FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :-

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb
(3) 3.67×10^{-4} Wb (4) 3.67×10^{-3} Wb

Sol. $\phi_q = \dfrac{\mu_0 i_1 R^2}{2(R^2+x^2)^{\frac{3}{2}}} \times \pi r^2 = 10^{-3}$

$\phi_p = \dfrac{\mu_0 i_2 r^2}{2(r^2+x^2)^{\frac{3}{2}}} \times \pi R^2$

$\dfrac{\phi_P}{\phi_Q} = \dfrac{i_2}{i_1} \cdot \dfrac{(R^2+x^2)^{\frac{3}{2}}}{(r^2+x^2)^{\frac{3}{2}}} = \dfrac{\phi_P}{10^{-3}}$

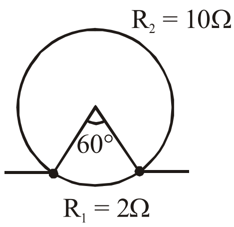
$\dfrac{2}{3} = \dfrac{\phi_P}{10^{-3}}$

$\phi_P = 6.67 \times 10^{-4}$.

2. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :-

(1) $\dfrac{12}{5}\Omega$ (2) $\dfrac{5}{3}\Omega$ (3) $\dfrac{5}{2}\Omega$ (4) $\dfrac{7}{2}\Omega$

Sol.



$R = \dfrac{\rho\ell^2}{A\ell D} d = \dfrac{\rho d \ell^2}{m}$

$R \propto \ell^2$

$R = 12\Omega$ (new resistance of wire)

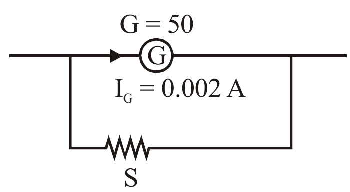
$R_1 = 2\Omega \quad R_2 = 10\Omega$

$R_{eq} = \dfrac{10\times 2}{10+2} = \dfrac{5}{3}\Omega$.

3. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A ?

(1) 0.2 ohm (2) 0.002 ohm
(3) 0.02 ohm (4) 0.5 ohm

Sol.



$S(0.5 - 0.002) = 50 \times 0.002$

$S = \dfrac{50\times 0.002}{(0.5-0.002)} = \dfrac{0.1}{0.498} = 0.2$

4. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \dfrac{b^2}{4c}$ (2) $a + \dfrac{b^2}{c}$

(3) $a + \dfrac{b^2}{2c}$ (4) $a + \dfrac{b^2}{3c}$

Sol. $x = at + bt^2 - ct^3$

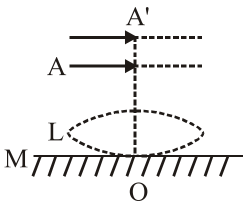
$v = \dfrac{dx}{dt} = a + 2bt - 3ct^2$

$a = \dfrac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \dfrac{b}{3c}$

$$v_{\left(\text{at } t=\frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)$$

$$= a + \frac{b^2}{3c}.$$

5. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, The pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_1 will be :-



(1) $\sqrt{2}$ (2) $\frac{4}{3}$

(3) $\sqrt{3}$ (4) $\frac{3}{2}$

Sol. $\frac{1}{f_1} = \frac{1}{2} \times \frac{2}{18} = \frac{1}{18}$

$$\frac{1}{f_2} = \frac{(\mu_1 - 1)}{-18}$$

when μ_1 is filled between lens and mirror

$$P = \frac{2}{18} - \frac{2}{18}(\mu_1 - 1) = \frac{2 - 2\mu_1 + 2}{18}$$

$$= F_m = -\left(\frac{18}{2 - \mu_1}\right)$$

$2 = 6 - 3\mu_1$

$3\mu_1 = 4$

$\mu_1 = 4/3$.

6. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a maganetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :-

(1) 10^{-3} (2) 10^{-1}

(3) 10^{-4} (4) 10^{-2}

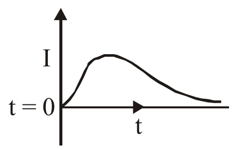
Sol. $\tau = \vec{M} \times \vec{B}$

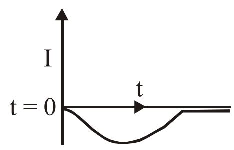
$C\theta = i\,N\,A\,B$

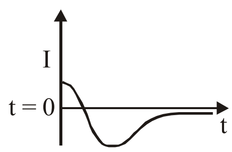
$$10^{-6} \times \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175 \times B$$

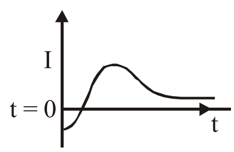
$B = 10^{-3}$ Tesla.

7. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t} (k > 0)$, as a function of time $(t \geq 0)$. counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by :-

(1) 

(2) 

(3) 

(4) 

Sol. $\phi_{outer} = (\mu_0 nKte^{-\alpha t})4\pi R^2$

$$\varepsilon = \frac{-d\phi}{dt} = -Ce^{-\alpha t}[1 - \alpha t]$$

$$i_{induced} = \frac{-Ce^{-\alpha t}\,[1 - at]}{(\text{Resistance})}$$

At t = 0 $i_{induced}$ = –ve.

8. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely ? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(1) 10^{-3}K (2) 10^{-4} K (3) 10^{-1} K (4) 10^{-5}K

Sol. By law of conservation of energy

$$\frac{1}{2}kx^2 = (m_1s_1 + m_2s_2)\ \Delta T$$

$$\Delta T = \frac{16\times10^{-2}}{4384} = 3.65 \times 10^{-5}.$$

9. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is :-

(1) $\lambda_x + \lambda_y$ (2) $\frac{\lambda_x\lambda_y}{\lambda_x+\lambda_y}$

(3) $\frac{\lambda_x\lambda_y}{|\lambda_x-\lambda_y|}$ (4) $\lambda_x - \lambda_y$

Sol. $(x)\longrightarrow \quad \longleftarrow(y) \quad = \quad (P)\longrightarrow$

By momentum conservation

$P_x - P_y = P_P$

$$\frac{h}{\lambda_x}-\frac{h}{\lambda_y}=\frac{h}{\lambda_p}$$

$$\lambda_p = \frac{\lambda_x\lambda_y}{|\lambda_y-\lambda_x|}.$$

10. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :-

(1) 5 : 3 (2) 2 : 1

(3) 4 : 3 (4) 3 : 1

Sol. Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$$\frac{x_1}{x_2}=3:1.$$

11. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600nm. coming from a distant object, the limit of resolution of the telescope is close to :-

(1) 1.5×10^{-7} rad (2) 2.0×10^{-7} rad

(3) 3.0×10^{-7} rad (4) 4.5×10^{-7} rad

Sol. Limit of resolution $= \frac{1.22\ \lambda}{d}$

$$= \frac{1.22\times600\times10^{-9}}{250\times10^{-2}}$$

$= 2.9 \times 10^{-7}$ rad.

12. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular accleration of 20 rad/s^2 must be applied about the axis for a duration of :-

(1) 2 s (2) 5s

(3) 2.5 s (4) 3 s

Sol. Given moment of inertia 'I' = 1.5 kgm^2

Angular Acc. "α" = 20 Rad/s^2

$KE = \frac{1}{2} I\omega^2$

$1200 = \frac{1}{2} 1.5 \times \omega^2$

$\omega^2 = \frac{1200 \times 2}{1.5} = 1600$

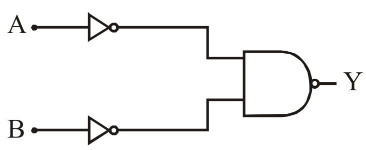
$\omega = 40 \text{ rad/s}^2$

$\omega = \omega_0 + \alpha t$

$40 = 0 + 20\ t$

$t = 2\text{sec.}$

13. The logic gate equivalent to the given logic circuit is :-



(1) OR (2) AND

(3) NOR (4) NAND

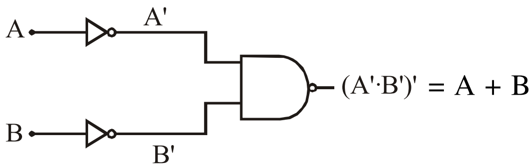
Sol. **Method 1**

Truth table can be formed as

A	B	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the Equivalent is "OR" gate.

Method 2



(OR GATE)

14. 50 W/m² energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1m² surface area will be close to ($c = 3 \times 10^8$ m/s) :-

(1) 15×10^{-8} N

(2) 35×10^{-8} N

(3) 10×10^{-8} N

(4) 20×10^{-8} N

Sol. Force on the surface (25% reflecting and rest absorbing)

$F = \frac{25}{100}\left(\frac{2I}{C}\right) + \frac{75}{100}\left(\frac{I}{C}\right) = \frac{125}{100}\left(\frac{I}{C}\right)$

$= \frac{125}{100} \times \left(\frac{50}{3 \times 10^8}\right) = 20.83 \times 10^{-8}$ N.

15. The area of a square is 5.29 cm². The area of 7 such squares taking into account the significant figures is :-

(1) 37 cm²

(2) 37.0 cm²

(3) 37.03 cm²

(4) 37.030 cm²

Sol. Total Area = $A_1 + A_2 + \ldots\ldots A_7$

$= A + A + \ldots\ldots\ldots$ 7 times

$= 37.03$ m².

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits.)

16. The physical sizes of the transmitter and receiver antenna in a communication system are :-

(1) proportional to carrier frequency

(2) inversely proportional to modulation frequency

(3) inversely proportional to carrier frequency

(4) independent of both carrier and modulation frequency

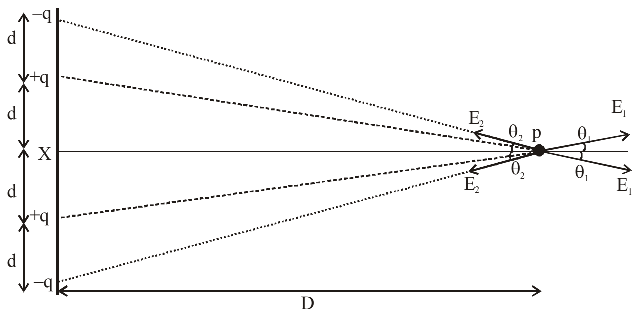
Sol. The physical size of antenna of reciver and transmitter both inversely proportional to carrier frequency.

17. Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x = D$, with $D >> d$, will behave as :-

(1) $E \propto \frac{1}{D}$ (2) $E \propto \frac{1}{D^3}$

(3) $E \propto \frac{1}{D^2}$ (4) $E \propto \frac{1}{D^4}$

Sol.



Electric field at p = $2E_1\cos\theta_1 - 2E_1\cos\theta_2$

$$= \frac{2Kq}{(d^2+D^2)} \times \frac{D}{(d^2+D^2)^{1/2}} - \frac{2Kq}{[(2d)^2+D^2]} \times \frac{D}{[(2d)^2+D^2]^{1/2}}$$

$$= 2KqD\left[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation $\because$ d << D

$$= \frac{2KqD}{D^3}\left[1-\frac{3}{2}\frac{d^2}{D^2} - \left(1-\frac{3\times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9kqd^2}{D^4}$$

18. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :-

(1) A has one vibrational mode and B has two

(2) Both A and B have a vibrational mode each

(3) A is rigid but B has a vibrational mode

(4) A has a vibrational mode but B has none

Sol. For A

$R = C_p - C_v = 7$

$C_v = \frac{fR}{2} = 22 \Rightarrow f = \frac{44}{7} = 6.3$

$f \simeq 6$ → 5 (Rotation + Translational), 1 (Vibration)

For B

$R = C_p - C_v = 9$

$C_v = \frac{fR}{2} = 21 \Rightarrow f = \frac{42}{9}$

$f \simeq 5$ → 5 (Rotation + Translational), 0 (Vibration)

19. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4-20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1 ?

(1) 40 (2) 100

(3) 25 (4) 50

Sol. $\vec{r} = 15t^2\hat{i} + (4-20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} + (-40t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$$

$$|\vec{a}| = 50 m/s^2.$$

20. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T :

(1) T/R^2 is a constant

(2) TR is a constant

(3) T^2/R^3 is a constant

(4) T/R is a constant

Sol. $m = \int_0^R \rho\, 4\pi r^2 dr$

$m = 4\pi KR$

$v \propto \sqrt{4\pi K}$

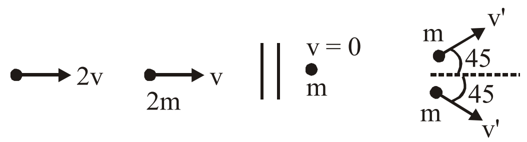
$$\frac{T}{R} = \frac{2\pi}{\sqrt{4\pi K}}.$$

21. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the origianl direction.

The speed of each of the moving particle will be :-

(1) $v/(2\sqrt{2})$ (2) $2\sqrt{2}v$

(3) $\sqrt{2}v$ (4) $v/\sqrt{2}$

Sol. 

Linear momentum conservation

$$m\ 2v + 2m\ v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$v' = 2\sqrt{2}\ v.$

22. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :-

(1) 0.5 (2) 0.7 (3) 0.6 (4) 0.8

Sol. In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \text{...(i)}$$

here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil

finally in equilibrium condition

$$V_b\rho_b\ g = \frac{V_b}{2}\rho_O g + \frac{V_b}{2}\rho_w g$$

$2\rho_b = \rho_O + \rho_w$

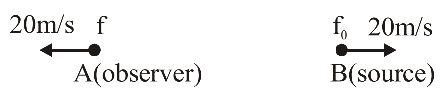
$$\Rightarrow \frac{\rho_O}{\rho_w} = \frac{3}{5} = 0.6$$

23. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ?

(speed of sound in air = 340 ms^{-1}) :-

(1) 2250 Hz (2) 2060 Hz

(3) 2150 Hz (4) 2300 Hz

Sol. 

$f = \frac{v+v_0}{v-v_s} f_0$ (v_0 & v_s is taken ⊕ when approaching each other)

$2000 = \frac{340+(-20)}{340-(-20)} f_0$

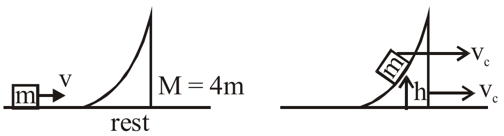
$f_0 = 2250$ Hz.

24. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :-

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{g}$

(3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{2g}$

Sol.



Applying Linear momentum conservation

$mv = (m + M)v_c$

$v_c = \frac{v}{5}$

applying work energy theorem

$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2} mv^2$

solve, $h = \frac{2v^2}{5g}$

25. A He^+ ion is in its first excited state. Its ionization energy is :-

(1) 6.04 eV (2) 13.60 eV

(3) 54.40 eV (4) 48.36 eV

Sol. Energy levels in Hydrogen like atom is given by

$E = -13.6 \frac{z^2}{n^2}$ eV

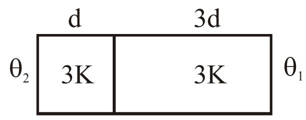
As He^+ is 1st excited state

$\therefore$ z = 2, n = 2

E = –13.6 eV

As total energy of He^+ in 1st excited state is –13.6 eV, ionisation energy should be +13.6eV.

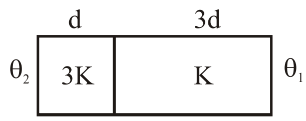
26. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is :-



(1) $\frac{\theta_2+\theta_1}{2}$ (2) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

(3) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$ (4) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$

Sol.



Let the temperature of interface be "θ"

$i_1 = i_2$ {Steady state conduction}

$\frac{3KA(\theta_2-\theta)}{d} = \frac{KA(\theta-\theta_1)}{3d}$

$\theta = \frac{9\theta_2}{10}+\frac{\theta_1}{10}$

27. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system , when the beads reach the opposite ends of the rod, will be :-

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+2m}$ (4) $\frac{M\omega_0}{M+6m}$

Sol. Applying angular momentum conservation, about axis of rotation

$L_i = L_f$

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + m\left(\frac{L}{2}\right)^2 \times 2\right)\omega$$

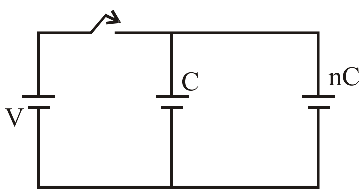
$$\Rightarrow \omega = \frac{M\omega_0}{M+6m}.$$

28. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :-

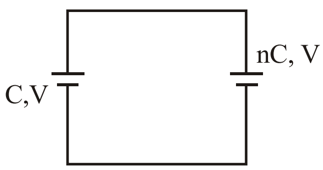
(1) $\frac{V}{K+n}$ (2) V

(3) $\frac{(n+1)V}{(K+n)}$ (4) $\frac{nV}{K+n}$

Sol.



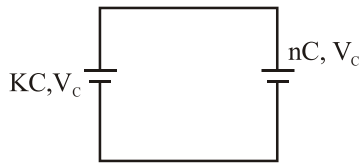
After fully charging, battery is disconnected



Total charge of the system = CV + nCV

= (n + 1)CV

After the insertion of dielectric of constant K



New potential (common)

$$V_C = \frac{\text{total charge}}{\text{total capacitance}}$$

$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}.$$

29. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25fs (femto second), it's approximate resistivity is :-

($m_e = 9.1 \times 10^{-31}$ kg)

(1) 10^{-5} Ωm (2) 10^{-6} Ωm

(3) 10^{-7} Ωm (4) 10^{-8} Ωm

Sol. $\rho = \frac{m}{ne^2\tau}$

$= 1.67 \times 10^{-8}$Ωm

30. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :-

(1) 320m/s, 120 Hz (2) 180m/s, 80 Hz

(3) 180m/s, 120 Hz (4) 320m/s, 80 Hz

Sol. $3\left(\frac{v}{2\ell}\right) = 240$

$$3\frac{v}{2\times 2} = 240$$

v = 320 m/s

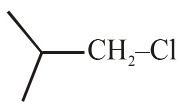
fundamental frequency = $\frac{v}{2\ell} = \frac{320}{2\times 2}$ = 80 Hz.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

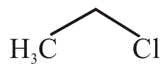
(Held On Tuesday 09th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

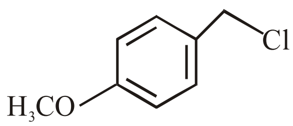
1. Increasing order of reactivity of the following compounds for S_N1 substitution is:



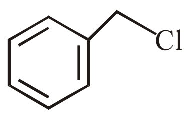
(A)



(B)



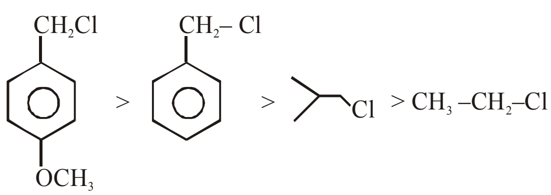
(C)



(D)

(1) (B) < (C) < (D) < (A)
(2) (A) < (B) < (D) < (C)
(3) (B) < (A) < (D) < (C)
(4) (B) < (C) < (A) < (D)

Sol. S_N1 Reactivity order



(C) (D) (A) (B)

Order C> D> A> B

2. The one that is not a carbonate is :

(1) bauxite (2) siderite
(3) calamine (4) malachite

Sol. 1. Bauxite–$AlO_x(OH)_{3-2x}$ where $0 < x < 1$
2. Siderite – $FeCO_3$
3. Calamine – $ZnCO_3$
4. Malachite – $CuCO_3.Cu(OH)_2$

3. During compression of a spring the work done is 10kJ and 2kJ escaped to the surroundings as heat. The change in internal energy, ΔU(inkJ) is:

(1) 8 (2) 12
(3) – 12 (4) –8

Sol. $\Delta U = q + w$

$q = -2kJ,\ W = 10kJ$

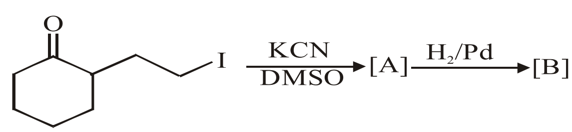
$\Delta U = 8kJ$

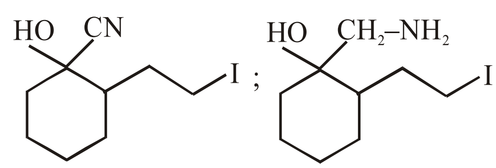
4. The amorphous form of silica is :

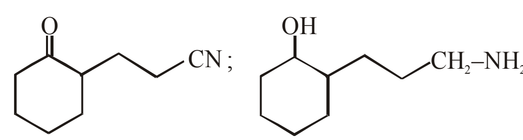
(1) quartz (2) kieselguhr
(3) cristobalite (4) tridymite

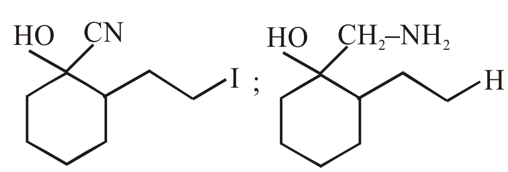
Sol. Kieselguhr is amorphous form of silica, it's a fact

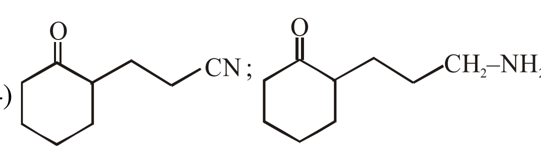
5. The major products A and B for the following reactions are, respectively:



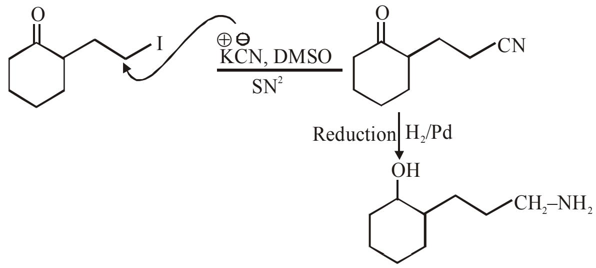
(1) 

(2) 

(3) 

(4) 

Sol.



6. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect ? (The Bohr radius is represented by a_0)

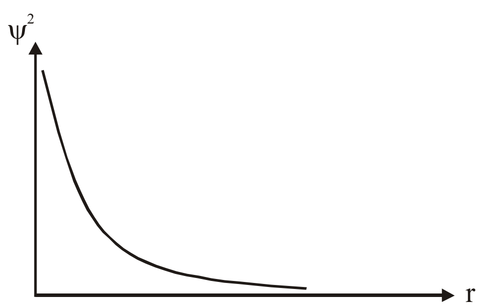
(1) The electron can be found at a distance $2a_0$ from the nucleus

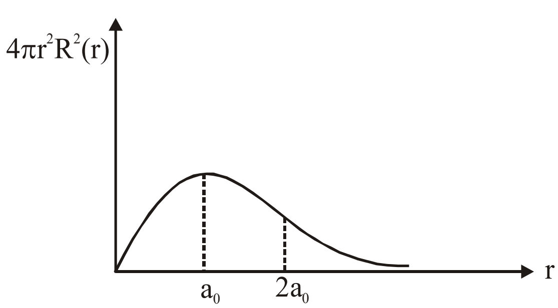
(2) The probability density of finding the electron is maximum at the nucleus.

(3) The magnitude of potential energy is double that of its kinetic energy on an average.

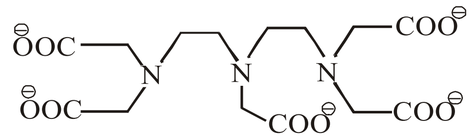
(4) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus.

Sol.





7. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are :



(1) 6 and 8 (2) 8 and 6

(3) 8 and 8 (4) 6 and 6

Sol. Towards common transition element and inner transition metal ion given ligand can have maximum denticities of 6 and 8 respectively.

8. The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

(II) Oxides of aluminium and gallium are amphoteric.

(III) Oxides of indium and thalliumare basic.

(1) (I), (II) and (III) (2) (II) and (III) only

(3) (I) and (III) only (4) (I) and (II) only

Sol. All statements are correct

$B_2O_3 \rightarrow$ acidic

Al_2O_3 & Ga_2O_3 are amphoteric

oxides of In & Tl are basic

9. Among the following species, the diamagnetic molecule is

(1) O_2 (2) NO (3) B_2 (4) CO

Sol. O_2, NO, B_2 are paramagnetic according to M.O.T. where as CO is diamagnetic.

10. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is :

(1) Lys-Asp (2) Ser-Lys

(3) Gln-Asp (4) Asp-Gln

Sol. Serine $\Rightarrow$ $HO-\underset{\underset{O}{\|}}{C}-\underset{\underset{NH_2}{|}}{CH}-CH_2-OH$

Lysine $\Rightarrow$ $H_2N-CH_2-CH_2-CH_2-CH_2-\underset{\underset{NH_2}{|}}{CH}-\overset{\overset{O}{\|}}{C}-OH$

Lysine has $-NH_2$ group hence gives ⊕ve carbyl amine test and serine has –OH group hence gives ⊕ve serric ammonium nitrate test

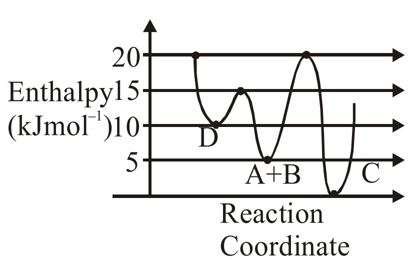
11. **Assertion:** For the extraction of iron, haematite ore is used.
Reason: Haematite is a carbonate ore of iron.
(1) Only the reason is correct.
(2) Both the assertion and reason are correct and the reason is the correct explanation for the assertion.
(3) Only the assertion is correct.
(4) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion.

Sol. Assertion is correct as Haemetite ore is used for extraction of Fe.
Haemetite is an oxide ore so reason is incorrect

12. 10 mL of 1mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as cube, what is its edge length?
(1) 2.0 pm (2) 2.0 nm
(3) 1.0 pm (4) 0.1 nm

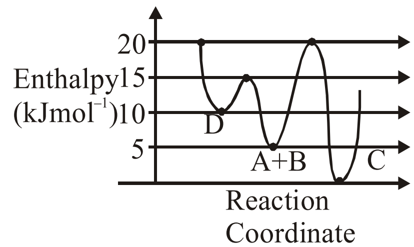
Sol. Millimoles $= 10 \times 10^{-3} = 10^{-2}$
Moles $= 10^{-5}$
No. of molecules $= 6 \times 10^{23} \times 10^{-5} = 6 \times 10^{+18}$
surface area occupied by one molecule
$= \dfrac{0.24}{6\times10^{18}} = 0.04\times10^{-18}cm^2$
$4 \times 10^{-20} = a^2$
$a = 2 \times 10^{-10}cm = 2pm$

13. Consider the given plot of enthalpy of the following reaction between A and B.
$A + B \rightarrow C + D$
Identify the incorrect statement.



(1) C is the thermodynamically stable product.
(2) Formation of A and B from C has highest enthalpy of activation.
(3) D is kinetically stable product.
(4) Activation enthalpy to form C is 5kJ mol^{-1} less than that to form D.

Sol. $A + B \rightarrow C + D$



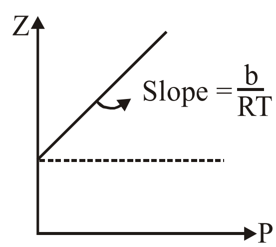
Activation enthalpy for C = 20 – 5 = 15kJ/mol
Activation enthalpy for D = 15 – 5 = 10kJ/mol

14. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as

$p = \dfrac{RT}{V-b}$ at T.

Here, b is the van der Waals constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) vs p?
(1) Ne (2) Ar
(3) Xe (4) Kr

Sol.



As b ↑ ⇒ slope ↑
Hence, Xe, will have highest slope

15. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?
(1) 0.20 (2) 0.05
(3) 0.10 (4) 0.15

Sol. 0.1 eq. of Ni^{+2} will be discharged.
No. of eq = (No of moles) × (n-factor)
0.1 = (No. of moles) × 2

No. of moles of Ni = $\dfrac{0.1}{2} = 0.05$

16. In the following reaction

carbonyl compound + MeOH $\xrightleftharpoons{HCl}$ acetal

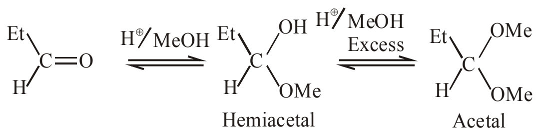
Rate of the reaction is the highest for :

(1) Acetone as substrate and methanol in stoichiometric amount

(2) Propanal as substrate and methanol in stoichiometric amount.

(3) Acetone as substrate and methanol in excess

(4) Propanal as substrate and methanol in excess

Sol. 

Acetone as substrate is less rective than propanal towards neucleophilic addition.

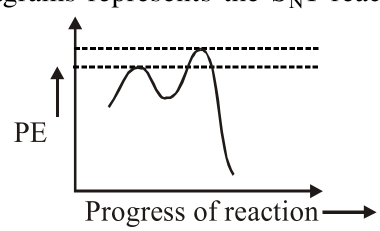
17. The structures of beryllium chloride in the solid state and vapour, phase, respectively,are :

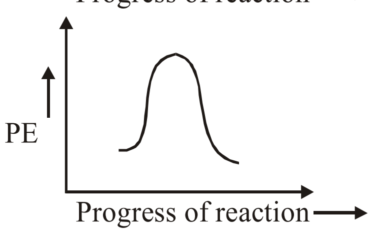
(1) chain and dimeric (2) chain and chain

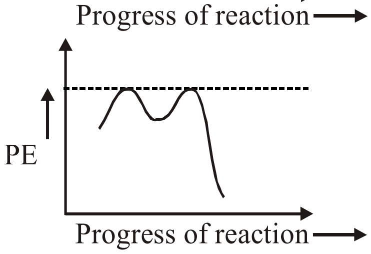
(3) dimeric and dimeric (4) dimeric and chain

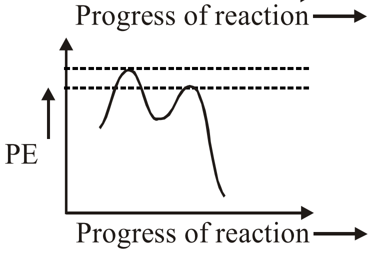
Sol. $BeCl_2$ exist as $(BeCl_2)_n$ polymeric chain in solid form, while $BeCl_2$ exist as dimer $(BeCl_2)_2$ in vapour phase.

18. Which of the following potential energy (PE) diagrams represents the S_N1 reaction?

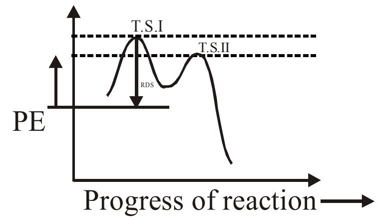
(1) 

(2) 

(3) 

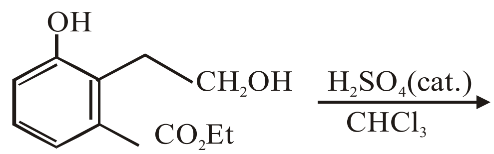
(4) 

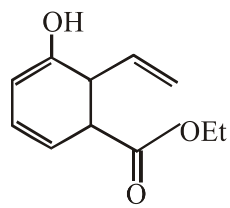
Sol. PE diagram for S_N1

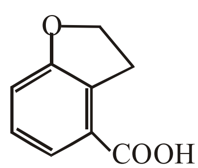


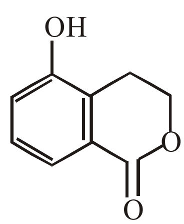
$S_N1$is two step reaction where in step (1) formation of carbocation is RDS

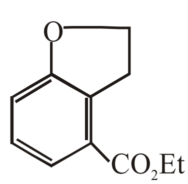
19. The major product of the following reaction is:



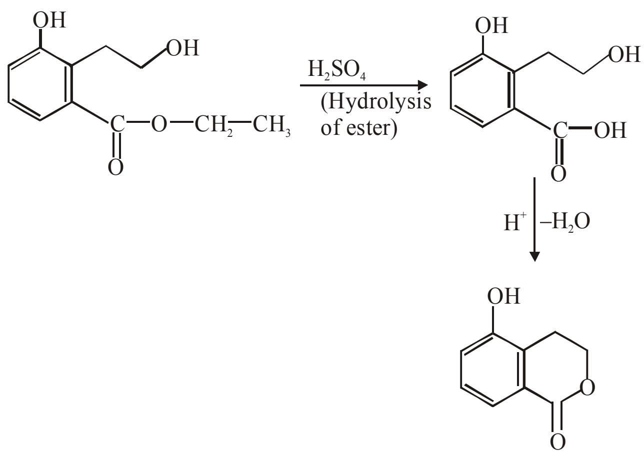
(1) 

(2) 

(3) 

(4) 

Sol.



20. The maximum number of possible oxidation states of actinoides are shown by

(1) berkelium (Bk) and californium (Cf)

(2) nobelium (No) and lawrencium (Lr)

(3) actinium (Ac) and thorium (Th)

(4) neptunium (Np) and plutonium (Pu)

Sol. Np and Pu show maximum no. of oxidations states starting from +3 to +7 all oxidation states.

21. Molal depression constant for a solvent is 4.0 kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is :

(Assume complete dissociation of the electrolyte)

(1) 0.12 K (2) 0.36 K

(3) 0.18 K (4) 0.24 K

Sol. K_f = 4 K-kg/mol

m = 0.03 mol/kg

i = 3

$\Delta T_f = iK_f \times m$

$\Delta T_f = 3 \times 4 \times 0.03 = 0.36K$

22. Noradrenaline is a /an

(1) Neurotransmitter

(2) Antidepressant

(3) Antihistamine

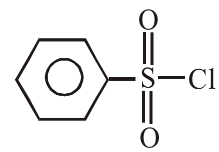
(4) Antacid

Sol. Nor adrenaline is a neutro transmitter and it belongs to catecholamine family that fanctions in brain & body as a hermone & neutro transmitter.

23. Hinsberg's reagent is :

(1) $C_6H_5SO_2Cl$ (2) C_6H_5COCl

(3) $SOCl_2$ (4) $(COCl)_2$

Sol. Hinsberg Reagenl ⇒ 

[Benzene Sulphonyl chloride]

24. The layer of atmosphere between 10 km to 50 km above the sea level is called as :

(1) troposphere (2) mesosphere

(3) stratosphere (4) thermosphere

Sol. It's a fact, the layer of atmosphere between 10km to 50km above sea level is called as stratosphere.

25. HF has highest boiling point among hydrogen halides, because it has :

(1) lowest dissociation enthalpy

(2) strongest van der Waals' interactions

(3) strongest hydrogen bonding

(4) lowest ionic character

Sol. HF has highest boiling point among hydrogen halides because it has strongest hydrogen bonding

26. What would be the molality of 20% (mass/mass) aqueous solution of KI?

(molar mass of KI = 166 g mol^{-1})

(1) 1.08 (2) 1.48

(3) 1.51 (4) 1.35

Sol. $\frac{w}{w}\% = 20$

100 gm solution has 20 gm KI

80 gm solvent has 20 gm KI

$$m = \frac{\frac{20}{166}}{\frac{80}{1000}} = \frac{20\times1000}{166\times80} = 1.506 \simeq 1.51\,\text{mol/kg}$$

27. Which of the following compounds is a constituent of the polymer

$$\left[HN - \overset{\overset{\displaystyle O}{\|}}{C} - NH - CH_2 \right]_n \ ?$$

(1) Formaldehyde (2) Ammonia

(3) Methylamine (4) N-Methyl urea

Sol. $\underset{\text{Urea}}{H_2N - \underset{\underset{\displaystyle O}{\|}}{C} - NH_2} + \underset{\text{formaldehyde}}{H - \underset{\underset{\displaystyle O}{\|}}{C} - H} \longrightarrow \left[HN - \underset{\underset{\displaystyle O}{\|}}{C} - NH - CH_2 \right]_n$

28. The correct statements among I to III are :

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes.

(II) Valence bond theory can predict quantitatively the magnetic properties of transtition metal complexes.

(III) Valence bond theory cannot distinguish ligands as weak and strong field ones.

(1) (I) and (II) only

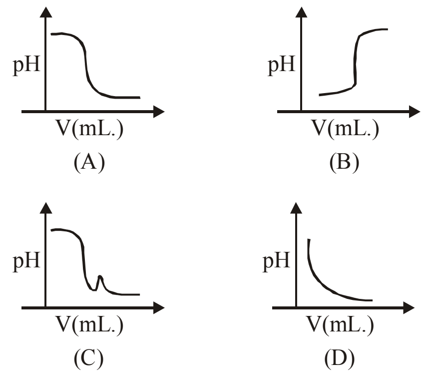
(2) (I), (II) and (III)

(3) (I) and (III) only

(4) (II) and (III) only

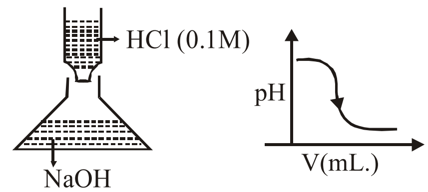
Sol. Based on NCERT, statement of limitations of VBT, I & III are correct

29. In an acid-base titration, 0.1 M HCl solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titraction mixture in this experiment?



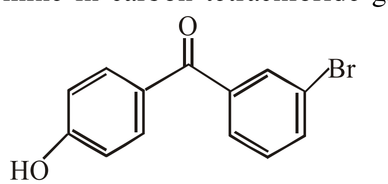
(1) (A) (2) (C) (3) (D) (4) (B)

Sol.



30. p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives:

(1)



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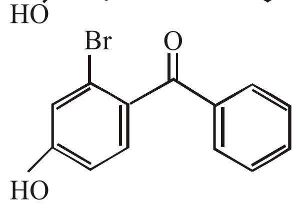

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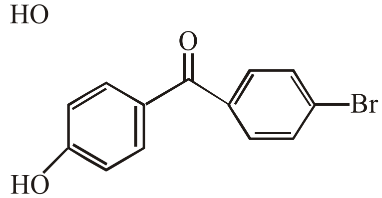
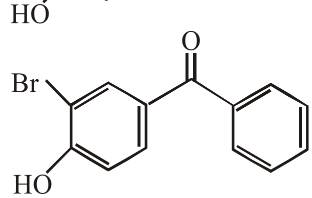
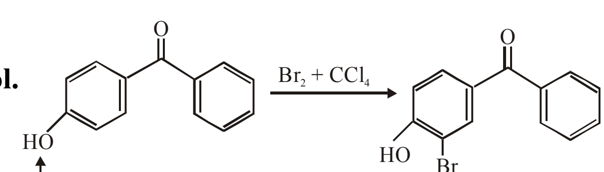


(4)



Sol.



⊕M group

OH is activating and ortho para directing group towards ESR

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS | TEST PAPER WITH ANSWER & SOLUTION

1. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

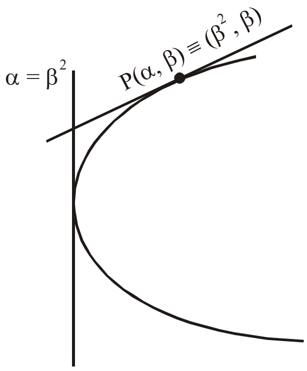
(1) $2\sqrt{2}+1$ (2) $\sqrt{2}-1$

(3) $\sqrt{2}+1$ (4) $2\sqrt{2}-1$

Sol. $T: y(\beta) = \frac{1}{2}(x+\beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$



$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$

$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2}+\frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2}+\frac{1}{2}$

$\frac{\beta^2}{4} = \frac{1+2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2}+1$

2. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :-

(1) 190 (2) 262 (3) 225 (4) 157

Sol. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$n^2 + n + 198 = 2(n^2 + 4 - 4n)$

$n^2 - 9n - 190 = 0$

$n^2 - 19n + 10 - 190 = 0$

$n(n - 19) + 10(n - 19) = 0$

$n = 19$

3. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim\limits_{x\to 2}\int_6^{f(x)}\frac{2t\,dt}{(x-2)}$ is :-

(1) 0 (2) $2f'(2)$

(3) $12f'(2)$ (4) $24f'(2)$

Sol. $\lim\limits_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{x-2}$

L Hopital Rule

$\lim\limits_{x\to 2}\frac{2f(x)f'(x)}{1} = 2f(2) = f'(2) = 12f'(2)$

4. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trival solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:-

(1) $\frac{3}{4}$ (2) -4 (3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$

By solving $K = \frac{9}{2}$

$2x + 3y - z = 0$ ...(1)

$x + \frac{9}{2}y - 2z = 0$ ...(2)

$2x - y + z = 0 \quad \ldots(3)$

$(1)-(3) \Rightarrow 4y - 2z = 0$

$2y = z \quad \ldots(4)$

$\boxed{\frac{y}{z} = \frac{1}{2}} \quad \ldots(5)$

put z from eqn. (4) into (1)

$2x + 3y - 2y = 0$

$2x + y = 0$

$\boxed{\frac{x}{y} = -\frac{1}{2}} \quad \ldots(6)$

$\frac{(6)}{(5)} \boxed{\frac{z}{x} = -4}$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + K = \frac{1}{2}$

5. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :-

(1) (–4, 6) (2) (6, –2)

(3) (–6, 4) (4) (4, –2)

Sol. Circle touches internally

$C_1(0, 0);\ r_1 = 2$

$C_2 : (-3, -4);\ r_2 = 7$

$C_1C_2 = |r_1 - r_2|$

$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent

$6x + 8y - 20 = 0$

$3x + 4y = 10$

(6, –2) satisfy it

6. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11th term is :-

(1) –25 (2) 25

(3) –36 (4) –35

Sol. $a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If d = 4 then Ist term = 7

If d = –4 then Ist term = 15

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

7. The value of the integral $\int_0^1 x \cot^{-1}(1 - x^2 + x^4)dx$ is :-

(1) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$

(3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \log_e 2$

Sol. $I = \int_0^1 x \tan\left(\frac{1}{1 + x^2(x^2 - 1)}\right)dx$

$I = \int_0^1 x\left(\tan^{-1} x^2 - \tan^{-1}(x^2 - 1)\right)dx$

$x^2 = t \Rightarrow 2xdx = dt$

$I = \frac{1}{2}\int_0^1\left(\tan^{-1} t - \tan^{-1}(t - 1)\right)dx$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1}(t - 1)\, dt$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt$

$\tan^{-1}t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta d\theta$

$\int_0^{\pi/4} \theta \cdot \sec^2\theta\, d\theta$

$I = (\theta.\tan\theta)\Big|_0^{\pi/4} - \int_0^{\pi/4} \tan\theta\, d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - \left(\ell n\sqrt{2} - 0\right)$

$= \frac{\pi}{4} - \frac{1}{2}\ell n 2$

8. The value of sin 10º sin30º sin50º sin70º is :-

(1) $\frac{1}{36}$ (2) $\frac{1}{32}$

(3) $\frac{1}{18}$ (4) $\frac{1}{16}$

Sol. (sin 10º sin 30º sin 70º) sin 30º

$$\frac{1}{4}(\sin 30^\circ)^2 = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$

9. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then:-

(1) $5\text{Im}(\omega) < 1$ (2) $4\text{Im}(\omega) > 5$

(3) $5\text{Re}(\omega) > 1$ (4) $5\text{Re}(\omega) > 4$

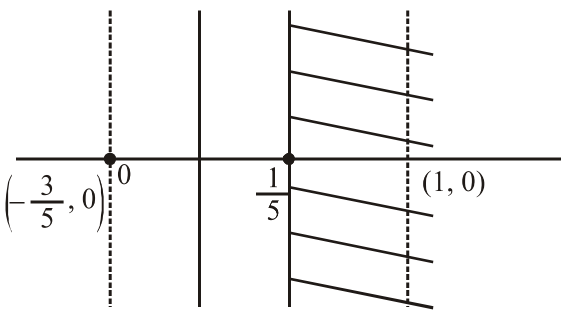
Sol. $|z| < 1$

$5\omega(1 - z) = 5 + 3z$

$5\omega - 5\omega z = 5 + 3z$

$$z = \frac{5\omega - 5}{3 + 5\omega}$$

$$|z| = 5\left|\frac{\omega - 1}{3 + 5\omega}\right| < 1$$



$5|\omega - 1| < |3 + 5\omega|$

$$5|\omega - 1| < 5\left|\omega + \frac{3}{5}\right|$$

$$|\omega - 1| < 5\left|\omega - \left(-\frac{3}{5}\right)\right|$$

10. If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :-

(1) 964 (2) 625

(3) 227 (4) 232

Sol. $$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$15r = 2n - 2r + 2$

$$\boxed{17r = 2n + 2}$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$$\frac{r+1}{n-r} = \frac{3}{14}$$

$14r + 14 = 3n - 3r$

$$\begin{array}{c} 3n - 17r = 14 \\ 2n - 17r = -2 \\ \hline n = 16 \end{array}$$

$17r = 34$, $r = 2$

${}^{16}C_1$, ${}^{16}C_2$, ${}^{16}C_3$

$$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$$

$$\frac{680 + 16}{3} = \frac{696}{3} = 232$$

11. If $\cos x \dfrac{dy}{dx} - y\sin x = 6x, (0 < x < \dfrac{\pi}{2})$ and

$y\left(\dfrac{\pi}{3}\right) = 0$, then $y\left(\dfrac{\pi}{6}\right)$ is equal to :-

(1) $-\dfrac{\pi^2}{4\sqrt{3}}$ (2) $-\dfrac{\pi^2}{2}$

(3) $-\dfrac{\pi^2}{2\sqrt{3}}$ (4) $\dfrac{\pi^2}{2\sqrt{3}}$

Sol. $\dfrac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\dfrac{\pi}{3}\right) = 0; y\left(\dfrac{\pi}{6}\right) = ?$

$e^{\int pdx} = e^{-\int \tan x dx} = e^{\ell n \cos x} = \cos x$

$y \cdot \cos x = \int 6x \sec x \cos x\, dx$

$y.\cos x = \dfrac{6x^2}{2} + C$

$y = 3x^2\sec x + C \sec x$

$0 = 3\cdot\dfrac{\pi^2}{9}\cdot(2) + C(2)$

$2C = \dfrac{-2\pi^2}{3} \Rightarrow \boxed{C = -\dfrac{\pi^2}{3}}$

$y(\pi/6) = 3\cdot\dfrac{\pi^2}{36}\cdot\left(\dfrac{2}{\sqrt{3}}\right) + \left(\dfrac{2}{\sqrt{3}}\right)\cdot\left(-\dfrac{\pi^2}{3}\right)$

$\Rightarrow y = -\dfrac{\pi^2}{2\sqrt{3}}$

12. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1 (a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is :-

(1) $\dfrac{2}{5}$ (2) $\dfrac{2}{\sqrt{5}}$ (3) $\dfrac{\sqrt{2}}{5}$ (4) $\sqrt{\dfrac{2}{5}}$

Sol. $\left(\dfrac{-1}{a-1}\right)\left(\dfrac{-2}{a^2}\right) = -1$

$2 = -(a^2)(a - 1)$

$a^3 - a^2 + 2 = 0$

$(a + 1)(a^2 - 2a + 2) = 0$

$\therefore\ a = -1$

$\left.\begin{aligned} L_1 &: x - 2y + 1 = 0 \\ L_2 &: 2x + y - 1 = 0 \end{aligned}\right\}$

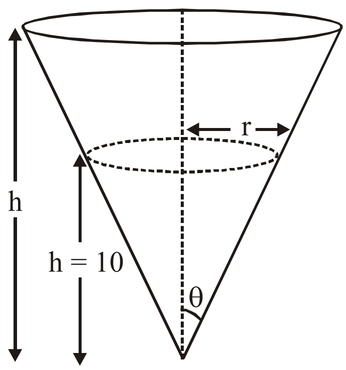
$0(0, 0)$ $P\left(\dfrac{1}{5}, \dfrac{3}{5}\right)$

$OP = \sqrt{\dfrac{1}{25} + \dfrac{9}{25}} = \sqrt{\dfrac{10}{25}} = \sqrt{\dfrac{2}{5}}$

13. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\dfrac{1}{2}\right)$. Water is poured into it at a constant rage of 5 cubic meter per minute. The the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is :-

(1) $2/\pi$ (2) $1/5\pi$ (3) $1/10\pi$ (4) $1/15\pi$

Sol. $\tan\theta = \dfrac{1}{2} = \dfrac{r}{h}$



$r = \dfrac{h}{2}$

$V = \dfrac{1}{3}\pi r^2 h$

$V = \dfrac{1}{3}\pi.\dfrac{h^3}{4}$

$\dfrac{dV}{dt} = \dfrac{\pi}{12}(3h)^2\left(\dfrac{dh}{dt}\right)$

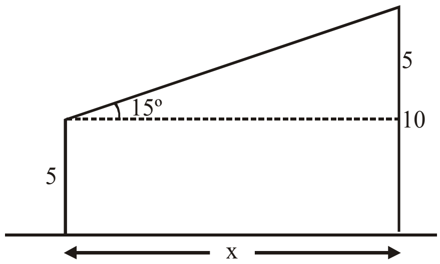
$5 = \dfrac{\pi}{4}.(100)\dfrac{dh}{dt} \Rightarrow \dfrac{dh}{dt} = \dfrac{1}{5\pi}$

14. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15° with ground. Then the distance (in m) between the poles, is :-

(1) $\frac{5}{2}(2+\sqrt{3})$ (2) $5(\sqrt{3}+1)$

(3) $5(2+\sqrt{3})$ (4) $10(\sqrt{3}-1)$

Sol. $\tan 15^\circ = \frac{5}{x}$



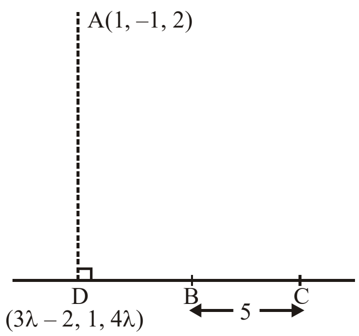
$2-\sqrt{13}=\frac{5}{x}$

$x=5(2+\sqrt{3})$

15. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :-

(1) $2\sqrt{34}$ (2) $\sqrt{34}$ (3) 6 (4) $5\sqrt{17}$

Sol.



$\overrightarrow{AD}\cdot(3\hat{i}+4\hat{k})=0$

$3(3\lambda-3)+0+4(4\lambda-2)=0$

$(9\lambda-9)+(16\lambda-8)=0$

$25\lambda=17 \Rightarrow \lambda=\frac{17}{25}$

$\therefore \overrightarrow{AD}=\left(\frac{51}{25}-3\right)\hat{i}+2\hat{j}+\left(\frac{68}{25}-2\right)\hat{k}$

$=\frac{24}{25}\hat{i}+2\hat{j}+\frac{18}{25}\hat{k}$

$|\overrightarrow{AD}|=\sqrt{\frac{576}{625}+4+\frac{324}{625}}$

$=\sqrt{\frac{900}{625}+4}=\sqrt{\frac{3400}{625}}$

$=\sqrt{34}\cdot\frac{10}{25}=\frac{2}{5}\sqrt{34}$

Area of $\Delta=\frac{1}{2}\times 5\times\frac{2\sqrt{34}}{5}=\sqrt{34}$

16. The total number of matrices

$$A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}, (x, y \in R, x \neq y) \text{ for which}$$

$A^TA = 3I_3$ is :-

(1) 6 (2) 2 (3) 3 (4) 4

Sol. $A^TA = 3I_3$

$$\begin{pmatrix}0 & 2x & 2x\\ 2y & y & -y\\ 1 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}=\begin{pmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{pmatrix}$$

$$\begin{pmatrix}8x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3\end{pmatrix}=\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

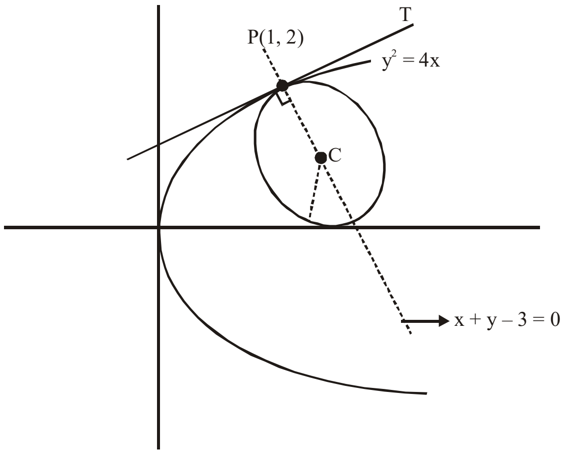
$x=\pm\sqrt{\frac{3}{8}};y=\pm\sqrt{\frac{1}{2}}$

17. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :-

(1) $4\pi(2-\sqrt{2})$ (2) $8\pi(3-2\sqrt{2})$

(3) $4\pi(3+\sqrt{2})$ (4) $8\pi(2-\sqrt{2})$

Sol.



Equation of circle is

$(x-1)^2 + (y-2)^2 + \lambda(x - y + 1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda - 2) + y(-4 - \lambda) + (5 + \lambda) = 0$

As cirlce touches x axis then $g^2 - c = 0$

$$\frac{(\lambda-2)^2}{4} = (5+\lambda)$$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

$\lambda^2 - 8\lambda - 16 = 0$

$$\lambda = \frac{8 \pm \sqrt{128}}{2}$$

$$\lambda = 4 \pm 4\sqrt{2}$$

$$\text{Radius} = \left|\frac{(-4-\lambda)}{2}\right|$$

Put λ and get least radius.

18. If the function $f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|x - \pi| + 3, x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is :-

(1) $\frac{2}{5-\pi}$ (2) $\frac{2}{\pi-5}$

(3) $\frac{2}{\pi+5}$ (4) $\frac{-2}{\pi+5}$

Sol. $f(x) \begin{cases} a|\pi - x| + 1; x \ge 5 \\ b|\pi - x| + 3; x > 5 \end{cases}$

$a|\pi - 5| + 1 = b|5 - \pi| + 3$

$a(5 - \pi) + 1 = b(5 - \pi) + 3$

$(a - b)(5 - \pi) = 2$

$$a - b = \frac{2}{5-\pi}$$

19. If $f(x) = [x] - \left[\frac{x}{4}\right], x \in R$, where [x] denotes the greatest integer function, then :

(1) Both $\lim\limits_{x\to4-} f(x)$ and $\lim\limits_{x\to4+} f(x)$ exist but are not equal

(2) $\lim\limits_{x\to4-} f(x)$ exists but $\lim\limits_{x\to4+} f(x)$ does not exist

(3) $\lim\limits_{x\to4+} f(x)$ exists but $\lim\limits_{x\to4-} f(x)$ does not exist

(4) f is continuous at x = 4

Sol. $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to4+} f(x) = \lim_{x\to4+}\left(\left([x] - \left[\frac{x}{4}\right]\right)\right) = 4 - 1 = 3$$

$$\lim_{x\to4-} f(x) = \lim_{x\to4-}\left([x] - \frac{x}{4}\right) = 3 - 0 = 3$$

$f(x) = 3$

$\therefore$ continuous at x = 4

20. If $\int e^{\sec x}(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)dx =$ $e^{\sec x}f(x) + C$, then a possible choice of f(x) is :-

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $x\sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x\tan x - \frac{1}{2}$ (4) $\sec x + \tan x + \frac{1}{2}$

Sol. $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x)\, dx$

$= e^{\sec x} f(x) + C$

Diff. both sides w.r.t. 'x'

$e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))$

$= e^{\sec x} \cdot \sec x \tan x\, f(x) + e^{\sec x} f'(x)$

$f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec x + c$

21. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :-

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$

(3) $10\sqrt{5}$ (4) $8\sqrt{5}$

Sol. $\text{SOR} = \frac{3}{m^2+1} \Rightarrow (\text{S.O.R})_{max} = 3$

when m = 0

$$x^2 - 3x + 1 = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^2| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha-\beta)^2 - \alpha\beta} \quad ((\alpha+\beta)^2 - \alpha\beta)\right|$

$= \left|\sqrt{9-4}\ (9-1)\right|$

$= \sqrt{5} \times 8$

22. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :-

(1) 12.8 (2) 13.5 (3) 13.9 (4) 13

Sol. Let population = 100

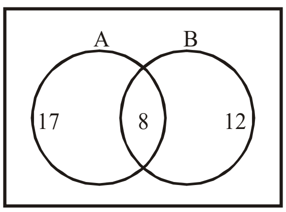
n(A) = 25

n(B) = 20

$n(A \cap B) = 8$

$n(A \cap \bar{B}) = 17$

$n(\bar{A} \cap B) = 12$



$\frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$

5.1 + 4.8 + 4 = 13.9

23. Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :-

(1) $63\sqrt{5}$ (2) $205\sqrt{5}$

(3) $17/\sqrt{5}$ (4) $11/\sqrt{5}$

Sol. $\lambda(x + y + z - 6) + 2x + 3y + z + 5 = 0$

$(\lambda + 2)x + (\lambda + 3)y + (\lambda + 1)z + 5 - 6\lambda = 0$

$\lambda + 1 = 0 \Rightarrow \lambda = -1$

P : x + 2y + 11 = 0

perpendicular distance $= \frac{11}{\sqrt{5}}$

24. If $P \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :-

(1) F, T, T (2) T, F, F

(3) T, T, F (4) F, F, F

Sol. $P \Rightarrow (q \vee r)$: F

P : T $q \vee r$: F

P : T : q : F : r : F

25. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :-

(1) $(1, 2) \cup (2, \infty)$

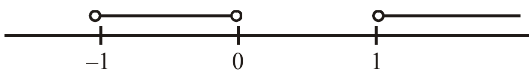
(2) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2$ $\quad x(x - 1)(x + 1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

26. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ...$ upto 11^{th} term is :-

(1) 915 (2) 946 (3) 945 (4) 916

Sol. $T_r = r(2r - 1)$

$S = \Sigma 2r^2 - \Sigma r$

$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

$S_{11} = \frac{2}{6}.(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$

27. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :-

(1) 7/3 (2) 9/4 (3) 7/2 (4) 8/3

Sol. $\frac{34+x}{2}=35$

x = 36

$$42=\frac{10+22+26+29+34+36+42+67+70+y}{10}$$

420 – 336 = y ⇒ y = 84

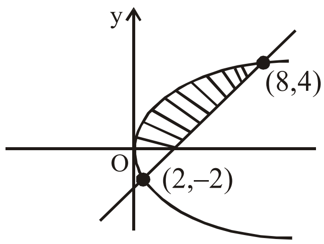
$$\frac{y}{x}=\frac{84}{36}=\frac{7}{3}$$

28. The area (in sq. units) of the region

$A=\{(x,y):\frac{y^2}{2}\le x\le y+4\}$ is :-

(1) $\frac{53}{3}$ (2) 18 (3) 30 (4) 16

Sol.



$y^2 = 2x$

$x - y - 4 = 0$

$(x-4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

x = 8, 2

y = 4, –2

$$A=\int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy$$

$$=\frac{y^2}{2}\bigg|_{-2}^{4}+4y\big|_{-2}^{4}-\frac{y^3}{6}\bigg|_{-2}^{4}$$

$$=(8-2)+4(6)-\frac{1}{6}(64+8)$$

= 6 + 24 – 12 = 18

29. If a unit vector $\vec{a}$ makes angles π/3 with $\hat{i}$, π/4 with $\hat{j}$ and θ∈(0, π) with $\hat{k}$, then a value of θ is :-

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$ (3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{4}$

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1$$

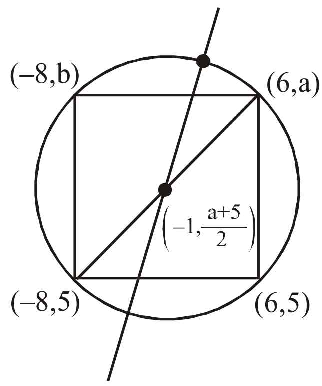
$$\cos^2\gamma=1-\frac{3}{4}=\frac{1}{4}$$

$$\cos^2\gamma=\pm\frac{1}{2}\Rightarrow\gamma=\frac{\pi}{3}\text{ or }\frac{2\pi}{3}$$

30. A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :-

(1) 72 (2) 84 (3) 98 (4) 56

Sol.



$$\frac{3(a+5)}{2}=-1+7$$

$$a+5=\frac{2(6)}{3}$$

a = – 1

sides = 6 and 14

⇒ A = 84