FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS

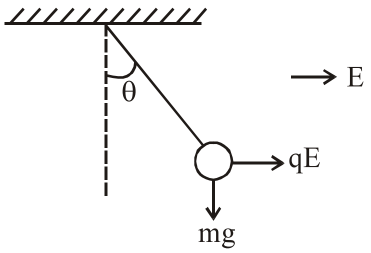
TEST PAPER WITH ANSWER & SOLUTION

1. The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : (take g = 10 m/s^2)

(1) $\tan^{-1}(5.0)$ (2) $\tan^{-1}(2.0)$

(3) $\tan^{-1}(0.5)$ (4) $\tan^{-1}(0.2)$

Sol.



$$\tan\theta = \frac{qE}{mg} = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}$$

$$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$$

2. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas)

(1) 10^6 (2) 10^3

(3) 10^4 (4) 10^2

Sol. Reynolds Number = $\dfrac{\rho v d}{\eta}$

Volume flow rate = $v \times \pi r^2$

$$v = \frac{100\times10^{-3}}{60}\times\frac{1}{\pi\times25\times10^{-4}}$$

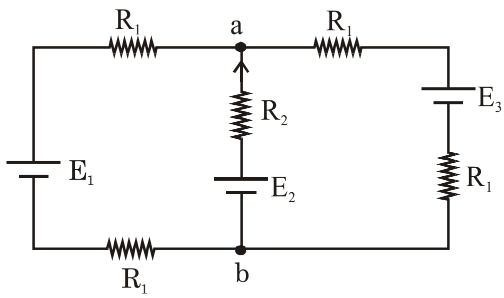
$$v = \frac{2}{3\pi}\,m/s$$

$$\text{Reynolds Number} = \frac{10^3\times2\times10\times10^{-2}}{10^{-3}\times3\pi}$$

$$\simeq 2\times10^4$$

order 10^4

3. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\ \Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points 'a' and 'b' is approximately (in V) :



(1) 2.7 (2) 3.3

(3) 2.3 (4) 3.7

Sol. $$E_{eq} = \frac{\frac{E_1}{2R_1}+\frac{E_2}{R_2}+\frac{E_3}{2R_1}}{\frac{1}{2R_1}+\frac{1}{R_2}+\frac{1}{2R_1}}$$

$$= \frac{\frac{2}{2}+\frac{4}{2}+\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}$$

$$= \frac{5}{\frac{3}{2}} = \frac{10}{3} = 3.3$$

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4. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be :

(1) 100 Ω (2) 400 Ω

(3) 500 Ω (4) 300 Ω

Sol. When red is replaced with green 1st digit changes to 5 so new resistance will be 500Ω

5. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to:

(1) 10^4 Nm^{-2} (2) 10^8 Nm^{-2}

(3) 10^6 Nm^{-2} (4) 10^3 Nm^{-2}

Sol. Energy of catapult $=\frac{1}{2}\times\left(\frac{\Delta\ell}{\ell}\right)^2\times Y\times A\times\ell$

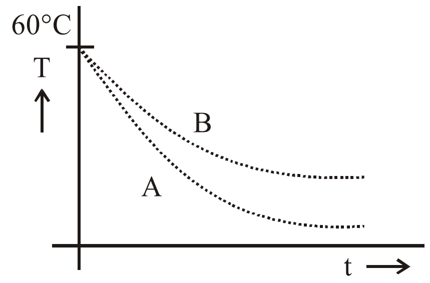
= Kinetic energy of the ball $=\frac{1}{2}mv^2$

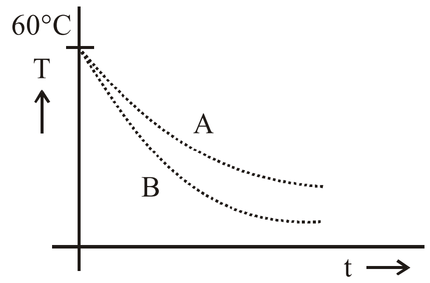
therefore,

$$\frac{1}{2}\times\left(\frac{20}{42}\right)^2\times Y\times\pi\times3^2\times10^{-6}\times42\times10^{-2}=\frac{1}{2}\times2\times10^{-2}\times(20)^2$$

$Y \simeq 3\times10^6$ Nm^{-2}

6. Two identical breakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

(1)


(2)


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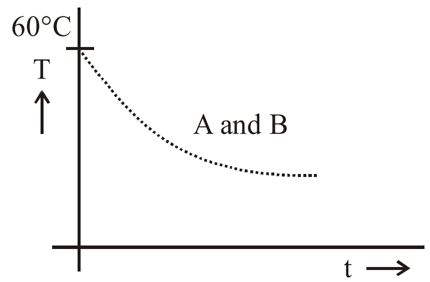

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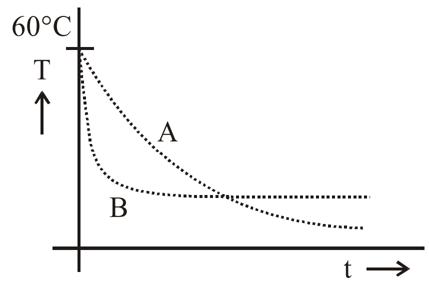

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Sol. $-ms\dfrac{dT}{dt} = e\sigma A\left(T^4 - T_0^4\right)$

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_0^4\right)$$

$$-\frac{dT}{dt} = \frac{4e\sigma AT_0^3}{ms}(\Delta T)$$

$T = T_0 + (T_i - T_0)e^{-kt}$

where $k = \dfrac{4e\sigma AT_0^3}{ms}$

$$k = \frac{4e\sigma AT_0^3}{\rho vs}$$

$$\left|\frac{dT}{dt}\right| \propto k$$

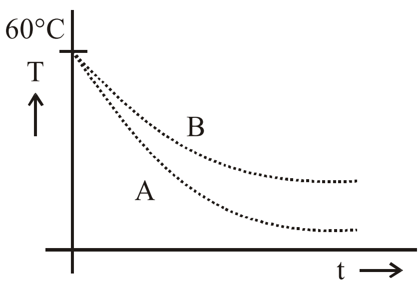
$$\therefore \left|\frac{dT}{dt}\right| \propto \frac{1}{\rho s}$$

$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

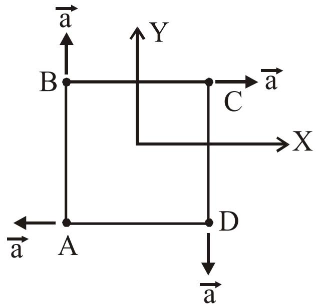
$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A s_A < \rho_B s_B$

$$\left|\frac{dT}{dt}\right|_A > \left|\frac{dT}{dt}\right|_B$$



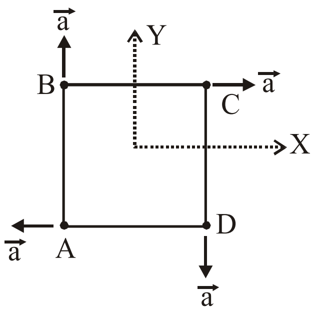
7. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



(1) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$ (2) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$

(3) Zero (4) $a\left(\hat{i}+\hat{j}\right)$

Sol.



$\vec{a}_A = -a\hat{i}$

$\vec{a}_B = a\hat{j}$

$\vec{a}_C = a\hat{i}$

$\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j}$$

$$= \frac{a}{5}\left(\hat{i}-\hat{j}\right)$$

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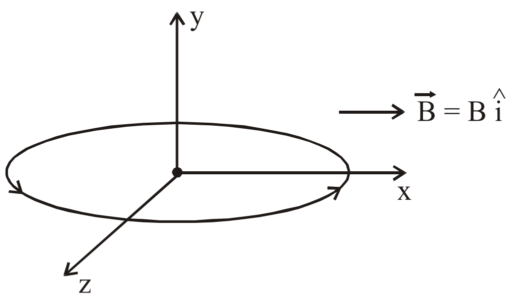
8. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is :

(1) $B\pi r^2 IN$ (2) $\frac{Br^2 I}{\pi N}$

(3) Zero (4) $\frac{B\pi r^2 I}{N}$

Sol.



Magnetic moment of coil = $NIA\hat{j}$

$= NI(\pi r^2)\hat{j}$

Torque on loop (coil) = $\vec{M} \times \vec{B}$

$= NI(\pi r^2)B\sin 90°(-\hat{k})$

$= NI\pi r^2 B(-\hat{k})$

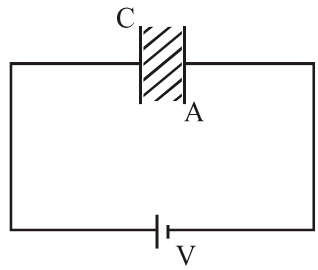
9. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant is the capacitance is 15 pF?

(given $\epsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(1) 3.8 (2) 4.5

(3) 6.2 (4) 8.5

Sol.



$A = 10^{-4}$ m^2

$E_{max} = 10^6$ V/m

$C = 15$ μF

$$C = \frac{k\varepsilon_0 A}{d}$$

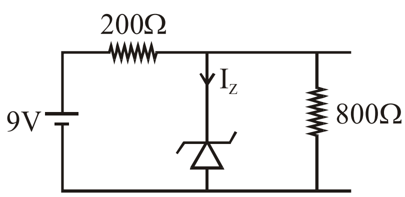
$$\frac{Cd}{\varepsilon_0 A} = k$$

$$k = \frac{15\times10^{-12}\times500\times10^{-6}}{8.86\times10^{-12}\times10^{4}}$$

$$= \frac{15\times5}{8.86} = 8.465$$

$k \approx 8.5$

10. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.

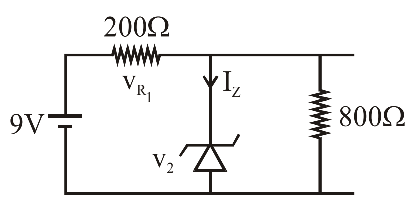


The current I_Z through the Zener is :

(1) 7 mA (2) 17 mA

(3) 10 mA (4) 15mA

Sol.



$9 = V_Z + V_{R_1}$

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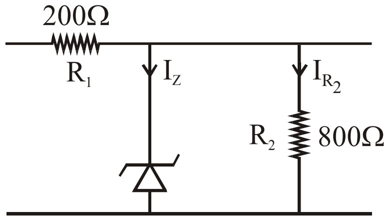
$V_Z = 5.6$ V

$V_{R_1} = 9 - 5.6$

$V_{R_1} = 3.4$

$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200}$

$I_{R_1} = 17$ mA



$V_z = V_{R_2} = I_{R_2}(R_2)$

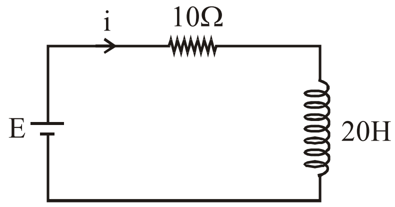
$\frac{5.6}{800} = I_{R_2}$

$I_{R_2} = 7\text{mA}$

$I_Z = (17 - 7)$ mA

= 10 mA

11. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is :



(1) $\frac{2}{\ell n2}$ (2) $\ell n2$

(3) $2\ell n2$ (4) $\frac{1}{2}\ell n2$

Sol. LIdI = I²R

$$L \times \frac{E}{10}\left(-e^{-t/2}\right) \times \frac{-1}{2} = \frac{E}{10}\left(1 - e^{-t/2}\right) \times 10$$

$e^{-t/2} = 1 - e^{-t/2}$

$t = 2\ell n2$

12. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be :

(1) 40 cm from the convergent mirror, same size as the object

(2) 20 cm from the convergent mirror, same size as the object

(3) 20 cm from the convergent mirror, twice the size of the object

(4) 40 cm from the convergent lens, twice the size of the object

Sol. **Note :**

There will be 3 phenomenon

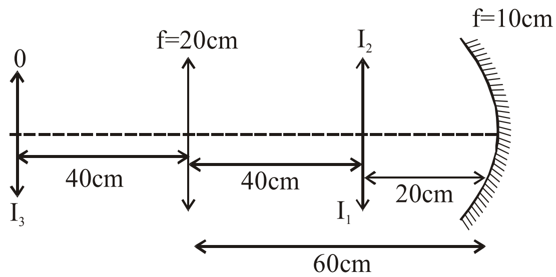
(i) Refraction from lens

(ii) Reflection from mirror

(iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

None of the option depicts so this question is Bonus



1st refraction u = –40cm; f = +20cm

⇒ v = +40 cm (image I_1)

and $m_1 = -1$

for reflection

u = –20cm; f = –10cm

⇒ v = –20cm (image I_2)

and $m_2 = -1$

2nd refraction

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$u = -40cm;\ f = +20cm$

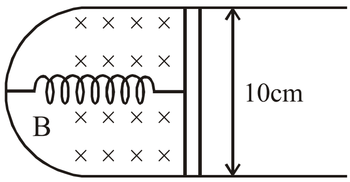
$\Rightarrow v = +40cm$ (image I_3)

and $m_3 = -1$

Total magnification $= m_1 \times m_2 \times m_3 = -1$

and final image is formed at distance 40cm from convergent lens and is of same size as the object

13. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to :



(1) 50000 (2) 5000

(3) 10000 (4) 1000

Sol. $T_0 = 2\pi\sqrt{\dfrac{m}{k}}$

$= \dfrac{2\pi}{\sqrt{10}}$

$A = A_0 e^{-t/\gamma}$

$\therefore$ for $A = \dfrac{A_0}{e}$, $t = \gamma$

$$t = \gamma = \frac{2m}{b} = \frac{2m}{\dfrac{B^2\ell^2}{R}} = 10^4 s$$

$\therefore$ No of oscillation $\dfrac{t}{T_0} = \dfrac{10^4}{2\pi/\sqrt{10}} \approx 5000$.

14. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is :

(1) $\dfrac{3}{2}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{3}{5}$ (4) $\dfrac{8}{5}$

Sol. $M = \int_0^R \rho_0 r(2\pi r dr) = \dfrac{\rho_0 \times 2\pi \times R^3}{3}$

$\underset{\text{(MOI about COM)}}{I_0} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \dfrac{\rho_0 \times 2\pi R^5}{5}$

by parallel axis theorem

$I = I_0 + MR^2$

$$= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15}$$

$$= MR^2 \times \frac{8}{5}$$

15. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in :

(1) 4.2 hrs. (2) 2.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

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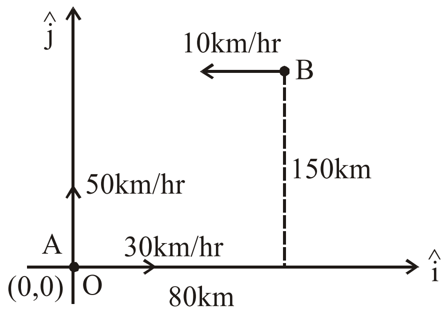
Sol. If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :



$\vec{v}_A = \left(30\hat{i} + 50\hat{j}\right)\text{km/hr}$

$\vec{v}_B = -10\hat{i}\ \text{km/hr}$

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = \left(80\hat{i} + 150\hat{j}\right)\text{km}$

Time after which distance between them will be minimum

$t = -\dfrac{\vec{r}_{BA}\cdot\vec{v}_{BA}}{\left|\vec{v}_{BA}\right|^2}$;

where $\vec{r}_{BA} = \left(80\hat{i} + 150\hat{j}\right)\text{km}$

$\vec{v}_{BA} = -10\hat{i} - \left(30\hat{i} + 50\hat{j}\right)$

$\left(-40\hat{i} - 50\hat{j}\right)\text{km/hr}$

$$\therefore t = -\frac{\left(80\hat{i} + 150\hat{j}\right)\cdot\left(-40\hat{i} - 50\hat{j}\right)}{\left|-40\hat{i} - 50\hat{j}\right|^2}$$

$$= \frac{3200 + 7500}{4100}\text{hr} = \frac{10700}{4100}\text{hr} = 2.6\text{hrs}$$

16. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 π ms^{-2}, what will be the tensile stress that would be developed in the wire ?

(1) $4.8 \times 10^6\ Nm^{-2}$

(2) $5.2 \times 10^6\ Nm^{-2}$

(3) $6.2 \times 10^6\ Nm^{-2}$

(4) $3.1 \times 10^6\ Nm^{-2}$

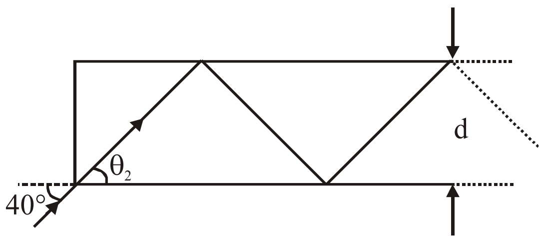
Sol. Tensile stresss in wire will be

$$= \frac{\text{Tensile force}}{\text{Cross section Area}}$$

$$= \frac{mg}{\pi R^2} = \frac{4\times 3.1\pi}{\pi\times 4\times 10^{-6}}\text{Nm}^{-2} = 3.1 \times 10^6\ \text{Nm}^{-2}$$

17. In figure, the optical fiber is ℓ = 2m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle θ_1 = 40°, the number of reflection it makes before emerging from the other end is close to:

(refractive index of fibre is 1.31 and sin 40° = 0.64)



(1) 55000 (2) 57000

(3) 66000 (4) 45000

Sol. **Note :**

If we approximate the angle θ_2 as 30° initially then answer will be closer to 57000. But if we solve thorougly, answer will be close to 55000.

So both the answers must be awarded.

Detailed solution is as following.

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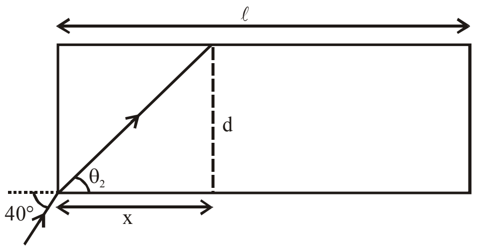


Exact solution

by Snells' law 1.sin40° = (1.31)sinθ_2

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

Now $\tan\theta_2 = \frac{64}{\sqrt{(131)^2 - (64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{d}{x}$

Now no. of reflections

$$= \frac{2\times 64}{114.3\times 20\times 10^{-6}} = \frac{64\times 10^5}{114.3}$$

≈ 55991 ≈ 55000

Approximate solution

By Snells' law 1.sin40° = (1.31)sinθ_2

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

If assume $\Rightarrow \theta_2 \approx 30°$

$$\tan 30° = \frac{d}{x} \Rightarrow x = \sqrt{3}d$$

Now number of reflections

$$= \frac{\ell}{\sqrt{3}d} = \frac{2}{\sqrt{3}\times 20\times 10^{-6}} = \frac{10^5}{\sqrt{3}}$$

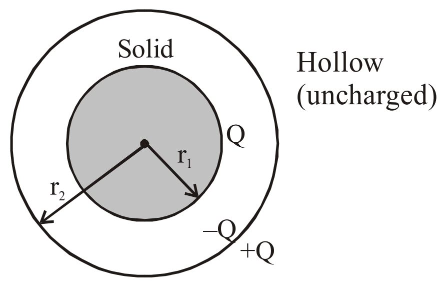
≈ 57735 ≈ 57000

18. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is :

(1) V (2) 2V

(3) –2V (4) 4V

Sol. As given in the first condition :

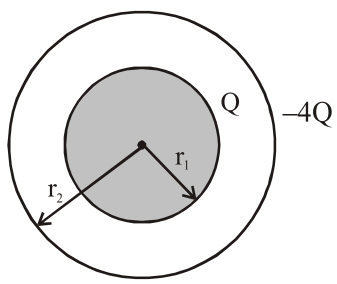


Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

In the second condition :



Shell is now given charge –4Q.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

Hence, we also obtain that potential difference does not depend on charge of outer sphere.

∴ P.d. remains same

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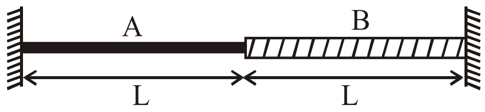
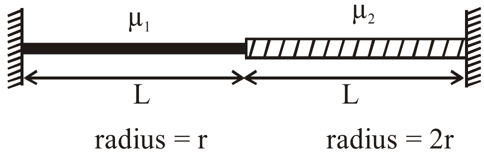
19. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is :



(1) 4 : 9 (2) 3 : 5 (3) 1 : 4 (4) 1 : 2

Sol.



Let mass per unit length of wires are μ_1 and μ_2 respectively.

$\because$ Materials are same, so density ρ is same.

$$\therefore\ \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu \ \text{ and } \ \mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension in both are same = T, let speed of wave in wires are V_1 and V_2

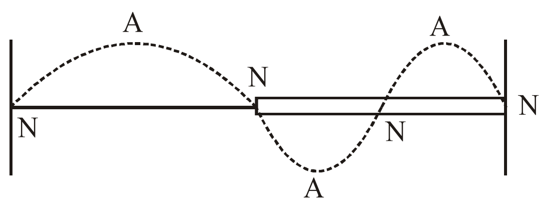
$$V_1 = \sqrt{\frac{T}{\mu}} = V\ ;\ V_2 = \sqrt{\frac{T}{4\mu}} = \frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_{01} = \frac{V_1}{2L} = \frac{V}{2L}\ \ \&\ \ f_{02} = \frac{V_2}{2L} = \frac{V}{4L}$$

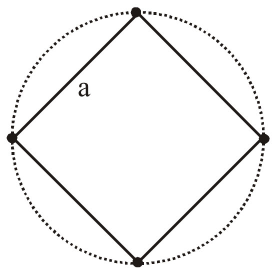
Frequency at which both resonate is L.C.M of both frequencies i.e. $\frac{V}{2L}$.

Hence no. of loops in wires are 1 and 2 respectively.



So, ratio of no. of antinodes is 1 : 2.

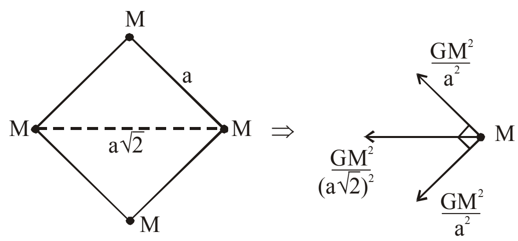
20. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square?



(1) $1.21\sqrt{\frac{GM}{a}}$ (2) $1.41\sqrt{\frac{GM}{a}}$

(3) $1.16\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

Sol.



Net force on particle towards centre of circle

is $F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$

$$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force. Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

$$r = \frac{a}{\sqrt{2}},\ F_C = \frac{mv^2}{r}$$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

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$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}}+1\right)$$

$$v^2 = \frac{GM}{a}(1.35)$$

$$v = 1.16\sqrt{\frac{GM}{a}}$$

21. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\ V\ m^{-1}$ along y-direction. Its corresponding magnetic field component, B would be :

(1) 6×10^{-8} T along z-direction
(2) 6×10^{-8} T along x-direction
(3) 2×10^{-8} T along z-direction
(4) 2×10^{-8} T along y-direction

Sol. The direction of propogation of an EM wave is direction of $\vec{E} \times \vec{B}$.

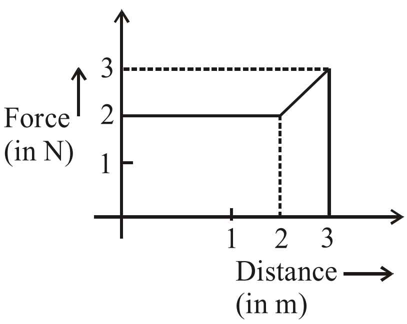
$$\hat{i} = \hat{j} \times \hat{B}$$

$$\Rightarrow \hat{B} = \hat{k}$$

$$C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{6}{3 \times 10^8}$$

$B = 2 \times 10^{-8}$ T along z direction.

22. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is :



(1) 6.5 J (2) 2.5 J (3) 4 J (4) 5 J

Sol. According to work energy theorem

Work done by force on the particle = Change in KE

Work done = Area under F-x graph

$$= \int F.dx$$

$$= 2 \times 2 + \frac{(2+3) \times 1}{2}$$

$W = KE_{final} - KE_{initial} = 6.5$

$KE_{initial} = 0$

$KE_{final} = 6.5$ J

23. In SI units, the dimesions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is :

(1) $A^{-1}\ TML^3$ (2) $A^2T^3M^{-1}L^{-2}$
(3) $AT^2M^{-1}L^{-1}$ (4) $AT^{-3}ML^{3/2}$

Sol. dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2}$

$= [M^{-1}L^{-2}T^3A^2]$

24. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is :

(1) n = 1 → n = 4 (2) n = 2 → n = 4
(3) n = 2 → n = 5 (4) n = 2 → n = 3

Sol. Energy released for transition n = 2 to n = 1 of hydrogen atom

$$E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$Z = 1,\ n_1 = 1,\ n_2 = 2$

$$E = 13.6 \times 1 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

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$$E = 13.6 \times \frac{3}{4} \text{eV}$$

For He^+ ion z = 2

(1) n = 1 to n = 4

$$E = 13.6 \times 2^2 \times \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{15}{4} \text{eV}$$

(2) n = 2 to n = 4

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{3}{4} \text{eV}$$

(3) n = 2 to n = 5

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 13.6 \times \frac{21}{25} \text{eV}$$

(4) n = 2 to n = 3

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{9} \text{eV}$$

Energy required for transition of He^+ for n = 2 to n = 4 matches exactly with energy released in transition of H for n = 2 to n = 1.

25. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ, of the final particle, is given by :

(1) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (2) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(3) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (4) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Sol.  $\frac{h}{\lambda_1} = P_1$  $P_2 = \frac{h}{\lambda_2}$

$$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i}$$

& $\vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$

Using momentum conservation

$$\vec{P} = \vec{P}_1 + \vec{P}_2$$

$$= \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$$

$$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

26. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to :

(Latent heat of vaporization of water = 2.10×10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36×10^5 J kg^{-1})

(1) 130 g (2) 35 g

(3) 20 g (4) 150 g

Sol. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{req} = mL_v$

Mass that converts into ice = (150 – m)

So, heat released in this process

$\Delta Q_{rel} = (150 - m) L_f$

Now,

$\Delta Q_{rel} = \Delta Q_{req}$

$(150 - m) L_f = mL_V$

$m(L_f + L_v) = 150 L_f$

$$m = \frac{150L_f}{L_f + L_v}$$

m = 20g

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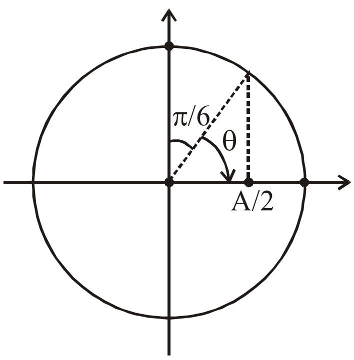
27. An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is :

(1) 2.2 ms (2) 5 ms

(3) 3.3 ms (4) 7.2 ms

Sol.



V(t) = 220 sin(100πt) volt

time taken,

$$t=\frac{\theta}{\omega}=\frac{\frac{\pi}{3}}{100\pi}=\frac{1}{300}\text{sec}$$

= 3.3 ms

28. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

(1) 600 nm (2) 900 nm

(3) 2400 nm (4) 1500 nm

Sol. To minimise attenuation, wavelength of carrier waves is close to 1500 nm

29. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2}=\frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be :

(1) 4 (2) 2

(3) 9 (4) 18

Sol. Given $\frac{a_1}{a_2}=\frac{1}{3}$

Ratio of intensities, $\frac{I_1}{I_2}=\left(\frac{a_1}{a_2}\right)^2=\frac{1}{9}$

Now, $\frac{I_{max}}{I_{min}}=\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=\left(\frac{1+3}{1-3}\right)^2=4$

30. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of :

(1) 10^8 N/m^2 (2) 10^4 N/m^2

(3) 10^3 N/m^2 (4) 10^{16} N/m^2

Sol. Note :

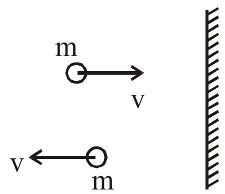
Pressure is defined as normal force per unit area.

Force is calculated as change in momentum/ time

By this answer is 2N/m^2

None of the option matches so this question must be Bonus

Detailed solution is as following.



Magnitude of change in momentum per collision = 2mv

Pressure = $\frac{\text{Force}}{\text{Area}}=\frac{N(2mv)}{1}$

$=\frac{10^{22}\times2\times10^{-26}\times10^4}{1}$

= 2N/m^2

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FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fraction of components A and B in vapour phase, respectively are-

(1) 500 mmHg, 0.5, 0.5
(2) 450 mmHg, 0.4, 0.6
(3) 450 mmHg, 0.5, 0.5
(4) 500 mmHg, 0.4, 0.6

Sol. $P_{total} = X_A . P_A^0 + X_B . P_B^0 = 0.5 \times 400 + 0.5 \times 600$

$= 500$ mmHg

Now, mole fraction of A in vapour,

$$Y_A = \frac{P_A}{P_{total}} = \frac{0.5 \times 400}{500} = 0.4$$

and mole fraction of B in vapour,

$Y_B = 1 - 0.4 = 0.6$

Correct option : (4)

2. If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relation between S and K_{sp} is correct

(1) $S = \left(\frac{K_{sp}}{929}\right)^{1/9}$ (2) $S = \left(\frac{K_{sp}}{216}\right)^{1/7}$

(3) $S = \left(\frac{K_{sp}}{144}\right)^{1/6}$ (4) $S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons \underset{3S\,M}{3Zr^{4+}(aq.)} + \underset{4S\,M}{4PO_4^{3-}(aq.)}$

$K_{sp} = [Zr^{4+}]^3 [PO_4^{3-}]^4 = (3S)^3.(4S)^4 = 6912\ S^7$

$$\therefore S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$$

Correct option : (4)

3. In order to oxidise a mixture one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium , the number of moles of $KMnO_4$ required is -

(1) 3 (2) 2
(3) 1 (4) 1.5

Sol. $n_{eq.}\ KMnO_4 = n_{eq.}[FeC_2O_4 + Fe_2(C_2O_4)_3 + FeSO_4]$

or $n \times 5 = 1 \times 3 + 1 \times 6 + 1 \times 1$

$\therefore n = 2$

Correct option : (2)

4. In the following compounds, the decreasing order of basic strength will be -

(1) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$
(2) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$
(3) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$
(4) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$

Sol. Basic strength order

$\underset{2^\circ\ \text{amine}}{(CH_3CH_2)_2NH} > \underset{1^\circ\text{amine}}{CH_3CH_2NH_2} > NH_3$

Correct option : (1)

5. Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively

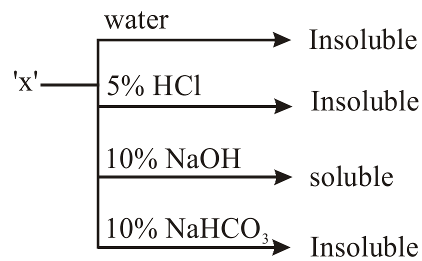
(1) HBO_2 and H_3BO_3 (2) H_3BO_3 and B_2O_3
(3) B_2O_3 and H_3BO_3 (4) B_2O_3 and $[BH_4]^-$

Sol. $B_2H_6 + 3H_2O \longrightarrow 2H_3BO_3 + 3H_2$

$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

Correct option : (3)

6. An organic compound 'X' showing the following solubility profile is -



(1) m-Cresol (2) Oleic acid
(3) o-Toluidine (4) Benzamide

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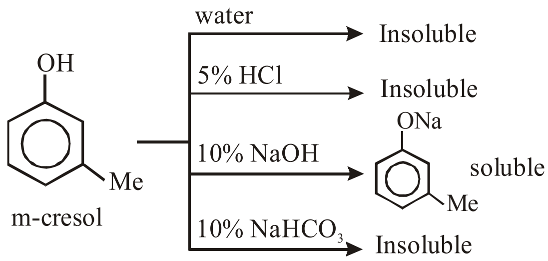
Sol.

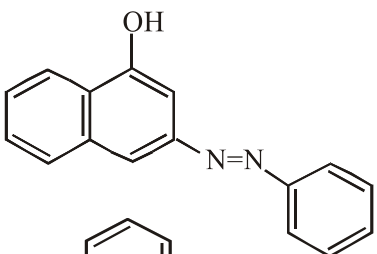


* Oleic acid is also soluble in $NaHCO_3$
* o-toluidine is not soluble in NaOH as well as $NaHCO_3$
* Benzamide is also not soluble in NaOH & $NaHCO_3$.

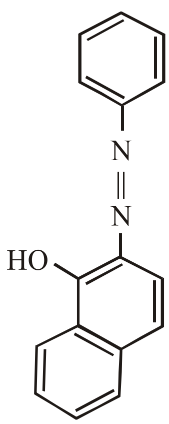
Correct option : (1)

7. Coupling of benzene diazonium chloride with 1-napthol in alkaline medium will give

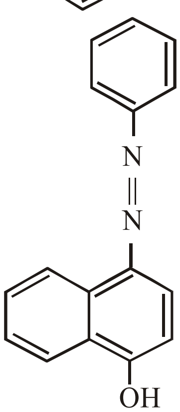
(1)



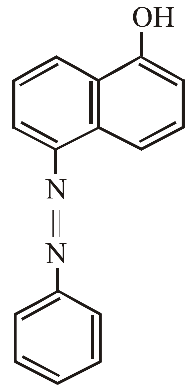
(2)



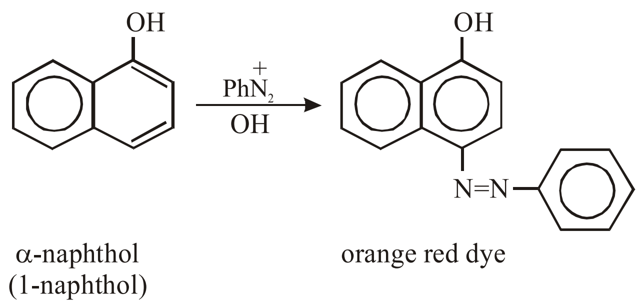
(3)



(4)



Sol.



8. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas ? (Assume non-expansion work is zero)

(1) Cyclic process : $q = -w$

(2) Isothermal process : $q = -w$

(3) Adiabatic process : $\Delta U = -w$

(4) Isochoric process : $\Delta U = q$

Sol. For cyclic process : $\Delta U = 0 \Rightarrow q = -w$

For isothermal process : $\Delta U = 0 \Rightarrow q = -w$

For adiabatic process : $q = 0 \Rightarrow \mathbf{\Delta U = W}$

For isochoric process : $w = 0 \Rightarrow \Delta U = q$

Correct option : (3)

9. The lanthanide ion that would show colour is-

(1) Sm^{3+} (2) La^{3+}

(3) Lu^{3+} (4) Gd^{3+}

Sol. Sm^{3+} $(4f^5)$ = yellow colour

Correct option : (1)

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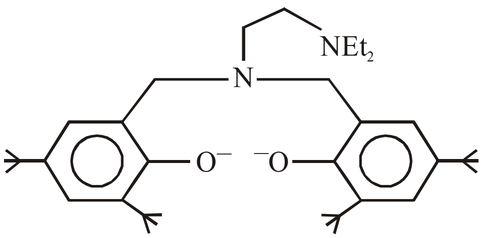
10. With respect to an ore, Ellingham diagram helps to predict the feasibility of its -
(1) Vapour phase refining
(2) Zone refining
(3) Electrolysis
(4) Thermal reduction

Sol. Ellingham diagram helps in predicting the feasibiltiy of thermal reduction of ores.
Correct option : (4)

11. The following ligand is



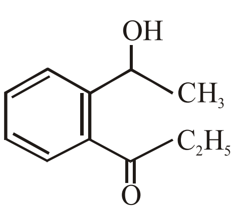
(1) Bidentate (2) Hexadentate
(3) Tetradentate (4) Tridentate

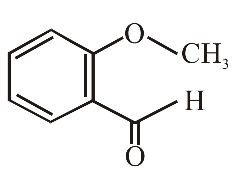
Sol. Donating atoms are both nitrogen & oxygen.
Correct option : (3)

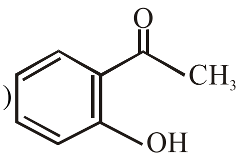
12. The correct order of hydration enthalpies of alkali metal ions is -
(1) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$
(2) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$
(3) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$
(4) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

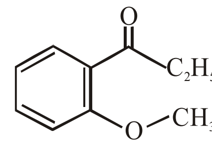
Sol. Hydration enthalpy depends upon ionic potential (charge / size). As ionic potential increases hydration enthalpy increases.
Correct option : (1)

13. An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution, It however, reacts with Grignard reagent and gives positive iodoform test. The compound is -

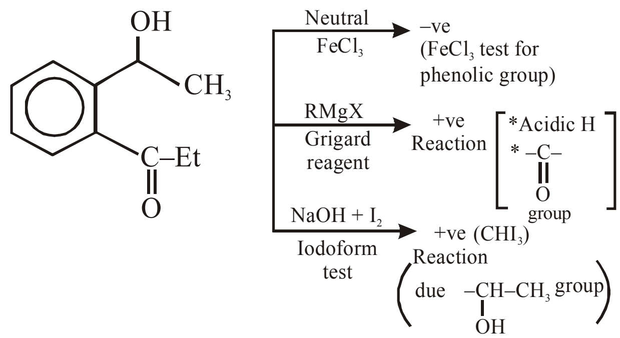
(1) 

(2) 

(3) 

(4) 

Sol.



Correct option : (1)

14. The quantum number of four electrons are given below -
I. $n = 4, l = 2, m_l = -2, m_s = -½$
II. $n = 3, l = 2, m_l = 1, m_s = +½$
III. $n = 4, l = 1, m_l = 0, m_s = +½$
IV. $n = 3, l = 1, m_l = 1, m_s = -½$
The correct order of their increasing energies will be -
(1) IV < III < II < I (2) IV < II < III < I
(3) I < II < III < IV (4) I < III < II < IV

Sol. According to $(n+\ell)$ rule : 3p < 3d < 4p < 4d
Correct option : (2)

15. **Assertion :** Ozone is destroyed by CFCs in the upper stratosphere
Reason : Ozone holes increase the amount of UV radiation reaching the earth.
(1) Assertion and reason are correct, but the reason is not the explanation for the assertion
(2) Assertion is false, but the reason is correct
(3) Assertion and reason are incorrect, Assertion and reason are both correct
(4) And the reason is the correct explanation for the assertion

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Sol. The upper stratosphere consists of ozone (O_3), which protect us from harmful ultraviolet (UV) radiations coming from sun.

Correct option : (1)

16. The size of the iso-electronic species Cl^-, Ar and Ca^{2+} is affected by -

(1) Principal quantum number of valence shell

(2) Nuclear charge

(3) Azimuthal qunatum number of valence shell

(4) Electron-electron interaction in the outer orbitals

Sol. For isoelectronic species the size is compared by nuclear charge.

Correct option: (2)

17. Given that : $E^0_{O_2/H_2O} = +1.23V$,

$E^0_{S_2O_8^{2-}/SO_4^{2-}} = +2.05V$

$E^0_{Br_2/Br^-} = +1.09V$

$E^0_{Au^{3+}/Au} = +1.4V$

The strongest oxidizing agent is -

(1) O_2 (2) Br_2

(3) $S_2O_8^{2-}$ (4) Au^{3+}

Sol. For strongest oxidising agent, standard reduction potential should be highest.

Correct option : (3)

18. For silver, $C_p(JK^{-1}mol^{-1}) = 23 + 0.01T$. If the temperature (T) of 3 moles of silver is raised from 300K to 1000 K at 1 atm pressure, the value of ΔH will be close to

(1) 21 kJ (2) 16 kJ

(3) 13 kJ (4) 62 kJ

Sol. $\Delta H = n\int_{T_1}^{T_2} C_{p,m}dT = 3\times\int_{300}^{1000}(23+0.01T)dT$

$= 3\left[23(1000-300) + \dfrac{0.01}{2}(1000^2 - 300^2)\right]$

$= 61950\ J \approx 62\ kJ$

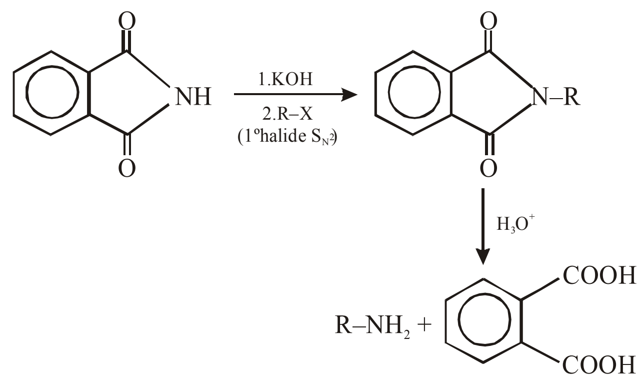
Correct option : (4)

19. Which of the following amines can be prepared by Gabriel phthalimide reaction ?

(1) Neo-pentylamine (2) n-butylamine

(3) triethylamine (4) t-butylamine

Sol. Gabriel phthalimide synthesis :



Correct option : (2)

20. Which is wrong with respect to our responsibility as a human being to protect our environment ?

(1) Avoiding the use of floodlighted facilities

(2) Restricting the use of vehicles

(3) Using plastic bags

(4) Setting up compost tin in gardens

Sol. Correct option : (3)

21. Maltose on treatment with dilute HCI gives :

(1) D-Galactose

(2) D-Glucose

(3) D-Glucose and D-Fructose

(4) D-Fructose

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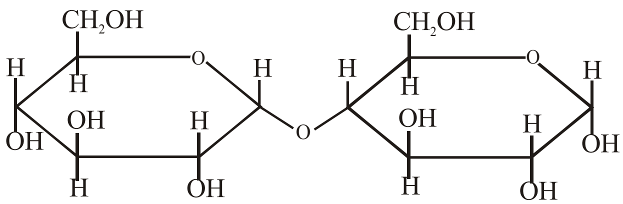
Sol.



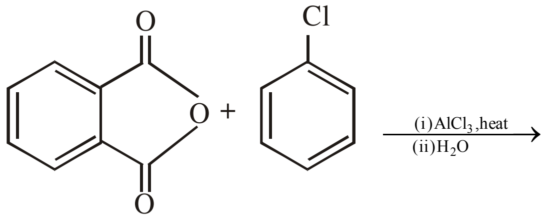
Maltose

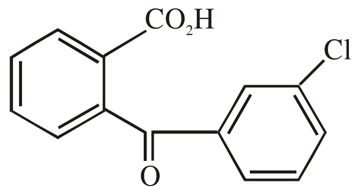
$\xrightarrow{H_3O^+}$

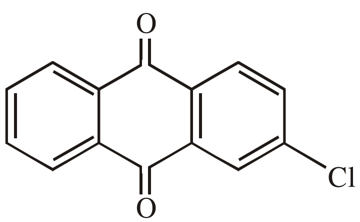
2 D-glucose

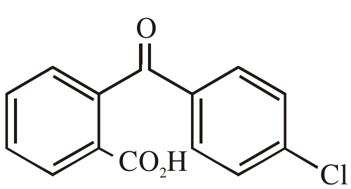
Correct option : (2)

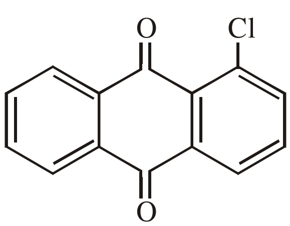
22. The major product of the following reaction is:



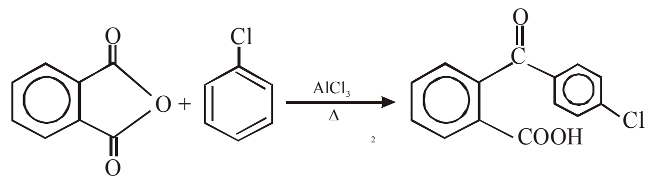
(1) 

(2) 

(3) 

(4) 

Sol.



Fridel-craft acylation. –Cl group is an ortho & para directing

Correct option : (3)

23. The correct order of the spin-only magnetic moment of metal ions in the following low spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$, and $[Cr(NH_3)_6]^{2+}$, is :

(1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

(2) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(3) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

(4) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$

Sol. According to question all the complexes are **low spin.**

Complex	Configuration	No. of unpaired electrons
$[V(CN)_6]^{4-}$	$t_{2g}^3e_g^0$	3
$[Cr(NH_3)_6]^{2+}$	$t_{2g}^4e_g^0$	2
$[Ru(NH_3)_6]^{3+}$	$t_{2g}^5e_g^0$	1
$[Fe(CN)_6]^{4-}$	$t_{2g}^6e_g^0$	0

Correct option : (1)

24. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is: (molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 $gmol^{-1}$)

(1) 1,000 ppm (2) 10,000 ppm

(3) 100 ppm (4) 5,000 ppm

Sol. $n_{eq.}CaCO_3 = n_{eq}Ca(HCO_3)_2 + n_{eq}Mg(HCO_3)_2$

or , $\frac{W}{100}\times 2 = \frac{0.81}{162}\times 2 + \frac{0.73}{146}\times 2$

$\therefore$ w = 1.0

$\therefore$ Hardness = $\frac{1.0}{100}\times 10^6 = 10000ppm$

Correct option : (2)

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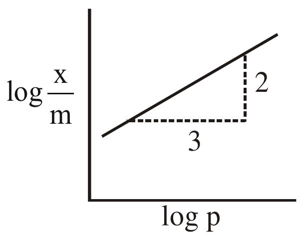
25. Adsorption of a gas follows Freundlich adsorption isotherm x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\log \frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to :



(1) $p^{3/2}$ (2) p^3

(3) $p^{2/3}$ (4) p^2

Sol. $\frac{x}{m} = K.p^{1/n}$

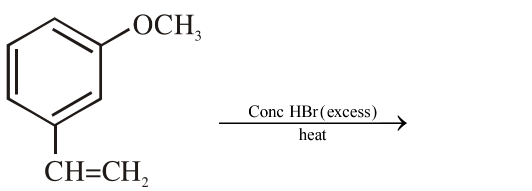
$\therefore \log \frac{x}{m} = \log K + \frac{1}{n}.\log P$

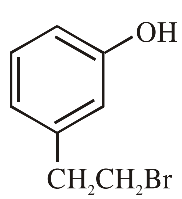
slope $= \frac{1}{n} = \frac{2}{3}$

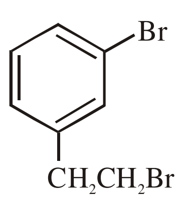
$\therefore \frac{x}{m} = K.p^{2/3}$

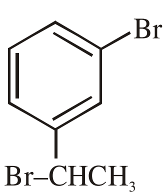
Correct option : (3)

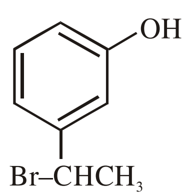
26. The major product of the following reactions:



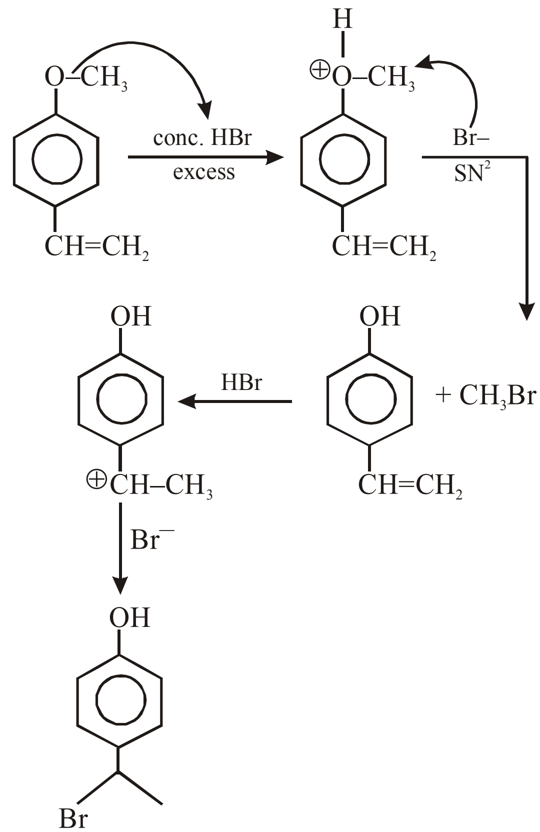
(1) 

(2) 

(3) 

(4) 

Sol.



Correct option : (4)

27. For the reaction 2A +B → C, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is :

[A] (mol L^{-1})	[B] (mol L^{-1})	Initial Rate (mol $L^{-1}s^{-1}$)
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

(1) Rate = k [A][B] (2) Rate = k $[A]^2[B]^2$

(3) Rate = k $[A][B]^2$ (4) Rate = k $[A]^2[B]$

Sol. $r = K [A]^x [B]^y$

$0.045 = K (0.05)^x (0.05)^y$ ....(1)

$0.090 = K (0.10)^x (0.05)^y$ ....(2)

$0.72 = K (0.20)^x (0.10)^y$ ....(3)

From (1) ÷ (2), $\frac{0.045}{0.090} = \left(\frac{0.05}{0.10}\right)^x \Rightarrow x = 1$

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From (2) ÷ (3), $\frac{0.090}{0.720}=\left(\frac{0.10}{0.20}\right)^{x}\cdot\left(\frac{0.05}{0.10}\right)^{y} \Rightarrow y=2$

Hence, r = K [A] $[B]^2$

Correct option : (3)

28. The IUPAC name of the following compound is :

$$H_3C-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-\overset{OH}{\overset{|}{C}H}-CH_2-COOH$$

(1) 2-Methyl-3Hydroxypentan-5-oic acid
(2) 4,4-Dimethyl-3-hydroxy butanoic acid
(3) 3-Hydroxy-4 -methylpentanoic acid
(4) 4-Methyl-3-hydroxypentanoic acid

Sol.

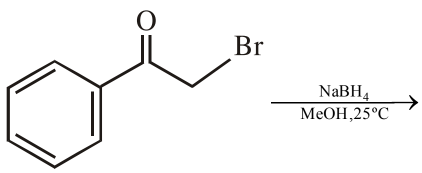
$$\underset{5}{CH_3}-\underset{4}{\overset{CH_3}{\overset{|}{C}H}}-\underset{\underset{OH}{|}}{\overset{3}{C}H}-\overset{2}{CH_2}-\overset{1}{C}OOH$$

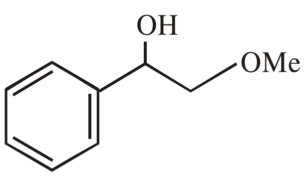
3-Hydroxy-4-methylpentanoic acid

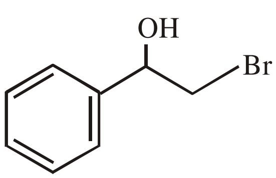
–COOH principal functional group

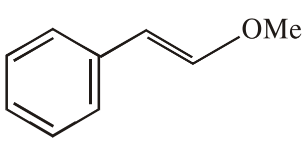
Correct option : (3)

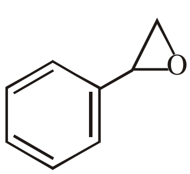
29. The major product of the following reaction is:



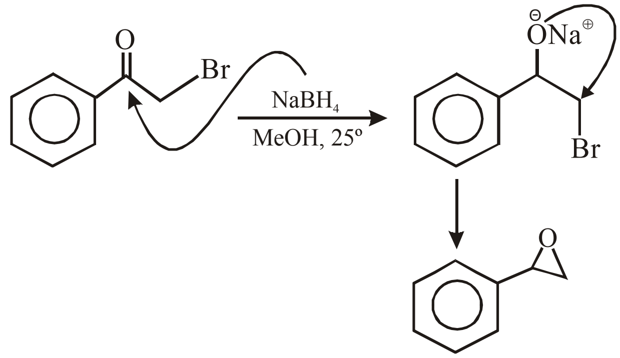
(1) 

(2) 

(3) 

(4) 

Sol.



30. Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is :

(1) A_2BO_4 (2) A_2B_2O
(3) A_4B_2O (4) AB_2O_4

Sol. $Z_B = 4$, $Z_A = 4\times\frac{1}{2}=2, Z_O = 8$

Formula ; $A_2B_2O_8 \equiv AB_2O_4$

Correct option : (4)

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FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

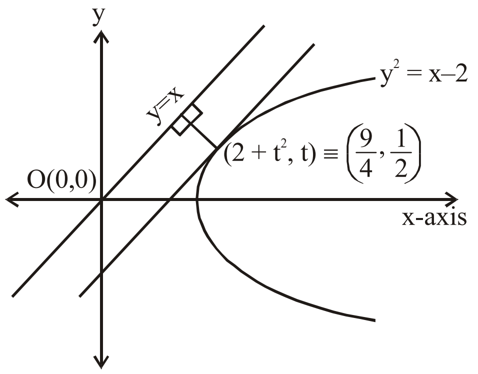
MATHEMATICS — TEST PAPER WITH ANSWER & SOLUTION

1. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) $\frac{11}{4\sqrt{2}}$ (4) 2

Sol.



we have, $2y.\frac{dy}{dx} = 1 \Rightarrow \left.\frac{dy}{dx}\right]_{P(2+t^2,t)} = \frac{1}{2t} = 1$

$\Rightarrow \quad t = \frac{1}{2}$

$\therefore \quad P\left(\frac{9}{4}, \frac{1}{2}\right)$

So, shortest distance

$= \frac{\left|\frac{9}{4} - \frac{2}{4}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$

2. Let y = y(x) be the solution of the differential equation, $\left(x^2+1\right)^2 \frac{dy}{dx} + 2x\left(x^2+1\right)y = 1$ such that y(0) = 0. If $\sqrt{a}\,y(1) = \frac{\pi}{32}$, then the value of 'a' is :

(1) $\frac{1}{2}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) 1

Sol. $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore \quad$ I.F. $= e^{\ell n\left(x^2+1\right)} = (x^2+1)$

So, general solution is $y.(x^2 + 1) = \tan^{-1}x + c$

As y(0) = 0 $\Rightarrow$ c = 0

$\therefore \quad y(x) = \frac{\tan^{-1}x}{x^2+1}$

As, $\sqrt{a}.\,y(1) = \frac{\pi}{32}$

$\Rightarrow \quad \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

3. A point on the straight line, 3x + 5y = 15 which is equidistant from the coordinate axes will lie only in :

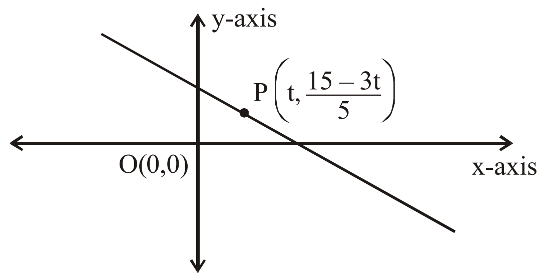
(1) 1st and 2nd quadrants

(2) 4th quadrant

(3) 1st, 2nd and 4th quadrant

(4) 1st quadrant

Sol.



Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \quad \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

$\therefore \quad t = \frac{15}{8}$ or $t = \frac{-15}{2}$

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So, $P\left(\frac{15}{8},\frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2},\frac{15}{2}\right) \in II^{nd}$ quadrant

4. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

(1) 2 (2) 3

(3) 4 (4) 5

Sol. $(x-1)^2 + 1 = 0 \Rightarrow x = 1 + i,\ 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$

$\therefore$ n (least natural number) = 4

5. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals :

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$

(3) $\sqrt{2}$ (4) 4

Sol. $\lim\limits_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$

$$= \frac{(1)^2.\left(2\sqrt{2}\right)}{\frac{1}{2}} = 4\sqrt{2}$$

6. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

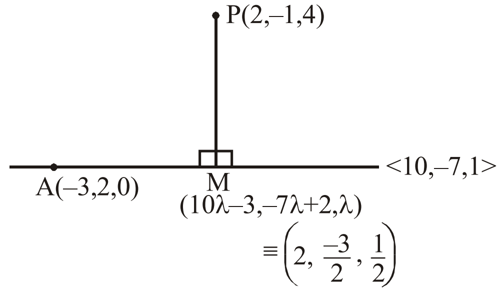
(1) less than 2

(2) greater than 3 but less than 4

(3) greater than 4

(4) greater than 2 but less than 3

Sol.



Now, $\overrightarrow{MP}.\left(10\hat{i} - 7\hat{j} + \hat{k}\right) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore$ Length of perpendicular

$$(= PM) = \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}},$$

which is greater than 3 but less than 4.

7. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$

(3) $\sqrt{6}$ (4) $3\sqrt{6}$

Sol. Vector perpendicular to plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ & $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Required magnitude of projection

$$= \frac{\left|(2\hat{i} + 3\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k})\right|}{\left|\hat{i} - 2\hat{j} + \hat{k}\right|}$$

$$= \frac{|2 - 6 + 1|}{\left|\sqrt{6}\right|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

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8. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not a citizen of India, then you are not born in India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not born in India, then you are not a citizen of India.

Sol. The contrapositive of statement
$p \rightarrow q$ is $\sim q \rightarrow \sim p$
Here, p : you are born in India.
q : you are citizen of India.
So, contrapositive of above statement is
" If you are not a citizen of India, then you are not born in India".

9. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

(1) 40 (2) 49

(3) 48 (4) 45

Sol. Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56 \qquad ......(1)$$

Also $\sigma^2 = 16$

$$\Rightarrow \quad 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$$

$$\Rightarrow \quad 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$$

$$\Rightarrow \quad \left(\sum_{i=1}^{7} x_i^2\right) = 560 \qquad .....(2)$$

Now, $x_1 = 2$, $x_2 = 4$, $x_3 = 10$, $x_4 = 12$, $x_5 = 14$

$\Rightarrow \quad x_6 + x_7 = 14$ (from (1))

& $\quad x_6^2 + x_7^2 = 100$ (from (2))

$\therefore \quad x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6.x_7 \Rightarrow x_6.x_7 = 48$

10. If $f(x) = \dfrac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x, (x > 0)$ then the value of integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))\,dx$ is :

(1) $\log_e 3$ (2) $\log_e 2$

(3) $\log_e e$ (4) $\log_e 1$

Sol. $g(f(x)) = \ell n(f(x)) = \ell n\left(\dfrac{2 - x.\cos x}{2 + x.\cos x}\right)$

$$\therefore \quad I = \int_{-\pi/4}^{\pi/4} \ell n\left(\frac{2 - x.\cos x}{2 + x.\cos x}\right)dx$$

$$= \int_{0}^{\pi/4}\left(\ell n\left(\frac{2 - x.\cos x}{2 + x.\cos x}\right) + \ell n\left(\frac{2 + x.\cos x}{2 - x.\cos x}\right)\right)dx$$

$$= \int_{0}^{\pi/2} (0)dx = 0 = \log_e(1)$$

11. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{128}{17}$ (4) $\frac{4}{17}$

Sol. $4a^2 + b^2 = 8 \qquad ....(1)$

also $\left.\dfrac{dy}{dx}\right)_{(1,2)} = -\dfrac{4x}{y} = -2$

$\Rightarrow \quad -\dfrac{4a}{b} = \dfrac{1}{2}$

$b = -8a$

$\Rightarrow \quad b^2 = 64a^2$

$68a^2 = 8$

$$a^2 = \frac{2}{17}$$

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12. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right), \beta = \tan^{-1}\left(\frac{1}{3}\right)$,

where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$

(3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Sol. $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$\Rightarrow \tan\alpha = \frac{4}{3}$

$\Rightarrow \tan(\alpha - \beta) = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3}\cdot\frac{1}{3}} = \frac{9}{13}$

$\Rightarrow \sin(\alpha - \beta) = \frac{9}{5\sqrt{10}}$

$\Rightarrow \alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

13. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then :

(1) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(2) $S_1 = \{-2, 0\}; S_2 = \{1\}$

(3) $S_1 = \{-2\}; S_2 = \{0, 1\}$

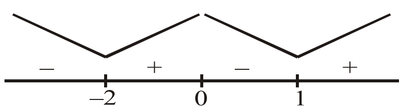
(4) $S_1 = \{-1\}; S_2 = \{0, 2\}$

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$



Points of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$

14. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

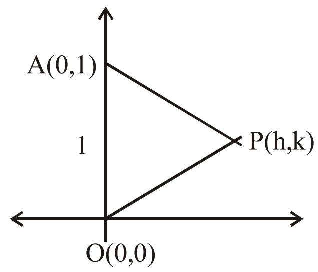
(1) $8x^2 - 9y^2 + 9y = 18$

(2) $9x^2 + 8y^2 - 8y = 16$

(3) $8x^2 + 9y^2 - 9y = 18$

(4) $9x^2 - 8y^2 + 8y = 16$

Sol.



AP + OP + AO = 4

$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} + 1 = 4$

$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} = 3$

$h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2 + k^2}$

$-2k - 8 = -6\sqrt{h^2 + k^2}$

$k + 4 = 3\sqrt{h^2 + k^2}$

$k^2 + 16 + 8k = 9(h^2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

15. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that

$A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is

(1) $\frac{\pi}{16}$ (2) 0

(3) $\frac{\pi}{32}$ (4) $\frac{\pi}{64}$

Sol. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

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$$=\begin{bmatrix}\cos 2\alpha & -\sin 2\alpha\\ \sin 2\alpha & \cos 2\alpha\end{bmatrix}$$

$$A^3=\begin{bmatrix}\cos 2\alpha & -\sin 2\alpha\\ \sin 2\alpha & \cos 2\alpha\end{bmatrix}\begin{bmatrix}\cos \alpha & -\sin \alpha\\ \sin \alpha & \cos \alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos 3\alpha & -\sin 3\alpha\\ \sin 3\alpha & \cos 3\alpha\end{bmatrix}$$

Similarly $A^{32}=\begin{bmatrix}\cos 32\alpha & -\sin 32\alpha\\ \sin 32\alpha & \cos 32\alpha\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$\Rightarrow$ $\cos 32\alpha = 0$ & $\sin 32\alpha = 1$

$\Rightarrow$ $32\alpha=(4n+1)\frac{\pi}{2}, n\in I$

$\alpha=(4n+1)\frac{\pi}{64}, n\in I$

$\alpha=\frac{\pi}{64}$ for $n=0$

16. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) $2f(x)$ (2) $2f(x^2)$

(3) $(f(x))^2$ (4) $-2f(x)$

Sol. $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$

$$f\left(\frac{2x}{1+x^2}\right)=\ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$=\ell n\left(\frac{(x-1)^2}{(x+1)^2}\right)=2\ell n\left|\frac{1-x}{1+x}\right|=2f(x)$$

17. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

(1) $x + 3y + z = 4$ (2) $x - y - z = 0$

(3) $x - 3y - 2z = -2$ (4) $2x - z = 2$

Sol. The required plane is

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

it passes through (1, 1, 0)

$\Rightarrow$ $(2 - 1 - 4) + \lambda(1 - 4) = 0$

$\Rightarrow$ $-3 - 3\lambda = 0$ $\Rightarrow$ $\lambda = -1$

$\Rightarrow$ $x - y - z = 0$

18. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is :

(1) 3221 (2) 3121

(3) 3203 (4) 3303

Sol. S_A = sum of numbers between 100 & 200 which are divisible by 7.

$\Rightarrow$ $S_A = 105 + 112 + ... + 196$

$S_A=\frac{14}{2}[105+196]=2107$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.

$S_B=104+117+....+195=\frac{8}{2}[104+195]=1196$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.

$S_C = 182$

$\Rightarrow$ H.C.F. $(91, n) > 1$ $= S_A + S_B - S_C = 3121$

19. The sum of the series

$2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$

is equal to :

(1) 2^{24} (2) 2^{25}

(3) 2^{26} (4) 2^{23}

Sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r.\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r\left(\frac{20}{r}\right){}^{19}C_{r-1}+2.2^{20}$$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

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20. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0,\ (x>0)$ is equal to :

(1) 4 (2) 9

(3) 10 (4) 12

Sol. $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

$\left|\sqrt{x}-2\right|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left|\sqrt{x}-2\right|=-2$ (not possible) or $\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}=3,1$

x = 9, 1

Sum = 10

21. Let A and B be two non-null events such that $A\subset B$. Then, which of the following statements is always correct ?

(1) P(A|B) = 1

(2) P(A|B) = P(B) – P(A)

(3) P(A|B) $\leq$ P(A)

(4) P(A|B) $\geq$ P(A)

Sol. $P(A\,|\,B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{P(A)}{P(B)}$

$\left(\text{as } A\subset B\Rightarrow P(A\cap B)=P(A)\right)$

$\Rightarrow\ P(A\,|\,B)\geq P(A)$

22. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6,(x>1)$ is equal to :

(1) 32 (2) 26

(3) 29 (4) 24

Sol. $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$

$= 2[{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 2\,{}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x

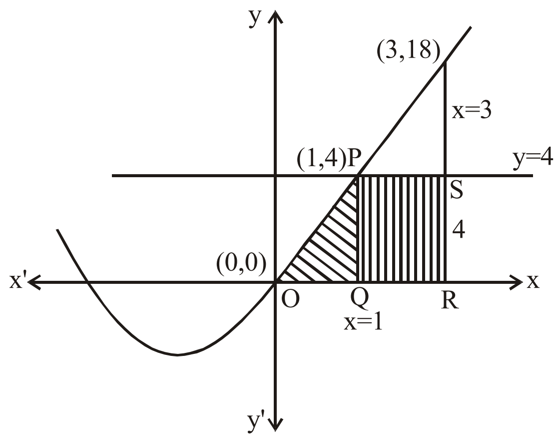
= 2[1 – 15 + 15 + 15 – 1 – 3] = 24

23. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \leq x \leq 3,\ 0 \leq y \leq 4,\ y \leq x^2 + 3x\}$ is :

(1) $\dfrac{53}{6}$ (2) $\dfrac{59}{6}$

(3) 8 (4) $\dfrac{26}{3}$

Sol.



Required Area

$=\int_0^1(x^2+3x)dx+\text{Area of rectangle PQRS}$

$=\dfrac{11}{6}+8=\dfrac{59}{6}$

24. Let f : [0, 2] $\rightarrow$ R be a twice differentiable function such that $f''(x) > 0$, for all x $\in$ (0, 2). If $\phi(x) = f(x) + f(2 - x)$, then ϕ is :

(1) decreasing on (0, 2)

(2) decreasing on (0, 1) and increasing on (1, 2)

(3) increasing on (0, 2)

(4) increasing on (0, 1) and decreasing on (1, 2)

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Sol. $\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f'(x) - f'(2-x)$ ......(1)

Since $f''(x) > 0$

$\Rightarrow f'(x)$ is increasing $\forall\ x \in (0, 2)$

Case-I : When $x > 2 - x \quad \Rightarrow \quad x > 1$

$\Rightarrow \phi'(x) > 0\ \forall\ x \in (1, 2)$

$\therefore \phi(x)$ is increasing on $(1, 2)$

Case-II : When $x < 2 - x \quad \Rightarrow \quad x < 1$

$\Rightarrow \phi'(x) < 0\ \forall\ x \in (0,1)$

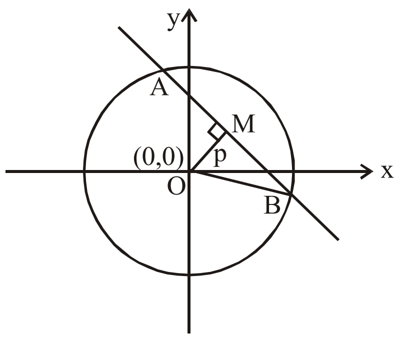
$\therefore \phi(x)$ is decreasing on $(0, 1)$

25. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

(1) 320 (2) 160

(3) 105 (4) 210

Sol.



$p = \frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}} < 4 \Rightarrow n = 1, 2, 3, 4, 5.$

Length of chord AB $= 2\sqrt{16 - \frac{n^2}{2}}$

$= \sqrt{64 - 2n^2} = \ell(\text{say})$

For $n = 1$, $\ell^2 = 62$

$n = 2$, $\ell^2 = 56$

$n = 3$, $\ell^2 = 46$

$n = 4$, $\ell^2 = 32$

$n = 5$, $\ell^2 = 14$

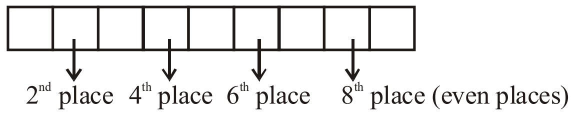
$\therefore$ Required sum $= 62 + 56 + 46 + 32 + 14 = 210$

26. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 175 (2) 162

(3) 160 (4) 180

Sol.



Number of such numbers $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!} = 180$

27. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

Sol. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx = \int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$= \int \frac{\sin 3x + \sin 2x}{\sin x}dx$

$= \int \frac{3\sin x - 4\sin^3 x - 2\sin x\cos x}{\sin x}dx$

$= \int (3 - 4\sin^2 x + 2\cos x)dx$

$= \int \left(3 - 2\left(1 - \cos 2x\right) + 2\cos x\right)dx$

$= \int \left(1 + 2\cos 2x + 2\cos x\right)dx$

$= x + \sin 2x + 2\sin x + c$

28. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$,

then $\frac{dy}{dx}$ is equal to :

(1) $2x - \frac{\pi}{3}$ (2) $\frac{\pi}{3} - x$

(3) $\frac{\pi}{6} - x$ (4) $x - \frac{\pi}{6}$

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Sol. Consider $\cot^{-1}\left(\dfrac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\sin x}\right)$

$$=\cot^{-1}\left(\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)}\right)$$

$$=\cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)=\frac{\pi}{2}-\tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases}\frac{\pi}{2}-\left(x+\frac{\pi}{3}\right)=\left(\frac{\pi}{6}-x\right); & 0<x<\frac{\pi}{6}\\ \frac{\pi}{2}-\left(\left(x-\frac{\pi}{3}\right)-\pi\right)=\left(\frac{7\pi}{6}-x\right); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore \quad 2y=\begin{cases}\left(\frac{\pi}{6}-x\right)^2; & 0<x<\frac{\pi}{6}\\ \left(\frac{7\pi}{6}-x\right)^2; & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore \quad 2\frac{dy}{dx}=\begin{cases}2\left(\frac{\pi}{6}-x\right).(-1); & 0<x<\frac{\pi}{6}\\ 2\left(\frac{7\pi}{6}-x\right).(-1); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

29. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) $\frac{1}{2}$ (2) -1

(3) 0 (4) 2

Sol. For non-trivial solution

$D = 0$

$$\begin{vmatrix}1 & -c & -c\\ c & -1 & c\\ c & c & -1\end{vmatrix}=0\Rightarrow 2c^3-3c^2-1=0$$

$\Rightarrow \quad (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

30. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha, \beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

Sol. $0<\alpha+\beta=\frac{\pi}{2}$ and $\frac{-\pi}{4}<\alpha-\beta<\frac{\pi}{4}$

if $\cos(\alpha+\beta)=\frac{3}{5}$ then $\tan(\alpha+\beta)=\frac{4}{3}$

and if $\sin(\alpha-\beta)=\frac{5}{13}$ then $\tan(\alpha-\beta)=\frac{5}{12}$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha)=\tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}=\frac{63}{16}$$

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