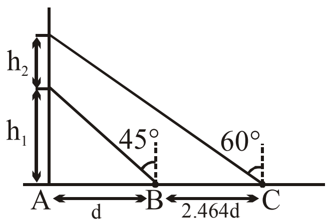
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Saturday 05th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

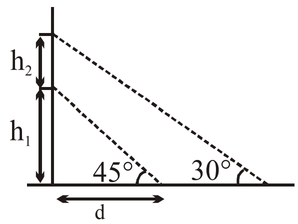
1. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) :



(1) d (2) 0.732d

(3) 1.464d (4) 0.464d

Sol.



$$\frac{h_1}{d} = \tan 45° \Rightarrow h_1 = d \ ...\ (1)$$

$$\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$$

$$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46\ d$$

$$(h_1 + h_2) = \frac{3.46d}{\sqrt{3}}$$

$$\Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$$

$h_2 = d$

2. In a resonance tube experiment when the tube is filled with water up to height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is:

(1) 1100 Hz (2) 3300 Hz

(3) 2200 Hz (4) 550 Hz

Sol. $\Rightarrow \lambda = 2\ (l_2 - l_1) \Rightarrow 2 \times (24.5 - 17)$

$\Rightarrow 2 \times 7.5 = 15$ cm

& $v = f\lambda \ \Rightarrow 330 = \lambda \times 15 \times 10^{-2}$

$$\lambda = \frac{330}{15} \times 100 \ \Rightarrow \ \frac{1100 \times 100}{5}$$

$\Rightarrow$ 2200 Hz

3. A helicopter reises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

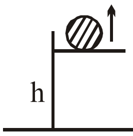
(1) $t = \sqrt{\frac{2h}{3g}}$

(2) $t = 1.8\sqrt{\frac{h}{g}}$

(3) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$

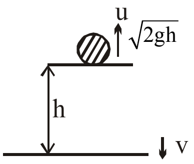
(4) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

Sol.



$u^2 = 0 + 2gh$

$\Rightarrow u = \sqrt{2gh}$



$v^2 = u^2 + 2as$

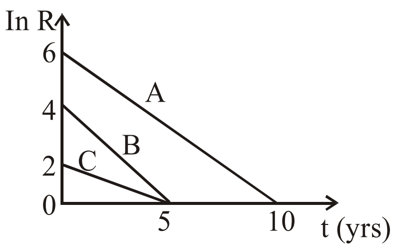
$v^2 = 2gh + 2gh$

$v = \sqrt{4gh}$

$$\Rightarrow \sqrt{4gh} = \sqrt{2gh} + gt$$

$$\Rightarrow t = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \Rightarrow 3.4\sqrt{\frac{h}{g}}$$

4. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A):T_{\frac{1}{2}}(B):T_{\frac{1}{2}}(C)$ are in the ratio :



(1) 3 : 2 : 1 (2) 4 : 3 : 1

(3) 2 : 1 : 3 (4) 2 : 1 : 1

Sol. $R = R_0 e^{-\lambda t}$

$\ln R = \ln R_0 - \lambda t$

$$\lambda_A = \frac{6}{10} \Rightarrow T_A = \frac{10}{6} \ln 2$$

$$\lambda_B = \frac{6}{5} \Rightarrow T_B = \frac{5 \ln 2}{6}$$

$$\lambda_C = \frac{2}{5} \Rightarrow T_C = \frac{5 \ln 2}{2}$$

$$\frac{10}{6} : \frac{5}{6} : \frac{15}{6} :: 2 : 1 : 3$$

5. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t. water, the value of r is :

(1) $\frac{4}{9}R$ (2) $\frac{8}{9}R$

(3) $\frac{1}{3}R$ (4) $\frac{2}{3}R$

Sol. $\frac{4}{3}\pi\left(R^3 - r^3\right) \rho_m g = \frac{4}{3}\pi R^3 \rho_w g$

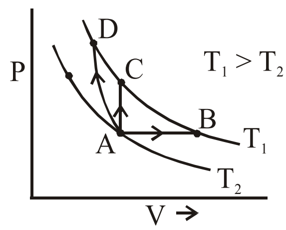
$$1 - \left(\frac{r}{R}\right)^3 = \frac{8}{27}$$

$$\Rightarrow \frac{r}{R} = \left(\frac{19}{27}\right)^{1/3} = \frac{19^{1/3}}{3}$$

$$= 0.88 \simeq \frac{8}{9}$$

6. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A → D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and W_{AD}.

The correct relation between these parameters are :



(1) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$

(2) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$

(3) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

(4) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

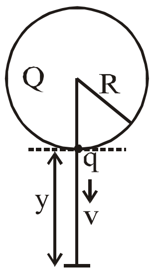
Sol. $\Delta U = nC_v\,\Delta T$ = same

AB $\rightarrow$ volume is increasing $\Rightarrow$ W > 0

AD $\rightarrow$ volume is decreasing $\Rightarrow$ W < 0

AC $\rightarrow$ volume is constant $\Rightarrow$ W = 0

7. A solid sphere of radius R carries a charge (Q + q) distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).



(1) $v^2 = 2y\left[\dfrac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$

(2) $v^2 = y\left[\dfrac{qQ}{4\pi\epsilon_0 R^2 ym} + g\right]$

(3) $v^2 = 2y\left[\dfrac{qQR}{4\pi\epsilon_0 (R+y)^3 m} + g\right]$

(4) $v^2 = y\left[\dfrac{qQ}{4\pi\epsilon_0 R(R+y)m} + g\right]$

Sol. $\dfrac{kQq}{R} + mgy$

$= \dfrac{kQq}{R+y} + \dfrac{1}{2}mv^2$

$v^2 = 2gy + \dfrac{2kQqy}{mR(R+y)}$

8. With increasing biasing voltage of a photodiode, the photocurrent magnitude :

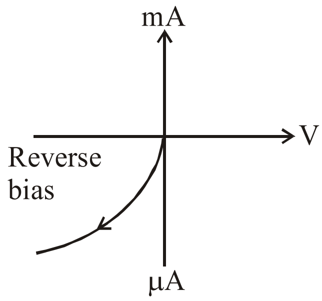
(1) increases initially and saturates finally

(2) increases initially and after attaining certain value, it decreases

(3) increases linearly

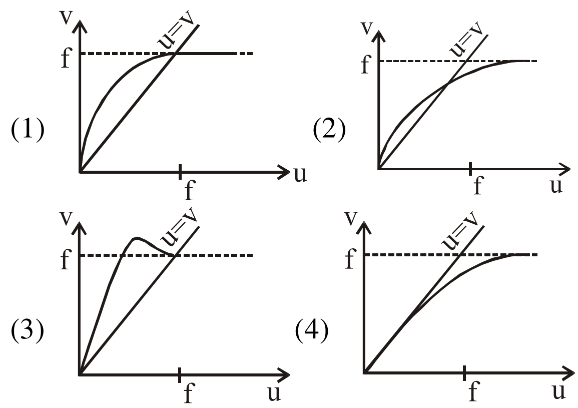
(4) remains constant

Sol. I-V characteristic of a photodiode is as follows:



On increasing the potential difference the current first increases and then attains a saturation.

9. For a concave lens of focal length f, the relation between object and image distance u and v, respectively, from its pole can best be represented by (u = v is the reference line):



Sol. $v = \dfrac{uf}{u+f}$

Case-I

If v = u

$\Rightarrow$ f + u = f

$\Rightarrow$ u = 0

Case-II

If u = ∞

then v = f

Only option (4) satisfies this condition.

10. An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately:
(1) 72% (2) 96% (3) 91% (4) 85%

Sol. $vi = 10^3$

$i = \frac{1000}{220}$

$\text{loss} = i^2R = \left(\frac{50}{11}\right)^2 \times 2$

$\text{efficiency} = \frac{1000}{1000 + i^2R} \times 100 = 96\%$

11. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$Pa, $v \sim 300$ m/s, $p \sim 1$ kg/m^3 and $f \sim 1000$Hz, then s will be the order of (take multiplicative constant to be 1)

(1) 10 mm (2) $\frac{3}{100}$ mm
(3) 1 mm (4) $\frac{1}{10}$ mm

Sol. $\Delta p = BkS_0$

$= \rho v^2 \times \frac{\omega}{v} \times S_0$

$\Rightarrow S_0 = \frac{\Delta p}{\rho v \omega}$

$\approx \frac{10}{1 \times 300 \times 1000}$ m

$= \frac{1}{30}$ mm $\approx \frac{3}{100}$ mm

12. A bullet of mass 5g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g–°C) (1 cal = 4.2×10^7 ergs) close to :
(1) 83.3°C (2) 87.5°C
(3) 119.2°C (4) 38.4°C

Sol. $\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$

$\Delta T = \frac{v^2}{4 \times 5} = \frac{210^2}{4 \times 30 \times 4.200}$

$= 87.5°C$

13. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to ? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, $g = 980$ cm/s^2, density of mercury = 13.6 g/cm^3)
(1) 5.8×10^{18} (2) 5.8×10^{16}
(3) 4.0×10^{18} (4) 4.0×10^{16}

Sol. $n = \frac{PV}{RT}, \frac{3}{2}kT = 4 \times 10^{-14}$

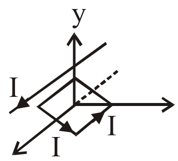
$N = \frac{PV}{RT} \times Na$

$= \frac{2 \times 13.6 \times 980 \times 4}{\frac{8}{3} \times 10^{-14}} = 3.99 \times 10^{18}$

14. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by:

(1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (2) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$
(3) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (4) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

Sol.



$\vec{\tau} = \vec{M} \times \vec{B}$

$= 4a^2 I \times \frac{\mu_0 I}{2\pi b}$

15. A physical quantity z depends on four observables a, b, c and d, as $z = \dfrac{a^2 b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentage of error in the measurement of a, b, c and d 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is:

(1) 12.25% (2) 14.5%

(3) 16.5% (4) 13.5%

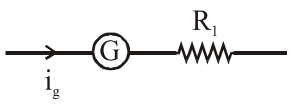
Sol. $\dfrac{\Delta Z}{Z} = \dfrac{2\Delta a}{a} + \dfrac{2}{3}\dfrac{\Delta b}{b} + \dfrac{1}{2}\dfrac{\Delta c}{c} + \dfrac{3\Delta d}{d} = 14.5\%$

16. A galvanometer of resistance G is converted into a voltmeter of range 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be :

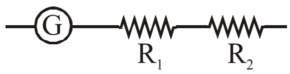
(1) R_1 (2) $R_1 + G$

(3) $R_1 - G$ (4) G

Sol.



$\Rightarrow 1 = i_g(G + R_1)$ ... (1)



$\Rightarrow 2 = i_g(R_1 + R_2 + G)$ ...(2)

(1) % (2)

$\Rightarrow \dfrac{1}{2} = \dfrac{G + R_1}{G + R_1 + R_2}$

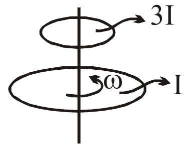
$G + R_1 + R_2 = 2G + 2h_1$

$(R_2 = G + R_1)$

17. A wheel is rotaing freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of momet of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

(1) 0 (2) $\dfrac{1}{4}$ (3) $\dfrac{3}{4}$ (4) $\dfrac{5}{6}$

Sol.



By anglar momentum conservation

$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \dfrac{\omega}{4}$

$(KE)_i = \dfrac{1}{2} I\omega^2$

$(KE)_f = \dfrac{1}{2} \times (4I) \times \left(\dfrac{\omega}{4}\right)^2 = \dfrac{I\omega^2}{8}$

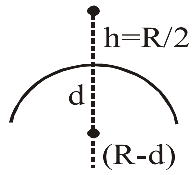
$\Delta KE = \dfrac{3}{8} I\omega^2$

fractional loss $= \dfrac{\Delta KE}{KE_1} = \dfrac{\frac{3}{8} I\omega^2}{\frac{1}{2} I\omega^2} = \dfrac{3}{4}$

18. The value of the acceleration due to gravity is g_1 at a height $h = \dfrac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\dfrac{d}{R}\right)$ equals :

(1) $\dfrac{7}{9}$ (2) $\dfrac{4}{9}$ (3) $\dfrac{1}{3}$ (4) $\dfrac{5}{9}$

Sol.



$g_1 = \dfrac{GM}{\left(R + \dfrac{R}{2}\right)^2}$ ... (1)

$g_2 = \dfrac{GM(R - d)}{R^3}$ ... (2)

$g_1 = g_2$

$\dfrac{GM}{\left(\dfrac{3R}{2}\right)^2} = \dfrac{GM(R - d)}{R^3}$

$\Rightarrow \dfrac{4}{9} = \dfrac{(R - d)}{R}$

$4R = 9R - 9d$

$5R = 9d \Rightarrow \dfrac{d}{R} = \dfrac{5}{9}$

19. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is

$\vec{E} = 30\hat{j}\ \sin(1.5\times10^{7}t - 5\times10^{-2}x)V/m$.

The maximum magnetic force experienced by the electron will be :

(given $c = 3 \times 10^8\ ms^{-1}$ and electron charge $= 1.6 \times 10^{-19}$ C)

(1) 1.6×10^{-19} N (2) 4.8×10^{-19} N

(3) 3.2×10^{-18} N (4) 2.4×10^{-18} N

Sol. $\Rightarrow E = \vec{E} = 30\hat{j}\sin(1.5\times10^{7}t - 5\times10^{-2}x)V/m$

$\Rightarrow B \Rightarrow E/V \Rightarrow \dfrac{30}{1.5\times10^{7}}\times5\times10^{-2}$

$\Rightarrow 10^{-7}$ Tesla

$\Rightarrow F_{mag} = q\left(\vec{V}\times\vec{B}\right) = |qVB|$

$= 1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 10^{-7}$

$= 4.8 \times 10^{-19}$ N

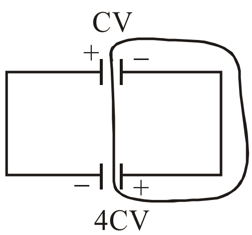
20. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is:

(1) $\dfrac{9}{2}CV^2$ (2) $\dfrac{25}{6}CV^2$

(3) zero (4) $\dfrac{3}{2}CV^2$

Sol.

$Q_1 = CV$ $\quad Q_2 = 2C \times 2V = 4CV$



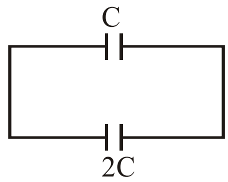
$\Rightarrow$ By conservation of charge

$q_i = q_f$

$Q_1 + Q_2 = q_1 + q_2$

$4CV - CV = (C + 2C)\ V_C$

$V_C = \dfrac{3CV}{3C} \Rightarrow V$

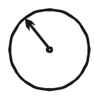


$\Rightarrow \dfrac{1}{2}\times(3C)\times V_C^2$

$= \dfrac{1}{2} \times 3C \times V^2 = \dfrac{3}{2}CV^2$

21. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1s is $\dfrac{4}{x}$. The value of x is ____.

Sol.



$B = \dfrac{\mu_0 NI}{2R}$

$\phi = \dfrac{\mu_0 NN'I}{2R}\pi r^2$

$\varepsilon = \dfrac{d\phi}{dt} = \dfrac{2\pi\times10^{-7}\times10^{5}\times\pi\times10^{-4}}{0.2}$

$= 8 \times 10^{-4} = 0.8$ mV

22. A force $\vec{F}=(\hat{i}+2\hat{j}+3\hat{k})N$ acts at a point $(4\hat{i}+3\hat{j}-\hat{k})m$. Then the magnitude of torque about the point $(\hat{i}+2\hat{j}+\hat{k})m$ will be $\sqrt{x}$ N-m. The value of x is _____.

Sol. $\vec{\tau}=(\vec{r}_2-\vec{r}_1)\times\vec{F}$

$=[(4\hat{i}+3\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})]\times\vec{F}$

$=(3\hat{i}+\hat{j}-2\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})$

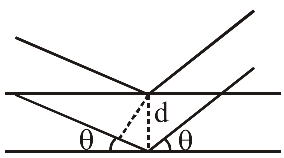
$$\tau=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & 2 & 3\end{vmatrix}$$

$=7\hat{i}-11\hat{j}+5\hat{k}$

$|\vec{\tau}|=\sqrt{195}$

23. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$. Then E (in eV) is ______.
(Planck constant h = 6.64×10^{-34} Js, 1eV = 1.6×10^{-19} J, electron mass m = 9.1×10^{-31} kg)

Sol.



$$2d\sin\theta=\lambda=\frac{h}{\sqrt{2mE}}$$

$$2\times10^{-10}\times\frac{\sqrt{3}}{2}=\frac{6.6\times10^{-34}}{\sqrt{2mE}}$$

$$E=\frac{1}{2}\times\frac{6.64^2\times10^{-48}}{9.1\times10^{-31}\times3\times1.6\times10^{-19}}=50.47$$

24. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is :
(Given the mass of the hydrogen atom to be 1 GeV/c^2) ________.

Sol. $mV_0 = MV = p$

$$10.2=\frac{p^2}{2m}-\frac{p^2}{2M}=\frac{p^2}{2m}\left(1-\frac{m}{M}\right)$$

$$=\frac{p^2}{2m}(1-0.2)$$

$$\Rightarrow \frac{p^2}{2m}=K=\frac{10.2}{0.8}$$

25. A compound microscope consists of an objective lens of focal length 1cm and an eye piece of focal length 5 cm with a separation of 10 cm.
The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is ____.

Sol. Final image at ∞
⇒ obj. for eye piece at 5cm
⇒ image for objective at 5 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{5}+\frac{1}{x}=1$$

$$\frac{1}{x}=1-\frac{1}{5}=\frac{4}{5}\Rightarrow x=\frac{5}{4}$$

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(Held On Saturday 05th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The equation that represents the water-gas shift reaction is :

(1) $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673K} CO_2(g) + H_2(g)$

(2) $CH_4(g) + H_2O(g) \xrightarrow[\text{Ni}]{1270K} CO(g) + 3\,H_2(g)$

(3) $C(s) + H_2O(g) \xrightarrow{1270K} CO(g) + H_2(g)$

(4) $2C(s)+O_2(g)+4N_2(g) \xrightarrow{1273K} 2CO(g)+4N_2(g)$

Sol. (1) Water gas shift reaction

$$CO_{(g)} + H_2O_{(g)} \xrightarrow[\text{catalyst}]{673K} CO_{2(g)} + H_{2(g)}$$

(2) Water gas is produced by this reaction.

$$CH_{4(g)} + H_2O_{(g)} \xrightarrow[\text{Ni}]{1270K} CO_{(g)} + 3H_{2(g)}$$

(3) Water gas is produced by this reaction

$$C_{(s)} + H_2O_{(g)} \xrightarrow{1270K} CO_{(g)} + H_{2(g)}$$

(4) producer gas is produced by this reaction.

$$2C_{(s)} + O_{2(g)} + 4N_{2(g)} \xrightarrow{1270K} 2CO_{(g)} + 4N_{2(g)}$$

2. Consider the following reaction

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$; $\Delta H^0 = +58$ kJ

For each of the following cases (a, b), the direction in which the equilibrium shifts is:

(a) Temperature is decreased

(b) Pressure is increased by adding N_2 at constant T

(1) (a) towards reactant, (b) no change

(2) (a) towards product, (b) towards reactant

(3) (a) towards product, (b) no change

(4) (a) towards reactant, (b) towards product

Sol. $\Delta H^o > 0$ T$\downarrow$ equation shifts back ward.

N_2 is treated as inert gas in this case hence no effect on equilibrium.

3. The values of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are :

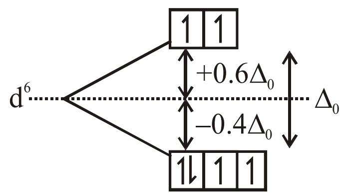
(1) $-0.4\ \Delta_0$ and $-0.27\ \Delta_t$

(2) $-1.6\ \Delta_0$ and $-0.4\ \Delta_t$

(3) $-0.4\ \Delta_0$ and $-0.6\ \Delta_t$

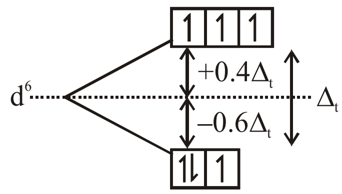
(4) $-2.4\ \Delta_0$ and $-0.6\ \Delta_t$

Sol. For high spin octahedral field



CFSE = (4) $(-0.4\Delta_0) + 2(0.6\ \Delta_0) = -\ 0.4\ \Delta_0$

For high spin tetrahedral field



CFSE = $3(-0.6\Delta_t) + 3(0.4\ \Delta_t) = -\ 0.6\ \Delta_t$

4. Which of the following is not an essential amino acid :

(1) Valine

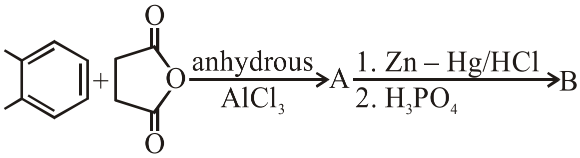
(2) Leucine

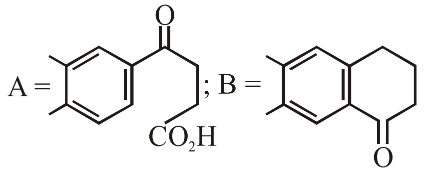
(3) Lysine

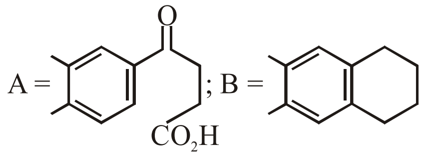
(4) Tyrosine

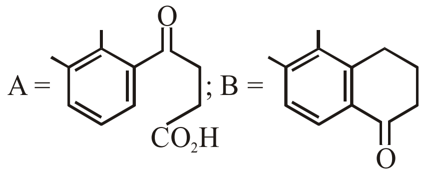
Sol. Tyrosine is not an essential amino acid.

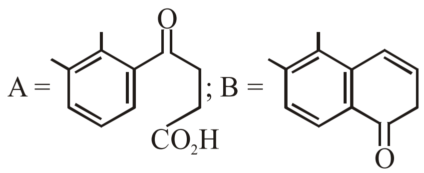
5. In the following reaction sequence the major products A and B are :



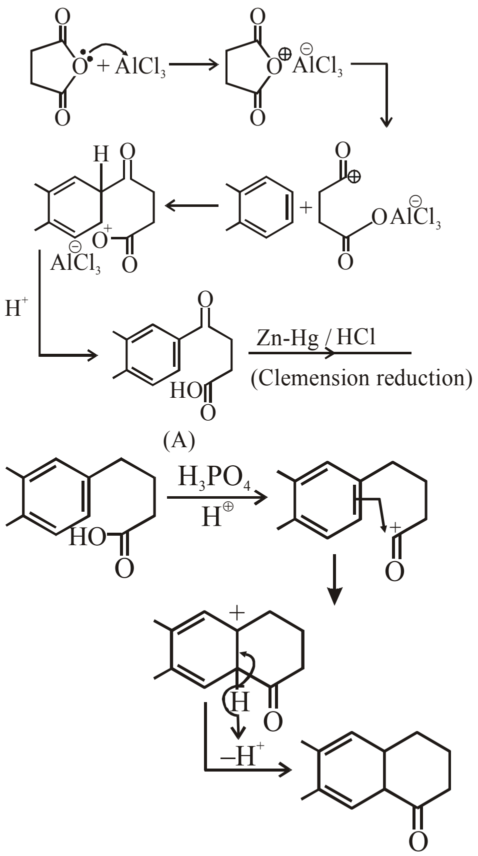
(1) 

(2) 

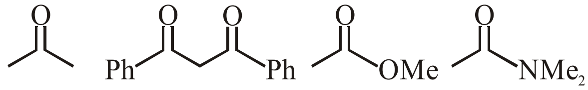
(3) 

(4) 

Sol.



6. The increasing order of the acidity of the α-hydrogen of the following compounds is :



(A) (B) (C) (D)

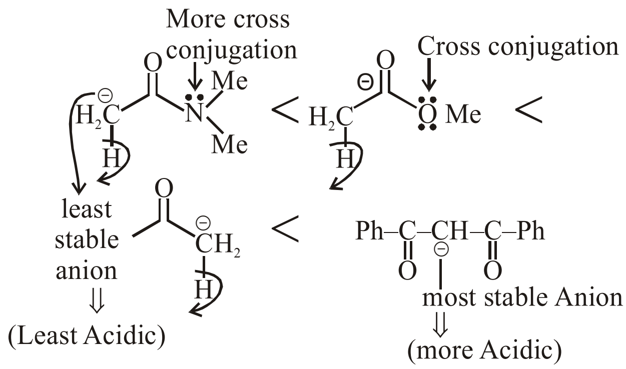
(1) (C) < (A) < (B) < (D)

(2) (B) < (C) < (A) < (D)

(3) (A) < (C) < (D) < (B)

(4) (D) < (C) < (A) < (B)

Sol. $\boxed{D < C < A < B}$



7. An Ellingham diagram provides information about :

(1) the pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.

(2) the kinetics of the reduction process.

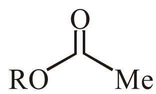
(3) the temperature dependence of the standard Gibbs energies of formation of some metal oxides.

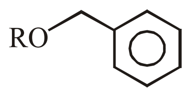
(4) the conditions of pH and potential under which a species is thermodynamically stable.

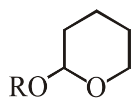
Sol. Ellingham diagram provides information about temperature dependence of the standard gibbs energies of formation of some metal oxides.

8. Which of the following derivatives of alcohols is unstable in an aqueous base ?

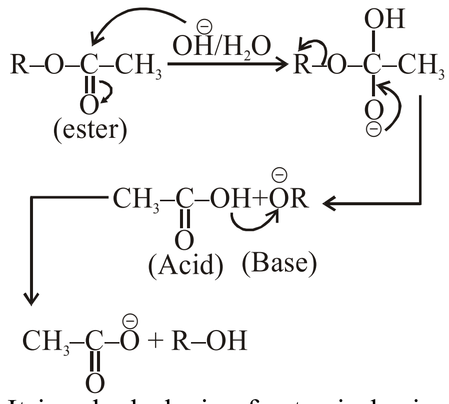
(1) $RO - CMe_3$

(2) 

(3) 

(4) 

Sol.



It is a hydrolysis of ester in basic medium.

9. The structure of PCl_5 in the solid state is

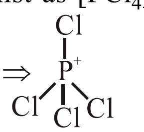
(1) square pyramidal

(2) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$

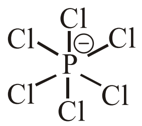
(3) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$

(4) trigonal bipyramidal

Sol. $PCl_{5(s)}$ exist as $[PCl_4]^+$ and $[PCl_6]^-$

$[PCl_4]^+ \Rightarrow$  (sp^3 hybridisation)

Tetrahedral

$[PCl_6]^- \Rightarrow$ 

octahedral

sp^3d^2 hybridization

10. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is :

(1) $Na(CN)BH_3$ (2) $LiAlH_4$

(3) $NaBH_4$ (4) CaH_2

Sol. $CH_3–CH_2–C \equiv N \xrightarrow{?} CH_3–CH_2–CH_2–NH_2$

$CH_3–CH_2–C \equiv N \xrightarrow{LiAlH_4} CH_3–CH_2–CH_2–NH_2$

11. The difference between the radii of 3rd and 4th orbits of Li^{2+} is ΔR_1. The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2. Ratio $\Delta R_1 : \Delta R_2$ is :

(1) 8 : 3 (2) 3 : 2

(3) 3 : 8 (4) 2 : 3

Sol. $$\frac{\Delta R_1}{\Delta R_2} = \frac{(r_4 - r_3)_{Li^{2+}}}{(r_4 - r_3)_{He^+}} = \frac{\frac{4^2}{3} - \frac{3^2}{3}}{\frac{4^2}{2} - \frac{3^2}{2}} = \frac{7/3}{7/2} = \frac{2}{3}$$

12. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B(in s) : (Use ln 2 = 0.693)

(1) 180 (2) 120

(3) 300 (4) 900

Sol. $[A]_t = 4[B]_t$

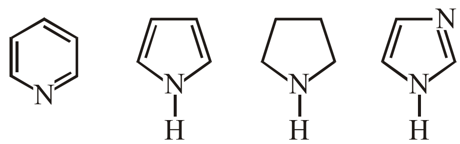
$$[A]_0\ e^{-(\ln^2/300)^t} = 4[B]_0 e^{(-\ln 2/180)t}$$

$$e^{\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)} = 4$$

$$\left(\frac{\ln^2}{180} - \frac{\ln^2}{300}\right)t = \ln 4$$

$$\left(\frac{1}{180} - \frac{1}{300}\right)t = 2 \Rightarrow t = \frac{2 \times 180 \times 300}{120} = 900\,\text{sec}.$$

13. The increasing order of basicity of the following compounds is



(A) (B) (C) (D)

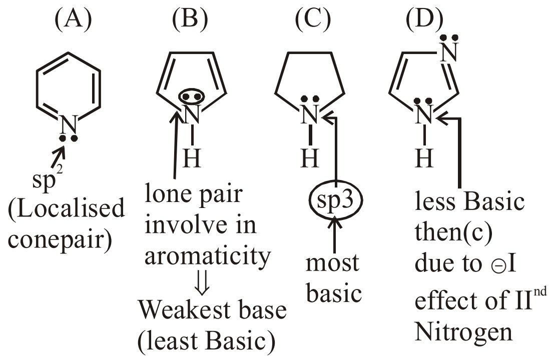
(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (C) < (D)

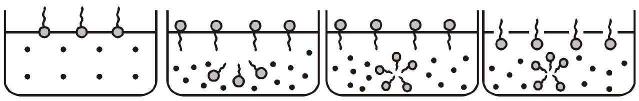
(3) (D) < (A) < (B) < (C)

(4) (B) < (A) < (D) < (C)

Sol.



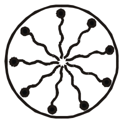
14. Identify the correct molecular picture showing that happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (○ polar head; ∼non-polar tail; • water).



(A) (B) (C) (D)

(1) (B) (2) (A) (3) (D) (4) (C)

Sol. 



Micelles formed at CMC.

15. If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested ?

(1) Anti-inflammatory (2) Analgesic
(3) Antihistamine (4) Antidepressant

Sol. Anti depressant → drug which enhance the mood. Non adrenaline is neurotransmitter and its level is low in body due to some reason then person suffers from depression and in that situation anti depressant drug is required.

16. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} (Z = 64), respectively, are

(1) $[Xe]5f^7$ and 8.9 (2) $[Xe]4f^7$ and 7.9
(3) $[Xe]5f^7$ and 7.9 (4) $[Xe]4f^7$ and 8.9

Sol. Electronic configuration of Gd^{3+} is

${}_{64}Gd^{3+} = [Xe]4f^7$

[Xe] $\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}$

Gd^{3+} having 7 unpaired electrons.

Magnetic moment (μ) = $\sqrt{n(n+2)}$B.M.

$\mu = \sqrt{7(7+2)}$B.M.

= 7.9 B.M.

n ⇒ Number of unpaired electrons.

17. The condition that indicates a polluted environment is

(1) BOD value of 5 ppm
(2) eutrophication
(3) 0.03% of CO_2 in the atmosphere
(4) pH of rain water to be 5.6

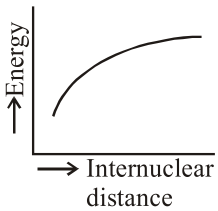
Sol. In Eutrophication nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity. If indicates polluted environment.

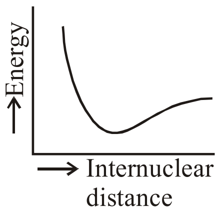
18. In the sixth period, the orbitals that are filled are

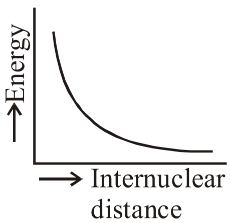
(1) 6s, 5f, 6d, 6p (2) 6s, 6p, 6d, 6f
(3) 6s, 5d, 5f, 6p (4) 6s, 4f, 5d, 6p

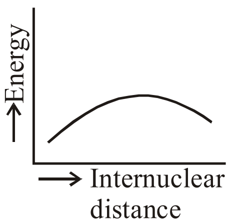
Sol. As per (n + ℓ) rule in 6th period, order of orbitals filling is 6s, 4f, 5d, 6p.

19. The potential energy curve for the H_2 molecule as a function of internuclear distance is :

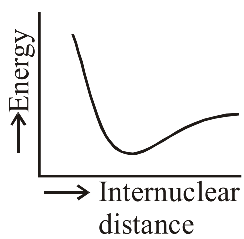
(1) 

(2) 

(3) 

(4) 

Sol. Potential energy curve for H_2 molecule is.



20. A diatomic molecule X_2 has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is 6.17 g cm^{-3}. The number of molecules present in 200 g of X_2 is

(Avogadro constant $(N_A) = 6 \times 10^{23}$ mol^{-1})

(1) 8 N_A

(2) 40 N_A

(3) 4 N_A

(4) 2 N_A

Sol. $$p = \frac{2 \times \frac{M}{N_A}}{a^3} \Rightarrow 6.17 = \frac{2 \times \frac{M}{N_A}}{(3 \times 10^{-8}\,cm)^3}$$

$$\Rightarrow M \simeq 50\ gm/mol$$

$$No = \frac{w}{M} \times N_A = \frac{200}{50} \times N_A = 4N_A$$

21. an oxidation-reduction reaction in which 3 electrons are transferred has a ΔG^o of 17.37 kJ mol^{-1} at 25^{o}C. The value of E^o_{cell} (in V) is _____ $\times 10^{-2}$

(1 F = 96,500 C mol^{-1})

Sol. $$\Delta G^\circ = -AFE^\circ = -3 \times 96500 \times E^\circ$$

$$\Rightarrow E^\circ = -6 \times 10^{-2}\ V$$

22. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is ______.

Sol. $$\underset{1mole}{C_3H_8} + \underset{5mole}{SO_2} \rightarrow 3Co_2 + 4H_2O$$

For 1 mole propane combustion 5 mole O_2 required

$$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4Co_2 + 5H_2O$$

1 mole 6.5 mole

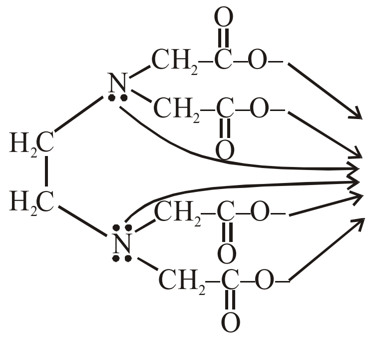
2 mole 13 mole

For 2 moles of butane 13 mole of O_2 is required

total moles = 13 + 5 = 18

23. The total number of coordination sites in ethylenediaminetetraacetate ($EDTA^{4-}$) is ________.

Sol. $EDTA^{4-}$ is hexadentate ligand, so its donation sites are six.



24. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is _____.

25. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is ___________ $\times 10^{-1}$.

(First dissociation constant of $H_2CO_3 = 4.0 \times 10^{-7}$; log 2 = 0.3; density of the soft drink = 1 g mL^{-1})

Sol. $$P_{CO_2} = K_H \times CO_2$$

$$\frac{3}{30} = \frac{K_H . n_{CO_2}}{K_H 1} \Rightarrow n_{CO_2 = 0.1} mol$$

$$pH = \frac{1}{2}(pka_1 - \log c) = \frac{1}{2}(6.4 \times 1) = 3.7$$

pH = 37 $\times 10^{-1}$

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Saturday 05th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P. is :

(1) 66 (2) 65

(3) 81 (4) 78

Sol. Given that

$3^{4-\sin 2\alpha} + 3^{2\sin 2\alpha - 1} = 28$

Let $3^{2\sin 2\alpha} = t$

$$\frac{81}{t} + \frac{t}{3} = 28$$

$t = 81, 3$

$3^{2\sin 2\alpha} = 3^1, 3^4$

$2\sin 2\alpha = 1, 4$

$\sin 2\alpha = \frac{1}{2}, 2$ (rejected)

First term $a = 3^{2\sin 2\alpha - 1}$

$a = 1$

Second term = 14

$\therefore$ common difference d = 13

$T_6 = a + 5d$

$T_6 = 1 + 5 \times 13$

$T_6 = 66$

2. If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$

is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

(1) $\left(\frac{1}{2}, 1\right)$ (2) (1, 1)

(3) $\left(\frac{1}{2}, -1\right)$ (4) (1, 0)

Sol. f(x) is continuous and differentiable

$f(\pi^-) = f(\pi) = f(\pi^+)$

$-1 = -k_2$

$\boxed{k_2 = 1}$

$$f'(x) = \begin{cases} 2k_1(x-\pi) \ ; \ x \le \pi \\ -k_2 \sin x \ \ ; \ x > \pi \end{cases}$$

$f'(\pi^-) = f'(\pi^+)$

$0 = 0$

so, differentiable at x = 0

$$f''(x) = \begin{cases} 2k_1 \ ; \ x \le \pi \\ -k_2 \cos x \ ; \ x > \pi \end{cases}$$

$f''(\pi^-) = f''(\pi^+)$

$2k_1 = k_2$

$\boxed{k_1 = \frac{1}{2}}$

$$(k_1, k_2) = \left(\frac{1}{2}, 1\right)$$

3. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4}$

Sol. $y = mx + \frac{1}{m}$ (tangent at $y^2 = 4x$)

$y = mx - m^2$ (tangent at $x^2 = 4y$)

$\frac{1}{m} = -m^2$ (for common tangent)

$m^3 = -1$

$\boxed{m = -1}$

$y = -x - 1$
$x + y + 1 = 0$
This line touches circle
$\therefore$ apply $p = r$

$c = \left|\frac{0+0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

4. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(2) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(3) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

Sol. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$x \leftrightarrow \sim y \equiv (x \rightarrow \sim y) \wedge (\sim y \rightarrow x)$

$\because (p \rightarrow q \equiv \sim p \vee q)$

$x \leftrightarrow \sim y \equiv (\sim x \vee \sim y) \wedge (y \vee x)$

$\sim (x \leftrightarrow \sim y) \equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

5. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158 cu. units, then :

(1) $\vec{a} \cdot \vec{c} = 17$ (2) $\vec{b} \cdot \vec{c} = 10$

(3) $n = 7$ (4) $n = 9$

Sol. $v = [\vec{a}\ \vec{b}\ \vec{c}]$

$$158 = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}, \ n \geq 0$$

$158 = 1\,(12 + n^2) - (6 + n) + n(2n - 4)$
$158 = n^2 + 12 - 6 - n + 2n^2 - 4n$
$3n^2 - 5n - 152 = 0$

$n = 8, \ -\frac{38}{6}$ (rejected)

$\vec{a} \cdot \vec{c} = 1 + n + 3n = 1 + 4n = 33$

$\vec{b} \cdot \vec{c} = 2 + 4n - 3n = 2 + n = 10$

6. If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is :

(1) 1 (2) –1

(3) 2 (4) 0

Sol. $\frac{(5+e^x)}{2+y}\frac{dy}{dx} = -e^x$

$$\int \frac{dy}{2+y} = \int \frac{-e^x}{e^x+5}\,dx$$

$\ln (y + 2) = -\ln(e^x + 5) + k$
$(y + 2)(e^x + 5) = C$
$\because \ y(0) = 1$
$\Rightarrow C = 18$

$y + 2 = \frac{18}{e^x+5}$

at $x = \ln 13$

$y + 2 = \frac{18}{13+5} = 1$

$\boxed{y = -1}$

7. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

(1) 63 (2) 38

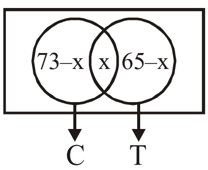
(3) 54 (4) 36

Sol. $C \rightarrow$ person like coffee

$T \rightarrow$ person like Tea

$n(C) = 73$

$n(T) = 65$



$n(C \cup T) \leq 100$

$n(C) + n(T) - n(C \cap T) \leq 100$

$73 + 65 - x \le 100$

$x \ge 38$

$73 - x \ge 0 \Rightarrow x \le 73$

$65 - x \ge 0 \Rightarrow x \le 65$

$\boxed{38 \le x \le 65}$

8. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$

(3) $\frac{5}{27}$ (4) $\frac{5}{9}$

Sol. $9x^2 - 18|x| + 5 = 0$

$9|x|^2 - 15|x| - 3|x| + 5 = 0$ ($\because x^2 = |x|^2$)

$3|x|(3|x| - 5) - (3|x| - 5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

$x = \pm\frac{1}{3}, \pm\frac{5}{3}$

Product of roots $= \frac{25}{81}$

9. If $\int (e^{2x} + 2e^{x} - e^{-x} - 1)e^{(e^x + e^{-x})}dx$

$= g(x)e^{(e^x+e^{-x})} + c$, where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e^2

(3) e (4) 1

Sol. $e^{2x} + 2e^x - e^{-x} - 1$

$= e^x(e^x + 1) - e^{-x}(e^x + 1) + e^x$

$= [(e^x + 1)(e^x - e^{-x}) + e^x]$

so $I = \int (e^x + 1)(e^x - e^{-x})e^{e^x+e^{-x}} + \int e^x \cdot e^{e^x+e^{-x}}dx$

$= (e^x + 1)e^{e^x+e^{-x}} - \int e^x.e^{e^x+e^{-x}}dx + \int e^x.e^{e^x+e^{-x}}dx$

$= (e^x + 1)e^{e^x+e^{-x}} + C$

$\therefore g(x) = e^x + 1 \Rightarrow g(0) = 2$

10. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by :

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to :

(1) (0, 4) (2) (–4, 4)

(3) $(0, 2\sqrt{2})$ (4) (–4, 0)

Sol. $C_3 \to C_3 - (C_1 - C_2)$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 0 \\ -\cos^2\theta & -1-\cos^2\theta & 0 \\ 12 & 10 & -4 \end{vmatrix}$$

$= -4[(1 + \cos^2\theta)\sin^2\theta - \cos^2\theta(1 + \sin^2\theta)]$

$= -4[\sin^2\theta + \sin^2\theta\cos^2\theta - \cos^2\theta - \cos^2\theta\sin^2\theta]$

$f(\theta) = 4\cos 2\theta$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$2\theta \in \left[\frac{\pi}{2}, \pi\right]$

$f(\theta) \in [-4, 0]$

$(m, M) = (-4, 0)$

11. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for :

(1) exactly one negative value of λ.

(2) exactly one positive value of λ.

(3) every value of λ.

(4) exactly two values of λ.

Sol. $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$

$= 2(3\lambda + 2)(\lambda - 3)$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them is zero so equations are inconsistant

$\boxed{\therefore \lambda = -\frac{2}{3}}$

12. If S is the sum of the first 10 terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \ldots,$$

then tan(S) is equal to :

(1) $\frac{5}{11}$ (2) $-\frac{6}{5}$

(3) $\frac{10}{11}$ (4) $\frac{5}{6}$

Sol. $S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \ldots$

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\times 3}\right) + \tan^{-1}\left(\frac{4-3}{1+3\times 4}\right) + \ldots + \tan^{-1}\left(\frac{11-10}{1+10\times 11}\right)$

$S = (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + \ldots + (\tan^{-1}(11) - \tan^{-1}(10))$

$S = \tan^{-1}11 - \tan^{-1}1 = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

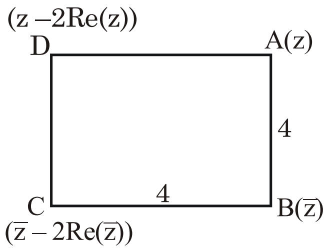
$\tan(S) = \frac{11-1}{1+11\times 1} = \frac{10}{12} = \frac{5}{6}$

13. If the four complex numbers z, $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :

(1) 4 (2) 2

(3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Sol. Let $z = x + iy$



Length of side = 4

AB = 4

$|z - \bar{z}| = 4$

$|2y| = 4$; $|y| = 2$

BC = 4

$|\bar{z} - (\bar{z} - 2\text{Re}(\bar{z})| = 4$

$|2x| = 4$; $|x| = 2$

$|z| = \sqrt{x^2 + y^2} = \sqrt{4+4} = 2\sqrt{2}$

14. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, –4), then PQ^2 is equal to :

(1) 21 (2) 36

(3) 48 (4) 29

Sol. Given ellipse is $\frac{x^2}{5}+\frac{y^2}{4}=1$

Let point P is $(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2$

$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$

$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

will be maximum when $\sin\theta = 1$

$\Rightarrow (PQ)^2_{max} = 85 - 49 = 36$

15. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :

(1) 2 (2) 4

(3) 3 (4) 1

Sol. $\overline{x} = \frac{2+4+10+12+14+x+y}{7} = 8$

$x + y = 14$ .....(i)

$(\sigma)^2 = \frac{\sum(x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - 8^2$

$16 + 64 = \frac{460+x^2+y^2}{7}$

$560 = 460 + x^2 + y^2$

$x^2 + y^2 = 100$ ....(ii)

Clearly by (i) and (ii), $|x - y| = 2$

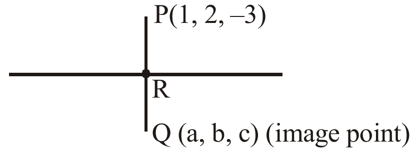
Ans. 1

16. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b + c is equal to

(1) –1 (2) 2

(3) 3 (4) 1

Sol.



Line is $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda$: Let point R is

$(2\lambda - 1, -2\lambda + 3, -\lambda)$

Direction ratio of PQ$\equiv(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is (1, 1, –1)

$\frac{a+1}{2}=1$ | $\frac{b+2}{2}=1$ | $\frac{c-3}{2}=-1$

$a = 1$ | $b = 0$ | $c = 1$

$\Rightarrow a + b + c = 2$

17. The value of $\int_{-\pi/2}^{\pi/2}\frac{1}{1+e^{\sin x}}dx$ is

(1) π (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. $I = \int_{-\pi/2}^{\pi/2}\frac{1}{1+e^{\sin x}}dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2}\frac{1}{1+e^{-\sin x}}dx = \int_{-\pi/2}^{\pi/2}\frac{e^{\sin x}}{1+e^{\sin x}}dx$ ...(2)

Add (1) & (2)

$$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$$

$$I = \frac{\pi}{2}$$

18. If $2^{10} + 2^9\cdot 3^1 + 2^8\cdot 3^2 + ... + 2\cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to :

(1) $\frac{3^{11}}{2} + 2^{10}$ (2) $3^{11} - 2^{12}$

(3) 3^{11} (4) $2\cdot 3^{11}$

Sol. $a = 2^{10}$; $r = \frac{3}{2}$; $n = 11$ (G.P.)

$$S' = (2^{10})\frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$$

$S' = 3^{11} - 2^{11} = S - 2^{11}$ (Given)

$\therefore$ $S = 3^{11}$

19. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

(1) 8 (2) 6

(3) 16 (4) 9

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a = 4$; $b = 3$; $e = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$

A and B are foci

$\Rightarrow$ PA + PB = 2a = 2 × 4 = 8

20. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x \to \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$ is equal to

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Sol. $x^2 - x - 2 = 0$

roots are 2 & –1

$$\Rightarrow \lim_{x \to 2^+} \frac{\sqrt{1 - \cos(x^2 - x - 2)}}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{\sqrt{2\sin^2\frac{(x^2 - x - 2)}{2}}}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)}$$

$$= \frac{3}{\sqrt{2}}$$

21. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ____.

Sol. 4 dice are independently thrown. Each die has probability to show 3 or 5 is

$$p = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ $q = 1 - \frac{1}{3} = \frac{2}{3}$ (not showing 3 or 5)

Experiment is performed with 4 dices independently.

$\therefore$ Their binomial distribution is

$(q + p)^4 = (q)^4 + {}^4C_1\, q^3p + {}^4C_2\, q^2p^2 + {}^4C_3\, qp^3 + {}^4C_4p^4$

$\therefore$ In one throw of each dice probability of showing 3 or 5 at least twice is

$= p^4 + {}^4C_3\, qp^3 + {}^4C_2 q^2 p^2$

$= \frac{33}{81}$

$\therefore$ Such experiment performed 27 times

$\therefore$ so expected out comes = np

$= \frac{33}{81} \times 27$

= 11

22. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

Sol. Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$\left|\frac{\alpha - 6}{255}\right| = \frac{1}{55}$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$\left|\frac{\beta - 9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}}$

$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

sum = 18

sum of all values of α and β is = 30

23. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ________.

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r} \left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m-mr-2r}$

$= {}^{22}C_r\, x$

$\because {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore$ r = 3 or 19

$22m - mr - 2r = 1$

$m = \frac{2r+1}{22-5}$

$r = 3, \quad m = \frac{7}{19} \notin N$

$r = 19, \ m = \frac{38+1}{22-19} = \frac{39}{3} = 13$

m = 13

24. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ________.

Sol. S_2YL_2ABU

ABCC type words

$= \underbrace{{}^2C_1}_{\text{selection of two alike letters}} \times \underbrace{{}^5C_2}_{\text{selection of of two distinct letters}} \times \underbrace{\frac{\lfloor 4}{\lfloor 2}}_{\text{arrangement of selected letters}}$

= 240

25. Let $f(x) = x \cdot \left[\frac{x}{2}\right]$, for $-10 < x < 10$, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ________.

Sol. $x \in (-10, 10)$

$\frac{x}{2} \in (-5, 5) \rightarrow 9$ integers

check continuity at x = 0

$\left.\begin{aligned} f(0) &= 0 \\ f(0^+) &= 0 \\ f(0^-) &= 0 \end{aligned}\right\}$ continuous at x = 0

function will be distcontinuous when

$\frac{x}{2} = \pm 4, \ \pm 3, \ \pm 2, \ \pm 1$

8 points of discontinuity