

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

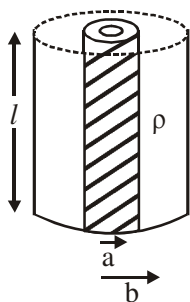
PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

- (1) 2.123 cm (2) 2.125 cm
(3) 2.121 cm (4) 2.124 cm

2. Model a torch battery of length l to be made up of a thin cylindrical bar of radius ' a ' and a concentric thin cylindrical shell of radius ' b ' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R , the maximum Joule heating in R will take place for:-



- (1) $R = \frac{2\rho}{\pi l} \ln\left(\frac{b}{a}\right)$ (2) $R = \frac{\rho}{\pi l} \ln\left(\frac{b}{a}\right)$
(3) $R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right)$ (4) $R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right)$

3. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

- (1) 0.61 eV (2) 0.52 eV
(3) 0.81 eV (4) 1.02 eV

4. Moment of inertia of a cylinder of mass M , length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of material, the ratio L/R for it to have minimum possible I is :-
- (1) $\sqrt{\frac{2}{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{2}{3}$
5. The magnetic field of a plane electromagnetic wave is $\vec{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{i}$ T. Where $c = 3 \times 10^8 \text{ ms}^{-1}$ is the speed of light. The corresponding electric field is :
- (1) $\vec{E} = -10^{-6} \sin[200\pi(y + ct)]\hat{k}$ V / m
 (2) $\vec{E} = -9 \sin[200\pi(y + ct)]\hat{k}$ V / m
 (3) $\vec{E} = 9 \sin[200\pi(y + ct)]\hat{k}$ V / m
 (4) $\vec{E} = 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{k}$ V / m
6. A charged particle carrying charge $1 \mu\text{C}$ is moving with velocity $(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ ms}^{-1}$. If an external magnetic field of $(5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3} \text{ T}$ exists in the region where the particle is moving then the force on the particle is $\vec{F} \times 10^{-9} \text{ N}$. The vector \vec{F} is :
- (1) $-0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$
 (2) $-300\hat{i} + 320\hat{j} - 90\hat{k}$
 (3) $-30\hat{i} + 32\hat{j} - 9\hat{k}$
 (4) $-3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$

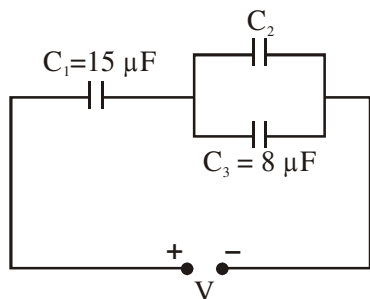
7. A 750 Hz, 20 V (rms) source is connected to a resistance of $100\ \Omega$, an inductance of 0.1803 H and a capacitance of $10\ \mu\text{F}$ all in series. The time in which the resistance (heat capacity $2\text{J/}^\circ\text{C}$) will get heated by 10°C . (assume no loss of heat to the surroundings) is close to :
(1) 418 s (2) 245 s (3) 348 s (4) 365 s
8. In a radioactive material, fraction of active material remaining after time t is $9/16$. The fraction that was remaining after $t/2$ is :
(1) $\frac{3}{4}$ (2) $\frac{7}{8}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$
9. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :
(1) Irreversible isothermal
(2) Irreversible adiabatic
(3) Reversible adiabatic
(4) Reversible isothermal
10. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :
(1) 8 : 1 (2) 0.8 : 1
(3) 2 : 1 (4) 4 : 1

11. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e . By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that

is become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R , value of R is :

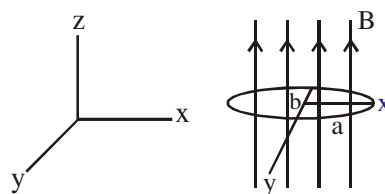
- (1) $4R_e$ (2) $3R_e$ (3) $2R_e$ (4) $2.5R_e$

12. In the circuit shown in the figure, the total charge is $750 \mu\text{C}$ and the voltage across capacitor C_2 is 20 V . Then the charge on capacitor C_2 is :



- (1) $590 \mu\text{C}$ (2) $450 \mu\text{C}$
(3) $650 \mu\text{C}$ (4) $160 \mu\text{C}$

13. An elliptical loop having resistance R , of semi major axis a , and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x -axis with angular frequency ω , the average power loss in the loop due to Joule heating is :



- (1) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$ (2) Zero
(3) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$ (4) $\frac{\pi a b B \omega}{R}$

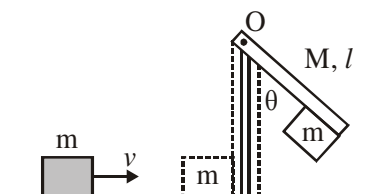
14. When a diode is forward biased, it has a voltage drop of 0.5 V . The safe limit of current through the diode is 10 mA . If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

- (1) 100Ω (2) 50Ω
(3) 300Ω (4) 200Ω

15. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 9 (2) 12 (3) 6 (4) 3

16. A block of mass $m = 1$ kg slides with velocity $v = 6$ m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2$ kg, and length $l = 1$ m, the value of θ is approximately :
(Take $g = 10$ m/s²)



(1) 69° (2) 63° (3) 55° (4) 49°

17. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

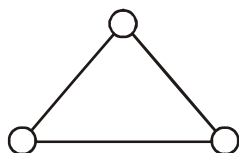
(1) 0.07° (2) 0.17° (3) 1.7° (4) 0.57°

18. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in

units of $\frac{\mu_0 I}{\pi}$ is :

- (1) $250\sqrt{3}$ (2) $5\sqrt{3}$
(3) $500\sqrt{3}$ (4) $50\sqrt{3}$

19.



Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

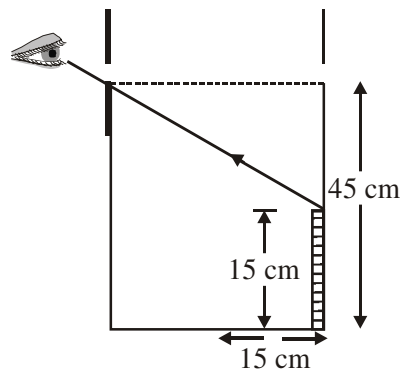
- (1) $\frac{9}{2}RT$ (2) $\frac{3}{2}RT$
(3) $\frac{5}{2}RT$ (4) $3RT$

20. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have $12 \mu C$ and $-3 \mu C$ charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

- (1) $6 \mu C$ and $3 \mu C$
(2) $+4.5 \mu C$ and $-4.5 \mu C$
(3) $3 \mu C$ and $6 \mu C$
(4) $4.5 \mu C$ on both

21. A bakelite beaker has volume capacity of 500 cc at $30^\circ C$. When it is partially filled with V_m volume (at 30°) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})} = 6 \times 10^{-6} ^\circ C^{-1}$ and $\gamma_{(\text{mercury})} = 1.5 \times 10^{-4} ^\circ C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____.

22. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is ($g = 10 \text{ ms}^{-2}$)_____.
23. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass is close to 0° , the surface tension of the liquid, in milliNewton m^{-1} , is [$\rho_{\text{liquid}} = 900 \text{ kgm}^{-3}$, $g = 10 \text{ ms}^{-2}$] (Give answer in closest integer)_____.
24. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis as 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre_____.
25. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, the value of N is_____.



FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The complex that can show optical activity is:

- (1) $\text{trans-}[\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$
- (2) $\text{cis-}[\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$
- (3) $\text{cis-}[\text{CrCl}_2(\text{ox})_2]^{3-}$ (ox = oxalate)
- (4) $\text{trans-}[\text{Cr}(\text{Cl}_2)(\text{ox})_2]^{3-}$

2. An organic compound [A], molecular formula $\text{C}_{10}\text{H}_{20}\text{O}_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and alcohol [C]. Oxidation of [C] with $\text{CrO}_3 - \text{H}_2\text{SO}_4$ produced [B]. Which of the following structures are not possible for [A] ?

- (1) $(\text{CH}_3)_3\text{C}-\text{COOCH}_2\text{C}(\text{CH}_3)_3$
- (2) $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOCH}_2\text{CH}_2\text{CH}_2\text{CH}_3$
- (3)
$$\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3-\text{CH}_2-\text{CH}-\text{OCOCH}_2-\text{CH}-\text{CH}_2\text{CH}_3 \\ | \\ \text{CH}_3 \end{array}$$
- (4)
$$\begin{array}{c} \text{CH}_3 \\ | \\ \text{CH}_3-\text{CH}_2-\text{CH}-\text{COOCH}_2-\text{CH}-\text{CH}_2\text{CH}_3 \\ | \\ \text{CH}_3 \end{array}$$

3. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be :

- (1) Greater than 300 K but less than 373 K
- (2) Less than 300 K
- (3) Equal to 373 K
- (4) More than 373 K

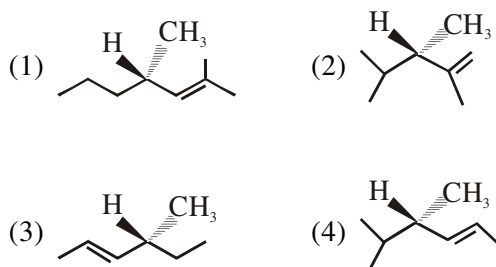
4. In a molecule of pyrophosphoric acid, the number of P–OH, P=O and P–O–P bonds/moiety(ies) respectively are :

- (1) 3, 3 and 3 (2) 2, 4 and 1
(3) 4, 2 and 0 (4) 4, 2 and 1

5. It is true that :

- (1) A zero order reaction is a single step reaction
(2) A second order reaction is always a multistep reaction
(3) A first order reaction is always a single step reaction
(4) A zero order reaction is a multistep reaction

6. Which of the following compounds produces an optically inactive compound on hydrogenation ?



7. Henry's constant (in kbar) for four gases α , β , γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that :

- (1) The pressure of a 55.5 molal solution of γ is 1 bar
(2) The pressure of a 55.5 molal solution of δ is 250 bar
(3) Solubility of γ at 308 K is lower than at 298 K
(4) α has the highest solubility in water at a given pressure

8. Tyndall effect is observed when :

- (1) The diameter of dispersed particles is much smaller than the wavelength of light used
(2) The diameter of dispersed particles is much larger than the wavelength of light used
(3) The diameter of dispersed particles is similar to the wavelength of light used
(4) The refractive index of dispersed phase is greater than that of the dispersion medium

9. Thermal power plants can lead to :

- (1) Ozone layer depletion
- (2) Eutrophication
- (3) Acid rain
- (4) Blue baby syndrome

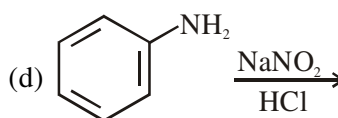
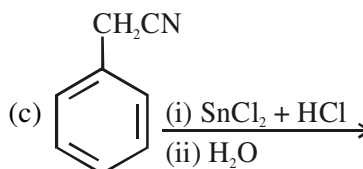
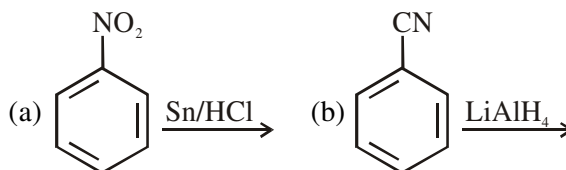
10. The electronic spectrum of $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ shows a single broad peak with a maximum at $20,300 \text{ cm}^{-1}$. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1} , is :

- (1) 242.5
- (2) 83.7
- (3) 145.5
- (4) 97

11. Aqua regia is used for dissolving noble metals (Au, Pt, etc). The gas evolved in this process is :

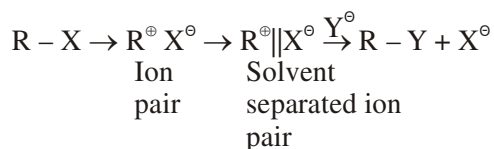
- (1) N_2
- (2) N_2O_3
- (3) NO
- (4) N_2O_5

12. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products ?



- (1) a and d
- (2) c and d
- (3) a, c and d
- (4) b and c

13. The mechanism of S_N1 reaction is given as :



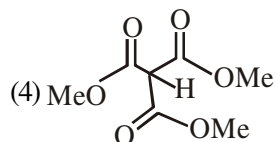
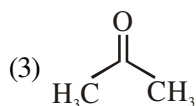
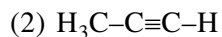
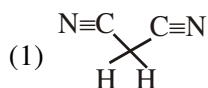
A student writes general characteristics based on the given mechanism as :

- (a) The reaction is favoured by weak nucleophiles
- (b) R^{\oplus} would be easily formed if the substituents are bulky
- (c) The reaction is accompanied by racemization
- (d) The reaction is favoured by non-polar solvents.

Which observations are correct ?

- (1) b and d (2) a and c
- (3) a, b and c (4) a and b

14. Which one of the following compounds possesses the most acidic hydrogen ?



15. Glycerol is separated in soap industries by :

- (1) Steam distillation
- (2) Differential extraction
- (3) Distillation under reduced pressure
- (4) Fractional distillation

16. Of the species, NO, NO⁺, NO²⁺, NO⁻, the one with minimum bond strength is :

- (1) NO²⁺ (2) NO⁺ (3) NO (4) NO⁻

17. The atomic number of the element unnilennium is :

- (1) 119 (2) 108 (3) 102 (4) 109

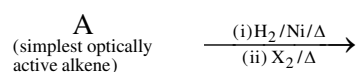
18. An acidic buffer is obtained on mixing :

- (1) 100 mL of 0.1 M CH₃COOH and 200 mL of 0.1 M NaOH
- (2) 100 mL of 0.1 M CH₃COOH and 100 mL of 0.1 M NaOH
- (3) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH₃COONa
- (4) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl

- 19.** Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solutions of NaCl and BaSO_4 , respectively, at a temperature T. Which of the following is false ?
- (1) Ionic mobilities of ions from both salts increase with T
 - (2) $C_{\text{NaCl}} \gg C_{\text{BaSO}_4}$ at a given T
 - (3) $C_{\text{NaCl}}(T_2) > C_{\text{NaCl}}(T_1)$ for $T_2 > T_1$
 - (4) $C_{\text{BaSO}_4}(T_2) > C_{\text{BaSO}_4}(T_1)$ for $T_2 > T_1$
- 20.** The antifertility drug 'Novestrol' can react with :
- (1) Br_2/water ; ZnCl_2/HCl ; FeCl_3
 - (2) Alcoholic HCN; NaOCl ; ZnCl_2/HCl
 - (3) Br_2/water ; ZnCl_2/HCl ; NaOCl
 - (4) ZnCl_2/HCl ; FeCl_3 ; Alcoholic HCN
- 21.** The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. ($R=0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$) (rounded off to the nearest integer)
- 22.** The mole fraction of glucose ($\text{C}_6\text{H}_{12}\text{O}_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _____.
- 23.** The photoelectric current from Na (work function, $w_0 = 2.3 \text{ eV}$) is stopped by the output voltage of the cell
 $\text{Pt(s)}|\text{H}_2(\text{g}, 1\text{bar})|\text{HCl(aq., pH} = 1)|\text{AgCl(s)}|\text{Ag(s)}$
 The pH of aq. HCl required to stop the photoelectric current from K ($w_0 = 2.25\text{eV}$), all other conditions remaining the same, is _____ $\times 10^{-2}$ (to the nearest integer).
 Given, $2.303 \frac{RT}{F} = 0.06\text{V}$; $E_{\text{AgCl}|\text{Ag}|\text{Cl}^-}^0 = 0.22\text{V}$

24. An element with molar mass $2.7 \times 10^{-2} \text{ kg mol}^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is $2.7 \times 10^3 \text{ kg m}^{-3}$, the radius of the element is approximately $\text{---} \times 10^{-12} \text{ m}$ (to the nearest integer).

25. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is --- .



FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Thursday 03rd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

1. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

- (1) $\frac{1}{8}$ (2) $\frac{1}{9}$
(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

2. The lines

$$\vec{r} = (\hat{i} - \hat{j}) + \ell(2\hat{i} + \hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$$

- (1) Intersect when $\ell = 1$ and $m = 2$
(2) Intersect when $\ell = 2$ and $m = \frac{1}{2}$
(3) Do not intersect for any values of ℓ and m
(4) Intersect for all values of ℓ and m

3. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, -2, 3) and (1, 1, 0) lies on the plane :

- (1) $x + 2y - z = 1$ (2) $x - 2y + z = 1$
(3) $x - y - 2z = 1$ (4) $2x + y - z = 1$

4. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

5. The area (in sq. units) of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$ is :

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$

(3) $\frac{79}{24}$ (4) $\frac{23}{16}$

6. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

- (1) $\frac{1}{4}$ (2) $\frac{1}{5}$
(3) $\frac{1}{7}$ (4) $\frac{1}{6}$

7. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

- (1) $MQ = \frac{1}{3}$ (2) $PN = 3$
(3) $MQ = \frac{1}{4}$ (4) $PN = 4$

8. For the frequency distribution :

Variate (x) : x_1 x_2 x_3 x_{15}

Frequency (f) : f_1 f_2 f_3 f_{15}

where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and

$\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

- (1) 2 (2) 1
(3) 4 (4) 6

9. $\int_{-\pi}^{\pi} |\pi - |x|| dx$ is equal to :

- (1) π^2 (2) $2\pi^2$
 (3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

10. Consider the two sets :

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m + 1)x + m + 4 = 0 \text{ are real}\}$ and
 $B = [-3, 5)$.

Which of the following is not true ?

- (1) $A - B = (-\infty, -3) \cup (5, \infty)$
 (2) $A \cap B = \{-3\}$
 (3) $B - A = (-3, 5)$
 (4) $A \cup B = \mathbb{R}$

11. If $y^2 + \log_e (\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then :

- (1) $|y''(0)| = 2$ (2) $|y'(0)| + |y''(0)| = 3$
 (3) $|y'(0)| + |y''(0)| = 1$ (4) $y''(0) = 0$

12. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in \mathbb{R}$, is increasing for all x lying in :

- (1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$
 (2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$
 (3) $\left(-\infty, \frac{14}{15}\right)$
 (4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

13. The value of $(2.^1P_0 - 3.^2P_1 + 4.^3P_2 - \dots$ up to 51^{th} term) $+ (1! - 2! + 3! - \dots$ up to 51^{th} term) is equal to :

- (1) $1 + (51)!$ (2) $1 - 51(51)!$
 (3) $1 + (52)!$ (4) 1

14. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to :

- (1) -1 (2) 1
 (3) -3 (4) 9

15. The solution curve of the differential equation,

$(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$, which passes through the point $(0, 1)$, is :

- (1) $y^2 = 1 + y \log_e \left(\frac{1+e^x}{2} \right)$
 (2) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^x}{2} \right) + 2 \right)$
 (3) $y^2 = 1 + y \log_e \left(\frac{1+e^{-x}}{2} \right)$
 (4) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^{-x}}{2} \right) + 2 \right)$

16. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

(1) 264 (2) 256
(3) 128 (4) 248

17. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to:

(1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 - q^2)$
(3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 + q^2)$

18. Let $[t]$ denote the greatest integer $\leq t$. If for some

$$\lambda \in \mathbb{R} - \{0, 1\}, \lim_{x \rightarrow 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L, \text{ then } L \text{ is}$$

equal to :

(1) 1 (2) 2
(3) $\frac{1}{2}$ (4) 0

19. $2\pi - \left(\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} \right)$ is equal to:

(1) $\frac{7\pi}{4}$

(2) $\frac{5\pi}{4}$

(3) $\frac{3\pi}{2}$

(4) $\frac{\pi}{2}$

20. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$

(2) q

(3) $(\sim p) \wedge q$

(4) $(\sim p) \vee (\sim q)$

21. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If

$a_{11} = 109$, then a_{22} is equal to _____ .

22. If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$,

then the value of k is _____ .

23. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _____ .

24. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty\right)}$ is equal to _____ .

25. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, ($m, n \in \mathbb{N}$) then the greatest common divisor of the least values of m and n is _____ .