TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

PHYSICS

1. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is :-

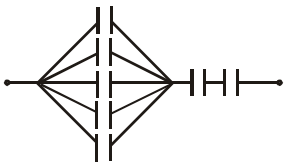
(1) 2.3×10^{-2} (2) 3.3×10^{-2}

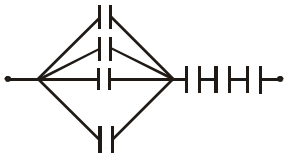
(3) 3.3×10^{-4} (4) 4.3×10^{-2}

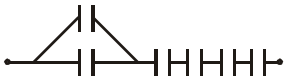
2. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :-

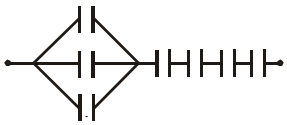
(1) $2\sqrt{\frac{p}{k}}$ (2) $\sqrt{\frac{2p}{k}}$ (3) $\sqrt{\frac{2k}{p}}$ (4) $2\sqrt{\frac{k}{p}}$

3. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value ?

(1) 

(2) 

(3) 

(4) 

4. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole ?

(1) -9×10^{-20} J

(2) -7×10^{-27} J

(3) -10×10^{-29} J

(4) -20×10^{-18} J

5. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(1) 10^{-3} rad/s

(2) 10^{-1} rad/s

(3) 1 rad/s

(4) 10^{-5} rad/s

6. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is :-

(1) 270°C (2) 230°C

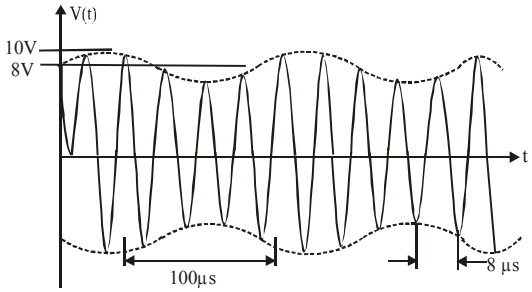
(3) 250°C (4) 200°C

7. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :-

(1) 09 (2) 10

(3) 04 (4) 05

8. An amplitude modulated signal is plotted below :-



Which one of the following best describes the above signal ?

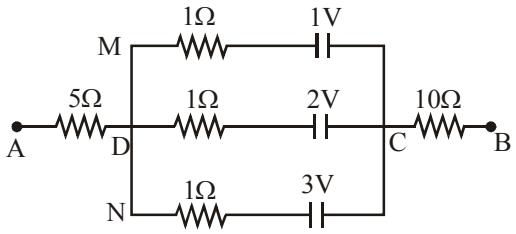
(1) $(9 + \sin(2.5\pi \times 10^5 t))\sin(2\pi \times 10^4 t)V$

(2) $(9 + \sin(4\pi \times 10^4 t))\sin(5\pi \times 10^5 t)V$

(3) $(1 + 9\sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)V$

(4) $(9 + \sin(2\pi \times 10^4 t))\sin(2.5\pi \times 10^5 t)V$

9. In the circuit , the potential difference between A and B is :-



(1) 6 V (2) 1 V (3) 3 V (4) 2 V

10. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\epsilon_0 = 9 \times 10^{-12}$ SI units, Speed of light $c = 3 \times 10^8$ m/s]:-

(1) 1 kV/m (2) 2 kV/m

(3) 1.4 kV/m (4) 0.7 kV/m

11. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :-

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$

(3) $K_2 = 2K_1$ (4) $K_2 = K_1$

12. In a hydrogen like atom, when an electron jumps from the M - shell to the L - shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be :-

(1) $\frac{27}{20}\lambda$ (2) $\frac{16}{25}\lambda$ (3) $\frac{20}{27}\lambda$ (4) $\frac{25}{16}\lambda$

13. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be :-

(1) $V^{-2}A^2F^2$ (2) $V^{-4}A^2F$

(3) $V^{-4}A^{-2}F$ (4) $V^{-2}A^2F^{-2}$

14. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$ m, at t = 0, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)ms^{-2}$. What is the distance of the particle from the origin at time 2 s ?

(1) $20\sqrt{2}$ m (2) $10\sqrt{2}$ m

(3) 5 m (4) 15 m

15. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is :-

(1) 30° (2) 45°

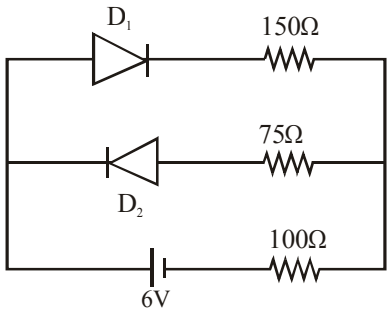
(3) 90° (4) 60°

16. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :-

(1) 80 Ω (2) 120 Ω

(3) 125 Ω (4) 100 Ω

17. The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in Amperes) is :-



(1) 0.027 (2) 0.020
(3) 0.030 (4) 0.036

18. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :-

(1) 80°C (2) 60°C
(3) 70°C (4) 85°C

19. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be :-

(1) $\frac{2}{\sqrt{3}}$s (2) $2\sqrt{3}$ s

(3) $\frac{\sqrt{3}}{2}$s (4) $\frac{3}{2}$s

Ans. (2)

Sol. $\because g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_e}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also $T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3}s$$

20. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If d = $\frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :-

(1) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{\sqrt{2}}\hat{j}\right)$

(3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

(4) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

Ans. (BONUS)

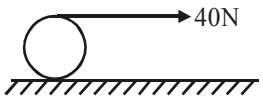
21. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice.

What is the temperature of an object in 0 ºC, if this thermometer in the contact with the object reads $x_0/2$?

(1) 35 (2) 25

(3) 60 (4) 40

22. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :-



(1) 12 rad/s^2 (2) 16 rad/s^2

(3) 10 rad/s^2 (4) 20 rad/s^2

23. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is incresed by ΔT. The amount of heat absorbed by gas is (R is gas constant) :

(1) $\frac{1}{2}R\Delta T$ (2) $\frac{3}{2}R\Delta T$

(3) $\frac{1}{2}KR\Delta T$ (4) $\frac{2K}{3}\Delta T$

24. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to : $\left(\frac{hc}{e} = 1240\,\text{nm} - \text{V}\right)$

(1) 0.5 V (2) 1.0 V
(3) 2.0 V (4) 1.5 V

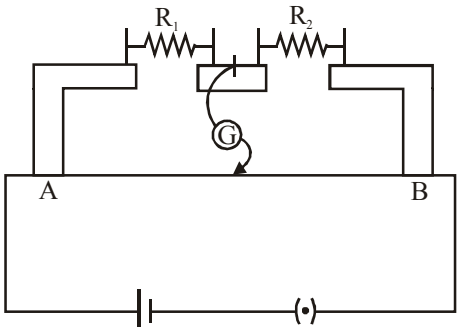
25. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water ? [Specific Heat Capacities of water and metal are, respectively, 4200 $Jkg^{-1}K^{-1}$ and 400 $JKg^{-1}K^{-1}$]

(1) 30% (2) 20%
(3) 25% (4) 15%

26. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :-

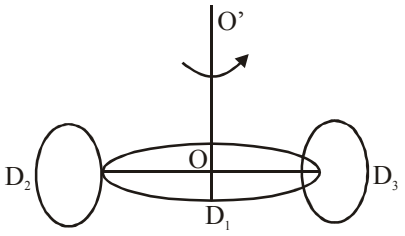
(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

27. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is



(1) 40 Ω (2) 60 Ω (3) 20 Ω (4) 30Ω

28. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



(1) $3MR^2$ (2) $\frac{2}{3}MR^2$

(3) MR^2 (4) $\frac{4}{5}MR^2$

29. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil :

(1) Decreases by a factor of $9\sqrt{3}$

(2) Increases by a factor of 3

(3) Decreases by a factor of 9

(4) Increases by a factor of 27

30. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i} + 3\hat{j}$; $\vec{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from the origin to the point P(x = 1 ; y = 1) along a straight path. The magnitude of the total work done is :-

(1) (0.35)q (2) (0.15)q

(3) (2.5)q (4) 5q

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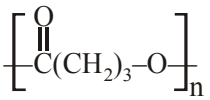
CHEMISTRY

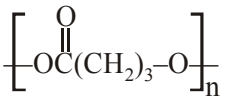
1. The correct option with respect to the Pauling electronegativity values of the elements is :-

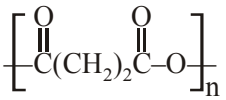
(1) Ga < Ge (2) Si < Al

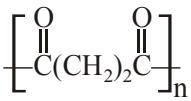
(3) P > S (4) Te > Se

2. The homopolymer formed from 4-hydroxy-butanoic acid is :-

(1) 

(2) 

(3) 

(4) 

3. The correct match between Item I and Item II is :-

Item I		Item II	
(A)	Ester test	(P)	Tyr
(B)	Carbylamine test	(Q)	Asp
(C)	Phthalein dye test	(R)	Ser
		(S)	Lys

(1) (A)→(Q); (B)→(S); (C)→(P)

(2) (A)→(R); (B)→(Q); (C)→(P)

(3) (A)→(Q); (B)→(S); (C)→(R)

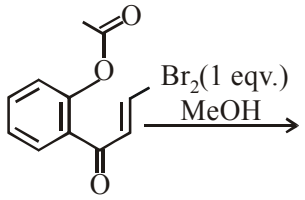
(4) (A)→(R); (B)→(S); (C)→(Q)

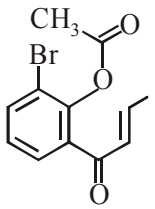
4. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to :-

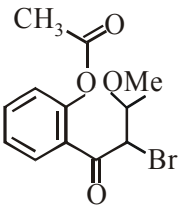
(1) Water pollution (2) Global warming

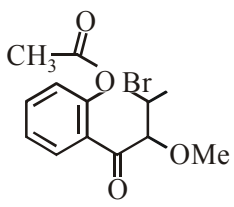
(3) Soil pollution (4) Acid rain

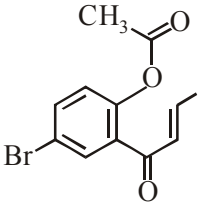
5. The major product obtained in the following conversion is :-



(1) 

(2) 

(3) 

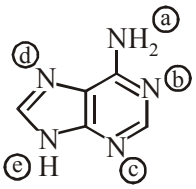
(4) 

6. The number of bridging CO ligand (s) and Co-Co bond (s) in $CO_2(CO)g$, respectively are :-

(1) 0 and 2 (2) 2 and 0

(3) 4 and 0 (4) 2 and 1

7. In the following compound,



the favourable site/s for protonation is/are :-

(1) (b), (c) and (d) (2) (a)

(3) (a) and (e) (4) (a) and (d)

8. The higher concentration of which gas in air can cause stiffness of flower buds ?

(1) SO_2 (2) NO_2

(3) CO_2 (4) CO

9. The correct match between item I and item II is :-

Item I		Item II	
(A)	Allosteric effect	(P)	Molecule binding to the active site of enzyme
(B)	Competitive inhibitor	(Q)	Molecule crucial for communication in the body
(C)	Receptor	(R)	Molecule binding to a site other than the active site of enzyme
(D)	Poison	(S)	Molecule binding to the enzyme covalently

(1) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)

(2) (A)→(R); (B)→(P); (C)→(S); (D)→(Q)

(3) (A)→(P); (B)→(R); (C)→(Q); (D)→(S)

(4) (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

10. The radius of the largest sphere which fits properly at the centre of the edge of body centred cubic unit cell is : (Edge length is represented by 'a') :-

(1) 0.134 a (2) 0.027 a

(3) 0.067 a (4) 0.047 a

11. Among the colloids cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is :-

(1) C : solid in liquid; M : solid in liquid; S : solid in gas

(2) C : solid in liquid; M : liquid in liquid; S : gas in solid

(3) C : liquid in solid; M : liquid in solid; S : solid in gas

(4) C : liquid in solid; M : liquid in liquid; S : solid in gas

12. The reaction that does NOT define calcination is:-

(1) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$

(2) $Fe_2O_3 \cdot XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

(3) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2\ CO_2$

(4) $2\ Cu_2S + 3\ O_2 \xrightarrow{\Delta} 2\ Cu_2O + 2\ SO_2$

13. The reaction,

$MgO(s) + C(s) \rightarrow Mg(S) + CO(g)$, for which $\Delta_r H^o = +491.1$ kJ mol^{-1} and $\Delta_r S^o = 198.0$ JK^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaction will be feasible is :-

(1) 1890.0 K (2) 2480.3 K

(3) 2040.5 K (4) 2380.5 K

14. Given the equilibrium constant :

KC of the reaction :

$Cu(s) + 2Ag^+(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s)$ is 10×10^{15}, calculate the E^0_{cell} of this reaction at 298 K

$\left[2.303\frac{RT}{F}\text{at}\,298\,K = 0.059V\right]$

(1) 0.04736 V (2) 0.4736 V

(3) 0.4736 mV (4) 0.04736 mV

15. The hydride that is NOT electron deficient is:-

(1) B_2H_6 (2) AlH_3

(3) SiH_4 (4) GaH_3

16. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$$\Delta_r G^o = A - Bt$$

Where A and B are non-zero constants. Which of the following is TRUE about this reaction ?

(1) Exothermic if $B < 0$

(2) Exothermic if $A > 0$ and $B < 0$

(3) Endothermic if $A < 0$ and $B > 0$

(4) Endothermic if $A > 0$

17. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is :-

(1) 1.8 (2) 2.2 (3) 2.0 (4) 1.6

18. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (ν) of the incident radiation as, [ν_0 is thrshold frequency] :

(1) $\lambda \propto \frac{1}{(\nu - \nu_0)^{\frac{3}{2}}}$ (2) $\lambda \propto \frac{1}{(\nu - \nu_0)^{\frac{1}{2}}}$

(3) $\lambda \propto \frac{1}{(\nu - \nu_0)^{\frac{1}{4}}}$ (4) $\lambda \propto \frac{1}{(\nu - \nu_0)}$

19. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be :-

(1) 18.0 h (2) 7.2 h (3) 9.0 h (4) 12.0 h

20. A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is :-

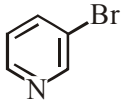
(1) $CH_3COCH_2NHCH_3$

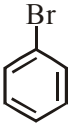
(2) $CH_3CH_2COCH_2NH_2$

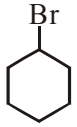
(3) $CH_3CH_2CH_2CONH_2$

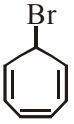
(4) $CH_3CON(CH_3)_2$

21. Which of the following compounds will produce a precipitate with $AgNO_3$?

(1) 

(2) 

(3) 

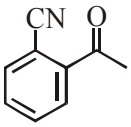
(4) 

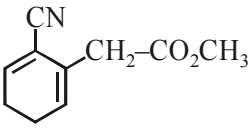
22. The relative stability of +1 oxidation state of group 13 elements follows the order :-

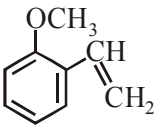
(1) Al < Ga < Tl < In (2) Tl < In < Ga < Al

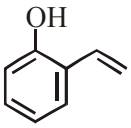
(3) Al < Ga < In < Tl (4) Ga < Al < In < Tl

23. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution :-

(1) 

(2) 

(3) 

(4) 

24. Match the following items in column I with the corresponding items in column II.

Column I		Column II	
(i)	$Na_2CO_3 \cdot 10\ H_2O$	(P)	Portland cement ingredient
(ii)	$Mg(HCO_3)_2$	(Q)	Castner-Keller process
(iii)	NaOH	(R)	Solvay process
(iv)	$Ca_3Al_2O_6$	(S)	Temporary hardness

(1) (i)→(C); (ii)→(B); (iii)→(D); (iv)→(A)

(2) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

(3) (i)→(D); (ii)→(A); (iii)→(B); (iv)→(C)

(4) (i)→(B); (ii)→(C); (iii)→(A); (iv)→(D)

25. 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solution?

(1) 25 mL (2) 50 mL (3) 12.5 mL(4) 75 mL

26. $\underline{A} \xrightarrow{4\,KOH,\,O_2} \underset{(Green)}{2\underline{B}} + 2\,H_2O$

$3\,\underline{B} \xrightarrow{4\,HCl} \underset{(Purple)}{2\,\underline{C}} + MnO_2 + 2\,H_2O$

$2\,\underline{B} \xrightarrow{H_2O,\,KI} 2\,\underline{A} + 2KOH + \underline{D}$

In the above sequence of reactions,

$\underline{A}$ and $\underline{D}$ respectively, are :-

(1) KIO_3 and MnO_2 (2) KI and K_2MnO_4

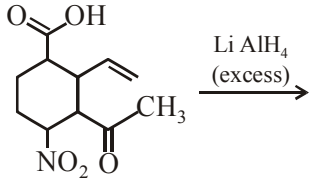
(3) MnO_2 and KIO_3 (4) KI and $KMnO_4$

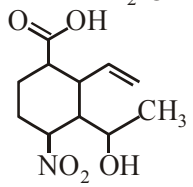
27. The coordination number of Th in $K_4[Th(C_2O_4]_4(OH_2)_2]$ is :-

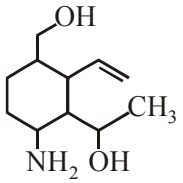
$\left(C_2O_4^{2-} = \text{Oxalato}\right)$

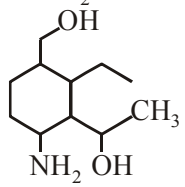
(1) 6 (2) 10 (3) 14 (4) 8

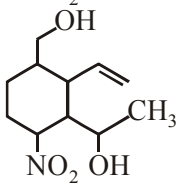
28. The major product obtained in the following reaction is :-



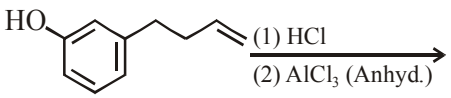
(1) 

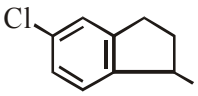
(2) 

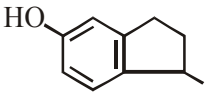
(3) 

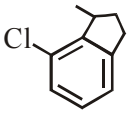
(4) 

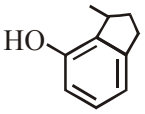
29. The major product of the following reaction is :-



(1) 

(2) 

(3) 

(4) 

30. For the equilibrium,

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG° at 298 K is approximately :-

(1) –80 kJ mol^{-1} (2) –100 kJ mol^{-1}

(3) 100 kJ mol^{-1} (4) 80 kJ mol^{-1}

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

(1) 5 (2) 17 (3) 12 (4) 7

2. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :-

(1) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(2) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

3. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$ is not differentiable. Then the set K is equal to :-

(1) $\{\pi\}$ (2) $\{0\}$

(3) ϕ (an empty set) (4) $\{0, \pi\}$

4. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

5. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :-

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

6. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\,(\tan^5 x + \cot^5 x)}$ equals :-

(1) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{10}$

(4) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

7. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + ...$ $+ a_{50}\,x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:-

(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25

8. Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is :-

(1) Injective only

(2) Not injective but it is surjective

(3) Both injective as well as surjective

(4) Neither injective nor surjective

9. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :-

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

10. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

(1) (2, 4, 7) (2) (–2, 4, 7)

(3) (2, –4, –7) (4) (2, –4, 7)

11. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :-

(1) $(15)! \times 6!$ (2) $5^6 \times 15$

(3) $5! \times 6!$ (4) $6^5 \times (15)!$

12. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

13. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when $y(1) = 1$, is :-

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

14. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :-

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

15. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to :-

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

16. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to :-

(1) 4 (2) $\frac{4\sqrt{3}}{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

17. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:-

(1) $5x + 3y - 11 = 0$ (2) $3x - 5y + 7 = 0$

(3) $3x + 5y - 13 = 0$ (4) $5x - 3y + 1 = 0$

18. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :-

(1) 2 (2) $\frac{13}{6}$ (3) $\frac{13}{8}$ (4) $\frac{13}{12}$

19. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :-

(1) $\frac{14}{3}$ (2) $\frac{187}{24}$ (3) $\frac{37}{24}$ (4) $\frac{8}{3}$

20. Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is :-

(1) 2 (2) 1 (3) 3 (4) 4

21. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $= (a + b + c)(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to :-

(1) $-(a + b + c)$ (2) $2(a + b + c)$

(3) abc (4) $-2(a + b + c)$

22. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + ... + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then α is equal to :-

(1) 2^{100} (2) 200 (3) 2^{99} (4) 202

23. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :-

(1) A hyperbola (2) A parabola
(3) A straight line (4) An ellipse

24. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :-

(1) 3 : 1 (2) 4 : 1 (3) 2 : 1 (4) 1 : 3

25. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{d - x}{\sqrt{b^2 + (d - x)^2}}$, $x \in R$, where a, b and d are non-zero real constants. Then :-

(1) f is a decreasing function of x
(2) f is neither increasing nor decreasing function of x
(3) f' is not a continuous function of x
(4) f is an increasing function of x

26. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to :-

(1) $\frac{5}{4}$ (2) $\frac{\sqrt{41}}{4}$ (3) $\frac{\sqrt{34}}{3}$ (4) $\frac{5}{3}$

27. All x satisfying the inequality $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, lie in the interval:-

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) $(\cot 5, \cot 4)$

(3) $(\cot 2, \infty)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

28. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :-

(1) (3, 4, 5) (2) (19, 7, 25)

(3) (7, 19, 25) (4) (5, 12, 13)

29. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

30. $\lim\limits_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :-

(1) 2 (2) 0 (3) 4 (4) 1