FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. if the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :

(1) 0.4 ms^{-1} (2) 0.1 ms^{-1}

(3) 0.3 ms^{-1} (4) 0.7 ms^{-1}

Sol. m = 20 g , u = 1 m/s, v = ?

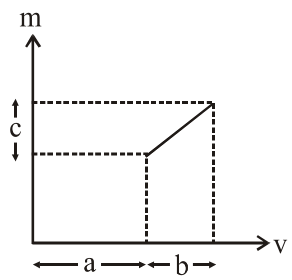
$S = 20 \times 10^{-2}$ m $\qquad a = \dfrac{-2.5\times10^{-2}}{20\times10^{-3}} m/s^2$

$v^2 = u^2 + 2as$

$$v^2 = 1 - 2\times\frac{2.5\times10^{-2}}{20\times10^{-3}}\times\frac{20}{100}$$

$$v = \frac{1}{\sqrt{2}} \simeq 0.7 \text{ m/s}$$

2. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used ?



(1) $\dfrac{b^2c}{a}$ (2) $\dfrac{b^2}{ac}$ (3) $\dfrac{a}{c}$ (4) $\dfrac{b}{c}$

Sol. $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$$1-\frac{v}{u}=\frac{v}{f}$$

$$1 - m = \frac{v}{f}$$

$$m = 1 - \frac{v}{f}$$

At v = a, $m_1 = 1 - \dfrac{a}{f}$

At v = a + b, $m_2 = 1 - \dfrac{a+b}{f}$

$$m_2 - m_1 = c = \left[1-\frac{a+b}{f}\right]-\left[1-\frac{a}{f}\right]$$

$$c = \frac{b}{f}$$

$$f = \frac{b}{c}$$

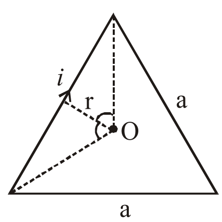
3. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1m which is carrying a current of 10 A is : [Take $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}]

(1) 18 μT (2) 3 μT

(3) 1 μT (4) 9 μT

Sol. $B = 3\left[\dfrac{\mu_0 i}{4\pi r}(\sin60^\circ+\sin60^\circ)\right]$

Here, $r = \dfrac{a}{2\sqrt{3}}=\dfrac{1}{2\sqrt{3}}$



$$B = 3\left[\frac{4\pi\times10^{-7}\times10\times2\sqrt{3}}{4\pi\times1}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right]\right]$$

$B = 18 \times 10^{-6} = 18$ μT

4. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. ,Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2})

(1) 500 m (2) 400 m

(3) 300 m (4) 600 m

Sol. $P_0 + \rho g d_1 = P_1$

$P_0 + \rho g d_2 = P_2$

$\rho g(d_2 - d_1) = P_2 - P_1$

$10^3 \times 10\ (d_2 - d_1) = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

$\simeq 300$ m

5. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be :

(1) $\frac{3m}{\pi}$ (2) $\frac{4m}{\pi}$

(3) $\frac{2m}{\pi}$ (4) $\frac{m}{\pi}$

Sol. $m = NIA = 1 \times I \times a^2$

here a = side of square

Now,

$4a = 2\pi r$

$r = \frac{2a}{\pi}$

For circular loop

$m' = 1 \times I \times \pi r^2$

$= 1 \times I \times \pi \times \left(\frac{2a}{\pi}\right)^2$

$m' = \frac{4m}{\pi}$

6. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit ?

(1) 1.16 mm (2) 0.90 mm

(3) 1.36 mm (4) 1.00 mm

Sol. $\frac{F}{A} = \text{stress}$

$\frac{400 \times 4}{\pi d^2} = 379 \times 10^6$

$d^2 = \frac{1600}{\pi \times 379 \times 10^6} = 1.34 \times 10^{-6}$

$d = \sqrt{1.34} \times 10^{-3} = 1.15 \times 10^{-3}$ m

7. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ?

[Given : Mass of planet = 8×10^{22} kg ;

Radius of planet = 2×10^6 m,

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2]

(1) 9 (2) 11

(3) 13 (4) 17

Sol. $F_g = \frac{mv^2}{r}$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$V = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(8 \times 10^{22})}{2.02 \times 10^6}}$

$V = 1.625 \times 10^3$

$T = \frac{2\pi r}{V}$

$n \times T = 24 \times 60 \times 60$

$n\left[\frac{2\pi(2.02 \times 10^6)}{1.625 \times 10^3}\right] = 24 \times 3600$

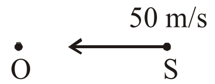
$n = \frac{24 \times 3600 \times 1.625 \times 10^3}{2\pi(2.02 \times 10^6)}$

$n = 11$

8. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? (Take velocity of sound in air is 350 m/s)

(1) 857 Hz (2) 807 Hz

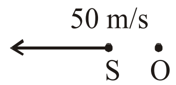
(3) 750 Hz (4) 1143 Hz

Sol. 

$$f_{app} = \left(\frac{V-0}{V-50}\right) f_{source}$$

$$1000 = \left(\frac{350}{300}\right) f_{source}$$

$$f_{source} = \frac{1000 \times 300}{350}$$

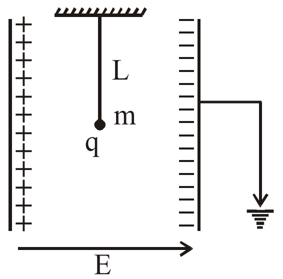


$$f_{app} = \left(\frac{V}{V+50}\right) . f_{source}$$

$$= \frac{350}{400} \times 1000 \times \frac{300}{350}$$

= 750 Hz

9. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by :



(1) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (2) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$

(3) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ (4) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

Sol. $g_{eff} = \sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}$

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

$$= 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$$

10. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. if the surface has an area of 25 cm^{-2}, the momentum transferred to the surface in 40 min time duration will be :

(1) 5.0×10^{-3} Ns (2) 3.5×10^{-6} Ns
(3) 1.4×10^{-6} Ns (4) 6.3×10^{-4} Ns

Sol. Pressure = $\dfrac{I}{C}$

Force = Pressure × Area = $\dfrac{I}{C}$. Area

Momentum transferred = Force . Δt

$$= \frac{I}{C} . \text{Area} . \Delta t$$

$$= \frac{25 \times 10^4}{3 \times 10^8} \times 25 \times 10^{-4} \times 40 \times 60$$

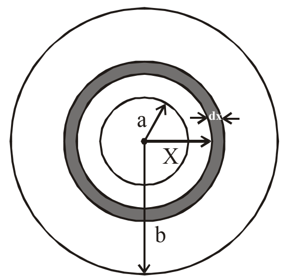
= 5×10^{-3} N-s

11. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be :

(1) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$ (2) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$

(3) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ (4) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$

Sol. $dR = \rho . \dfrac{dx}{4\pi x^2}$



$$\int dR = \rho . \int_a^b \frac{dx}{4\pi x^2}$$

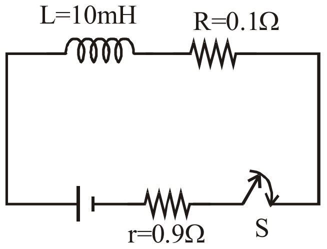
$$R = \frac{\rho}{4\pi}\left[-\frac{1}{x}\right]_a^b$$

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$

12. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is : (Take *l*n5 = 1.6)

(1) 0.103 s (2) 0.016 s
(3) 0.002 s (4) 0.324 s

Sol.



$i = i_0\,(1 - e^{-t/\tau})$

$\frac{80}{100} i_0 = i_0(1 - e^{-t/\tau})$

$0.8 = 1 - e^{-t/\tau}$

$e^{-t/\tau} = 0.2 = \frac{1}{5}$

$-\frac{t}{\tau} = \ln\left(\frac{1}{5}\right)$

$-\frac{t}{\tau} = -\ln(5)$

$t = \tau.\ln(5)$

$= \frac{L}{R_{eq}}.\ln(5)$

$= \frac{10 \times 10^{-3}}{(0.1 + 0.9)} \times 1.6$

$t = 1.6 \times 10^{-2}$

$t = 0.016$ s

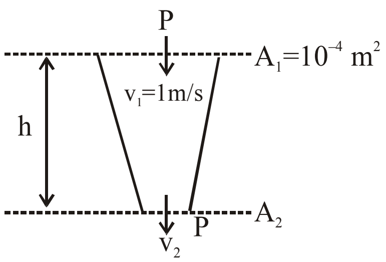
13. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be :

(Take g = 10 ms^{-2})

(1) 1×10^{-5} m^2 (2) 5×10^{-5} m^2

(3) 2×10^{-5} m^2 (4) 5×10^{-4} m^2

Sol.



$A_1v_1 = A_2v_2$

$10^{-4} \times 1 = A_2v_2$

$A_2v_2 = 10^{-4}$ ....(1)

$P + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho gh = P$

$v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{v_1^2 + 2gh}$

$= \sqrt{1 + 2 \times 10 \times 0.15}$

$\frac{10^{-4}}{A_2} = 2$

$A_2 = 5 \times 10^{-5}$ m^2

14. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ?

(1) $[M^{-2}\,L^{-2}\,T^{6}\,A^{3}]$ (2) $[M^{-1}\,L^{-2}\,T^{4}\,A^{2}]$

(3) $[M^{-3}\,L^{-2}\,T^{8}\,A^{4}]$ (4) $[M^{-2}\,L^{0}\,T^{-4}\,A^{-2}]$

Sol. $X = 5\,YZ^2$

$Y = \frac{X}{5Z^2}$

$[Y] = \frac{[X]}{[Z^2]}$

$= \frac{A^2.M^{-1}L^{-2}.T^4}{(MA^{-1}T^{-2})^2}$

$= M^{-3}\,.\,L^{-2}\,.\,T^{8}\,.\,A^{4}$

15. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is :

[Given Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m/s]

(1) 2×10^{16} (2) 1.5×10^{16}

(3) 5×10^{15} (4) 1×10^{16}

Sol. $P = \frac{n.\,hc}{\lambda}$

$n = \frac{P\lambda}{h.c}$

$= \frac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}}$

$= 1.5 \times 10^{16}$

16. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is :

(1) $\frac{7}{5}Q$ (2) $\frac{3}{2}Q$ (3) $\frac{5}{3}Q$ (4) $\frac{2}{3}Q$

Sol. $Q = nC_v \Delta T$

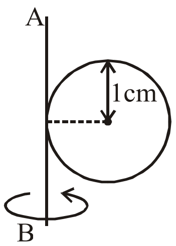
$Q' = nC_p \Delta T$

$\therefore \quad \frac{Q'}{Q} = \frac{C_p}{C_v}$

For diatomic gas : $\frac{C_p}{C_v} = \gamma = \frac{7}{5}$

$Q' = \frac{7}{5}Q$

17. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to :



(1) 4.0×10^{-6} Nm (2) 2.0×10^{-5} Nm

(3) 1.6×10^{-5} Nm (4) 7.9×10^{-6} Nm

Sol. $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{25 \times 2\pi}{5} = 10\pi \text{ rad/sec}^2$

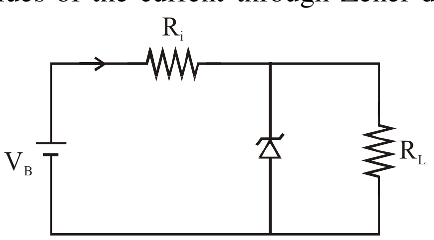
$\tau = \left(\frac{5}{4}MR^2\right)\alpha$

$= \frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 10\pi$

$= 1.9625 \times 10^{-5}$ Nm

$\simeq 2.0 \times 10^{-5}$ Nm

18. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6V and the load resistance is $R_L = 4\ k\Omega$. The series resistance of the circuit is $R_i = 1\ k\Omega$. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode ?



(1) 0.5 mA ; 6 mA (2) 0.5 mA ; 8.5 mA

(3) 1.5 mA ; 8.5 mA (4) 1 mA ; 8.5 mA

Sol. At $V_B = 8V$

$i_L = \frac{6 \times 10^{-3}}{4} = 1.5 \times 10^{-3}$ A

$i_R = \frac{8 - 6 \times 10^{-3}}{1} = 2 \times 10^{-3}$ A

$\therefore \quad i_{\text{zener diode}} = i_R - i_{\text{load}}$

$= 0.5 \times 10^{-3}$ A

At $V_B = 16$ V

$i_L = 1.5 \times 10^{-3}$ A

$i_R = \frac{(16-6) \times 10^{-3}}{1} = 10 \times 10^{-3}$ A

$\therefore \quad i_{\text{zener diode}} = i_R - i_L$

$= 8.5 \times 10^{-3}$ A

19. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. when the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ?

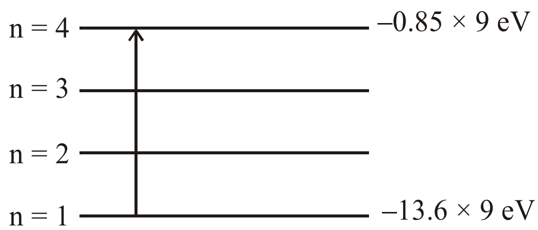
(Given : $h = 6.63 \times 10^{-34}$ Js ;

$c = 3 \times 10^8\ ms^{-1}$)

(1) 9.4 nm (2) 12.3 nm

(3) 10.8 nm (4) 11.4 nm

Sol.



$\Delta E = \frac{hc}{\lambda}$

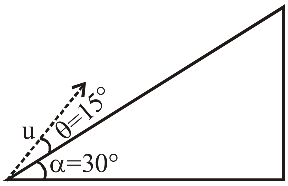
$13.6 \times 9 - 0.85 \times 9 = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{9 \times (13.6 - 0.85)\text{eV}}$

$= \frac{1240 \text{eV.nm}}{9 \times 12.75 \text{eV}}$

$\lambda = 10.8$ nm

20. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :
(Take $g = 10\ ms^{-2}$)



(1) 14 cm (2) 20 cm
(3) 18 cm (4) 26 cm

Sol. $t = \dfrac{2\times 2\times \sin 15°}{g\cos 30°}$

$S = 2\cos 15° \times t - \dfrac{1}{2} g \sin 30° t^2$

Put values and solve

$S \simeq 20cm$

21. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. if B is released, then its velocity at a distance of 9 mm from P is :

$\left[\text{Take } \dfrac{1}{4\pi\varepsilon_0} = 9\times 10^9 Nm^2\ C^{-2}\right]$

(1) 2.0×10^3 m/s
(2) 3.0×10^4 m/s
(3) 1.5×10^2 m/s
(4) 1.0 m/s

Sol. $W_E = -[\Delta U] = U_i - U_F = \dfrac{1}{2}mv^2$

$U = \dfrac{kq_1q_2}{r}$

$\dfrac{(9\times 10^9)\times 10^{-12}}{10^{-3}} - \dfrac{(9\times 10^9)\times 10^{-12}}{9\times 10^{-3}} = \dfrac{1}{2}\times(4\times 10^{-6})v^2$

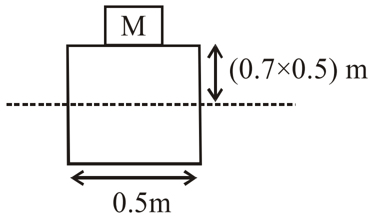
$v^2 = 4 \times 10^6$

$v = 2 \times 10^3$ m/s

22. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? (Take density of water = $10^3\ kg/m^3$)

(1) 65.4 kg
(2) 87.5 kg
(3) 30.1 kg
(4) 46.3 kg

Sol.



$M = \rho_L [0.5 \times 0.5 \times 0.35]$

$= 10^3 [0.0875]$

$M = 87.5$ kg

23. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

(1) $36\hat{k}$

(2) $-34(\hat{k}-\hat{i})$

(3) $48(\hat{i}+\hat{j})$

(4) $-48\hat{k}$

Sol. $\vec{L} = m[\vec{r}\times\vec{v}]$

m = 2 kg

$\vec{r} = 2t\,\hat{i} - 3t^2\hat{j}$

$= 4\,\hat{i} - 12\,\hat{j}$ (At t = 2 sec)

$\vec{v} = \dfrac{d\vec{r}}{dt} = 2\hat{i} - 6t\,\hat{j} = 2\hat{i} - 12\hat{j}$

$\vec{r}\times\vec{v} = (4\hat{i} - 12\hat{j})\times(2\hat{i} - 12\hat{j})$

$= -24\,\hat{k}$

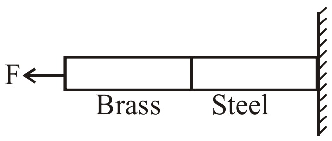
$\vec{L} = m(\vec{r}\times\vec{v})$

$= -48\hat{k}$

24. In an experiment, bras and steel wires of length 1m each with areas of cross section 1 mm² are used. teh wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is :
(Given, the Young's Modulus for steel and brass are respectively, 120×10^9 N/m² and 60×10^9 N/m²)

(1) 0.2×10^6 N/m² (2) 4.0×10^6 N/m²
(3) 1.8×10^6 N/m² (4) 1.2×10^6 N/m²

Sol.



$k_1 = \dfrac{y_1A_1}{\ell_1} = \dfrac{120\times10^9\times A}{1}$

$k_2 = \dfrac{y_2A_2}{\ell_2} = \dfrac{60\times10^9\times A}{1}$

$k_{eq} = \dfrac{k_1k_2}{k_1\times k_2} = \dfrac{120\times60}{180}\times10^9\times A$

$k_{eq} = 40 \times 10^9 \times A$

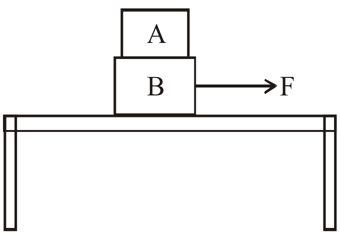
$F = k_{eq}\,(x)$

$F = (40 \times 10^9)A \,.\, (0.2 \times 10^{-3})$

$\dfrac{F}{A} = 8 \times 10^6$ N/m²

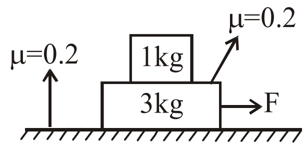
No option is matching. Hence question must be bonus.

25. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is :
(Take g = 10 m/s²)

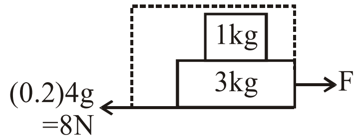


(1) 16 N (2) 40 N (3) 12 N (4) 8 N

Sol.



$a_{Amax} = \mu g = 2$ m/s²



$F - 8 = 4 \times 2$

$F = 16$ N

26. In a Young's doubble slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1) $(\sqrt{3}+1)^4 : 16$ (2) 9 : 1
(3) 4 : 1 (4) 25 : 9

Sol. $I_1 = 4I_0$

$I_2 = I_0$

$I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2$

$= (2\sqrt{I_0}+\sqrt{I_0})^2 = 9I_0$

$I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$

$= (2\sqrt{I_0}-\sqrt{I_0})^2 = I_0$

$\therefore \quad \dfrac{I_{max}}{I_{min}} = \dfrac{9}{1}$

27. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\dfrac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by :

(1) 185 (2) 65 (3) 285 (4) 140

Sol. $I_1 = \dfrac{\left(\frac{7M}{8}\right)(2R)^2}{2} = \left(\frac{7}{16}\times 4\right)MR^2 = \frac{7}{4}MR^2$

$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)R_1^2 = \frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4} = \frac{MR^2}{80}$

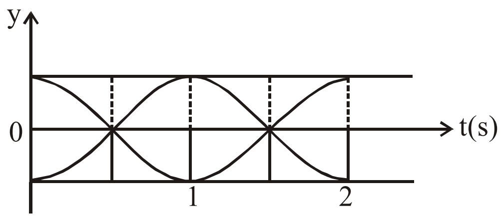
$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi R_1^3\right)$

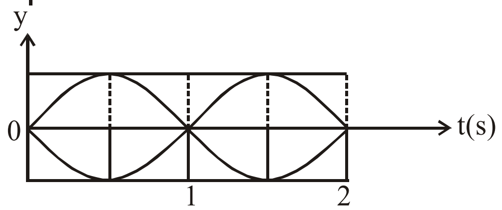
$R^3 = 8R_1^3$

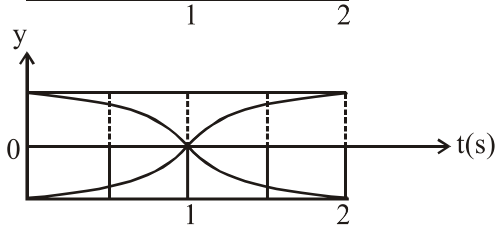
$R = 2R_1$

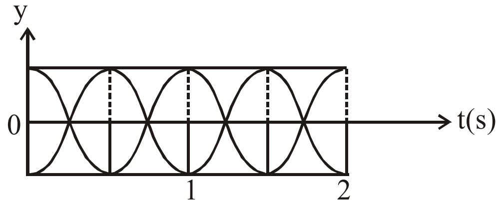
$\therefore \frac{I_1}{I_2} = \dfrac{7/4\, MR^2}{\frac{MR^2}{80}} = \frac{7}{4}\times 80 = 140$

28. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is :

(1) 

(2) 

(3) 

(4) 

Sol. $f_{beat} = 11 - 9 = 2$ Hz

$\therefore$ Time period of oscillation of amplitude

$= \frac{1}{f_{beat}} = \frac{1}{2}$ Hz

Although the graph of oscillation is not given, the equation of envelope is given by option (4)

29. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be :

(1) $1/4\lambda$ (2) $1/\lambda$

(3) $1/2\lambda$ (4) $2/\lambda$

Sol. $N_A = N_0 e^{-5\lambda t}$

$N_B = N_0 e^{-\lambda t}$

$\frac{N_A}{N_B} = \frac{e^{-5\lambda t}}{e^{-\lambda t}} = \frac{1}{e^2}$

$\Rightarrow e^{-4\lambda t} = e^{-2}$

$\Rightarrow 4\lambda t = 2$

$\Rightarrow t = \frac{1}{2\lambda}$

30. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_o\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_o and V_o are constants. Calculate the change in the temperature of the gas if its volume changes from V_o to $2V_o$.

(1) $\frac{1}{2}\frac{P_oV_o}{R}$ (2) $\frac{3}{4}\frac{P_oV_o}{R}$

(3) $\frac{5}{4}\frac{P_oV_o}{R}$ (4) $\frac{1}{4}\frac{P_oV_o}{R}$

Sol. $P = P_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$

Pressure at V_0 = $P_0\left(1-\frac{1}{2}\right) = \frac{P_0}{2}$

Pressure at $2V_0$ = $P_0\left(1-\frac{1}{2}\times\frac{1}{4}\right) = \frac{7}{8}P_0$

Temperature at V_0 = $\dfrac{\frac{P_0}{2}V_0}{nR} = \frac{P_0V_0}{2nR}$

Temperature at $2V_0$ = $\dfrac{\left(\frac{7}{8}P_0\right)(2V_0)}{nR} = \dfrac{\frac{7}{4}P_0V_0}{nR}$

Change in temperature = $\left(\frac{7}{4}-\frac{1}{2}\right)\frac{P_0V_0}{nR}$

$= \frac{5}{4}\frac{P_0V_0}{nR} = \frac{5P_0V_0}{4R}$

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. The correct match between Item-I and Item-II is:

	Item-I		Item-II
(a)	High density polythene	(I)	Peroxide catalyst
(b)	Polyacrylonitrile	(II)	Condensation at high temperature & pressure
(c)	Novolac	(III)	Ziegler-Natta Catalyst
(d)	Nylon 6	(IV)	Acid or base catalyst

(1) (a)→(III), (b)→(I), (c)→(II), (d)→(IV)

(2) (a)→(IV), (b)→(II), (c)→(I), (d)→(III)

(3) (a)→(II), (b)→(IV), (c)→(I), (d)→(III)

(4) (a)→(III), (b)→(I), (c)→(IV), (d)→(II)

Sol.

(a)	High density polythene	(III)	Ziegler-Natta Catalyst
(b)	Polyacrylonitrile	(I)	Peroxide catalyst
(c)	Novolac	(IV)	Acid or base catalyst
(d)	Nylon 6	(II)	Condensation at high temperature & pressure

2. Which of the following is NOT a correct method of the preparation of benzylamine from cyanobenzene ?

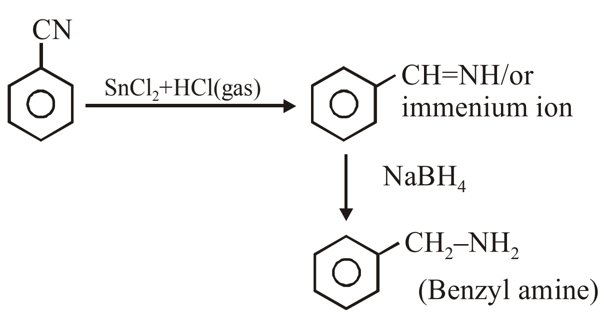
(1) (i) HCl/H_2O (ii) $NaBH_4$

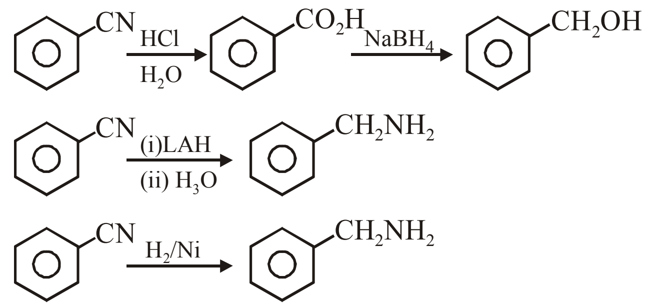
(2) (i) $LiAlH_4$ (ii) H_3O^+

(3) (i) $SnCl_2+HCl(gas)$ (ii) $NaBH_4$

(4) H_2/Ni

Sol.





3. Which of these factors does not govern the stability of a conformation in acyclic compounds ?

(1) Torsional strain

(2) Angle strain

(3) Steric interactions

(4) Electrostatic forces of interaction

Sol. in acyclic compounds angle strain does not govern the stability of a conformation.

4. The difference between ΔH and ΔU (ΔH–ΔU), when the combustion of one mole of heptane (l) is carried out at a temperature T, is equal to:

(1) 3RT (2) –3RT (3) –4RT (4) 4RT

Sol. $C_7H_{16}(\ell) + 11O_2(g) \longrightarrow 7CO_2(g) + 8H_2O(\ell)$

$\Delta n_g = n_p - n_r = 7 - 11 = -4$

$\because \Delta H = \Delta U + \Delta n_g RT$

$\therefore \Delta H - \Delta U = -4\ RT$

5. For the reaction of H_2 with I_2, the rate constant is $2.5\times10^{-4} dm^3\ mol^{-1}\ s^{-1}$ at 327°C and $1.0\ dm^3\ mol^{-1}\ s^{-1}$ at 527°C. The activation energy for the reaction, in kJ mol^{-1} is: ($R=8.314 J\ K^{-1}\ mol^{-1}$)

(1) 72 (2) 166 (3) 150 (4) 59

Sol. $H_2(g) + I_2(g) \rightarrow 2HI(g)$

Apply Arrhenius equation

$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{600} - \frac{1}{800}\right)$$

$$\log\frac{1}{2.5\times10^{-4}} = \frac{E_a}{2.303\times8.31}\left(\frac{200}{600\times800}\right)$$

$\therefore E_a \approx 166 kJ/mol$

6. The correct statements among (a) to (b) are:

(a) saline hydrides produce H_2 gas when reacted with H_2O.

(b) reaction of $LiAH_4$ with BF_3 leads to B_2H_6.

(c) PH_3 and CH_4 are electron - rich and electron-precise hydrides, respectively.

(d) HF and CH_4 are called as molecular hydrides.

(1) (c) and (d) only

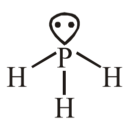
(2) (a), (b) and (c) only

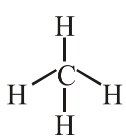
(3) (a), (b), (c) and (d)

(4) (a), (c) and (d) only

Sol. (a) $\underset{\text{Ionic hydride/saline hydride}}{\overset{\oplus}{M}\overset{\ominus}{H}} + HOH \longrightarrow MOH + H_2$

(b) $4BF_3 + 3LiAlH_4 \longrightarrow 2B_2H_6 + 3LiF + 3AlF_3$

(c)  $\rightarrow$ phosphorous is electron rich hydride due to presence of lone pair

 $\rightarrow$ It is electron precise hydride.

(d) HF & CH_4 are molecular hydride due to they are covalent molecules.

7. The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^{\ominus}$ (b) H_2O

(c) $CH_3SO_3^{\ominus}$ (d) $\overset{\ominus}{O}H$

(1) (b) < (c) < (a) < (d)

(2) (a) < (d) < (c) < (b)

(3) (d) < (a) < (c) < (b)

(4) (b) < (c) < (d) < (a)

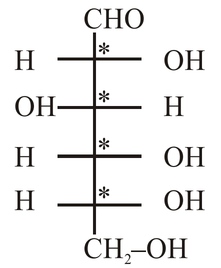
Sol. $\underbrace{\bar{O}H > CH_3-\underset{\underset{O}{\|}}{C}-O^- > CH_3-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-O^-}_{\text{Charged ion}} > \underbrace{H_2O}_{\text{neutral system}}$

ione pair donating tendency on oxygen is reduced, nucleophilicity reduced b < c < a < d

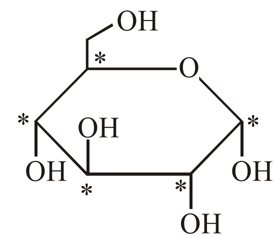
8. Number of stereo centers present in linear and cyclic structures of glucose are respectively :

(1) 4 & 5 (2) 5 & 5 (3) 4 & 4 (4) 5 & 4

Sol.



D–Glucose
(Linear structure)



α-D–Glucose
(cyclic structure)

∗ :- Stereocenter

9. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373K leads to an anhydrous white powder Z. X and Z, respectively, are:

(1) Washing soda and soda ash.

(2) Washing soda and dead burnt plaster.

(3) Baking soda and dead burnt plaster.

(4) Baking soda and soda ash.

Sol. $\underset{\substack{\text{washing soda}\\(X)}}{Na_2CO_3.10H_2O(s)} \xrightarrow{\Delta} \underset{(Y)}{Na_2CO_3.H_2O}$

$\downarrow \Delta\ T > 373k$

$\underset{\substack{\text{(soda ash)}\\(Z)}}{Na_2CO_3}$

10. The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are:

(1) 12 and 3 (2) 20 and 4

(3) 12 and 4 (4) 20 and 3

Sol. Total No. of pentagons in C_{60} = 12

Total no. of trigons (triangles) in white phosphorus (P_4) = 4

11. The correct order of the first ionization enthalpies is:

(1) Mn < Ti < Zn < Ni

(2) Ti < Mn < Ni < Zn

(3) Zn < Ni < Mn < Ti

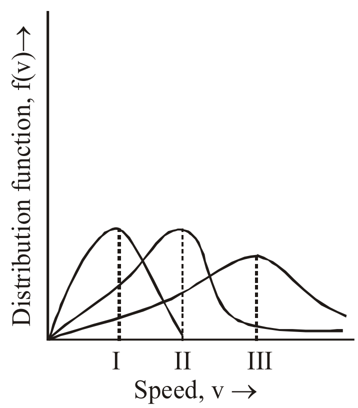
(4) Ti < Mn < Zn < Ni

Sol. Ti $\rightarrow$ |Ar| $3d^2\ 4s^2$
Mn $\rightarrow$ |Ar| $3d^5\ 4s^2$
Ni $\rightarrow$ |Ar| $3d^8\ 4s^2$
Zn $\rightarrow$ |Ar| $3d^{10}\ 4s^2$
Correct order of I.P. is
[Ti < Mn < Ni < Zn]

12. The correct option among the following is :
(1) Colloidal particles in lyophobic sols can be precipitated by electrophoresis.
(2) Brownian motion in colloidal solution is faster the viscosity of the solution is very high.
(3) Colloidal medicines are more effective because they have small surface area.
(4) Addition of alum to water makes it unfit for drinking.

Sol. In electrophoresis precipitation occurs at the electrode which is oppositely charged therefore (1) is correct.

13. Points I, II and III in the following plot respectively correspond to
(V_{mp} : most probable velocity)



(1) V_{mp} of N_2 (300K); V_{mp} of H_2(300K); V_{mp} of O_2(400K)
(2) V_{mp} of H_2 (300K); V_{mp} of N_2(300K); V_{mp} of O_2(400K)
(3) V_{mp} of O_2 (400K); V_{mp} of N_2(300K); V_{mp} of H_2(300K)
(4) V_{mp} of N_2 (300K); V_{mp} of O_2(400K); V_{mp} of H_2(300K)

Sol. $V_{mp} = \sqrt{\dfrac{2RT}{M}} \Rightarrow V_{mp} \propto \sqrt{\dfrac{T}{M}}$
For N_2, O_2, H_2
$\sqrt{\dfrac{300}{28}} < \sqrt{\dfrac{400}{32}} < \sqrt{\dfrac{300}{2}}$
V_{mp} of N_2(300K) < V_{mp} of O_2(400K) < V_{mp} of H_2(300K)

14. The INCORRECT statement is :
(1) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and and $[Cr(H_2O)_6]^{2+}$ are nearly similar.
(2) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83BM.
(3) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.
(4) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

Sol. (1) $[Fe(H_2O)_6]^{2+}$, $Fe^{2+} \rightarrow 3d^6 \rightarrow$ 4 unpaired electron
$[Cr(H_2O)_6]^{2+}$, $Cr^{2+} \rightarrow 3d^4 \rightarrow$ 4 unpaired electron
(2) $[Ni(NH_3)_4(H_2O)_2]^{2+}$ = $Ni^{2+} \rightarrow 3d^8$
$\rightarrow$ 2 unpaired electron
μ_m = 2.83 B.M
(3) In gemstone, ruby has Cr^{3+} ion occupying the octahedral sites of aluminium oxide (Al_2O_3) normally occupied by Al^{3+} ion.
(4) Complimenry color of violet is yellow

15. For the reaction,
$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$,
$\Delta H = -57.2$kJ mol^{-1} and
$K_c = 1.7 \times 10^{16}$.
Which of the following statement is INCORRECT?
(1) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
(2) The equilibrium will shift in forward direction as the pressure increase.
(3) The equilibrium constant decreases as the temperature increases.
(4) The addition of inert gas at constant volume will not affect the equilibrium constant.

Sol. In option (2)- Δn_g is –ve therfore increase in pressure will bring reaction in forward direction.
In option (3)- as the reaction is exothermic therefore increase in temperature will decrease the equilibrium constant.
In option (4)- Equillibrium constant changes only with temperature.
Hence, option (2), (3) and (4) are correct therefore option (1) is incorrect choice.

16. The pH of a 0.02M NH_4Cl solution will be [given $K_b(NH_4OH)=10^{-5}$ and log2=0.301]

(1) 4.65 (2) 5.35

(3) 4.35 (4) 2.65

Sol. For the salt of strong acid and weak base

$$H^+ = \sqrt{\frac{K_w \times C}{K_b}}$$

$$\left[H^+\right] = \sqrt{\frac{10^{-14} \times 2 \times 10^{-2}}{10^{-5}}}$$

$$-\log\left[H^+\right] = 6 - \frac{1}{2}\log 20$$

$\therefore$ pH = 5.35

17. The noble gas that does NOT occur in the atmosphere is:

(1) He (2) Ra

(3) Ne (4) Kr

Sol. In question noble gas asked, which does not exist in the atmosphere and answer is given Ra. Ra is a alkaline earth metal not noble gas it should be Rn. It is printing error in JEE Main paper

18. 1 g of non-volatile non-electrolyte solute is dissolved in 100g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\frac{\Delta T_b(A)}{\Delta T_b(B)}$, is :

(1) 5 : 1

(2) 10 : 1

(3) 1 : 5

(4) 1 : 0.2

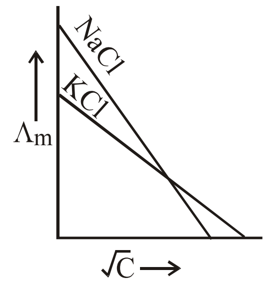
Sol. $\Delta T_b = K_b \times m$

$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{K_{b(A)}}{K_{b(B)}} \text{ as } m_A = m_B$$

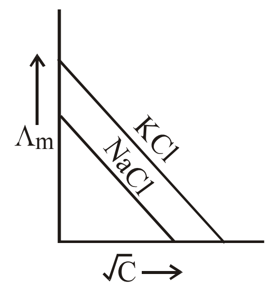
$$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{1}{5}$$

19. Which one of the following graphs between molar conductivity (Λ_m) versus $\sqrt{C}$ is correct?

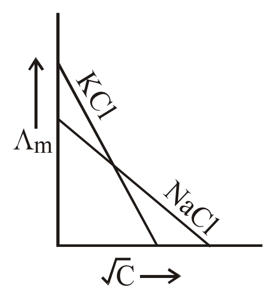
(1)



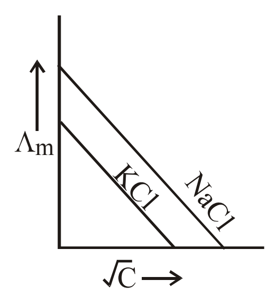
(2)



(3)



(4)



Sol. Both NaCl and KCl are strong electrolytes and as $Na^+(aq.)$ has less conductance than $K^+(aq.)$ due to more hydration therefore the graph of option (2) is correct.

20. The correct statement is :

(1) zincite is a carbonate ore

(2) aniline is a froth stabilizer

(3) zone refining process is used for the refining of titanium

(4) sodium cyanide cannot be used in the metallurgy of silver

Sol. (1) Zincite is ZnO

(2) Aniline is the forth stablizer.

(3) Zone refining process is not used for refining of 'Ti'

(4) Sodium cyanide is used in the metallurgy of silver

21. The minimum amount of $O_2(g)$ consumed per gram of reactant is for the reaction :

(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

(1) $C_3H_8(g) + 5\ O_2(g) \rightarrow 3\ CO_2(g) + 4\ H_2O(l)$

(2) $P_4(s) + 5\ O_2(g) \rightarrow P_4O_{10}(s)$

(3) $4\ Fe(s) + 3\ O_2(g) \rightarrow 2\ FeO_3(s)$

(4) $2\ Mg(s) + O_2(g) \rightarrow 2\ MgO(s)$

Sol. $C_3H_8(g) + 5O_2(g) \longrightarrow 3CO_2(g) + 4H_2O(\ell)$

Each 1g of C_3H_8 requires 3.63 g of O_2

$P_4(s) + 5O_2(g) \longrightarrow P_4O_{10}(s)$

Each 1g of P_4 requires 1.29 g of O_2

$4Fe(s) + 3O_2(g) \longrightarrow 2Fe_2O_3(s)$

Each 1g of Fe requires 0.428 g of O_2

$2Mg(s) + O_2(g) \longrightarrow 2MgO(s)$

Each 1g of Mg requires 0.66 g of O_2

therefore least amount of O_2 is required in option (3).

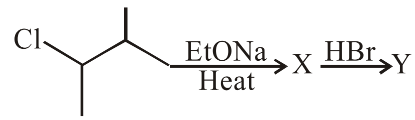
22. Air pollution that occurs in sunlight is :

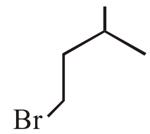
(1) oxidising smog (2) acid rain

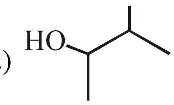
(3) reducing smog (4) fog

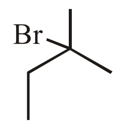
Sol. Photochemical smog occurs in warm (sunlight) and has high concentration of oxidising agent therefore it is called photochemical smog/oxidising smog.

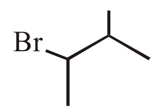
23. The major product 'Y' in the following reaction is:

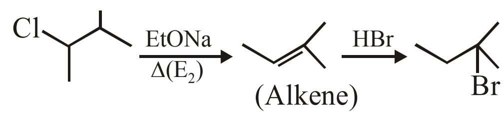


(1) 

(2) 

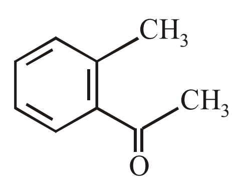
(3) 

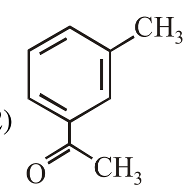
(4) 

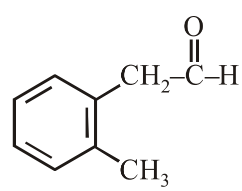
Sol. 

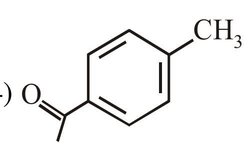
(Alkene)
(Saytzeff prod.)

24. Compound A ($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH gives acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is :-

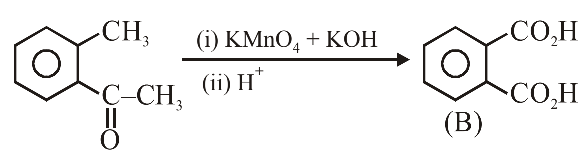
(1) 

(2) 

(3) 

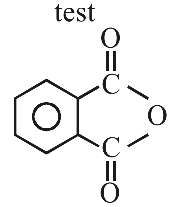
(4) 

Sol.



+ve iodoform test

($C_8H_6O_4$) Pthleic acid



is used for prepareation of phenolphthalein indicator

Pthleic anhydride

25. The crystal fied stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are :-

(1) $-0.4\Delta_O$ and $-0.8\Delta_t$

(2) $-0.4\Delta_O$ and $-1.2\Delta_t$

(3) $-2.4\Delta_O$ and $-1.2\Delta_t$

(4) $-0.6\Delta_O$ and $-0.8\Delta_t$

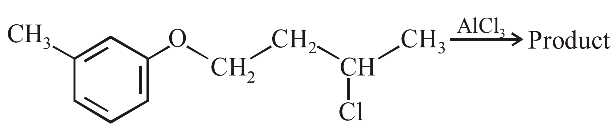
Sol. $[Fe(H_2O)_6]Cl_2$, $Fe^{2+} \rightarrow 3d^6 \rightarrow (t_{2g})^4(e_g)^2$

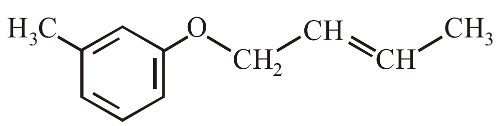
C.F.S.E. $= 4 \times(-0.4\Delta_o) + 2 \times 0.6\Delta_o = -0.4\Delta_o$

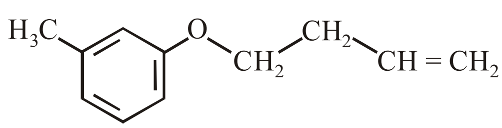
$K_2[NiCl_4]$, $Ni^{2+} \rightarrow 3d^8 \rightarrow (e)^4(t_2)^4$

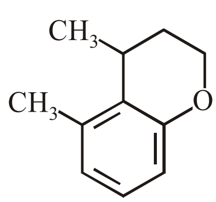
C.F.S.E. $= 4\times(-0.6\Delta_t) + 4 \times (0.4\Delta_t) = -0.8\Delta_t$

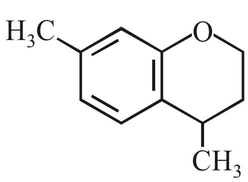
26. The major product obtained in the given reaction is :-

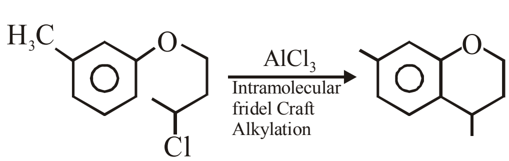


(1) 

(2) 

(3) 

(4) 

Sol. 

27. The highest possible oxidation states of uranium and plutonium, respectively, are :-

(1) 6 and 4 (2) 7 and 6

(3) 4 and 6 (4) 6 and 7

Sol. The highest oxidation state of U and Pu is 6+ and 7+ respectively

28. In chromatography, which of the following statements is INCORRECT for R_f?

(1) R_f value depends on the type of chromatography.

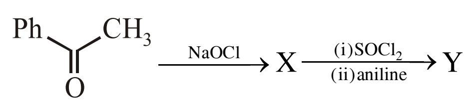
(2) The value of R_f can not be more than one.

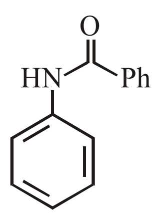
(3) Higher R_f value means higher adsorption.

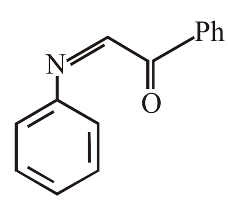
(4) R_f value is dependent on the mobile phase.

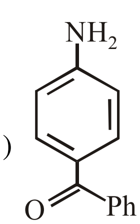
Sol. Except (3) all are correct

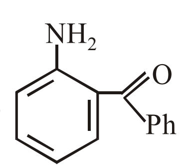
29. The major product 'Y' in the following reaction is:-



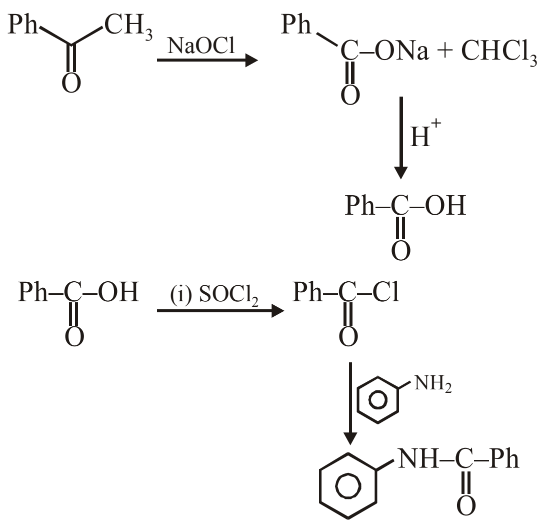
(1) 

(2) 

(3) 

(4) 

Sol.



30. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are:

(1) Paschen and P fund

(2) Lyman and Paschen

(3) Brackett and Piund

(4) Balmer and Brackett

Sol.
$$\frac{\dfrac{1}{\lambda_2}=R_H\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)Z^2}{\dfrac{1}{\lambda_1}=R_H\left(\dfrac{1}{m_1^2}-\dfrac{1}{m_2^2}\right)Z^2}$$

as for shortest wavelengths both n_2 and m_2 are ∞

$$\therefore \quad \frac{\lambda_1}{\lambda_2}=\frac{9}{1}=\frac{m_1^2}{n_1^2}$$

Now if $m_1 = 3$ & $n_1 = 1$ it will justify the statement hence Lyman and Paschen (2) is correct.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

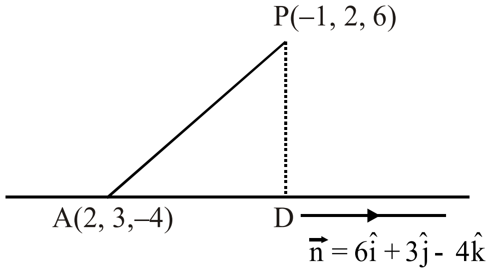
TEST PAPER WITH ANSWER & SOLUTION

1. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) $2\sqrt{13}$ (4) 6

Sol.



$$AD = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$$

$$\Rightarrow \quad PD=\sqrt{AP^2-AD^2}=\sqrt{110-61}=7$$

2. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots\ldots (x_{50} - 4)^2$ is :

(1) 525 (2) 380

(3) 480 (4) 400

Sol. Mean $(\mu) = \dfrac{\sum x_i}{50}=16$

standard deviation $(\sigma) = \sqrt{\dfrac{\sum x_i^2}{50}-(\mu)^2}=16$

$\Rightarrow (256) \times 2 = \dfrac{\sum x_i^2}{50}$

$\Rightarrow$ New mean

$$=\frac{\sum\left(x_i-4\right)^2}{50}=\frac{\sum x_i^2+16\times 50-8\sum x_i}{50}$$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

3. A perpendicular is drawn from a point on the line $\dfrac{x-1}{2}=\dfrac{y+1}{-1}=\dfrac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

(1) (2, 0, 1) (2) (4, 0, –1)

(3) (–1, 0, 4) (4) (1, 0, 2)

Sol. Let point P on the line is $(2\lambda +1, -\lambda-1, \lambda)$
foot of perpendicular Q is given by

$$\frac{x-2\lambda-1}{1}=\frac{y+\lambda+1}{1}=\frac{z-\lambda}{1}=\frac{-(2\lambda-3)}{3}$$

$\because$ Q lies on $x + y + z = 3$ & $x - y + z = 3$
$\Rightarrow x + z = 3$ & $y = 0$

$y = 0 \Rightarrow \lambda + 1 = \dfrac{-2\lambda+3}{3} \Rightarrow \lambda = 0$

$\Rightarrow$ Q is (2, 0, 1)

4. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\dfrac{14}{3}$ (2) $\dfrac{16}{3}$ (3) $\dfrac{68}{15}$ (4) $\dfrac{34}{15}$

Sol. $3x^2 + 5y^2 =32$

$$\left.\frac{dy}{dx}\right|_{(2,2)}=-\frac{3}{5}$$

Tangent : $y - 2 = -\dfrac{3}{5}(x - 2) \Rightarrow Q\left(\dfrac{16}{3},0\right)$

Normal : $y - 2 = \dfrac{5}{3}(x - 2) \Rightarrow R\left(\dfrac{4}{5},0\right)$

Area is $= \dfrac{1}{2}(QR)\times 2 = QR = \dfrac{68}{15}$.

5. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation.

(1) $\lambda^2 - 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + 3\lambda - 4 = 0$ (4) $\lambda^2 + \lambda - 6 = 0$

Sol. D = 0

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

6. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 35 (2) 38

(3) 23 (4) 58

Sol. $T_r = \sum_{r=0}^{n} {}^nC_r x^{2n-2r} . x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$

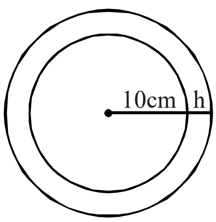
for r = 15. n = 38

smallest value of n is 38.

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

Sol. $V = \frac{4}{3}\pi\left((10+h)^3 - 10^3\right)$



$\frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$

$-50 = 4\pi\,(10 + 5)^2 \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = -\frac{1}{18}\frac{cm}{min}$

8. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(-\frac{5}{3}, 0\right)$ (2) (5, 0)

(3) (–5, 0) (4) $\left(\frac{5}{3}, 0\right)$

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a = 3,\ b = 4\ \&\ e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

corresponding focus will be (–ae, 0) i.e., (–5, 0).

9. The sum $1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + ...$

$+ \frac{1^3 + 2^3 + 3^3 + ... + 15^3}{1+2+3+....+15} - \frac{1}{2}(1+2+3+....+15)]$

(1) 1240 (2) 1860

(3) 660 (4) 620

Sol. Sum $= \sum_{n=1}^{15} \frac{1^3 + 2^3 + ...n^3}{1+2+....+n} - \frac{1}{2}.\frac{15.16}{2}$

$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$

$= \sum_{n=1}^{15} \frac{n(n+1)(n+2-(n-1))}{6} - 60$

$= \frac{15.16.17}{6} - 60 = 620$

10. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then | c | is equal to :

(1) 1/2 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol. Tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \quad \Rightarrow m = \pm 1$

$\Rightarrow$ Tagent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$

compare with y = – ax + C

$\Rightarrow a = \pm 1$ & $C = \pm\sqrt{2}$

11. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$,

then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $4\sin^2\alpha - 2x^2y^2$ (2) $4\cos^2\alpha + 2x^2y^2$

(3) $4\sin^2\alpha$ (4) $2\sin^2\alpha$

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos\alpha$

$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = \left(1-x^2\right)\left(1-\frac{y^2}{4}\right)$

$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

12. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

(1) $-\frac{5}{2}$ (2) 1

(3) $-\frac{1}{2}$ (4) –1

Sol. Let $x^2 = t$ $\quad 2xdx = dt$

$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt = \frac{1}{2}\left[-t^2.e^{-t} + \int 2t.e^{-t}.dt\right]$

$= \frac{1}{2}\left(-t^2.e^{-t}\right) + \left(-t.e^{-t} + \int 1.e^{-t}.dt\right)$

$= -\frac{t^2e^{-t}}{2} - te^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + C$

$g(x) = -1 - x^2 - \frac{x^4}{2} + ke^{x^2}$

for k = 0

$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$

13. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

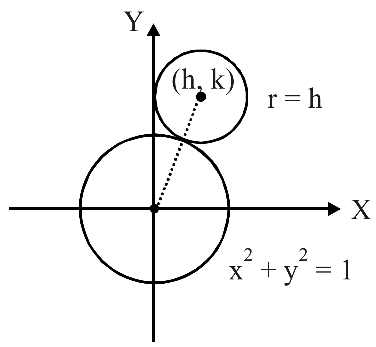
(1) $y = \sqrt{1+4x}$, $x \ge 0$

(2) $x = \sqrt{1+4y}$, $y \ge 0$

(3) $x = \sqrt{1+2y}$, $y \ge 0$

(4) $y = \sqrt{1+2x}$, $x \ge 0$

Sol.



$\sqrt{h^2 + k^2} = |h| + 1$

$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y = \sqrt{1+2x}$; $x \ge 0$.

14. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin.

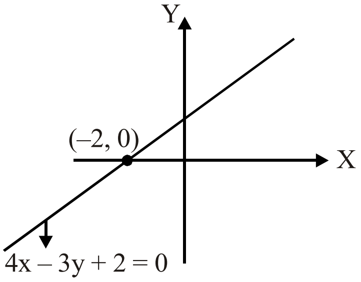
Then which one of the following points lies on any of these lines ?

(1) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$

(3) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$ (4) $\left(\frac{1}{4}, -\frac{1}{3}\right)$

Sol. Required line is $4x - 3y + \lambda = 0$

$$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$$



$\Rightarrow \lambda = \pm 3$.

So, required equation of line is $4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$

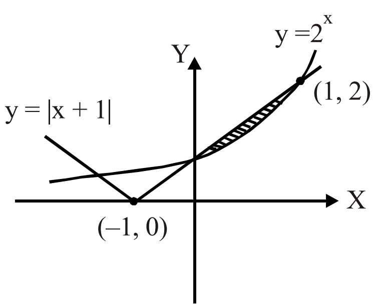
15. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

Sol. Required Area

$$\int_0^1 \left((x+1) - 2^x\right)dx$$



$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

16. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 15 (2) 5

(3) 13 (4) 9

Sol. $4x - 2y + 4z + 6 = 0$

$$\frac{|\lambda - 6|}{\sqrt{16 + 4 + 16}} = \left|\frac{\lambda - 6}{6}\right| = \frac{1}{3}$$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

$$\frac{|\mu - 3|}{\sqrt{4 + 4 + 1}} = \frac{2}{3}$$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore$ Maximum value of $(\mu + \lambda) = 13$.

17. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

(1) $\bar{z}w = i$ (2) $\bar{z}w = -i$

(3) $z\bar{w} = \frac{1 - i}{\sqrt{2}}$ (4) $z\bar{w} = \frac{-1 + i}{\sqrt{2}}$

Sol. $|z|.\ |w| = 1 \quad z = re^{i(\theta + \pi/2)}$ and $w = \frac{1}{r}e^{i\theta}$

$$\bar{z}.w = e^{-i(\theta + \pi/2)}.e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$z.\bar{w} = e^{i(\theta + \pi/2)}.e^{-i\theta} = e^{i(\pi/2)} = i$$

18. Let a, b and c be in G. P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

Sol. b = ar

$c = ar^2$

3a, 7b and 15 c are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\, ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \quad \Rightarrow (3r-1)(5r-3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$.

Only acceptable value is $r = \frac{1}{3}$, because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore$ c. d = 7b – 3a = 7ar – 3a = $\frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ 4th term = 15 c – $\frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

19. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ equal to :

(1) $3^{7/6} - 3^{5/6}$

(2) $3^{5/3} - 3^{1/3}$

(3) $3^{4/3} - 3^{1/3}$

(4) $3^{5/6} - 3^{2/3}$

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x . \sin x} dx$

$= \int \frac{\tan^{2/3} x}{\tan^2 x} . \sec^2 x . dx$

$= \int \frac{\sec^2 x}{\tan^{4/3} x} . dx \qquad \{\tan x = t, \sec^2 x dx = dt\}$

$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3\left(t^{-1/3}\right)$

$\Rightarrow I = -3\tan(x)^{-1/3}$

$\Rightarrow I = \frac{3}{(\tan x)^{1/3}}\Bigg|_{\pi/6}^{\pi/3} = -3\left(\frac{1}{\left(\sqrt{3}\right)^{1/3}} - \left(\sqrt{3}\right)^{1/3}\right)$

$= 3\left(3^{1/3} - \frac{1}{3^{1/6}}\right) = 3^{7/6} - 3^{5/6}$

20. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 1. Then :

(1) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Sol. $\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

I.F $= e^{\int \tan x\, dx} = e^{\ln .\sec x} = \sec x$

$\therefore$ y . secx $= \int (2x + x^2 \tan x)\sec x.dx$

$= \int 2x \sec x\, dx + \int x^2 (\sec x . \tan x) dx$

y secx = x^2 sec x + λ

$\Rightarrow y = x^2 + \lambda \cos x$

$y(0) = 0 + \lambda = 1 \qquad \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right) = \frac{-\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

21. Let a_1, a_2, a_3,......be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1 a_4 a_5$, is :

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Sol. Let a is first term and d is common difference then, a + 5d = 2 (given) ...(1)

f(d) = (2 – 5d) (2 – 2d) (2 – d)

f '(d) = 0 $\quad \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

f "(d) < 0 at d = 8/5

$\Rightarrow d = \frac{8}{5}$

22. The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

Sol. $\angle B = \frac{\pi}{3}$, by sine Rule

sin A = $\frac{1}{2}$

$\Rightarrow A = 30°, a = 2$, b = $2\sqrt{3}$, c = 4

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$ sq. cm

23. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\Rightarrow$ n = 7.

24. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :

(1) 210 (2) 190

(3) 170 (4) 180

Sol. Total cases = number of diagonals

= ${}^{20}C_2 - 20 = 170$

25. The sum of the real roots of the equatuion

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$, is equal to :

(1) 6 (2) 1
(3) 0 (4) – 4

Sol. By expansion, we get
$-5x^3 + 30x - 30 + 5x = 0$
$\Rightarrow -5x^3 + 35x - 30 = 0$
$\Rightarrow x^3 - 7x + 6 = 0$, All roots are real
So, sum of roots = 0

26. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :
(1) $a\alpha^2 - b\alpha - a = 0$
(2) $a\alpha^2 + b\alpha - a = -2\alpha^2$
(3) $a\alpha^2 + b\alpha + a = 0$
(4) $a\alpha^2 - b\alpha - a = 1$

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$
$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

27. If the tangent to the curve $y = \dfrac{x}{x^2-3}$, $x \in R$, $\left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :
(1) $|6\alpha + 2\beta| = 19$
(2) $|2\alpha + 6\beta| = 11$
(3) $|6\alpha + 2\beta| = 9$
(4) $|2\alpha + 6\beta| = 19$

Sol. $\left.\dfrac{dy}{dx}\right|_{(\alpha,\beta)} = \dfrac{-\alpha^2-3}{\left(\alpha^2-3\right)^2}$

Given that :

$$\frac{-\alpha^2-3}{\left(\alpha^2-3\right)^2} = -\frac{1}{3}$$

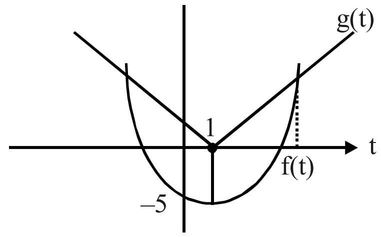
$\Rightarrow \alpha = 0, \pm 3$ $(\alpha \neq 0)$

$\Rightarrow \beta = \pm \dfrac{1}{2}$. $(\beta \neq 0)$

$|6\alpha + 2\beta| = 19$

28. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :
(1) 2 (2) 3
(3) 4 (4) 1

Sol. Let $2^x = t$
$5 + |t - 1| = t^2 - 2t$
$\Rightarrow \underbrace{|t - 1|}_{g(t)} = \underbrace{(t^2 - 2t - 5)}_{f(t)}$
From the graph



$\because t > 0$

So, number of real root is 1.

29. If $\lim\limits_{x\to1} \dfrac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to :-
(1) –7 (2) – 4
(3) 5 (4) 1

Sol. $\lim\limits_{x\to1} \dfrac{x^2 - ax + b}{x - 1} = 5$
$1 - a + b = 0$ ...(i)
$2 - a = 5$ ...(ii)
$\Rightarrow a + b = -7$.

30. The negation of the boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :
(1) r (2) $s \wedge r$
(3) $s \vee r$ (4) $\sim s \wedge \sim r$

Sol. $\sim\left(\sim s \vee \left(\sim r \wedge s\right)\right)$

$s \wedge \left(r \vee \sim s\right)$

$\left(s \wedge r\right) \vee \left(s \wedge \sim s\right)$

$\left(s \wedge r\right) \vee \left(c\right)$

$\left(s \wedge r\right)$