solution to give NaBr, $NaBrO_4$ and H_2O

Ans. (4)

Sol. Br_2 reacts with hot and strong NaOH to give NaBr, $NaBrO_3$ and H_2O.
Hence, (4) is incorrect statement.

116. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizinig order of the polarizing power of the cationic species, K^+, Ca^{2+}, Mg^{2+}, Be^{2+}?

(1) $Mg^{2+}, Be^{2+}, K^+, Ca^{2+}$
(2) $Be^{2+}, K^+, Ca^{2+}, Mg^{2+}$
(3) $K^+, Ca^{2+}, Mg^{2+}, Be^{2+}$
(4) $Ca^{2+}, Mg^{2+}, Be^{2+}, K^+$

Ans. (3)

Sol. Higher the charge/size ratio, more is the polarizing power.

$\therefore K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

Hence, (3) is correct.

117. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (Molar mass = 98 g mol^{-1}) by mass will be

(1) 1.64
(2) 1.88
(3) 1.22
(4) 1.45

Ans. (3)

Sol. Let the density of solution be 'd'
Molarity of solution given = 3.6
i.e. 1 litre of solution contains 3.6 moles of H_2SO_4
or 1 litre of solution contains 3.6 × 98 gms of H_2SO_4
Since, the solution is 29% by mass.

100 gm solution contains 29 gm H_2SO_4

$\frac{100}{d}$ ml solution contains 29 gm of H_2SO_4

1000 ml solution contains 3.6 × 98 gm of H_2SO_4

$\therefore 3.6 \times 98 = \frac{29 \times d}{100} \times 1000$

d = 1.22

Hence, (3) is correct.

118. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be

(1) 5.0×10^{-5} (2) 5.0×10^{15}

(3) 5.0×10^{-15} (4) 0.0×10^{5}

Ans. (3)

Sol. $H_2A \rightleftharpoons HA^- + H^+$ $\quad K_1 = \frac{[HA^-][H^+]}{[H_2A]}$ ...(1)

$HA^- \rightleftharpoons H^+ + A^{2-}$ $\quad K_2 = \frac{[H^+][A^{2-}]}{[HA^-]}$ ...(2)

For the reaction

$H_2A \rightleftharpoons 2H^+ + A^{2-}$

$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} =$ $K_1 \times K_2$

$= 1 \times 10^{-5} \times 5 \times 10^{-10}$

$= 5 \times 10^{-15}$

Hence, (3) is correct.

119. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 mm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be

(1) 350 (2) 300

(3) 700 (4) 360

Ans. (1)

Sol. Let the vapour pressure of pure ethyl alcohol be P,

According to Raoult's law

290 = 200 × 0.4 + P × 0.6

P = $\frac{290 - 80}{0.6} = 350$ mm Hg

Hence, (1) is correct.

120. In conversion of lime-stone to lime,

$CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$

the vales of ΔH° and ΔS° are +179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that ΔH° do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is

(1) 1008 K (2) 1200

(3) 845 K (4) 1118 K

Ans. (4)

Sol. We know, $\Delta G = \Delta H - T\Delta S$

So, lets find the equilibrium temperature, i.e. at which

$\Delta G = 0$

$\Delta H = T\Delta S$

$T = \frac{179.1 \times 1000}{160.2}$

= 1118 K

So, at temperature above this, the reaction will become spontaneous.

Hence, (4) is correct answer.