FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is :

(1) 0.10 kg/m^3 (2) 0.31 kg/m^3

(3) 0.07 kg/m^3 (4) 0.01 kg/m^3

Sol. $\rho = \frac{m}{v}$

maximum % error in S will be given by

$$\frac{\Delta\rho}{\rho}\times 100\% = \left(\frac{\Delta m}{m}\right)\times 100\% + 3\left(\frac{\Delta L}{L}\right)\times 100\% \quad ...(i)$$

which is only possible when error is small which is not the case in this question.

Yet if we apply equation (i), we get

$\Delta\rho = 3100\ kg/m^3$

Now, we will calculate error, without using approximation.

$$\rho_{min} = \frac{m_{min}}{v_{max}} = \frac{9.9}{(0.11)^3} = 7438 kg/m^3$$

$$\& \ \rho_{max} = \frac{m_{max}}{v_{min}} = \frac{10.1}{(0.09)^3} = 13854.6 kg/m^3$$

$\Delta\rho = 6416.6\ kg/m^3$

No option is matching.

Therefore this question should be awarded bonus

2. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\overline{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be :

(1) $\frac{m\overline{v}^2}{6k_B}$ (2) $\frac{m\overline{v}^2}{5k_B}$

(3) $\frac{m\overline{v}^2}{3k_B}$ (4) $\frac{m\overline{v}^2}{7k_B}$

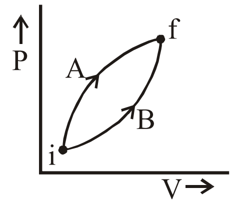
Sol. According to equipartion energy theorem

$$\frac{1}{2}m\left(v_{rms}^2\right) = 3\times\frac{1}{2}K_bT$$

$$T = \frac{m\overline{v}_{rms}^2}{3k}$$

$\therefore$ correct option should be (3)

3. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then :



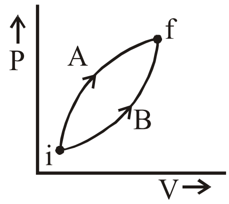
(1) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

(2) $\Delta Q_A > \Delta Q_B$; $\Delta U_A = \Delta U_B$

(3) $\Delta Q_A > \Delta Q_B$; $\Delta U_A > \Delta U_B$

(4) $\Delta Q_A < \Delta Q_B$; $\Delta U_A < \Delta U_B$

Sol.



Initial and final states for both the processes are same.

$\therefore \Delta U_A = \Delta U_B$

Work done during process A is greater than in process B.

By First Law of thermodynamics

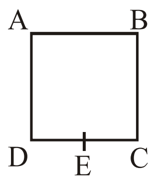
$\Delta Q = \Delta U + W$

$\Rightarrow \Delta Q_A > \Delta Q_B$

Option (2)

4. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is :

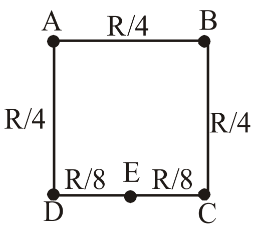
(E is mid-point of arm CD)

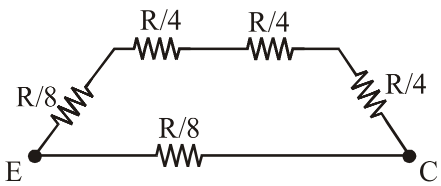


(1) R (2) $\frac{1}{16}R$

(3) $\frac{7}{64}R$ (4) $\frac{3}{4}R$

Sol.





$$\frac{1}{R_{eq}} = \frac{8}{7R} + \frac{8}{R}$$

$$\frac{1}{R_{eq}} = \frac{8+56}{7R}$$

$$R_{eq} = \frac{7R}{64}$$

Option (3)

5. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to :

(1) $1/L^2$ (2) $1/L$

(3) L (4) L^2

Sol. $\phi = NBA = LI$

$N \mu_0 n I \pi R^2 = LI$

$N \mu_0 \frac{N}{\ell} I \pi R^2 = LI$

N & R constant

self inductance (L) $\propto \frac{1}{\ell} \propto \frac{1}{\text{length}}$

Option (2)

6. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

Sol. For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

situation 1 : when pendulum is in air $\rightarrow g_{eff} = g$

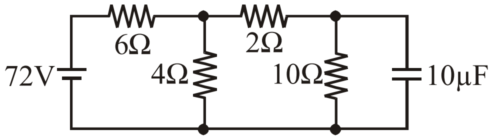
situation 2 : when pendulum is in liquid

$$\rightarrow g_{eff} = g\left(1 - \frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15g}{16}$$

So, $$\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$$

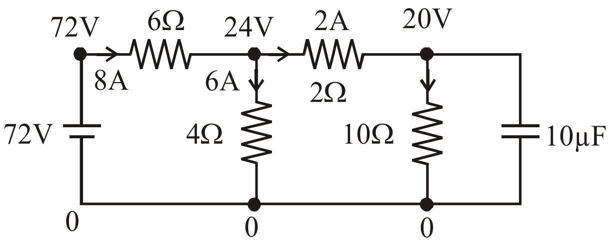
Option (1)

7. Determine the charge on the capacitor in the following circuit :



(1) 2μC (2) 60μC
(3) 200μC (4) 10μC

Sol. Applying point potential method

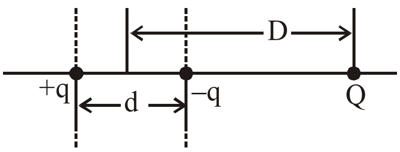


$q = cV$

$q = 10\mu F \times 20 = 200\mu C$

Option (3)

8. A system of three charges are placed as shown in the figure :



If D >> d, the potential energy of the system is best given by :

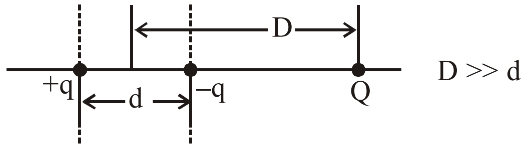
(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$

(2) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$

(4) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

Sol.



$U_{total} = U_{self\ of\ dipole} + U_{interaction}$

$= -\frac{kq^2}{d} - \left(\frac{kQ}{D^2}\right)qd$

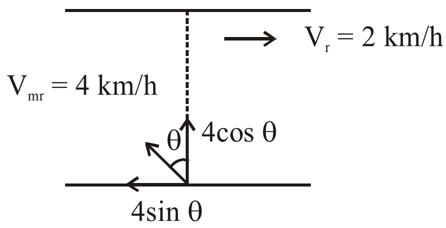
$= -\ k\left[\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

Option (4)

9. The stream of a river is flowing with a speed of 2km/h. A swimmer can swim at a speed of 4km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

(1) 60° (2) 150° (3) 90° (4) 120°

Sol.



For swimmer to cross the river straight

$\Rightarrow\ 4\sin\theta = 2$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

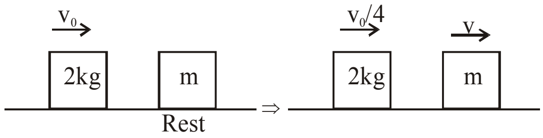
So, angle with direction of river flow = 90° + θ = 120°

Option (4)

10. A body of mass 2 kg makes an eleastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body ?

(1) 1.8 kg (2) 1.2 kg
(3) 1.5 kg (4) 1.0 kg

Sol.



By conservation of linear momentum :-

$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv$$

$$\Rightarrow \frac{3v_0}{2} = mv \text{ ....(1)}$$

Since collision is elastic →

$V_{separation} = v_{approch}$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow \frac{5v_0}{4} = v \text{ .....(2)}$$

equating (2) and (1)

$$\frac{3v_0}{2} = m\left(\frac{5v_0}{4}\right) \Rightarrow m = \frac{6}{5} = 1.2 \text{ kg}$$

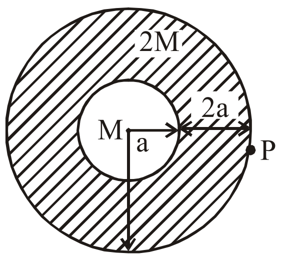
Option (2)

11. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be :

(1) $\frac{2GM}{9a^2}$ (2) $\frac{GM}{3a^2}$

(3) $\frac{GM}{9a^2}$ (4) $\frac{2GM}{3a^2}$

Sol.



We use gauss's Law for gravitation

$g \cdot 4\pi r^2$ = (Mass enclosed) $4\pi G$

$$g = \frac{3M4\pi G}{4\pi(3a)^2}$$

$$= \frac{MG}{3a^2}$$

Option (2)

12. The pressure wave, P = 0.01 sin [1000t – 3x] Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 15°C (2) 12°C

(3) 4°C (4) 11°C

Sol. Speed of wave from wave equation

$$v = -\frac{(\text{coeffecient of } t)}{(\text{coeffecient of } x)}$$

$$v = -\frac{1000}{(-3)} = \frac{1000}{3}$$

since speed of wave $\propto \sqrt{T}$

$$\text{so} = \frac{\frac{1000}{3}}{336} = \sqrt{\frac{273}{T}}$$

$\Rightarrow$ T = 277.41 K

T = 4.41°C

Option (3)

13. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15μA change in the base current of the amplifier. The input resistance and voltage gain are :

(1) 0.33 kΩ, 1.5

(2) 0.67 kΩ, 200

(3) 0.33 kΩ, 300

(4) 0.67 kΩ, 300

Sol. input current = 15×10^{-6}

output current = 3×10^{-3}

resistance output = 1000

$V_{input} = 10 \times 10^{-3}$

Now $V_{input} = r_{input} \times i_{input}$

$10 \times 10^{-3} = r_{input} \times 15 \times 10^{-6}$

$$r_{input} = \frac{2000}{3} = 0.67\ K\Omega.$$

$$\text{voltage gain} = \frac{V_{output}}{V_{input}} = \frac{1000 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

Option (4)

14. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4mA current. A voltmeter is made using this galvanometer and a 5 $k\Omega$ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to :

(1) 10 V (2) 20 V

(3) 40 V (4) 15 V

Sol. $G = 50\ \Omega$

$S = 5000\ \Omega$

$i_g = 4 \times 10^{-3}$

$V = i_g (G + S)$

$V = 4 \times 10^{-3} (50 + 5000)$

$= 4 \times 10^{-3}(5050)$

= 20.2 volt

Option (2)

15. The electric field of light wave is given as $\vec{E} = 10^{-3} \cos\left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right)\hat{x}\frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is :

$$\text{Given, E (in eV)} = \frac{12375}{\lambda(\text{in Å})}$$

(1) 0.48 V (2) 2.0 V

(3) 2.48 V (4) 0.72 V

Sol. $\omega = 6 \times 10^{14} \times 2\pi$

$f = 6 \times 10^{14}$

$C = f\lambda$

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5000 Å$$

$$\text{energy of photon} \Rightarrow \frac{12375}{5000}$$

= 2.475 eV

from Einstein's equation

$KE_{max} = E - \phi$

$eV_s = E - \phi$

$eV_s = 2.475 - 2$

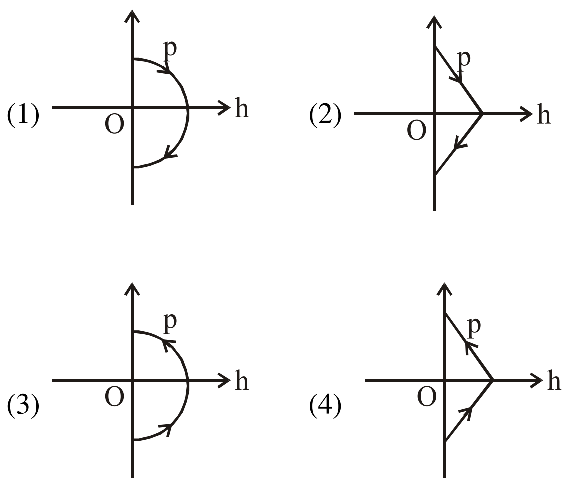
$eV_s = 0.475 - 2$

$eV_s = 0.475$ eV

$V_s = 0.475$ V $= 0.48$ volt

Option (1)

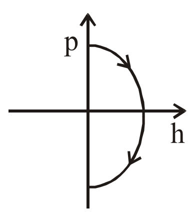
16. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is :



Sol. Momentum $p = mv$ .....(1)

and for motion under gravity $h = \frac{u^2 - v^2}{2g}$ ...(2)

$$h = \frac{u^2 - p^2/m^2}{2g}$$



Option (1)

17. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done ?

(1) 3.75×10^{-6} J

(2) 2.55×10^{-6} J

(3) 2.16×10^{-6} J

(4) 6.25×10^{-6} J

Sol. Work done = ΔU

$$= U_f - U_i$$

$$= \frac{q^2}{2C_f} - \frac{q^2}{2C_i}$$

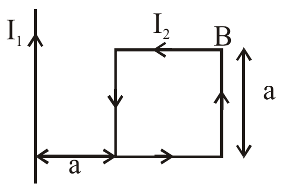
$$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$$

$$= \frac{15}{4} \times 10^{-6}$$

$$= 3.75 \times 10^{-6} \text{ J}$$

Option (1)

18. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be :



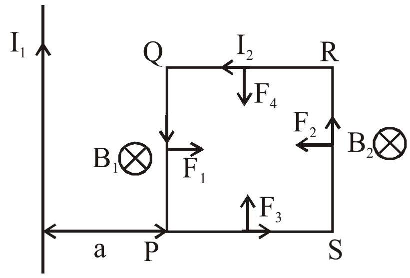
(1) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

(2) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(3) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$

(4) Zero

Sol.



F_3 & F_4 cancel each other

Force on PQ will be $F_1 = I_2 B_1 a$

$$= I_2 \frac{\mu_0 I_1}{2\pi a} a$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}$$

Force on RS will be $F_2 = I_2 B_2 a$

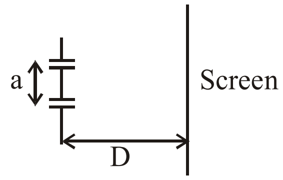
$$= I_2 \frac{\mu_0 I_1}{2\pi 2a} a$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}$$

Net force = $F_1 - F_2 = \frac{\mu_0 I_1 I_2}{4\pi}$ repulsion

Option (2)

19. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gest shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be :



(1) $\frac{2D\lambda}{a(\mu-1)}$ (2) $\frac{D\lambda}{a(\mu-1)}$

(3) $\frac{2nD\lambda}{a(\mu-1)}$ (4) $\frac{nD\lambda}{a(\mu-1)}$

Sol. Path difference at central maxima $\Delta x = (\mu - 1)t$, whole pattern will shift by same amount which will be given by

$(\mu - 1)t\ \dfrac{D}{d} = n\dfrac{\lambda D}{d}$, according to the question

$t = \dfrac{n\lambda}{(\mu - 1)}$

no option is matching, therefore question should be awarded bonus.

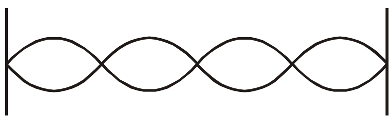
$\therefore$ Correct Option should be (Bonus)

20. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is : (All quantities are in SI units.)

(1) 20 m (2) 80 m

(3) 60 m (4) 40 m

Sol. 4th harmonic



$4\dfrac{\lambda}{2} = \ell$

$2\lambda = \ell$

From equation $\dfrac{2\pi}{\lambda} = 0.157$

$\lambda = 40$

$\ell = 2\lambda$

$= 80$ m

Option (2)

21. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane. : (i) a ring of radius R, (ii) a solid cylinder of radius $\dfrac{R}{2}$ and (iii) a solid sphere of radius $\dfrac{R}{4}$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 4 : 3 : 2 (2) 14 : 15 : 20

(3) 10 : 15 : 7 (4) 2 : 3 : 4

Sol. $\dfrac{1}{2}\left(m + \dfrac{I}{R^2}\right)v^2 = mgh$

if radius of gyration is k, then

$h = \dfrac{\left(1 + \dfrac{k^2}{R^2}\right)v^2}{2g}$, $\dfrac{k_{ring}}{R_{ring}} = 1$, $\dfrac{k_{solid\ cylinder}}{R_{solid\ cylinder}} = \dfrac{1}{\sqrt{2}}$

$\dfrac{k_{solid\ sphere}}{R_{solid\ sphere}} = \sqrt{\dfrac{2}{5}}$

$h_1 : h_2 : h_3 :: (1 + 1) : \left(1 + \dfrac{1}{2}\right) : \left(1 + \dfrac{2}{5}\right) :: 20 : 15 : 14$

Therefor **most appropriate option is (2)** athough which in not in correct sequence

22. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is :

(1) $\dfrac{k}{2I}\theta$ (2) $\dfrac{k}{I}\theta$ (3) $\dfrac{k}{4I}\theta$ (4) $\dfrac{2k}{I}\theta$

Sol. Kinetic energy KE = $\frac{1}{2}I\omega^2 = k\theta^2$

$$\Rightarrow \omega^2 = \frac{2k\theta^2}{I} \Rightarrow \omega = \sqrt{\frac{2k}{I}}\,\theta \qquad ....(1)$$

Differentiate (1) wrt time →

$$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \alpha = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\,\theta \ \{\text{by (1)}\}$$

$$\Rightarrow \alpha = \frac{2k}{I}\theta$$

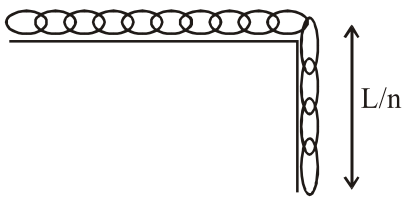
Option (4)

23. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be :

(1) $\frac{MgL}{n^2}$ (2) $\frac{MgL}{2n^2}$

(3) $\frac{2MgL}{n^2}$ (4) nMgL

Sol. Mass of the hanging part = $\frac{M}{n}$



$h_{COM} = \frac{L}{2n}$

work done $W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

Option (2)

24. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) 4M (2) M

(3) 2M (4) $\frac{M}{2}$

Sol. Height of liquid rise in capillary tube $h = \frac{2T\cos\theta_C}{\rho r g}$

$$\Rightarrow h \propto \frac{1}{r}$$

when radius becomes double height become half

$$\therefore h' = \frac{h}{2}$$

Now, $M = \pi r^2 h \times \rho$

and $M' = \pi\,(2r)^2\,(h/2) \times \rho = 2M$

Option (3)

25. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm (2) 642.7 nm

(3) 488.9 nm (4) 388.9 nm

Sol. $$\frac{1}{660} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \qquad ....(1)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16} \qquad ....(2)$$

divide equation (1) with (2)

$$\frac{\lambda}{660} = \frac{5\times16}{36\times3}$$

$$\lambda = \frac{4400}{9} = 488.88 = 488.9 \text{ nm}$$

Option (3)

26. The magnetic field of a plane electromagnetic wave is given by :

$\vec{B} = B_0\hat{i}[\cos(kz - \omega t)] + B_1\hat{j}\cos(kz + \omega t)$ where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to :

(1) 0.9 N (2) 0.1 N

(3) 3×10^{-2} N (4) 0.6 N

Official Ans. by NTA (4)

Sol. Maximum Electric field E = (B) (c)

$\vec{E}_0 = (3 \times 10^{-5})c\left(-\hat{j}\right)$

$\vec{E}_1 = (2 \times 10^{-6})c\left(-\hat{i}\right)$

Maximum force

$\vec{F}_{net} = q\vec{E} = qc\left(-3\times10^{-5}\hat{j} - 2\times10^{-6}\hat{i}\right)$

$\vec{F}_{0max} = 10^{-4} \times 3 \times 10^8 \sqrt{(3\times10^{-5})^2 + (2\times10^{-6})^2}$

= 0.9N

$F_{rms} = \dfrac{F_0}{\sqrt{2}} = 0.6$ N (approx)

Option (4)

27. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is :

(1) 0.55 Nm (2) 0.27 Nm

(3) 0.38 Nm (4) 0.42 Nm

Sol. $|\vec{\tau}| = |\vec{M} \times \vec{B}|$

τ = NI × A × B × sin 45°

τ = 0.27 Nm

Option (2)

28. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is :

(1) 1.60 m (2) 0.24 m

(3) 0.16 m (4) 0.32 m

Sol. $m = \dfrac{f}{f-u}$

$5 = \dfrac{-40}{-40-u}$

u = –32 cm

Option (4)

29. A signal $A\cos\omega t$ is transmitted using $v_0 \sin \omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is :

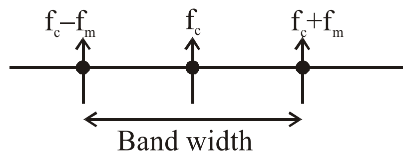
(1) $v_0 \sin\omega_0 t + A\cos\omega t$

(2) $v_0 \sin\omega_0 t + \dfrac{A}{2}\sin(\omega_0 - \omega)t + \dfrac{A}{2}\sin(\omega_0 + \omega)t$

(3) $(v_0 + A)\cos\omega t \sin\omega_0 t$

(4) $v_0 \sin[\omega_0(1 + 0.01A\sin\omega t)t]$

Sol.



Option (2)

30. For a given gas at 1 atm pressure, rms speed of the molecule is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be :

(1) 80 m/s (2) $100\sqrt{5}$ m/s

(3) $80\sqrt{5}$ m/s (4) 100 m/s

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M_w}} \Rightarrow v_{rms} \propto \sqrt{T}$

Now, $\dfrac{v}{200} = \sqrt{\dfrac{500}{400}} \Rightarrow \dfrac{v}{200} = \dfrac{\sqrt{5}}{2}$

$\Rightarrow v = 100\sqrt{5}$ m/s

Option (2)

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is :

(1) $NO_2 < N_2O_3 < NO < N_2O$

(2) $NO_2 < NO < N_2O_3 < N_2O$

(3) $N_2O < N_2O_3 < NO < NO_2$

(4) $N_2O < NO < N_2O_3 < NO_2$

Sol. Correct order of oxidation state of nitrogen in oxides of nitrogen is following

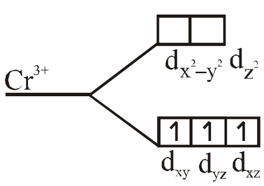
$$\overset{+1}{N_2}O < \overset{+2}{N}O < \overset{+3}{N_2}O_3 < \overset{+4}{N}O_2$$

2. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are :

(1) d_{yz} and d_{z^2} (2) d_{z^2} and d_{xz}

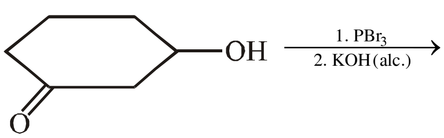
(3) d_{xz} and d_{yz} (4) $d_{x^2-y^2}$ and d_{xy}

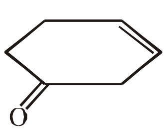
Sol. Degenerate orbitals of $[Cr(H_2O)_6]^{3+}$

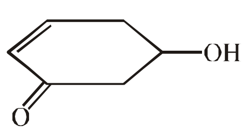


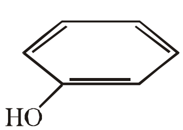
Hence according to the options given, degenerate orbitals are d_{xz} & d_{yz}

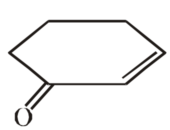
3. The mojor product of the following reaction is :

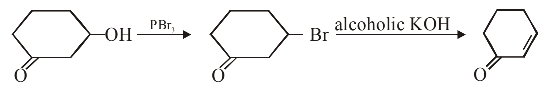


(1) 

(2) 

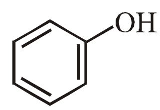
(3) 

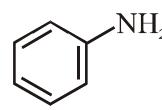
(4) 

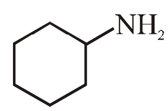
Sol. 

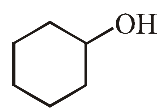
4. The organic compound that gives following qualitative analysis is :

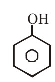
Test	Inference
(a) Dil. HCl	Insoluble
(b) NaOH solution	soluble
(c) Br_2/water	Decolourization

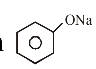
(1) 

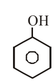
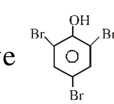
(2) 

(3) 

(4) 

Sol.  is insoluble in dil. HCl but soluble in NaOH

to form 

 decolorise Br_2 water to give 

(2,4,6- tribromophenol)

5. Which of the following statements is not true about sucrose?

(1) On hydrolysis, it produces glucose and fructose

(2) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose

(3) It is also named as invert sugar

(4) It is a non reducing sugar

Sol. Sucrose $\xrightarrow{H_2O}$ α-D-glucose + β-D-fructose

also named as invert sugar & it is a example of non-reducing sugar.

The glycosidic linkage is present between C_1 of α-glucose & C_2 of β-fructose.

6. Excessive release of CO_2 into the atomosphere results in :

(1) polar vortex (2) depletion of ozone
(3) formation of smog (4) global warming

Sol. Excessive release of CO_2 into the atmosphere results in **global warming**.

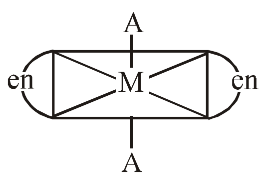
7. Among the following, the molecule expected to be stabilized by anion formation is :

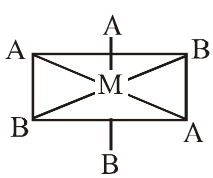
C_2, O_2, NO, F_2

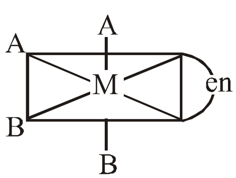
(1) NO (2) C_2 (3) F_2 (4) O_2

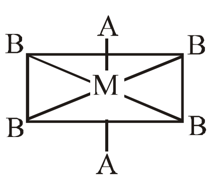
Sol. In case of only C_2, incoming electron will enter in the bonding molecular orbital which increases the bond order and stability too. Whereas rest of all takes electron in their antibonding molecular orbital which decreases bond order and stability.

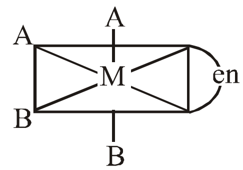
8. The one that will show optical activity is :
(en = ethane-1,2-diamine)

(1) 

(2) 

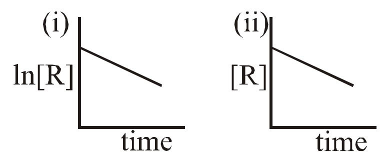
(3) 

(4) 

Sol. 

This structure does not contain plane of symmetry hence it is optically active, rest of all options has plane of symmetry and they are optically inactive.

9. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are :



(1) 1,0 (2) 1,1 (3) 0,1 (4) 0,2

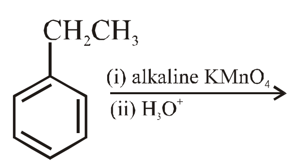
Sol. (i) $\ln[R] = \ln[R]_0 - Kt$ (I[st] order)
$[R] = [R]_0 - Kt$ (zero order)
$\therefore$ Ans.(1)

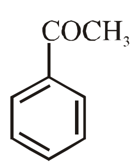
10. The aerosol is a kind of colloid in which :

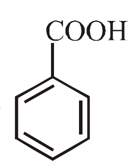
(1) gas is dispersed in solid
(2) solid is dispersed in gas
(3) liquid is dispersed in water
(4) gas is dispersed in liquid

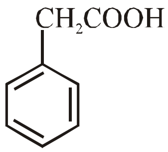
Sol. Aerosol is suspension of fine solid or liquid particles in air or other gas.
Ex. Fog, dust, smoke etc
$\therefore$ Ans.(2)

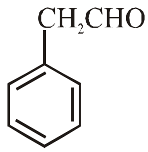
11. The major product of the following reaction is :

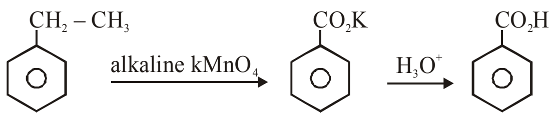


(1) 

(2) 

(3) 

(4) 

Sol. 

12. For a reaction,
$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$;
identify dihydrogen (H_2) as a limiting reagent in the following reaction mixtures.
(1) 14g of N_2 + 4g of H_2
(2) 28g of N_2 + 6g of H_2
(3) 56g of N_2 + 10g of H_2
(4) 35g of N_2 + 8g of H_2

Sol. $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

(1) $\underset{\text{(LR)}}{0.5\text{ mol}}$ 2 mol

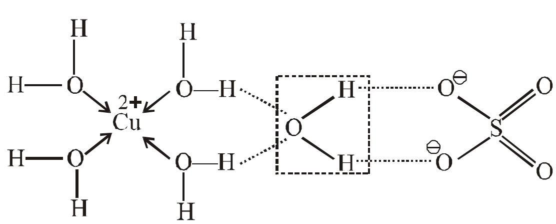
(2) 1 mol 3 mol (completion)

(3) 2 mol $\underset{\text{(LR)}}{5\text{ mol}}$

(4) $\underset{\text{(LR)}}{1.25\text{ mol}}$ 4 mol

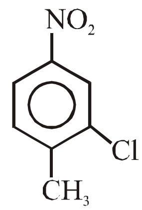
$\therefore$ Ans.(3)

13. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4.5H_2O$, is :
(1) 4 (2) 3 (3) 1 (4) 2

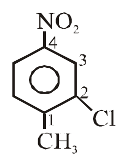
Sol. 

One water molecule as shown in the diagram, is not coordinated to copper ion directly.

14. The correct IUPAC name of the following compound is :



(1) 5-chloro-4-methyl-1-nitrobenzene
(2) 2-methyl-5-nitro-1-chlorobenzene
(3) 3-chloro-4-methyl-1-nitrobenzene
(4) 2-chloro-1-methyl-4-nitrobenzene

Sol. 

2-chloro–1-methyl–4–nitrobenzene

15. C_{60}, an allotrope of carbon contains :
(1) 20 hexagons and 12 pentagons.
(2) 12 hexagons and 20 pentagons.
(3) 18 hexagons and 14 pentagons.
(4) 16 hexagons and 16 pentagons.

Sol. In C_{60} molecule there are 20 hexagons and 12 pentagons
$\therefore$ Ans.(1)

16. Among the following, the set of parameters that represents path function, is :
(A) q + w
(B) q
(C) w
(D) H–TS
(1) (A) and (D) (2) (B), (C) and (D)
(3) (B) and (C) (4) (A), (B) and (C)

Sol. (A) $q + w = \Delta U \leftarrow$ definite quantity
(B) $q \rightarrow$ Path function
(C) $w \rightarrow$ Path function
(D) $H - TS = G \rightarrow$ state function
$\therefore$ Ans.(3)

17. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is :

(1) 6×10^{-2} (2) 4×10^{-4}

(3) 16×10^{-4} (4) 4×10^{-2}

Sol. $\pi_{XY} = 4\pi_{BaCl_2}$

$2 \times [XY] = 4 \times 3 \times 0.01$

(Assuming same temperature)

$\Rightarrow [XY] = 0.06$ M

$\therefore$ Ans. is (1)

18. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is :

$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$,

$E^\circ = 2$ V at 298 K

(Faraday's constant, F = 96000 C mol^{-1})

(1) –384 (2) –192

(3) 192 (4) 384

Sol. $\Delta G^\circ = -nFE^\circ_{cell}$

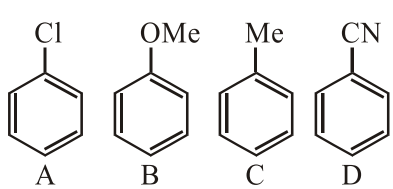
$= -2 \times 96000 \times 2$

$= -384000$ J

$= -384$ kJ

$\therefore$ Ans. is (1)

19. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is :

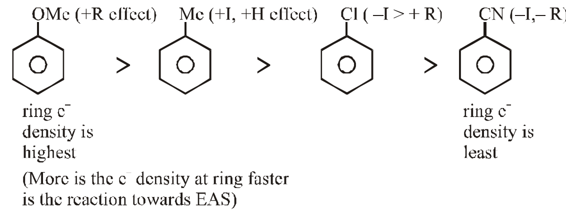


(1) D < B < A < C (2) A < B < C < D

(3) D < A < C < B (4) B < C < A < D

Official Ans. by NTA (3)

Sol.



20. The ore that contains the metal in the form of fluoride is :

(1) magnetite (2) sphalerite

(3) malachite (4) cryolite

Sol. $Na_3AlF_6 \rightarrow$ Cryolite is the fluoride ore.

Magnetite Fe_3O_4

Sphalerite ZnS

Malachite $Cu(OH)_2.CuCO_3$

21. Consider the van der Waals constants, a and b, for the following gases.

Gas	Ar	Ne	Kr	Xe
a/ (atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/ (10^{-2} dm^3 mol^{-1}	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(1) Kr (2) Ne

(3) Ar (4) Xe

Sol. $T_c = \dfrac{8a}{27Rb}$

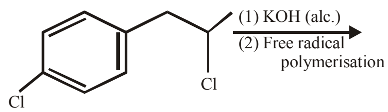
Greater value of $\dfrac{a}{b} \Rightarrow$ higher is 'T_c'

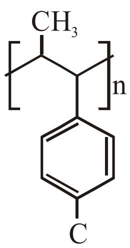
Gas	a/b
Ar	$\dfrac{1.3}{3.2} = 0.406$
Ne	$\dfrac{0.2}{1.7} = 0.118$
Kr	$\dfrac{5.1}{1} = 5.1$
Xe	$\dfrac{4.1}{5} = 0.82$

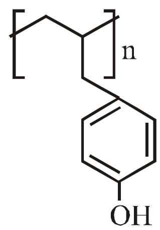
$\therefore$ T_c has order : Kr > Xe > Ar > Ne

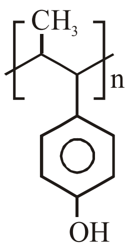
$\therefore$ Ans. is (1)

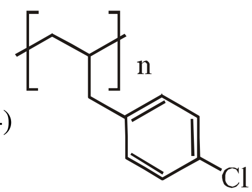
22. The major product of the following reaction is :



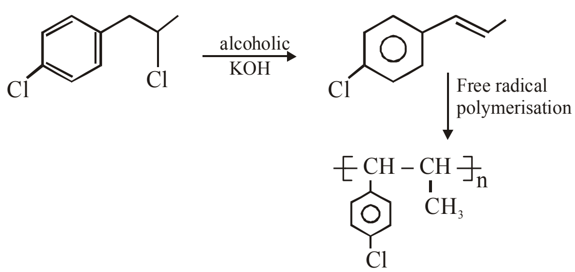
(1) 

(2) 

(3) 

(4) 

Sol.



23. The major product of the following reaction is :

$$CH_3CH=CHCO_2CH_3 \xrightarrow{LiAlH_4}$$

(1) $CH_3CH_2CH_2CHO$

(2) $CH_3CH = CHCH_2OH$

(3) $CH_3CH_2CH_2CO_2CH_3$

(4) $CH_3CH_2CH_2CH_2OH$

Sol. $CH_3-CH=CH-\underset{\underset{O}{\|}}{C}-OCH_3$

$\downarrow LiAlH_4$

$CH_3-CH=CH-CH_2OH$

24. Magnisium powder burns in air to give:

(1) MgO only

(2) MgO and $Mg(NO_3)_2$

(3) MgO and Mg_3N_2

(4) $Mg(NO_3)_2$ and Mg_3N_2

Sol. $Mg+\underset{O_2/N_2}{air}\xrightarrow{\Delta}MgO+Mg_3N_2$

25. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:
(x_M = Mole fraction of 'M' in solution ;
x_N = Mole fraction of 'N' in solution ;
y_M = Mole fraction of 'M' in vapour phase ;
y_N = Mole fraction of 'N' in vapour phase)

(1) $(x_M - y_M) < (x_N - y_N)$

(2) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$

(3) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

(4) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$

Sol. $\because P_N^\circ > P_M^\circ$

$\therefore y_N > X_N$

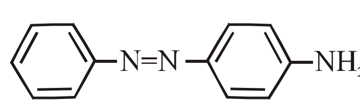
& $X_M > y_M$

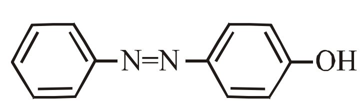
Multiply we get

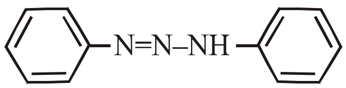
$y_N X_M > X_N\ y_M$

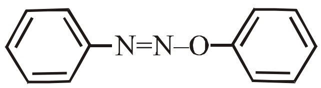
$\therefore$ Ans. is (3)

26. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0ºC. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is :

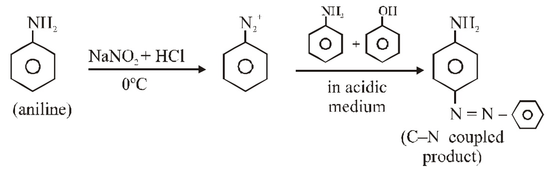
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

Aniline undergoes diazo coupling in acidic medium with $Ph\overset{+}{N}_2$

27. The element having greatest difference between its first and second ionization energies, is :

(1) Ca (2) K (3) Ba (4) Sc

Sol. K = 2, 8, 8 ,1

After removal of one electron, second electron we have to remove from another shell, hence there is large difference between first and second ionization energies.

28. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman}/\Delta\bar{v}_{Balmer}$ is :

(1) 27 : 5 (2) 4 : 1

(3) 5 : 4 (4) 9 : 4

Sol. For Lyman

$$\bar{v}_{max} = R_H\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R_H$$

$$\bar{v}_{min} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R_H$$

$$\Delta\bar{v}_{Lyman} = \frac{R_H}{4}$$

For Balmer

$$\bar{v}_{max} = R_H\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R_H}{4}$$

$$\bar{v}_{min} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R_H$$

$$\Delta\bar{v}_{Balmer} = \frac{R_H}{4} - \frac{5R_H}{36} = \frac{4R_H}{36} = \frac{R_H}{9}$$

$$\frac{\Delta\bar{v}_{Lyman}}{\Delta\bar{v}_{Balmer}} = \frac{R_H/4}{R_H/9} = \frac{9}{4}$$

$\therefore$ Ans. is (4)

29. The major product of the following reaction is :

$$CH_3C\equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1 equiv.)}}$$

(1) $CH_3CD(Cl)CHD(I)$

(2) $CH_3CD_2CH(Cl)(I)$

(3) $CH_3CD(I)CHD(Cl)$

(4) $CH_3C(I)(Cl)CHD_2$

Sol. $CH_3-C\equiv CH \xrightarrow{DCl} H_3C-\underset{\underset{Cl}{|}}{C}=CHD \xrightarrow{DI} H_3C-\overset{\overset{I}{|}}{\underset{\underset{Cl}{|}}{C}}-CHD_2$

30. Match the catalysts **(Column I)** with products **(Column II)**.

Column I	**Column II**
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) ethanal
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron Oxide	(iv) NH_3

(1) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

(2) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(3) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(4) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

Sol. V_2O_5 is catalyst $\rightarrow$ contact process for H_2SO_4

$TiCl_4/Al(Me)_3 \rightarrow$ Ziegler Natta salt used as catalyst for polymerisation of ethene.

$PdCl_2 \rightarrow$ used as catalyst for ethanal (Wacker process).

Iron oxide $\rightarrow$ is used as catalyst in Haber's synthesis.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS — TEST PAPER WITH ANSWER & SOLUTION

1. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(3) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (4) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

Sol. $\vec{\alpha} = 3\hat{i} + \hat{j}$

$\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\vec{\beta}_1 = \lambda\left(3\hat{i} + \hat{j}\right), \vec{\beta}_2 = \lambda\left(3\hat{i} + \hat{j}\right) - 2\hat{i} + j - 3\hat{k}$

$\vec{\beta}_2 . \vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$$\text{Now } \vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$$

$$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$$

$$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$$

$$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$$

Aliter :

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \Rightarrow \vec{\beta}.\hat{\alpha} = \vec{\beta}_1.\hat{\alpha} = \left|\vec{\beta}_1\right|$

$\Rightarrow \vec{\beta}_1 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha}$

$\Rightarrow \vec{\beta}_2 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} - \vec{\beta}$

$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = -\left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} \times \vec{\beta}$

$= \frac{-5}{10}\left(3\hat{i} + \hat{j}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

2. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is

(1) $p \wedge q$ (2) $p \leftrightarrow q$

(3) $\sim p \vee \sim q$ (4) $\sim p \wedge \sim q$

Sol. $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q)$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

3. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is

(1) $\frac{\pi - 2}{4}$ (2) $\frac{\pi - 2}{8}$ (3) $\frac{\pi - 1}{4}$ (4) $\frac{\pi - 1}{2}$

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$

$= \int_0^{\pi/4} \left(1 - \sin x \cos x\right) dx$

$$= \left(x - \frac{\sin^2 x}{2} \right)_0^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{1}{4}$$

$$= \frac{\pi - 1}{4}$$

4. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$

Contains exactly :

(1) four irrational numbers.

(2) two irrational and one rational number.

(3) four rational numbers.

(4) two irrational and two rational numbes.

Sol. $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu$$

Now $f(x) = f(0)$

$$\Rightarrow \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu = \mu$$

$\Rightarrow x = 0, 0, \pm\sqrt{2}$

Two irrational and one rational number

5. If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$ where $k > 0$, then k is equal to

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

Sol. $S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$$\bar{x} = \frac{\Sigma x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$$

Now $\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$

$$\Rightarrow 5 \times 4 = 2\left(1+\frac{k}{16}\right)^2 + \frac{5k^2}{8}$$

$$\Rightarrow 18 = \frac{3k^2}{4}$$

$\Rightarrow k^2 = 24$

$\Rightarrow k = 2\sqrt{6}$

6. All the points in the set $S = \left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\}$ $\left(i = \sqrt{-1}\right)$ lie on a

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is -1

(4) circle whose radius is $\sqrt{2}$.

Sol. Let $\frac{\alpha+i}{\alpha-i} = z$

$$\Rightarrow \frac{|\alpha+i|}{|\alpha-i|} = |z|$$

$\Rightarrow 1 = |z|$

$\Rightarrow$ circle of radius 1

7. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to :

(1) $\left\{-\frac{1}{3}, -1\right\}$ (2) $\left\{\frac{1}{3}, -1\right\}$

(3) $\left\{-\frac{1}{3}, 1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$$m = \frac{f(1) - f(-1)}{1+1} = \frac{-2-0}{2} = -1$$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$\Rightarrow x = 1, -\frac{1}{3}$

8. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) {5,10,15,20} (2) {10,15}

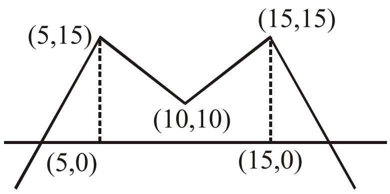
(3) {5,10,15} (4) {10}

Sol. $f(x) = 15 - |x - 10|, x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



x = 5, 10, 15 are points of non differentiability

Aliter :

At x = 10 $f(x)$ is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow x = 5, 15$

$\therefore$ non differentiability points are {5, 10, 15}

9. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $q^2 + 4p + 14 = 0$ (2) $p^2 - 4q - 12 = 0$

(3) $q^2 - 4p - 16 = 0$ (4) $p^2 - 4q + 12 = 0$

Sol. In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2 - (\alpha + 2 - \sqrt{3})x + \alpha.\left(2-\sqrt{3}\right) = 0$

Now, we can have none or any of the options can be correct depending upon 'α'

Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2+\sqrt{3}$

$\Rightarrow p = -\left(2+\sqrt{3}-2-\sqrt{3}\right) = -4$

and $q = \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (2) is correct

10. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is

(1) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\dfrac{1-\sqrt{5}}{1+\sqrt{5}}$

(3) $\dfrac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\dfrac{\sqrt{7}-1}{\sqrt{7}+1}$

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \dfrac{2}{r} = \dfrac{2}{\pm 4} = \pm\dfrac{1}{2}$

$\Rightarrow 1 + \sin 2\theta = \dfrac{1}{4}$

$\Rightarrow \sin 2\theta = -\dfrac{3}{4}$

$\Rightarrow \dfrac{2m}{1+m^2} = -\dfrac{3}{4}$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$\Rightarrow m = \dfrac{-4\pm\sqrt{7}}{1-7}$

$$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$$

11. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m = n = 78 (2) n = m – 8

(3) m + n = 68 (4) m = n = 68

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13\times 12}{2} = 78$$

12. If the fourth term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8 (2) 8^2 (3) 8^{-2} (4) 8^3

Sol. $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 . \left(x^{\log_8 x}\right)^3$

$$20\times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$$

$$8^6 = x^{\log_2 x} - 3$$

$$2^{18} = x^{\log_2 x - 3}$$

$$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$$

Let $\log_2 x = t$

$$\Rightarrow t^2 - 3t - 18 = 0$$

$$\Rightarrow (t-6)(t+3) = 0$$

$$\Rightarrow t = 6, -3$$

$$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$$

$$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$$

13. The solution of the differential equation $x\frac{dy}{dx}+2y = x^2$ ($x \neq 0$) with y(1) = 1, is

(1) $y=\frac{x^3}{5}+\frac{1}{5x^2}$ (2) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

(3) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$ (4) $y=\frac{x^2}{4}+\frac{3}{4x^2}$

Sol. $x\frac{dy}{dx}+2y = x^2 : y(1) = 1$

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y = x \quad \text{(LDE in y)}$$

$$\text{IF } = e^{\int \frac{2}{x}dx} = e^{2\ell n x} = x^2$$

$$y.(x^2) = \int x.x^2 dx = \frac{x^4}{4}+C$$

y(1) =1

$$1 = \frac{1}{4}+C \Rightarrow C = 1-\frac{1}{4} = \frac{3}{4}$$

$$yx^2 = \frac{x^4}{4}+\frac{3}{4}$$

$$y = \frac{x^2}{4}+\frac{3}{4x^2}$$

14. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point

(1) $\left(-\sqrt{2},1,-4\right)$ (2) $\left(\sqrt{2},1,4\right)$

(3) $\left(\sqrt{2},-1,4\right)$ (4) $\left(-\sqrt{2},-1,-4\right)$

Sol. Let ax + by + cz = 1 be the equation of the plane

$$\Rightarrow 0 - b + 0 = 1$$

$$\Rightarrow b = -1$$

0 + 0 + c = 1

$$\Rightarrow c = 1$$

$$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$$

$$\Rightarrow a^2 + 2 = 4$$

$$\Rightarrow a = \pm\sqrt{2}$$

$$\Rightarrow \pm\sqrt{2}x - y + z = 1$$

Now for –sign

$$-\sqrt{2}.\sqrt{2} - 1 + 4 = 1$$

option (2)

15. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to (Hence C is a constant of integration)

(1) $3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$

(3) $-3\cot^{-1/3}x + C$ (4) $-3\tan^{-1/3}x + C$

Sol. $I=\int \frac{dx}{(\sin x)^{4/3}.(\cos x)^{2/3}}$

$I=\int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$

$\Rightarrow I=\int \frac{\sec^2 x}{(\tan x)^{4/3}}dx$

put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I=\int \frac{dt}{t^{4/3}} \Rightarrow I=\frac{-3}{t^{1/3}}+c$

$\Rightarrow I=\frac{-3}{(\tan x)^{1/3}}+c$

16. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be $50n+\frac{n(n-7)}{2}A$, wherre A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

(1) (A, 50+46A) (2) (A, 50+45A)
(3) (50, 50+46A) (4) (50, 50+45A)

Sol. $S_n=50n+\frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$=50n+\frac{n(n-7)}{2}A-50(n-1)-\frac{(n-1)(n-8)}{2}A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n - 4)$

$d = T_n - T_{n-1}$

$= 50 + A(n - 4) - 50 - A(n - 5)$

$= A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50+46A)$

17. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$ (3) $\frac{31}{6}$ (4) $\frac{13}{6}$

Sol. $x^2 \le y \le x + 2$

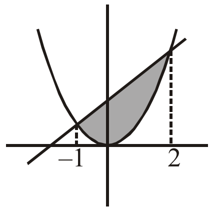
$x^2 = y$; $y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2, -1$



Area $= \int_{-1}^{2}(x+2)-x^2\,dx=\frac{9}{2}$

18. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is

(1) $\frac{9}{2}$ (2) $2\sqrt{5}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{7}{2}$

Sol. Any point on the given line can be $(1 + 2\lambda , -1 + 3\lambda, 2 + 4\lambda)$; $\lambda \in R$

Put in plane

$1 + 2\lambda + (-2 + 6\lambda) + (6 + 12\lambda) = 15$

$20\lambda + 5 = 15$

$20\lambda = 10$

$\lambda = \frac{1}{2}$

$\therefore$ Point $\left(2,\frac{1}{2},4\right)$

Distance from origin

$=\sqrt{4+\frac{1}{4}+16}=\frac{\sqrt{16+1+64}}{2}=\frac{\sqrt{81}}{2}$

$=\frac{9}{2}$

19. Let $\sum_{k=1}^{10} f(a+k)=16\left(2^{10}-1\right)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. then the natural number 'a' is

(1) 4 (2) 3 (3) 16 (4) 2

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall\ x \in N$

$2^{a+1} + 2^{a+2}+....+2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2+....+2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.(2^{10}-1)}{1}=16(2^{10}-1)$$

$2^{a+1} = 16 = 2^4$

a = 3

20. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(1) y^3 (2) $y^3 - 1$

(3) $y(y^2 - 1)$ (4) $y(y^2 - 3)$

Sol. Roots of the equation $x^2 + x + 1 = 0$ are $\alpha = \omega$ and $\beta = \omega^2$

where ω, ω^2 are complex cube roots of unity

$$\therefore \quad \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \ \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta = y.y^2 \Rightarrow D = y^3$

21. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?

(1) (–2, 2) (2) (2, –2)

(3) (2, –1) (4) (–2, 1)

Sol. $y = x^3 + ax - b$

(1, –5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'_{(1, -5)} = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

By (i) and (ii) : a = –4, b = 2

$\therefore \ y = x^3 - 4x - 2$.

(2,–2) lies on this curve.

22. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. if all hit at the target independently, then the probability that the target would be hit, is

(1) $\frac{25}{192}$ (2) $\frac{1}{192}$ (3) $\frac{25}{32}$ (4) $\frac{7}{32}$

Sol. Let persons be A,B,C,D

P(Hit) = 1 – P(none of them hits)

$$=1-P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D})$$

$$=1-P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$$

$$=1-\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$$

$$=\frac{25}{32}$$

23. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{15}}{2}$

Sol. $\frac{x^2}{24}-\frac{y^2}{18}=1 \Rightarrow a=\sqrt{24}; b=\sqrt{18}$

Parametric normal :

$$\sqrt{24}\cos\theta.x+\sqrt{18}.y\cot\theta=42$$

At x = 0 : $y=\frac{42}{\sqrt{18}}\tan\theta=7\sqrt{3}$ (from given equation)

$$\Rightarrow \tan\theta=\sqrt{\frac{3}{2}} \Rightarrow \sin\theta=\pm\sqrt{\frac{3}{5}}$$

slope of parametric normal $=\frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta}=m$

$$\Rightarrow m=-\sqrt{\frac{4}{3}}\sin\theta=-\frac{2}{\sqrt{5}} \text{ or } \frac{2}{\sqrt{5}}$$

24. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is

(1) $\frac{13\pi}{6}$ (2) π (3) 2π (4) $\frac{5\pi}{3}$

Sol. $2\left(1-\sin^2\theta\right)+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2};\sin\theta=2(\text{reject})$

roots : $\pi+\frac{\pi}{6}, 2\pi-\frac{\pi}{6}, -\frac{\pi}{6}, -\pi+\frac{\pi}{6}$

$\Rightarrow$ sum of values $=2\pi$

25. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

(1) $\frac{3}{2}(1+\cos 20°)$ (2) $\frac{3}{4}$

(3) $\frac{3}{4}+\cos 20°$ (4) $\frac{3}{2}$

Sol. $\frac{1}{2}\left(2\cos^2 10°-2\cos 10°\cos 50°+2\cos^2 50°\right)$

$\Rightarrow \frac{1}{2}\left(1+\cos 20°-(\cos 60°+\cos 40°)+1+\cos 100°\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20°+2\sin 70°\sin(-30°)\right)$

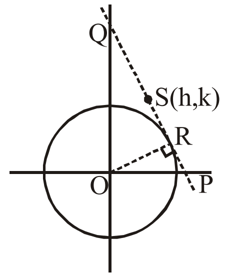
$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20°-\sin 70°\right)$

$\Rightarrow \frac{3}{4}$ Ans. (2)

26. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

(1) $x^2 + y^2 - 2xy = 0$
(2) $x^2 + y^2 - 16x^2y^2 = 0$
(3) $x^2 + y^2 - 4x^2y^2 = 0$
(4) $x^2 + y^2 - 2x^2y^2 = 0$

Sol.



Let the mid point be S(h,k)

$\therefore$ P(2h,0) and Q(0,2k)

equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

$\because$ PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\Rightarrow \frac{1}{4h^2}+\frac{1}{4k^2}=1$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

Aliter :

tangent to circle

$x\cos\theta + y\sin\theta = 1$

P : $(\sec\theta, 0)$

Q : $(0, \operatorname{cosec}\theta)$

$2h = \sec\theta \Rightarrow \cos\theta=\frac{1}{2h}$ & $\sin\theta=\frac{1}{2k}$

$\frac{1}{(2x)^2}+\frac{1}{(2y)^2}=1$

27. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by

$f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$ is continuous,

then k is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) 2

Sol. $\therefore$ function should be continuous at $x = \frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\text{cosec}^2 x} = k \quad \text{(Using L'H}\hat{o}\text{pital rule)}$$

$$\lim_{x\to\frac{\pi}{4}} \sqrt{2}\sin^3 x = k$$

$$\Rightarrow k = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2}$$

28. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}......\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is

(1) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

Sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}.....\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}1 & 1+2+3+.....+n-1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

$$\Rightarrow \frac{n(n-2)}{2} = 78 \Rightarrow n = 13, -12(\text{reject})$$

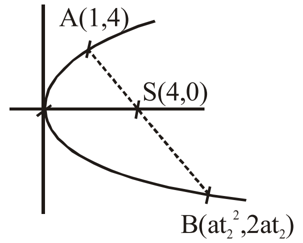
$\therefore$ We have to find inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

$$\therefore \begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$$

29. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is

(1) 25 (2) 24 (3) 20 (4) 22

Sol.



$$y^2 = 4ax = 16x \Rightarrow a = 4$$

$$A(1,4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$$

$$\therefore \text{ length of focal chord } = a\left(t + \frac{1}{t}\right)^2$$

$$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$$

30. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to

(1) R – [–1, 0) (2) R – (–1, 0)
(3) R – {–1} (4) [0, ∞)

Sol. $y = \frac{x^2}{1-x^2}$

Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.