FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. A beam of plane polarised light of large cross sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to :

(1) 1.0×10^{-5} J (2) 5.0×10^{-4} J

(3) 1.0×10^{-4} J (4) 1.5×10^{-4} J

Sol. Intensity, I = 3.3 Wm^{-2}

Area, A = 3 × 10–4 m^2

Angular speed, ω = 31.4 rad/s

$\because \langle\cos^2\theta\rangle = \frac{1}{2}$, in one time period

$\therefore$ Average energy = $I_0A \times \frac{1}{2}$

$= \frac{(3.3)(3\times10^{-4})}{2}$

$\simeq 5\times10^{-4}$J

2. Match the C_P/C_V ratio for ideal gases with different type of molecules :

Molecular type	**C_P/C_V**
(A) Monoatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-II, C-I, D-III

(3) A-III, B-IV, C-II, D-I

(4) A-II, B-III, C-I, D-IV

Sol. $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

where 'f' is degree of freedom

(A) Monoatomic f = 3, $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$

(B) Diatomic rigid molecules,

f = 5, $\gamma = 1 + \frac{2}{3} = \frac{7}{5}$

(C) Diatomic non-rigid molecules

f = 7, $\gamma = 1 + \frac{2}{7} = \frac{9}{7}$

(D) Triatomic rigid molecules

f = 6, $\gamma = 1 + \frac{2}{6} = \frac{4}{3}$

3. Choose the correct option relating wavelengths of differnet parts of electromagnetic wave spectrum :

(1) $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

(2) $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

(3) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

(4) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$

Sol. Information based

$\lambda_{\text{radiowaves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

4. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2)

(1) 3.15 gm (2) 4.51 gm

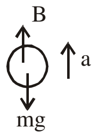
(3) 4.15 gm (4) 1.52 gm

Sol. Volume $V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3}\times(1)^3 = 4.19\text{cm}^3$

$a = 9.8\ \text{cm/s}^2$

$B - mg = ma$

$m = \frac{B}{g+a}$



$m = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\frac{a}{g}}$

$= \frac{(4.19)\times 1}{1+\frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15\text{gm}$

5. Dimensional formula for thermal conductivity is (here K denotes the temperature)

(1) $MLT^{-3}K$ (2) $MLT^{-2}K$

(3) $MLT^{-2}K^{-2}$ (4) $MLT^{-3}K^{-1}$

Sol. $\because \frac{d\theta}{dt} = kA\frac{dT}{dx}$

$k = \frac{\left(\frac{d\theta}{dt}\right)}{A\left(\frac{dT}{dx}\right)}$

$[k] = \frac{[ML^2T^{-3}]}{[L^2][KL^{-1}]} = [MLT^{-3}K^{-1}]$

6. On the x-axis and a dsitance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

(1) $\frac{A}{(x^2+a^2)^{1/2}}$ (2) $\frac{A}{(x^2+a^2)^{3/2}}$

(3) $A(x^2 + a^2)^{3/2}$ (4) $A(x^2 + a^2)^{1/2}$

Sol. Given $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}$, $V_\infty = 0$

$$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G.\vec{d}_x$$

$$V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}}dx$$

put $x^2 + a^2 = z$

$2x\ dx = dz$

$$V_x - 0 = -\int_\infty^x \frac{A\,dz}{2(z)^{3/2}} = \left[\frac{A}{z^{1/2}}\right]_\infty^x = \left[\frac{A}{(x^2+a^2)^{1/2}}\right]_\infty^x$$

$$V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

7. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\ \text{ms}^{-1}$, a particle moves in the x-y plane with a constant acceleration of $(10\hat{i}+4\hat{j})\text{ms}^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0, are respectively :

(1) 4s and 52 m (2) 2s and 24 m

(3) 2s and 18 m (4) 5s and 25 m

Sol. Given $\vec{u} = 5\hat{j}\,m/s$, $\vec{a} = 10\hat{i}+4\hat{j}$, final coordinate (20, y_0) in time t

$S_x = 4_x t + \frac{1}{2}a_x t^2$

$20 - 0 = 0 + \frac{1}{2} \times 10 \times t^2$

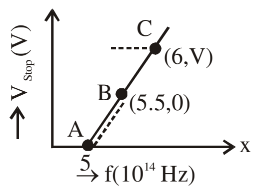
$t = 2\text{sec}$

$S_y = u_y \times t + \frac{1}{2}a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2}4 \times 2^2 = 18\text{m}$

2 sec and 18 m

8. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surfface is : (Plancks constant $h = 6.62 \times 10^{-34}$ J.s)



(1) 2.27 eV (2) 2.59 eV

(3) 1.93 eV (4) 2.10 eV

Sol. Graph of V_s and f given (B 5.5, 0)

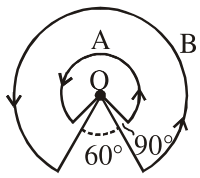
$h\nu = \phi + eV_s$

at B $V_s = 0, \quad \nu = 5.5$

$\Rightarrow \quad h \times 5.5 \times 10^{14} = \phi$

$$\phi = \frac{6.62\times10^{-34}\times5.5\times10^{14}}{1.6\times10^{-19}}eV = 2.27eV$$

9. A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is :



(1) 4 : 6 (2) 6 : 4

(3) 6 :5 (4) 2 : 5

Sol. Given $i_A = 2$, $r_A = 2$ cm, $\theta_A = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

$i_B = 3$, $r_B = 4$ cm, $\theta_B = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$B = \frac{\mu_0 I\theta}{4\pi R}$$

$$\frac{B_A}{B_B} = \frac{I_A}{I_B} \times \frac{\theta_A R_B}{\theta_B R_A} = \frac{6}{5}$$

10. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are :

(1) 1, 2, 3, ... (2) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots$

(3) 1, 3, 5, ... (4) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \dots$

Sol. Given T to C 1.5 m

C to C 5m

T to C = $(2n_1 + 1)\frac{\lambda}{2}$

C to C = $n_2\lambda$

$$\frac{1.5}{5} = \frac{(2n_1+1)}{2n_2} \Rightarrow 3n_2 = 10n_1 + 5$$

$n_1 = 1, n_2 = 5 \rightarrow \lambda = 1$

$n_1 = 4, n_2 = 15 \rightarrow \lambda = 1/3$

$n_1 = 7, n_2 = 25 \rightarrow \lambda = 1/5$

11. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is :

(1) 9.2×10^{-3} J (2) 6.4×10^{-2} J

(3) 11.7×10^{-3} J (4) 7.2×10^{-2} J

Sol. Torque on a bar magnet : I = MB sin θ

Here, θ = 30°, I = 0.018 N-m, B = 0.06 T

$\Rightarrow 0.018 = M \times 0.06 \times \sin 30°$

$\Rightarrow 0.018 = M \times 0.06 \times \frac{1}{2}$

$\Rightarrow M = 0.6$ A-m²

Now v = – MB cos θ

Position of stable equilibrium (θ = 0°) : $u_i = -MB$

Position of unstable equilibrium (θ = 180°) : $u_f = MB$

⇒ work done : ΔU

⇒ W = 2MB

$\Rightarrow W = 2 \times 0.6 \times 0.06$

$\Rightarrow W = 7.2 \times 10^{-2}$ J

option (4) is correct

12. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change Δλ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is :

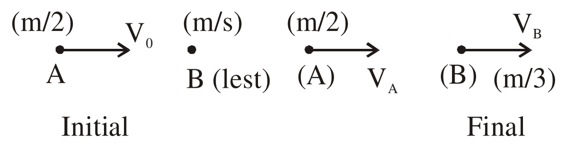
(1) $\Delta\lambda = 4\lambda_0$

(2) $\Delta\lambda = \frac{5}{2}\lambda_0$

(3) $\Delta\lambda = 2\lambda_0$

(4) $\Delta\lambda = \frac{3}{2}\lambda_0$

Sol.



Applying momentum conservation

$$\frac{m}{2}\times V_0 + \frac{m}{3}\times(0) = \frac{m}{2}V_A + \frac{m}{3}V_B$$

$$= \frac{V_0}{2} = \frac{V_A}{2} + \frac{V_B}{3} \quad .... (1)$$

Since, collision is elastic (e = 1)

$$e = 1 = \frac{V_B - V_A}{V_0} \Rightarrow V_0 = V_B - V_A \quad ... (2)$$

On solving (1) & (2) : $V_A = \frac{V_0}{5}$

Now, De-Broglie wavelength of A before collision :

$$\lambda_0 = \frac{h}{m_A V_0} = \frac{h}{\left(\frac{m}{2}\right)V_0}$$

$$\Rightarrow \lambda_0 = \frac{2h}{mV_0}$$

Final De-Broglie wavelength :

$$\lambda_f = \frac{h}{m_A V_0} = \frac{h}{\frac{m}{2}\times\frac{V_0}{5}} \Rightarrow \lambda_f = \frac{10h}{mV_0}$$

Now $\Delta\lambda = \lambda_f - \lambda_0$

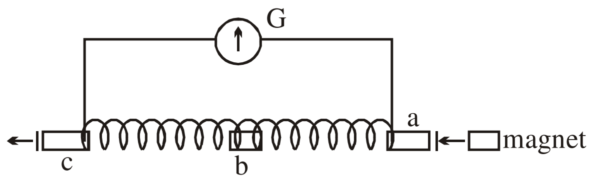
$$\Delta\lambda = \frac{10h}{mV_0} - \frac{2h}{mV_0}$$

$\Rightarrow \Delta\lambda = \dfrac{8h}{mv_0} \quad \Rightarrow \Delta\lambda = 4 \times \dfrac{2h}{mv_0}$

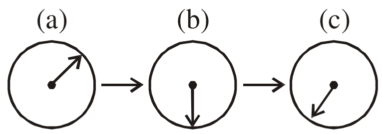
$\Rightarrow \Delta\lambda = 4\lambda_0$

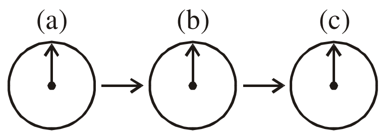
option (1) is correct.

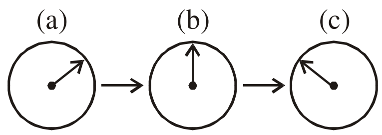
13. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations wil be seen on the galvanometer G attached across the coil ?

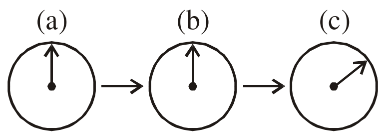


Three positions shown describe : (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.

(1) 

(2) 

(3) 

(4) 

Sol. When bar magnet is entering with constant speed, flux will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.

When magnet is completely inside, flux will not change, so reading of galvanometer will be zero.

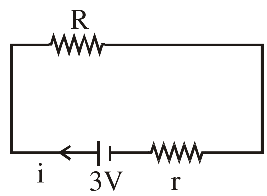
When bar magnet is making on exit, again flux will change and on e.m.f. is induced in opposite direction to not of (a), so galvanometer will deflect in negative direction.

Looking at options, option (3) is correct.

14. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is :

(1) 0.50 W (2) 0.125 W

(3) 0.072 W (4) 0.10 W

Sol. 

$P_R = 0.5W$

$\Rightarrow i^2R = 0.5W$

Also, $V = E - ir$

$2.5 = 3 - ir$

$\Rightarrow ir = 0.5$

Power dissipated across 'r' : $P_r = i^2r$

Now $iR = 2.5$

$ir = 0.5$

On dividing : $\dfrac{R}{r} = 5$

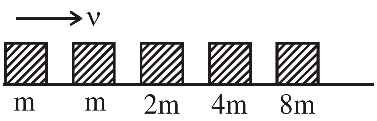
Now $\frac{P_R}{P_r} = \frac{i^2R}{i^2r} \Rightarrow \frac{P_R}{P_r} = \frac{R}{r} \Rightarrow \frac{P_R}{P_r} = 5$

$\Rightarrow P_r = \frac{P_R}{5}$

$\Rightarrow P_r = \frac{0.50}{5} \Rightarrow P_r = 0.10$ W

option (4) is correct.

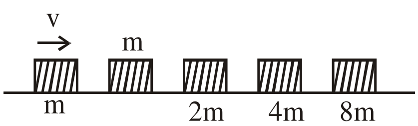
15. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to :



(1) 77 (2) 37

(3) 87 (4) 94

Sol.



All collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So let the final velocity be v', so on applying momentum conservation:

mv = 16m v' $\Rightarrow$ v' = v/16

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2} \times 16m \times \left(\frac{v}{16}\right)^2$

$\Rightarrow E_f = \frac{1}{2}m\frac{v^2}{16}$

Energy loss : $E_i - E_f$

$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$\Rightarrow \frac{1}{2}mv^2\left[1 - \frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$

$\%p = \frac{\text{Energy loss}}{\text{Original energy}} \times 100$

$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2} \times 100 = 93.75\%$

$\Rightarrow$ Value of P is close to 94.

16. The specific heat of water = 4200 J kg^{-1} K^{-1} and the latent heat of ice = 3.4 × 10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) :

(1) 61.7 (2) 63.8

(3) 69.3 (4) 64.6

Sol. Here the water will provide heat for ice to melt therefore

$m_w s_w \Delta\theta = m_{ice} L_{ice}$

$m_{ice} = \frac{0.2 \times 4200 \times 25}{3.4 \times 10^5}$

= 0.0617 kg

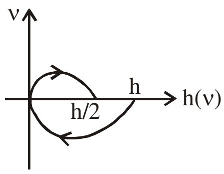
= 61.7 gm

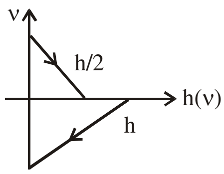
Remaining ice will remain un-melted

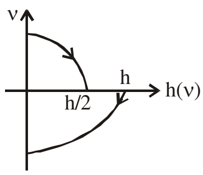
so correct answer is 1

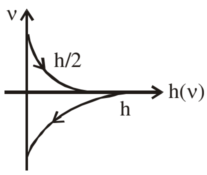
17. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :

(graph are drawn schematically and on not to scale)

(1) 

(2) 

(3) 

(4) 

Sol. Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

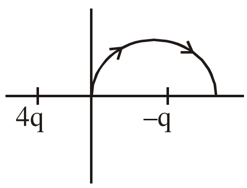
$v^2 = 2gh$

$$v\frac{dv}{dh} = 2g = \text{const.}$$

$$\frac{dv}{dh} = \frac{\text{constant}}{v}$$

Here we can see slope is very high when velocity is low therefore at Maximum height the slope should be very large which is in option 3 and as velocity increases slope must decrease there for option 3 is correct.

18. A two point charges 4q and –q are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will :



(1) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$

(2) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

(3) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

(4) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

Sol. Potential of –q is same as initial and final point of the path therefore potential due to 4q will only change and as potential is decreasing the energy will decrease

Decrease in potential energy = q (V_i – V_f)

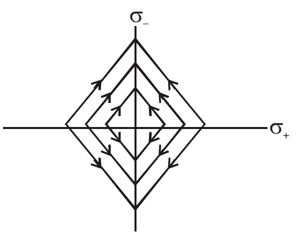
Decrease in potential energy

$$= q\left[\frac{k4q}{d/2} - \frac{k4q}{3d/2}\right] = \frac{4q^2}{3\pi\varepsilon_0 d}$$

Therefore correct answer is 3.

19. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_- where $|\sigma_+| > |\sigma_-|$ intersect at right angle. Which of the following best represents the electric field lines for this system :

(1)



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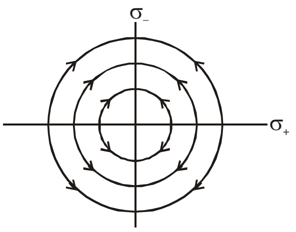

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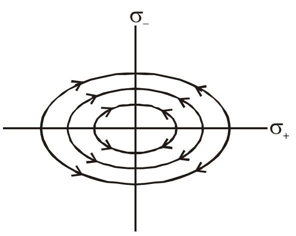

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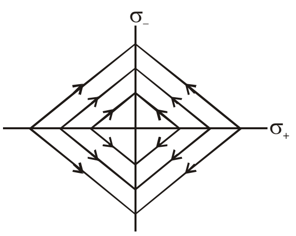
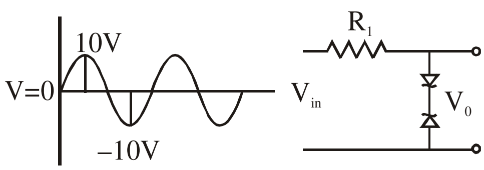


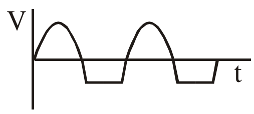
Sol. Thin infinite uniformly charged planes produces uniform electric field therefore option 2 and option 3 are obviously wrong.

And as positive charge density is bigger in magnitude so its field along Y direction will be bigger than field of negative charge in X direction and this is evident in option 1 so it is correct.

20. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale)



(1)



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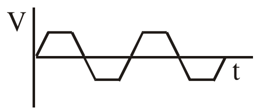
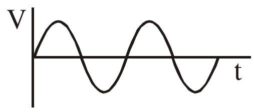


(3)



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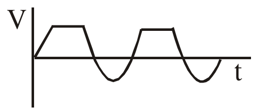
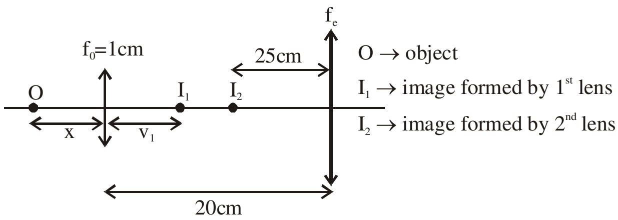


Sol. As there are two zener diodes in reverse polarity so if one is in forward bias the other will be in reverse bias and above 6V the reverse bias will too be in conduction mode. Therefore when voltage is more than 6V the output will be constant. And when it is less than 6V it will follow the input voltage so correct answer is two.

21. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________.

Sol.



for first lens = $\frac{1}{v_1}-\frac{1}{-x}=\frac{1}{1} \Rightarrow v_1=\frac{x}{x-1}$

also magnification $|m_1| = \left|\frac{v_1}{u_1}\right| = \frac{1}{x-1}$

for 2nd lens this is acting as object

so $u_2 = -(20 - v_1) = -\left(20-\frac{x}{x-1}\right)$

and $v_2 = -25\text{cm}$

angular magnification $|m_A| = \left|\frac{D}{u_2}\right| = \frac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$

$$\left(\frac{1}{x-1}\right)\left(\frac{25}{20-\frac{x}{x-1}}\right)=100$$

$$\frac{25}{20(x-1)-x}=100 \Rightarrow 1=80(x-1)-4x$$

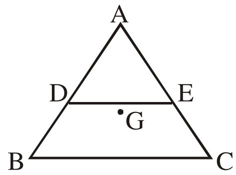
$$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76}$$

$$\Rightarrow u_2 = -\left(20-\frac{81/76}{81/76-1}\right)=\frac{-19}{5}$$

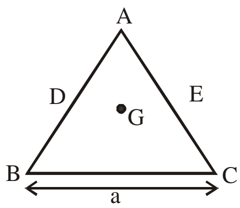
now by lens formula

$$\frac{1}{-25}-\frac{1}{-19/5}=\frac{1}{f_e} \Rightarrow f_e=\frac{25\times 19}{106} \approx 4.48\text{cm}$$

22. ABC is a plane lamina of the shape of an equilateral triagnle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _____.



Sol. Let side of triangle is a and mass is m



MOI of plate ABC about centroid

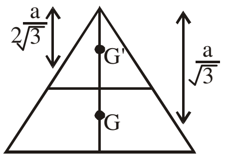
$$I_0 = \frac{m}{3}\left(\left(\frac{a}{2\sqrt{3}}\right)^2\times 3\right)=\frac{ma^2}{12}$$

triangle ADE is also an equilateral triangle of side a/2.

Let moment of inertia of triangular plate ADE about it's centroid (G') is I_1 and mass is m_1

$$m_1 = \frac{m}{\frac{\sqrt{3}a^2}{4}} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4\times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



distance GG' $= \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

so MOI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}.\frac{a^2}{12}$$

$$= \frac{5ma^2}{192}$$

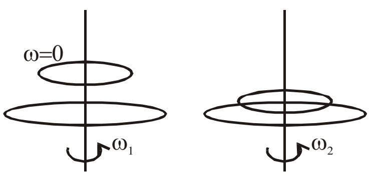
now MOI of remaining part

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12\times 16} = \frac{11I_0}{16}$$

$\Rightarrow$ N = 11

23. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is ______.

Sol.



Let moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

$\Rightarrow$ MOI of small disc $I_2 = \frac{M\left(\frac{R}{2}\right)^2}{2} = \frac{I}{4}$

by angular momentum conservation

$$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2 \Rightarrow \omega_2 = \frac{4\omega_1}{5}$$

initial kinetic energy $K_1 = \frac{1}{2}I\omega_1^2$

final kinetic energy K_2

$$= \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$P\% = \frac{K_1 - K_2}{K_1}\times 100\% = \frac{1-4/5}{1}\times 100 = 20\%$$

24. A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be closed to ______.

Sol. As work done on gas and heat supplied to the gas are zero,

total internal energy of gases remain same

$u_1 + u_2 = u_1' + u_2'$

$(0.1)\ C_v\ (200) + (0.05)\ C_v\ (400) = (0.15)C_v\ T$

$$T = \frac{800}{3}k = 266.67\,k$$

25. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschan series in Å is : ______.

Sol. $\lambda = \dfrac{c}{\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)}$

for lyman series

$$\lambda_1 = \frac{c}{\frac{1}{1^2} - \frac{1}{\infty^2}} = c \text{ (n = } \infty \text{ to n = 1)}$$

$$\lambda_2 = \frac{c}{\frac{1}{1^2} - \frac{1}{2^2}} = \frac{4c}{3} \text{ (n = 2 to n = 1)}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{c}{3} = 304\text{Å} \Rightarrow c = 912\text{Å}$$

for paschen series

$$\lambda_1 = \frac{c}{\frac{1}{3^2} - \frac{1}{\infty^2}} = 9c \quad \text{(n = } \infty \text{ to n = 3)}$$

$$\lambda_2 = \frac{c}{\frac{1}{3^2} - \frac{1}{4^2}} = \frac{144c}{7} \text{ (n = 4 to n = 3)}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{144c}{7} - 9c = \frac{81c}{7} = \frac{81 \times 912}{7}$$

= 10553.14 Å

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. On heating, lead(II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

(1) +5 (2) +2

(3) +4 (4) +3

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + \underset{\text{Brown gas (A)}}{2NO_2} + \frac{1}{2}O_2(g)$

$NO_2(g) \xrightarrow{\text{Cooling}} \underset{(B)}{N_2O_4}$

$N_2O_4 + NO \xrightarrow{\Delta} \underset{\text{Blue Solid (C)}}{N_2O_3}$

O.S. of nitrogen in N_2O_3 is + 3

N_2O_3 $2x + 3(-2) = 0$

$x = +3$

2. Which of the following will react with $CHCl_3$ + alc. KOH ?

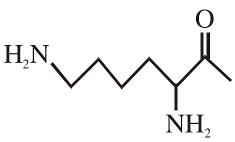
(1) Adenine and lysine

(2) Adenine and thymine

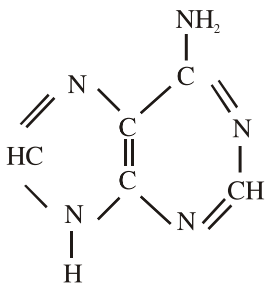
(3) Adenine and proline

(4) Thymine and proline

Sol. Adenine and lysine Both have primary amine react with $CHCl_3$ + alc. KOH

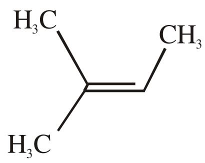
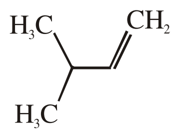


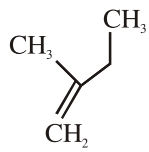
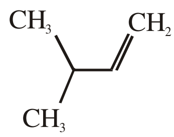
Lysine

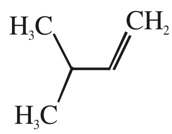
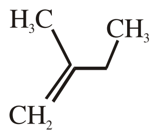


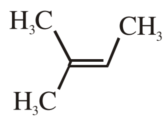
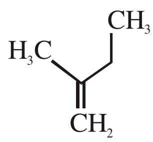
Adenine

3. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?

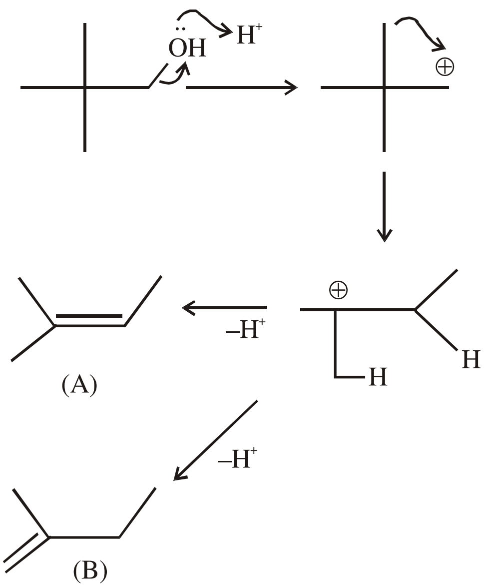
(1)  and 

(2)  and 

(3)  and 

(4)  and 

Sol.



4. Among statements (a) -(d), the correct ones are :

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

(d) Zr and Ti are purified by Van Arkel method.

(1) (c) and (d) only

(2) (a), (c) and (d) only

(3) (b), (c) and (d) only

(4) (a), (b), (c) and (d)

Sol. (a) $\underset{\text{lime stone}}{CaCO_3} \xrightarrow{\Delta} CaO + CO_2$ {In Blast furnace}

(b) Ag form cyanide complex $[Ag(CN)_2]^-$ during cyaride process

$Ag / Ag_2S + CN^{\ominus} \rightarrow [Ag(CN)_2]^-$

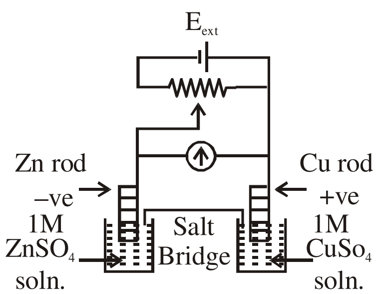
(c) Ni is purified by mond's process

(d) Zr and Ti are purified by van arkel method

All (a), (b), (c), (d) are correct statements

Thus correct option is (4)

5.



$E^o_{Cu^{2+}|Cu} = +0.34V$

$E^o_{Zn^{2+}|Zn} = -0.76V$

Identify the incorrect statement from the options below for the above cell :

(1) If $E_{ext} > 1.1$ V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

(2) If $E_{ext} > 1.1$ V, e^- flows from Cu to Zn

(3) If $E_{ext} = 1.1$ V, no flow of e^- or current occurs

(4) If $E_{ext} < 1.1$ V, Zn dissolves at anode and Cu deposits at cathode

Sol. $E^o_{cell} = 0.34 - (-0.76)$

$= 1.10$ volt

If $E_{ext} > 1.10$ volt

Cu $\rightarrow$ Anode

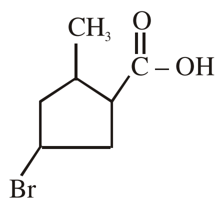
Zn $\rightarrow$ Cathode

If $E_{ext} = 1.10$ volt

Zn $\rightarrow$ Anode

Cu $\rightarrow$ Cathode

6. The IUPAC name of the following compound is :



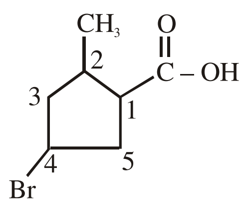
(1) 4-Bromo-2-methylcyclopentane carboxylic acid

(2) 5-Bromo-3-methylcyclopentanoic acid

(3) 3-Bromo-5-methylcyclopentane carboxylic acid

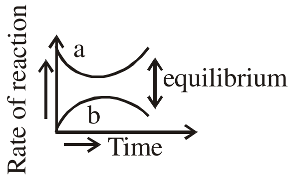
(4) 3-Bromo-5-methylcyclopentanoic acid

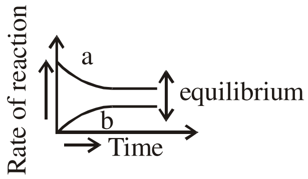
Sol.

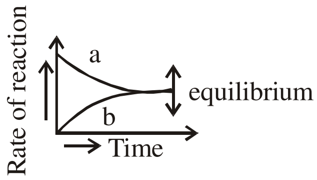


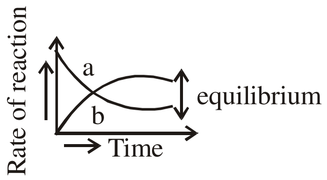
4-bromo-2-methyl cyclopentane carboxylic Acid

7. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) and reverse (b) reaction with time is given by

(1) 

(2) 

(3) 

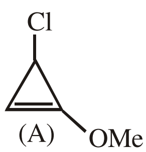
(4) 

Sol. at equilibrium

$r_a = r_b$

8. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is :

(A)



(A)

CH_3CHCH_3 with Cl on the central carbon

(C)

$CH_3CHCH_2NO_2$ with Cl on the central carbon

(D)

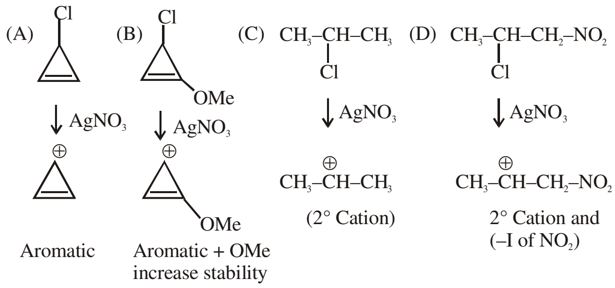
(1) (A) > (B) > (D) > (C)

(2) (A) > (B) > (C) > (D)

(3) (C) > (D) > (A) > (B)

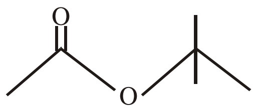
(4) (B) > (A) > (C) > (D)

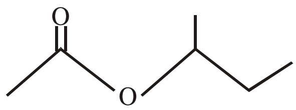
Sol.

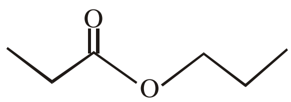


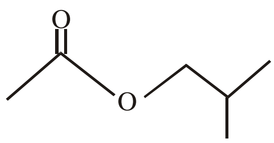
∴ Stability Cation B > A > C > D

9. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' give white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is :

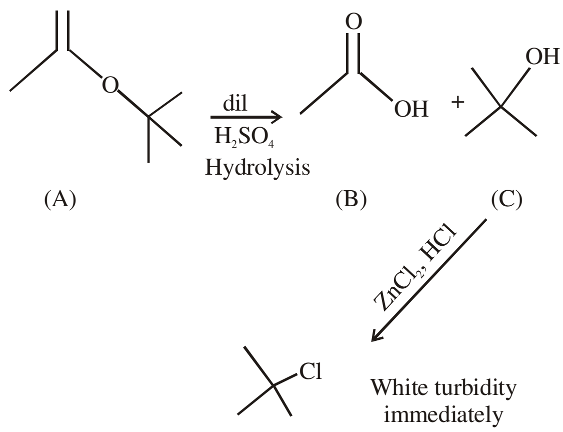
(1) 

(2) 

(3) 

(4) 

Sol.



10. Match the following :

(i) Foam	(a) smoke
(ii) Gel	(b) cell fluid
(iii) Aerosol	(c) jellies
(iv) Emulsion	(d) rubber
	(e) froth
	(f) milk

(1) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)
(2) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)
(3) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)
(4) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)

Sol. Foam - Froth
Gel $\rightarrow$ Jellies
Aerosol $\rightarrow$ Smoke
Sol $\rightarrow$ Cell fluids
Solid sol $\rightarrow$ rubber

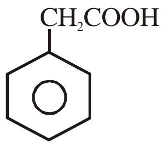
11. The elements with atomic numbers 101 and 104 belong to, respectively :
(1) Group 11 and Group 4
(2) Actinoids and Group 4
(3) Actinoids and Group 6
(4) Group 6 and Actinoids

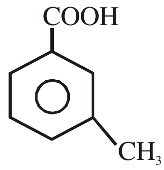
Sol. Element with atomic no. 101 is an Actinoid element.

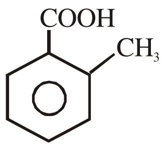
12. On combustion Li, Na and K in excess of air, the major oxides formed, respectively, are :
(1) Li_2O, Na_2O and K_2O_2
(2) Li_2O, Na_2O_2 and K_2O
(3) Li_2O, Na_2O_2 and KO_2
(4) Li_2O_2, Na_2O_2 and K_2O_2

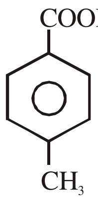
Sol. $Li + \underset{\text{excess}}{O_2} \rightarrow Li_2O$ (Major Oxides)
$Na + " \rightarrow Na_2O_2$ (")
$K + " \rightarrow KO_2$ (")

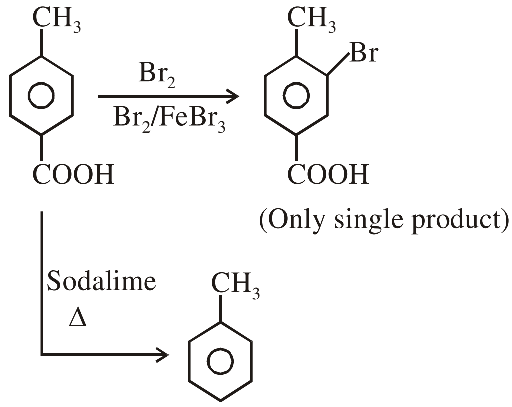
13. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2$ Br while heating [P] with sodalime gave toluene.
The compound [P] is :

(1) 

(2) 

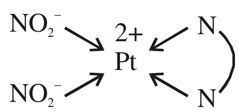
(3) 

(4) 

Sol. 

14. The number of isomers possible for $[Pt(en)(NO_2)_2]$ is :
(1) 3 (2) 2
(3) 1 (4) 4

Sol. $[Pt\ (en)\ (NO_2)_2] \Rightarrow$ does not show G.I. as well as optical isomerism.



This complex will have three linkage isomers as follows :-
$[Pt\ (en)\ (NO_2)2]$ I
$[Pt\ (en)\ (NO_2)(ONO)]$ II
$[Pt\ (en)\ (ONO)_2]$ III

15. The ionic radii of O_2^-, F^-, Na^+ and Mg^{2+} are in the order :

(1) $F^- > O^{2-} > Na^+ > Mg^{2+}$
(2) $Mg^{2+} > Na^+ > F^- > O^{2-}$
(3) $O^{2-} > F^- > Mg^{2+} > Na^+$
(4) $O^{2-} > F^- > Na^+ > Mg^{2+}$

Sol.

	O^{-2}	F^-	Na^+	Mg^{2+}
z	8	9	11	12
e^-	10	10	10	10
$\frac{z}{e}$	0.8	0.9	1.1	1.2

as $\frac{z}{e}$ ratio increases size decreases.

Thus correct ionic radii order is

$O^{-2} > F^- > Na^+ > Mg^{2+}$

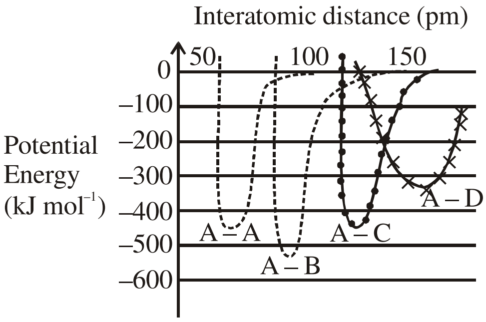
Therefore correct option is (4)

16. The region in the electromagnetic spectrum where the Balmer series lines appear is

(1) Visible
(2) Microwave
(3) Ultraviolet
(4) Infrared

Sol. Balmer series give visible lines For H-atom

17. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :



(1) D is more electronegative than other atoms
(2) A-D has the shortest bond length
(3) A-B has the stiffest bone
(4) A–A has the largest bond enthalpy

Sol. From the given graph, potential energy of A-B molecule is minimum.

Thus A-B bond is most stable and have strongest bond amongst these.

B $\rightarrow$ Most electronegative

D $\rightarrow$ Least electronegative

A-B $\rightarrow$ Shortest bond length

A-B $\rightarrow$ Largest bond enthalpy

Therefore correct option is (3).

18. What are the functional groups present in the structure of maltose ?

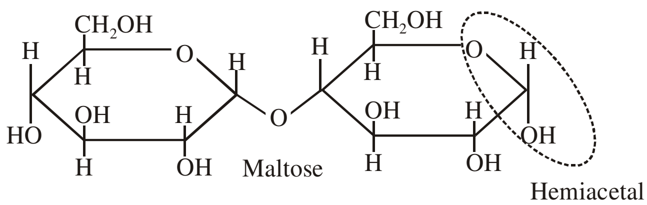
(1) One ketal and one hemiketal

(2) One acetal and one hemiacetal

(3) Two acetals

(4) One acetal and one ketal

Sol.



19. For one mole of an ideal gas, which of these statements must be true ?

(a) U and H each depends only on temperature
(b) Compressibility factor z is not equal to 1

(c) $C_{P,m} - C_{V,m} = R$

(d) $dU = C_V dT$ for any process

(1) (a), (c) and (d) (2) (b), (c) and (d)

(3) (c) and (d) (4) (a) and (c)

Sol. For ideal Gas

$U = f(T)$, $H = f(T)$

$Z = 1$

$C_P - C_V = R$

$dU = C_V\, dT$

20. The pair in which both the species have the same magnetic moment (spin only) is :

(1) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)]^{2+}$

(2) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(3) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Co(OH)_4]^{2-}$ and $[Fe(NH_3)_6]^{2+}$

Sol.

Complex	e^- configuration	no. of unpaired e^-
$[Mn(H_2O)_6]^{2+}$ WFL	↑ ↑ eg ↑ ↑ ↑ t2g	5
$[Cr(H_2O)_6]^{2+}$ WFL	↑ eg ↑ ↑ ↑	4
$[COCl_4]^{2-}$ Tetrahedral	↑ ↑ ↑ t_2 ↑↓ ↑↓ e	3
$[Fe(H_2O)_6]^{2+}$ WFL	↑ ↑ eg ↑↓ ↑ ↑ t_2g	4
$[Co(OH)_4]^{2-}$ WFL	↑ ↑ ↑ t_2 ↑↓ ↑↓ e	3
Tetrahedral $[Fe(NH_3)_6]^{2+}$	↑ ↑ ↑↓ ↑ ↑	4

Thus complex $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have same no. of unpaired e^- and hence same magnetic moment (spin only).

21. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is ________.

Sol. $N_2 + 3H_2 \rightarrow 2NH_3$

$\frac{2.8}{28}$ K mol $\qquad \frac{1}{2}$ K mol

= 0.1 K mol $\qquad$ 0.5 K mol $\qquad$ —

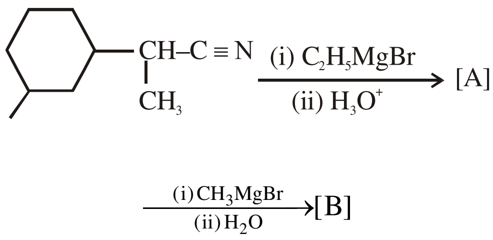
0 $\qquad$ 0.2 K mol $\qquad$ 0.2 K mol

mass (NH_3) = 0.2 × 17 Kg

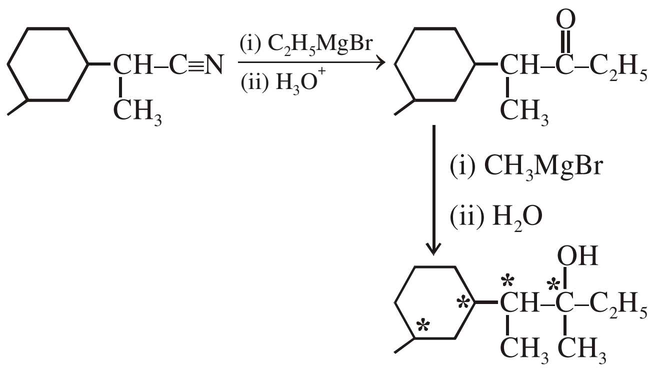
= 3.4 Kg

= 3400 gm

22. The number of chiral centres present in [B] is ________.



Sol.



23. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is ______ (mol. wt. of H_2O_2 = 34; mol. wt. of $KMnO_4$ = 158)

Sol. Eq of H_2O_2 = Eq of $KMnO_4$

$$x \times 2 = \frac{0.316}{158} \times 5$$

$x = 5 \times 10^{-3}$ mol

$$m_{H_2O_2} = 5 \times 10^{-3} \times 34 = 0.17 \text{gm}$$

$$\%H_2O_2 = \frac{0.17}{0.2} \times 100 = 85$$

24. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) ________.

(Take : log 2 = 0.30; log 2.5 = 0.40)

Sol. $t_{0.75} = 2 \times \frac{\ln 2}{k} = 90$

$k = \frac{\ln 2}{45} \text{min}^{-1}$

$kt = \ln \frac{1}{1-0.6} = \ln 2.5$

$\frac{\ln 2}{45} \times t = \ln 2.5$

$t = 45 \times \frac{\log 2.5}{\log 2} = 45 \times \frac{0.4}{0.3} = 60 \text{ min}$

25. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state _________ ?

Sol. $550 = P_A^o \times \frac{1}{4} + P_B^o \times \frac{3}{4}$

$2200 = P_A^o + 3P_B^o$ .....(i)

$2800 = P_A^o + 4P_B^o$ ....(ii)

$560 = P_A^o \times \frac{1}{5} + P_B^o \times \frac{4}{5}$

$P_B^o = 600, P_A^o = 400$

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

1. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, $\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true?

(1) $0 \le a^2 + b^2 \le 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Sol. $A^2=\begin{pmatrix}\cos 2\theta & i\sin 2\theta\\ i\sin 2\theta & \cos 2\theta\end{pmatrix}$

Similarly, $A^5=\begin{pmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos 75°$

(2) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(4) $a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

2. Let [t] denote the greatest integer $\le$ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution

(2) exactly four integral solutions

(3) exactly two solutions

(4) infinitely many solutions

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

3. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If α, β, γ, δ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :

(1) 3 : 1 (2) 33 : 31

(3) 9 : 7 (4) 5 : 3

Sol. $x^2 - 3x + p = 0 \begin{cases}\alpha\\ \beta\end{cases}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ....(1)

$x^2 - 6x + q = 0 \begin{cases}\gamma\\ \delta\end{cases}$

$\alpha r^2 + \alpha r^3 = 6$ ...(2)

(2) $\div$ (1)

$r^2 = 2$

So, $\frac{2q+p}{2q-p}=\frac{2r^5+r}{2r^5-r}=\frac{2r^4+1}{2r^4-1}=\frac{9}{7}$

4. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :

(1) 126 (2) 135

(3) 145 (4) 116

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$; $\frac{2b^2}{a}=10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t)=\frac{5}{12}+t-t^2=\frac{8}{12}-\left(t-\frac{1}{2}\right)^2$

$\phi(t)_{max}=\frac{8}{12}=\frac{2}{3}=e \Rightarrow e^2=1-\frac{b^2}{a^2}=\frac{4}{9}$ ... (ii)

$\Rightarrow a^2 = 81$ (from (i) & (ii))

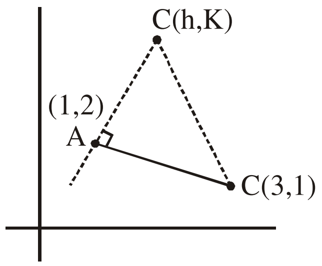
So, $a^2 + b^2 = 81 + 45 = 126$

5. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and ar(ΔABC) = $5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

(1) $2+\sqrt{5}$ (2) $1+\sqrt{5}$

(3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$

Sol.



$\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K = 2h$ ...(1)

$\sqrt{5}\,|h-1| = 10$

$\because$ [ΔABC] = $5\sqrt{5}$

$\Rightarrow \frac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2+(K-2)^2}=5\sqrt{5}$ ....(2)

$\Rightarrow$ h = $2\sqrt{5}+1$ (h > 0)

6. Let f(x) = |x – 2| and g(x) = f(f(x)), x $\in$ [0, 4].

Then $\int_0^3 \left(g(x)-f(x)\right)dx$ is equal to :

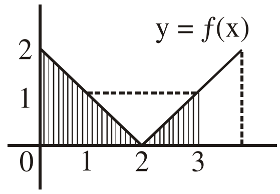
(1) $\frac{3}{2}$ (2) 0

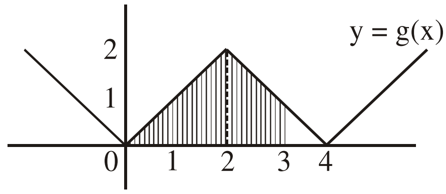
(3) $\frac{1}{2}$ (4) 1

Sol. $\int_0^3 g(x)-f(x) = \int_0^3 \left||x-2|-2\right|dx - \int_0^3 |x-2|\,dx$

$=\left(\frac{1}{2}\times2\times2+1+\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times1\right)$

$= \left(2+1+\frac{1}{2}\right)-\left(2+\frac{1}{2}\right) = 1$





7. Given the following two statements :

(S_1) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S_1) is correct.

(2) both (S_1) and (S_2) are correct.

(3) both (S_1) and (S_2) are not correct.

(4) only (S_2) is correct.

Sol. Let TV(r) denotes truth value of a statement r.
Now, if TV(p) = TV(q) = T

$\Rightarrow$ TV(S_1) = F

Also, if TV(p) = T & TV(q) = F

$\Rightarrow$ TV(S_2) = T

8. Let P(3, 3) be a point on the hyperbola,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1. If the normal to it at P intesects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$

(3) $\left(\frac{3}{2}, 2\right)$ (4) (9, 3)

Sol. Since, (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$ ....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through (9, 0) $\Rightarrow b^2 = 2a^2$ ....(2)

So, $e^2 = 1 + \frac{b^2}{a^2} = 3$

Also, $a^2 = \frac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2) = \left(\frac{9}{2}, 3\right)$

9. Let $f(x) = \int\frac{\sqrt{x}}{(1+x)^2}dx\ (x\ge 0)$. Then f(3) – f(1) is equal to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Sol. $f(x) = \int_1^3\frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}}\frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x}=t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1}t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4}-\frac{1}{2}\right)+\frac{\pi}{3}-\frac{\pi}{4}$

$= \frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{\pi}{12}$

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

Sol. $n(B) \le n(A\cup B) \le n(U)$

$\Rightarrow 76 \le 76 + 63 - x \le 100$

$\Rightarrow -63 \le -x \le -39$

$\Rightarrow 63 \ge x \ge 39$

11. Let $u=\frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is :

(1) 3/2 (2) 4

(3) 2 (4) 1/2

Sol. $u = \frac{2z+i}{z-ki}$

$= \frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2} + i\frac{\left(x(2y+1)-2x(y-k)\right)}{x^2+(y-k)^2}$

Since Re(u) + Im(u) = 1

$\Rightarrow 2x^2 + (2y+1)(y-k)+x(2y+1)-2x(y-k)$

$= x^2 + (y-k)^2$

$\left.\begin{matrix}P(0,y_1)\\ Q(0,y_2)\end{matrix}\right\rangle \Rightarrow y^2+y-k-k^2=0 \left\langle\begin{matrix}y_1+y_2=-1\\ y_1y_2=-k-k^2\end{matrix}\right.$

$\because$ PQ = 5

$\Rightarrow |y_1-y_2|=5 \Rightarrow k^2 + k - 6 = 0$

$\Rightarrow k = -3, 2$

So, k = 2 (k > 0)

12. Let x_0 be the point of local maxima of $f(x)=\vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x = x_0$ is :

(1) –30 (2) 14

(3) –4 (4) –22

Sol. $f(x)=\vec{a}.\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a}=-3\hat{i}-2\hat{j}+3\hat{k}$,

$\vec{b}=-2\hat{i}-3\hat{j}-\hat{k}$,

and $\vec{c}=7\hat{i}-2\hat{j}-3\hat{k}$

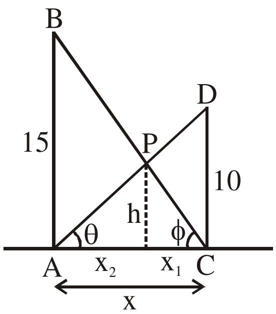
$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=9-5-26=-22$

13. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

(1) 20/3 (2) 5

(3) 10/3 (4) 6

Sol.



$\tan\theta=\dfrac{10}{x}=\dfrac{h}{x_2}\Rightarrow x_2=\dfrac{hx}{10}$

$\tan\phi=\dfrac{15}{x}=\dfrac{h}{x_1}\Rightarrow x_1=\dfrac{hx}{15}$

Now, $x_1 + x_2 = x = \dfrac{hx}{15}+\dfrac{hx}{10}$

$\Rightarrow 1 = \dfrac{h}{10}+\dfrac{h}{15} \Rightarrow h = 6$

14. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3

(3) 5 (4) 9

Sol. $\bar{x}=10$

$\Rightarrow \bar{x}=\dfrac{63+a+b}{8}=10 \Rightarrow a + b = 17$ ...(1)

Since, variance is independent of origin.

So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \dfrac{79+(a-10)^2+(b-10)^2}{8}-(10-10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ...(2)

From (i) & (ii) ; a = 12 & b = 5

15. The integral $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to :

(where C is a constant of integration)

(1) $\sec x+\dfrac{x\tan x}{x\sin x+\cos x}+C$

(2) $\sec x-\dfrac{x\tan x}{x\sin x+\cos x}+C$

(3) $\tan x+\dfrac{x\sec x}{x\sin x+\cos x}+C$

(4) $\tan x-\dfrac{x\sec x}{x\sin x+\cos x}+C$

Sol. $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx=\int\left(\dfrac{x}{\cos x}\right).\dfrac{x\cos x\, dx}{(x\sin x+\cos x)^2}$

$=\dfrac{x}{\cos x}\left(-\dfrac{1}{x\sin x+\cos x}\right)$

$+\int\left(\dfrac{\cos x+x\sin x}{\cos^2 x}\right)\left(\dfrac{1}{x\sin x+\cos x}\right)dx$

$=-\dfrac{x\sec x}{x\sin x+\cos x}+\int\sec^2 x\, dx$

$=-\dfrac{x\sec x}{x\sin x+\cos x}+\tan x+C$

16. If

$1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :

(1) (10, 97) (2) (11, 103)

(3) (10, 103) (4) (11, 97)

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) +...... + (1 - 20^2.19) = \alpha - 220\,\beta$

$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r-1) = 11 - 4\left(\frac{110^2}{2} - 35\times 11\right)$

$= 11 - 220(103)$

$\Rightarrow \alpha = 11,\ \beta = 103$

17. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $2+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}$

(3) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

Sol. $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x),\ x > 0$

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$

so, I.F. $= e^{\int -\frac{1}{x}dx} = \frac{1}{|x|} = \frac{1}{x}$ $(x > 0)$

Thus, $\frac{y}{x} = \int \frac{1}{x}\big(x(x\cos x + \sin x)\big)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C$

$\because\ y(\pi) = \pi \Rightarrow C = 1$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$

Also, $\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\Rightarrow \frac{d^2y}{dx^2} = -x^2\sin x + 4x\cos x + 2\sin x$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$

Thus, $y_{\left(\frac{\pi}{2}\right)} + \frac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \frac{\pi}{2} + 2$

18. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

(1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$

(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + ... + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

19. Let f be a twice differentiable function on (1, 6). If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then :

(1) $f(5) \leq 10$ (2) $f'(5) + f''(5) \leq 20$

(3) $f(5) + f'(5) \geq 28$ (4) $f(5) + f'(5) \leq 26$

Sol. $f(2) = 8,\ f'(2) = 5,\ f'(x)\geq 1,\ f''(x) \geq 4,\ \forall x\in(1,6)$

$f''(x) = \frac{f'(5)-f'(2)}{5-2} \geq 4 \Rightarrow f'(5) \geq 17$ ...(1)

$f'(x) = \frac{f(5)-f(2)}{5-2} \geq 1 \Rightarrow f(5) \geq 11$ ...(2)

$\overline{f'(5)+f(5) \geq 28}$

20. If $\left(a+\sqrt{2}\,b\cos x\right)\left(a-\sqrt{2}\,b\cos y\right)=a^2-b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is :

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$

(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

Sol. $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right)=a^2-b^2$

$\Rightarrow a^2-\sqrt{2}\,ab\cos y+\sqrt{2}\,ab\cos x$

$-2b^2\cos x\cos y=a^2-b^2$

Differentiating both sides :

$0-\sqrt{2}\,ab\left(-\sin y\frac{dy}{dx}\right)+\sqrt{2}\,ab(-\sin x)$

$-2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right)+\cos y\left(-\sin x\right)\right]=0$

At $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$:

$ab\frac{dy}{dx}-ab-2b^2\left(-\frac{1}{2}\frac{dy}{dx}-\frac{1}{2}\right)=0$

$\Rightarrow \frac{dx}{dy}=\frac{ab+b^2}{ab-b^2}=\frac{a+b}{a-b}$; a, b > 0

21. If the system of equations

x – 2y + 3z = 9

2x + y + z = b

x – 7y + az = 24,

has infinitely many solutions, then a – b is equal to ________ .

Sol. $D=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & 1\\ 1 & -7 & a\end{vmatrix}=0\Rightarrow a=8$

also, $D_1=\begin{vmatrix}9 & -2 & 3\\ b & 1 & 1\\ 24 & -7 & 8\end{vmatrix}=0\Rightarrow b=3$

hence, a – b = 8 – 3 = 5

22. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$\Rightarrow 1-\left(\frac{9}{10}\right)^n>\frac{1}{4}$

$\Rightarrow \frac{3}{4}>\left(\frac{9}{10}\right)^n\Rightarrow n\geq 3$

23. Suppose a differentiable function f(x) satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real x and y. If $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$, then f'(3) is equal to ________.

Sol. Since, $\lim\limits_{x\to 0}\frac{f(x)}{x}$ exist $\Rightarrow f(0)=0$

Now, $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\frac{f(h)+xh^2+x^2h}{h}$ (take y = h)

$=\lim\limits_{h\to 0}\frac{f(h)}{h}+\lim\limits_{h\to 0}(xh)+x^2$

$\Rightarrow f'(x)=1+0+x^2$

$\Rightarrow f'(3)=10$

24. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _________.

Sol. Given $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}(2x^2+3x+4)^{10}}{x^{20}} = \sum_{r=0}^{20} \frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10} \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{(20-r)} \text{ (from (i))}$$

now, comparing coefficient of x^7 from both sides

(take r = 7 in L.H.S. & r = 13 in R.H.S.)

$$2^{10} a_7 = a_{13} 2^{13} \Rightarrow \frac{a_7}{a_{13}} = 2^3 = 8$$

25. If the equation of a plane P, passing through the intesection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in R$, then the distance of the point (3, 2, –1) from the plane P is _________.

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0 \quad .... (1)$$

P : $2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$