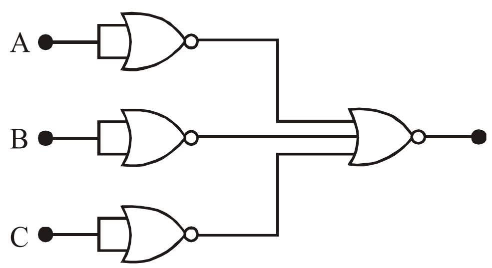
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

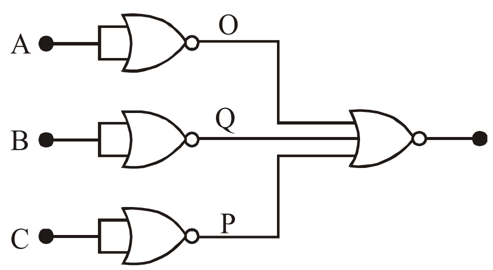
PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. Identify the operation performed by the circuit given below :



(1) AND (2) NAND
(3) OR (4) NOT

Sol.



A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

2. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is :

(1) $\sqrt{2}$ (2) 2 (3) 4 (4) $2\sqrt{2}$

Sol. $I_1 = \dfrac{MR^2}{2} = \dfrac{\rho(\pi R^2)t.R^2}{2}$

$I \propto R^4$

$$\frac{I_1}{I_2} = \frac{R_1^4}{R_2^4} = \frac{1}{16}$$

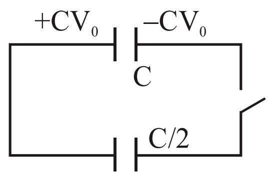
$$\therefore \frac{R_1}{R_2} = \frac{1}{2}$$

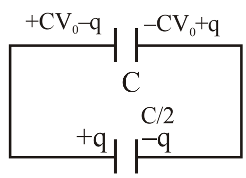
3. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\dfrac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is :

(1) $\frac{1}{6}CV_0^2$ (2) $\frac{1}{2}CV_0^2$

(3) $\frac{1}{3}CV_0^2$ (4) $\frac{1}{4}CV_0^2$

Sol.





$$\frac{CV_0 - q}{C} = \frac{q}{C/2} = \frac{2q}{C}$$

$$V_0 = \frac{3q}{C} \quad \Rightarrow q = \frac{CV_0}{3}$$

$$U_i = \frac{1}{2}CV_0^2$$

$$U_f = \frac{\left(\frac{2CV_0}{3}\right)^2}{2C} + \frac{\left(\frac{CV_0}{3}\right)^2}{2\left(\frac{C}{2}\right)}$$

$$= \frac{1}{2}CV_0^2\left[\frac{4}{9}+\frac{2}{9}\right] = \frac{1}{2}CV_0^2\left(\frac{2}{3}\right)$$

$$\text{Heat loss} = \frac{1}{2}CV_0^2 - \left(\frac{2}{3}\right)\left(\frac{1}{2}CV_0^2\right)$$

$$= \frac{1}{6}CV_0^2$$

4. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance of 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ?

(1) 5690 J (2) 5250 J

(3) 3280 J (4) 2780 J

Sol. F = 200 N for $0 \le x \le 15$

$= 200 - \frac{100}{15}(x-15)$ for $15 \le x < 30$

$W = \int F\,dx$

$= \int_0^{15} 200\,dx + \int_{15}^{30}\left(300 - \frac{100}{15}x\right)dx$

$= 200 \times 15 + 300 \times 15 - \frac{100}{15} \times \frac{(30^2 - 15^2)}{2}$

= 3000 + 4500 – 2250

= 5250 J

5. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$

Its magnetic field will be given by :

(1) $\frac{E_0}{c}(\hat{x} - \hat{y})\cos(kz - \omega t)$

(2) $\frac{E_0}{c}(-\hat{x} + \hat{y})\sin(kz - \omega t)$

(3) $\frac{E_0}{c}(\hat{x} - \hat{y})\sin(kz - \omega t)$

(4) $\frac{E_0}{c}(\hat{x} + \hat{y})\sin(kz - \omega t)$

Sol. $\vec{E} = E_0(\hat{x} + \hat{y})\sin(kz - \omega t)$

direction of propagation = $+\hat{k}$

$\hat{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

$\hat{k} = \hat{E} \times \hat{B}$

$\hat{k} = \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) \times \hat{B} \quad \Rightarrow \hat{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$

$\therefore \vec{B} = \frac{E_0}{C}(-\hat{x} + \hat{y})\sin(kz - \omega t)$

6. Find the bindng energy per nucleon for ${}^{120}_{50}\text{Sn}$.

Mass of proton m_P = 1.00783 U, mass of neutron m_n = 1.00867 U and mass of tin nucleus m_{Sn} = 119.902199 U. (take 1U = 931 MeV)

(1) 8.5 MeV (2) 7.5 MeV

(3) 8.0 MeV (4) 9.0 MeV

Sol. B.E. = $[\Delta m].c^2$

$M_{expected} = ZM_p + (A - Z)M_n$

= 50 [1.00783] + 70 [1.00867]

M_{actual} = 119.902199

B.E. = $\left[50[1.00783] + 70[1.00867] - 119.902199\right] \times 931$

= 1020.56

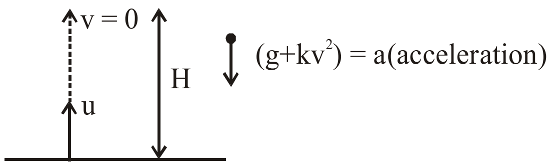
$\frac{BE}{\text{nucleon}} = \frac{1020.56}{120} = 8.5$ MeV

7. A small ball of mass is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is :

(1) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ (2) $\frac{1}{2k}\ln\left(1 + \frac{ku^2}{g}\right)$

(3) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (4) $\frac{1}{k}\ln\left(1 + \frac{ku^2}{2g}\right)$

Sol.



$$\vec{F} = mkv^2 - mg$$

$$\vec{a} = \frac{\vec{F}}{m} = -\left[kv^2 + g\right]$$

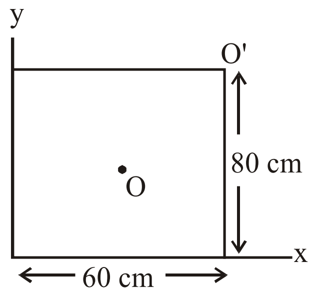
$$\Rightarrow \quad v.\frac{dv}{dh} = -[kv^2 + g]$$

$$\Rightarrow \int_u^0 \frac{v.dv}{kv^2 + g} = -\int_0^H dh$$

$$\frac{1}{2K}\ln\left[kv^2 + g\right]_u^0 = -H$$

$$\Rightarrow \frac{1}{2K}\ln\left[\frac{ku^2 + g}{g}\right] = H$$

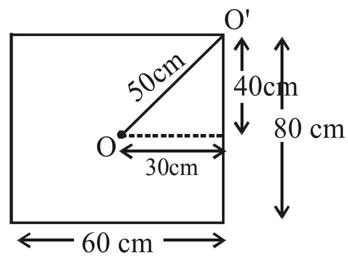
8.



For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is :

(1) 1/2 (2) 2/3 (3) 1/8 (4) 1/4

Sol.



Rectangular sheet

$$I_O = \frac{M}{12}[L^2 + B^2] = \frac{M}{12}[80^2 + 60^2]$$

$$I_{O'} = I_0 + Md^2 \{\text{parallel axis theorem}\}$$

$$= \frac{M}{12}\left[80^2 + 60^2\right] + M\,[50]^2$$

$$\frac{I_O}{I_{O'}} = \frac{M/12[80^2 + 60^2]}{\frac{M}{12}[80^2 + 60^2] + M[50]^2} = \frac{1}{4}$$

9. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system :

Process	Condition
(I) Adiabatic	(A) $\Delta W = 0$
(II) Isothermal	(B) $\Delta Q = 0$
(III) Isochoric	(C) $\Delta U \neq 0$, $\Delta W \neq 0$, $\Delta Q \neq 0$
(IV) Isobaric	(D) $\Delta U = 0$

(1) I-B, II-D, III-A, IV-C

(2) I-B, II-A, III-D, IV-C

(3) I-A, II-A, III-B, IV-C

(4) I-A, II-B, III-D, IV-D

Sol. (I) Adiabatic process $\Rightarrow \Delta Q = 0$

No exchange of heat takes place with surroundings

(II) Isothermal proess $\Rightarrow$ Temperature remains constant ($\Delta T = 0$)

$$\Delta u = \frac{F}{2}nR\Delta T \Rightarrow \Delta u = 0$$

No change in internal energy [$\Delta u = 0$]

(III) Isochoric process Volume remains constant $\Delta V = 0$

$$W = \int P.dV = 0$$

Hence work done is zero.

(IV) Isobaric process $\Rightarrow$ Pressure remains constant

$$W = P.\ \Delta V \neq 0$$

$$\Delta u = \frac{F}{2}nR\Delta T = \frac{F}{2}[P\Delta V] \neq 0$$

$$\Delta Q = nC_P\Delta T \neq 0$$

10. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be :

(1) 4 A/m (2) 0.75 A/m

(3) 2.25 A/m (4) 1 A/m

Sol. For paramagnetic material

According to curies law

$$\chi \propto \frac{1}{T}$$

$$\chi \propto \frac{1}{T} \quad \Rightarrow \quad \chi_1 T_1 = \chi_2 T_2$$

$$\Rightarrow \frac{6}{0.4} \times 4 = \frac{I}{0.3} \times 24$$

$$I = \frac{0.3}{0.4} = 0.75 \text{ A/m}$$

11. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is :

(1) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (2) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$

(3) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (4) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

Sol. $U_{max} = \frac{1}{2}LI_{max}^2$

$$i = I_{max}\left(1-e^{-Rt/L}\right)$$

For U to be $\frac{U_{max}}{n}$; i has to be $\frac{I_{max}}{\sqrt{n}}$

$$\frac{I_{max}}{\sqrt{n}} = I_{max}(1-e^{-Rt/L})$$

$$e^{-Rt/L} = 1-\frac{1}{\sqrt{n}} = \frac{\sqrt{n}-1}{\sqrt{n}}$$

$$-\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

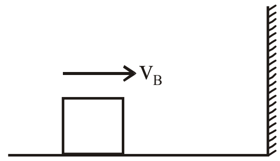
$$t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

12. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz, when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}.

(1) 91 kmh^{-1} (2) 71 kmh^{-1}

(3) 81 kmh^{-1} (4) 61 kmh^{-1}

Sol.



$$f_1 = \left(\frac{330}{330-v_B}\right)420$$

$$f_2 = \left(\frac{330+v_0}{330}\right)\left(\frac{330}{330-v_B}\right)420$$

$$490 = \left(\frac{330+v_B}{330-v_B}\right)420$$

$$\frac{7}{6} = \frac{330+v_B}{330-v_B}$$

$$v_B = \frac{330}{13} \text{ m/s}$$

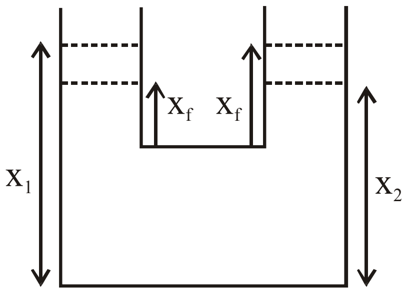
$$= \frac{330}{13}\times\frac{18}{5} \approx 91 \text{ km/hr}$$

13. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is :

(1) $gdS\ (x_2 + x_1)^2$ (2) $\frac{3}{4}gdS(x_2-x_1)^2$

(3) $\frac{1}{4}gdS(x_2-x_1)^2$ (4) $gdS(x_2^2+x_1^2)$

Sol.



$$U_i = (\rho S x_1)g \cdot \frac{x_1}{2} + (\rho S x_2)g \cdot \frac{x_2}{2}$$

$$U_f = (\rho S x_f)g \cdot \frac{x_f}{2} \times 2$$

By volume conservation

$Sx_1 + Sx_2 = S(2x_f)$

$$x_f = \frac{x_1 + x_2}{2}$$

$$\Delta U = \rho S g\left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2}\right) - x_f^2\right]$$

$$= \rho S g\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2}\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2\right]$$

$$= \frac{\rho S g}{4}(x_1 - x_2)^2$$

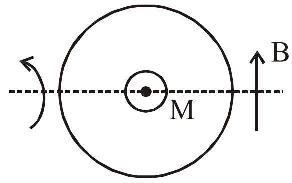
14. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be :

(1) 10 rad s^{-1} (2) 20 π rad s^{-1}

(3) 10 π rad s^{-1} (4) 20 rad s^{-1}

Sol. $I_{dia} = 0.8$ kg/m^2

$M = 20$ Am^2



$U_i + K_i = U_f + K_f$

$$0 + 0 = -MB\cos 30° + \frac{1}{2}I\omega^2$$

$$20 \times 4 \times \frac{\sqrt{3}}{2} = \frac{1}{2}(0.8)\,\omega^2$$

$$\omega = \sqrt{100\sqrt{3}} = 10(3)^{1/4}$$

15. A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is :

(1) $\sqrt{\frac{2}{a}}$ (2) $\sqrt{\frac{1}{a}}$ (3) a (4) $\sqrt{\frac{3}{a}}$

Sol. $E = E_0(1 - ax^2)$

$$W = \int qE\,dx = qE_0\int_0^{x_0}(1 - ax^2)\,dx$$

$$= qE_0\left[x_0 - \frac{ax_0^3}{3}\right]$$

For $\Delta KE = 0$, $W = 0$

Hence $x_0 = \sqrt{\frac{3}{a}}$

16. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to : (Given bulk modulus of metal, $B = 8 \times 10^{10}$ Pa)

(1) 0.6 (2) 1.67 (3) 5 (4) 20

Sol. $B = \dfrac{-\Delta P}{\dfrac{\Delta V}{V}}$

$\left|\dfrac{\Delta V}{V}\right| = \dfrac{\Delta P}{B}$

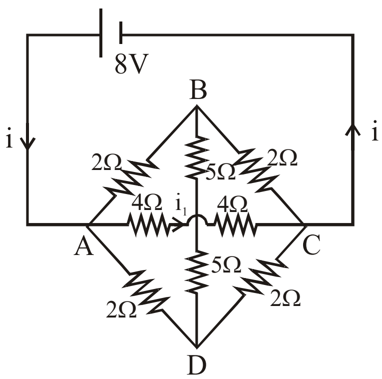
$= \dfrac{4\times10^9}{8\times10^{10}} = \dfrac{1}{20}$

$\dfrac{\Delta \ell}{\ell} = \dfrac{1}{3} \times \dfrac{\Delta V}{V} = \dfrac{1}{60}$

Percentage change $= \dfrac{\Delta \ell}{\ell}\times100\%$

$= \dfrac{100}{60}\% = 1.67\%$

17. The value of current i_1 flowing from A to C in the circuit diagram is :



(1) 5A (2) 2A (3) 4A (4) 1A

Sol. Voltage across AC = 8V

$R_{AC} = 4 + 4 = 8\Omega$

$i_1 = \dfrac{V}{R_{AC}} = \dfrac{8}{8} = 1\text{ A}$

18. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is :

(1) 1 (2) 2 (3) $\dfrac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Sol. $V_{orbit} = \sqrt{\dfrac{GM}{R}}$

$V_{escape} = \sqrt{\dfrac{2GM}{R}}$

$\dfrac{V_{orbit}}{V_{escape}} = \dfrac{1}{\sqrt{2}}$

19. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of :

(1) Planck's constant
(2) Force constant
(3) Energy density
(4) Coefficient of viscosity

Sol. $x = \dfrac{IFV^2}{WL^4}$

$[x] = \dfrac{[ML^2][MLT^{-2}][LT^{-1}]^2}{[ML^2T^{-2}][L]^4}$

$[x] = [ML^{-1}T^{-2}]$

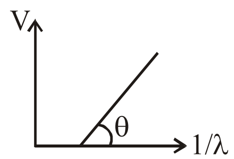
$[\text{Energy density}] = \left[\dfrac{E}{V}\right]$

$= \left[\dfrac{ML^2T^{-2}}{L^3}\right]$

$= [ML^{-1}T^{-2}]$

Same as x

20. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased :



(1) Slope of the straight line get more steep
(2) Straight line shifts to left
(3) Graph does not change
(4) Straight line shifts to right

Sol. $eV = \frac{hc}{\lambda} - \phi$

$$V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \phi$$

Slope of the line in above equation and all other terms are independent of intensity.

The graph does not change.

21. Orange light of wavelength 6000×10^{-10} m in illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is ______.

Sol. Condition for minimum,

$d\sin\theta = n\lambda$

$$\therefore \sin\theta = \frac{n\lambda}{d} < 1$$

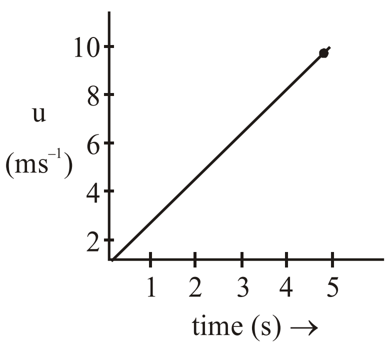
$$n < \frac{d}{\lambda} = \frac{6\times10^{-5}}{6\times10^{-7}} = 100$$

$\therefore$ Total number of minima on one side = 99

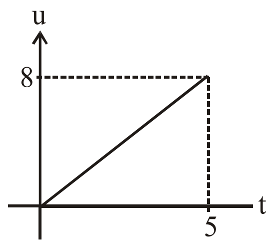
Total number of minima = 198

Correct Answer is 198

22. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5s will be ______ :



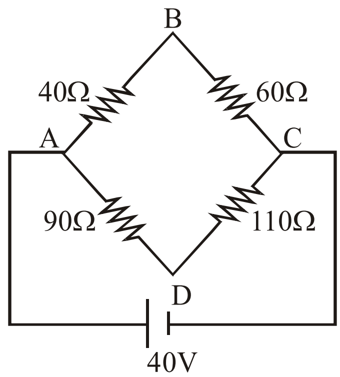
Sol.



Distance = $\int v\,dt$

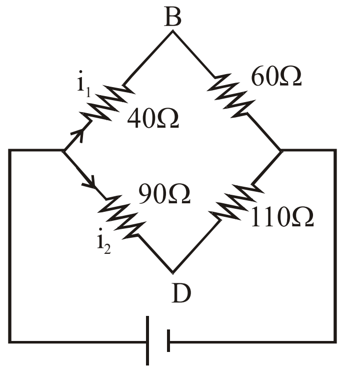
Area under graph $= \frac{1}{2}\times5\times8 = 20$

23.



Four resistances 40Ω, 60Ω, 90Ω and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40V and internal resistance negligible. The potential difference across BD is V is _______.

Sol.



$$i_1 = \frac{40}{40+60} = 0.4$$

$$i_2 = \frac{40}{90+110} = \frac{1}{5}$$

$v_B + i_1(40) - i_2(90) = v_D$

$$v_B - v_D = \frac{1}{5}(90) - \frac{4}{10}\times40$$

$v_B - v_D = 18 - 16 = 2$

24. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is ______.

Sol. Using displacement method

$$f = \frac{D^2 - d^2}{4D}$$

Here, D = 100 cm

d = 40 cm

$$f = \frac{100^2 - 40^2}{4(100)} = 21 \text{ cm}$$

$$P = \frac{1}{f} = \frac{100}{21} \text{ D} \qquad \frac{N}{100} = \frac{100}{21} \qquad N = 476$$

25. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm) is ______:

Sol. $PV = nRT$

$P\Delta V + V\Delta P = 0$ (for constant temp.)

$P\Delta V = nR\Delta T$ (for constant pressure)

$$\Delta T = \frac{P\Delta V}{nR}$$

$$\Delta P = -\frac{P\Delta V}{V} \qquad (\Delta V \text{ is same in both cases})$$

$$\frac{\Delta T}{\Delta P} = \frac{P\Delta V}{nR} \frac{V}{-P\Delta V} = \frac{-V}{nR} = -\frac{T}{P}$$

$(PV = nRT)$

$$\left(\frac{V}{nR} = \frac{T}{P}\right) \qquad \left|\frac{\Delta T}{\Delta P}\right| = \left|\frac{-300}{2}\right| = 150$$

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CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. If the equilibrium constant for $A \rightleftharpoons B+C$ is $K_{eq}^{(1)}$ and that of $B+C \rightleftharpoons P$ is $K_{eq}^{(2)}$, the equilibrium constant for $A \rightleftharpoons P$ is :-

(1) $K_{eq}^{(2)} - K_{eq}^{(1)}$ (2) $K_{eq}^{(1)}K_{eq}^{(2)}$

(3) $K_{eq}^{(1)} / K_{eq}^{(2)}$ (4) $K_{eq}^{(1)} + K_{eq}^{(2)}$

Sol. $A \rightleftharpoons B+C \quad K_{eq}^{(1)} = \dfrac{[B][C]}{[A]}$ .....(1)

$B+C \rightleftharpoons P \quad K_{eq}^{(2)} = \dfrac{[P]}{[B][C]}$ .....(2)

For

$A \rightleftharpoons P \quad K_{eq} = \dfrac{[P]}{[A]}$

Multiplying equation (1) & (2)

$$K_{eq}^{(1)} \times K_{eq}^{(2)} = \frac{[P]}{[A]} = K_{eq}$$

2. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is :-

(1) $C_V(T_2 - T_1)$ (2) $-RT \ln V_2/V_1$

(3) $-RT(V_2 - V_1)$ (4) zero

Sol. As the expansion is done in vaccum that is in absence of p_{ext} so

W = zero

3. The process that is NOT endothermic in nature is :-

(1) $Ar_{(g)} + e^- \rightarrow Ar^-_{(g)}$ (2) $H_{(g)} + e^- \rightarrow H^-_{(g)}$

(3) $Na_{(g)} \rightarrow Na^+_{(g)} + e^-$ (4) $O^-_{(g)} + e^- \rightarrow O^{2-}_{(g)}$

Sol. $H_{(g)} + e^- \rightarrow H^-$ is exothermic

rest of all endothermic process.

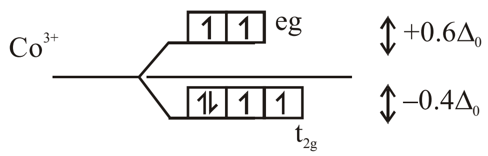
4. The crystal Field stabilization Energy (CFSE) of $[CoF_3(H_2O)_3](\Delta_0 < P)$ is :-

(1) $-0.8\ \Delta_0$ (2) $-0.4\ \Delta_0 + P$

(3) $-0.8\ \Delta_0 + 2P$ (4) $-0.4\ \Delta_0$

Sol. $[CoF_3(H_2O)_3]$ $\Delta_0 < P$

Means all ligands behaves as weak field ligands



$= [-0.4 \times 4 + 0.6 \times 2]\Delta_0$

$= [-1.6 + 1.2]\Delta_0$

$= [-0.4\Delta_0]$

5. The mechanism of action of "Terfenadine" (Seldane) is :-

(1) Activates the histamine receptor

(2) Inhibits the secretion of histamine

(3) Inhibits the action of histamine receptor

(4) Helps in the secretion of histamine

Sol. Seldane is an antihistamine drugs it inhibits the action of histamine receptor.

6. An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is :-

(1) Ca (2) Be (3) Mg (4) Sr

Sol. [Be]

$BeSO_4$ is water soluble

$Be(OH)_2$ is water insoluble

BeO is stable to heat

7. The reaction in which the hybridisation of the underlined atom is affected is :-

(1) $\underline{N}H_3 \xrightarrow{H^+}$

(2) $\underline{Xe}F_4 + SbF_5 \rightarrow$

(3) $H_2\underline{S}O_4 + NaCl \xrightarrow{420\ K}$

(4) $H_3\underline{P}O_2 \xrightarrow{\text{Disproportionation}}$

Sol. $XeF_4 + SbF_5 \rightarrow [XeF_3]^+[SbF_6]^-$

sp^3d^2 sp^3d sp^3d sp^3d^2

8. The one that can exhibit highest paramagnetic behaviour among the following is :-

gly = glycinato; bpy = 2, 2'-bipyridine

(1) $[Pd(gly)_2]$

(2) $[Ti(NH_3)_6]^{3+}$

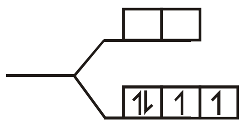
(3) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)

(4) $[Fe(en)(bpy)(NH_3)_2]^{2+}$

Sol. $[Co(OX)_2(OH)_2]^-$ $\Delta_0 > P$ [S.F.L]

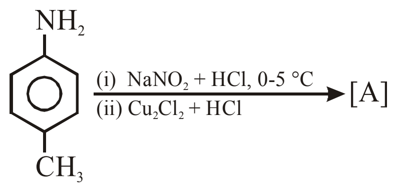
$Co = 3d^7\ 4s^2$

$Co^{+5} = 3d^4\ 4s^0$



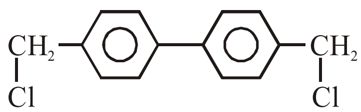
It has highest number of unpaired e⁻s. so it is most paramagnetic.

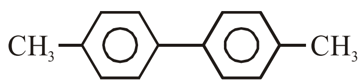
9. In the following reaction sequence, [C] is :-

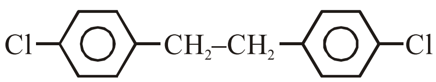


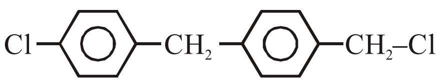
$\xrightarrow[hv]{Cl_2}$ [B] $\xrightarrow{\text{Na+dry ether}}$ [C]

(Major Product)

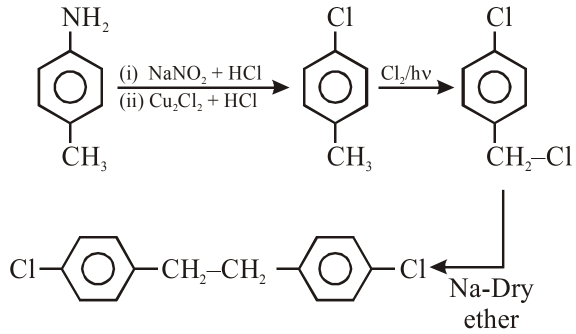
(1) 

(2) 

(3) 

(4) 

Sol.



10. A sample of red ink (a colloidal suspension) is prepared by mixing eosin dye, egg white, HCHO and water. The component which ensures stability of the ink sample is :-

(1) HCHO (2) Eosin dye

(3) Egg white (4) Water

11. The processes of calcination and roasting in metallurgical industries, respectively, can lead to :-

(1) Global warming and acid rain

(2) Photochemical smog and ozone layer depletion

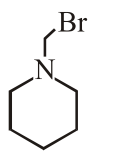
(3) Global warming and photochemical smog

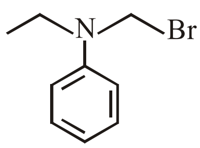
(4) Photochemical smog and global warming

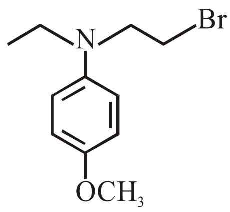
Sol. Due to industrial process SO_2 gas is released which is responsible for acid rain & global warming.

12. Which of the following compounds will form the precipitate with aq. $AgNO_3$ solution most readily ?

(1) 

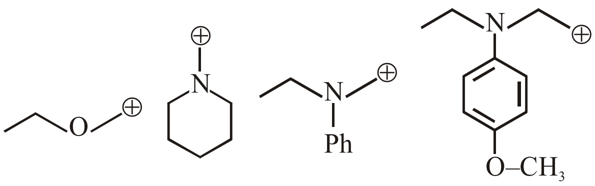
(2) 

(3) 

(4) 

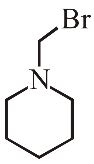
Sol. $R - x + aq.AgNO_3 \xrightarrow{R.D.S} R^{\oplus} + \underset{(PPT)}{Agx}$ (1)

So rate of P.P.T formation of Agx depend's on stability of carbocation (R^+)
In given question formed carbocation will be



(a) (b) (c) (d)

Most stable carbocation is (b) so



give fastest P.P.T of AgBr with aq $AgNO_3$

13. The molecule in which hybrid MOs involve only one d-orbital of the central atom is :-

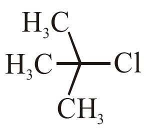
(1) $[Ni(CN)_4]^{2-}$ (2) $[CrF_6]^{3-}$

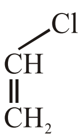
(3) BrF_5 (4) XeF_4

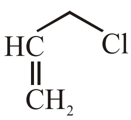
Sol. $[Ni(CN)_4]^{2-}$
dsp^2 hybridisation.

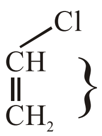
14. Among the following compounds, which one has the shortest C—Cl bond ?

(1) $H_3C–Cl$

(2) 

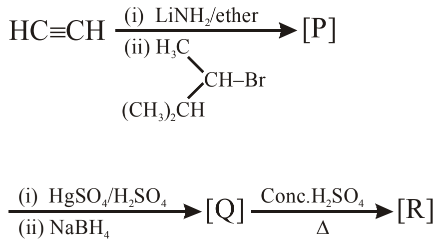
(3) 

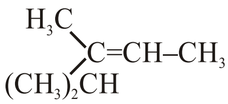
(4) 

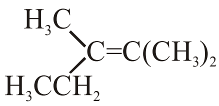
Sol. 

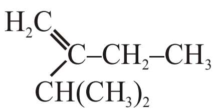
In option (3) C—Cl bond is shortest due to resonance of lone pair of –Cl.
Due to resonance C—Cl bond acquire partial double bond character.
Hence C—Cl bond length is least.

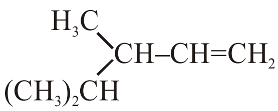
15. The major product [R] in the following sequence of reactions is :-

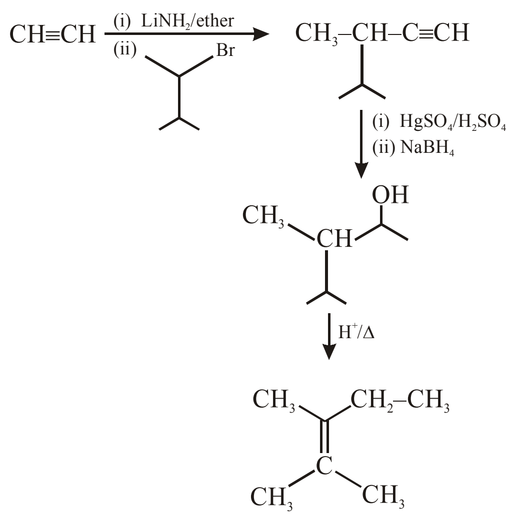


(1) 

(2) 

(3) 

(4) 

Sol. 

(major)

Now :- (i) $HgSO_4$/dil.H_2SO_4

(ii) $NaBH_4$

is convert triple bond into ketone and formed ketone is reduced by $NaBH_4$ and convert into Alcohol.

16. The incorrect statement(s) among (a) - (c) is (are) :-

(a) W(VI) is more stable than Cr(VI).

(b) in the presence of HCl, permanganate titrations provide satisfactory results.

(c) some lanthanoid oxides can be used as phosphors.

(1) (a) and (b) only (2) (a) only

(3) (b) and (c) only (4) (b) only

Sol. $KMnO_4$ will not give satisfactory result when it is titrated by HCl.

17. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M $AgNO_3$ and 0.1 M AuCl. The solution was electrolyzed at 2 V by passing a current of 1 A for 15 minutes. The metal/metals electrodeposited will be :-

$\left(E^0_{Ag^+/Ag} = 0.80V,\ E^0_{Au^+/Au} = 1.69V\right)$

(1) only silver

(2) only gold

(3) silver and gold in equal mass proportion

(4) silver and gold in proportion to their atomic weights

Sol. As voltage is '2V' so both Ag^+ & Au^+ will reduce and their equal gm equivalent will reduce so

gmeq Ag = gmeq of Au

$$\frac{Wt_{Ag}}{E_{qwt_{Ag}}} = \frac{Wt_{Au}}{E_{qwt_{Au}}}$$

$$\text{So } \frac{wt_{Ag}}{wt_{Au}} = \frac{E_{qwt_{Ag}}}{E_{qwt_{Au}}} = \frac{At\,wt_{Ag}}{At\,wt_{Au}}$$

18. The major product [B] in the following reactions is :-

$$CH_3-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_2-OCH_2-CH_3$$

$$\xrightarrow[Heat]{HI} [A]\text{ alcohol} \xrightarrow[\Delta]{H_2SO_4} [B]$$

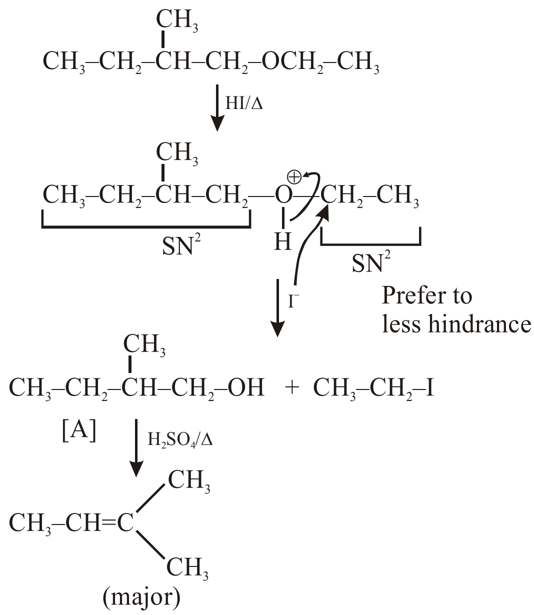
(1) $CH_3-CH_2-\overset{\overset{CH_3}{|}}{C}=CH_2$

(2) $CH_3-CH_2-CH=CH-CH_3$

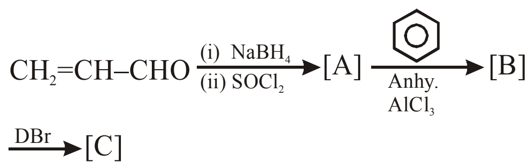
(3) $CH_2=CH_2$

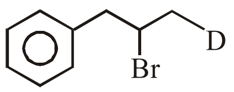
(4) $CH_3-CH=\overset{\overset{CH_3}{|}}{C}-CH_3$

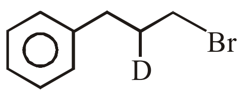
Sol.

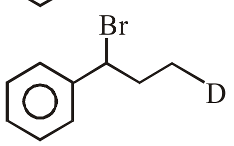


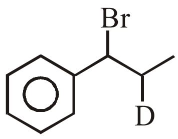
19. The major product [C] of the following reaction sequence will be :-



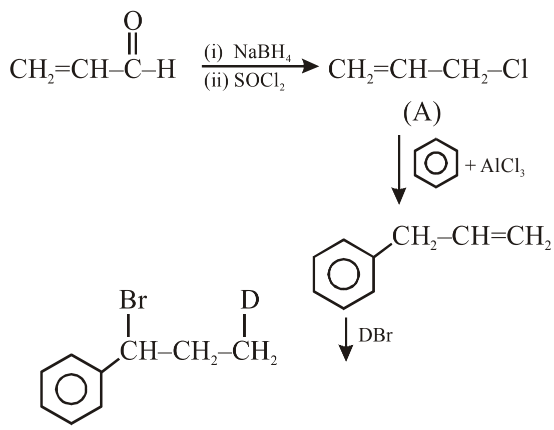
(1) 

(2) 

(3) 

(4) 

Sol.



20. The shortest wavelength of H atom is the Lyman series is λ_1. The longest wavelength in the Balmer series of He^+ is :-

(1) $\frac{5\lambda_1}{9}$ (2) $\frac{27\lambda_1}{5}$

(3) $\frac{9\lambda_1}{5}$ (4) $\frac{36\lambda_1}{5}$

Sol. As we know $\Delta E = \frac{hc}{\lambda}$

So $\lambda = \frac{hc}{\Delta E}$ for λ minimum i.e.

shortest; ΔE = maximum

for Lyman series n = 1 & for ΔE_{max}

Transition must be form n = ∞ to n = 1

So $\frac{1}{\lambda} = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda} = R_H Z^2(1-0)$

$\frac{1}{\lambda} = R \times (1)^2 \Rightarrow \lambda_1 = \frac{1}{R}$

For longest wavelength ΔE = minimum for Balmer series n = 3 to n = 2 will have ΔE minimum

for He^+ Z = 2

So $\frac{1}{\lambda_2} = R_H \times Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda_2} = R_H \times 4\left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{1}{\lambda_2} = R_H \times \frac{5}{9}$

$\lambda_2 = \lambda_1 \times \frac{9}{5}$

21. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3 \cdot xH_2O$. The normality of the solution is 0.1 N. The value of x is _______.

(The atomic mass of Na is 23g/mol) :-

Sol. Molar mass of $Na_2CO_3 \cdot xH_2O$

$\Rightarrow 23 \times 2 + 12 + 48 + 18x$

$\Rightarrow 46 + 12 + 48 + 18x$

$\Rightarrow (106 + 18x)$

$\text{Eqwt} = \frac{M}{2} = (53 + 9x)$

As n_{factor} in dissolution will be determined from net cationic or anionic charge; which is 2 so

$\text{Eqwt} = \frac{M}{2} = 53 + 9x$

$\text{Gmeq} = \frac{\text{wt}}{\text{Eqwt}} = \frac{1.43}{53 + 9x}$

$\text{Normality} = \frac{\text{Gmeq}}{V_{\text{litre}}}$

$\text{Normality} = 0.1 = \frac{\frac{1.43}{53+9x}}{0.1}$

As volume = 100 ml

= 0.1 Litre

So $10^{-2} = \frac{1.43}{53 + 9x}$

53 + 9x = 143

9x = 90

x = 10.00

22. The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm. The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is $x \times 10^{-3}$ atm. x is _____. (nearest integer) :-

Sol. Osmotic pressure = $\pi = i \times C \times RT$

For NaCl i = 2 so

$\pi_{NaCl} = i \times C_{NaCl} \times RT$ $\quad C_{NaCl}$ = conc. of NaCl

$0.1 = 2 \times C_{NaCl} \times RT$

$C_{NaCl} = \dfrac{0.05}{RT}$ $\quad C_{glucose}$ = conc. of glucose

For glucose i = 1 so

$\pi_{Glucose} = i \times C_{glucose} \times RT$

$0.2 = 1 \times C_{glucose} \times RT$

$C_{Glucose} = \dfrac{0.2}{RT}$ $\quad \eta_{NaCl}$ = No. of moles NaCl

η_{NaCl} in 1 L = $C_{NaCl} \times V_{Litre}$

$= \dfrac{0.05}{RT}$ $\quad \eta_{glucose}$ = No. of moles glucose

$\eta_{glucose}$ in 2 L = $C_{glucose} \times V_{Litre}$

$= \dfrac{0.4}{RT}$

$V_{Total} = 1 + 2 = 3L$

so Final conc. NaCl = $\dfrac{0.05}{3RT}$

Final conc. glucose = $\dfrac{0.4}{3RT}$

$\pi_{Total} = \pi_{NaCl} + \pi_{glucose}$

$= \left[i \times C_{NaCl} + C_{glucose}\right] \times RT$

$= \left(\dfrac{2 \times 0.05}{3RT} + \dfrac{0.4}{3RT}\right) \times RT$

$= \dfrac{0.5}{3}$ atm

= 0.1666 atm

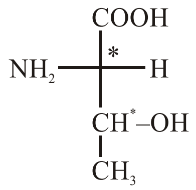
= 166.6×10^{-3} atm

$\Rightarrow 167.00 \times 10^{-3}$ atm

so x = 167.00

23. The number of chiral centres present in threonine is _________.

Sol. Structure of Threonine is :



S. 2-chiral center is present

24. Consider the following equations :

$2\ Fe^{2+} + H_2O_2 \rightarrow x\ A + y\ B$

(in basic medium)

$2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow x'\ C + y'\ D + z'\ E$

(in acidic medium)

The sum of the stoichiometric coefficients x, y, x', y' and z' for products A, B, C, D and E, respectively, is ___________.

Sol. $\left[Fe^{2+} \rightarrow Fe^{3+} + e^-\right] \times 2$

$$\frac{H_2O_2 + 2e^- \rightarrow 2HO^{\ominus}}{2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2HO^{\ominus}_{(aq)}}$$

x = 2 $\quad$ y = 2

$\left[8H^+ + MnO_4^- + 5e^- \rightarrow Mn^{2+} + 4H_2O\right] \times 2$

$\left[H_2O_2 \rightarrow O_{2(g)} + 2H^+ + 2e^-\right] \times 5$

$\Rightarrow 16H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)} + 10H^+$

$\Rightarrow 6H^+ + 2MnO_4^- + 5H_2O_2$

$\rightarrow 2Mn^{2+} + 8H_2O + 5O_{2(g)}$

So $\quad$ x' = 2 $\quad$ y' = 8 $\quad$ z' = 5

so $\quad$ x + y + x' + y' + z'

$\Rightarrow$ 2 + 2 + 2 + 8 + 5

$\Rightarrow$ 19

25. The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27 °C to 42 °C. Its energy of activation in J/mol is _________. (Take ln 5 = 1.6094; R = 8.314 J $mol^{-1}K^{-1}$)

Sol. T_1 = 300K T_2 = 315K

As per question $K_{T_2} = 5K_{T_1}$ as molecules activated are increased five times so k will increases 5 times

Now

$$\ln\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{Ea}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln 5 = \frac{Ea}{R}\left(\frac{15}{300 \times 315}\right)$$

So $$Ea = \frac{1.6094 \times 8.314 \times 300 \times 315}{15}$$

Ea = 84297.47 Joules/mole

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, |x| \le 1 \\ \frac{1}{2}(|x|-1), |x| > 1 \end{cases}$ is :

(1) continuous on R–{1} and differentiable on R – {–1, 1}.
(2) both continuous and differentiable on R – {–1}.
(3) continuous on R – {–1} and differentiable on R – {–1, 1}.
(4) both continuous and differentiable on R –{1}

Sol. $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & x \in (-\infty,-1] \cup [1,\infty) \\ -\frac{(x+1)}{2}, & x \in (-1,0] \\ \frac{x-1}{2}, & x \in (0,1) \end{cases}$

for continuity at x = –1

L.H.L. = $\frac{\pi}{4} - \frac{\pi}{4} = 0$

R.H.L. = 0
so, continuous at x = –1
for continuity at x = 1
L.H.L. = 0

R.H.L. = $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

so, not continuous at x = 1
For differentiability at x = –1

L.H.D. = $\frac{1}{1+1} = \frac{1}{2}$

R.H.D. = $-\frac{1}{2}$

so, non differentiable at x = –1

2. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to :

(1) 45 (2) 15
(3) 50 (4) 30

Sol. $n(X_i) = 10$. $\bigcup_{i=1}^{50} X_i = T, \Rightarrow n(T) = 500$

each element of T belongs to exactly 20 elements of $X_i \Rightarrow \frac{500}{20}$ = 25 distinct elements

so $\frac{5n}{6} = 25 \Rightarrow n = 30$

3. Let $\lambda \ne 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

(1) 36 (2) 27
(3) 9 (4) 18

Sol. $\alpha + \beta = 1, \ \alpha\beta = 2\lambda$

$\alpha + \beta = \frac{10}{3}, \quad \alpha\gamma = \frac{27\lambda}{3} = 9\lambda$

$\gamma - \beta = \frac{7}{3},$

$\frac{\gamma}{\beta} = \frac{9}{2} \Rightarrow \gamma = \frac{9}{2}\beta = \frac{9}{2} \times \frac{2}{3} \Rightarrow \gamma = 3$

$\frac{9}{2}\beta - \beta = \frac{7}{3}$

$\frac{9}{2}\beta = \frac{7}{3} \Rightarrow \beta = \frac{2}{3}$

$\alpha = 1 - \frac{2}{3} = \frac{1}{3}$

$2\lambda = \frac{2}{9} \Rightarrow \lambda = \frac{1}{9}$

$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$

4. The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is :-

(where C is a constant of integration.)

(1) $x-2\log_e(y+3x)=C$

(2) $x-\log_e(y+3x)=C$

(3) $x-\frac{1}{2}(\log_e(y+3x))^2 = C$

(4) $y + 3x-\frac{1}{2}(\log_e x)^2 = C$

Sol. $\ell n(y + 3x) = z$ (let)

$$\frac{1}{y+3x}\cdot\left(\frac{dy}{dx}+3\right) = \frac{dz}{dx} \quad ..(1)$$

$$\frac{dy}{dx}+3 = \frac{y+3x}{\ell n(y+3x)} \quad \text{(given)}$$

$$\frac{dz}{dx} = \frac{1}{z}$$

$$\Rightarrow z\,dz = dx \Rightarrow \frac{z^2}{2} = x + C$$

$$\Rightarrow \frac{1}{2}\ell n^2(y+3x) = x+C$$

$$\Rightarrow x - \frac{1}{2}(\ell n(y + 3x))^2 = C$$

5. Let $a_1, a_2, ..., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ..+ a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

(1) (2480, 249) (2) (2490, 249)

(3) (2490, 248) (4) (2480, 248)

Sol. $a_n = a_1 + (n - 1)d$

$\Rightarrow 300 = 1 + (n - 1) d$

$\Rightarrow (n - 1)d = 299 = 13 \times 23$

since, $n \in [15, 50]$

$\therefore$ n = 24 and d = 13

$a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$

$\Rightarrow a_{n-4} = 248$

$S_{n-4} = \frac{20}{2}\{1+248\} = 2490$

6. The distance of the point (1, – 2, 3) from the plane x–y+z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is :

(1) 7 (2) 1 (3) $\frac{1}{7}$ (4) $\frac{7}{5}$

Sol. equation of line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ passes through (1, –2, 3) is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$$

x = 2r + 1

y = 3r –2,

z = –6r + 3

So 2r + 1 – 3r + 2– 6r + 3 = 5

$\Rightarrow$ –7r + 1 = 0

$r = \frac{1}{7}$

$x = \frac{9}{7},\ y = \frac{-11}{7},\ z = \frac{15}{7}$

$$\text{Distance is} = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(2-\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2}$$

$$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{4+9+36}$$

$$= \frac{1}{7}\sqrt{49} = 1$$

7. Let f : (0, ∞) → (0, ∞) be a differentiable function such that f(1) = e and

$$\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$$

If f(x) = 1, then x is equal to :

(1) 2e (2) $\frac{1}{2e}$ (3) e (4) $\frac{1}{e}$

Sol. $L = \lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$

using L.H. rule

$L = \lim_{t\to x}\frac{2tf^2(x)-x^2\cdot 2f'(t).f(t)}{1}$

$\Rightarrow$ L = 2xf(x) (f(x) – x f'(x)) = 0 (given)

$\Rightarrow f(x) = xf'(x) \Rightarrow \int\frac{f'(x)dx}{f(x)}=\int\frac{dx}{x}$

$\Rightarrow \ell n\,|f(x)| = \ell n\,|x| + C$

$\because$ f(1) = e, x > 0, f(x) > 0

$\Rightarrow$ f(x) = ex, if f(x) = 1 $\Rightarrow x = \frac{1}{e}$

8. If the system of equations

x + y + z = 2

2x + 4y – z = 6

3x + 2y + λz = μ

has infinitely many solutions, then :

(1) λ – 2μ = –5 (2) 2λ – μ = 5

(3) 2λ + μ = 14 (4) λ + 2μ = 14

Sol. For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

Now $\Delta = 0 \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -1\\ 3 & 2 & \lambda\end{vmatrix}=0$

$\Rightarrow \lambda = \frac{9}{2}$

$\Delta_{x=0} \Rightarrow \begin{vmatrix}2 & 1 & 1\\ 6 & 4 & -1\\ \mu & 2 & -\frac{9}{2}\end{vmatrix} = 0$

$\Rightarrow \mu = 5$

For $\lambda = \frac{9}{2}$ & μ = 5, $\Delta_y = \Delta_z = 0$

Now check option 2λ + μ = 14

9. The minimum value of $2^{\sin x} + 2^{\cos x}$ is :-

(1) $2^{1-\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$

(3) $2^{1-\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

Sol. Usnign AM ≥ GM

$\Rightarrow \frac{2^{\sin x}+2^{\cos x}}{2}\geq\sqrt{2^{\sin x}\cdot 2^{\cos x}}$

$\Rightarrow 2^{\sin x}+2^{\cos x}\geq 2^{1+\left(\frac{\sin x+\cos x}{2}\right)}$

$\Rightarrow \min\left(2^{\sin x}+2^{\cos x}\right)=2^{1-\frac{1}{\sqrt{2}}}$

10. $\int_{\pi/6}^{\pi/3}\tan^3 x\cdot\sin^2 3x(2\sec^2 x\cdot\sin^2 3x+3\tan x\cdot\sin 6x)dx$

is equal to :

(1) $\frac{9}{2}$ (2) $-\frac{1}{9}$ (3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

Sol. $I = \int_{\pi/6}^{\pi/3}((2\tan^3 x\cdot\sec^2 x\cdot\sin^4 3x)+(3\tan^4 x\cdot\sin^3 3x\cdot\cos 3x))dx$

$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}d((\sin 3x)^4(\tan x)^4)$

$\Rightarrow I = ((\sin 3x)^4(\tan x)^4)_{\pi/6}^{\pi/3}$

$\Rightarrow I = -\frac{1}{18}$

11. The circle passing through the intersection of the circles, $x^2+y^2-6x=0$ and $x^2+y^2-4y=0$, having its centre on the line, 2x – 3y + 12 = 0, also passes through the point :

(1) (1, –3) (2) (–1, 3)

(3) (–3, 1) (4) (–3, 6)

Sol. Let S be the circle pasing through point of intersection of S_1 & S_2

$\therefore\ S = S_1 + \lambda S_2 = 0$

$\Rightarrow S : (x^2+y^2-6x)+\lambda(x^2+y^2-4y)=0$

$\Rightarrow S : x^2 + y^2 - \left(\frac{6}{1+\lambda}\right)x - \left(\frac{4\lambda}{1+\lambda}\right) y = 0$ ...(1)

Centre $\left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right)$ lies on

$2x - 3y + 12 = 0 \Rightarrow \lambda = -3$

put in (1) $\Rightarrow S : x^2 + y^2 + 3x - 6y = 0$

Now check options point $(-3, 6)$

lies on S.

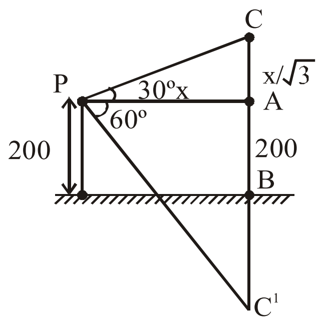
12. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to :

(1) 400 (2) $400\sqrt{3}$

(3) 100 (4) $200\sqrt{3}$

Sol. Let PA = x

For ΔAPC



$AC = \frac{PA}{\sqrt{3}} = \frac{x}{\sqrt{3}}$

$AC^1 = AB + BC^1$

$AC^1 = AB + BC$

$AC^1 = 400 + \frac{x}{\sqrt{3}}$

From ΔC^1PA : $AC^1 = \sqrt{3}\,PA$

$\Rightarrow \left(400 + \frac{x}{\sqrt{3}}\right) = \sqrt{3}x \Rightarrow x = (200)(\sqrt{3})$

from Δ APC : $PC = \frac{2x}{\sqrt{3}} \Rightarrow PC = 400$

13. If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then a + b is equal to :

(1) 57 (2) 33 (3) 24 (4) 9

Sol. $\alpha = \omega$ $(\omega^3 = 1)$

$\Rightarrow (2 + \omega)^4 = a + b\omega$

$\Rightarrow 2^4 + 4.2^3\,\omega + 6.2^2\omega^3 + 4.2\,.\,\omega^3 + \omega^4 = a + b\omega$

$\Rightarrow 16 + 32\,\omega + 24\,\omega^2 + 8 + \omega = a + b\omega$

$\Rightarrow 24 + 24\,\omega^2 + 33\omega = a + b\omega$

$\Rightarrow -24\omega + 33\omega = a + b\omega$

$\Rightarrow a = 0,\ b = 9$

14. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\frac{31}{61}$ (2) $\frac{5}{6}$

(3) $\frac{5}{31}$ (4) $\frac{30}{61}$

Sol. $P(6) = \frac{1}{6}$, $P(7) = \frac{5}{36}$

P(A) = W + FFW + FFFFW + ...

$= \frac{1}{6} + \frac{5}{6} \times \frac{31}{36} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{31}{36}\right)^2\frac{1}{6} + ...$

$= \frac{\frac{1}{6}}{1 - \frac{155}{216}} = \frac{36}{61}$

15. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P $(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :-

(1) $7x - 4y = 1$ (2) $4x - 2y = 1$

(3) $4x - 3y = 2$ (4) $8x - 2y = 5$

Sol. Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

directrix : $x = \frac{a}{e} = 4$ & $e = \frac{1}{2}$

$\Rightarrow a = 2$ & $b^2 = a^2(1-e^2) = 3$

$\Rightarrow$ Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

P is $\left(1, \frac{3}{2}\right)$

Normal is : $\frac{4x}{1}-\frac{3y}{3/2}=4-3$

$\Rightarrow 4x - 2y = 1$

16. Contrapositive of the statement:

'If a function f is differentiable at a, then it is also continuous at a', is :-

(1) If a function f is continuous at a, then it is not differentiable at a.

(2) If a function f is not continuous at a, then it is differentiable at a.

(3) If a function f is not continuous at a, then it is not differentiable at a.

(4) If a function f is continuous at a, then it is differentiable at a.

Sol. p = function is differantiable at a

q = function is continuous at a

contrapositive of statement $p \rightarrow q$ is

$\sim q \rightarrow \sim p$

17. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y= x^2 -1$ below the x-axis, is :

(1) $\frac{4}{3\sqrt{3}}$ (2) $\frac{1}{3\sqrt{3}}$ (3) $\frac{4}{3}$ (4) $\frac{2}{3\sqrt{3}}$

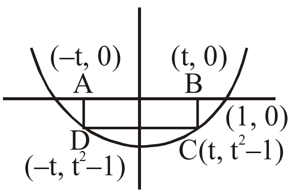
Sol. Area (A) = $2t \cdot (1 - t^2)$

$(0 < t < 1)$

$A = 2t - 2t^3$

$\frac{dA}{dt}=2-6t^2$

$t = \frac{1}{\sqrt{3}}$



$\Rightarrow A_{max} = \frac{2}{\sqrt{3}}\left(1-\frac{1}{3}\right)=\frac{4}{3\sqrt{3}}$

18. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :-

(1) 792 (2) 252 (3) 462 (4) 330

Sol. Let n + 5 = N

${}^{N}C_{r-1} : {}^{N}C_{r} : {}^{N}C_{r+1} = 5 : 10 : 14$

$\Rightarrow \frac{{}^{N}C_{r}}{{}^{N}C_{r-1}}=\frac{N+1-r}{r}=2$

$\frac{{}^{N}C_{r+1}}{{}^{N}C_{r}}=\frac{N-r}{r+1}=\frac{7}{5}$

$\Rightarrow$ r = 4, N = 11

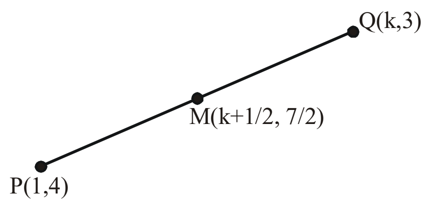
$\Rightarrow (1 + x)^{11}$

Largest coefficient = ${}^{11}C_6 = 462$

19. If the perpendicular bisector of the line segment joining the points P (1, 4) and Q (k, 3) has y-intercept equal to –4, then a value of k is :-

(1) $\sqrt{15}$ (2) –2 (3) $\sqrt{14}$ (4) –4

Sol.



Slope = $m=\frac{1}{1-k}$

Equation of $\perp^r$ bisector is

y + 4 = (k –1) (x– 0)

$\Rightarrow$ y + 4 = x(k –1)

$\Rightarrow \frac{7}{2}+4=\frac{k+1}{2}(k-1)$

$\Rightarrow \frac{15}{2}=\frac{k^2-1}{2}\Rightarrow k^2=16\Rightarrow k=4,-4$

20. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$$x = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to :-

(1) $\frac{1}{2}$ (2) 4 (3) $\frac{3}{2}$ (4) 2

Sol. $Ax_1 = b_1$
$Ax_2 = b_2$
$Ax_3 = b_3$

$$\Rightarrow \quad |A|\begin{vmatrix}1 & 0 & 0\\1 & 2 & 0\\1 & 1 & 1\end{vmatrix} = \begin{vmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{vmatrix}$$

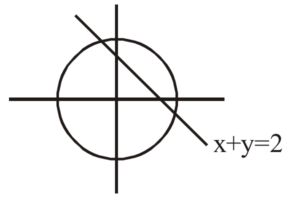
$$\Rightarrow |A| = \frac{4}{2} = 2$$

21. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ____

Sol. Ways $= {}^6C_4 \cdot 1^4 \cdot 3^2$
$= 15 \times 9$
$= 135$

22. Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ____

Sol.



Let P $(3\cos\theta, 3\sin\theta)$
Q $(-3\cos\theta, -3\sin\theta)$

$$\Rightarrow \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - 4|}{2}$$

$$\Rightarrow \alpha\beta = \frac{5 + 9\sin 2\theta}{2} \le 7$$

23. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\le$ x respectively of a real number x. If $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n)$, $(n \in N, n > 1)$ are three consecutive terms of a G.P., then n is equal to____

Sol. $$\int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\int_0^1 x\,dx = \frac{n}{2}$$

$$\int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$$

$$\Rightarrow \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2}\cdot 10\cdot n(n-1) \quad (\text{where } n > 1)$$

$$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$$

24. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\left|\hat{i}\times(\vec{a}\times\hat{i})\right|^2 + \left|\hat{j}\times(\vec{a}\times\hat{j})\right|^2 + \left|\hat{k}\times(\vec{a}\times\hat{k})\right|^2$ is equal to ____

Sol. $\Sigma|\vec{a} - (\vec{a}\cdot i)i|^2$

$\Rightarrow \quad \Sigma\left(|a|^2 + (\vec{a}\cdot i)^2 - 2(\vec{a}\cdot i)^2\right)$

$\Rightarrow \quad 3|\vec{a}|^2 - \Sigma(\vec{a}\cdot i)^2$

$\Rightarrow \quad 2|\vec{a}|^2$

$\Rightarrow \quad 18$

25. If the variance of the following frequency distribution :

Class :	10–20	20–30	30–40
Frequency :	2	x	2

is 50, then x is equal to ____

Sol. $\because$ Variance is independent of shifting of origin

$\Rightarrow$ x_i : 15 25 35 or –10 0 10
f_i : 2 x 2 2 x 2

$$\Rightarrow \text{Variance } (\sigma^2) = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\vec{x})^2$$

$$\Rightarrow 50 = \frac{200+0+200}{x+4} - 0 \qquad \{\bar{x} = 0\}$$

$\Rightarrow$ 200 + 50x = 200 + 200

$\Rightarrow$ x = 4