FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. A circuit connected to an ac source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this ?

(1) RC circuit with R = 1 kΩ and C = 1μF

(2) RL circuit with R = 1kΩ and L = 1mH

(3) RL circuit with R = 1 kΩ and L = 10 mH

(4) RC circuit with R = 1kΩ and C = 10 μF

Sol. Given phase difference = $\frac{\pi}{4}$

and ω = 100 rad/s

⇒ Reactance (X) = Resistance (R)

now by checking options

Option (1)

R = 1000 Ω and $X_C = \frac{1}{10^{-6}\times100} = 10^4\,\Omega$

Option (2)

R = 10^3 Ω and $X_L = 10^{-3}\times100 = 10^{-1}\,\Omega$

Option (3)

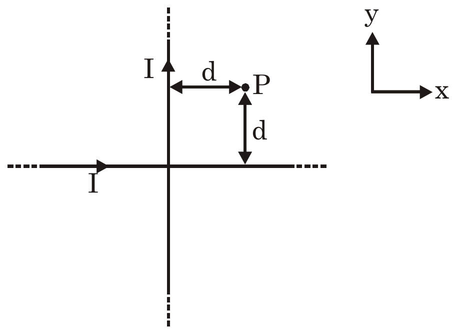
R = 10^3 Ω and $X_L = 10\times10^{-3}\times100 = 1\,\Omega$

Option (4)

R = 10^3 Ω and $X_C = \frac{1}{10\times10^{-6}\times100} = 10^3\,\Omega$

Clear option (4) matches the given condition

2. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be :



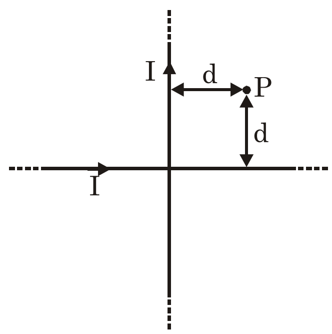
(1) Zero

(2) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

(3) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

(4) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

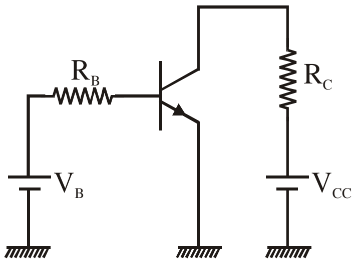
Sol.



Magnetic field at point P

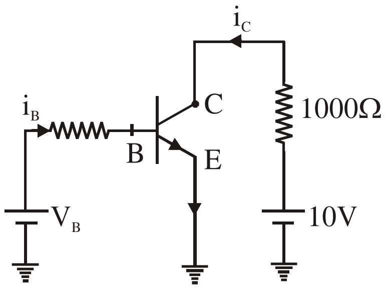
$$\vec{B}_{net} = \frac{\mu_0 i}{2\pi d}\left(-\hat{k}\right)+\frac{\mu_0 i}{2\pi d}\left(\hat{k}\right)=0$$

3. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, $R_C = 1k\Omega$ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation ?



(1) 100 μA (2) 7 μA (3) 40 μA (4) 10 μA

Sol.



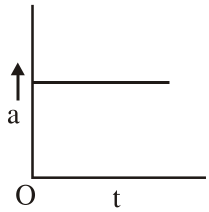
At saturation state, V_{CE} becomes zero

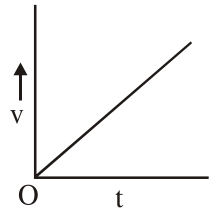
$$\Rightarrow i_C = \frac{10V}{1000\Omega} = 10mA$$

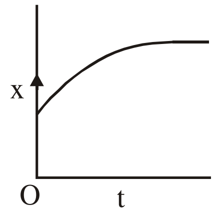
now current gain factor $\beta = \frac{i_C}{i_B}$

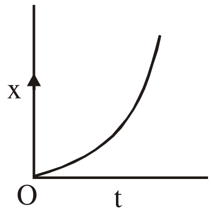
$$\Rightarrow i_B = \frac{10mA}{250} = 40\mu A$$

4. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

(A) 

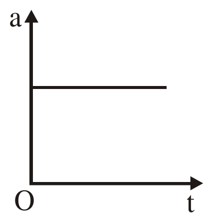
(B) 

(C) 

(D) 

(1) (A), (B), (C) (2) (A)
(3) (A), (B), (D) (4) (B), (C)

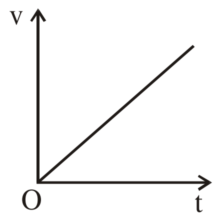
Sol. Given initial velocity u = 0 and acceleration is constant

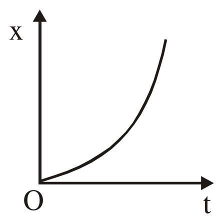


At time t

$v = 0 + at \Rightarrow v = at$

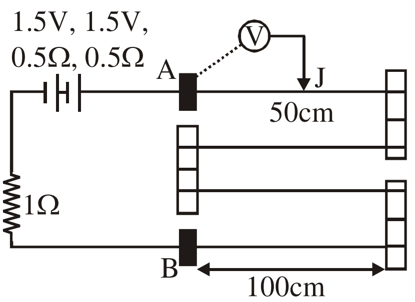
also $x = 0(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$





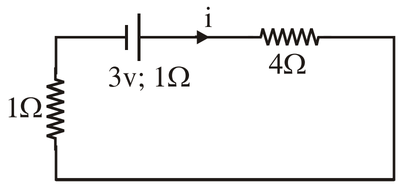
Graph (A) ; (B) and (D) are correct.

5. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :-



(1) 0.20 V (2) 0.25 V (3) 0.75 V (4) 0.50V

Sol.



Resistance of wire AB = 400 × 0.01 = 4Ω

i = $\frac{3}{6} = 0.5A$

Now voltmeter reading = i (Resistance of 50 cm length)

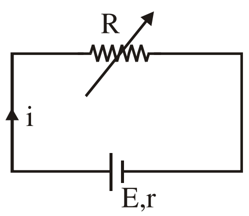
= (0.5A) (0.01 × 50) = 0.25 volt

6. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :-

(1) R = 1000 r (2) R = 0.001 r

(3) R = 2r (4) R = r

Sol.



Current i = $\frac{E}{r+R}$

Power generated in R

P = i^2R

P = $\frac{E^2R}{(r+R)^2}$

for maximum power $\frac{dP}{dR} = 0$

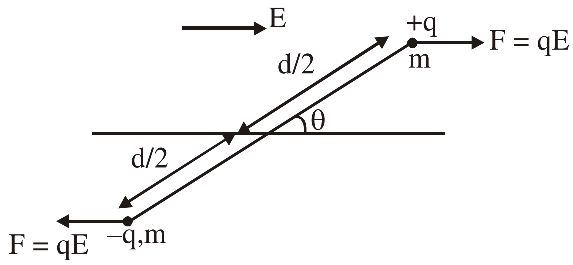
$$E^2\left[\frac{(r+R)^2 \times 1 - R \times 2(r+R)}{(r+R)^4}\right] = 0$$

⇒ r = R

7. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is :-

(1) $\sqrt{\frac{qE}{2md}}$ (2) $2\sqrt{\frac{qE}{md}}$ (3) $\sqrt{\frac{2qE}{md}}$ (4) $\sqrt{\frac{qE}{md}}$

Sol.



moment of inertia (I) = $m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by τ = Iα

(qE) (d sin θ) = $\frac{md^2}{2}.\alpha$

α = $\left(\frac{2qE}{md}\right)\sin\theta$

for small θ

⇒ α = $\left(\frac{2qE}{md}\right)\theta$

⇒ Angular frequency ω = $\sqrt{\frac{2qE}{md}}$

8. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70m, then the minimum height of the transmitting antenna should be :

(Radius of the Earth = 6.4×10^6 m).

(1) 40 m (2) 51 m (3) 32 m (4) 20 m

Sol. Range = $\sqrt{2Rh_T} + \sqrt{2Rh_R}$

$50 \times 10^3 = \sqrt{2 \times 6400 \times 10^3 \times h_T} + \sqrt{2 \times 6400 \times 10^3 \times 70}$

by solving h_T = 32 m

9. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to :-

(1) 1 (2) 2

(3) 0.1 (4) 5

Sol. mass densities of all nuclei are same so their ratio is 1.

10. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star :-

(1) 305×10^{-9} radian

(2) 152.5×10^{-9} radian

(3) 610×10^{-9} radian

(4) 457.5×10^{-9} radian

Sol. Limit of resolution of telescope = $\dfrac{1.22\lambda}{D}$

$\theta = \dfrac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = 305 \times 10^{-9}$ radian

11. The magnetic field of an electromagnetic wave is given by :-

$$\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right)\frac{Wb}{m^2}$$

The associated electric field will be :-

(1) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(\hat{i} - 2\hat{j}\right)\frac{V}{m}$

(2) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right)\frac{V}{m}$

(3) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right)\left(-2\hat{j} + \hat{i}\right)\frac{V}{m}$

(4) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(-\hat{i} + 2\hat{j}\right)\frac{V}{m}$

Sol. **If we use that direction of light propagation will be along $\vec{E} \times \vec{B}$. Then (4) option is correct.**

Detailed solution is as following.

magnitude of E = CB

$E = 3 \times 10^8 \times 1.6 \times 10^{-6} \times \sqrt{5}$

$E = 4.8 \times 10^2 \sqrt{5}$

$\vec{E}$ and $\vec{B}$ are perpendicular to each other

$\Rightarrow \vec{E}.\vec{B} = 0$

$\Rightarrow$ either direction of $\vec{E}$ is $\hat{i} - 2\hat{j}$ or $-\hat{i} + 2\hat{j}$

from given option

Also wave propagation direction is parallel to $\vec{E} \times \vec{B}$ which is $-\hat{k}$

$\Rightarrow \vec{E}$ is along $\left(-\hat{i} + 2\hat{j}\right)$

12. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :-

(1) 1.9 mm (2) 1.7 mm

(3) 1.5 mm (4) 1.3 mm

Sol. Given

$\frac{Y_A}{Y_B} = \frac{7}{4}$ $L_A = 2m$ $A_A = \pi R^2$

$L_B = 1.5m$ $A_B = \pi(2mm)^2$

$$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$$

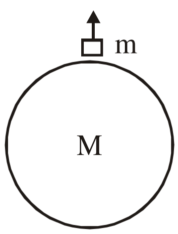
$$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2mm)^2 . Y_B}{1.5}$$

R = 1.74 mm

13. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon :-

(1) $\frac{E}{4}$ (2) $\frac{E}{16}$ (3) $\frac{E}{32}$ (4) $\frac{E}{64}$

Sol.



minimum energy required (E) = – (Potential energy of object at surface of earth)

$$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Now $M_{earth} = 64\ M_{moon}$

$$\rho.\frac{4}{3}\pi R_e^3 = 64.\frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$$

Now $\frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}} . \frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$

$$\Rightarrow E_{moon} = \frac{E}{16}$$

14. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio

$\frac{h_{sph}}{h_{cyl}}$ is given by :-

(1) $\frac{14}{15}$ (2) $\frac{4}{5}$ (3) 1 (4) $\frac{2}{\sqrt{5}}$

Sol. for solid sphere

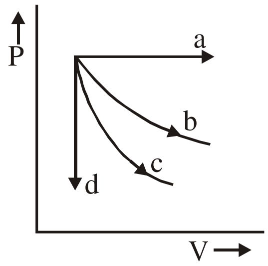
$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph.}$$

for solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{1}{2}mR^2.\frac{v^2}{R^2} = mgh_{cyl.}$$

$$\Rightarrow \frac{h_{sph.}}{h_{cyl.}} = \frac{7/5}{3/2} = \frac{14}{15}$$

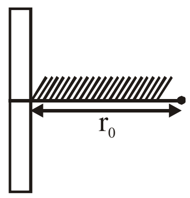
15. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :-



(1) d a c b (2) a d c b
(3) a d b c (4) d a b c

Sol. isochoric $\rightarrow$ Process d
isobaric $\rightarrow$ Process a
Adiabatic slope will be more than isothermal so
Isothermal $\rightarrow$ Process b
Adiabatic $\rightarrow$ Process c
order $\rightarrow$ d a b c

16. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :-



(1) $v \propto e^{+r/r_0}$ (2) $v \propto \ell n\left(\frac{r}{r_0}\right)$
(3) $v \propto \left(\frac{r}{r_0}\right)$ (4) $v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

Sol. $\frac{1}{2}mV^2 = -q(V_f - V_i)$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$$

17. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :-
(1) 100 s (2) 20 s (3) 10 s (4) 50 s

Sol. $A = A_0e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$$\frac{A_0}{2} = A_0e^{-\gamma(2)}$$

$2 = e^{2\gamma}$

$\ell n2 = 2\gamma$

$$\gamma = \frac{\ell n2}{2}$$

$\because$ $A = A_0e^{-\gamma t}$

$$\ell n\frac{A_0}{A} = \gamma t$$

$$\ell n1000 = \frac{\ell n2}{2}t$$

$$2\left(\frac{3\ell n10}{\ell n2}\right) = t$$

$$\frac{6\ell n10}{\ell n2} = t$$

t = 19.931 sec
t ≈ 20 sec

18. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is :-

(1) –48 V (2) –520 V

(3) 180 V (4) 320 V

Sol. $\vec{E} = (20x + 10)\hat{i}$

$$V_1 - V_2 - \int_{-5}^{1}(20x+10)dx$$

$$V_1 - V_2 = -\left(10x^2 + 10x\right)_{-5}^{1}$$

$V_1 - V_2 = 10(25 - 5 - 1 - 1)$

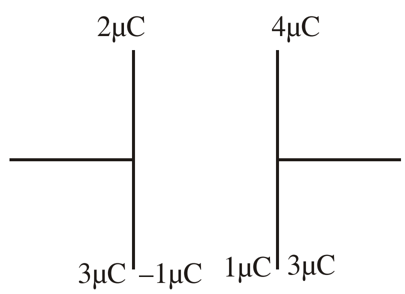
$V_1 - V_2 = 180$ V

19. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:-

(1) 5V (2) 2V

(3) 3V (4) 1V

Sol.



Charges at inner plates are 1μC and –1μC

∴ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\mu C}{1\mu F} = \frac{1\times10^{-6}C}{1\times10^{-6}\,\text{Farad}} = 1V$$

20. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to :-

(1) 0.7%

(2) 0.2%

(3) 3.5%

(4) 6.8%

Sol. $T = \frac{30\,\text{sec}}{20}$ $\Delta T = \frac{1}{20}$ sec.

L = 55 cm ΔL = 1mm = 0.1 cm

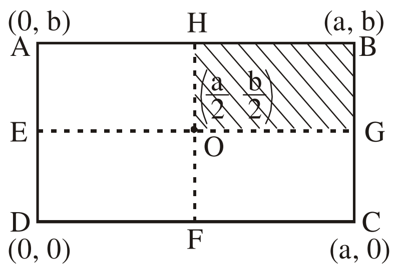
$$g = \frac{4\pi^2 L}{T^2}$$

percentage error in g is

$$\frac{\Delta g}{g}\times100\% = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right)100\%$$

$$= \left(\frac{0.1}{55} + \frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right)100\% \simeq 6.8\%$$

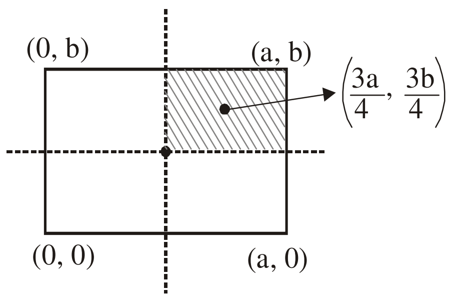
21. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :-



(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Sol.



$$x = \frac{M\frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}}$$

$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

22. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :

[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K

Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg

Radius of Earth : 6.4×10^6 m

Gravitational acceleration on Earth = $10 ms^{-2}$]

(1) 650 K (2) 3×10^5 K

(3) 10^4 K (4) 800 K

Sol. $v_{rms} = \sqrt{\frac{3RT}{m}}$ $v_{escape} = \sqrt{2gR_e}$

$v_{rms} = v_{escape}$

$$\frac{3RT}{m} = 2gR_e$$

$$\frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}}{2} \times T$$

$= 2 \times 10 \times 6.4 \times 10^6$

$$T = \frac{4 \times 10 \times 6.4 \times 10^6}{3 \times 1.38 \times 6.02 \times 10^3} = 10 \times 10^3 = 10^4 k$$

Note : Question gives avogadro Number N_A = 6.02 × 10^{26} /kg but we take N_A = 6.02 × 10^{26} /kmol.

23. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :-

(1) 20 cm (2) 10 cm

(3) 25 cm (4) 30 cm

Sol.

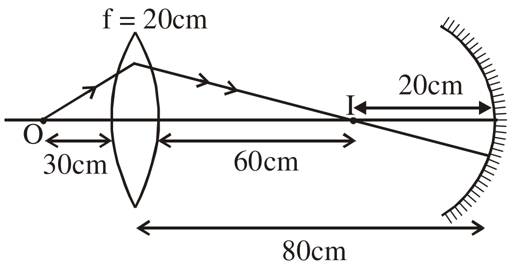


Image formed by lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{30}=\frac{1}{20}$$

$v = +60$ cm

If image position does not change even when mirror is removed it means image formed by lens is formed at centre of curvature of spherical mirror.

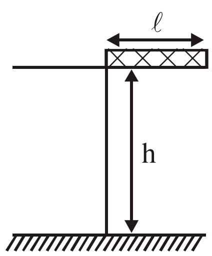
Radius of curvature of mirror = 80 – 60 = 20 cm

$\Rightarrow$ focal length of mirror f = 10 cm

for virtual image, object is to be kept between focus and pole.

$\Rightarrow$ maximum distance of object from spherical mirror for which virtual image is formed, is 10cm.

24. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :-



(1) 0.02 (2) 0.28

(3) 0.5 (4) 0.3

Sol. Angular impulse = change in angular momentum

$\tau\,\Delta t = \Delta L$

$$mg\frac{\ell}{2}\times .01 = \frac{m\ell^2}{3}\omega$$

$$\omega=\frac{3g\times 0.01}{2\ell}$$

$$=\frac{3\times 10\times .01}{2\times 0.3}$$

$$=\frac{1}{2}=0.5\text{ rad/s}$$

time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 5}{10}}=1\text{ sec.}$$

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

25. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :-

(1) $S^{3/2}I^{1/2}h^{0}$ (2) $S^{1/2}I^{1/2}h^{0}$

(3) $S^{1/2}I^{1/2}h^{-1}$ (4) $S^{1/2}I^{3/2}h^{-1}$

Sol. $p = k\, s^{a}I^{b}h^{c}$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^{a}(ML^{2})^{b}(ML^{2}T^{-1})^{c}$

$a + b + c = 1$

$2b + 2c = 1$

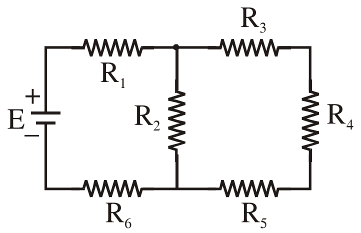
$-2a - c = -1$

$a = \frac{1}{2}$ $\quad b = \frac{1}{2}$ $\quad c = 0$

$s^{1/2}I^{1/2}h^{0}$

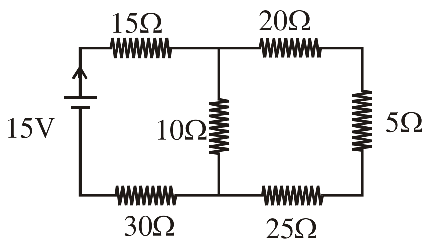
26. In the figure shown, what is the current (in Ampere) drawn from the battery ? You are given:

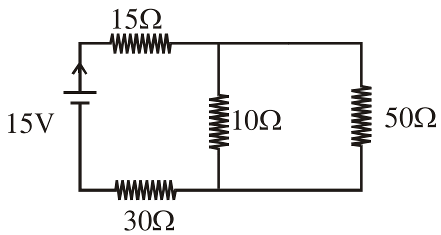
$R_1 = 15\Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\Omega$, $R_5 = 25\Omega$, $R_6 = 30\ \Omega$, $E = 15\ V$

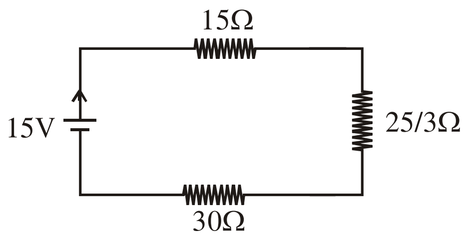


(1) 7/18 (2) 13/24 (3) 9/32 (4) 20/3

Sol.







$$R_{eq} = 15 + \frac{25}{3} + 30 = \frac{45+25+90}{3} = \frac{160}{3}$$

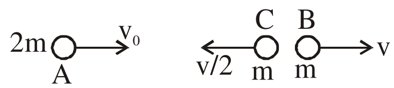
$$I = \frac{E}{R_{eq}} = \frac{15\times 3}{160} = \frac{9}{32}\text{amp.}$$

27. A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively :-

(1) $2\lambda_A, \lambda_A$ (2) $\lambda_A, 2\lambda_A$

(3) $\lambda_A, \frac{\lambda_A}{2}$ (4) $\frac{\lambda_A}{2}, \lambda_A$

Sol.



let mass of B and C is m each.

By momentum conservation

$$2mv_0 = mv - \frac{mv}{2}$$

$v = 4v_0$

$p_A = 2mv_0$ $p_B = 4mv_0$ $p_c = 2mv_0$

De-Broglie wavelength $\lambda = \frac{h}{p}$

$$\lambda_A = \frac{h}{2mv_0} ; \lambda_B = \frac{h}{4mv_0} ; \lambda_C = \frac{h}{2mv_0}$$

28. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.

If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :-

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - \frac{v_2}{2}$

(3) $v_4 - v_2$ (4) $v_4 + v_2$

Sol. Applying linear momentum conservation

$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$

$m_1v_1 + 0.5\ m_1v_2 = m_1(0.5\ v_1) + 0.5\ m_1v_4$

$0.5\ m_1v_1 = 0.5\ m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

29. Let $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$ and $|\vec{A}_1 + \vec{A}_2| = 5$. The value of $(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$ is :-

(1) –112.5 (2) –106.5 (3) –118.5 (4) –99.5

Sol. $|\vec{A}_1| = 3 \qquad |\vec{A}_2| = 5 \qquad |\vec{A}_1 + \vec{A}_2| = 5$

$$|\vec{A}_1 + \vec{A}_2| = \sqrt{|\vec{A}_1|^2 + |\vec{A}_2|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta}$$

$$5 = \sqrt{9 + 25 + 2\times3\times5\cos\theta}$$

$$\cos\theta = -\frac{9}{2\times3\times5} = -\frac{3}{10}$$

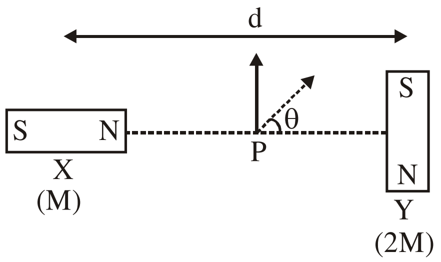
$$(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$$

$$= 6|\vec{A}_1|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6|\vec{A}_2|^2$$

$$54 + 5\times3\times5\left(-\frac{3}{10}\right) - 6\times25$$

$$= 54 - 150 - \frac{45}{2} = -118.5$$

30. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle θ = 45° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant ? (d is much larger than the dimensions of the dipole)



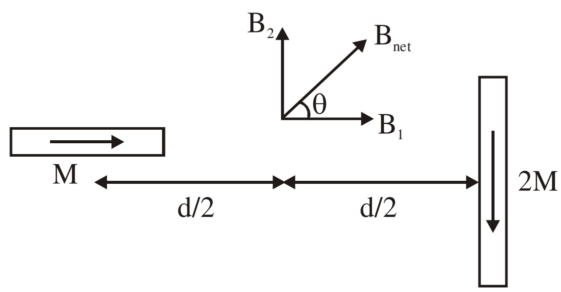
(1) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(2) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

(3) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(4) 0

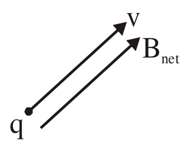
Sol.



$$B_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}; \qquad B_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}$$

$B_1 = B_2$

$\Rightarrow B_{net}$ is at 45° (θ = 45°)



velocity of charge and B_{net} are parallel so by $\vec{F} = q(\vec{v}\times\vec{B})$ force on charge particle is zero.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. Calculate the standard cell potential in(V) of the cell in which following reaction takes place : $Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

Given that

$E^{o}_{Ag^{+}/Ag} = xV$

$E^{o}_{Fe^{2+}/Fe} = yV$

$E^{o}_{Fe^{3+}/Fe} = zV$

(1) x + 2y – 3z (2) x – z

(3) x – y (4) x + y – z

Sol. $Fe^{+2}(aq) + Ag^{+}(aq) \rightarrow Fe^{+3}(aq) + Ag(s)$

Cell reaction

anode : $Fe^{+2}(aq) \rightarrow Fe^{+3}(aq) + e^{\ominus}$;

$$E^{o}_{Fe^{+2}/Fe^{+3}} = mV$$

cathode : $Ag^{+}(aq) + e^{\ominus} \rightarrow Ag(s)$;

$$E^{o}_{Ag^{+}/Ag} = xV$$

$\Rightarrow$ cell standard potential = (m + x)V

$\therefore$ to find 'm';

$Fe^{+2} + 2e^{\ominus} \rightarrow Fe$;

$$E^{o}_{1} = yV \Rightarrow \Delta^{o}_{1}G = -(2Fy)$$

$Fe^{+3} + 3e^{\ominus} \rightarrow Fe$;

$$E^{o}_{2} = zV \Rightarrow \Delta^{o}_{2}G = -(3Fz)$$

$Fe^{+2}(aq) \rightarrow Fe^{+3}(aq) + e^{\ominus}$;

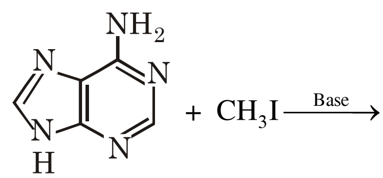
$$E^{o}_{3} = mV \Rightarrow \Delta^{o}_{3}G = -(1Fm)$$

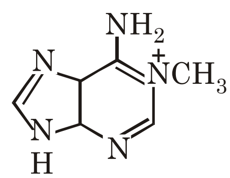
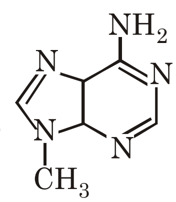
$\Delta^{o}_{3}G = \Delta G^{o}_{1} - \Delta G^{o}_{2} = (-2Fy + 3Fz) = -Fm$

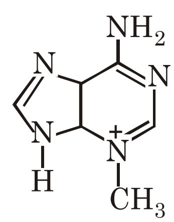
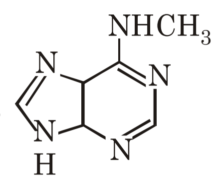
$\Rightarrow m = (2y - 3z)$

$\Rightarrow E^{o}_{cell} = (x + 2y - 3z)V$

2. The major product in the following reaction is :



(1)  (2) 

(3)  (4) 

Official Ans. by NTA (2)

ToraLabs Ans. (Bonus)

Sol. because one double bond is missing in all given option. So aromaticity is lost in both the ring.

3. For the following reactions, equilibrium constants are given :

$S(s) + O_2(g) \rightleftharpoons SO_2(g)$; $K_1 = 10^{52}$

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g)$; $K_2 = 10^{129}$

The equilibrium constant for the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is :

(1) 10^{181} (2) 10^{154} (3) 10^{25} (4) 10^{77}

Sol. $S(s) + O_2(g) \rightleftharpoons SO_2(g)$ $K_1 = 10^{52}$ ...(1)

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g)$ $K_2 = 10^{129}$ ...(2)

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ $K_3 = x$

multiplying equation (1) by 2;

$2SO(s) + 2O_2(g) \rightleftharpoons 2SO_2(g)$ $K'_1 = 10^{104}$ ...(3)

$\Rightarrow$ Substracting (3) from (2); we get

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$;

$$K_{eq} = 10^{(129-104)} = 10^{25}$$

4. The ion that has sp^3d^2 hybridization for the central atom, is :

(1) $[ICl_2]^-$ (2) $[IF_6]^-$

(3) $[ICl_4]^-$ (4) $[BrF_2]^-$

Sol.

Chemical species	Hybridisation of central atom
ICl_2^-	sp^3d
IF_6^-	sp^3d^3
ICl_4^-	sp^3d^2
BrF_2^-	sp^3d

5. The structure of Nylon-6 is :

(1) $\left[(CH_2)_6 - \overset{\overset{O}{\|}}{C} - \overset{\overset{H}{|}}{N} \right]_n$

(2) $\left[(CH_2)_4 - \overset{\overset{O}{\|}}{C} - \overset{\overset{H}{|}}{N} \right]_n$

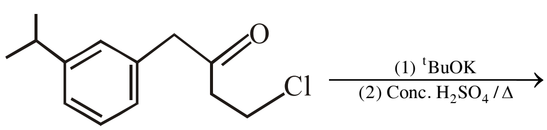
(3) $\left[\overset{\overset{O}{\|}}{C} - (CH_2)_5 - \overset{\overset{H}{|}}{N} \right]_n$

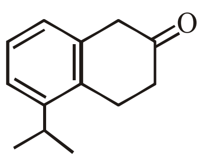
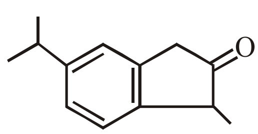
(4) $\left[\overset{\overset{O}{\|}}{C} - (CH_2)_6 - \overset{\overset{H}{|}}{N} \right]_n$

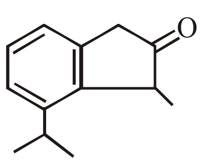
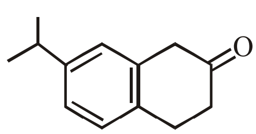
Sol. $\left[\overset{\overset{O}{\|}}{C} - (CH_2)_5 - \overset{\overset{H}{|}}{N} \right]_n$

Nylon-6

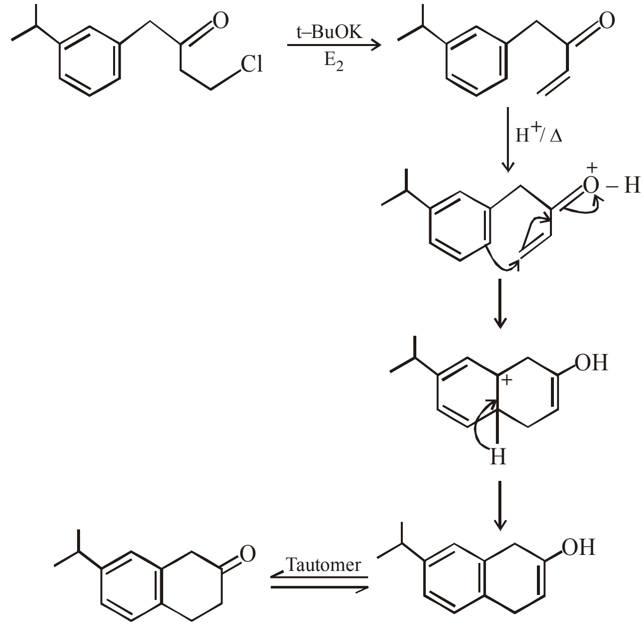
6. The major product of the following reaction is:



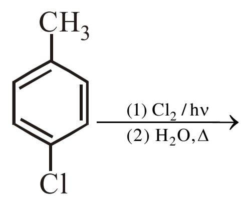
(1) 
(2) 

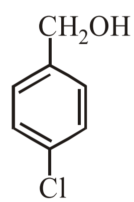
(3) 
(4) 

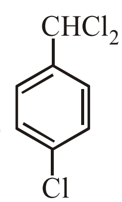
Sol.

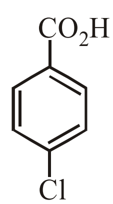


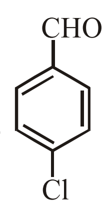
7. The major product of the following reaction is:

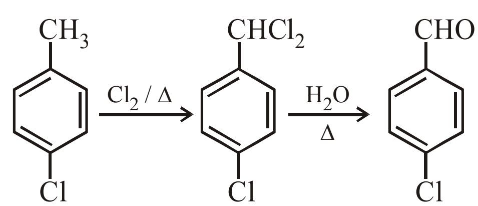


(1) 

(2) 

(3) 

(4) 

Sol. 

8. The percentage composition of carbon by mole in methane is :

(1) 80% (2) 25% (3) 75% (4) 20%

Sol. CH_4

$$\% \text{ by mole of carbon} = \frac{1 \text{ mol atom}}{5 \text{ mol atom}} \times 100$$

$$= 20\%$$

9. The IUPAC symbol for the element with atomic number 119 would be :

(1) unh (2) uun (3) une (4) uue

Sol.

Symbol	Atomic number
unh	106
uun	110
une	109
uue	119

10. The compound that inhibits the growth of tumors is :

(1) cis-$[Pd(Cl)_2(NH_3)_2]$

(2) cis-$[Pt(Cl)_2(NH_3)_2]$

(3) trans-$[Pt(Cl)_2(NH_3)_2]$

(4) trans-$[Pd(Cl)_2(NH_3)_2]$

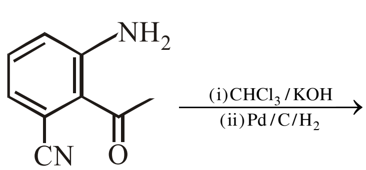
Sol. cis–$[PtCl_2(NH_3)_2]$ is used in chemotherapy to inhibits the growth of tumors.

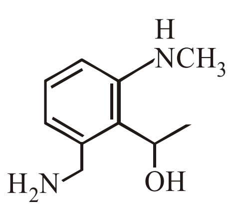
11. The covalent alkaline earth metal halide (X = Cl, Br, I) is :

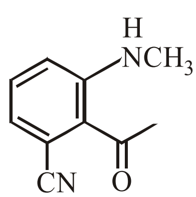
(1) CaX_2 (2) SrX_2 (3) BeX_2 (4) MgX_2

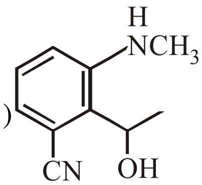
Sol. All halides of Be are predominantly covalent in nature.

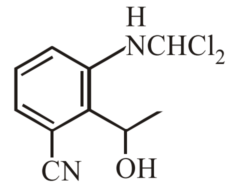
12. The major product obtained in the following reaction is :



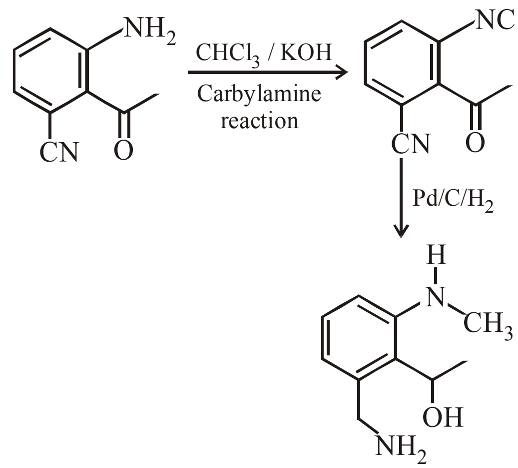
(1) 

(2) 

(3) 

(4) 

Sol.



13. The statement that is **INCORRECT** about the interstitial compounds is :

(1) They have high melting points

(2) They are chemically reactive

(3) They have metallic conductivity

(4) They are very hard

Sol. Generally interstitial compounds are chemicaly inert.

14. The maximum prescribed concentration of copper in drinking water is:

(1) 5 ppm (2) 0.5 ppm

(3) 0.05 ppm (4) 3 ppm

Sol. The maximum prescribed concentration of Cu in drinking water is 3 ppm.

15. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are :

(1) 4.9 and 0 (2) 2.84 and 5.92

(3) 0 and 4.9 (4) 0 and 5.92

Sol. Complex is $[Fe\,(H_2O)_6]_2\,[Fe(CN)_6]$

Complex ion	Configuration	No. of unpaired electrons	Magnetic moment
$[Fe(H_2O)_6]^{2+}$	$t_{2g}^4\ e_g^2$	4	4.9 BM
$[Fe(CN)_6]^{4-}$	$t_{2g}^6 e_g^0$	0	0

16. 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

[Density of fatty acid = 0.9 g cm^{-3}, π = 3]

(1) 10^{-8} m (2) 10^{-6} m

(3) 10^{-4} m (4) 10^{-2} m

Sol. Radius of watchglass= 10 cm

$\Rightarrow$ surface area = $\pi r^2 = 3 \times (10\text{ cm})^2$

$= 300\text{ cm}^2$

mass of fatty acid in 10 ml solution

$$= \frac{10 \times 0.27}{100} = 0.027\,\text{gm}$$

$$\text{volume of fatty acid} = \frac{0.027\,\text{g}}{0.9\,\text{g/ml}} = 0.03\text{ cm}^3$$

$$\Rightarrow \text{Height} = \frac{\text{volume of fatty acid}}{\text{surface area of watch glass}}$$

$$= \frac{0.03\text{ cm}^3}{300\text{ cm}^2} = 0.0001\text{ cm} = 10^{-6}\text{ m}$$

17. Among the following molecules / ions,

$C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2$

which one is diamagnetic and has the shortest bond length?

(1) C_2^{2-} (2) N_2^{2-} (3) O_2 (4) O_2^{2-}

Sol.

Chemical Species	Bond Order	Magnetic behaviour
C_2^{2-}	3	diamagnetic
N_2^{2-}	2	paramagnetic
O_2	2	paramagnetic
O_2^{2-}	1	diamagnetic

$$\text{B.O.} \propto \frac{1}{\text{bond length}}$$

18. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K.

If $C_V = 28\ JK^{-1}mol^{-1}$, calculate ΔU and ΔpV for this process. (R = 8.0 JK^{-1} mol^{-1}]

(1) ΔU = 14 kJ; $\Delta(pV)$ = 4 kJ

(2) ΔU = 14 kJ; $\Delta(pV)$ = 18 kJ

(3) ΔU = 2.8 kJ; $\Delta(pV)$ = 0.8 kJ

(4) ΔU = 14 kJ; $\Delta(pV)$ = 0.8 kJ

Sol. n = 5; T_i = 100 K; T_f = 200 K;

C_V = 28 J/mol K; Ideal gas

$\Delta U = nC_V\Delta T$

$= 5\text{ mol} \times 28\text{ J/mol K} \times (200 - 100)\text{ K}$

$= 14{,}000\text{ J} = 14\text{ kJ}$

$\Rightarrow C_p = C_v + R = (28 + 8)$ J/mol K

$= 36$ J/mol K

$\Rightarrow \Delta H = nC_p\Delta T = 5\text{ mol} \times 36\text{ J/mol K} \times 100\text{ K}$

$= 18000\text{ J} = 18\text{ kJ}$

$\Delta H = \Delta U + \Delta(PV)$

$\Rightarrow \Delta(PV) = \Delta H - \Delta U = (18 - 14)\text{ kJ} = 4\text{ kJ}$

19. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product?

(1) $F_3C - CH = CH_2$

(2) $Cl - CH = CH_2$

(3) $CH_3O - CH = CH_2$

(4) $H_2N - CH = CH_2$

Sol. $CF_3{-}CH{=}CH_2 \xrightarrow{HCl} CF_3 - \overset{H}{\overset{|}{C}}H - \overset{\oplus}{C}H_2$

$\downarrow$

$CF_3 - \overset{H}{\overset{|}{C}}H - \overset{Cl}{\overset{|}{C}}H_2$

Due to higher e^- withdrawing nature of CF_3 group.

It follow anti markovnikoff product

20. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by :

(1) $\left(\frac{k_1}{k_2}\right)[A]$ (2) $(k_1 + k_2)\ [A]$

(3) $k_1k_2[A]$ (4) $(k_1 - k_2)\ [A]$

Sol. $A \xrightarrow{K_1} B \xrightarrow{K_2} C$

$$\frac{d[B]}{dt} = 0 = K_1[A] - K_2[B]$$

$$\Rightarrow [B] = \frac{K_1}{K_2}[A]$$

21. Which of the following compounds will show the maximum enol content?

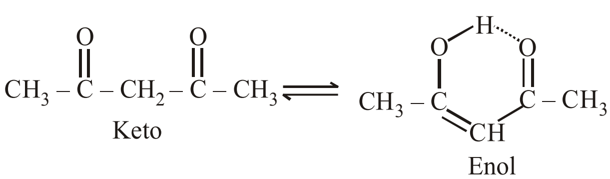
(1) $CH_3COCH_2COCH_3$

(2) CH_3COCH_3

(3) $CH_3COCH_2CONH_2$

(4) $CH_3COCH_2COOC_2H_5$

Sol. Solution



Due to intramolecular H-bonding and resonance stabilisation enol content is maximum

22. The correct statement about ICl_5 and ICl_4^- is

(1) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral.

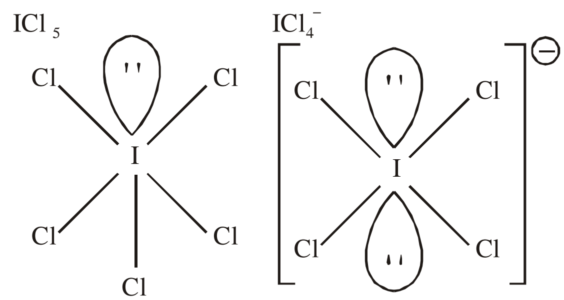
(2) ICl_5 is square pyramidal and ICl_4^- is tetrahedral.

(3) ICl_5 is square pyramidal and ICl_4^- is square planar.

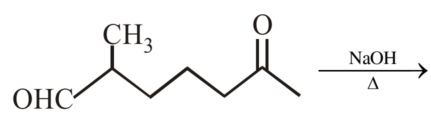
(4) Both are isostructural.

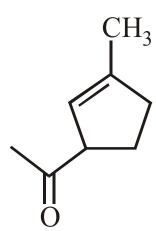
Sol.

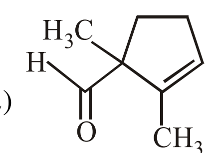
Chemical species	Hybridisation	Shape
ICl_5	sp^3d^2	Square pyramidal
ICl_4^-	sp^3d^2	Square planar

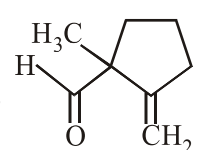


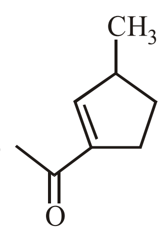
23. The major product obtained in the following reaction is



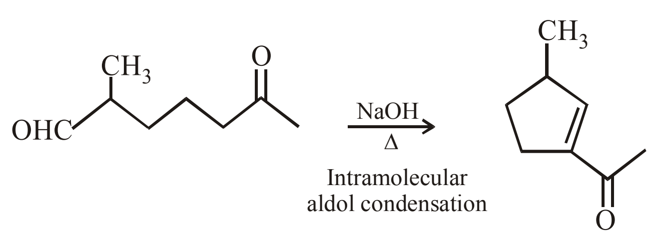
(1) 

(2) 

(3) 

(4) 

Sol.



24. Fructose and glucose can be distinguished by :

(1) Fehling's test

(2) Barfoed's test

(3) Benedict's test

(4) Seliwanoff's test

Sol. Seliwanoff's test is used to distinguished aldose and ketose group.

25. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be:

(Assume kinetic energy of ejected photoelectron to be very high in comparison to work function)

(1) $\frac{1}{2}\lambda$ (2) $\frac{3}{4}\lambda$

(3) $\frac{2}{3}\lambda$ (4) $\frac{4}{9}\lambda$

Sol. $h\nu - \phi = KE$

$$\Rightarrow \left(\frac{hc}{\lambda}\right)_{incident} = KE + \phi$$

$$\left(\frac{hc}{\lambda}\right)_{incident} \simeq KE$$

$$KE = \frac{p^2}{2m} = \frac{hc}{\lambda_{incident}} = \frac{hc}{\lambda} \quad ...(1)$$

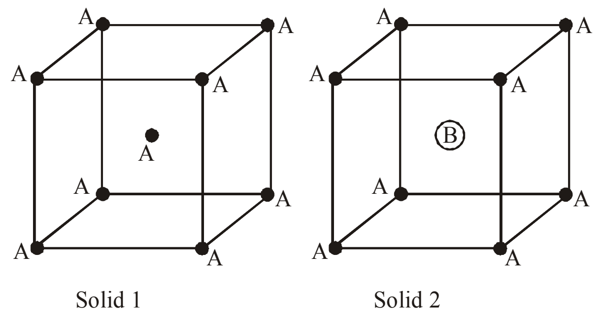
$$\Rightarrow \frac{p^2 \times (1.5)^2}{2m} = \frac{hc}{\lambda'} \quad ...(2)$$

divide (1) and (2)

$$(1.5)^2 = \frac{\lambda}{\lambda'}$$

$$\Rightarrow \lambda' = \frac{4\lambda}{9}$$

26. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?



(1) 45% (2) 65% (3) 90% (4) 75%

Sol. $$\text{p.f.} = \frac{\left(Z_{eff} \times \frac{4}{3}\pi r_A^3\right)_A + \left(Z_{eff} \times \frac{4}{3}\pi r_B^3\right)_B}{a^3}$$

$$2(r_A + r_B) = \sqrt{3}a$$

$$\Rightarrow 2(r_A + 2r_A) = \sqrt{3}a$$

$$\Rightarrow 2\sqrt{3}\, r_A = a$$

$$\Rightarrow \text{p.f.} = \frac{1 \times \frac{4}{3}\pi r_A^3 + \frac{4}{3}\pi\left(8r_A^3\right)}{8 \times 3\sqrt{3}\, r_A^3} = \frac{9 \times \frac{4}{3}\pi}{8 \times 3\sqrt{3}} = \frac{\pi}{2\sqrt{3}}$$

$$\text{p. efficiency} = \frac{\pi}{2\sqrt{3}} \times 100 \simeq 90\%$$

27. Polysubstitution is a major drawback in:

(1) Reimer Tiemann reaction

(2) Friedel Craft's acylation

(3) Friedel Craft's alkylation

(4) Acetylation of aniline

Sol. In Friedal crafts alkylation product obtained is more activated and hence polysubtitution will take place.

28. The Mond process is used for the

(1) extraction of Mo

(2) Purification of Ni

(3) Purification of Zr and Ti

(4) Extraction of Zn

Sol. Mond's process is used for the purification of Nickel.

29. The strength of 11.2 volume solution of H_2O_2 is : [Given that molar mass of H = 1 g mol^{-1} and O = 16 g mol^{-1}]

(1) 13.6% (2) 3.4%

(3) 34% (4) 1.7%

Sol. Volume strength = 11.2 × molarity = 11.2

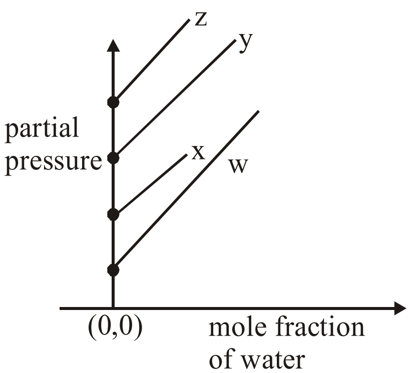
$\Rightarrow$ molarity = 1 M

$\Rightarrow$ strength = 34 g/L

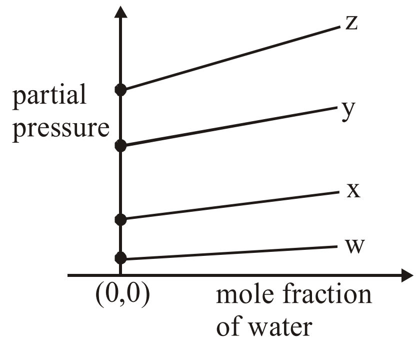
$\Rightarrow$ % w/w = $\frac{34}{1000} \times 100 = 3.4\%$

30. For the solution of the gases w, x, y and z in water at 298K, the Henrys law constants (K_H) are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is :-

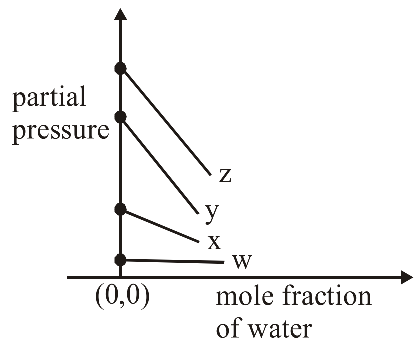
(1)



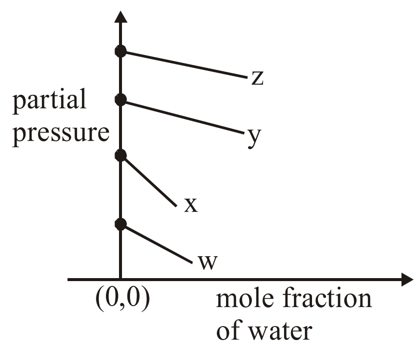
(2)



(3)



(4)



Sol. $p = k_H \times \left(\frac{n_{gas}}{n_{H_2O} + n_{gas}}\right)$

$= k_H \left(1 - \frac{n_{H_2O}}{n_{H_2O} + n_{gas}}\right)$

$\Rightarrow p = k_H - k_H \times \chi_{H_2O}$

$p = (-k_H) \times \chi_{H_2O} + k_H$

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 3 (3) 2 (4) 4

Sol. Probability of observing at least one head out of n tosses

$$=1-\left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$$

$\Rightarrow n \geq 4$

$\Rightarrow$ minimum number of tosses = 4

2. A student scores the following marks in five tests : 45,54,41,57,43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$ (3) $\frac{100}{3}$ (4) $\frac{10}{3}$

Sol. Let x be the 6^{th} observation

$\Rightarrow$ 45 + 54 + 41 + 57 + 43 + x = 48 × 6 = 288

$\Rightarrow x = 48$

$$\text{variance} = \left(\frac{\Sigma x_i^2}{6}-(\bar{x})^2\right)$$

$$\Rightarrow \text{variance} = \frac{14024}{6}-(48)^2$$

$$=\frac{100}{3}$$

$$\Rightarrow \text{standard deviation} = \frac{10}{\sqrt{3}}$$

3. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to-

(1) $2-\frac{3}{2^{17}}$ (2) $2-\frac{11}{2^{19}}$

(3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{21}{2^{20}}$

Sol. $S=\sum_{k=1}^{20} \frac{1}{2^k}$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{3^2}+\ldots+\frac{20}{2^{20}}$$

$$S\times\frac{1}{2}=\frac{1}{2^2}+\frac{2}{2^3}+\ldots+\frac{19}{2^{20}}+\frac{20}{2^{21}}$$

$$\Rightarrow \left(1-\frac{1}{2}\right)S=\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\Rightarrow S=2-\frac{11}{2^{19}}$$

4. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if :

(1) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (2) $0<r\leq\sqrt{\frac{3}{2}}$

(3) $\sqrt{\frac{3}{2}}<r\leq 3\sqrt{\frac{3}{2}}$ (4) $r\geq 5\sqrt{\frac{3}{2}}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$$

$$=(2+x)\hat{i}+(x-3)\hat{j}-5k$$

$$|\vec{a}\times\vec{b}|=\sqrt{4+x^2+4x+x^2+9-6x+25}$$

$$=\sqrt{2x^2-2x+38}$$

$$\Rightarrow |\vec{a}\times\vec{b}|\geq\sqrt{\frac{75}{2}}$$

$$\Rightarrow |\vec{a}\times\vec{b}|\geq 5\sqrt{\frac{3}{2}}$$

5. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x,y,z), $z \neq 0$, then (x,y) lies on the straight line whose equation is :

(1) $3x - 4y - 1 = 0$ (2) $3x - 4y - 4 = 0$

(3) $4x - 3y - 4 = 0$ (4) $4x - 3y - 1 = 0$

Sol. $x - 2y + kz = 1$ ...(1)

$2x + y + z = 2$ ...(2)

$3x - y - kz = 3$ ...(3)

(1) +(3)

$\Rightarrow 4x - 3y = 4$

6. If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is-

(1) $2x - y - 2 = 0$ (2) $3x - 2y = 0$

(3) $2x - 3y + 10 = 0$ (4) $x - 2y + 8 = 0$

Sol. Let us Suppose equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through (4,6) $\Rightarrow a^2 = 4,\ b^2 = 12$

$\Rightarrow$ equaiton of tangent

$$x - \frac{y}{2} = 1$$

$$\Rightarrow 2x - y - 2 = 0$$

7. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

(1) 5 : 9 : 13 (2) 5 : 6 : 7

(3) 4 : 5 : 6 (4) 3 : 4 : 5

Sol. $a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A$

$\Rightarrow \sin C = 2\sin A.\cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

8. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x)f_2(y)$

(3) $2f_1(x + y)f_2(x - y)$

(4) $2f_1(x + y)f_1(x - y)$

Sol. $f(x) = a^x$, $a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$= 2f_1(x)\ f_1(y)$

9. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is :

(1) 10^3 (2) 100 (3) 10^4 (4) 10

Sol. $200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{\frac{3}{2}} \times x^{\frac{1}{4}}$

$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$

$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$

where t = $\log_{10}x$

$\Rightarrow t^2+3t-4=0$

$\Rightarrow t = 1,-4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow$ x = 10 (As x > 1)

10. Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

Sol. $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$A(\alpha) = 2\int_0^{\alpha}\sqrt{x}dx = 2\alpha^{\frac{3}{2}}$

$A(4) = 2\times 4^{3/2} = 16$

$A(\lambda) = 2\times\lambda^{3/2}$

$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4.\left(\frac{4}{25}\right)^{1/3}$

11. Given that the slope of the tangent to a curve y = y(x) at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is :

(1) x $\log_e|y| = 2(x - 1)$
(2) x $\log_e|y| = x - 1$
(3) $x^2 \log_e|y| = -2(x - 1)$
(4) x $\log_e|y| = -2(x - 1)$

Sol. given $\frac{dy}{dx} = \frac{2y}{x^2}$

$\Rightarrow \int\frac{dy}{2y} = \int\frac{dx}{x^2}$

$\Rightarrow \frac{1}{2}\ell ny = -\frac{1}{x}+c$

passes through centre (1,1)

$\Rightarrow c = 1$

$\Rightarrow x\ell ny = 2(x-1)$

12. The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y+ 4z = 5 which is perpendicular to the plane x – y + z = 0 is :

(1) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

(2) $\vec{r}.(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}.(\hat{i}-\hat{k})+2=0$

(4) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

Sol. Let the plane be

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$

$\Rightarrow (2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0$

$\perp$ to the plane x – y + z = 0

$\Rightarrow \lambda = -\frac{1}{3}$

$\Rightarrow$ the required plane is x – z + 2 = 0

13. Which one of the following statements is not a tautology ?

(1) $(p\wedge q)\to p$

(2) $(p\wedge q)\to(\sim p)\vee q$

(3) $p\to(p\vee q)$

(4) $(p\vee q)\to(p\vee(\sim q))$

Sol. Tautology

(1)

p	q	$p\wedge q$	$(p\wedge q)\rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

(2)

p	q	$p\wedge q$	$\sim p\vee q$	$(p\wedge q)\rightarrow(\sim p)\vee q$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Tautology

(3)

p	q	$p\vee q$	$p\rightarrow p\vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology

(4)

p	q	$p\vee q$	$\sim p$	$p\vee\sim q$	$p\vee q\rightarrow p\wedge(\sim q)$
T	T	T	F	F	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	F	T

14. Let f : R $\rightarrow$ R be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

(1) e^2 (2) e

(3) e^{-1} (4) 1

Sol. $\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ (1^∞ form)

$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(2-x)-f(3)+f(2)}{x(1+f(2-x)-f(2))}}$

using L'Hopital

$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f'(3+x)+f'(2-x)}{-xf'(2-x)+(1+f(2-x)-f(2))}}$

$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}}=1$

15. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

(1) $\left(-\frac{1}{3},\frac{4}{3}\right)$ (2) $\left(-\frac{1}{4},\frac{1}{2}\right)$

(3) $\left(\frac{3}{4},\frac{7}{4}\right)$ (4) $\left(\frac{1}{4},\frac{3}{4}\right)$

Sol. Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

by (1) and (2)

$\Rightarrow x=1$ and $y=2$

equation of tangent at (1,2) to $y^2 = 4x$

is $y = x + 1$

16. Let the number 2,b,c be in an A.P. and $A=\begin{bmatrix}1 & 1 & 1\\ 2 & b & c\\ 4 & b^2 & c^2\end{bmatrix}$. If $\det(A)\in[2,16]$, then c lies in the interval :

(1) [2,3)

(2) $(2 + 2^{3/4},4)$

(3) $[3,2 + 2^{3/4}]$

(4) [4,6]

Sol. put $b=\dfrac{2+c}{2}$ in determinant of A

$|A|=\dfrac{c^3-6c^2+12c-8}{4}\in[2,16]$

$\Rightarrow (c-2)^3 \in [8, 64]$

$\Rightarrow c \in [4, 6]$

17. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d,e,f are in A.P.

(2) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P.

(3) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P.

(4) d,e,f are in G.P.

Sol. a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

18. The number of integral values of m for which the equation

$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

has no real root is :

(1) infinitely many (2) 2

(3) 3 (4) 1

Sol. $D < 0$

$4(1 + 3m)^2 - 4(1 + m^2)(1 + 8m) < 0$

$\Rightarrow m(2m - 1)^2 > 0 \Rightarrow m > 0$

19. If a point R(4,y,z) lies on the line segment joining the points P(2,–3,4) and Q(8,0,10), then the distance of R from the origin is :

(1) $2\sqrt{14}$ (2) 6

(3) $\sqrt{53}$ (4) $2\sqrt{21}$

Sol. $\frac{4}{2} = \frac{-y}{y+3} = \frac{10-z}{z-4}$

$\Rightarrow z = 6 \ \& \ y = -2$

$\Rightarrow R(4, -2, 6)$

dist. from origin = $\sqrt{16+4+36} = 2\sqrt{14}$

20. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$,

then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to

(1) -1 (2) 1

(3) 0 (4) $(-1+2i)^9$

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$$\Rightarrow z^5 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}+i}{2}$$

and $z^8 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\left(\frac{1+i\sqrt{3}}{2}\right)$

$$\Rightarrow \left(1+iz+z^5+iz^8\right)^9 = \left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{i}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$$

$$= \left(\frac{1+i\sqrt{3}}{2}\right)^9 = \cos 3\pi + i\sin 3\pi = -1$$

21. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$ equals-

(1) $\int_{x+5}^{5} g(t)dt$ (2) $5\int_{x+5}^{5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $2\int_{5}^{x+5} g(t)dt$

Sol. $f(x) = \int_0^x g(t)dt$

$f(-x) = \int_0^{-x} g(t)dt$

put $t = -u$

$= -\int_0^x g(-u)du$

$= -\int_0^x g(u)d(u) = -f(x)$

$\Rightarrow f(-x) = -f(x)$

$\Rightarrow f(x)$ is an odd function

Also $f(5 + x) = g(x)$
$f(5 - x) = g(-x) = g(x) = f(5 + x)$
$\Rightarrow f(5 - x) = f(5 + x)$
Now

$$I = \int_0^x f(t)dt$$

$t = u + 5$

$$I = \int_{-5}^{x-5} f(u+5)du$$

$$= \int_{-5}^{x-5} g(u)du$$

$$= \int_{-5}^{x-5} f'(u)du$$

$= f(x - 5) - f(-5)$
$= -f(5 - x) + f(5)$
$= f(5) - f(5+x)$

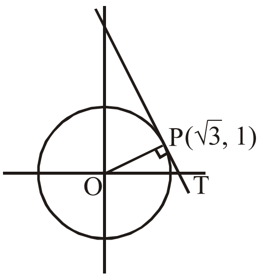
$$= \int_{5+x}^{5} f'(t)dt = \int_{5+x}^{5} g(t)dt$$

22. The tangent and the normal lines at the point $(\sqrt{3},1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{1}{3}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Sol.



Given $x^2 + y^2 = 4$
equation of tangent

$\Rightarrow \sqrt{3}x + y = 4$ ...(1)

Equation of normal

$x - \sqrt{3}y = 0$ ...(2)

Coordinate of $T\left(\frac{4}{\sqrt{3}},0\right)$

$\therefore$ Area of triangle = $= \frac{2}{\sqrt{3}}$

23. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0,5\sqrt{3}\right)$, then the length of its latus rectum is:

(1) 10 (2) 8 (3) 5 (4) 6

Sol. Let equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$2a - 2b = 10$ ...(1)

$ae = 5\sqrt{3}$ ...(2)

$\frac{2b^2}{a} = ?$

$b^2 = a^2(1 - e^2)$
$b^2 = a^2 - a^2e^2$
$b^2 = a^2 - 25 \times 3$
$\Rightarrow b = 5$ and $a = 10$

$\therefore$ length of L.R. = $\frac{2(25)}{10} = 5$

24. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is :
(1) 12 (2) 33 (3) 9 (4) 15

Sol. $y = f(f(f(x))) + (f(x))^2$

$\frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$
$= f'(1)f'(1)f'(1) + 2f(1)f'(1)$
$= 3 \times 5 \times 3 + 2 \times 1 \times 3$
$= 27 + 6$
$= 33$

25. If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}} + C$

where C is a constant of integration, then the function $f(x)$ is equal to-

(1) $-\frac{1}{6x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{2x^3}$

Sol. $\int \frac{dx}{x^3(1+x^6)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$

$\int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$

Let $t = \frac{1}{x^6} + 1$

$dt = \frac{-6}{x^7} dx$

$-\frac{1}{6}\int \frac{dt}{t^{2/3}} = -\frac{1}{2} t^{1/3}$

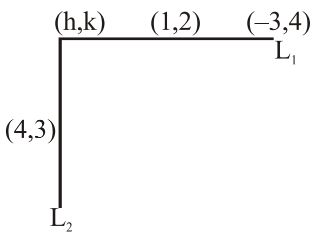
$= -\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} = -\frac{1}{2}\frac{(1+x^6)^{1/3}}{x^2}$

$\therefore\ f(x) = -\frac{1}{2x^3}$

26. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1. If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1, then $\frac{k}{h}$ equals :

(1) 3 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 0

Sol.



equation of L_1 is

$y = -\frac{1}{2}x + \frac{5}{2}$ ...(1)

equation of L_2 is

$y = 2x - 5$ ...(2)

by (1) and (2)

$x = 3$

$y = 1 \Rightarrow h = 3,\ k = 1$

$\frac{k}{h} = \frac{1}{3}$

27. Let $f : [-1,3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x] & , & -1 \le x < 1 \\ x + |x| & , & 1 \le x < 2 \\ x + [x] & , & 2 \le x \le 3 \end{cases},$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

(1) four or more points
(2) only one point
(3) only two points
(4) only three points

Sol.

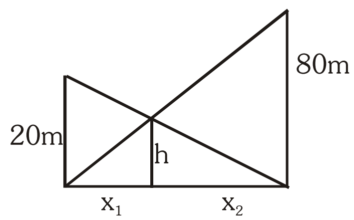
$$f(x) = \begin{cases} -(x+1) & , & -1 \le x < 0 \\ x & , & 0 \le x < 1 \\ 2x & , & 1 \le x < 2 \\ x+2 & , & 2 \le x < 3 \\ x+3 & , & x = 3 \end{cases}$$

function discontinuous at x = 0,1,3

28. Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :

(1) 12 (2) 15
(3) 16 (4) 18

Sol.



by similar triangle

$\frac{h}{x_1} = \frac{80}{x_1 + x_2}$ ...(1)

by $\frac{h}{x_2} = \frac{20}{x_1 + x_2}$ ..(2)

by (1) and (2)

$\frac{x_2}{x_1} = 4$ or $x_2 = 4x_1$

$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$

or h = 16m

29. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is :

(1) 288 (2) 306 (3) 360 (4) 310

Sol. (1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$

(2) Starting with 44 and 55 is equal to $36 \times 2 = 72$

(3) Starting with 433,434 and 435 is equal to $6 \times 3 = 18$

(3) Remaining numbers are 4322,4323,4324,4325is equal to 4

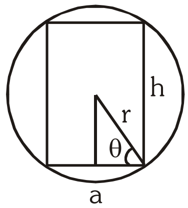
so total numbers are

$216 + 72 + 18 + 4 = 310$

30. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\sqrt{6}$ (4) $\frac{2}{3}\sqrt{3}$

Sol.



$h = 2r\sin\theta$

$a = 2r\cos\theta$

$v = \pi(r\cos\theta)^2(2r\sin\theta)$

$v = 2\pi r^3\cos^2\theta\sin\theta$

$$\frac{dv}{d\theta} = \pi r^3\left(-2\cos\theta\sin^2\theta + \cos^3\theta\right) = 0$$

$$\text{or } \tan\theta = \frac{1}{\sqrt{2}}$$

$$\because\ h = 2\times 3\times\frac{1}{\sqrt{3}}$$

$$= 2\sqrt{3}$$