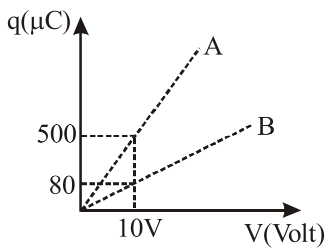
FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS | TEST PAPER WITH ANSWER & SOLUTION

1. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



(1) 50 μF and 30 μF (2) 20 μF and 30 μF
(3) 60 μF and 40 μF (4) 40 μF and 10 μF

Sol. As q = CV

Hence slope of graph will give capacitance. Slope will be more in parallel combination. Hence capacitance in parallel should be 50μF & in series combination must be 8μF.

Only in option 40μF & 10 μF

$$C_{parallel} = 40 + 10 = 50\mu F$$

$$C_{series} = \frac{40\times10}{40+10} = 8\ \mu F$$

2. A current of 5 A passes through a copper conductor (resistivity = 1.7×10^{-8} Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

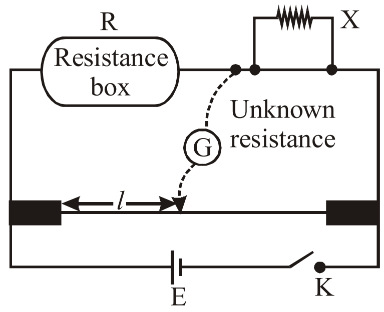
(1) 1.3 m^2/Vs (2) 1.5 m^2/Vs
(3) 1.8 m^2/Vs (4) 1.0 m^2/Vs

Sol. $\mu = \frac{V_d}{E}$ $\qquad E = \rho J$

$$= \frac{1.1\times10^{-3}}{1.7\times10^{-8}\times\frac{5}{\pi\times25\times10^{-6}}}$$

$$= \frac{1.1\times10^{-3}\times\pi\times25\times10^{-6}}{1.7\times10^{-8}\times5} \approx 1.01 m^2/Vs$$

3. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure



SI. No.	R(Ω)	l(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

(1) 4 (2) 1 (3) 2 (4) 3

Sol. as $x = \frac{R(100-\ell)}{\ell}$

for (1) $\quad x = \frac{1000\times(100-60)}{40} \approx 667$

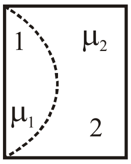
for (2) $\quad x = \frac{100\times(100-13)}{13} \approx 669$

for (3) $\quad x = \frac{10\times(100-1.5)}{98.5} \approx 656$

for (4) $\quad x = \frac{1\times(100-1)}{1} \approx 95$

So option (4) is completely different hence correct Ans. (4)

4. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\dfrac{R}{2-(\mu_1-\mu_2)}$ (2) $\dfrac{2R}{\mu_1-\mu_2}$

(3) $\dfrac{R}{2(\mu_1-\mu_2)}$ (4) $\dfrac{R}{\mu_1-\mu_2}$

Sol. For 1st lens $\dfrac{1}{f_1}=\left(\dfrac{\mu_1-1}{1}\right)\left(\dfrac{1}{\infty}-\dfrac{1}{-R}\right)=\dfrac{\mu_1-1}{R}$

for 2nd lens $\dfrac{1}{f_2}=\left(\dfrac{\mu_2-1}{1}\right)\left(\dfrac{1}{-R}-0\right)=-\dfrac{\mu_2-1}{R}$

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{f_{eq}}=\frac{R}{\mu_1-1}+\frac{R}{-(\mu_2-1)}\Rightarrow\frac{1}{f_{eq}}=\frac{R}{\mu_1-\mu_2}$$

Hence $f_{eq}=\dfrac{\mu_1-\mu_2}{R}$

5. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma\upsilon^2$ (where m is mass of the ball, υ is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

(1) $\dfrac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\dfrac{\gamma}{g}}V_0\right)$

(2) $\dfrac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\dfrac{\gamma}{g}}V_0\right)$

(3) $\dfrac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\dfrac{2\gamma}{g}}V_0\right)$

(4) $\dfrac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\dfrac{\gamma}{g}}V_0\right)$

Sol. $-(g+\gamma v^2)=\dfrac{dv}{dt}$

$$-gdt=\frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma}+v^2}\right)$$

Integrating $0\to t$ & $V_0\to 0$:-

$$-gt=-\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t=\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

6. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]

(1) 748 J (2) 374 J (3) 350 J (4) 700 J

Sol. $\Delta Q=nC_V\,\Delta T=n\dfrac{3}{2}R\Delta T$

$$=\left(\frac{67.2}{22.4}\right)\left(\frac{3}{2}\times 8.31\right)(20)$$

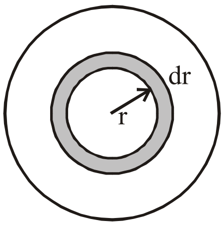
$\approx$ 748 J

7. A thin disc of mass M and radius R has mass per unit area $\sigma(r)=kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\dfrac{MR^2}{6}$ (2) $\dfrac{MR^2}{3}$

(3) $\dfrac{2MR^2}{3}$ (4) $\dfrac{MR^2}{2}$

Sol.



$$I_{Disc} = \int_0^R (dm)r^2 \Rightarrow I_{Disc} = \int_0^R (\sigma 2\pi r dr) r^2$$

$$I_{Disc} = \int_0^R (kr^2 2\pi r dr) r^2 \qquad \text{Mass of disc}$$

$$I_{Disc} = 2\pi k \int_0^R r^5 dr \qquad M = \int_0^R 2\pi r dr\ kr^2$$

$$I_{Disc} = 2\pi k \left(\frac{r^6}{6}\right)_0^R \qquad M = 2\pi k \int_0^R r^3 dr$$

$$I_{Disc} = 2\pi k \frac{R^6}{6} \qquad M = 2\pi k \left.\frac{r^4}{4}\right|_0^R$$

$$I_{Disc} = \frac{\pi k R^6}{3} = \left(\frac{\pi k R^4}{2}\right)\frac{R^2 2}{3} \qquad M = 2\pi k \frac{R^4}{4}$$

$$I_{Disc} = \frac{M2R^2}{3}$$

$$I_{Disc} = \frac{2}{3}MR^2$$

8. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

(1) $\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $\frac{I_1\omega_1^2}{24}$

Sol. $E_i = \frac{1}{2}I_1 \times \omega_1^2 + \frac{1}{2}\frac{I_1}{2} \times \frac{\omega_1^2}{4}$

$$= \frac{I_1\omega_1^2}{2}\left(\frac{9}{8}\right) = \frac{9}{16}I_1\omega_1^2$$

$$I_1\omega_1 + \frac{I_1\omega_1}{4} = \frac{3I_1}{2}\omega$$

$$\frac{5}{4}I_1\omega_1 = \frac{3I_1}{2}\omega$$

$$\omega = \frac{5}{6}\omega_1$$

$$E_f = \frac{1}{2} \times \frac{3I_1}{2} \times \frac{25}{36}\omega_1^2$$

$$= \frac{25}{48}I_1\omega_1^2$$

$$\Rightarrow E_f - E_i = I_1\omega_1^2\left(\frac{25}{48} - \frac{9}{16}\right) = \frac{-2}{48}I_1\omega_1^2$$

$$= \frac{-I_1\omega_1^2}{24}$$

9. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is :

(1) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$

(2) $+my_0 a\,\omega_1^2\hat{k}$

(3) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(4) Zero

Sol. $F = -m\ (a\omega_1^2\cos\omega_1 t\hat{i} + b\omega_2^2\sin\omega_2 t\hat{j})$

$$\vec{r} = (x_0 + a\cos\omega_1 t)\hat{i} + (y_0 + b\sin\omega_2 t)\hat{j}$$

$$\vec{T} = \vec{r} \times \vec{F} = -m(x_0 + a\cos\omega_1 t)b\omega_2^2\sin\omega_2 t\,\hat{k}$$

$$+m(y_0 + b\sin\omega_2 t)a\omega_1^2\cos\omega_1 t\,\hat{k}$$

$$= ma\omega_1^2 y_0\,\hat{k}$$

10. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicualr to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(1) $r_e > r_p > r_{He}$ (2) $r_e < r_p < r_{He}$

(3) $r_e < r_p = r_{He}$ (4) $r_e > r_p = r_{He}$

Sol. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$

$r_{He} = r_p > r_e$

11. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0 \hat{i} \cos(kz)\cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by :

(1) $\vec{B} = \dfrac{E_0}{C}\hat{j}\sin(kz)\cos(\omega t)$

(2) $\vec{B} = \dfrac{E_0}{C}\hat{j}\sin(kz)\sin(\omega t)$

(3) $\vec{B} = \dfrac{E_0}{C}\hat{k}\sin(kz)\cos(\omega t)$

(4) $\vec{B} = \dfrac{E_0}{C}\hat{j}\cos(kz)\sin(\omega t)$

Sol. $\because \vec{E} \times \vec{B} \parallel \vec{v}$

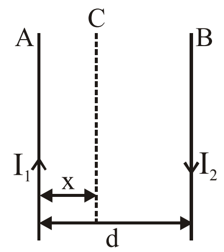
Given that wave is propagating along positive z-axis and $\vec{E}$ along positive x-axis. Hence $\vec{B}$ along y-axis.

From Maxwell equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

i.e. $\dfrac{\partial E}{\partial Z} = -\dfrac{\partial B}{dt}$ and $B_0 = \dfrac{E_0}{C}$

12. Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are :



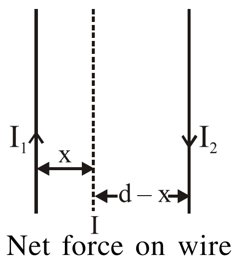
(1) $x = \left(\dfrac{I_1}{I_1 - I_2}\right)d$ and $x = \dfrac{I_2}{(I_1 + I_2)}d$

(2) $x = \pm\dfrac{I_1 d}{(I_1 - I_2)}$

(3) $x = \left(\dfrac{I_1}{I_1 + I_2}\right)d$ and $x = \dfrac{I_2}{(I_1 - I_2)}d$

(4) $x = \left(\dfrac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right)d$

Sol.



Net force on wire carrying current I per unit length is

$$\frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi(d-x)} = 0$$

$$\frac{I_1}{x} = \frac{I_2}{x-d}$$

$$\Rightarrow x = \frac{I_1 d}{I_1 - I_2}$$

13. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

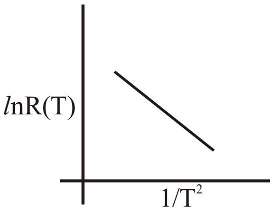
(1) 4; 1×10^8 Hz (2) 0.25; 1×10^8 Hz
(3) 4; 2×10^8 Hz (4) 0.25; 2×10^8 Hz

Sol. $f_m = 100$ MHz $= 10^8$ Hz, $(V_m)_0 = 100V$
$f_c = 300$ GHz , $(V_c)_0 = 400V$

Modulaton Index $= \dfrac{(V_m)_0}{(V_c)_0} = \dfrac{100}{400} = \dfrac{1}{4} = 0.25$

Upper band frequency (UBF) $= f_c + f_m$
Lower band frequency (LBF) $= f_c - f_m$
$\therefore$ UBF – LBF $= 2f_m = 2 \times 10^8$ Hz

14. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. One may conclude that :



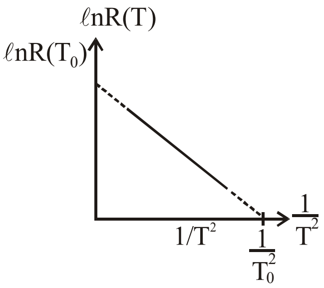
(1) $R(T) = \dfrac{R_0}{T^2}$

(2) $R(T) = R_0 e^{-T^2/T_0^2}$

(3) $R(T) = R_0 e^{-T_0^2/T^2}$

(4) $R(T) = R_0 e^{T^2/T_0^2}$

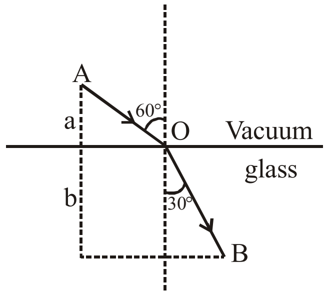
Sol. $$\frac{\frac{1}{T^2}}{\frac{1}{T_0^2}} + \frac{\ell n(T)}{\ell nR(T_0)} = 1$$



$\Rightarrow \ell nR(T) = [\ell nR(T_0)]\left(1 - \dfrac{T_0^2}{T^2}\right)$

$\Rightarrow R(T) = R_0 e^{\left(-\frac{T_0^2}{T^2}\right)}$

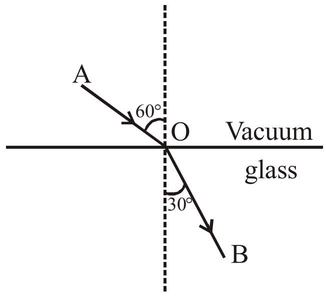
15. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :



(1) $2a + 2b$ (2) $2a + \dfrac{2b}{3}$

(3) $\dfrac{2\sqrt{3}}{a} + 2b$ (4) $2a + \dfrac{2b}{\sqrt{3}}$

Sol. From Snell's law
$1.\sin 60° = \mu \sin 30°$



$\Rightarrow \mu = \sqrt{3}$

Optical path = AO + μ(OB)

$= \dfrac{a}{\cos 60°} + \sqrt{3}\dfrac{b}{\cos 30°}$

$= 2a + 2b$

16. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are :

(1) 220 V and 10A
(2) 440 V and 5A
(3) 440 V and 20 A
(4) 220 V and 20 A

Sol. Given $N_P = 300$, $N_s = 150$, $P_0 = 2200W$
$I_s = 10A$
$P_0 = V_0 I_0 \Rightarrow 2200 = V_0 \times 10 \Rightarrow V_0 = 220V$

$\because \dfrac{V_i}{V_0} = \dfrac{N_p}{N_s} \Rightarrow V_i = 2 \times 220 = 440V$

Also $P_0 = V_i I_i$

$\Rightarrow I_i = \dfrac{2200}{440} = 5A$

17. In a photoelectric effect experiment the threshold wavelength of the light is 380 nm. If the wavelentgh of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be:

Given E (in eV) = $\frac{1237}{\lambda(\text{in nm})}$

(1) 1.5 eV (2) 4.5 eV
(3) 15.1 eV (4) 3.0 eV

Sol. $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$\Rightarrow K_{max} = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$

$\Rightarrow K_{max} = (1237)\left(\frac{380-260}{380\times 260}\right)$

$= 1.5$ eV

18. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-01.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(1) 13 s (2) 7 s (3) 27 s (4) 4 s

Sol. $A = A_0 e^{-0.1t} = \frac{A_0}{2}$

$ln2 = 0.1t$

$t = 10\, ln2 = 6.93 \approx 7$ sec

19. A moving coil galvanometer allows a full scale current of 10^{-4}A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is :

(1) 200 Ω (2) 100 Ω
(3) 10 Ω (4) 500 Ω

Sol. $200 + 10^{-4}\ G = 5$

$G = -ve$

So answer is Bonus

20. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s)

(1) 16, 14 (2) 12, 18
(3) 12, 16 (4) 8, 18

Sol. $f = 480 = \frac{300 - v_1}{300}\times 500$

$\frac{1440}{5} = 300 - v_1$

$v_1 = \frac{60}{5} = 12\text{m/s}$

$530 = \frac{300 + v_2}{300}\times 500$

$1590 = 1500 + 5v_2$

$5v_2 = 90$

$v_2 = 18$ m/s

21. Two radioactive materials A and B have decay constants 10λ and λ, respectively. It initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time :

(1) $\frac{11}{10\lambda}$ (2) $\frac{1}{9\lambda}$

(3) $\frac{1}{10\lambda}$ (4) $\frac{1}{11\lambda}$

Sol. $N_1 = N_0 e^{-10\lambda t}$

$N_2 = N_0 e^{-\lambda t}$

$\frac{1}{e} = \frac{N_1}{N_2} = e^{-9\lambda t}$

$\Rightarrow 9\lambda t = 1$

$t = \frac{1}{9\lambda}$

22. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is 10 kΩ. The common emitter current gain β is :

(1) 60 (2) 10^4

(3) 6×10^2 (4) 10^2

Sol. $A_v \times \beta = P_{gain}$

$$60 = 10\log_{10}\left(\frac{P}{P_0}\right)$$

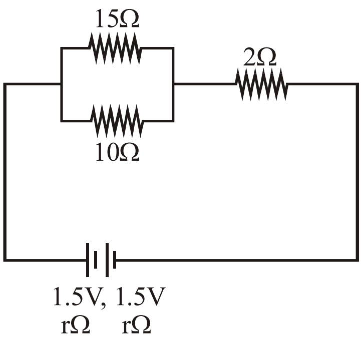
$$P = 10^6 = \beta^2 \times \frac{R_{out}}{R_{in}}$$

$$= \beta^2 \times \frac{10^4}{100}$$

$\beta^2 = 10^4$

$\beta = 100$

23. In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2V. The internal resistance r, of each cell is :



(1) 1Ω (2) 1.5Ω

(3) 0Ω (4) 0.5Ω

Sol. $R_{eq} = \frac{15\times 10}{25} + 2 + 2r$

$= 8 + 2r$

$$i = \frac{3}{8+2r}$$

$$2 = i\,R_{eq} = \frac{3}{8+2r}\times 6$$

$16 + 4r = 18$

$\Rightarrow r = 0.5\ \Omega$

24. A 25×10^{-3} m³ volume cylinder is filled with 1 mol of O_2 gas at room temperature (300K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ?

(1) $\sim 10^{11}$ (2) $\sim 10^{13}$

(3) $\sim 10^{10}$ (4) $\sim 10^{12}$

Sol. $\nu = \frac{V_{av}}{\lambda}$

$$\lambda = \frac{RT}{\sqrt{2}\pi\sigma^2 N_A P}$$

$\sigma = 2 \times .3 \times 10^{-9}$

$$P = \frac{RT}{V}$$

$$\Rightarrow \lambda = \frac{V}{\sqrt{2}\pi\sigma^2 N_A}$$

$$V_{av} = \sqrt{\frac{8}{3\pi}} \times V_{rms}$$

$$\therefore\ \nu = \frac{200 \times \sqrt{2}\pi \times \sigma^2 N_A}{25 \times 10^{-3}} \times \sqrt{\frac{8}{3\pi}}$$

$= 17.68 \times 10^8$/sec.

$= .1768 \times 10^{10}$/sec. $\sim 10^{10}$

This answer does not match with JEE-Answer key

25. n moles of an ideal gas with constant volume heat capcity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(1) $\frac{4nR}{C_V - nR}$ (2) $\frac{nR}{C_V - nR}$

(3) $\frac{nR}{C_V + nR}$ (4) $\frac{4nR}{C_V + nR}$

Sol. $w = nR\Delta T$

$\Delta H = (C_v + nR)\ \Delta T$

$$\frac{w}{\Delta H} = \frac{nR}{C_v + nR}$$

26. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed u at z = 4a. The minimum value of u such that it crosses the origin is :

(1) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(2) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(3) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(4) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Sol. $U_i + K_i = U_f + K_f$

$$\frac{kq^2}{\sqrt{16a^2+9a^2}}+\frac{1}{2}mv^2=\frac{kq^2}{3a}$$

$$\frac{1}{2}mv^2=\frac{kq^2}{a}\left(\frac{1}{3}-\frac{1}{5}\right)=\frac{2kq^2}{15a}$$

$$v=\sqrt{\frac{4kq^2}{15ma}}$$

27. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m)

(1) 1.6×10^6 m
(2) 6.4×10^6 m
(3) 9.0×10^6 m
(4) 2.6×10^6 m

Sol. $$\frac{GM}{(R+h)^2}=\frac{GM}{2R^2}$$

$R + h = \sqrt{2}R$

$h = (\sqrt{2}-1)R$

$\simeq 2.6\times10^6 m$

28. Given below in the the left column are different modes of communication using the kinds of waves given the right column.

A.	Optical Fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

(1) A-S, B-Q, C-R, D-P

(2) A-R, B-P, C-S, D-Q

(3) A-Q, B-S, C-R, D-P

(4) A-Q, B-S, C-P, D-R

Sol. Conceptual

29. The radtio of surface tensions of mercury and water is given to be 7.5 while the ratio of thier densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

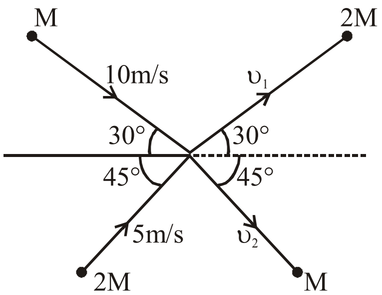
(1) 2/3
(2) 3/5
(3) 2/5
(4) 4/5

Sol. $$h=\frac{2S_1\cos\theta_1}{r_1\rho_1 g}$$

$$h=\frac{2S_2\cos\theta_2}{r_2\rho_2 g}$$

$$\Rightarrow \frac{r_1}{r_2}=\frac{2}{5}$$

30. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds υ_1 and υ_2, respectively. The values of υ_1 and υ_2 are nearly :



(1) 3.2 m/s and 6.3 m/s

(2) 3.2 m/s and 12.6 m/s

(3) 6.5 m/s and 6.3 m/s

(4) 6.5 m/s and 3.2 m/s

Sol. M × 10 cos 30° + 2M × 5 cos 45°

= 2M × v_1 cos 30° + M v_2 cos 45°

$$5\sqrt{3}+5\sqrt{2}=2v_1\frac{\sqrt{3}}{2}+\frac{v_2}{\sqrt{2}}$$

10 × M sin 30° – 2M × 5 sin 45°

= M v_2 sin 45° – 2M v_1 sin 30°

$$5-5\sqrt{2}=\frac{v_2}{\sqrt{2}}-v_1$$

Solving $v_1 = \dfrac{17.5}{2.7} \simeq 6.5\text{m / s}$

$v_2 \approx 6.3$ m/s

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

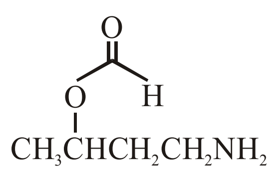
CHEMISTRY | TEST PAPER WITH ANSWER & SOLUTION

1. The major product of the following reaction is :

$$\underset{\quad\;\; |}{\overset{\quad\;\; OH}{CH_3CHCH_2CH_2NH_2}} \xrightarrow[\text{triethylamine}]{\text{ethyl formate (1equiv.)}}$$

(1) $\overset{OH}{CH_3CHCH_2CH_2NHCHO}$

(2) $CH_3CH=CH–CH_2NH_2$

(3) 

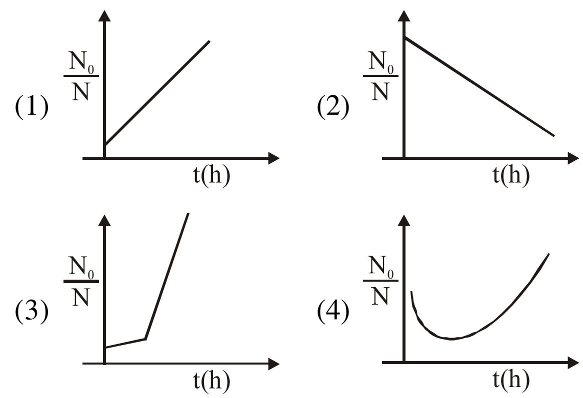
(4) $\overset{OH}{CH_3–CH–CH=CH_2}$

Sol. $\overset{OH}{CH_3–CH–CH_2–CH_2–NH_2} \xrightarrow[\text{triethylamine}]{\text{ethyl formate (1equiv.)}}$

$$\overset{OH}{CH_3–CH}–CH_2–CH_2–NH–\overset{O}{\overset{\|}{C}}–H$$

as NH_2 is a better nucleophile than OH.

2. A bacterial infection in an internal wound grows as $N'(t) = N_0\ \exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour ?



Sol. From 0 to 1 hour, $N' = N_0\ e^t$

From 1 hour onwards $\frac{dN}{dt} = -5N^2$

So at t = 1 hour, $N' = eN_0$

$$\frac{dN}{dt} = -5N^2$$

$$\int_{eN_0}^{N} N^{-2}\ dN = -5\int_1^t dt$$

$$\frac{1}{N} - \frac{1}{eN_0} = 5(t-1)$$

$$\frac{N_0}{N} - \frac{1}{e} = 5N_0\ (t-1)$$

$$\frac{N_0}{N} = 5N_0(t-1) + \frac{1}{e}$$

$$\frac{N_0}{N} = 5N_0t + \left(\frac{1}{e} - 5N_0\right)$$

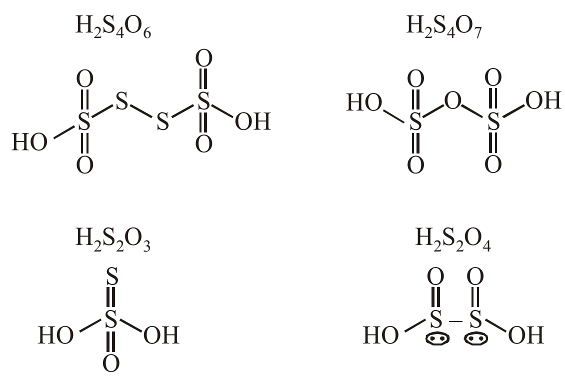
which is following y = mx + C

3. The correct order of catenation is :
(1) C > Si > Ge ≈ Sn (2) C > Sn > Si ≈ Ge
(3) Ge > Sn > Si > C (4) Si > Sn > C > Ge

Sol. As we move down the group, bond strength decreases, thereby decreasing the catenation tendency.
Hence the order is as expected
C > Si > Ge ≈ Sn

4. The oxoacid of sulphur that does not contain bond between sulphur atoms is :
(1) $H_2S_4O_6$ (2) $H_2S_2O_7$
(3) $H_2S_2O_3$ (4) $H_2S_2O_4$

Sol.



$H_2S_2O_7$ does not contain bond between sulphur atoms.

5. Consider the statements S1 and S2 :
S1 : Conductivity always increases with decrease in the concentration of electrolyte.
S2 : Molar conductivity always incrteases with decrease in the concentration of electrolyte.
The correct option among the following is :
(1) Both S1 and S2 are correct
(2) S1 is wrong and S2 is correct
(3) S1 is correct and S2 is wrong
(4) Both S1 and S2 are wrong

Sol. On dilution, no. of ions per ml decreases so conductivity decreases hence S1 is wrong.

$$\wedge_M = \frac{1000 \times \kappa}{C}$$

On dilution C and κ both decreases but effect of C is more dominating so $\wedge_M$ increases hence S2 is right.

6. Which of the following is a condensation polymer ?
(1) Buna - S (2) Nylon 6, 6
(3) Teflon (4) Neoprene

Sol. Nylon-6,6 is a condensation polymer of hexamethylene diamine and adipic acid.
Buna-S, Teflon and Neoprene are addition polymer.

7. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :
(1) C_4H_8 (2) C_4H_7Cl
(3) C_4H_{10} (4) C_4H_6

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$10 \qquad 10\left(x+\frac{y}{4}\right) \qquad 10x$

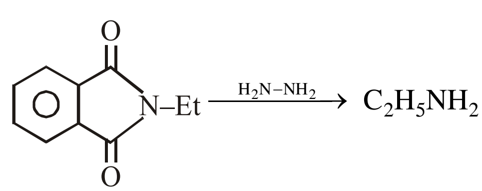
By given data, $10\left(x+\frac{y}{4}\right) = 55$ .... (1)

$10x = 40$ .... (2)

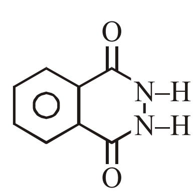
$\therefore$ x = 4, y = 6 ⇒ C_4H_6

8. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with :
(1) $NaBH_4$ (2) CaH_2
(3) H_2O (4) NH_2NH_2

Sol.



reagent is NH_2–NH_2 byproduct will be



9. The isoelectronic set of ions is :

(1) N^{3-}, Li^+, Mg^{2+} and O^{2-}

(2) Li^+, Na^+, O^{2-} and F^-

(3) F^-, Li^+, Na^+ and Mg^{2+}

(4) N^{3-}, O^{2-}, F^- and Na^+

Sol. $N^{-3} \rightarrow 1s^2\ 2s^2\ 2p^6$

$Li^+ \rightarrow 1s^2$

$Mg^{+2} \rightarrow 1s^2\ 2s^2\ 2p^6$

$O^{-2} \rightarrow 1s^2\ 2s^2\ 2p^6$

$F^- \rightarrow 1s^2\ 2s^2\ 2p^6$

$Na^+ \rightarrow 1s^2\ 2s^2\ 2p^6$

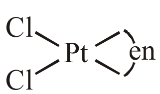
N^{-3}, O^{-2}, F^- and Na^+ are isoelectronic

10. The species that can have a trans-isomer is :

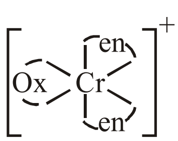
(en = ethane-1, 2-diamine, ox = oxalate)

(1) $[Pt(en)Cl_2]$ (2) $[Cr(en)_2(ox)]^+$

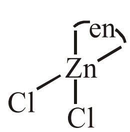
(3) $[Zn(en)Cl_2]$ (4) $[Pt(en)_2Cl_2]^{2+}$

Sol. (1) 

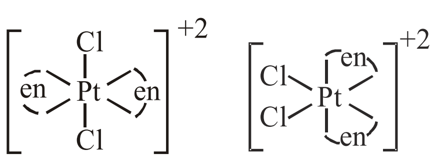
no trans isomer possible because bidentate ligand will be co-ordinating only at 90° angle in square planar complex

(2) 

no trans isomer possible

(3) 

sp^3 hybridized so no trans possible

(4) 

trans and cis both are possible

11. Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)	Column II (Metals)
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

(1) (I) – (b); (II) – (c); (III) – (d); (IV) – (a)

(2) (I) – (b); (II) – (d); (III) – (a); (IV) – (c)

(3) (I) – (c); (II) – (a); (III) – (b); (IV) – (d)

(4) (I) – (c); (II) – (d); (III) – (b); (IV) – (a)

Sol. Liquation is used for Sn.

Zone refining is used for Ga.

Mond's process is used for Ni.

Van arkel process is used for Zr.

12. Consider the following statements

(a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.

(b) Ionic product of water is temperature dependent.

(c) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.

(d) The Le Chatelier's principle is not applicable to common-ion effect.

the correct statement are :

(1) (a), (b) and (d) (2) (a), (b) and (c)

(3) (a) and (b) (4) (b) and (c)

Sol. (a) $H_2SO_4 \quad + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

$400 \times .1 = 40 \quad 400 \times .1 = 40$

$20 \qquad 0$

$\therefore [H^+] = \dfrac{20 \times 2}{800} = \dfrac{1}{20} \Rightarrow pH = -\log\left(\dfrac{1}{20}\right)$

$\therefore$ pH = 1.3 so (a) is correct

(b) $\log\left(\dfrac{Kw_2}{Kw_1}\right) = \dfrac{\Delta H}{2.303R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

so ionic product of water is temp. dependent hence (b) is correct.

(c) $K_a = 10^{-5}$, pH = 5 $\Rightarrow [H^+] = 10^{-5}$

$K_a = \dfrac{c\alpha^2}{(1-\alpha)} \Rightarrow K_a = \dfrac{[H^+].\alpha}{(1-\alpha)}$

$\therefore 10^{-5} = \dfrac{10^{-5}.\alpha}{(1-\alpha)} \Rightarrow 1 - \alpha = \alpha \Rightarrow \alpha = \dfrac{1}{2} = 50\%$

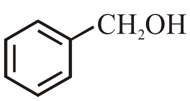
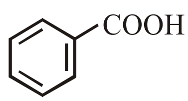
so (c) is correct.

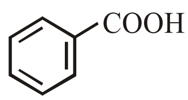
(d) Le-chatelier's principle is applicable to common –Ion effect so option (d) is wrong

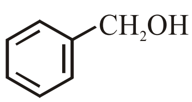
$\therefore$ correct answer (2)

13. Major products of the following reaction are :

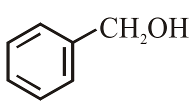
(1) CH_3OH and HCO_2H

(2)  and 

(3) CH_3OH and 

(4) HCOOH and 

Sol. 

This is cross cannizaro reaction so more reactive carbonyl compound is oxidized and less reactive is reduced so answer is  $+ HCO_2H$

14. The principle of column chromatography is :

(1) Capillary action.

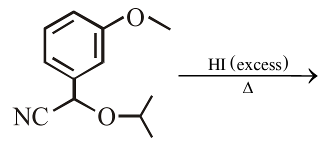
(2) Gravitational force.

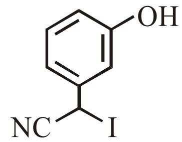
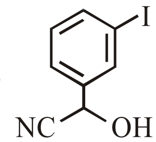
(3) Differential adsorption of the substances on the solid phase.

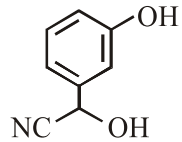
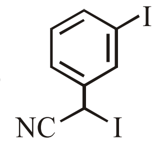
(4) Differential absorption of the substances on the solid phase.

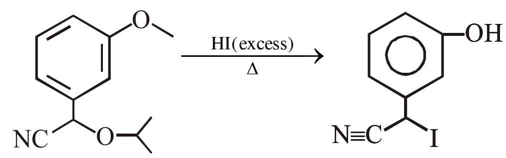
Sol. Main principle of column chromatography is differential adsorption of the substance on the solid phase.

15. The major product of the following reaction is :



(1)  (2) 

(3)  (4) 

Sol. 

Phenolic –OH does not react with HI and benzylic –O– having –CN attached will react with HI by S_N2 mechanism.

16. Amylopectin is composed of :

(1) α-D-glucose, C_1–C_4 and C_1–C_6 linkages

(2) α-D-glucose, C_1–C_4 and C_2–C_6 linkages

(3) β-D-glucose, C_1–C_4 and C_2–C_6 linkages

(4) β-D-Glucose, C_1–C_4 and C_1–C_6 linkages

Sol. Amylopetcin is a homopolymer of α-D-glucose where C_1–C_4 linkage and C_1 – C_6 linkage are present.

17. Consider the hydrates ions of Ti^{2+}, V^{2+}, Ti^{3+} and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(1) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

(2) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$

(3) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$

(4) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

Sol. $Ti^{+2} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^2$

unpaired electrons = 2.

spin only magnetic moment (μ) = $\sqrt{2(2+2)}$

$= \sqrt{8}$ B.M

$Ti^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1$

unpaired electrons = 1

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ B.M

$V^{+2} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3$

$\mu = \sqrt{3(3+2)} = \sqrt{15}$ B.M

$Sc^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

$\mu = 0$

18. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increases with :

(1) Decrease in p and decrease in T

(2) Increase in p and increase in T

(3) Increase in p and decrease in T

(4) Decrease in p and increase in T

Sol. Freundlich adsorption isotherm $\frac{x}{m} = Kp^{0.5}$

so on increasing pressure, $\frac{x}{m}$ increases

physical adsorption decreases with increase in temperature so option (3) is correct.

19. Three complexes,

$[CoCl(NH_3)_5]^{2+}$ (I),

$[Co(NH_3)_5H_2O]^{3+}$ (II) and

$[Co(NH_3)_6]^{3+}$ (III)

absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

(1) (III) > (I) > (II)　　(2) (I) > (II) > (III)

(3) (II) > (I) > (III)　　(4) (III) > (II) > (I)

Sol. A complex having strong field ligand has tendency to absorb light of highest energy. Among the three complexes.

$\left[Co(NH_3)_6\right]^{+3}$ will absorb radiation of highest energy and least wavelength.

$\left[Co(NH_3)_5H_2O\right]^{+3}$ has field weaker than the above compound and therefore absorb radiation of lesser energy and more wavelength.

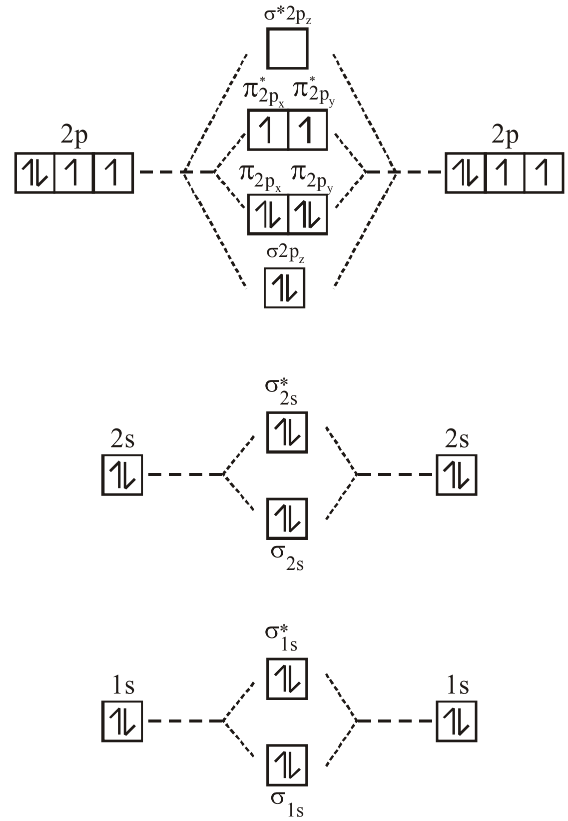
$\left[CoCl(NH_3)_5\right]^{+2}$ has the weakest field and therefore will absorb light of least energy and highest wavelength.

Strength of ligand $NH_3 > H_2O > Cl$.

20. During the change of O_2 to O_2^-, the incoming electron goes to the orbital :

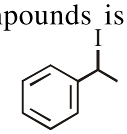
(1) $\sigma^*\ ^2P_z$　(2) $\pi\ ^2P_y$　(3) $\pi^*\ ^2P_x$　(4) $\pi\ ^2P_x$

Sol. Molecular orbital diagram of O_2 is

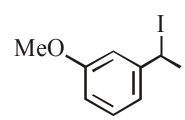


An incoming electron will go in $\pi^*_{2p_x}$ orbital.

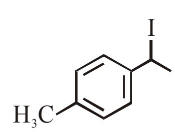
21. Increasing rate of S_N1 reaction in the following compounds is :



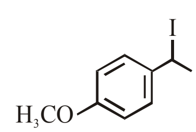
(A)



(B)



(C)



(D)

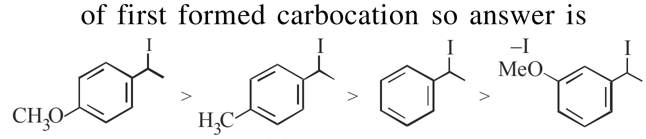
(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (D) < (C)

(3) (B) < (A) < (C) < (D)

(4) (A) < (B) < (D) < (C)

Sol. Rate of $S_{N'}$ is directly proposional to stability of first formed carbocation so answer is



(D)　(C)　(A)　(B)

22. Consider the following table :

Gas	$a/(k\ Pa\ dm^6\ mol^{-1})$	$b/(dm^3\ mol^{-1})$
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are vander waals constant. The correct statement about the gases is :

(1) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(2) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

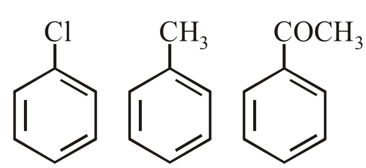
(3) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

(4) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

Sol.
- Gas A and C have same value of 'b' but different value of 'a' so gas having higher value of 'a' have more force of attraction so molecules will be more closer hence occupy less volume.
- Gas B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

so option (3) is correct.

23. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :-

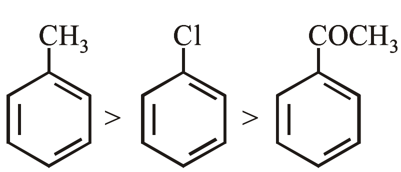


(I) (II) (III)

(1) I < III < II (2) II < I < III

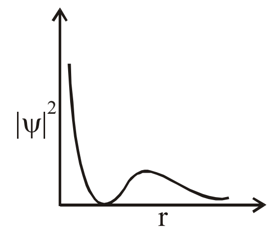
(3) III < I < II (4) III < II < I

Sol. Rate of aromatic electophilic substitution is



(II) (I) (III)

24. The graph betweeen $|\psi|^2$ and r(radial distance) is shown below. This represents :-



(1) 3s orbital (2) 1s orbital

(3) 2p orbital (4) 2s orbital

Sol. Graph of $|\psi^2|$ v/s r, touches r axis at 1 point so it has one radial node and since at r = 0, it has some value so it should be for 's' orbital.

$\therefore\ n - \ell - 1 = 1$ where $\ell = 0 \Rightarrow n - 1 = 1$

$\therefore\ n = 2 \Rightarrow$ '2s' orbital

25. At room temperature, a dilute soluton of urea is prepared by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be (molar mass of urea = 60 g mol^{-1}):-

(1) 0.027 mmHg (2) 0.028 mmHg

(3) 0.017 mmHg (4) 0.031 mmHg

Sol. Lowering of vapour pressure = $p^0 - p = p^0.x_{solute}$

$$\therefore\ \Delta p = 35 \times \frac{0.6/60}{\frac{0.6}{60} + \frac{360}{18}}$$

$$= 35 \times \frac{.01}{.01+20} = 35 \times \frac{.01}{20.01}$$

$= .017$ mm Hg

26. The synonym for water gas when used in the production of methanol is :-

(1) natural gas (2) laughing gas

(3) syn gas (4) fuel gas

Sol. water gas = $CO + H_2$

is also called syn gas because it is used for synthesis of methanol.

27. A process will be spontaneous at all temperatures if :-

(1) $\Delta H > 0$ and $\Delta S < 0$

(2) $\Delta H < 0$ and $\Delta S > 0$

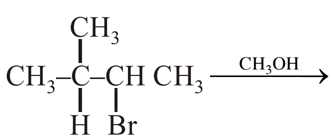
(3) $\Delta H > 0$ and $\Delta S > 0$

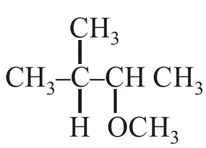
(4) $\Delta H < 0$ and $\Delta S < 0$

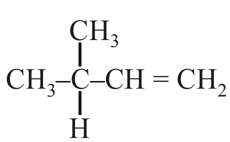
Sol. $\Delta G = \Delta H - T\Delta S$

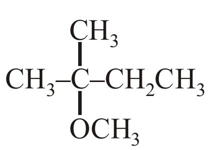
for spontaneous process at all temp. $\Delta G < 0$ and it is possible when $\Delta H < 0$ and $\Delta S > 0$.

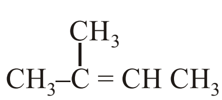
28. The major product of the following reaction is :-

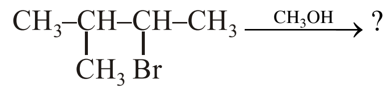


(1) 

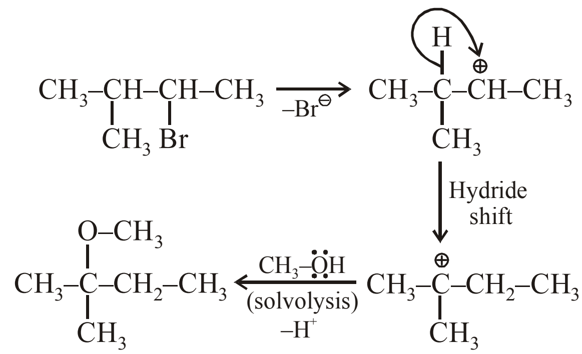
(2) 

(3) 

(4) 

Sol. 

In polar protic solvent S_{N^1} mechanism is favourable hence reaction complete via S_{N^1} mechanism



29. The regions of the atmosphere, where clouds form and where we line respectively, are :-

(1) Stratosphere and Troposphere

(2) Troposphere and Stratosphere

(3) Troposphere and Troposphere

(4) Stratosphere and Stratosphere

Sol. Troposphere is the lowest region of atmosphere bounded by Earth beneath and the stratosphere above where most of the clouds form and where life form exists.

30. The alloy used in the construction of aircrafts is :-

(1) Mg – Sn (2) Mg – Mn

(3) Mg – Al (4) Mg – Zn

Sol. Mg – Al alloy is used for construction of aircrafts.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS | TEST PAPER WITH ANSWER & SOLUTION

1. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2 (3) 3.0 (4) 2.5

Sol. $\sum f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$$

2. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

(2) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 - \Delta_2 = -2x^3$

(4) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

Sol. $\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$

and $\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$

So $\Delta_1 + \Delta_2 = -2x^3$

3. If $\lim\limits_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x\to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is :

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

Sol. $\lim\limits_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x\to k} \frac{x^3 - k^3}{x^2 - k^2}$

$\Rightarrow \lim\limits_{x\to 1} (x+1)(x^2+1) = \frac{k^2 + k^2 + k^2}{2k}$

$\Rightarrow k = 8/3$

4. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 12 (2) 10 (3) 9 (4) 7

Sol. $x + 3y + \lambda z - \mu = p(x + y + z - 5) + q(x + 2y + 2z - 6)$

on comparing the coefficient;

$p + q = 1$ and $p + 2q = 3$

$\Rightarrow (p,q) = (-1,2)$

Hence $x + 3y + \lambda z - \mu = x + 3y + 3z - 7$

$\Rightarrow \lambda = 3, \mu = 7$

5. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(1) exactly two values of K

(2) exactly one value of K

(3) no value of K.

(4) infinitely many values of K

Sol. Equation of common chord

$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$ ....(1)

and given line is $4x + 5y - k = 0$ .....(2)

On comparing (1) & (2), we get

$$k=\frac{1}{10}=\frac{k+\frac{1}{2}}{-k}$$

$\Rightarrow$ No real value of k exist

6. Le $f(x)=x^2$, $x\in R$. For any $A\subseteq R$, define $g(A)=\{x\in R, f(x)\in A\}$. If $S=[0,4]$, then which one of the following statements is not true ?

(1) $f(g(S))\neq f(S)$ (2) $f(g(S))=S$
(3) $g(f(S))=g(S)$ (4) $g(f(S))\neq S$

Sol. $g(S)=[-2,2]$
So, $f(g(S))=[0,4]=S$
And $f(S)=[0,16]\Rightarrow f(g(S)\neq f(S)$
Also, $g(f(S))=[-4,4]\neq g(S)$
So, $g(f(S)\neq S$

7. Let $f(x)=e^x-x$ and $g(x)\ x^2-x$, $\forall x\in R$. Then the set of all $x\in R$, where the function $h(x)=(fog)(x)$ is increasing, is :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$

(3) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$ (4) $[0,\infty)$

Sol. $h(x)=f(g(x))$
$\Rightarrow h'(x)=f'(g(x)).\ g'(x)$ and $f'(x)=e^x-1$
$\Rightarrow h'(x)=(e^{g(x)}-1)\ g'(x)$

$\Rightarrow h'(x)=\left(e^{x^2-x}-1\right)(2x-1)\geq 0$

Case-I $e^{x^2-x}\geq 1$ and $2x-1\geq 0$

$\Rightarrow x\in[1,\infty)$ ......(1)

Case-II $e^{x^2-x}\leq 1$ and $2x-1\leq 0$

$\Rightarrow x\in\left[0,\frac{1}{2}\right]$.....(2)

Hence, $x\in\left[0,\frac{1}{2}\right]\cup[1,\infty)$

8. Which one of the following Boolean expressions is a tautology ?

(1) $(P\vee q)\wedge(\sim p\vee\sim q)$ (2) $(P\wedge q)\vee(p\wedge\sim q)$

(3) $(P\vee q)\wedge(p\vee\sim q)$ (4) $(P\vee q)\vee(p\vee\sim q)$

Sol. (1) $(p\vee q)\wedge(\sim p\vee\sim q)\equiv(p\vee q)\wedge\sim(p\wedge q)\rightarrow$ Not tautology (Take both p and q as T)
(2) $(p\wedge q)\vee(p\wedge\sim q)\equiv p\wedge(q\vee\sim q)\equiv p\wedge t\equiv p$
(3) $(p\vee q)\wedge(p\vee\sim q)\equiv p\vee(q\wedge\sim q)\equiv p\vee c\equiv p$
(4) $(p\vee q)\vee(p\vee\sim q)\equiv p\vee(q\vee\sim q)\equiv p\vee t\equiv t$

9. All the pairs (x, y) that satisfy the inequality

$2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\leq 1$ also satisfy the eauation.

(1) $\sin x=|\sin y|$ (2) $\sin x=2\sin y$
(3) $2|\sin x|=3\sin y$ (4) $2\sin x=\sin y$

Sol. $2^{\sqrt{\sin^2x-2\sin x+5}}\ .\ 4^{-\sin^2y}\leq 1$

$\Rightarrow 2^{\sqrt{(\sin x-1)^2+4}}\leq 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\leq 2\sin^2 y$

$\Rightarrow$ sinx=1 and |siny| =1

10. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :
(1) 36 (2) 60 (3) 48 (4) 72

Sol. Sum of given digits 0, 1, 2, 5, 7, 9 is 24.
Let the six digit number be abcdef and to be divisible by 11
so $|(a+c+e)-(b+d+f)|$ is multiple of 11.
Hence only possibility is $a+c+e=12=b+d+f$
Case-I $\{a, c, e\}=\{9, 2, 1\}$ & $\{b, d, f\}=\{7, 5, 0\}$
So, Number of numbers $=3!\times 3!=36$
Case-II $\{a,c,e\}=\{7,5,0\}$ and $\{b,d,f\}=\{9,2,1\}$
So, Number of numbers $2\times 2!\times 3!=24$
Total = 60

11. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$ (3) $\frac{1}{10}$ (4) $\frac{1}{12}$

Sol. P(B) = P(G) = 1/2

Required Proballity =

$$\frac{\text{all 4 girls}}{(\text{all 4 girls}) + (\text{exactly 3 girls} + 1\text{boy}) + (\text{exactly 2 girls} + 2\text{boys})}$$

$$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4} = \frac{1}{11}$$

12. The sum

$$\frac{3\times 1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + ...$$

(1) 660 (2) 620 (3) 680 (4) 600

Sol. $T_n = \frac{(3+(n-1)\times 2)(1^3+2^3+...+n^3)}{(1^2+2^2+...+n^2)}$

$$= \frac{3}{2}n(n+1) = \frac{n(n+1)(n+2)-(n-1)n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{2}$$

$\Rightarrow S_{10} = 660$

13. If a directrix of a hyperbola centred at the origin and passing through the point $(4,-2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 35 = 0$

(2) $4e^4 + 8e^2 - 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

Sol. Hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{a}{e} = \frac{4}{\sqrt{5}}$ and $\frac{16}{a^2} - \frac{12}{b^2} = 1$

$a^2 = \frac{16}{5}e^2$ ....(1) and $\frac{16}{a^2} - \frac{12}{a^2(e^2-1)} = 1$ ....(2)

From (1) & (2)

$$16\left(\frac{5}{16e^2}\right) - \frac{12}{(e^2-1)}\left(\frac{5}{16e^2}\right) = 1$$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

14. If $f(x) = \begin{cases} \frac{\sin(p+1)+\sin x}{x} & , x<0 \\ q & , x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , x>0 \end{cases}$

is continuous at x = 0, then the ordered pair (p,q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$

(3) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

Sol. $\text{RHL} = \lim_{x\to 0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}} = \lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x} = \frac{1}{2}$

$\text{LHL} = \lim_{x\to 0}\frac{\sin(p+1)x+\sin x}{x} = (p+1)+1 = p+2$

for continuity LHL = RHL = f(0)

$\Rightarrow (p,q) = \left(\frac{-3}{2},\frac{1}{2}\right)$

15. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $2 + \frac{1}{e}$ (2) $\frac{1}{2} - e$ (3) $e - 2$ (4) $\frac{1}{2} - e$

Sol. $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

Now, put $\tan x = t \Rightarrow \frac{dt}{dx} = \sec^2 x$

So $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2$

16. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus recturm of the ellipse is :

(1) 9 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 5

Sol. Tangent at $\left(3, -\frac{9}{2}\right)$

$$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$$

Comparing this with $x - 2y = 12$

$$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$$

we get $a = 6$ and $b = 3\sqrt{3}$

$$L(LR) = \frac{2b^2}{a} = 9$$

17. The value of $\int_0^{2\pi} [\sin 2x(1 + \cos 3x)]\,dx$, where $[t]$ denotes the greatest integer function, is :

(1) -2π (2) π (3) $-\pi$ (4) 2π

Sol. $I = \int_0^{2\pi} \left[\sin 2x\left(1 + \cos 3x\right)\right]dx$

$$I = \int_0^{\pi} \left(\left[\sin 2x + \sin 2x\cos 3x\right] + \left[-\sin 2x - \sin 2x\cos 3x\right]\right)dx$$

$$= \int_0^{\pi} -dx = -\pi$$

18. The region represented by $|x-y| \le 2$ and $|x+y| \le 2$ is bounded by a :

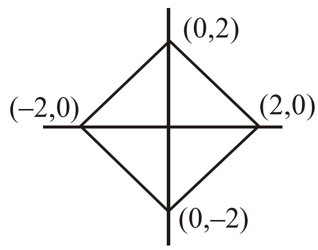
(1) square of side length $2\sqrt{2}$ units

(2) rhombus of side length 2 units

(3) square of area 16 sq, units

(4) rhombus of area $8\sqrt{2}$ sq. units

Sol. $|x-y| \le 2$ and $|x + y| \le 2$

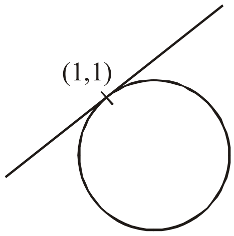


Square whose side is $2\sqrt{2}$

19. The line $x = y$ touches a circle at the point $(1, 1)$. If the circle also passes through the point $(1, -3)$, then its radius is :

(1) $3\sqrt{2}$ (2) 3 (3) $2\sqrt{2}$ (4) 2

Sol.



Equation of circle can be written as $(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

It passes through (1, –3)

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

So $(x-1)^2 + (y-1)^2 - 4(x-y) = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$

(correct key is 3)

20. Let A(3, 0, –1), B (2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos(\angle GOA)$ (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{30}}$ (2) $\frac{1}{6\sqrt{10}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{2\sqrt{15}}$

Sol. G is the centroid of Δ ABC

$G \equiv (2,4,2)$

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$$\cos(\angle GOA) = \frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$$

21. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at x = c, g is :

(1) differentiable if $f'(c) = 0$

(2) not differentiable

(3) differentiable if $f'(c) \neq 0$

(4) not differentiable if $f'(c) = 0$

Sol. $$g'(c) = \lim_{h\to 0} \frac{|f(c+h)| - |f(c)|}{h}$$

$$= \lim_{h\to 0} \frac{|f(c+h)|}{h} = \lim_{h\to 0} \frac{|f(c+h) - f(c)|}{h}$$

$$= \lim_{h\to 0} \left|\frac{f(c+h) - f(c)}{h}\right| \frac{|h|}{h}$$

$$= \lim_{h\to 0} |f'(c)| \frac{|h|}{h} = 0, \text{ if } f'(c) = 0$$

i.e., g(x) is differentiable at x = c, if f '(c) = 0

22. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then

$$\frac{\alpha^{12} + \beta^{12}}{\left(\alpha^{-12} + \beta^{-12}\right)(\alpha - \beta)^{24}}$$ is equal to :

(1) $\frac{2^6}{(\sin\theta + 8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta - 8)^6}$

(3) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$

Sol. $$\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$$

$$= \frac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2 - 4\alpha\beta\right]^{12}} = \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$$

$$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12} = \frac{2^{12}}{(\sin\theta + 8)^{12}}$$

23. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) –1 (2) 2 (3) –2 (4) 1

Sol. One of the point on line is P(0, 1, –1) and given point is Q(β, 0, β).

So, $\overrightarrow{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $$\beta^2 + 1 + (\beta+1)^2 - \frac{(\beta - \beta - 1)^2}{2} = \frac{3}{2}$$

$\Rightarrow 2\beta^2 + 2\beta = 0$

$\Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

24. If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration, then :

(1) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

Sol. $\int \frac{dx}{\left((x-1)^2+9\right)^2} = \frac{1}{27}\int \cos^2\theta\, d\theta$ (Put x–1= 3tanθ)

$= \frac{1}{54}\int (1+\cos 2\theta)d\theta = \frac{1}{54}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$

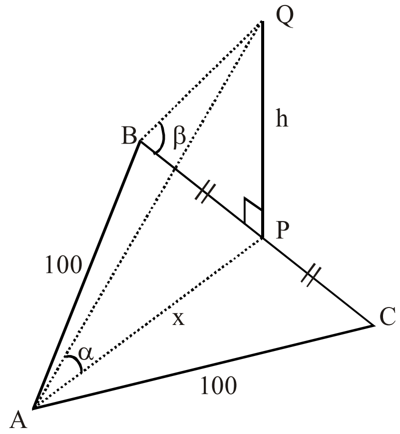
$= \frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right)+C$

25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$ (3) 20 (4) 25

Sol. $\cot\alpha = 3\sqrt{2}$

& $\operatorname{cosec}\beta = 2\sqrt{2}$



So, $\frac{x}{h} = 3\sqrt{2}$ ...(i)

And $\frac{h}{\sqrt{10^4-x^2}} = \frac{1}{\sqrt{7}}$ ...(ii)

So, from (i) & (ii)

$\Rightarrow \frac{h}{\sqrt{10^4-18h^2}} = \frac{1}{\sqrt{7}}$

$\Rightarrow 25h^2 = 100 \times 100$

$\Rightarrow h = 20.$

26. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98 (3) 76 (4) 64

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$

$\Rightarrow a_1 + a_{16} = 38$

So, $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 = 76$

27. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+......+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$

(2) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$

(4) $\frac{4}{3}(2)^{3/4}$

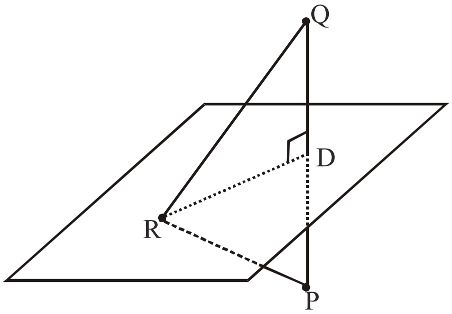
Sol. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3}$

$= \int_0^1 (1+x)^{1/3}dx = \frac{3}{4}\left(2^{4/3}-1\right)$

28. If Q(0, –1, –3) is the image of the point P in the plane 3x –y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

Sol. R lies on the plane.



DQ = $\frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

$\Rightarrow$ PQ = $\sqrt{26}$

Now, RQ = $\sqrt{9+1} = \sqrt{10}$

$\Rightarrow$ RD = $\sqrt{10-\frac{13}{2}} = \sqrt{\frac{7}{2}}$

Hence, ar(ΔPQR) = $\frac{1}{2}\times\sqrt{26}\times\sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$.

29. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (28, 315) (2) (–54, 315)
(3) (–21, 714) (4) (24, 861)

Sol. Coefiicient of x^2 = ${}^{15}C_2\times 9 - 3a\left({}^{15}C_1\right) + b = 0$

$\Rightarrow - 45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)

Also, $-27\times{}^{15}C_3 + 9a\times{}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24\, a + 672 = 0$

$\Rightarrow a = 28$

So, b = 315

30. If a > 0 and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{3}{5}-\frac{1}{5}i$ (2) $-\frac{1}{5}+\frac{3}{5}i$
(3) $-\frac{1}{5}-\frac{3}{5}i$ (4) $\frac{1}{5}-\frac{3}{5}i$

Sol. Given a > 0

$z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

Also $|z| = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

So $\bar{z} = \frac{-2i(3-i)}{10} = \frac{-1-3i}{5}$