TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS

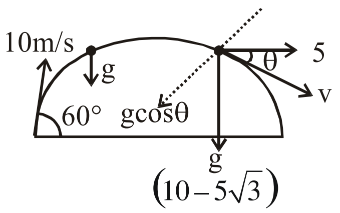
1. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is :

(1) 2.5 m (2) 10.3 m

(3) 2.8 m (4) 5.1 m

Ans. **(3)**

Sol.



v_x = 10cos60° = 5 m/s

v_y = 10cos30° = $5\sqrt{3}$ m/s

velocity after t = 1 sec.

v_x = 5 m/s

v_y = $\left|\left(5\sqrt{3}-10\right)\right| m/s = 10 - 5\sqrt{3}$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta}$$

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100\left(2-\sqrt{3}\right)}{10\cos 15} = 2.8m$$

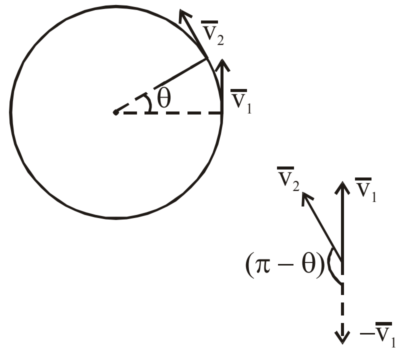
2. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(1) zero (2) 10 m/s

(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s

Ans. **(2)**

Sol.



$$|\Delta\overline{v}| = \sqrt{v_1^2+v_2^2+2v_1v_2\cos(\pi-\theta)}$$

$$= 2v\sin\frac{\theta}{2} \quad \text{since}\left[|\overline{v}_1| = |\overline{v}_2|\right]$$

= (2 × 10) × sin(30°)

= 10 m/s

3. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980Å. The radius of the atom in the excited state, it terms of Bohr radius a_0, will be :

(h_c = 12500 eV – Å)

(1) $9a_0$ (2) $25a_0$ (3) $4a_0$ (4) $16a_0$

Ans. **(4)**

Sol. Energy of photon = $\frac{12500}{980} = 12.75 eV$

∴ Electron will excite to n= 4

Since 'R' ∝ n^2

∴ Radius of atom will be $16a_0$

4. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be :

(1) pv^2 (2) $\frac{3}{4}pv^2$

(3) $\frac{1}{2}pv^2$ (4) $\frac{1}{4}pv^2$

Ans. **(2)**

Sol. Momentum per second carried by liquid per second is $\rho a v^2$

net force due to reflected liquid = $2\times\left[\frac{1}{4}\rho a v^2\right]$

net force due to stopped liquid = $\frac{1}{4}\rho a v^2$

Total force = $\frac{3}{4}\rho a v^2$

net pressure = $\frac{3}{4}\rho v^2$

5. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electrric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by :

(1) $\left(\frac{1}{\sqrt{n}},\frac{1}{\sqrt{n}}\right)$

(2) $\left(\sqrt{n},\frac{1}{\sqrt{n}}\right)$

(3) $\left(\sqrt{n},\sqrt{n}\right)$

(4) $\left(\frac{1}{\sqrt{n}},\sqrt{n}\right)$

Ans. (2)

Sol. $C=\frac{1}{\sqrt{\mu_0\in_0}}$

$V=\frac{1}{\sqrt{k\in_0\mu_0}}$ [For transparent medium $\mu_r \approx \mu_0$]

$\therefore\ \frac{C}{V}=\sqrt{k}=n$

$\frac{1}{2}\in_0 E_0^2 C$ = intensity = $\frac{1}{2}\in_0 kE^2 v$

$\therefore\ E_0^2 C = kE^2 v$

$\Rightarrow \frac{E_0^2}{E^2}=\frac{kV}{C}=\frac{n^2}{n} \Rightarrow \frac{E_0}{E}=\sqrt{n}$

similarly

$\frac{B_0^2 C}{2\mu_0}=\frac{B^2 v}{2\mu_0} \Rightarrow \frac{B_0}{B}=\frac{1}{\sqrt{n}}$

6. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given π = 22/7]

(1) 1785 and 1715

(2) 892.5 and 857.5

(3) 89.25 and 85.75

(4) 178.5 smf 171.5

Ans. (3)

Sol. $V(t)=10+\frac{3}{2}[2\cos A\sin B]$

$=10+\frac{3}{2}[\sin(A+B)-\sin(A-B)]$

$=10+\frac{3}{2}[\sin(57.2\times10^4 t)-\sin(52.8\times10^4 t)]$

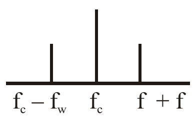
$\omega_1 = 57.2 \times 10^4 = 2\pi f_1$

$f_1=\frac{57.2\times10^4}{2\times\left(\frac{22}{7}\right)}=9.1\times10^4$

$\simeq 91\,KHz$

$f_2=\frac{52.8\times10^4}{2\times\left(\frac{22}{7}\right)}$

$\simeq 84\,KHz$



Side band frequency are

$f_1 = f_c - f_w = \frac{52.8\times10^4}{\pi} \approx 85.00$ kHz

$f_2 = f_c + f_w = \frac{57.2\times10^4}{\pi} \approx 90.00$ kHz

7. The force of interaction between two atoms is given by $F=\alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is :

(1) $M^2L^2T^{-2}$
(2) M^2LT^{-4}
(3) $M^0L^2T^{-4}$
(4) MLT^{-2}

Ans. (2)

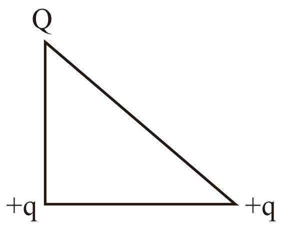
Sol. $F=\alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$

$$\left[\frac{x^2}{\alpha KT}\right]=M^\circ L^\circ T^\circ$$

$$\frac{L^2}{[\alpha]ML^2T^{-2}}=M^\circ L^\circ T^\circ$$

$\Rightarrow$ $[\alpha] = M^{-1}T^2$
$[F] = [\alpha]\ [\beta]$
$MLT^{-2} = M^{-1}T^2[\beta]$
$\Rightarrow$ $[\beta] = M^2LT^{-4}$

8. The charges Q + q and +q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, it the value of Q is:



(1) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (2) $-2q$

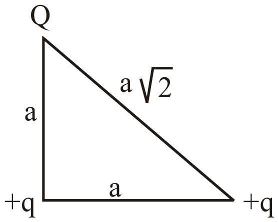
(3) $\frac{-q}{1+\sqrt{2}}$ (4) $+q$

Ans. (1)

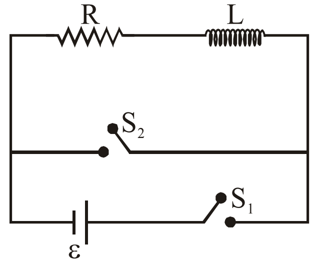
Sol. U = $K\left[\frac{q^2}{a}+\frac{Qq}{a}+\frac{Qq}{a\sqrt{2}}\right]=0$

$\Rightarrow$ $q=-Q\left[1+\frac{1}{\sqrt{2}}\right]$

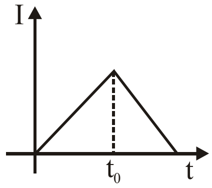
$\Rightarrow$ $Q=\frac{-q\sqrt{2}}{\sqrt{2}+1}$

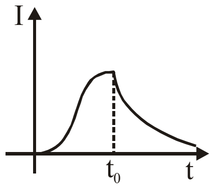


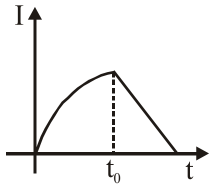
9. In the circuit shown,



the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behavious of the current I as a function of time 't' is given by :

(1)


(2)


(3)


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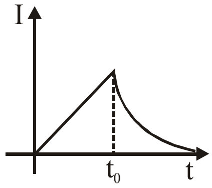
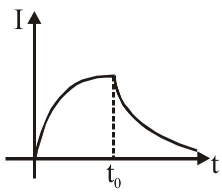


Ans. (2)

Sol. From time t = 0 to t = t_0, growth of current takes place and after that decay of current takes place.



most appropriate is (2)

10. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin(450 t – 9x) where distance and time are measured is SI units. The tension in the string is :

(1) 10 N (2) 12.5 N (3) 7.5 N (4) 5 N

Ans. **(2)**

Sol. y = 0.03 sin(450 t – 9x)

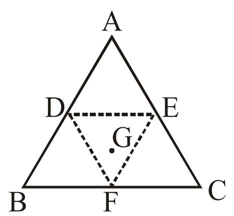
$$v=\frac{\omega}{k}=\frac{450}{9}=50\,m/s$$

$$v=\sqrt{\frac{T}{\mu}}\Rightarrow\frac{T}{\mu}=2500$$

$\Rightarrow$ T = 2500 × 5 × 10^{-3}

= 12.5 N

11. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. It the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



(1) $I=\frac{9}{16}I_0$ (2) $I=\frac{3}{4}I_0$

(3) $I=\frac{I_0}{4}$ (4) $I=\frac{15}{16}I_0$

Ans. **(4)**

Sol. Suppose M is mass and a is side of larger triangle, then $\frac{M}{4}$ and $\frac{a}{2}$ will be mass and side length of smaller triangle.

$$\frac{I_{removed}}{I_{original}}=\frac{\frac{M}{4}\cdot\left(\frac{a}{2}\right)^2}{M\ (a)^2}$$

$$I_{removed}=\frac{I_0}{16}$$

$$\text{So, } I=I_0-\frac{I_0}{16}=\frac{15I_0}{16}$$

12. There are two long co-axial solenoids of same length l. the inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is :

(1) $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$ (3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}$

Ans. **(4)**

Sol. $M=\mu_0 n_1 n_2 \pi r_1^2$

$L=\mu_0 n_1^2 \pi r_1^2$

$$\Rightarrow \frac{M}{L}=\frac{n_2}{n_1}$$

13. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature,. The relation between temperature and volume of this process is TV^x = constant, then x is :

(1) $\frac{5}{3}$ (2) $\frac{2}{5}$ (3) $\frac{2}{3}$ (4) $\frac{3}{5}$

Ans. **(2)**

Sol. For adiabatic process : $TV^{\gamma-1}$ = constant

For diatomic process : $\gamma-1=\frac{7}{5}-1$

$$\therefore x=\frac{2}{5}$$

14. The gas mixture constists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total inernal energy of the system is:

(1) 12 RT (2) 20 RT (3) 15 RT (4) 4 RT

Ans. **(3)**

Sol. $U=\frac{f_1}{2}n_1RT+\frac{f_2}{2}n_2RT$

$$=\frac{5}{2}(3RT)+\frac{3}{2}\times 5RT$$

U = 15RT

15. In a Young's double slit experiment, the path different, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a brigth fringe is close to :

(1) 0.94 (2) 0.74 (3) 0.85 (4) 0.80

Ans. **(3)**

Sol. $\Delta x = \frac{\lambda}{8}$

$$\Delta\phi = \frac{(2\pi)}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

$$I = I_0 \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

16. If the deBronglie wavelenght of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to :
(Speed of light = 3×10^8 m/s
Planck's constant = 6.63×10^{-34} J.s
Mass of electron = 9.1×10^{-31} kg)

(1) 1.45×10^6 m/s (2) 1.7×10^6 m/s
(3) 1.8×10^6 m/s (4) 1.1×10^6 m/s

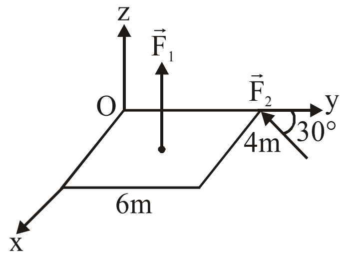
Ans. **(1)**

Sol. $\frac{h}{mv} = 10^{-3}\left(\frac{3\times10^8}{6\times10^{14}}\right)$

$$v = \frac{6.63\times10^{-34}\times6\times10^{14}}{9.1\times10^{-31}\times3\times10^5}$$

$v = 1.45 \times 10^6$ m/s

17. A slob is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i} + 3\vec{j})$. The moment of these forces about point O will be :



(1) $(3\hat{i} - 2\hat{j} - 3\hat{k})F$ (2) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

(3) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$ (4) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$

Ans. **(4)**

Sol. Torque for F_1 force

$$\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$$

$$\vec{r}_1 = 0\hat{i} + 6\hat{j}$$

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 3F\hat{k}$$

Torque for F_2 force

$$\vec{F}_2 = F\hat{k}$$

$$\vec{r}_2 = 2\hat{i} + 3\hat{j}$$

$$\vec{\tau}_{F_2} = \vec{r}_2 \times \vec{F}_2 = 3F\hat{i} + 2F(-\hat{j})$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2}$$

$$= 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$$

18. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h << R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed requried so that the satellite could escapte from the gravitational field of earth is :

(1) $\sqrt{gR}(\sqrt{2} - 1)$

(2) $\sqrt{2gR}$

(3) $\sqrt{gR}$

(4) $\sqrt{\frac{gR}{2}}$

Ans. **(1)**

Sol. $v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$

$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}$

$\Delta v = v_e - v_0 = \left(\sqrt{2}-1\right)\sqrt{gR}$

19. In an experiment electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.
[Charge of the electron = 1.6×10^{-19} C
Mass of the electron = 9.1×10^{-31} kg]

(1) 7.5×10^{-4} m (2) 7.5×10^{-3} m
(3) 7.5 m (4) 7.5×10^{-2} m

Ans. **(1)**

Sol. $r = \dfrac{\sqrt{2mk}}{eB} = \dfrac{\sqrt{2me\Delta v}}{eB}$

$r = \dfrac{\sqrt{\dfrac{2m}{e}.\Delta v}}{B} = \dfrac{\sqrt{\dfrac{2\times 9.1\times 10^{-31}}{1.6\times 10^{-19}}(500)}}{100\times 10^{-3}}$

$r = \dfrac{\sqrt{\dfrac{9.1}{0.16}\times 10^{-10}}}{10^{-1}} = \dfrac{3}{.4}\times 10^{-4} = 7.5\times 10^{-4}$

20. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\dfrac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

(1) 2 (2) $\dfrac{1}{9}$ (3) 3 (4) 1

Ans. **(3)**

Sol. $k = \dfrac{1}{2}m\omega^2A^2\cos^2\omega t$

$U = \dfrac{1}{2}m\omega^2A^2\sin^2\omega t$

$\dfrac{k}{U} = \cot^2\omega t = \cot^2\dfrac{\pi}{90}(210) = \dfrac{1}{3}$

Hence ratio is 3 (most appropriate)

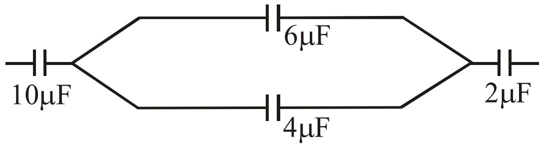
21. Ice at –20° C os added tp 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/°C)
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334 J/g)

(1) 50 g (2) 40 g
(3) 60 g (4) 100 g

Ans. **(2)**

Sol. Let amount of ice is m gm.
According to principal of calorimeter
heat taken by ice = heat given by water
$\therefore$ 20 × 2.1 × m + (m – 20) × 334
= 50 × 4.2 × 40
376 m = 8400 + 6680
m = 40.1
$\therefore$ correct answer is (2)

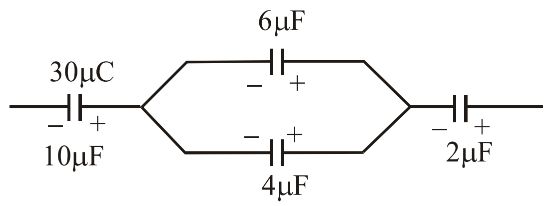
22. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. ?The charge on the right plate of the 6 μF capacitor is :



(1) –18 μC (2) –12 μC
(3) +12 μC (4) +18 μC

Ans. **(4)**

Sol.



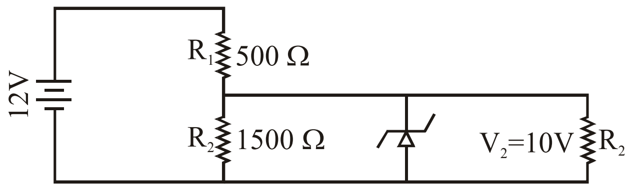
6μF & 4μF are in parallel & total charge on this combination is 30 μC

$\therefore$ Charge on 6μF capacitor = $\dfrac{6}{6+4}\times 30$

= 18 μC

Since charge is asked on right plate therefore is +18μC
Correct answer is (4)

23. In the given circuit the current through Zener Diode is close to :



(1) 6.0 mA (2) 4.0 mA

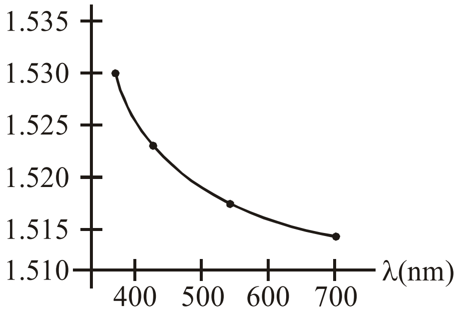
(3) 6.7 mA (4) 0.0 mA

Ans. **(4)**

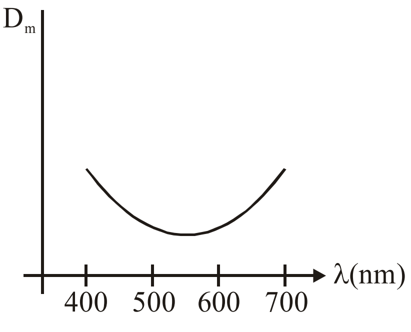
Sol. Since voltage across zener diode must be less than 10V therefore it will not work in breakdown region, & its resistance will be infinite & current through it = 0

$\therefore$ correct answer is (4)

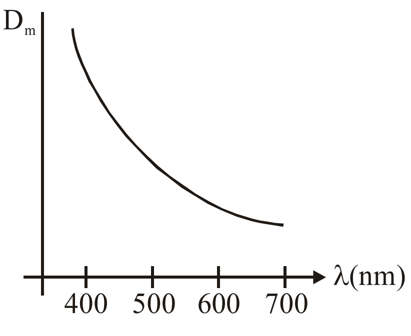
24. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



(1)



(2)



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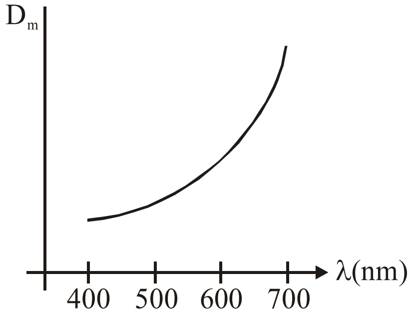
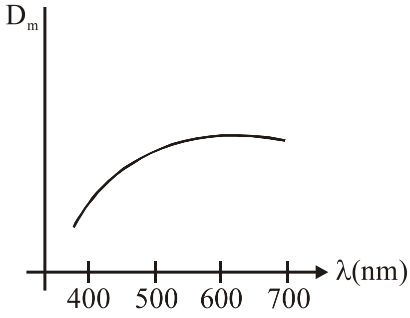


(4)

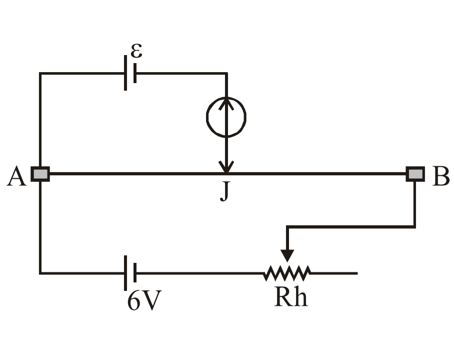


Ans. **(2)**

Sol. Since $D_m = (\mu - 1)A$

& on increasing the wavelength, μ decreases & hence D_m decreases. Therefore correct answer is (2)

25. The resistance of the meter bridge AB is given figure is 4Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2\ \Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6\ \Omega$. The emf ε_2 is;



(1) 0.6 V (2) 0.5 V

(3) 0.3 V (4) 0.4 V

Ans. **(3)**

Sol. Potential gradient with $R_h = 2\Omega$

is $\left(\frac{6}{2+4}\right)\times\frac{4}{L}=\frac{dV}{dL}$; $L = 100\,cm$

Let null point be at ℓ cm

thus $\varepsilon_1 = 0.5V = \left(\frac{6}{2+4}\right)\times\frac{4}{L}\times\ell$ ...(1)

Now with $R_h = 6\Omega$ new potential gradient is

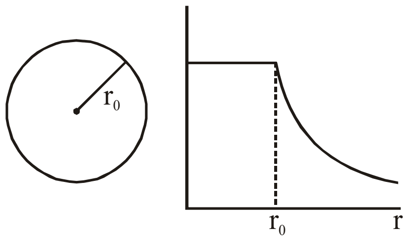
$\left(\frac{6}{4+6}\right)\times\frac{4}{L}$ and at null point

$$\left(\frac{6}{4+6}\right)\left(\frac{4}{L}\right)\times\ell=\varepsilon_2 \quad ...(2)$$

dividing equation (1) by (2) we get

$\frac{0.5}{\varepsilon_2}=\frac{10}{6}$ thus $\varepsilon_2 = 0.3$

26. The given graph shows variation (with distance r from centre) of :



(1) Potential of a uniformly charged sphere
(2) Potential of a uniformly charged spherical shell
(3) Electric field of uniformly charged spherical shell
(4) Electric field of uniformly charged sphere

Ans. (2)

27. Two equal resistance when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be :

(1) 60 W (2) 240 W
(3) 30 W (4) 120 W

Ans. (2)

Sol. In series condition, equivalent resistance is 2R

thus power consumed is $60W = \frac{\varepsilon^2}{2R}$

In parallel condition, equivalent resistance is R/2 thus new power is

$P'=\frac{\varepsilon^2}{(R/2)}$

or $P'=4P=240W$

28. An object is at a distacen of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be :

(1) 0.92×10^{-3} m/s away from the lens
(2) 2.26×10^{-3} m/s away from the lens
(3) 1.16×10^{-3} m/s towards the lens
(4) 3.22×10^{-3} m/s towards the lens

Ans. (3)

Sol. From lens equation

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}-\frac{1}{(-20)}=\frac{1}{(.3)}=\frac{10}{3}$

$\frac{1}{v}=\frac{10}{3}-\frac{1}{20}$

$\frac{1}{v}=\frac{197}{60}; v=\frac{60}{197}$

$m=\left(\frac{v}{u}\right)=\frac{\left(\frac{60}{197}\right)}{20}$

velocity of image wrt. to lens is given by $v_{I/L} = m^2 v_{O/L}$

direction of velocity of image is same as that of object

$v_{O/L} = 5$ m/s

$$v_{I/L} = \left(\frac{60\times1}{197\times20}\right)^2 (5)$$

$= 1.16 \times 10^{-3}$ m/s towards the lens

29. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\ ms^{-2}$, the value of x will be close to :

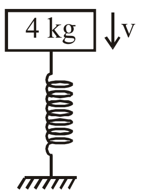
(1) 4 cm
(2) 8 cm
(3) 80 cm
(4) 40 cm

Ans. **(1)**

Sol. Velocity of 1 kg block just before it collides with 3kg block = $\sqrt{2gh} = \sqrt{2000}\ m/s$

Applying momentum conversation just before and just after collision.

$$1\times\sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4} m/s$$



initial compression of spring

$1.25 \times 10^6\ x_0 = 30 \Rightarrow x_0 \approx 0$

applying work energy theorem,

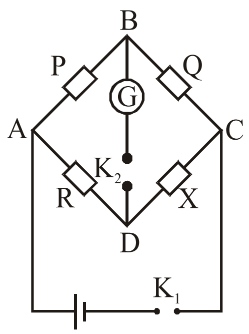
$W_g + W_{sp} = \Delta KE$

$$\Rightarrow 40 \times x + \frac{1}{2}\times1.25\times10^6(0^2 - x^2)$$

$$= 0 - \frac{1}{2}\times4\times v^2$$

solving $x \approx 4$ cm

30. In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is equal. When R = 400 Ω, the bridge is balanced. On inter-changing P and Q, the value of R, for balance, is 405 Ω. The value of X is close to :



(1) 403.5 ohm (2) 404.5 ohm
(3) 401.5 ohm (4) 402.5 ohm

Ans. **(4)**

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

1. For the cell Zn(s) | Zn^{2+}(aq) || M^{x+} (aq) | M(s), different half cells and their standard electrode potentials are given below :

$M^{x+}(aq/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{o}_{M^{x+}/M^{(v)}}$	1.40	0.80	0.77	–0.44

If $E^{o}_{Zn^{2+}/Zn} = -0.76\,V$, which cathode will give a mximum value of E^{o}_{cell} per electron transferred ?

(1) Fe^{3+} / Fe^{2+} (2) Ag^{+} / Ag

(3) Au^{3+} / Au (4) Fe^{2+} / Fe

Ans. (2)

2. The correct match between items-I and II is :

Item-I	***Item-II***
(Mixture)	***(Separation method)***
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

(1) A-Q, B-R, C-S (2) A-R, B-P, C-S

(3) A-S, B-R, C-P (4) A-Q, B-R, C-P

Ans. (1)

Sol. (Mixture) (Seperation method)

H_2O : Sugar $\Rightarrow$ Recrystallization

H_20 : Aniline $\Rightarrow$ Steam distillation

H_2O : Toluene $\Rightarrow$ Differential extraction

3. If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives straight line with a gradient (–y) unit. The energy required to activate the reactant is :

(1) y unit (2) –y unit

(3) yR unit (4) y/R unit

Ans. (1)

4. The concentration of dissolved oxygen (DO) in cold water can go upto :

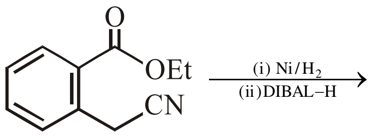
(1) 10 ppm (2) 14 ppm

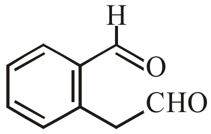
(3) 16 ppm (4) 8 ppm

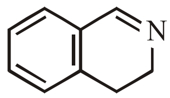
Ans. (1)

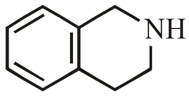
Sol. In cold water, dissolved oxygen (DO) can reach a concentration upto 10 ppm

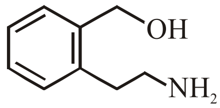
5. The major product of the following reaction is:



(1) 

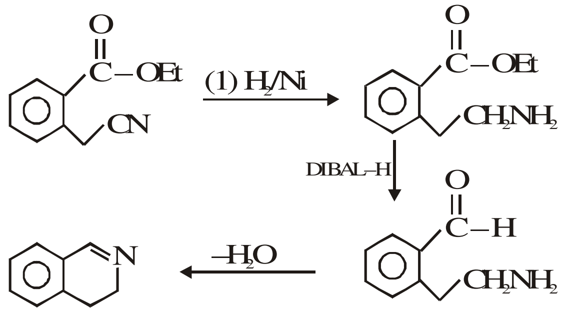
(2) 

(3) 

(4) 

Ans. (2)

Sol.



6. Th correct statements among (a) to (d) regarding H_2 as a fuel are :

(a) It produces less pollutant than petrol

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively

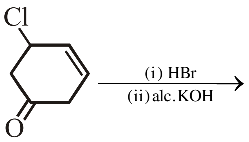
(1) b and d only (2) a, b and c only

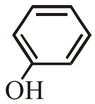
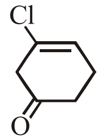
(3) b, c and d only (4) a and c only

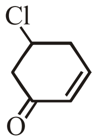
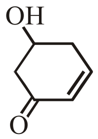
Ans. (2)

Sol. Option (a), (b) & (c) are correct answer (NCERT THEORY BASED)

7. The major poduct of the following reaction is:

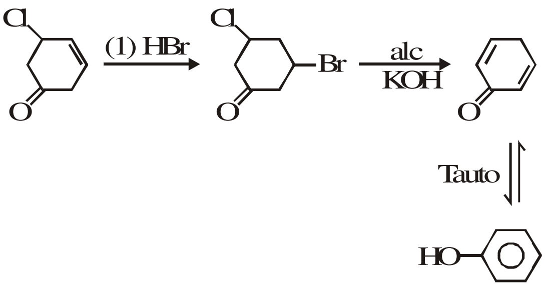


(1)  (2) 

(3)  (4) 

Ans. (1)

Sol.



8. The element that usually does not show variable oxidation states is :

(1) V (2) Ti (3) Sc (4) Cu

Ans. (3)

Sol. Usally Sc(Scandium) does not show variable oxidation states.

Most common oxidation states of :

(i) Sc : +3

(ii) V : +2, +3, +4, +5

(iii) Ti : +2, +3, +4

(iv) Cu : +1, +2

9. An organic compound is estimated through Dumus method and was found to evolve 6 moles of CO_2. 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is :

(1) $C_{12}H_8N$ (2) $C_{12}H_8N_2$

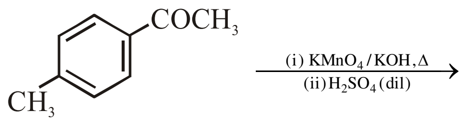
(3) C_6H_8N (4) $C_6H_8N_2$

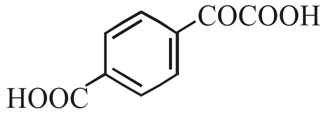
Ans. (4)

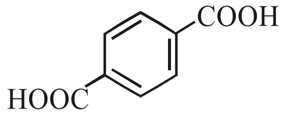
Sol. $[C_xH_yN_z] \xrightarrow[\text{Method}]{\text{Duma}} 6CO_2 + 4H_2O + N_2$

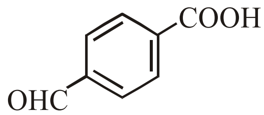
Hence, $C_6H_8N_2$

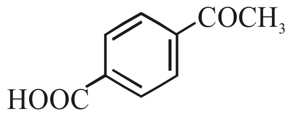
10. The major product of the following reaction is :



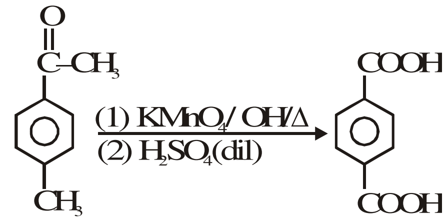
(1) 

(2) 

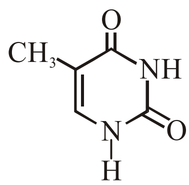
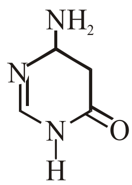
(3) 

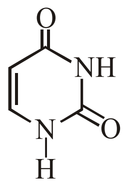
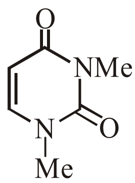
(4) 

Ans. (2)

Sol. 

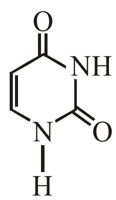
11. Among the following compound which one is found in RNA ?

(1)  (2) 

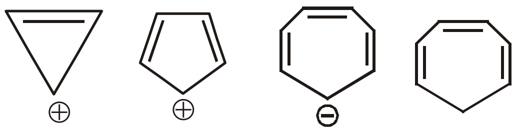
(3)  (4) 

Ans. (3)

Sol. For the given structure 'uracil' is found in RNA



12. Which compound(s) out of the following is/are not aromatic ?

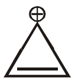


(A) (B) (C) (D)

(1) C and D (2) B, C and D

(3) A and C (4) B

Ans. (2)

Sol. out of the given options only  is aromatic.

Hence (B), (C) and (D) are not aromatic

13. The correct match between Item(I) and Item(II) is :

Item-I	**Item-II**
(A) Nortehindrone	(P) Anti-biotic
(B) Ofloxacin	(Q) Anti-fertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

(1) A-R, B-P, C-S (2) A-Q, B-P, C-R

(3) A-R, B-P, C-R (4) A-Q, B-R, C-S

Ans. (2)

Sol. (A) Norethindrone – Antifertility

(B) Ofloaxacin – Anti-Biotic

(C) Equanil – Hypertension (traiquilizer)

14. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose ?

[$R_H = 1 \times 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8\ ms^{-1}$]

(1) Paschen, $5 \to 3$ (2) Paschen, $\infty \to 3$

(3) Lyman, $\infty \to 1$ (4) Balmer, $\infty \to 2$

Ans. (2)

15. Consider the reaction,

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_P. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium)

(1) $\dfrac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{4}$ (2) $\dfrac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{16}$

(3) $\dfrac{K_P^{\frac{1}{2}} P^2}{16}$ (4) $\dfrac{K_P^{\frac{1}{2}} P^2}{4}$

Ans. (2)

16. Match the ores(Column A) with the metals (column B) :

Column-A Ores	***Column-B Metals***
(I) Siderite	(a) Zinc
(II) Kaolinite	(b) Copper
(III) Malachite	(c) Iron
(IV) Calamine	(d) Aluminium

(1) I-b ; II-c ; III-d ; IV-a

(2) I-c ; II-d ; III-a ; IV-b

(3) I-c ; II-d ; III-b ; IV-a

(4) I-a ; II-b ; III-c ; IV-d

Ans. (3)

Sol. Siderite : $FeCO_3$

Kaolinite : $Al_2(OH)_4Si_2O_5$

Malachite : $Cu(OH)_2.CuCO_3$

Calamine : $ZnCO_3$

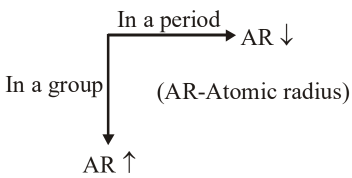
17. The correct order of the atomic radii of C, Cs, Al and S is :

(1) S < C < Al < Cs (2) S < C < Cs < Al

(3) C < S < Cs < Al (4) C < S < Al < Cs

Ans. (4)

Sol.



Atomic radii order : C < S < Al < Cs

Atomic radius of C : 170 pm

Atomic radius of S : 180 pm

Atomic radius of Al : 184 pm

Atomic radius of Cs : 300 pm

18. Match the metals (Column I) with the coordination compound(s) / enzyme(s) (Column II)

Column-I	***Column-II***
Metals	***Coordination compound(s) / Enzyme(s)***
(A) Co	(i) Wilkinson catalyst
(B) Zn	(ii) Chlorophyll
(C) Rh	(iii) Vitamin B_{12}
(D) Mg	(iv) Carbonic anhydrase

(1) A-ii ; B-i ; C-iv ; D-iii

(2) A-iii ; B-iv ; C-i ; D-ii

(3) A-iv ; B-iii ; C-i ; D-ii

(4) A-i ; B-ii ; C-iii ; D-iv

Ans. (2)

Sol. (i) Wilkinson catalyst : $RhCl(PPh_3)_3$

(ii) Chlorophyll : $C_{55}H_{72}O_5N_4Mg$

(iii) Vitamin B_{12}(also known as cyanocobalamin) contain cobalt.

(iv) Carbonic anhydrase contains a zinc ion.

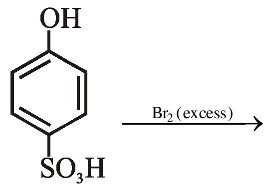
19. A 10 mg effervescent tablet contianing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet ? [Molar mass of $NaHCO_3$ = 84 g mol^{-1}]

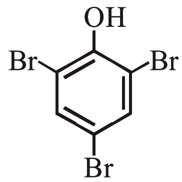
(1) 16.8 (2) 8.4

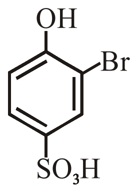
(3) 0.84 (4) 33.6

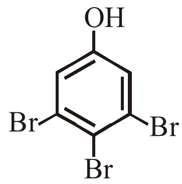
Ans. (1)

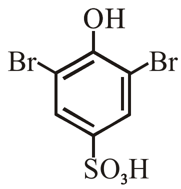
20. The major product of the following reaction is :



(1) 

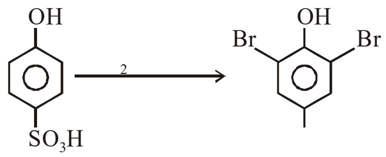
(2) 

(3) 

(4) 

Ans. (1)

Sol.



+ $SO_3\uparrow$

Due to Ipso attack

21. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively. are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is :

(1) $2C_P \ln\left(\frac{T_1+T_2}{4T_1T_2}\right)$ (2) $2C_P \ln\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

(3) $C_P \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (4) $2C_P \ln\left[\frac{T_1+T_2}{2T_1T_2}\right]$

Ans. (3)

22. The chloride that CANNOT get hydrolysed is :

(1) $SiCl_4$ (2) $SnCl_4$

(3) $PbCl_4$ (4) CCl_4

Ans. (4)

Sol. CCl_4 cannot get hydrolyzed due to the absence of vacant orbital at carbon atom.

23. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as :

$$\Delta_r G^o \text{ (in kJ mol}^{-1}) = 120 - \frac{3}{8}T$$

The major component of the reaction mixture at T is :

(1) X if T = 315 K

(2) X if T = 350 K

(3) Y if T = 300 K

(4) Y if T = 280 K

Ans. (1)

24. The freezing point of a diluted milk sample is found to be –0.2°C, while it should have been –0.5°C for pure milk. How much water has been added to pure milk to make the diluted sample ?

(1) 2 cups of water to 3 cups of pure milk
(2) 1 cup of water to 3 cups of pure milk

(3) 3 cups of water to 2 cups of pure milk
(4) 1 cup of water to 2 cups of pure milk

Ans. (3)

25. A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid ?

(Avogadro constant $\cong 6 \times 10^{23}$ mol^{-1}, $\pi \cong 3$)

(1) 0.0216 kg mol^{-1} (2) 0.0305 kg mol^{-1}

(3) 0.4320 kg mol^{-1} (4) 0.0432 kg mol^{-1}

Ans. (2)

26. The polymer obtained from the following reactions is :

(1) $\left[\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{H}{N} \right]_n$

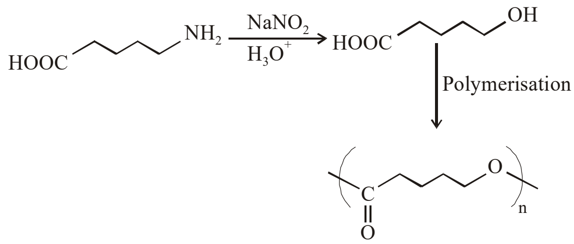
(2) $\left[O-(CH_2)_4-\overset{O}{\overset{\|}{C}} \right]_n$

(3) $\left[HN\overset{O}{\overset{\|}{C}}(CH_2)_4-\overset{O}{\overset{\|}{C}}-\overset{H}{N} \right]_n$

(4) $\left[O\overset{O}{\overset{\|}{C}}(CH_2)_4O \right]_n$

Ans. (2)

Sol.



27. An example of solid sol is :

(1) Butter (2) Gem stones

(3) Paint (4) Hair cream

Ans. (2)

28. Peoxyacetyl nitrate (PAN), an eye irritant is produced by :

(1) Acid rain (2) Photochemical smog

(3) Classical smog (4) Organic waste

Ans. (2)

Sol. Photochemical smog produce chemicals such as formaldehyde, acrolein and peroxyacetyl nitrate (PAN).

29. NaH is an example of :

(1) Electron-rich hydride (2) Molecular hydride

(3) Saline hydride (4) Metallic hydride

Ans. (3)

Sol. NaH is an example of ionic hydride which is also known as saline hydride.

30. The amphoteric hydroxide is :

(1) $Ca(OH)_2$ (2) $Be(OH)_2$

(3) $Sr(OH)_2$ (4) $Mg(OH)_2$

Ans. (2)

Sol. $Be(OH)_2$ is amphoteric in nature while rest all alkaline earth metal hydroxide are basic in nature.

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then $|p|$ is :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{\sqrt{5}}$

(3) $\frac{1}{\sqrt{6}}$

(4) $\frac{1}{\sqrt{3}}$

Ans. (1)

Sol. A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$

2. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

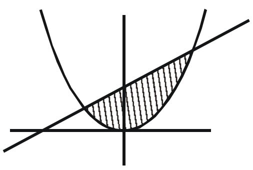
(1) $\frac{5}{4}$

(2) $\frac{9}{8}$

(3) $\frac{3}{4}$

(4) $\frac{7}{8}$

Ans. (2)

Sol. 

$x = 4y - 2$ & $x^2 = 4y$

$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$x = 2, -1$

So, $\int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{9}{8}$

3. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

(1) 2 (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{3}$ (4) $\sqrt{2}$

Ans. (4)

Sol. Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

4. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

(1) $\frac{4}{9}$ (2) $\frac{2}{9}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $\frac{a}{1-r} = 3$ ...(1)

$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ as $|r| < 1$

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is :

(1) $-14\hat{i} - 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-10\hat{i} + 5\hat{j}$ (4) $-14\hat{i} + 5\hat{j}$

Ans. (3)

Sol. $\left[\vec{a} \ \ \vec{b} \ \ \vec{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

Hence $\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$

$= -10\hat{i} + 5\hat{j}$

6. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27} (i = \sqrt{-1})$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85

(3) –91 (4) 91

Ans. (4)

Sol. $\left(-2 - \frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$

$= \frac{-198 - 107i}{27} = \frac{x + iy}{27}$

Hence, y – x = 198 – 107 = 91

7. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval (–2, 2), g is :-

(1) differentiable at all points

(2) not differentiable at two points

(3) not continuous

(4) not differentiable at one point

Ans. (4)

Sol. $|f(x)| = \begin{cases} 1 & , \ -2 \le x < 0 \\ 1 - x^2 & , \ 0 \le x < 1 \\ x^2 - 1 & , \ 1 \le x \le 2 \end{cases}$

and $f(|x|) = x^2 - 1$, $x \in [-2, 2]$

Hence $g(x) = \begin{cases} x^2 & , \ x \in [-2, 0) \\ 0 & , \ x \in [0, 1) \\ 2(x^2 - 1) & , \ x \in [1, 2] \end{cases}$

It is not differentiable at x = 1

8. Let f : R → R be defined by $f(x) = \frac{x}{1 + x^2}$, $x \in R$. Then the range of f is :

(1) $(-1, 1) - \{0\}$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $R - [-1, 1]$

Ans. (2)

Sol. $f(0) = 0$ & $f(x)$ is odd.

Further, if x > 0 then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

9. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

(1) 6 (2) 8 (3) 0 (4) 4

Ans. (3)

Sol. $T_5 = {}^8C_4\frac{x^{12}}{81}\times\frac{81}{x^4}=5670$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x=\pm\sqrt{3}$

10. The value of r for which

${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+....\,{}^{20}C_0\,{}^{20}C_r$ is maximum, is

(1) 20 (2) 15

(3) 11 (4) 10

Ans. (1)

Sol. Given sum = coefficient of x^r in the expansion of $(1+x)^{20}(1+x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when r = 20

11. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1}=25$, then $\frac{a_9}{a_5}$ equals :

(1) $2(5^2)$ (2) $4(5^2)$

(3) 5^4 (4) 5^3

Ans. (3)

Sol. $a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$\frac{a_3}{a_1}=25 \Rightarrow \frac{a_1r^2}{a_1}=25 \Rightarrow r^2 = 25$

$\frac{a_9}{a_5}=\frac{a_1r^8}{a_1r^4}=r^4=5^4$

12. If $\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$

(3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

Ans. (2)

Sol. $\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m+C$

$\int\frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx$,

Put $\frac{1}{x^2}-1=t \Rightarrow \frac{dt}{dx}=\frac{-2}{x^3}$

Case-1 $x \geq 0$

$-\frac{1}{2}\int\sqrt{t}\ dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$

$\Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$

$A(x)=-\frac{1}{3x^3}$ and m = 3

$(A(x))^m=\left(-\frac{1}{3x^3}\right)^3=-\frac{1}{27x^9}$

Case-II $x \leq 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x)=\frac{1}{-3x^3}$, m = 3

$(A(x))^m=\frac{-1}{27x^9}$

13. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

Ans. **(2)**

Sol. Given a + b = x and ab = y

If $x^2 - c^2 = y \Rightarrow (a+b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

14. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$

(where [x] denotes the greatest integer less than ^{20}Cr or equal to x) is :

(1) 4 (2) 4 – sin4

(3) sin 4 (4) 0

Ans. **(4)**

Sol. $$I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right] = -1 \text{ as } \quad x \neq n\pi\right)$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx = 0$$

15. If the system of linear equations

2x + 2y + 3z = a

3x – y + 5z = b

x – 3y + 2z = c

where a, b, c are non-zero real numbers, has more then one solution, then :

(1) b – c – a = 0 (2) a + b + c = 0

(3) b + c – a = 0 (4) b – c + a = 0

Ans. **(1)**

Sol. P_1 : 2x + 2y + 3z = a

P_2 : 3x – y + 5z = b

P_3 : x – 3y + 2z = c

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

16. A square is inscribed inthe circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the corrdinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2) $\sqrt{137}$

(3) 6 (4) $\sqrt{41}$

Ans. **(4)**

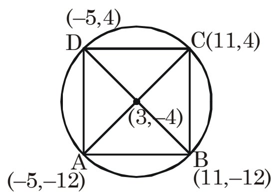
Sol. $R = \sqrt{9+16+103} = 8\sqrt{2}$

OA = 13

$OB = \sqrt{265}$

$OC = \sqrt{137}$

$OD = \sqrt{41}$



17. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :-

(1) $\frac{5}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

Ans. **(4)**

Sol. $f_4(x) - f_6(x)$

$$= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4}\left(1-\frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1-\frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

18. Let [x] denote the greatest integer less than or equal to x. Then :-

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$$

(1) equals π

(2) equals 0

(3) equals $\pi + 1$

(4) does not exist

Ans. (4)

Sol. R.H.L. $= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+x^2}{x^2}$$

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)}{\left(\pi\sin^2 x\right)}+1=\pi+1$$

L.H.L. $= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$$\lim_{x\to 0+}\frac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x}.\frac{\pi\sin^2 x}{x^2}+\left(-1+\frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L. $\neq$ L.H.L.

19. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an anlge $\frac{\pi}{4}$ with the plane y–z+5=0 are:

(1) $2\sqrt{3}$, 1, –1

(2) 2, $\sqrt{2}$, $-\sqrt{2}$

(3) 2, –1, 1

(4) $\sqrt{2}$, 1, –1

Ans. (2, 4)

Sol. Let the equation of plane be

$a(x - 0) + b(y + 1) + c(z - 0) = 0$

It passes through (0,0,1) then

$b + c = 0$ ...(1)

Now $\cos\frac{\pi}{4}=\frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow a=\pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $\left(\sqrt{2}, -1, 1\right)$ or $\left(\sqrt{2}, 1, -1\right)$

20. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then dy/dx at x = e is equal to :

(1) $\frac{e}{\sqrt{4+e^2}}$

(2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(4) $\frac{(1+2e)}{\sqrt{4+e^2}}$

Ans. (3)

Sol. Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \text{ as } y(e)=\sqrt{4+e^2}$$

21. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(1) $\frac{\sqrt{5}}{4}$

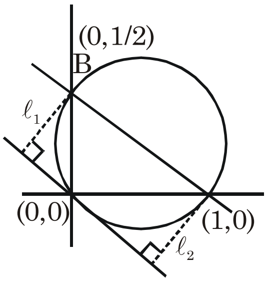
(2) $\frac{\sqrt{5}}{2}$

(3) $2\sqrt{5}$

(4) $4\sqrt{5}$

Ans. **(2)**

Sol.



Equation of circle

$$(x-1)(x-0)+(y-0)\left(y-\frac{1}{2}\right)=0$$

$$\Rightarrow x^2+y^2-x-\frac{y}{2}=0$$

Equation of tangent of origin is 2x + y = 0

$$\ell_1+\ell_2=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}}$$

$$=\frac{4+1}{2\sqrt{5}}=\frac{\sqrt{5}}{2}$$

22. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(1) $(p \vee r) \rightarrow (p \wedge r)$

(2) $p \vee r$

(3) $p \wedge r$

(4) $(p \wedge r) \rightarrow (p \vee r)$

Ans. **(4)**

Sol. Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow$ q is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

23. If y(x) is the solution of the differential equation

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x},\ x>0,$$

where $y(1)=\frac{1}{2}e^{-2}$, then :

(1) y(x) is decreasing in (0, 1)

(2) y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

(3) $y(\log_e 2)=\frac{\log_e 2}{4}$

(4) $y(\log_e 2)=\log_e 4$

Ans. **(2)**

Sol. $$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$$

$$\text{I.F.}=e^{\int\left(\frac{2x+1}{x}\right)dx}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ell nx}=e^{2x}.x$$

So, $y(xe^{2x})=\int e^{-2x}.xe^{2x}+C$

$\Rightarrow xye^{2x}=\int x\,dx+C$

$\Rightarrow 2xye^{2x}=x^2+2C$

It passess through $\left(1, \frac{1}{2}e^{-2}\right)$ we get C = 0

$$y=\frac{xe^{-2x}}{2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}e^{-2x}(-2x+1)$$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

$$y(\log_e 2)=\frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

$$=\frac{1}{8}\log_e 2$$

24. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is :

(1) 122 (2) –222
(3) –122 (4) 222

Ans. **(1)**

Sol. $S = \{x \in R, x^2 + 30 - 11x \le 0\}$
$= \{x \in R, 5 \le x \le 6\}$
Now $f(x) = 3x^3 - 18x^2 + 27x - 40$
$\Rightarrow f'(x) = 9(x - 1)(x - 3)$,
which is positive in [5, 6]
$\Rightarrow f(x)$ increasing in [5, 6]
Hence maximum value = $f(6) = 122$

25. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –81 (2) 100 (3) –300 (4) 144

Ans. **(3)**

Sol. $81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27}$

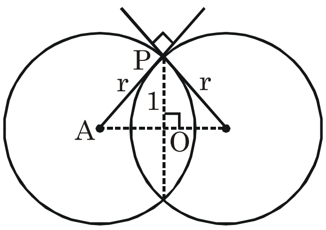
$\Rightarrow k = \pm 300$

26. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) $\sqrt{2}$
(3) $2\sqrt{2}$ (4) 2

Ans. **(4)**

Sol. In ΔAPO



$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$

So distance between centres = $\sqrt{2}\,r = 2$

27. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbole $xy = 2$ is :

(1) $x + 2y + 4 = 0$
(2) $x - 2y + 4 = 0$
(3) $x + y + 1 = 0$
(4) $4x + 2y + 1 = 0$

Ans. **(1)**

Sol. Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \frac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$$

$$\Rightarrow x^2 m + \frac{x}{m} - 2 = 0$$

$D = 0 \Rightarrow m = -\frac{1}{2}$

So tangent is $2y + x + 4 = 0$

28. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(1) (2, 0, –2) (2) (–2, 2, 2)
(3) (0, –2, 2) (4) (2, 2, 0)

Ans. **(1)**

Sol. The normal vector of required plane

$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right) \times \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)
So required plane is
$-(x - 3) + (y + 2) + (z - 1) = 0$
$\Rightarrow -x + y + z + 4 = 0$
Which is satisfied by (2, 0, –2)

29. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted betwen the coordinate axes lie on the curve :

(1) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (2) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(3) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (4) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

Ans. (3)

Sol. Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta}+\frac{y}{b\operatorname{cosec}\theta}=1$$

$a=\sqrt{2}$, b = 1

$$\Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\operatorname{cosec}\theta}=1$$

Let the midpoint be (h, k)

$$h=\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$$

and $k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta=\frac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$$

$$\Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$$

30. Two integers are selected at random from the set {1, 2,...., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$

(2) $\frac{1}{2}$

(3) $\frac{3}{5}$

(4) $\frac{7}{10}$

Ans. (1)

Sol. Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^5C_2 + {}^6C_2$

so required probability $= \frac{{}^5C_2}{{}^5C_2+{}^6C_2}$