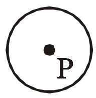
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. A perfectly dimagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is:



(1) Zero

(2) $\vec{B}$

(3) much large than $|\vec{B}|$ but opposite to $\vec{B}$

(4) much large than $|\vec{B}|$ and parallel to $\vec{B}$

Sol. A perfect diamagnetic substance will completely expel the magnetic field. Therefore, there will be no magnetic field inside the cavity of sphere. Hence the paramagnetic substance kept inside the cavity will experience no force.

2. The radius of R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of:

$$\left(M_{\text{prot.}} \cong M_{\text{neut.}} \simeq 1.67 \times 10^{-27}\,\text{kg}\right)$$

(1) 10^{24} kg m^{-3} (2) 10^{3} kg m^{-3}

(3) 10^{17} kg m^{-3} (4) 10^{10} kg m^{-3}

Sol.

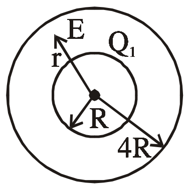
$$\rho_{\text{nucleus}} = \frac{\text{mass}}{\text{volume}} = \frac{A}{(4/3)\pi r_0^3 A} = \frac{3}{4\pi r_0^3} = 2.3 \times 10^{17}\,\text{kg}/\text{m}^3$$

3. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is:

(1) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (2) $\frac{Q_2}{4\pi\varepsilon_0 R}$

(3) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ (4) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

Sol.



$$E = \frac{KQ_1}{r^2}$$

$$\Delta V = \int_R^{4R} E\,dr = \frac{3KQ_1}{4R}$$

4. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to:

(1) 5 : 7 (2) 1 : 2

(3) 10 : 7 (4) 2 : 1

Sol. $q\Delta V = \frac{1}{2}mV^2 \Rightarrow v = \sqrt{\frac{2q\Delta V}{m}}$

$$\therefore \frac{V_1}{V_2} = \sqrt{\frac{e}{m}\,\frac{4m}{e}} = 2$$

5. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at:

(1) $r=\frac{1}{\sqrt{3}}R$ (2) $r=\sqrt{\frac{5}{9}}R$

(3) $r=\sqrt{\frac{3}{4}}R$ (4) r = R

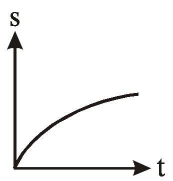
Sol. $E\ 4\pi r^2 = \int \rho_0 4\pi r^2 dr$

$$\Rightarrow Er^2 = 4\pi G\int_0^r \rho_0\left(1-\frac{r^2}{R^2}\right)r^2 dr$$

$$\Rightarrow E = 4\pi G\rho_0\left(\frac{r^3}{3}-\frac{r^5}{5R^2}\right)$$

$$\frac{dE}{dr}=0 \quad \therefore \quad r=\sqrt{\frac{5}{9}}\ R$$

6. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

(1)


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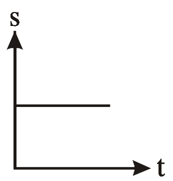

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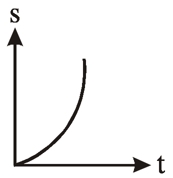

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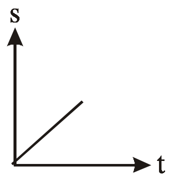


Sol. $\frac{dK}{dE}=P=\text{cost}\Rightarrow K=Pt=\frac{1}{2}mV^2$

$$\therefore \quad V=\sqrt{\frac{2Pt}{m}}=\frac{ds}{dt}\therefore S=\sqrt{\frac{2P}{m}}\frac{2}{3}t^{\frac{3}{2}}$$

7. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is:

Planck's constant $h = 6.63 \times 10^{-34}$ J.s.

Speed of ligh $c = 3 \times 10^8$ m/s

(1) 2.0 eV (2) 1.5 eV

(3) 3.1 eV (4) 1.1 eV

Sol. $\Delta E=\frac{\lambda c}{\lambda e}=3.1\text{eV}$

8. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal) ?

(1) 5 (2) 3 (3) 6 (4) 7

Sol. $nC_P(50)=160$

$nC_v\ (100) = 240$

$$\Rightarrow \frac{C_p}{2C_v}=\frac{160}{240}=\frac{\gamma}{2}$$

$$\therefore \ \gamma=\frac{4}{3} \text{ and } f=\frac{2}{\gamma-1}=6$$

9. A block of mass m attached to massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is:

(1) $\frac{1}{2}$ (2) $\sqrt{2}$ (3) 1 (4) $\frac{1}{\sqrt{2}}$

Sol. At equilibrium position

$$V_0 = \omega_0 A = \sqrt{\frac{K}{m}}A \qquad .....(i)$$

$$V = \omega A^1 = \sqrt{\frac{K}{\frac{m}{2}}}A^1 \qquad .....(ii)$$

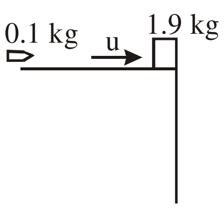
$$\therefore \quad A^1 = \frac{A}{\sqrt{2}}$$

10. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s². Assume there is no rotational motion and loss of energy after the collision is negligable.]

(1) 21 J (2) 23 J

(3) 19 J (4) 20 J

Sol.



$p_i = p_f \Rightarrow 0.1 \times 20 = 2v$

$\therefore$ v = 1 m/s

$KE_f = mgh + \frac{1}{2}mv^2 = 213$

11. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is:

(1) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (2) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$

(3) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$ (4) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$

Sol. $\Delta p = n_1 L_1 - n_2 L_2$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p$$

12. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is :

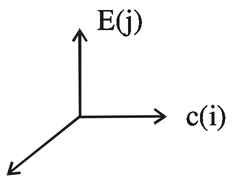
(1) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{j}$

(2) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{k}$

(3) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{k}$

(4) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{j}$

Sol.



$\therefore \vec{B}(\hat{k})$

$\Rightarrow \vec{B} = B_0 \cos(wt - kx)\hat{k}$

Now put t = 0.

13. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :

(1) 33°C

(2) 35°C

(3) 31°C

(4) 28°C

Sol. $\frac{50-40}{300} = \beta\left(\frac{50+40}{2} - 20\right)$

$$\frac{40-T}{300} = \beta\left(\frac{40+T}{2} - 20\right)$$

$$\therefore \ T = \frac{100}{3}$$

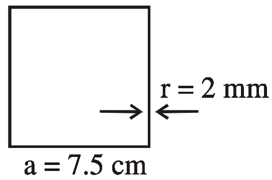
14. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}.The induced current in the loop is close to

(Resistivity of the metal wire is 1.23×10^{-8} Ωm)

(1) 0.61 A (2) 0.34 A

(3) 0.43 A (4) 0.53 A

Sol.



$$q_i = \frac{d(Ba^2)}{dt} = a^2\frac{dB}{dt}$$

$$i = \frac{q}{R} = \frac{a^2 dB/dt}{\frac{\rho(40)}{\pi r^2}}$$

15. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is:

(1) ML^2T^{-2} (2) MLT^{-2}

(3) $M^2L^0T^{-1}$ (4) ML^0T^{-3}

Sol. $S = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$

16. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?

(1) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

(2) Multimeter shows a deflection,accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

(3) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen compoenent is metal wire.

(4) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

Sol. (1) Multimeter shows deflection when it connects with capacitor

(2) If we assume that LED has negligiable resistance then multimeter shows no deflection for the forward blas but when it connects in reverse direction, it break down occurs so splash of light out.

(3) The resistance of metal wire may be taken zero, so no deflection in multimeter

(4) No matter, how we connect the resistance across multimeter It shows same deflection.

17. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400 Ω resistor is close to:

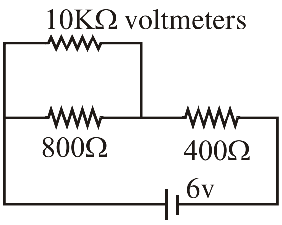
(1) 2 V

(2) 1.95V

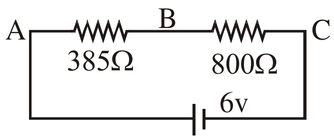
(3) 2.05 V

(4) 1.8 V

Sol.



$\Rightarrow$



So the potential difference in voltmeter across the points A and B is $\frac{6}{1185} \times 385 = 1.949$ V

18. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number densitty of photons of the visible light of the given wavelengths is :

(1) $\frac{1}{500}$

(2) 500

(3) 250

(4) $\frac{1}{250}$

Sol. $P = \frac{nhc}{\lambda t}$

$\therefore \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{5}$

19. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixure is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} $°C^{-1}$)

(1) 2.6

(2) 2

(3) 4

(4) 3.2

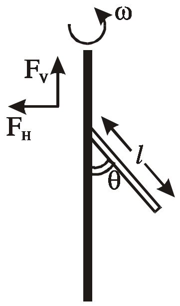
Sol.

Cal	H_2O	Sterm
20gm	180gm	m
25°C	25°C	100°C

$200 \times 1 \times (31 - 25)$

$= m \times 540 + m \times 1 \times (100 - 31)$

20.

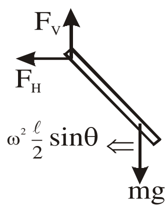


A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{m\ell^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:

(1) $\cos\theta = \frac{g}{2\ell\omega^2}$ (2) $\cos\theta = \frac{3g}{2\ell\omega^2}$

(3) $\cos\theta = \frac{2g}{3\ell\omega^2}$ (4) $\cos\theta = \frac{g}{\ell\omega^2}$

Sol.



$F_V = mg$

$F_H = m\omega^2\frac{\ell}{2}\sin\theta$

$$mg\frac{\ell}{2}\sin\theta - m\omega^2\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$$

$$\cos\theta = \frac{3}{2}\frac{g}{\omega^2\ell} \quad .....(ii)$$

21. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _______.

Sol. $$\left|\left(\frac{dv}{dt}\right)\right| = \left|\frac{v^2}{4^2}\right|\left|\frac{du}{dt}\right|$$

$$= \left(\frac{10}{30}\right)2\times 9 = 1m/s$$

22. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _______ .

Sol. $\vec{\tau} = \vec{m}\times\vec{B}$

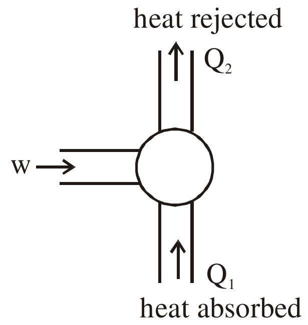
$\tau = NI \times A \times B$

$105 = 500 \times 3\times10^{-4} \times \frac{1}{2}\times B$

$B = 20$

23. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surrounding at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer?

Sol.



$w + Q_1 = Q_2$

$w = Q_2 - Q_1$

$$\text{C.O.P.} = \frac{Q_1}{w} = \frac{Q_1}{Q_2 - Q_1} = \frac{273}{300-273} = \frac{Q_1}{W}$$

$$w = \frac{27}{273} \times 80 \times 100 \times 4.2$$

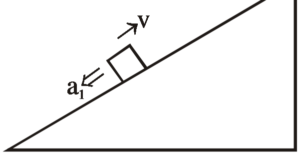
$Q_2 = w + \theta_1$

$$Q_2 = \frac{27}{273} \times 80 \times 100 \times 4.2 + 80 \times 100 \times 4.2$$

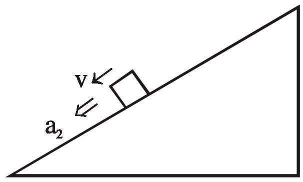
$$Q_2 = \frac{300}{273} \times 80 \times 100 = 8791.2 \text{ cal}$$

24. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, The nearest integer to I is __________ .

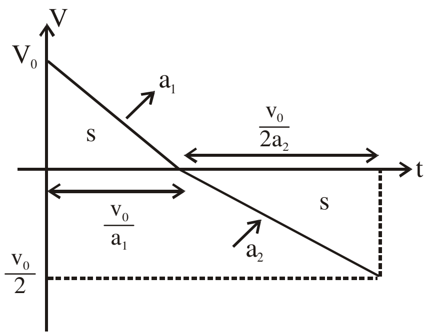
Sol.



$a_1 = g(\sin\theta + \mu \cos\theta)$



$a_2 = g(\sin\theta + \mu \cos\theta)$

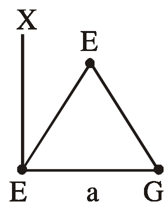


$$\therefore \frac{1}{2} v_0 \frac{v_0}{a_1} = \frac{1}{2}\left(\frac{v_0}{2}\right)\left(\frac{v_0}{2a_1}\right)$$

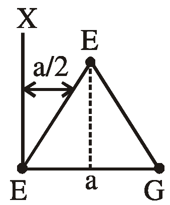
$\Rightarrow 3 \sin\theta = 5 \mu \cos\theta$

$\therefore \mu = \sqrt{3}/5$

25. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of intertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20} ma^2$ where N is an integer. The value of N is ________.



25.



$$I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$$

$$= \frac{5}{4} ma^2$$

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. Among the statements (I – IV), the correct ones are:

(I) Be has smaller atomic radius compared to Mg.

(II) Be has higher ionization enthalpy than Al.

(III) Charge/radius ratio of Be is greater than that of Al.

(IV) Both Be and Al form mainly covalent compounds.

(1) (I), (II) and (IV)

(2) (II), (III) and (IV)

(3) (I), (II) and (III)

(4) (I), (III) and (IV

Sol. I, A_N : Be < Mg

II IE : Be > Al

III Charge/radius ratio of Be w less than that of Al

IV Be, Al mainly form covalent compounds

2. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are:

(Take molar mass of hydrogen peroxide as 34 g/mol)

(1) 1.7 and 0.25 (2) 1.7 and 0.5

(3) 0.85 and 0.5 (4) 0.85 and 0.25

Sol. Volume strength = 11.2 × molarity

$$\Rightarrow \text{ molarity } = \frac{5.6}{11.2} = 0.5$$

Assuming 1 litre solution;

mass of solution = 1000 ml × 1 g/ml = 1000 g

mass of solute = moles × molar mass

= 0.5 mol × 34 g/mol

= 17 gm.

$$\Rightarrow \text{ mass\% } = \frac{17}{1000} \times 100 = 1.7\%$$

3. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2, ... n + 1, where n is the principal quantum number. Then, the element with atomic number :

(1) 13 has a half-filled valence subshell

(2) 9 is the first alkali metal

(3) 8 is the first noble gas

(4) 6 has a 2p-valence subshell

Sol. l = 0 to (n + 1)

n = 1 — l = 0, 1, 2

n = 2 — l = 0, 1, 2, 3

$$(n + l) \Rightarrow \frac{1s}{1}\ \frac{1p}{2}\ \frac{1d}{3} \qquad \frac{2s}{2}\ \frac{2p}{3}\ \frac{2d}{4}\ \frac{2f}{5}$$

n = 3

l = 0, 1, 2, 3, 4

$$\frac{3s}{3}\ \frac{3p}{4}\ \frac{3d}{5}\ \frac{3f}{6}\ \frac{3g}{7}$$

Now, in order to write electronic configuration, we need to apply (n + l) rule

Energy order : 1s < 1p < 2s < 1d < 2p < 3s < 2d...

Option 1) 13 : $1s^2 1p^6 2s^2 1d^3$ is not half filled

Option 2) 9 : $1s^2 1p^6 2s^1$ is the first alkali metal because after losing one electron, it will achieve first noble gas configuration

Option 3) 8 : $1s^2 1p^6$ is the first noble gas because after $1p^6$ e^- will enter 2s hence new period

Option 4) 6 : $1s^2\ 1p^4$ has 1p valence subshell.

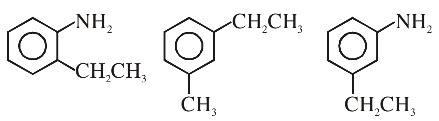
4. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results :

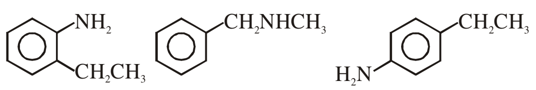
A and C $\xrightarrow{\text{Diazotization}}$ P + Q $\xrightarrow[\text{(ii) oxidation } (KMnO_4 + H^+)]{\text{(i) Hydrolysis}}$ R(product of A) + S(product of C)

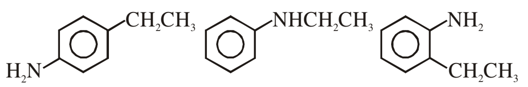
R has lower boiling point than S

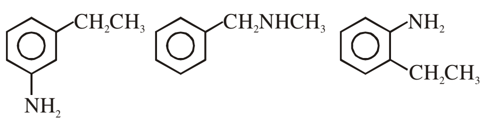
B $\xrightarrow{C_6H_5SO_2Cl}$ alkali-insoluble product

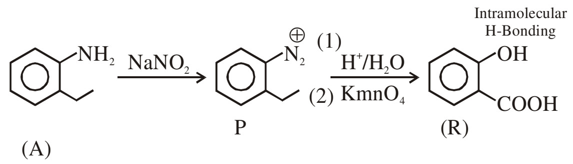
A, B and C, respectively are :

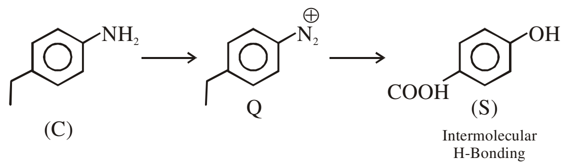
(1) 

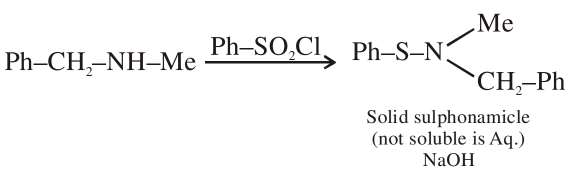
(2) 

(3) 

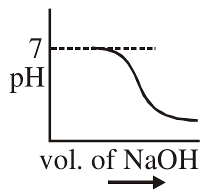
(4) 

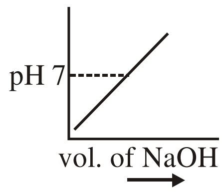
Sol. 

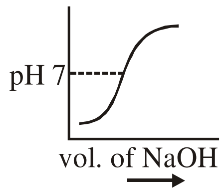


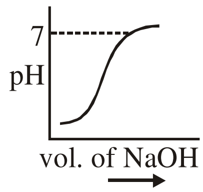


5. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?

(1) 

(2) 

(3) 

(4) 

Sol. Steep rise in pH around the equivalence point for titration of strong acid with strong base.

6. The incorrect statement(s) among (a) – (d) regarding acid rain is (are) :

(a) It can corrode water pipes.

(b) It can damage structures made up of stone.

(c) It cannot cause respiratory ailments in animals.

(d) It is not harmful for trees

(1) (c) and (d)

(2) (a), (b) and (d)

(3) (c) only

(4) (a), (c) and (d)

Sol. (1) Acid rain corrodes water pipes resulting in the leaching of heavy of heavy metals such as iron, lead and copper into the drinking water.

(2) Acid rain damages buildings and other structures made of stone or metal.

(3) It causes respiratory aliments in human beings and animals.

(4) It is harmful for agriculture, trees and plants as it wasshes down the nutrients needed for its growth.

7. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is :

(1) 2 (2) 3

(3) 4 (4) 5

Sol. Let suppose element X $\Rightarrow$

$$X_{(g)} \xrightarrow[800]{IE_1} \overset{+}{X}(g) \xrightarrow[2427]{IE_2} \overset{+2}{X}(g) \xrightarrow[3658]{IE_3}$$

$$\overset{+3}{X}(g) \xrightarrow[25024]{IE_4} \overset{+4}{X}(g) \xrightarrow[32824]{IE_5} \overset{+5}{X}(g)$$

X^{+3} has stable inert gas configuration as there is high jump after IE_3

So valence electrons are 3

8. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is :

(1) 14 atm (2) 22 atm

(3) 6 atm (4) 38 atm

Sol. According to Dalton's law of partial pressure

$p_i = x_i \times P_T$

p_i = partial pressure of the ith component

x_i = mole fraction of the ith component

p_T = total pressure of mixture

$$\Rightarrow \ 2 \text{ atm} = \left(\frac{n_{H_2}}{n_{H_2} + n_{H_e} + n_{O_2}}\right) \times p_T$$

$$\Rightarrow \ p_T = 2 \text{ atm} \times \frac{3}{1} = 6 \text{ atm}$$

9. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]Cl_2$, respectively are :

(1) $t_{2g}^4 e_g^2$ and $t_{2g}^6 e_g^0$

(2) $t_{2g}^6 e_g^0$ and $t_{2g}^6 e_g^0$

(3) $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$

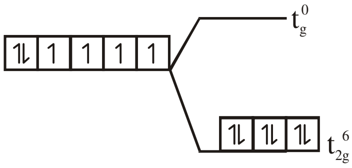
(4) $t_{2g}^4 e_g^2$ and $t_{2g}^4 e_g^2$

Sol. $[Ru(en)_3]Cl_2$ $R_u \Rightarrow$ 4d series

en $\Rightarrow$ chelating ligand

CN = 6, octahedral splitting hence laye splitting of d-subshell

$R_u^{+2} \Rightarrow [Kr]4d^6 5s^0$

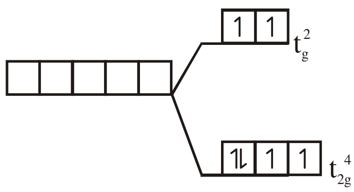


$[Fe(H_2O)_6]Cl_2$ $\Rightarrow H_2O \Rightarrow$ Weak filled ligand

$Fe^{+2} \Rightarrow [Ar]\ 3d^6 4s^0$

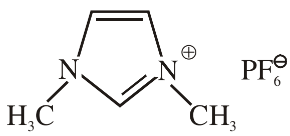
less plitting

CN = 6 octahedral splitting

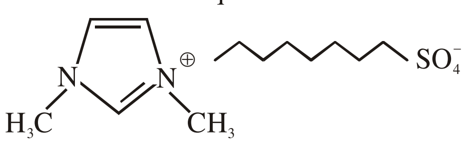


10. An ionic micelle is formed on the addition of :

excess water to liquid

(1) 

excess water to liquid

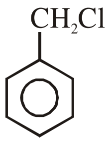
(2) 

(3) liquid diethyl ether to aqueous NaCl solution

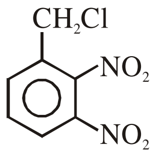
(4) sodium stearate to pure toluene

Sol. Correct Ans. is (2)

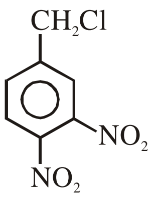
11. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is :



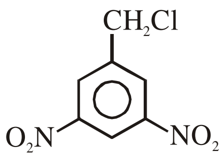
(I)



(II)



(III)



(IV)

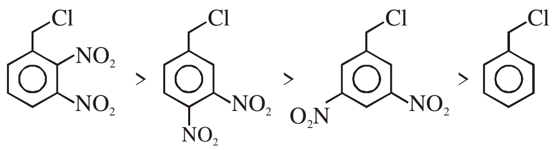
(1) (IV) > (II) > (III) > (I)

(2) (II) > (III) > (IV) > (I)

(3) (II) > (III) > (I) > (IV)

(4) (III) > (II) > (IV) > (I)

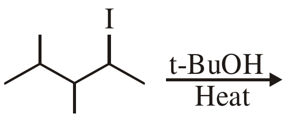
Sol.

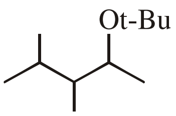


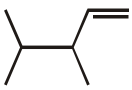
–I & –R   less –I & –R   Only 2 – I

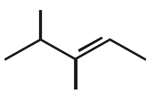
rate of SN[2] reaction

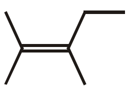
12. The major product in the following reaction is :



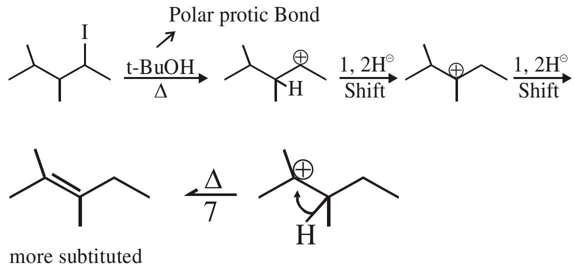
(1) 

(2) 

(3) 

(4) 

Sol.



more subtituted
Alkene (major)

13. The increasing order of the reactivity of the following compound in nucleophilic addition reaction is :

Propanal, Benzaldehyde, Propanone, Butanone

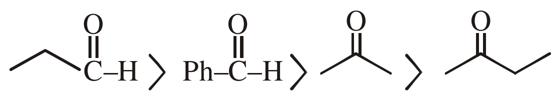
(1) Butanone < Propanone < Benzaldehyde < Propanal

(2) Benzaldehyde < Butanone < Propanone < Propanal

(3) Propanal < Propanone < Butanone < Benzaldehyde

(4) Benzaldehyde < Propanal < Propanone < Butanone

Sol. Reactivity order of various carbonyl compounds → Aldehydes > Ketones



14. The incorrect statement is :

(1) In manganate and permanganate ions, the π-bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese

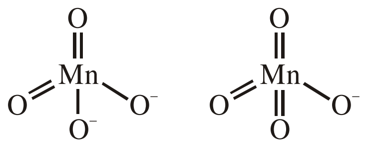
(2) Manganate ion is green in colour and permanganate ion in purple in colour

(3) Manganate and permanganate ions are paramagnetic

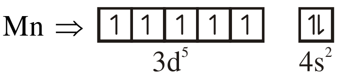
(4) Manganate and permanganate ions are tetrahedral

Sol. Option 1) Manganate $\Rightarrow$ MnO_4^{2-} ,

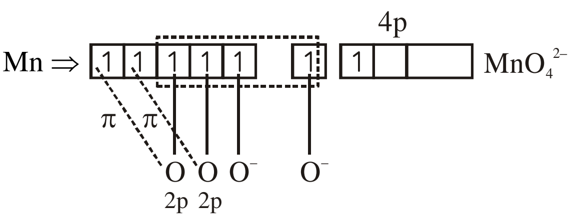
Permanganate $\Rightarrow$ MnO_4^-



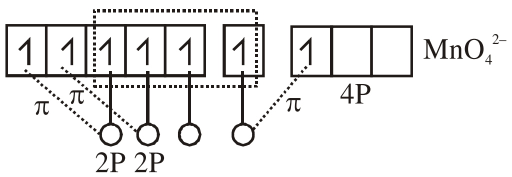
hybridisation of Mn $\Rightarrow d^3s$ hybridisation of Mn $\Rightarrow d^3s$



After excitation



$2 \times 2p_\pi - 3d_{\pi\sigma}$



$2 \times 2P_\pi - 3d_\pi$

$1 \times 2P_\pi - 4P_\pi$

(2) $MnO_4^{2-} \Rightarrow$ green

$MnO_4^- \Rightarrow$ purple/violet

(3) Manganate contains 1 unpaired electron hence it is paramagnetic

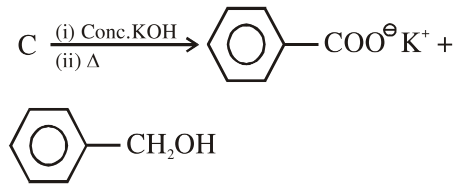
where as permanganetic contains no unpaired electrons hence it is diamagnetic.

(4) Both have d^3s hybridisation hence both have tetrahedral geometry.

15. The compound A in the following reaction is :

$$A \xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } CH_3MgBr/H_2O}$$

$$B \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i)} O_3} C + D$$



$$D \xrightarrow[\Delta]{Ba(OH)_2} H_3C-\overset{CH_3}{\overset{|}{C}}=CH-\overset{O}{\overset{||}{C}}-CH_3$$

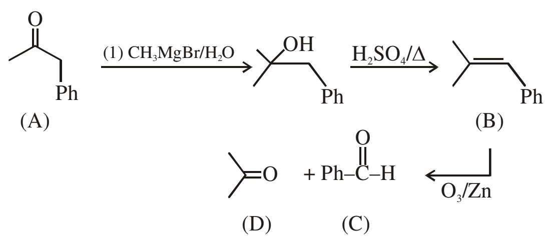
(1) $C_6H_5-\overset{O}{\overset{||}{C}}-CH\begin{matrix}CH_3\\CH_3\end{matrix}$

(2) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_2CH_3$

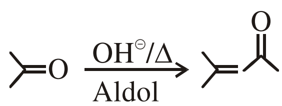
(3) $C_6H_5-CH_2-\overset{O}{\overset{||}{C}}-CH_3$

(4) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_3$

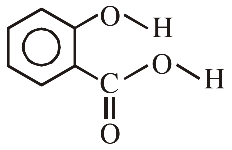
Sol.

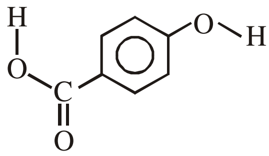


$$Ph-\overset{O}{\overset{||}{C}}-H \xrightarrow[\text{NaOH}]{\text{Conc.}} Ph-COOK + Ph-CH_2OH$$
(cannizaro)



16. Consider the following molecules and statements related to them :

(A) 

(B) 

(a) (B) is more likely to be crystalline than (A)

(b) (B) has higher boiling point than (A)

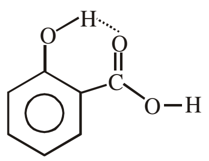
(c) (B) dissolves more readily than (A) in water

Identify the correct option from below :

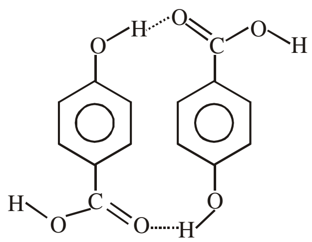
(1) only (a) is true (2) (a) and (c) are true

(3) (b) and (c) are true (4) (a) and (b) are true

Sol.



O-salicylic acid
intra molecular
H-bonding



p-salicylic acid
inter molecular H-bonding

(a) B will be more crystalline due to more inter molecular interactions hence more efficient packing.

(b) B will have higher boiling point due to higher intermolecular interactions.

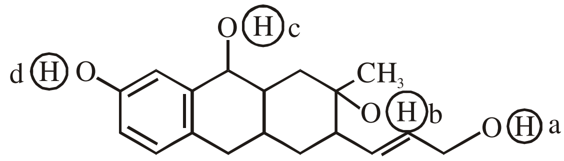
(c) B will be more soluble in water than A as B will have more extent of H-bonding in water

So all three statements are correct

{Solubility date ⇒ O-salicylic acid = 2g/L

P-salicylic acid = 5g/L}

17. Consider the following reaction :



$\xrightarrow[\text{anhydride}]{\text{Chromic}}$ 'P'

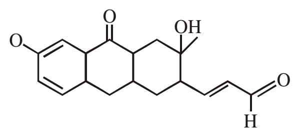
The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s) ?

(1) (c) and (d)

(2) (b) only

(3) (d) only

(4) (b) and (d)

Sol. Compound $\xrightarrow[\text{anhydride}]{\text{Chromic}}$ 

due to pressure of b

18. Match the following drugs with their therapeutic actions :

(i) Ranitidine	(a) Antidepressant
(ii) Nardil (Phenelzine)	(b) Antibiotic
(iii)Chloramphenicol	(c) Antihistamine
(iv)Dimetane (Brompheniramine)	(d) Antacid
	(e) Analgesic

(1) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e)

(2) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d)

(3) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c)

(4) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e)

Sol. Raniticline → Antacid

Nard1 → Antidepressant

Chloramphenicol → Antibiotic

Dimetane → Antihistamine

19. For the reaction $2A + 3B + \frac{3}{2}C \rightarrow 3P$, which statement is correct ?

(1) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$

(2) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(3) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(4) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

Sol. For $aA + bB \rightarrow cC$;

$$\frac{-1}{a}\frac{d[A]}{dt} = \frac{-1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt}$$

$$\therefore \frac{-1}{2}\frac{d[A]}{dt} = \frac{-1}{3}\frac{d[B]}{dt} = \frac{-2}{3}\frac{d[C]}{dt} = \frac{1}{3}\frac{d[p]}{dt}$$

20. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc. H_2SO_4 loses 13.5% of its original mass, the correct molecular formula of A is :

[Given : atomic mass of Cr = 52 amu and Cl = 35 amu]

(1) $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

(2) $[Cr(H_2O)_3Cl_3] \cdot 3H_2O$

(3) $[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(4) $[Cr(H_2O)_6]Cl_3$

Sol. % mass of water

$$= \frac{x \times 18}{(12 + 6 \times 16 + 35 \times 3 + 52)} \times 100 = 13.5$$

$$\Rightarrow \quad x = \frac{265 \times 13.5}{18 \times 100} \simeq 2$$

21. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process(in%) is

(Take : F = 96000 C, At. mass of chromium = 52)

______.

Sol. Moles of $e^{\ominus} = \left(\frac{8 \times 60 \times 2}{96000}\right)$

Using stoichiometry; theoritically

$$\frac{n_{e^{\ominus}} \text{ used}}{6} = \frac{n_{cr^{+3}} \text{ produced}}{2}$$

$$\Rightarrow \quad n_{cr^{+3}} \text{ produced} = \frac{2}{6} \times \frac{8 \times 60 \times 2}{96000}$$

$$= \frac{0.02}{6}$$

$\Rightarrow$ $wt_{cr^{+3}}$ theoritically produced

$$= \left(\frac{0.02}{6} \times 52\right) g$$

$$\Rightarrow \text{ \% efficiency } = \frac{0.104g}{\left(\frac{0.02 \times 52}{6}\right) g} \times 100$$

= 60%

22. 6.023×10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is ______ $\times 10^{-3}$.

$$\text{moles} = \frac{\text{number of molecules}}{6 \times 10^{23}} = \frac{\text{given mass}}{\text{molar mass}}$$

$$\Rightarrow \text{ molar mas } = \frac{10 \times 6.023 \times 10^{23}}{6.023 \times 10^{22}} = 100 g/mol$$

$$\Rightarrow \text{ molarity } = \frac{\text{moles of solute}}{\text{volume of sol}^n (\ell)} = \frac{(5/100)}{2}$$

= 0.025

23. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is ________ .

Sol. $H_3PO_2 + NaOH \rightarrow NaH_2PO_2 + H_2O$

$$\frac{n_{H_3PO_2}\text{ reacted}}{1} = \frac{n_{NaOH}\text{ reacted}}{1}$$

$$\Rightarrow \frac{0.1\times 10}{1} = 0.1\times V_{NaOH}$$

$\Rightarrow$ V_{NaOH} = 10 ml.

24. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular mases of A and B is ________ $\times 10^{-2}$ (to the nearest integer).

Sol. Let molar mass of protein A = x g/mol
Let molar mass of protein B = y g/mol

$$\pi_A = \text{osmotic pressure of protein A} = \frac{\left(\frac{0.73}{x}\right)}{0.25}RT$$

$$\pi_B = \text{osmotic pressure of protein B} = \frac{\left(\frac{1.65}{y}\right)}{1}RT$$

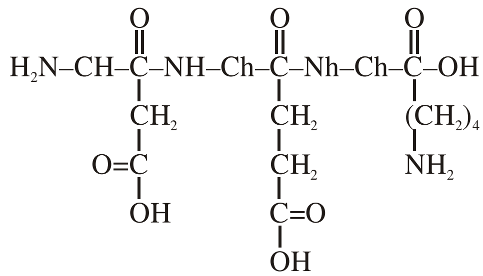
$$\pi_A = \pi_B$$

$$\Rightarrow \left(\frac{0.73}{x\times 0.25}\right)RT = \left(\frac{1.65}{y}\right)RT$$

$$\Rightarrow \left(\frac{x}{y}\right) = \frac{0.73}{0.25\times 1.65} = 1.769 \cong 1.77$$

25. The number of $\rangle C=O$ groups present in a tripeptide Asp – Glu – Lys is ________ .

Sol. Structure of Tri peptide Asp – Glu – Lys



FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :

(1) 9 (2) 18

(3) 10 (4) 20

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt} = 3.6$

$a\frac{da}{dt} = 0.3$

$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$

$= 3 \times 10 \times 0.3 = 9$

2. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3}-\pi$ (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}+\pi$

Sol. $\int_0^{1/2}\frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$\int_0^{1/2}\frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2}\frac{dx}{\sqrt{1-x^2}}$

$\int_0^{1/2}\frac{x^{-3}}{(x^{-2}-1)^{3/2}}dx - (\sin^{-1}x)_0^{1/2}$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -\,t\,dt$

$\int_{\infty}^{\sqrt{3}}\frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty}\frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$

$k = 2\sqrt{3} - \pi$

3. Let R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then:

(1) R_2 is transitive but R_1 is not transitive

(2) R_1 is transitive but R_2 is not transitive

(3) R_1 and R_2 are both transitive

(4) Neither R_1 nor R_2 is transitive

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt{3}$ & $b = 2 - \sqrt{3}$

$a^2 + b^2 = 14 \in Q$

Let $c = \left(1+2\sqrt{3}\right)$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = \left(2+\sqrt{3}\right)^2 + \left(1+2\sqrt{3}\right)^2 \notin Q$

for R_2 Let $a^2 = 1$, $b^2 = \sqrt{3}$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

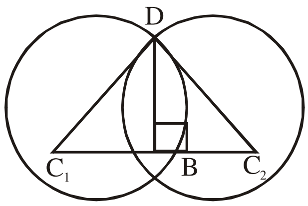
But $a^2 + c^2 \in Q$

4. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 8 (2) $4\sqrt{5}$

(3) 12 (4) $8\sqrt{5}$

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

5. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$,

where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1, \sqrt{x}\right)$ (2) $\left(x+1, \sqrt{x}\right)$

(3) $\left(x+1, -\sqrt{x}\right)$ (4) $\left(x-1, -\sqrt{x}\right)$

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\int \underset{\mathrm{I}}{\theta}.\underset{\mathrm{II}}{(2\tan\theta\cdot\sec^2\theta)}d\theta \quad \text{(By parts)}$$

$$= \theta.\tan^2\theta - \int \tan^2\theta\, d\theta$$

$$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$$

$$= \theta(1 + \tan^2\theta) - \tan\theta + C$$

$$= \tan^{-1}\left(\sqrt{x}\right)(1+x) - \sqrt{x} + C$$

6. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$

(3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Now, number of 5-digit numbers containing both digits = $2^5 - 2$

Second Case: Choose one non-zero & one zero as digit ${}^9C_1$.

Number of 5-digit numbers containg one non zero and one zero both = $(2^4 - 1)$

Required prob.

$$= \frac{\left({}^9C_2 \times \left(2^5 - 2\right) + {}^9C_1 \times \left(2^4 - 1\right)\right)}{9\times 10^4}$$

$$= \frac{36\times(32-2) + 9\times(16-1)}{9\times 10^4}$$

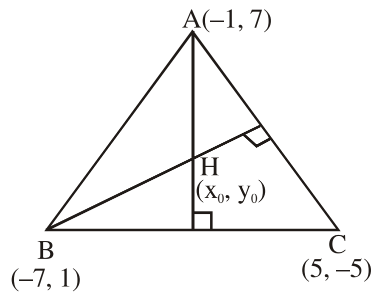
$$= \frac{4\times 30 + 15}{10^4} = \frac{135}{10^4}$$

7. If a ΔABC has vertices A(–1, 7), B(–7, 1) and C(5, –5), then its orthocentre has coordinates:

(1) (3, –3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3) (–3, 3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH}.m_{BC} = -1$

$$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right)\left(\frac{1+5}{-7-5}\right) = -1$$

$\Rightarrow$ $2x_0 - y_0 + 9 = 0$ .... (1)

and $m_{BH}.m_{AC} = -1$

$\Rightarrow \quad \left(\frac{y_0-1}{x_0+7}\right)\left(\frac{7-(-5)}{-1-5}\right)=-1$

$\Rightarrow \quad x_0 - 2y_0 + 9 = 0$ .... (2)

Solving equation (1) and (2) we get

$(x_0, y_0) \equiv (-3, 3)$

8. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to:

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}$

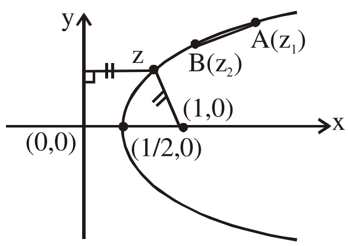
Sol. $\text{Re}(z) = |z - 1|$

$\Rightarrow \quad x=\sqrt{(x-1)^2+(y-0)^2} \quad (x > 0)$

$\Rightarrow \quad y^2 = 2x - 1 = 4.\frac{1}{2}\left(x-\frac{1}{2}\right)$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$\therefore \quad \text{Vertex} = \left(\frac{1}{2}, 0\right)$



$A(z_1)$ & $B(z_2)$ are two points on it such that slope of AB = $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(\arg(z_1 - z_2) = \frac{\pi}{6})$

for $y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

$$m_{AB}=\frac{2}{t_1+t_2}=\frac{4a}{y_1+y_2}=\frac{1}{\sqrt{3}}$$

$$\left(\text{Here } a=\frac{1}{2}\right)$$

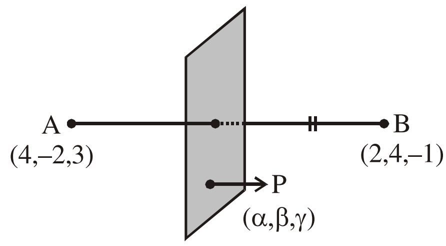
$\Rightarrow \quad y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

9. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (4, 0, –1) (2) (4, 0, 1)

(3) (0, 1, –1) (4) (0, –1, 1)

Sol.



$PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow \quad -4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow \quad 2x - 6y + 4z = 4$

10. $\lim\limits_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}(a\neq 0)$ is equal to :

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Sol. Required limit

$$L = \lim_{h\to 0}\frac{(a+2(a+h))^{1/3}-(3(a+h))^{1/3}}{(3a+a+h)^{1/3}-(4(a+h))^{1/3}}$$

$$= \lim_{h\to 0}\frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3}-(3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3}-(4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right)-\left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right)-\left(1+\frac{h}{3a}\right)}\right]$$

$$= \left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9}-\frac{1}{3}\right)}{\left(\frac{1}{12}-\frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

11. Let A be a 3 × 3 matrix such that

adj A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and

B = adj (adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{9}\right)$ (2) $\left(9, \frac{1}{81}\right)$

(3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

Sol. C = adj A = $\begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2, 4)$

$= +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

B = adj C

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

12. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R|\ f(x) = f(0)\}$, then the sum of squares of all the elements of T is :

(1) 6 (2) 8

(3) 4 (4) 2

Sol. $f'(x) = x(x + 1)(x - 1) = x^3 - x$

$$\int d f(x) = \int x^3 - x\, dx$$

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$$

$f(x) = f(0)$

$$\frac{x^4}{4} - \frac{x^2}{2} = 0$$

$x^2(x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

13. Let a, b, c $\in$ R be such that $a^2 + b^2 + c^2 = 1$.

If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$,

where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :

(1) $\frac{\pi}{2}$ (2) 0

(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

Sol. $\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab+bc+ca}{a^2+b^2+c^2} = \frac{\Sigma ab}{1}$

$= abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)\right)$

$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta + \pi)\cos\frac{\pi}{3}\right)$

$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$

$\phi = \frac{\pi}{2}$

14. If the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$ upto n^{th} term is 488

and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41

(3) n^{th} term is –4 (4) n = 60

Sol. $S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + ...n$

$S_n = \frac{n}{2}\left(2 \times \frac{100}{5} + (n-1)\left(-\frac{2}{5}\right)\right) = 188$

$n(100 - n + 1) = 488 \times 5$

$n^2 - 101n + 488 \times 5 = 0$

n = 61, 40

$T_n = a + (n-1)d = \frac{100}{5} - \frac{2}{5} \times 60$

= 20 – 24 = –4

15. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$

(3) $\frac{9}{10}$ (4) $\frac{4}{5}$

Sol. $\text{Variance} = \frac{\Sigma(x_i - p)^2}{n} - \left(\frac{\Sigma(x_i - p)}{n}\right)^2$

$= \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{81}{100}$

S.D. = $\frac{9}{10}$

16. If $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$; y(2) = e and x > 1, then y(4) is equal to :

(1) $\frac{3}{2} + \sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$

(3) $\frac{1}{2} + \sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow$ $dy(x^3 - x^2) = dx\,(2y - xy)$

$$\Rightarrow \quad -\int \frac{1}{y}dy = \int \frac{x-2}{x^2(x-1)}dx$$

$$\Rightarrow \quad -\ell ny = \int\left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}\right)dx$$

Where A = 1, B = +2, C = –1

$$\Rightarrow \quad -\ell ny = \ell n\,x - \frac{2}{x} - \ell n(x-1) + \lambda$$

$\Rightarrow$ $y(2) = e$

$\Rightarrow$ $-1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore$ $\lambda = -\ \ell n\,2$

$$\Rightarrow \quad \ell n\,y = -\ \ell nx + \frac{2}{x} + \ell n(x-1) + \ell n\,2$$

Now put x = 4 in equation

$$\Rightarrow \quad \ell n\,y = -\ell n\,4 + \frac{1}{2} + \ell n\,3 + \ell n\,2$$

$$\Rightarrow \quad \ell n\,y = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ell n\,e$$

$$\Rightarrow \quad y = \frac{3}{2}\sqrt{e}$$

17. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(1) (8, 10) (2) (8, 12)

(3) $\left(\frac{20}{3}, 12\right)$ (4) $\left(\frac{24}{5}, 10\right)$

Sol. For ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore$ $b^2 = 25\,(1 - e_1^2)$ .... (1)

Again for hyperbola

$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$$

Let e_2 is eccentricity of hyperbola.

$\therefore$ $b^2 = 16(e_2^2 - 1)$ .... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$$\therefore \quad 25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$$

or $e_1 = \frac{4}{5}$ $\therefore$ $e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse $= 2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola $= 2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore$ $(\alpha, \beta) \equiv (8, 10)$

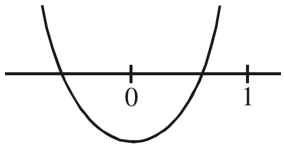
18. The set of all real values of λ for which the quadratic equations,

$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (1, 3]

(3) (0, 2) (4) (2, 4]

Sol. If exactly one root in (0, 1) then



$\Rightarrow \quad f(0).f(1) < 0$

$\Rightarrow \quad 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow \quad 1 < \lambda < 3$

Now for $\quad \lambda = 1,\ 2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0,\ x = 1,\ 1$

So both roots doesn't lie between (0, 1)

$\therefore\ \lambda \neq 1$

Again for $\quad \lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow \quad x = 1, \dfrac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \quad \lambda \in (1, 3]$.

19. If the term independent of x in the expansion of

$\left(\dfrac{3}{2}x^2 - \dfrac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(1) 9 (2) 11

(3) 5 (4) 7

Sol. $T_{r+1} = {}^9C_r\left(\dfrac{3}{2}x^2\right)^{9-r}\left(-\dfrac{1}{3x}\right)^r$

$$T_{r+1} = {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

For independent of x

$18 - 3r = 0,\ r = 6$

$\therefore \quad T_7 = {}^9C_6\left(\dfrac{3}{2}\right)^3\left(-\dfrac{1}{3}\right)^6 = \dfrac{21}{54} = k$

$\therefore\ 18k = \dfrac{21}{54} \times 18 = 7$

20. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T (2) F, T, F

(3) T, T, F (4) T, T, T

Sol. $(p \wedge q) \rightarrow (\sim q \vee r)$ = false

when $(p \wedge q)$ = T

and $(\sim q \vee r)$ = F

So $(p \wedge q)$ = T is possible when p = q = true

$\therefore \quad \sim q$ = False (q = true)

So $(\sim q \vee r)$ = False is possible if r is false

$\therefore$ p = T, q = T, r = F

21. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to _______.

Sol. 3, A_1, A_2 .... A_m, 243

$$d = \frac{243-3}{m+1} = \frac{240}{m+1}$$

Now 3, G_1, G_2, G_3, 243

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$$

$\therefore \quad A_4 = G_2$

$\Rightarrow \quad a + 4d = ar^2$

$$3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$$

m = 39

22. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point $(1, 2)$ intersect at the same point on the x-axis, then the value of c is _______.

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$$m = \left(\frac{dy}{dx}\right)_{(c,e^c)} = e^c$$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c(x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$ ......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,2)} = 1$

$\Rightarrow$ Slope of normal $= -1$

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$ ......(2)

Points are same

$\Rightarrow$ $x = c - 1 = 3$

$\Rightarrow$ $c = 4$

23. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$, $\lambda \in R$ and

$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k})$, $\mu \in R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P, then $3(\alpha + \beta + \gamma)$ equals _______.

Sol. Dr's normal to plane

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Equation of plane

$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$x - y - z - 1 = 0$ ......(1)

Now $\frac{\alpha - 1}{1} = \frac{\beta - 0}{-1} = \frac{\gamma - 1}{-1} = -\frac{(1 - 0 - 1 - 1)}{3}$

$$\frac{\alpha - 1}{1} = \frac{\beta}{-1} = \frac{\gamma - 1}{-1} = \frac{1}{3}$$

$$\alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$$

$$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

24. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _______.

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0,\ y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k,\ y = \frac{k}{2},\ z = 0$ put in condition

$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$

$12 \le k^2 \le 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

25. The total number of 3-digit numbers, whose sum of digits is 10, is _______.

Sol. Let three digit number is xyz

$x + y + z = 10\ ;\ \ x \ge 1,\ y \ge 0\ z \ge 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \ge 0$

Put in (1)

$T + y + z = 9\ ;\ \ 0 \le T \le 8,\ 0 \le y, z \le 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when T = 9)

$= 55 - 1 = 54$