JEE Advanced Exam 2017
(Paper & Solution)

Date : 21 / 05 / 2017

PAPER-1
PART-I (PHYSICS)

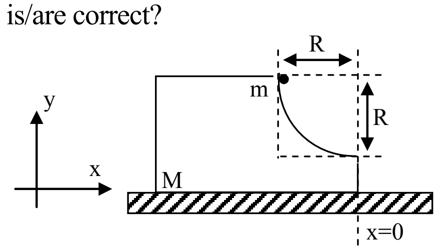
SECTION – 1 (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: + 4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: – 2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get + 4 marks; darkening only (A) and (D) will get + 2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

Q.1 A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct?



(A) The x component of displacement of the center of mass of the block M is : $-\frac{mR}{M+m}$

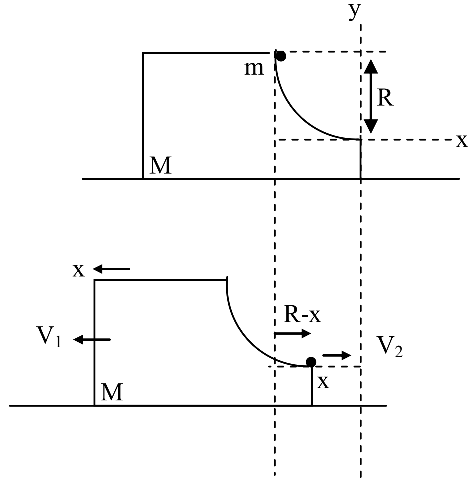
(B) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

(C) The velocity of the block M is: $V = -\frac{m}{M}\sqrt{2gR}$

(D) The velocity of the point mass m is: $v = \sqrt{\frac{2gR}{...}}$

Ans. **[A, D]**

Sol.



$\Delta X_{cm} = 0$ ($\because$ $\sum \vec{F}_{ext}$ Horizontal = 0, Initial $\vec{V}_{cm} = 0$ So COM remain at rest)

$Mx - m(R-x) = 0$

$(M+m)x = mR \Rightarrow x = \dfrac{mR}{(M+m)}$

x component of displacement of block $= -x = \dfrac{-mR}{(M+m)}$

Position of point mass also $= -x = \dfrac{-mR}{(M+m)}$.

Applying momentum conservation.

$Mv_1 - mV_2 = 0 \Rightarrow Mv_1 = mV_2$

Energy conservation.

$$\frac{1}{2}MV_1^2 + \frac{1}{2}mv_2^2 - mgR = 0 + 0 + 0$$

$$\frac{1}{2}M(V_1^2) + \frac{1}{2}m\left(\frac{Mv_1}{m}\right)^2 = mgR$$

$$\frac{1}{2}MV_1^2 + \frac{1}{2}\frac{M^2}{m}v_1^2 = mgR$$

$$\frac{M}{m}v_1^2 + \left(\frac{M}{m}\right)^2 v_1^2 = 2gR$$

$$\frac{M}{m}\left(1+\frac{M}{m}\right)v_1^2 = 2gR$$

$$V_1 = +\sqrt{\frac{2gR}{\frac{M}{m}\left(1+\frac{M}{m}\right)}} \qquad \left\{ \underset{\text{in direction}}{V_1} = -\sqrt{\frac{2gR}{\frac{M}{m}\left(1+\frac{M}{m}\right)}} \right.$$

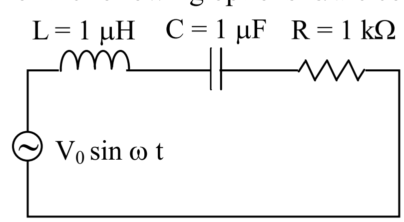
$$V_2 = \frac{M}{m} V_1$$

$$V_2 = \frac{M}{m}\sqrt{\frac{2gR}{\frac{M}{m}\left(1+\frac{M}{m}\right)}}$$

$$V_2 = \sqrt{\frac{M}{m}\frac{2gR}{\left(1+\frac{M}{m}\right)}}$$

$$V_2 = +\sqrt{\frac{2gR}{\left(1+\frac{M}{m}\right)}}$$

Q.2 In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ.. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(A) At ω~0 the current flowing through the circuit becomes nearly zero

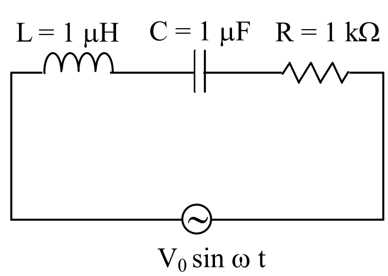
(B) The current will be in phase with the voltage if $\omega = 10^4$ rad. s^{-1}

(C) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

(D) The frequency at which the current will be in phase with the voltage is independent of R

Ans. **[A,D]**

Sol.



When $\omega = 0$ $X_L = \infty$ $z = \infty$ $\therefore$ $i = 0$ Ans. (A)

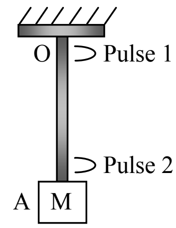
Current will be in phase with voltage at resonance (It is independent of resistance)

$$X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rad/sec.}$$

Q.3 A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The wavelength of Pulse 1 becomes longer when it reaches point A

(B) The velocities of the two pulses (Pulse 1 and 2) are the same at the midpoint of rope

(C) The velocity of any pulse along the rope is independent of its frequency and wavelength

(D) The time $T_{AO} = T_{OA}$

Ans. **[C,D]**

Sol. Tension at O is greater than tension at A

$\boxed{V_0 > V_A}$ frequency remains constant

So $v = n\lambda$

Wave pulse 1 when reach at A its speed decreases hence wavelength gets shorter.

Velocity of two pulse have same speed at all position & direction opposite.

So $T_{OA} = T_{AO}$

Q.4 A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K, For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460\ Wm^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct?

(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

(D) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

Ans. **[B,D]**

Sol. Body Temp.T $\longleftrightarrow$ T_0 (surrounding Temp.)

Heat Radiated $\rightarrow \sigma AT^4$ $\quad \sigma AT_0^4$ Heat absorbed

Net Heat Radiated = $\sigma A(T^4 - T_0^4)$

By Newton's cooling law :-

$$\frac{\Delta Q}{\Delta t} = 4\sigma A\, T_0^{\,3}(\Delta\theta)$$

$\Delta\theta = (T - T_0) = 10$

If surrounding temperature reduce by ΔT

$(\Delta Q)_{new} \Rightarrow (\Delta\theta) + \Delta T = 10 + \Delta T$

Net Heat Released $\Rightarrow 4\sigma A\, T_0^{\,3}.(\Delta\theta + \Delta T)$

Extra Heat Released $\Rightarrow 4\sigma A\, T_0^{\,3}\, \Delta T$

Q.5 For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

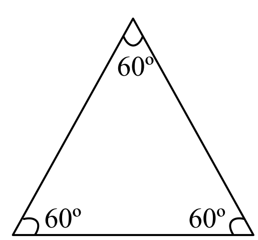
(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(B) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(C) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(D) For this prism the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

Ans. [A,B,D]



$\delta_{min.} = A$

$\delta_{min} = 2i_1 - A \Rightarrow A = 2i_1 - A \Rightarrow \underline{i_1 = A.}$

At i = A (angle of incidence for min. deviation)

$r_1 = r_2$ So ray parallel to base for isosceles and equilateral prism.

As $r_1 + r_2 = A \quad \therefore \quad r_1 + r_1 = A \Rightarrow r_1 = \frac{A}{2}$

$\therefore \quad r_1 = \dfrac{i_1}{2}$

$$\mu = \frac{\sin\left(\dfrac{\delta_{min} + A}{2}\right)}{\sin A/2}$$

$$\mu = \frac{\sin\left(\dfrac{A + A}{2}\right)}{\sin A/2} \Rightarrow \mu = \frac{\sin A}{\sin A/2}$$

$\boxed{\mu = 2\cos A/2} \quad \Rightarrow A = 2\cos^{-1}(\mu/2)$

For grazing emergence

$r_2 = \theta_c \Rightarrow r_1 + r_2 = A \Rightarrow r_1 = A - \theta_c$

1. $\sin i = \mu \sin (A - \theta_c)$

$\sin i = \mu (\sin A \cos \theta_c - \cos A \sin \theta_c)$

$$\sin i = \mu \left(\sin A \sqrt{1 - \frac{1}{\mu^2}} - \cos A \times \frac{1}{\mu} \right)$$

$$i = \sin^{-1}\left[\sin A \sqrt{4\cos^2 \frac{A}{2} - 1} - \cos A\right]$$

Q.6 A flat plate in moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate *v* is much less that the average speed u of the gas molecules. Which of the following options is/are true?

(A) The pressure difference between the leading and trailing faces of the plate is proportional to *uv*

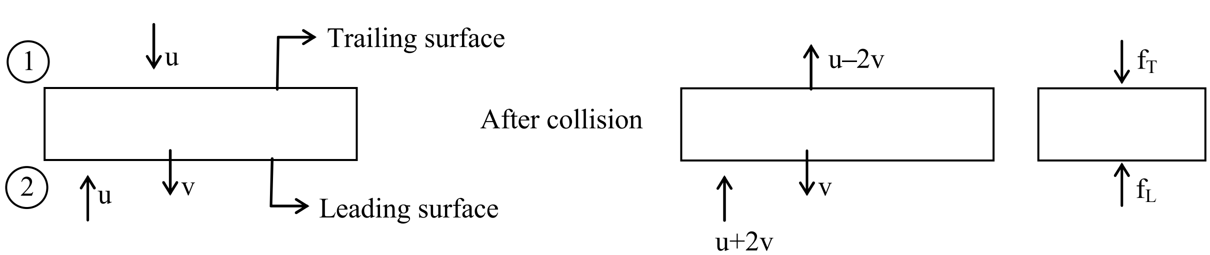
(B) The resistive force experienced by the plate is proportional to *v*

(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force

Ans. **[A,B,D]**

Sol.



Change in momentum at leading surface :

$\Delta P = 2m(u + v)$

$$F = \frac{\Delta P}{\Delta t} \Rightarrow \frac{2m(u+v)}{(d/u)} \rightarrow \qquad \{\text{Assuming particles return after 'd' distance' } \Delta t = \frac{d}{u}\}$$

$$F_L = \frac{2m(u+v).u}{d}$$

<u>Trailing surface:</u>

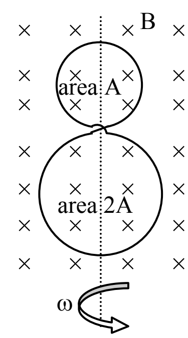
Similarly:

$F_T = \frac{2m(u-v).u}{d}$

Pressure Diff. $\Rightarrow \frac{2m(u+v).u}{(A)d} - \frac{2m(u-v).u}{d(A)}$

$\Delta P \Rightarrow \frac{2muv}{Ad}$

Q.7 A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated form each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct ?



(A) The emf induced in the loop is proportional to the sum of the areas of the two loops

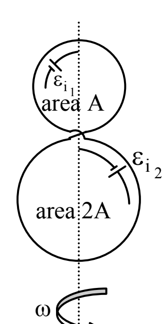
(B) The net emf induced due to both the loops is proportional to $\cos \omega t$

(C) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

Ans. **[C,D]**

Sol.



emf = $\varepsilon_{i_2} - \varepsilon_{i_1}$

emf = $-\frac{d}{dt}(BA_2 \cos \omega t) + \frac{d}{dt}(BA_1 \cos \omega t)$

emf = $BA_2\omega \sin \omega t - BA_1\omega \sin \omega t$

Rate of change of flux or (emf) maximum when $\sin\omega t = 1 \Rightarrow \omega t = \frac{\pi}{2}$ means loop is perpendicular to plane of paper

emf = $B\omega \sin \omega t\,(A_2 - A_1)$

emf = $B\omega\,(2A - A) \sin \omega t$

emf = $BA\omega \sin \omega t$

In smaller loop emf = $BA_1\omega \sin \omega t$

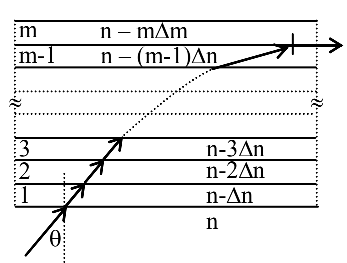
$= BA\omega \sin \omega t$

SECTION – 2 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	In all other cases.

Q.8 A monochromatic light is traveling in a medium of refractive index n = 1.6. If enters a stack of glass layer from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layer are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m ?



Ans. **[8]**

Sol. $n \sin 30° = n_m \sin 90°$

$n \times \frac{1}{2} = (n - m\Delta n)\,(1)$

$\frac{1.6}{2} = [1.6 - m\,(0.1)]$

$m\,(0.1) = \frac{1.6}{2}$

$m = 8$

Q.9 ^{131}I is an isotope of iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum lebelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$).

Ans. **[5]**

Sol. Let the total volume V

Initial activity of 2.5 ml = $\left(\dfrac{2.5}{V}\right)$ (2.4×10^5) Bq

After 11.5 hrs A = 115 Bq

$A = A_0\, e^{-\lambda t}$

$$115 = \frac{2.5}{V} \times 2.4 \times 10^5\, e^{-(\lambda t)}$$

$$115 = \frac{2.5}{V} \times 2.4 \times 10^5\, (1 - \lambda t)$$

$$\lambda = \left(\frac{0.693}{8 \times 24}\right)$$

$$115 = \frac{2.5}{V} \times 2.4 \times 10^5 \left(1 - \frac{0.693}{8 \times 24} \times 11\right)$$

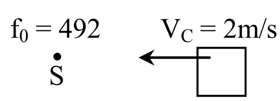
$V \approx 5000\ m\ell$

$\nu = 5$ lt.

Q.10 A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz ? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

Ans. **[6]**

Sol. $f_0 = 492$



Received by car $n_{r_{car}} = \left(\dfrac{V + V_C}{V}\right) f_0$

$$n_{r_{source}} = \left(\frac{V}{V - V_C}\right) n_{r_{car}}$$

$$n_{r_s} = \left(\frac{V + V_C}{V - V_C}\right) f_0$$

$$n_{r_s} = \left(\frac{330 + 2}{330 - 2}\right) \times f_0$$

beats = $n_{r_s} - f_0$

$\Rightarrow \left(\dfrac{332}{328} - 1\right) f_0 \qquad \Rightarrow \left(\dfrac{4}{328} \times 492\right) = 6$

Q.11 An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{v_i}{v_f} = 6.25$, then the smallest possible n_f is.

Ans. **[5]**

Sol. $V = \text{P.E.} = 2E = -2\,(13.6)\,\frac{z^2}{n^2}$

$$\frac{V_i}{V_f} = \left(\frac{n_f}{n_i}\right)^2 = 6.25$$

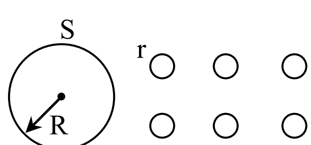
$$\frac{n_f}{n_i} = 2.5 = \frac{5}{2} \text{ or } \frac{10}{4} \text{ or } \frac{30}{6}$$

$\therefore$ Min. value of $n_f = 5$

Q.12 A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of a is.

Ans. **[6]**

Sol.



$$\frac{4}{3}\pi R^3 = k\,\frac{4}{3}\,\pi r^3$$

$$r = \left(\frac{1}{K}\right)^{\frac{1}{3}} R$$

$$\Delta U = k4\pi r^2 S - 4\pi R^2 S$$

$$\Delta U = -4\pi r^2 S - k4\pi R^2 S$$

$$\Delta U = -4\pi r^2 S - k4\pi \left(\frac{R^2}{K^{2/3}}\right) S$$

$$\Delta U = 4\pi r^2 S - (-1 + k^{1/3})$$

$$10^{-3} = 4\pi(10^{-2})^2 \left(\frac{0.1}{4\pi}\right) (-1 + k^{1/3})$$

$$k^{1/3} = 101$$

$$k \approx 10^6$$

Section 3 (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions.
- Each question has FOUR options (A), (B), (C) and (D). **Only one** of these four options is correct.
- For each questions, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –1	In all other cases.

Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, x = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in column 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Culumn-1	Column-2	Column-3
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = -B_0\hat{x}$
(III) Electron with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

Q.13 In which case will the particle move in a straight line with constant velocity ?

(A) (IV) (i) (S) (B) (II) (iii) (S) (C) (III) (iii) (P) (D) (III) (ii) (R)

Ans. **[B]**

Sol. Electron for straight path

$q(\vec{v}\times\vec{B}) + q\vec{E} = 0$

$\vec{E} = -\vec{V}\times\vec{B}$

(B) II (iii) S

$\vec{v} = \frac{E_0}{B_0}\hat{y}$; $\vec{E} = -E_0\hat{x}$; $\vec{B} = B_0\hat{z}$

Q.14 In which case will the particle describe a helical path with axis along the positive z direction ?

(A) (IV) (ii) (R) (B) (IV) (i) (S) (C) (III) (iii) (P) (D) (II) (ii) (R)

Ans. **[B]**

Sol. (IV (i) (S))

$\vec{v} = 2\frac{E_0}{B_0}\hat{x}$; $\vec{E} = E_0\hat{z}$, $\vec{B} = B_0\hat{z}$

Q.15 In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$) ?

(A) (IV) (ii) (S) (B) (III) (ii) (P) (C) (III) (ii) (R) (D) (II) (iii) (Q)

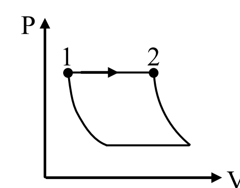
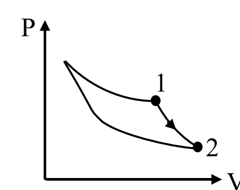
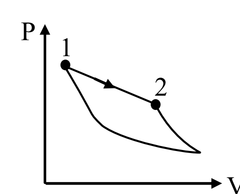
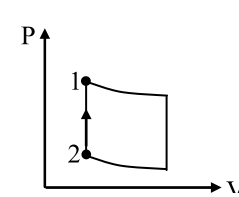
Ans. **[C]**

Sol. (C) III, (ii), R

Magnetic force always zero and electric field will take it to – y direction.

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P – V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system . The equation and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column 1	**Column 2**	**Column 3**
(I) $W_{1\to 2} = \frac{1}{\gamma - 1}(P_2 V_2 - P_1 V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to 2} = -\,nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

Q.16 Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iii) (P) (B) (II) (iii) (S) (C) (II) (iv) (R) (D) (III) (iii) (P)

Ans. [A]

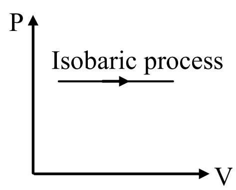
Sol. [16, 17 & 18]

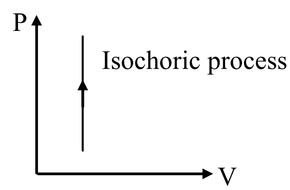
Work done in Adiabatic process = $\frac{1}{\gamma - 1}$ $[P_2 V_2 - P_1 V_1]$

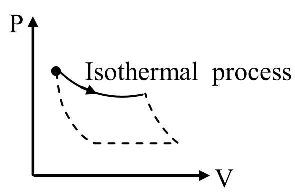
Work done in Isobaric process = $-PV_2 + PV_1$

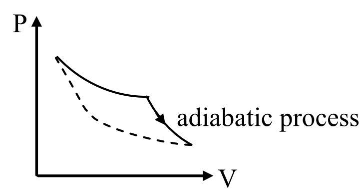
Work done in Isochoric process = 0

Work done in Isothermal process = $-nRT \ln\left(\frac{V_2}{V_1}\right)$









Q.17 Which one of the following options is the correct combination ?

(A) (III) (ii) (S) (B) (IV) (ii) (S) (C) (II) (iv) (R) (D) (II) (iv) (P)

Ans. [A]

Q.18 Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas ?

(A) (I) (ii) (Q) (B) (IV) (ii) (R) (C) (III) (iv) (R) (D) (I) (iv) (Q)

Ans. [D]

Process of compression and rarefaction is adiabatic process.

PART-II (CHEMISTRY)

SECTION – 1 (Maximum Marks : 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

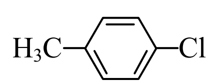
Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

Q.19 The IUPAC name(s) of the following compound is(are)

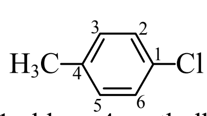


(A) 1-methyl-4-chlorobenzene (B) 1-chloro-4-methylbenzene

(C) 4-chlorotoluene (D) 4-methylchlorobenzene

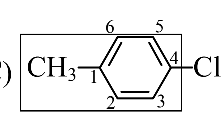
Ans. **[B,C]**

Sol. (B)



1-chloro-4-methylbenzene

; (C)

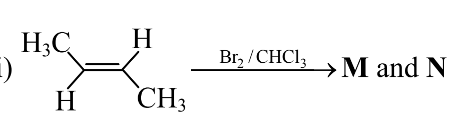


4-chloro Toluene

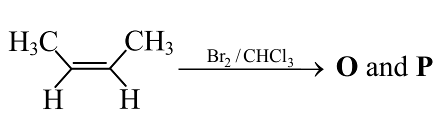
(Accepted name)

Q.20 The correct statement(s) for the following addition reactions is(are)

(i)



(ii)



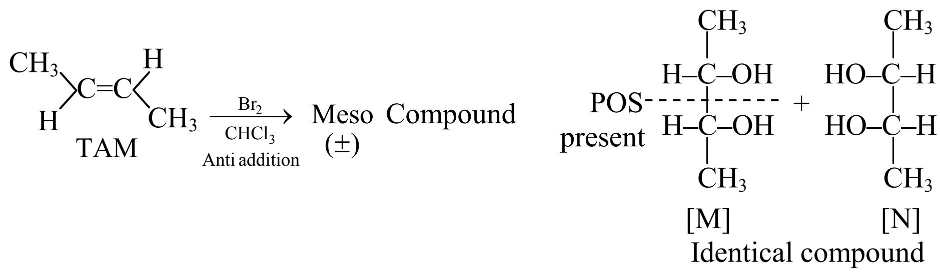
(A) **O** and **P** are identical molecules

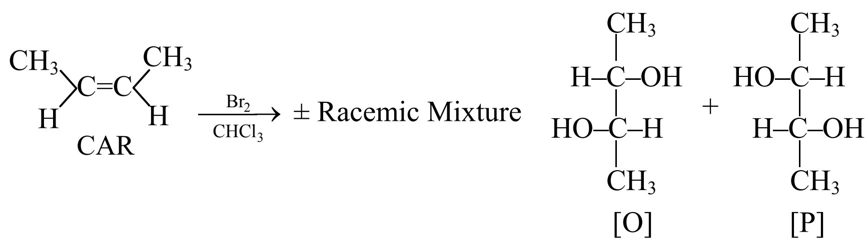
(B) (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers

(C) Bromination proceeds through trans-addition in both the reactions

Ans. **[B,C]**

Sol.





M and O $\Rightarrow$ Diastereomer

If N and P are diastereomer

$\Rightarrow$ Addition of halogen is Trans addition

Q.21 The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

(A) the physical state of X_2 at room temperature changes from gas to solid down the group

(B) decrease in π^*-σ^* gap down the group

(C) decrease in ionization energy down the group

(D) decrease in HOMO-LUMO gap down the group

Ans. **[B,D]**

Sol. X_2 molecules of group - 17 elements

$\xrightarrow[\text{gradually}]{\text{changes}}$ from yellow to violet

This is due to $\longrightarrow$ let $F_2 \rightarrow 9 \times 2 = 18$

By M.O.T

$\Rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \ \sigma 2p_x^2\ \pi 2p_y^2 = \pi 2p_z^2\ \pi^*2p_y^2 = \pi^*2p_z^2\ \sigma^*2p_x$

$\Rightarrow$ energy group decrease $\downarrow$ Down the group

Due to $\downarrow$ $\pi^* - \sigma^*$ gap

Q.22 An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)

(A) The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions

(B) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

(C) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

(D) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

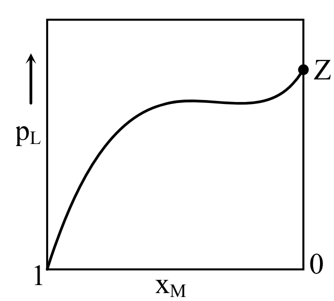
Ans. **[A,B,D]**

Sol. Work done by the gas is more in isothermal expansion than in adiabatic expansion

$\Rightarrow$ If $P_{ext} = 0$, $\therefore$ W = 0, & $\Delta E = 0$ $\therefore$ q = 0

$\Rightarrow$ Work done is maximum when compression is carried out irreversibly.

Q.23 For a solution formed by mixing liquids **L** and **M**, the vapour pressure of **L** plotted against the mole fraction of **M** in solution is shown in the following figure. Here x_L and x_M represent mole fractions of **L** and **M**, respectively, in the solution. The correct statement(s) applicable to this system is(are)



(A) The point **Z** represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \to 1$

(B) Attractive intermolecular interactions between **L-L** in pure liquid **L** and **M-M** in pure liquid **M** are stronger than those between **L-M** when mixed in solution

(C) The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law a obeyed when $x_L \to 0$

(D) The point Z represents vapour pressure of pure liquid **M** and Raoult's law is obeyed from $x_L = 0$ to $x_L = 1$

Ans. **[A,B]**

Sol. According to Raoult's law

Q.24 The Correct statement(s) about the oxoacids, $HClO_4$ and HClO, is(are)

(A) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

(B) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(C) The conjugate base of $HClO_4$ is weaker base than H_2O

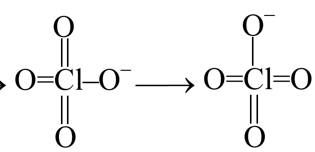
(D) The central atom in both $HClO_4$ and HClO is sp^3 hybridized

Ans. **[A,C,D]**

Sol. $HClO_4 \longrightarrow H^+ + ClO_4^-$

$HOCl \longrightarrow H^+ + OCl^-$

i.e.

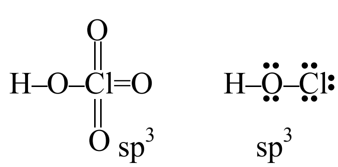
(A) $ClO_4^- \longrightarrow$  etc. ; H–O–Cl $\rightarrow$ $\overset{\ominus}{O}Cl$ No resonance stabilisation

Four equivalent resonating structure hence $HClO_4$ is more acidic than HOCl

(B) $Cl_2 + H_2O \longrightarrow HOCl + HCl$

$HOCl \longrightarrow HCl + \frac{1}{2} O_2$

(C) $\underset{\text{S.A.}}{HClO_4} \longrightarrow$ Conjugate is W.B. than H_2O



Q.25 Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2{\cdot}6H_2O$ (X) and NH_4Cl gives an octahedral complex Y in the presence of air. In aqueous solution, complex Y behaves as 1 : 3 electrolyte. The reaction of X with excess HCl at room temperature results in the formation of a blue coloured complex Z. The calculated spin only magnetic moment of X and Z is 3.87 B.M., whereas it is zero for complex Y.

(A) When X and Z are in equilibrium at 0ºC, the colour of the solution is pink

(B) Z is a tetrahedral complex

(C) The hybridization of the central metal ion in Y is d^2sp^3

(D) Addition of silver nitrate to Y gives only two equivalents of silver chloride

Ans. **[A,B,C]**

Sol.

$[M(H_2O)_4]Cl_2$ or $MCl_2.2H_2O$ / $CoCl_2.2H_2O$ (X) $\xrightarrow[O_2]{NH_4Cl}$ $[M(NH_3)_6]Cl_3$ (Y) ← Strong field ligand

where M is cobalt

Hybridisation is $\rightarrow d^2sp^3$ $\mu = 0$

(X) $\underset{\text{at room temp.}}{\overset{\text{H–Cl}}{\rightleftharpoons}}$ $[CoCl_4]^{2-}$ (Z) sp^3 – Hybridisation

(Y) $\xrightarrow{\text{In Aqueous medium}}$ $[M(NH_3)_6]^{3+} + 3Cl^{\ominus}$

electrolyte $\rightarrow$ 1 : 3

By spectro chemical series $H_2O > Cl^-$ So equilibrium between X and Z shifted toward X

$_{27}Co \longrightarrow 4s^2\ 3d^7$

$Co^{+2} \longrightarrow 4s^0\ 3d^7$

↑↓	↑↓	↑	↑	↑

n = 3

$\mu = \sqrt{n(n+2)}$ B.M.

$= 3.87$ B.M.

Similar for (X) of Co^{+2}

SECTION – 2 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories **:**

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 If none of the bubble is darkened.

Negative Marks : –2 In all other cases.

Q.26 The sum of the number of lone pairs of electrons on each central atom in the following species is

$$[TeBr_6]^{2-},\ [BrF_2]^+,\ SNF_3,\ \text{and}\ [XeF_3]^-$$

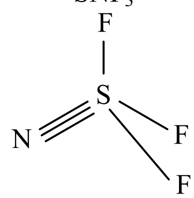
(Atomic numbers : N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

Ans. **[6]**

Sol.

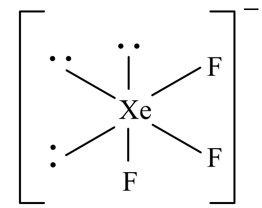
$[TeBr_6]^{2-}$
sp^3d^3
6 B.P.
1 L.P.

$[BrF_2]^+$
sp^3
B.P. = 2
L.P. = 2

SNF_3


No-lene Pair
of Central Atom

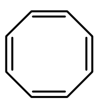
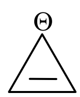
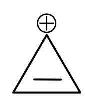
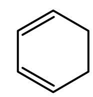
ions

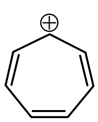
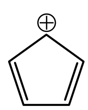
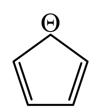
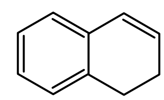
XeF_3^-
SP^3d^2


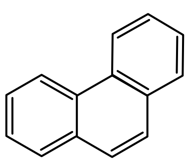
Three lone Pair

Ans. : 1 + 2 + 0 + 3 = 6

Q.27 Among the following, the number of aromatic compound(s) is

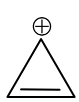
   

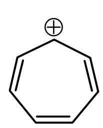
   

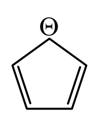


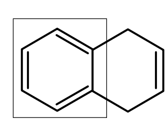
Ans. **[5]**

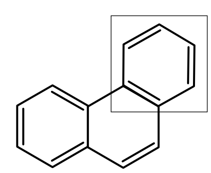
Sol.


Cyclo Propenyl
Carbo Cation


Tropolium


Cyclo Penta
Dienyl Anion


Aromatic Ring


Aromatic Ring

If one ring is aromatic and other is non aromatic then compound is aromatic

Total = 5

[All obey Huckel's Rule]

Q.28 A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is $N \times 10^{24}$. The value of N is

Ans. **[2]**

Sol. $\rho = \dfrac{Z \times M}{a^3 . N_A}$

$a = 400\text{pm}$

$a = 400 \times 10^{-10}\,\text{cm}$

$a = 4 \times 10^{-8}\,\text{cm}$

$8 = \dfrac{4 \times M}{64 \times 10^{-24} \times 6 \times 10^{23}}$

$8 = \dfrac{4 \times M}{64 \times 6 \times 10^{-1}}$

$M = \dfrac{8 \times 64 \times 6}{4 \times 10} = \dfrac{12 \times 64}{10} = 76.8$ gm/mol

76.8 gm contains = 6×10^{23} atoms

256 gm contains = $\dfrac{6 \times 10^{23}}{76.8} \times 256$

$= 20 \times 10^{23}$

$= 2 \times 10^{24}$

value of N = 2

Q.29 The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7} S. The pH of the solution is 4. The value of limiting molar conductivity (Λ_m^0) of this weak monobasic acid in aqueous solution is $Z \times 10^2$ S cm^{-1} mol^{-1}. The value of Z is

Ans. **[6]**

Sol. $\ell = 120$ cm

$A = 1\ cm^2$

$G = 5 \times 10^{-7}$ S

pH = 4, $[H^+] = 10^{-4}$

$\alpha = \dfrac{\Lambda_m^C}{\Lambda_m^\infty}$

$$\begin{array}{cccccc} HA & \rightleftharpoons & H^+ & + & A^- \\ C & & 0 & & 0 \\ C-C\alpha & & C\alpha & & C\alpha \end{array}$$

$[H^+] = C\alpha$

$10^{-4} = 0.0015 \times \alpha$

$\alpha = \dfrac{10^{-4}}{0.0015}$

$\Lambda_m^o = \dfrac{\kappa \times 1000}{M}$

$120 \times 5 \times 10^{-7} \times 1000$

$$\frac{120}{1} = \frac{1}{5\times10^{-7}} \times \kappa$$

$$\kappa = 120 \times 5 \times 10^{-7}$$

$$\lambda_m^{\infty} = \frac{\Lambda_m^C}{\alpha} = \frac{\frac{120\times5\times10^{-4}}{0.0015}}{\frac{10^{-4}}{0.0015}}$$

$$\lambda_m^{\infty} = 600 = 6 \times 10^2$$

Q.30 Among H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^-, and F_2, the number of diamagnetic species is (Atomic numbers: H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

Ans. **[6]**

Sol.

				Magnetic behaviour
H_2	$\longrightarrow$ $1 \times 2 = 2$	$\longrightarrow$	$\sigma(1s)^2$	(D)
$H_{e_2}^+$	$\longrightarrow$ $2 \times 2 - 1 = 3$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^1$	(P)
Li_2	$\longrightarrow$ $3 \times 2 = 6$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2$	(D)
Be_2	$\longrightarrow$ $4 \times 2 = 8$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma^*(2s)^2$	(D)
B_2	$\longrightarrow$ $5 \times 2 = 10$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma^*(2s)^2,\pi(2p_y)^1 = \pi(2p_z)^1$	(P)
C_2	$\longrightarrow$ $6 \times 2 = 12$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma^*(2s)^2,\pi(2p_y)^2 = \pi(2p_z)^2$	(D)
N_2	$\longrightarrow$ $7 \times 2 = 14$		$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma^*(2s)^2,\pi^*2(p_y)^2 = \pi^*(2p_z)^2,\sigma^*(2p_x)^2$	(D)
O_2^-	$\longrightarrow$ $8 \times 2 + 1 = 17$	$\longrightarrow$	$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma^*(2s)^2,\sigma(2p_x)^2,\pi(2p_y)^2 = \pi(2P_z)^2,$ $\pi(2p_y)^1 = \pi^*(2P_z)^2, \pi^*(2p_x)^1$	(P)
F_2	$\longrightarrow$ $9 \times 2 = 18 =$		$\sigma(1s)^2,\sigma^*(1s)^2,\sigma(2s)^2,\sigma*(2s)^2,\pi(2p_x)^2,\pi(2p_y)^2 = \pi(2p_z)^2$ $\pi^*(2p_y)^2 = \pi^*(2p_z)^2$	(D)

SECTION – 3 (Maximum Marks : 18)

- This section contains **SIX** question of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

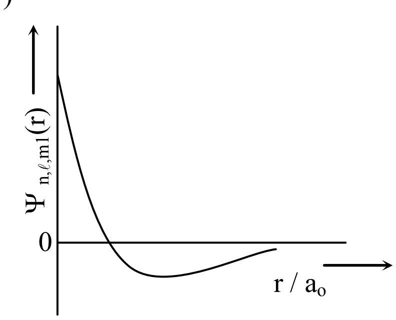
Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : –1 In all other cases

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table

The wave function, Ψ_{n,ℓ,m_l} is a mathematical function whose value depends upon spherical polar coordinates (r, θ, ϕ) of the electron and characterized by the quantum numbers n, ℓ and m_l Here r is distance form nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the Table, Z is atomic number and a_o is Bohr radius

Column 1	**Column 2**	**Column 3**
(I) 1s orbital	(i) $\Psi_{n,\ell,m_l} \propto \left(\frac{Z}{a_o}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_o}\right)}$	(P) 
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_o^3}$
(III) $2p_z$ orbital	(iii) $\Psi_{n,\ell,m_l} \propto \left(\frac{Z}{a_o}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{2a_o}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(IV) $3d_z^2$ orbital	(iv) xy- plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from

Q.31 For the given orbital in Column 1, the only **CORRECT** combination for any hydrogen-like species is

(A) (III) (iii) (P) (B) (II) (ii) (P) (C) (IV) (iv) (R) (D) (I) (ii) (S)

Ans. **[B]**

Sol. 2s orbital

No. of radial node = $n - \ell - 1 = 2 - 0 - 1 = 1$

Q.32 For He^+ ion, the only **INCORRECT** combination is

(A) (I) (i) (R) (B) (I) (i) (S) (C) (I) (iii) (R) (D) (II) (ii) (Q)

Ans. **[C]**

Sol. For He^+ ion

$He^+ = 1s^1$ it has non directional characteristics

Q.33 For hydrogen atom, the only **CORRECT** combination is

(A) (I) (i) (S) (B) (I) (iv) (R) (C) (I) (i) (P) (D) (II) (i) (Q)

Ans. **[A]**

Sol. For 1s orbital

There is no radial node

So Ψ function will be $\psi_{n,\ell,m} \propto \left(\frac{Z}{a_o}\right)^{3/2} e^{-\frac{Zr}{a_o}}$

Energy $\frac{E_{2\to 4}}{E_{2\to 6}} = \frac{27}{32}$

Answer Q.34, Q.35 and Q.36 by appropriately matching the information given in the three columns of the following table

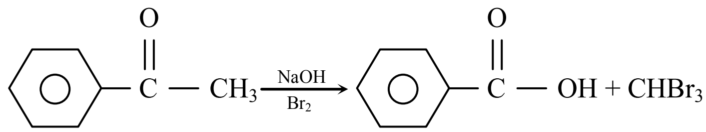
Columns 1, 2 and 3 contain starting materials , reaction conditions, and type of reactions respectively.		
Column 1	**Column 2**	**Column 3**
(I) Toluene	(i) $NaOH/Br_2$	(P) Condensation
(II) Acetophenone	(ii) Br_2 *hv*	(Q) Carboxylation
(III) Benzaldehyde	(iii) $(CH_3CO)_2$ O/CH_3COOK	(R) Substitution
(IV) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

Q.34 For the synthesis of benzoic acid the only **CORRECT** combination is

(A) (IV) (ii) (P) (B) (I) (iv) (Q) (C) (III) (iv) (R) (D) (II) (i) (S)

Ans. **[D]**

Sol.



Acetophenone　　Benzoic Acid

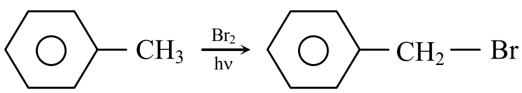
This is Haloform reaction

Q.35 The only **CORRECT** combination in which the reaction proceeds through radical mechanism is

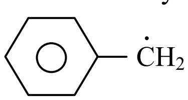
(A) (I) (ii) (R)　(B) (III) (ii) (P)　(C) (IV) (i) (Q)　(D) (II) (iii) (R)

Ans. **[A]**

Sol.



Mechanism by FR Formation



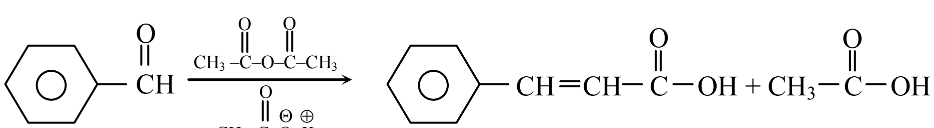
[Benzyl Free radical]

Q.36 The only **CORRECT** combination that gives two different carboxylic acid is

(A) (I) (i) (S)　(B) (III) (iii) (P)　(C) (IV) (iii) (Q)　(D) (II) (iv) (R)

Ans. **[B]**

Sol. It is Perkin's Reaction



Cinnemic Acid　　Acetic Acid

PART-III (MATHEMATICS)

SECTION – 1 (Maximum Marks : 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

Q.37 Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value(s) of x ?

(A) $-1 + \sqrt{1-y^2}$ (B) $1 - \sqrt{1+y^2}$ (C) $-1 - \sqrt{1-y^2}$ (D) $1 + \sqrt{1+y^2}$

Ans. **[A,C]**

Sol. $\text{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$\text{Im}\left(\frac{ax+b+iay}{(x+1)+iy}\right) = y$

$\text{Im}\left(\frac{(ax+b)+iay)((x+1)-iy)}{(x+1)^2+y^2}\right) = y$

$\frac{-y(ax+b)+ay(x+1)}{(x+1)^2+y^2} = y$

$-ax - b + ax + a = (x+1)^2 + y^2$

$a - b = (x+1)^2 + y^2$

$1 = (x+1)^2 + y^2$

$x + 1 = \pm\sqrt{1-y^2}$

$x = -1 \pm \sqrt{1-y^2}$

Q.38 Which of the following is(are) NOT the square of a 3×3 matrix real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. **[C,D]**

Sol. Det $(A^2) > 0$

Q.39 Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

(A) $P(X \cap Y) = \frac{1}{5}$ (B) $P(Y) = \frac{4}{15}$ (C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(X'|Y) = \frac{1}{2}$

Ans. **[B,D]**

Sol. $P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)}$

$\therefore P(X \cap Y) = P\left(\frac{Y}{X}\right) P(X) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)}$

$\therefore P(Y) = \frac{P(X \cap Y)}{P(X|Y)} = \frac{2/15}{1/2} = \frac{4}{15}$

Now $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

$P\left(\frac{X'}{Y}\right) = \frac{P(X' \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)} = \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$

Q.40 Let $f : R \to (0, 1)$ be a continuous function. Then, which of the following function(s) has (have) the value zero at some point in the interval (0, 1) ?

(A) $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$ (B) $e^x - \int_0^x f(t) \sin t \, dt$

(C) $x^9 - f(x)$ (D) $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$

Ans. **[C,D]**

Sol. $f : R \to (0, 1)$

(A) $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt > 0$

$\downarrow \quad \downarrow$

$\oplus \quad \oplus$

(B) $g(x) = e^x - \int_0^x f(t) \sin t \, dt$

$g'(x) = e^x - f(x) \sin x > 0 \quad ; \qquad x \in (0, 1)$

$g(0) = e^0 = 1$

$g(x) > 1$

(C) $g(x) = x^9 - f(x)$

$g(0)\, g(1) = (0 - f(0))\, (1 - f(1)) < 0$

$x^9 - f(x)$ is zero in $x \in (0, 1)$

(D) $g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$

$g(0) = -\int_0^{\frac{\pi}{2}} f(t) \cos t \, dt < 0$

$g(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t) \cos t \, dt > 0$

$g(0)\, g(1) < 0$

So $g(x) = 0$ in $x \in (0, 1)$

Q.41 Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous ?

(A) $x = 1$ (B) $x = 2$ (C) $x = 0$ (D) $x = -1$

Ans. **[A,B,D]**

Sol. $f(x) = x \cos (\pi (x + [x]))$

At $x = 1 \quad ; \qquad f(1) = 1$

LHL $= \lim_{h\to 0} (1 - h) \cos (\pi (1 - h + 0))$

$= \lim_{h\to 0} (1 - h) \cos (\pi - \pi h) = -1$

RHL $= \lim_{h\to 0} (1 + h) \cos (\pi (1 + h + 1)) = 1$

So discontinuous at $x = 1$

At $x = 2 \quad ; \quad f(2) = 2$

LHL $= \lim_{h\to 0} (2 - h) \cos (\pi (2 - h + 1)) = -2$

So discontinuous at x = 2

At x = 0 ; $f(0) = 0$

LHL $= \lim_{h \to 0} (0 - h) \cos(\pi(0 - h - 1)) = 0$

RHL $= \lim_{h \to 0} (0 + h) \cos(\pi(0 + h + 0)) = 0$

So continuous at x = 0

At x = –1 ; $f(-1) = -1$

LHL $= \lim_{h \to 0} (-1 - h) \cos(\pi(-1 - h - 2)) = 1$

RHL $= \lim_{h \to 0} (-1 + h) \cos(\pi(-1 + h - 1)) = -1$

So discontinuous at x = –1

Q.42 If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

(A) p = 5, h = 4, k = – 3
(B) p = 2, h = 3, k = – 4
(C) p = –2, h = 2, k = – 4
(D) p = –1, h = 1, k = – 3

Ans. **[B]**

Sol. $y^2 = 16x$

$T = S_1$

$ky - 8x - 8h = k^2 - 16h$

$ky - 8x = k^2 - 8h$

$y + 2x = p$

Comparing

$$\frac{k}{1} = \frac{-8}{2} = \frac{k^2 - 8h}{p}$$

$k = -4$ and $kp = k^2 - 8h$

$-4p = 16 - 8h$

$p = 2h - 4$

$k = -4, p = 2, h = 3$ satisfies the condition.

Q.43 If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle ?

(A) a, 4 , 2
(B) a, 4, 1
(C) 2a, 4, 1
(D) 2a, 8, 1

Ans. **[A,B,D]**

Sol. $c^2 = a^2 m^2 - b^2$

$1 = 4a^2 - 16$

$a^2 = \frac{17}{4}$

$\Rightarrow a = \sqrt{17}$

SECTION – 2 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

Q.44 Let $f : R \to R$ be a differentiable functions such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t \; f(t)]\, dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$

Ans. **[2]**

Sol. $$g(x) = \int_x^{\frac{\pi}{2}} [\underset{(II)}{f'(t)}\underset{(I)}{\operatorname{cosec} t}\, dt - \int_x^{\frac{\pi}{2}} \cot t \operatorname{cosec} t\, f(t)\, dt$$

$$g(x) = [f(t)\operatorname{cosec} t]_x^{\pi/2} + \int_x^{\frac{\pi}{2}} \cot t \operatorname{cosec} t\, f(t)\, dt - \int_x^{\frac{\pi}{2}} \cot t \operatorname{cosec} t\, f(t)\, dt$$

$$\therefore\; g(x) = f\left(\frac{\pi}{2}\right) - f(x) \operatorname{cosec} x$$

$$g(x) = 3 - f(x) \operatorname{cosec} x$$

$$\therefore\; \lim_{x\to 0} g(x) = \lim_{x\to 0} (3 - f(x) \operatorname{cosec} x)$$

$$= \lim_{x\to 0}\left(3 - \frac{f(x)}{\sin x}\right)$$

$$= 3 - \lim_{x\to 0} \frac{f(x)}{\sin x}$$

$$= 3 - \lim_{x\to 0} \frac{f'(x)}{\cos x}$$

$$= 3 - f'(0)$$

$$= 3 - 1$$

$$= 2$$

Q.45 Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated, and let y be the number of such words where exactly one letter is repeated twice and no other is repeated. Then, $\frac{y}{9x}=$

Ans. **[5]**

Sol. $x = \lfloor 10$

$$y = {}^{10}C_1 \times {}^{9}C_8 \times \frac{\lfloor 10}{\lfloor 2}$$

$$= 45 \times \lfloor 10$$

$$\therefore \frac{y}{9x} = \frac{45 \times \lfloor 10}{9 \times \lfloor 10} = 5$$

Q.46 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Ans. **[6]**

Sol. Let the sides are $a - d$, a, $a + d$

Given $(a - d)^2 + a^2 = (a + d)^2$

$$a^2 = 4ad$$

$$a = 4d \qquad \ldots(i) \qquad (\text{as } a \neq 0)$$

Also $\frac{1}{2}a(a - d) = 24$

$$4d\,(4d - d) = 48$$

$$d^2 = 4$$

$$d = 2$$

$\Rightarrow a = 8$

So sides are 6, 8, 10

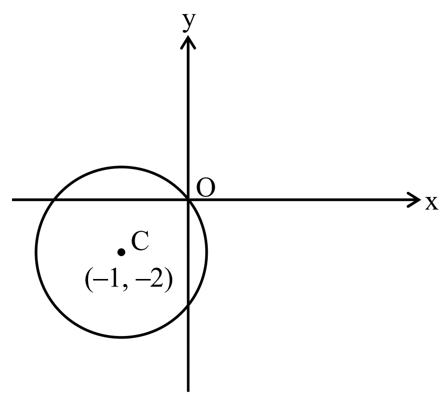
Q.47 For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Ans. **[2]**

Sol. Case (i)

Circle passes through origin

$\therefore\ p = 0$

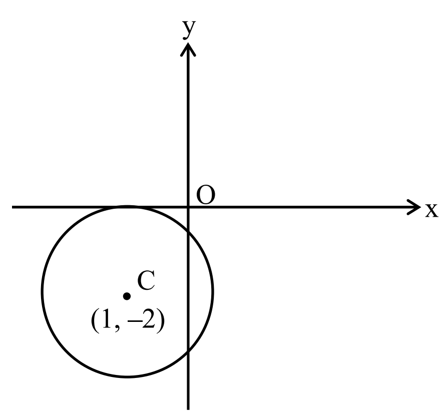


Case (ii)

$r = 2$

$\sqrt{5+p} = 2$

$p = -1$



Q.48 For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equation, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. **[1]**

Sol. $x + \alpha y + \alpha^2 z = 1$

$\alpha x + y + \alpha z = -1$

$$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 1 + \alpha^4 + \alpha^4 - \alpha^4 - \alpha^2 - \alpha^2 = 0$$

$\alpha^4 - 2\alpha^2 + 1 = 0$

$(\alpha^2 - 1)^2 = 0$

$\alpha^2 = 1$

$\alpha = \pm 1$

$\alpha = 1$

Not possible

2 plane coincident

1 plane parallel

$\alpha = -1$

$$\left.\begin{array}{l} x - y + z = 1 \\ -x + y - z = -1 \\ x - y + z = 1 \end{array}\right\} \infty \text{ solution}$$

$1 + \alpha + \alpha^2 = 1 - 1 + 1 = 1$

SECTION – 3 (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each questions, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories.

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –1	In all other cases.

Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q.49 If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only **CORRECT** combination ?

(A) (II) (iv) (R) (B) (III) (i) (P) (C) (III) (ii) (Q) (D) (I) (ii) (Q)

Ans. **[B]**

Sol. $y = x + 8$

$m = 1, c = 8$

For $y^2 = 4ax$

$c = \frac{a}{m}$

$\therefore a = 8$

So $y^2 = 32x$

Point of contact for given tangent is (8, 16).

Q.50 For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only **CORRECT** combination for obtaining its equation ?

(A) (I) (ii) (Q) (B) (I) (i) (P) (C) (III) (i) (P) (D) (II) (ii) (Q)

Ans. **[A]**

Sol. $a = \sqrt{2}$, and point of contact is (–1, 1).

Clearly option (A) is satisfied.

Q.51 The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}\,x + 2y = 4$, then which of the following options is the only **CORRECT** combination ?

(A) (IV) (iii) (S) (B) (II) (iii) (R) (C) (II) (iv) (R) (D) (IV) (iv) (S)

Ans. **[C]**

Sol. $\sqrt{3}\,x + 2y = 4$

$m = -\frac{\sqrt{3}}{2}, c = 2$

Point of contact is $\left(\sqrt{3}, \frac{1}{2}\right)$

Condition of ellipse is $c^2 = a^2m^2 + b^2$

$$4 = a^2 \times \frac{3}{4} + 1$$

$$a^2 = 4$$

$P\left(\sqrt{3}, \frac{1}{2}\right)$ satisfies equation of ellipse $\frac{x^2}{4} + y^2 = 1$.

Answer Q.52, Q.53 and Q.54 by approximately matching the information given in the three columns of the following table.

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behaviour of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim\limits_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim\limits_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim\limits_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim\limits_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

Q.52 Which of the following options is the only **CORRECT** combination ?

(A) (I) (i) (P) (B) (III) (iii) (R) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

Ans. **[D]**

Q.53 Which of the following options is the only **INCORRECT** combination ?

(A) (II) (iv) (Q) (B) (I) (iii) (P) (C) (II) (iii) (P) (D) (III) (i) (R)

Ans. **[D]**

Q.54 Which of the following options is the only **CORRECT** combination ?

(A) (III) (iv) (P) (B) (IV) (i) (S) (C) (II) (iii) (S) (D) (I) (ii) (R)

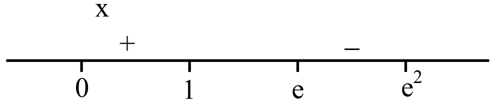
Ans. **[C]**

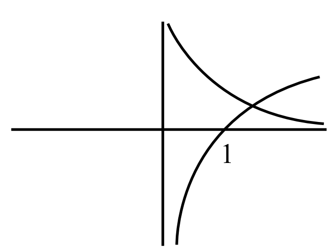
Sol. **(Q.51 to Q.53)**

$f(x) = x + \ell nx - x\ell nx$, $x \in (0, \infty)$

$$f'(x) = 1 + \frac{1}{x} - \ell nx - 1$$

$$= \frac{1}{x} - \ell nx$$





$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = -\frac{(1+x)}{x^2} < 0$ for $\forall\ x > 0$

$f'(x)$ decreasing (e, e^2) S

$f(x)$ decreasing (e, e^2) Q

Increasing $(0, 1)$ P

$$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty}\left(\frac{1}{x} - \ell nx\right) \to -\infty \text{ (iii)}$$

$$\lim_{x\to\infty} f''(x) = -0 - 0 = 0 \text{ (iv)}$$

$$\lim_{x\to\infty} f(x) \to -\infty \text{ (ii)}$$

$$\because\ f(1)\, f(e^2) = (1 + \ell n1 - \ell n1)\,(e^2 + 2 - e^2.2)$$
$$= 1\,(2 - e^2) < 0$$

So some $x \in (1, e^2)$, $f(x) = 0$ (I)

$$f'(1)\, f'(e) = (1 - \ell n1)\left(\frac{1}{e} - \ell ne\right) = \left(\frac{1}{e} - 1\right) < 0$$

So some $x \in (1, e)$, $f'(x) = 0$ (II)

$$f''(1)\, f''(e) = (-1 - 1)\left(-\frac{1}{e^2} - \frac{1}{e}\right) > 0.$$