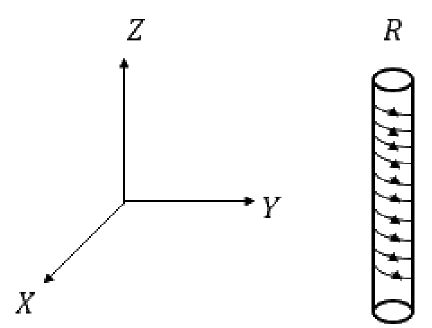
JEE Main 2020 Paper

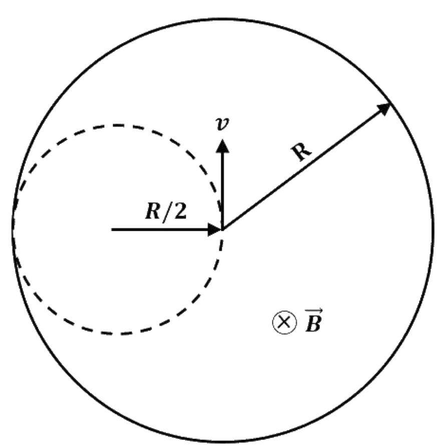
Date of Exam: **9th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

1. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has $n\frac{\text{turns}}{\text{length}}$ and carries a current i. The electron gun shoots an electron along the radius of solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning):



a. $\frac{2Re\mu_0 in}{m}$
b. $\frac{Re\mu_0 in}{2m}$
c. $\frac{Re\mu_0 in}{m}$
d. $\frac{Re\mu_0 in}{4m}$

Solution: (b)
Looking at the cross-section of the solenoid, R_{max} of the particle's motion has to be $\frac{R}{2}$ for it not to strike the solenoid.

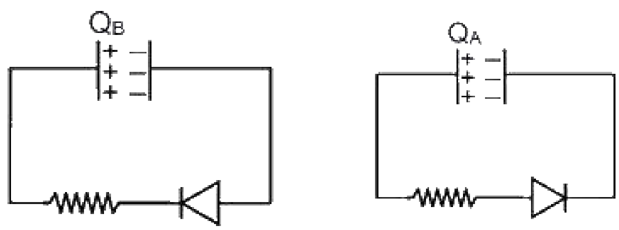


$$evB = \frac{mv^2}{\frac{R}{2}}$$

$$R_{max} = \frac{R}{2} = \frac{mv_{max}}{e\mu_0 in}$$

$$V_{max} = \frac{Re\mu_0 in}{2m}$$

2. Two identical capacitors A and B, charged to the same potential $5\,V$ are connected in two different circuit as shows below at time $t = 0$. If the charges on capacitors A and B at time $t = CR$ is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)



(a) (b)

a. $CV, \frac{CV}{e}$

b. $\frac{CV}{e}, \frac{CV}{2e}$

c. $\frac{CV}{e}, \frac{VC}{2}$

d. $\frac{CV}{e}, CV$

Solution: (a)

Charge on capacitor = CV

(b) is forward biased and (a) is reverse biased

For case (b)

$q = q_{max}(1 - e^{\frac{-t}{RC}}) = CV$

$$Q_B = CVe^{-1}$$

For case (b)

$$Q_A = CV$$

3. For the four sets of three measured physical quantities as given below. Which of the following options is correct?

$(i)\ A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$
$(ii)\ A_2 = 24.44, B_2 = 16.08, C_2 = 240.2$
$(iii)\ A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183$
$(iv)\ A_4 = 25, B_4 = 236.191, C_4 = 19.5$

a. $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$

b. $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

c. $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

d. $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

Solution: (a)

$$A_1 + B_1 + C_1 = 280.632$$
$$A_2 + B_2 + C_2 = 280.72$$
$$A_3 + B_3 + C_3 = 280.664$$
$$A_4 + B_4 + C_4 = 280.691$$

Hence, option (a) is correct.

4. A particle starts from the origin at $t = 0$ with an initial velocity of $\vec{u} = 3\hat{\imath}$ from origin and moves in the x-y plane with a constant acceleration $\vec{a} = (6\hat{\imath} + 4\hat{\jmath})\ \mathrm{m/s^2}$. The x-coordinate of the particle at the instant when its y –cordinated is 32 m is D meters. The value of D is:

a. 60
b. 50
c. 32
d. 40

Solution: (a)

$$S_y = u_y t + \frac{1}{2} a_y t^2$$
$$32 = 0 + \frac{1}{2} \times 4t^2 \qquad \rightarrow \quad t = 4\, sec$$
$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$$
$$= 60\ m$$

5. A spring mass system (mass m, spring constant k and natural length l) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the center of the disc. If the disc together with spring mass system, rotates about its axis with an angular velocity ω ($k >>> m\omega^2$), the relative change in the length of the spring is best given by the option :

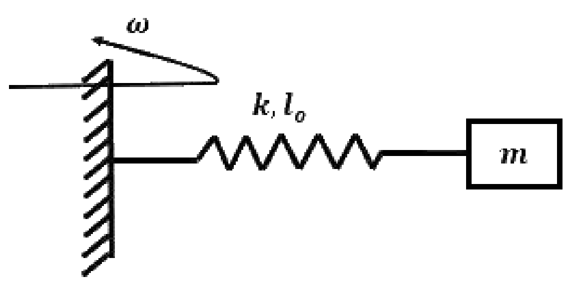
a. $\frac{m\omega^2}{3k}$

b. $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

c. $\frac{m\omega^2}{k}$

d. $\frac{2m\omega^2}{k}$

Solution: (c)



Using Newton's second law of dynamics,

$$m\omega^2(l_o + x) = kx$$
$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$
$$x = \frac{l_o m\omega^2}{k - m\omega^2}$$
$$k >> m\omega^2$$

So, $\frac{x}{l_o}$ is equal to $\frac{m\omega^2}{k}$

6. A small circular loop of conducting wire has radius a and carries current i. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, its starts performing simple harmonic motion of time period T. If the mass of the loop is m then :

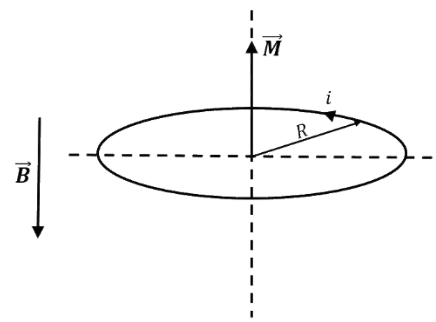
a. $T = \sqrt{\frac{\pi M}{iB}}$

b. T=$\sqrt{\frac{2\pi M}{iB}}$

c. $T = \sqrt{\frac{\pi M}{2iB}}$

d. $T = \sqrt{\frac{2M}{iB}}$

Solution: (b)



Considering the torque situation on the loop,

$$\tau = MBsin\theta = -I\alpha$$

$$\pi R^2 iB\theta = -\frac{mR^2}{2}\alpha$$

The above equation is analogous to $\theta = -C\alpha$, where $C = \omega^2 = \frac{2\pi iB}{M}$

$$\omega = \sqrt{\frac{2\pi iB}{M}} = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{2\pi M}{iB}}$$

7. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of droplet is (take note that the surface tension applied an upward force on droplet)

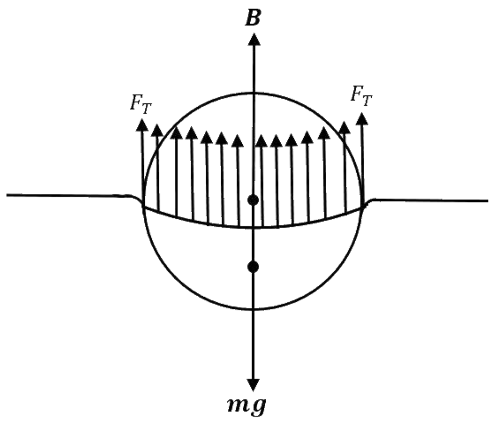
a. $r = \sqrt{\frac{2T}{3(\rho+d)g}}$

b. $r = \sqrt{\frac{T}{(\rho+d)g}}$

c. $r = \sqrt{\frac{T}{(d-\rho)g}}$

d. $r = \sqrt{\frac{3T}{(2d-\rho)g}}$

Solution: (d)



In equilibrium, net external force acting on the sphere is zero.

$$mg = F_T + B$$

$$\rho V g = d\left(\frac{V}{2}\right)g + T2\pi R$$

$$\rho\frac{4}{3}\pi R^3 g = d\frac{2}{3}\pi R^3 g + T2\pi R$$

$$R = \sqrt{\frac{3T}{(2\mathrm{d}-\rho)\mathrm{g}}}$$

8. A wire of length L and mass per unit length $6 \times 10^{-3}\ kg/m$ is put under tension of $540\ N$. Two consecutive frequencies that it resonates at are: $420\ Hz$ and $490\ Hz$. Then L in meter is

a. 8.1 m
b. 2.1 m
c. 1.1 m
d. 5.1 m

Solution: (b)
Key Idea: The difference of two consecutive resonant frequencies is the fundamental resonant frequency.
Fundamental frequency= $490 - 420 = 70\ Hz$

$$70 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow 70 = \frac{1}{2l}\sqrt{\frac{540}{6 \times 10^{-3}}}$$

$$\Rightarrow l = \frac{1}{2 \times 70}\sqrt{90 \times 10^{-3}} = \frac{300}{140}$$

$$\Rightarrow l \approx 2.1\, m$$

9. A plane electromagnetic wave is propagating along the direction $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$, with the polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant)

   a. $B_0 \frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   b. $B_0 \frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   c. $B_0 \frac{j-\hat{\imath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   d. $B_0 \hat{k} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$

Solution: (a)

EM wave is in direction $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Electric field is in direction $\hat{k}$

Direction of propagation of EM wave is given by $\vec{E} \times \vec{B}$

10. Two gases-Argon (atomic radius 0.07 nm, atomic weight 40) and Xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free time is closest to

   a. 4.67
   b. 2.04
   c. 1.83
   d. 3.67

Solution: (challenge question)

Mean free time = $\frac{1}{\sqrt{2}n\pi d^2}$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \left(\frac{0.1}{0.07}\right)^2 = \left(\frac{10}{7}\right)^2 = 2.04$$

11. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1: 4, the ratio of their diameters is:

   a. $\sqrt{2}: 1$
   b. $1: \sqrt{2}$
   c. $1: 2$
   d. $2: 1$

Solution: (a)

$$\frac{dU}{dV} = \frac{1}{2} \times stress \times \frac{stress}{Y}$$
$$= \frac{1}{2} \times \frac{F^2}{A^2 Y}$$
$$\frac{dU}{dV} \propto \frac{1}{D^4}$$
$$\frac{\left(\frac{dU}{dV}\right)_1}{\left(\frac{dU}{dV}\right)_2} = \frac{{D_2}^4}{{D_1}^4} = \frac{1}{4}$$
$$\frac{D_1}{D_2} = (4)^{\frac{1}{4}}$$

$\therefore D_1 : D_2 = \sqrt{2} : 1$

12. Planets A has a mass M and radius R. Planet B has the mass and half the radius of planet A. If the escape velocities from the planets A and B are v_A and v_B respectively, then surfaces is $\frac{v_A}{v_B} = \frac{n}{4}$, the value of n is :

a. 3
b. 2
c. 4
d. 5

Solution: (c)

We know that the escape velocity is given by,

$$V_e = \sqrt{\frac{2GM}{R}}$$

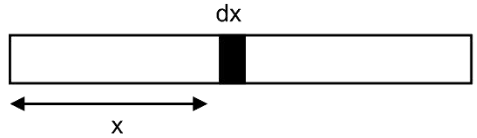
Now,

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{\frac{2GM}{2}}{\frac{R}{2}}}} = 1$$

We are given that $\frac{V_1}{V_2} = \frac{n}{4}$

$$\Rightarrow \frac{n}{4} = 1$$
$$\Rightarrow n = 4$$

13. A rod of length L has non-uniform linear mass density given by $\rho(x) = \left(a + b\left(\frac{x}{L}\right)^2\right)$, Where a and b are constants and $0 \le x \le L$. The value of x for the center of mass of the rod is at :



a. $\frac{3L}{2}\left(\frac{2a+b}{3a+b}\right)$
b. $\frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$
c. $\frac{3L}{4}\left(\frac{a+b}{3a+b}\right)$
d. $\frac{4L}{3}\left(\frac{a+b}{3a+b}\right)$

Solution: (b)
Here we take a small element along the length as dx at a distance x from the left end as shown.

$$x_{cm} = \frac{1}{M}\int_0^L x.dm$$

$$\Rightarrow dM = \lambda.dx = \left(a + b\left(\frac{x}{L}\right)^2\right).dx$$

$$x_{cm} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int_0^L x\left(a + \frac{bx^2}{L^2}\right)dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{a\left(\frac{x^2}{2}\right)_0^L + \frac{b}{L^2}\left(\frac{x^4}{4}\right)_0^L}{a(x)_0^L + \frac{b}{L^2}\left(\frac{x^3}{3}\right)_0^L}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{4}}{aL + \frac{bL}{3}}$$

$$= \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$$

14. A particle of mass m is projected with a speed u from the ground at angle $\theta = \frac{\pi}{3}$ is w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\,\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is :

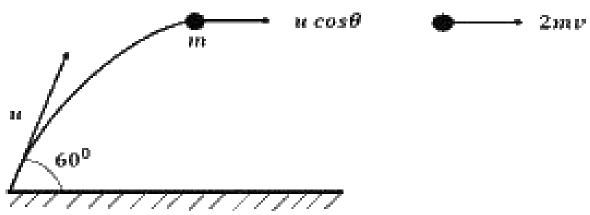
a. $\frac{3\sqrt{3}u^2}{8g}$

b. $\frac{2\sqrt{2}u^2}{g}$

c. $\frac{5u^2}{8g}$

d. $\frac{3\sqrt{2}u^2}{4g}$

Solution: (a)



The only external force acting on the colliding system during the collision is the gravitational force. Since gravitational force is non- impulsive, the linear momentum of the system is conserved just before and just after the collision.

$$p_i = p_f$$

$$mu + mu\,cos\theta = 2mv$$

$$\Rightarrow \quad v = \frac{u(1 + cos60^0)}{2} = \frac{3}{4}u$$

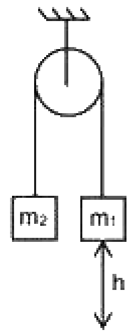
So the horizontal range after the collision $= vt$

$$= v\sqrt{\frac{2H_{\max}}{g}}$$

$$= \frac{3}{4}u\sqrt{\frac{2u^2\sin^2 60^0}{2g^2}}$$

$$= \frac{3}{4}u^2\frac{\sqrt{\frac{3}{4}}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

15. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its center of mass (see fig). A massless string is wrapped over its rim and two blocks of massless string is wrapped over its rim and two blocks of masses m_1 and m_2 $(m_1 > m_2)$ are attached to the ends of string. The system is released from rest. The angular speed of the wheel when m_1 descend by a distance h is :



a. $\frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2)+\frac{I}{R^2}}}$

b. $\frac{gh}{R}\sqrt{\frac{(m_2+m_1)}{m_1+m_2+\frac{I}{R^4}}}$

c. $\frac{gh}{R}\sqrt{\frac{(m_1-m_2)}{m_1+m_2+\frac{I}{R}}}$

d. $\frac{1}{R}\sqrt{\frac{2(m_2+m_1)gh}{m_1+m_2+\frac{I}{R^2}}}$

Solution: (a)

Assume initial potential energy of the blocks to be zero. Initial kinetic energy is also zero since the blocks are at rest.

When block m_1 falls by h, m_2 goes up by h (because of length constraint)

Final P.E $= m_2gh - m_1gh$

Let the final speed of the blocks be v and angular velocity of the pulley be ω

Final K.E $= \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}I\omega^2$

Total energy is conserved. Hence,

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}I\omega^2$$

$v = \omega r$ (due to no slip condition)

$$\Rightarrow \frac{1}{2}(m_1+m_2)\omega^2R^2 + \frac{1}{2}I\omega^2 = (m_1-m_2)gh$$

$$\Rightarrow \omega^2\left[\frac{1}{2}(m_1+m_2)R^2 + \frac{1}{2}I\right] = (m_1-m_2)gh$$

$$\Rightarrow \omega^2 = \frac{2(m_1-m_2)gh}{R^2\left[(m_1+m_2)+\frac{I}{R^2}\right]}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{\left[(m_1+m_2)+\frac{I}{R^2}\right]}}$$

16. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground stare ?

a. 8.6
b. 11.4
c. 24.2
d. 35.8

Solution: (b)

$$\frac{hc}{\lambda} = (13.6\ eV)Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$n_1 = 1$$
$$n_2 = 3$$

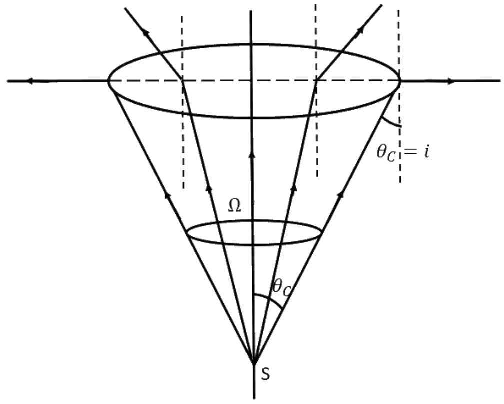
For an H-like atom, ionization energy is $(R)Z^2$.
This gives $Z = 3$

$$\frac{hc}{\lambda} = (13.6\ eV)(3^2)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$
$$\Longrightarrow \frac{hc}{\lambda} = (13.6\ eV)(9) \times \frac{8}{9}$$
$$\text{Wavelength} = \frac{1240}{8 \times 13.6}\ nm$$
$$\lambda = 11.4\ nm$$

17. There is a small source of light at some depth below the surface of water (refractive index $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly): [Use the fact that surface area of a spherical cap of height h and radius of curvature r is $2\pi rh$]

a. 17 %
b. 34 %
c. 50 %
d. 21 %

Solution: (a)



The portion of light escaping into the air from the liquid will form a cone. As long as the angle of incidence on the liquid – air interface is less than the critical angle, i.e. $i < \theta_C$, the light rays will undergo refraction and emerge into the air.

For $i > \theta_C$, the light rays will suffer TIR. So, these rays will not emerge into the air.

The portion of light rays emerging into the air from the liquid will form a cone of half angle = θ_C

$$\sin\theta_C = \frac{1}{\mu_{Liq}} = \frac{3}{4}, \quad \cos\theta_C = \frac{\sqrt{7}}{4}$$

Solid angle contained in this cone is

$\Omega = 2\pi(1 - \cos\theta_C)$

Percentage of light that escapes from liquid $= \frac{\Omega}{4\pi} \times 100$

Putting values we get

Percentage $= \frac{4-\sqrt{7}}{8} \times 100 \approx 17\%$

18. An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field E. The of charge of de-Broglie wavelength of this electron at time t ignoring relativistic effects is

a. $-\frac{|e|Et}{h}$

b. $\frac{h}{|e|E\sqrt{t}}$

c. $-\frac{h}{|e|Et^2}$

d. $-\frac{2ht^2}{|e|E}$

Solution: (c)

$$\lambda_D = \frac{h}{mv}$$

$where, v = at$

$$v = \frac{|e|E}{M}t \quad (a = \frac{|e|E}{M})$$

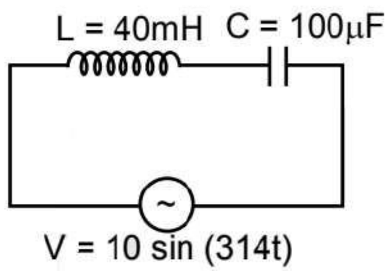
$$\lambda_D = \frac{h}{m\frac{|e|E}{M}t}$$

$$\lambda_D = \frac{h}{|e|Et}$$

$$\frac{d\lambda_D}{dt} = -\frac{h}{|e|Et^2}$$

19. In LC circuit the inductance $L = 40\, mH$ and $C = 100\, \mu F$. If a voltage $V(t) = 10\, sin\,(314t)$ is applied to the circuit, the current in the circuit is given as
 a. $10\, cos\,(314t)$
 b. $0.52\, cos\,(314t)$
 c. $0.52\, sin\,(314\text{t})$
 d. $5.2\, cos\,(314t)$

Solution: (b)



Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$= \sqrt{(X_C - X_L)^2}$$

$$= X_C - X_L$$

$$= \frac{1}{\omega C} - \omega L$$

$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 31.84 - 12.56 = 19.28\, \Omega$$

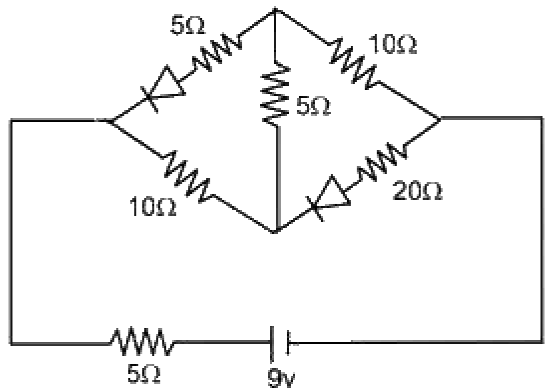
For $X_C > X_L$, current leads voltage by $\frac{\pi}{2}$

$$\therefore i = \frac{V}{Z} = \frac{10\sin(314t + \frac{\pi}{2})}{19.28}$$

$$= 0.52 \cos\ (314t)$$

20. The current (i) in the network is

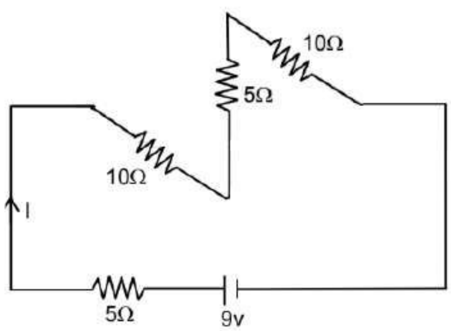
a. $0\ A$
b. $0.3\ A$
c. $0.2\ A$
d. $0.6\ A$



Solution: (b)

Since the diodes are reverse biased, they will not conduct.

Hence, the circuit will look like

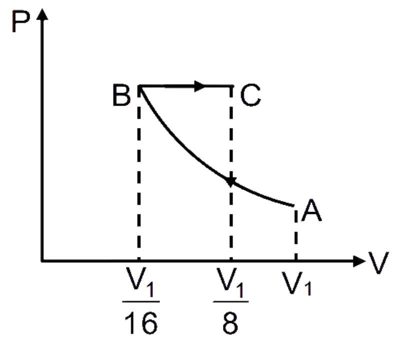


$$R_{eff} = 5 + 10 + 5 + 10 = 30\ \Omega$$

$$I = \frac{9}{30} = 0.3\ A$$

21. Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2=\frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (^{0}K) is (to the nearest integer)

Solution: (1818 K)



PV^{γ} =Constant

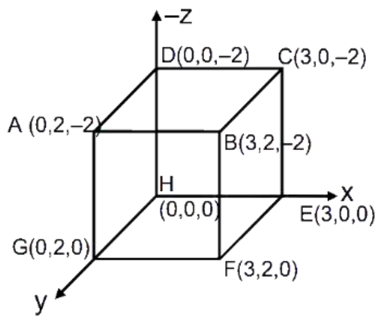
$TV^{(\gamma-1)}$=constant

$$300(V_1)^{(1.4-1)} = T_B\left(\frac{V_1}{16}\right)^{\frac{2}{5}}$$
$$T_B = 300 \times 2^{\left(\frac{8}{5}\right)}$$

Now for BC process

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$
$$T_C = \frac{V_C T_B}{T_B} = 2 \times 300 \times 2^{\left(\frac{8}{5}\right)}$$
$$T_C = 1818\ K$$

22. An electric field $\vec{E} = 4x\hat{\imath} - (y^2 + 1)\hat{\jmath}\ N/C$, passes through the box shown in figure. The flux of the electric field through surface $ABCD$and $BCGF$ are marked as ϕ_1 and ϕ_2, then difference between $(\phi_1 - \phi_2)$ is $(\frac{Nm^2}{C})$



Solution: (challenge question)

Electric flux through a surface is $\emptyset = \int \vec{E} = d\vec{A}$

For surface $ABCD$,

$d\vec{A}$ is along $(-\hat{k})$

So, at all the points of this surface,

$$\vec{E}.d\vec{A} = 0$$

Because, $\emptyset_{ABCD} = \emptyset_1 = 0$

For surface $BCEF$,

$d\vec{A}$ is along $(\hat{\imath})$

So,

$$\vec{E}.d\vec{A} = E_x dA$$

$$\emptyset_{BCEF} = \emptyset_2 = 4x(2 \times 2)$$

If $if\ x = 3$

$$\emptyset_2 = 48\ \frac{N - m^2}{C}$$

Hence, $\emptyset_1 - \emptyset_2 = -48\ \frac{N-m^2}{C}$

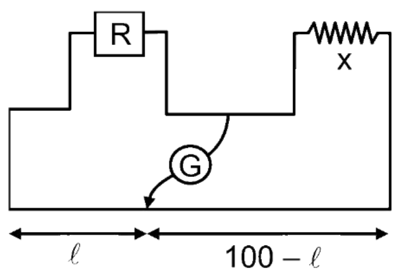
23. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. 10 fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (nm))

Solution: (750)

If the length of the segment is y,
Then $y = n\,\beta$
$n =$ no. of fringes,
$\beta =$ fringe width
$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$

$$\lambda_2 = 15 \times 50\ nm$$
$$\lambda_2 = 750\ nm$$

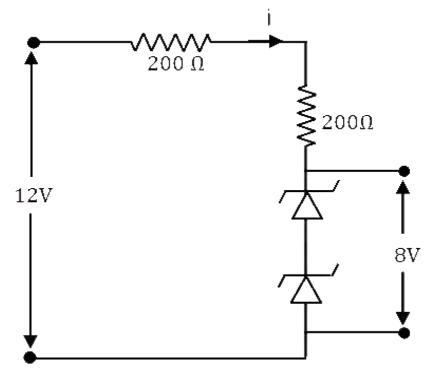
24. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It is found that balancing length is $l = 25\ cm$. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing l $(in\ cm)$ will now be



Solution: (40)

$$\frac{X}{R} = \frac{75}{25} = 3$$
$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}$$
$$R' = \frac{4\rho \frac{l}{2}}{\pi \left(\frac{d}{2}\right)^2} = 2R$$
$$\text{Then } \frac{X}{R'} = \left(\frac{100-l}{l}\right)$$
$$\frac{100 - l}{l} = \frac{X}{2R} = \frac{3}{2}$$
$$l = 40.00\ cm$$

25. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode id; (considering both zener diode are identical).



Solution: (challenge question)

$i = \left(\frac{12-8}{200+200}\right) A = \frac{4}{400} = 10^{-2} A$

Power loss in each diode = $(4)(10^{-2})\ W = 40\ mW$

JEE Main 2020 Paper

Date: 9th January 2020

Time: 02:30 PM – 05:30 PM

Subject: Chemistry

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1. 5 g of Zinc is treated separately with an excess of
   I. dilute hydrochloric acid and
   II. aqueous sodium hydroxide.
   The ratio of the volumes of H_2 evolved in these two reactions is:
   a. 2:1 b. 1:2
   c. 1:1 d. 1:4

Answer: c

Solution:
$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$
$Zn + 2HCl \rightarrow ZnCl_2 + H_2$
So, the ratio of volume of H_2 released in both the cases is 1:1.

2. The solubility product of $Cr(OH)_3$ at 298 K is 6×10^{-31}. The concentration of hydroxide ions in a saturated solution $Cr(OH)_3$ will be :
   a. $(18 \times 10^{-31})^{1/4}$ b. $(18 \times 10^{-31})^{1/2}$
   c. $(2.22 \times 10^{-31})^{1/4}$ d. $(4.86 \times 10^{-29})^{1/4}$

Answer: a

Solution:
$Cr(OH)_{3(s)} \rightarrow Cr^{3+}_{(aq.)} + 3OH^-_{(aq.)}$
1-S S 3S

$K_{sp} = 27S^4$
$6\times10^{-31} = 27S^4$
$S=\left[\frac{6}{27}\times10^{-31}\right]^{1/4}$
$[OH^-] = 3S = 3\times\left[\frac{6}{27}\times10^{-31}\right]^{1/4} = (18\times10^{-31})^{1/4}M$

3. Among the statements (a)-(d), the correct ones are :
   a) Lithium has the highest hydration enthalpy among the alkali metals.
   b) Lithium chloride is insoluble in pyridine.
   c) Lithium cannot form ethynide upon its reaction with ethyne.
   d) Both lithium and magnesium react slowly with H_2O.

a. (a), (b) and (d) only
b. (b) and (c) only
c. (a), (c) and (d) only
d. (a) and (d) only

Answer: a

Solution:
Only LiCl amongst the first group chlorides dissolve in pyridine because the solvation energy of lithium is higher than the other salts of the same group.
Lithium does not react with ethyne to form ethynilide due to its small size and high polarizability.
Lithium and Magnesium both have very small sizes and very high ionization potentials so, they react slowly with water.
Amongst all the alkali metals, Li has the smallest size hence, the hydration energy for Li is maximum.

4. The first and second ionization enthalpies of a metal are 496 and 4560 kJ mol^{-1} respectively. How many moles of HCl and H_2SO_4 , respectively, will be needed to react completely with 1 mole of metal hydroxide?

a. 1 and 2
b. 1 and 0.5
c. 1 and 1
d. 2 and 0.5

Answer: b

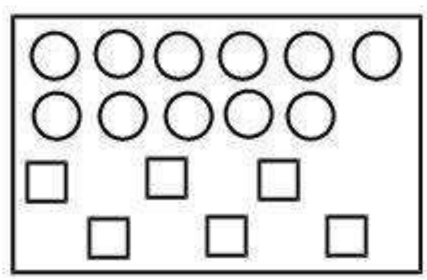
Solution:
The given data for ionization energies clearly shows that $IE_2 \gg IE_1$. So, the element belongs to the first group. Therefore, we can say that this element will be monovalent and hence forms a monoacidic base of the type MOH.

$MOH + HCl \rightarrow MCl + H_2O$

$2MOH + H_2SO_4 \rightarrow M_2SO_4 + 2H_2O$

So, from the above equation we can say that,
1 mole of metal hydroxide requires 1 mole of HCl and 0.5 mole of H_2SO_4.

5. In the figure shown below reactant A (represented by the square) is in equilibrium with product B (represented by circle). The equilibrium constant is:



a. 1
b. 2
c. 8
d. 4

Answer: b

Solution:

Let us assume the equation to be $A \rightleftharpoons B$,
Number of particles of A = 6
Number of particles of B = 11
$K = \frac{11}{6} \approx 2$

6. The correct order spin-only magnetic moments of the following complexes is :

I. $[Cr(H_2O)_6]Br_2$
II. $Na_4[FeCN_6]$
III. $Na_3[Fe(C_2O_4)_3]$ ($\Delta_0 > P$)
IV. $(Et_4N)_2[CoCl_4]$

a. (III)>(I)>(II)>(IV)
b. (III)>(I)>(IV)>(II)
c. (I)>(IV)>(III)>(II)
d. (II)$\approxeq$(I)>(IV)>(III)

Answer: c

Solution:

Complex (I) has the central metal ion as Fe^{2+} with strong field ligands.
Configuration of Fe^{2+}= [Ar] $3d^6$
Strong field ligands will pair up all the electrons and hence the magnetic moment will be zero.

Complex (II) has the central metal ion as $Cr^{2+}$with weak field ligands.
Configuration of Cr^{2+}= [Ar] $3d^4$
As weak field ligands are present, pairing does not take place. There will be 4 unpaired electrons and hence the magnetic moment $= \sqrt{24}$ B.M.

Complex (III) has the central metal ion as Co^{2+} with weak field ligands.
Configuration of Co^{2+}= [Ar] $3d^7$
As weak field ligands are present no pairing can occur. There will be 3 unpaired electrons and hence the magnetic moment $= \sqrt{15}$ B.M.

Complex (IV) has the central metal ion as Fe^{3+} with strong field ligands.
Configuration of Fe^{3+} =[Ar] $3d^5$
Strong field ligands will pair up the electrons but as we have a [Ar] $3d^5$ configuration, one electron will remain unpaired and hence the magnetic moment will be $\sqrt{3}$ B.M.

7. The true statement amongst the following
   a. S is a function of temperature but ΔS is not a function of temperature.
   b. Both ΔS and S are functions of temperature.
   c. Both S and ΔS are not functions of temperature.
   d. S is not a function of temperature but ΔS is a function of temperature.

Answer: b

Solution:
Entropy is a function of temperature, at any temperature, the entropy can be given as:
$S_T = \int_0^T \frac{nCdT}{T}$

Change in entropy is also a function of temperature, at any temperature, the entropy change can be given as:
$\Delta S = \int \frac{dq}{T}$

8. The reaction of $H_3N_3B_3Cl_3$ (A) with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Furthur, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively, are:
   a. Boron nitride, MeBr
   b. Diborane, MeMgBr
   c. Borazine, MeBr
   d. Borazine, MeMgBr

Answer: d

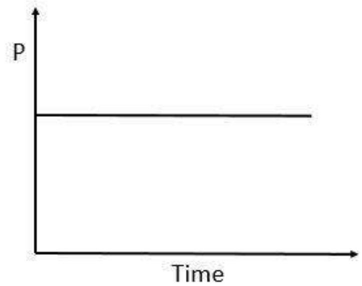
Solution:
$B_3N_3H_3Cl_3 + LiBH_4 \rightarrow B_3N_3H_6 + LiCl + BCl_3$

$B_3N_3H_3Cl_3 + 3CH_3MgBr \rightarrow B_3N_3H_3(CH_3)_3 + 3MgBrCl$

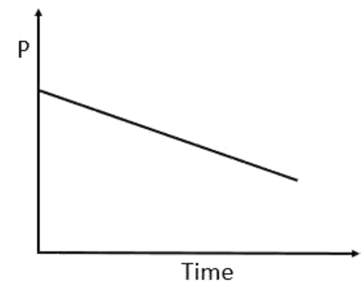
So, we can say that,
B is $B_3N_3H_6$
C is CH_3MgBr

9. A mixture of gases O_2 , H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behaviour of pressure with time is :

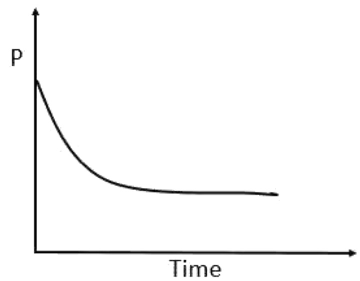
a.



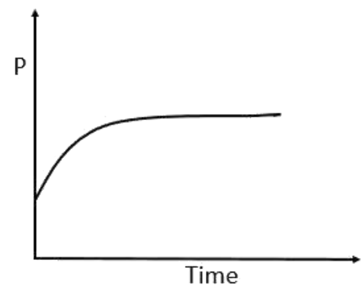
b.



c.



d.



Answer: c

Solution:
As H_2, O_2 and CO gets adsorbed on the surface of charcoal, the pressure decreases. So, option (a) and (d) can be eliminated. After some time, as almost all the surface sites are occupied, the pressure becomes constant.

10. The isomer(s) of $[Co(NH_3)_4Cl_2]$ that has/have a Cl–Co–Cl angle of 90^o, is/are :

a. cis only
b. trans only
c. meridional and trans
d. cis and trans

Answer: a

Solution:
In cis-isomer, similar ligands are at an angle of 90^o.

11. Amongst the following, the form of water with lowest ionic conductance at 298 K is :

a. distilled water
b. sea water
c. saline water used for intra venous injection
d. water from a well

Answer: a

Solution:
In distilled water there are no ions present except H^+ and OH^- ions, both of which are immensely minute in concentration, that renders their collective conductivity negligible.

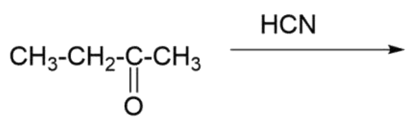
12. The number of sp^2 hybrid orbitals in molecule of benzene is:

a. 18
b. 24
c. 6
d. 12

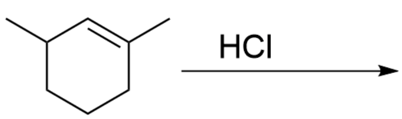
Answer: a

Solution:
Benzene (C_6H_6) has 6 sp^2 hybridized carbons. Each carbon has 3 σ-bonds and 1 π-bond. 3 σ-bonds means that there are 3 sp^2 hybrid orbitals for each carbon. Hence, the total number of sp^2 hybrid orbitals is 18.

13. Which of the following reactions will not produce a racemic product?

a.


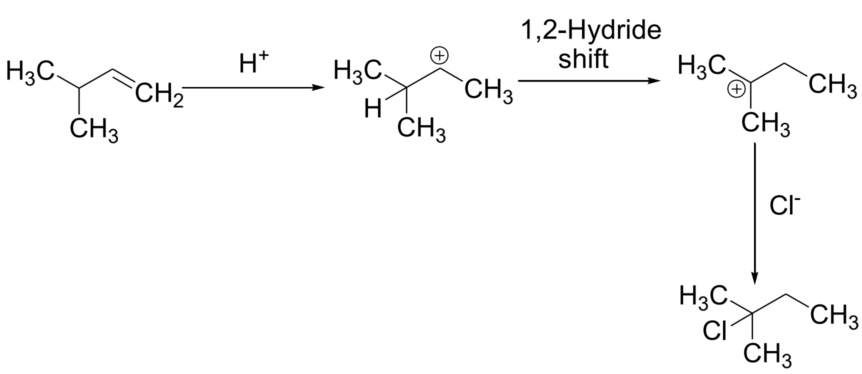
b. $(CH_3)_2\text{-}CH\text{-}CH{=}CH_2 \xrightarrow{HCl}$

c.


d. $CH_3CH_2\text{-}CH{=}CH_2 \xrightarrow{HBr}$

Answer: b

Solution:



14. Which of the following has the shortest C–Cl bond?

a. $Cl - CH = CH_2$

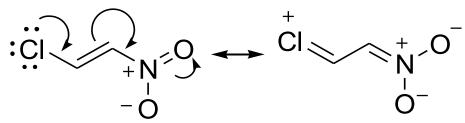
b. $Cl - CH = CH - CH_3$

c. $Cl - CH = CH - OCH_3$

d. $Cl - CH = CH - NO_2$

Answer: d

Solution:
There is extended conjugation present in option (d), which will reduce the length of C-Cl bond to the greatest extent which can be represented as follows:



15. Biochemical oxygen demand (BOD) is the amount of oxygen required (in ppm) :

a. for the photochemical breakdown of waste present in $1m^3$ volume of a water body

b. by anaerobic bacteria to break-down inorganic waste present in a water body.

c. by bacteria to break-down organic waste in a certain volume of water sample.

d. for sustaining life in a water body

Answer:c

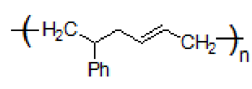
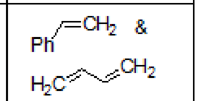
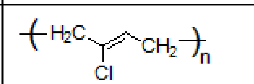
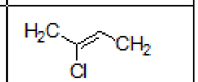
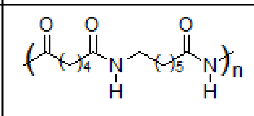
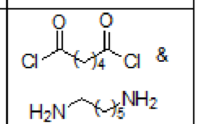
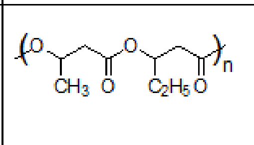
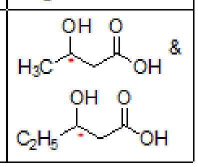
Solution:
Biochemical oxygen demand (BOD) is the amount of dissolved oxygen used by microorganisms in the biological process of metabolizing organic matter in water.

16. Which polymer has chiral, monomer(s)?

a. Buna-N
b. Neoprene
c. Nylon 6,6
d. PHBV

Answer: d

Solution:

	Polymers	Monomers
Buna-S		
Neoprene		
Nylon-6,6		
PHBV		

17. A, B and C are three biomolecules. The results of the tests performed on them are given below :

	Molisch's Test	**Barfoed Test**	**Biuret Test**
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

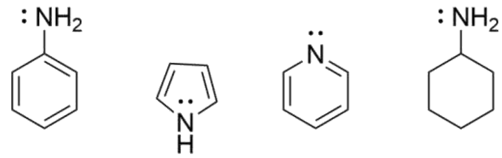
A, B and C are respectively

a. A=Lactose B=Glucose C=Albumin
b. A=Lactose B=Glucose C=Alanine
c. A=Lactose B=Fructose C=Alanine
d. A=Glucose B=Sucrose C=Albumin

Answer: a

Solution:
Lactose, glucose and fructose gives positive Molisch's test.
Glucose gives positive Barfoed's test whereas sucrose gives a negative for Barfoed's test.
Albumin gives positive for Biuret test whereas alanine gives a negative Biuret test.

18. The decreasing order of basicityof the following amines is:



(I) (II) (III) (IV)

a. I > II > III > IV

b. IV > III > I > II

c. II > I > III > IV

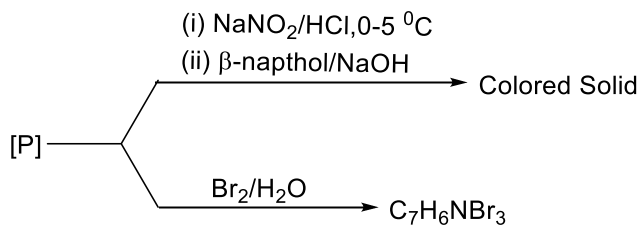
d. IV > I > II > III

Answer: b

Solution:

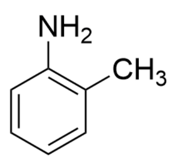
The basicity of the compound depends on the availability of the lone pairs.
In compound IV, Nitrogen is sp^3 hybridized.
In compound III, Nitrogen is sp^2 hybridized and the lone pairs are not involved in resonance.
In compound I, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance.
In compound II, Nitrogen is sp^2 hybridized and the lone pairs are involved in resonance such that, they are contributing to the aromaticity of the ring.
From the above points we can conclude that the basicity order should be IV > III > I > II.

19.

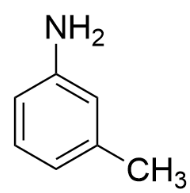


The compound [P] is :

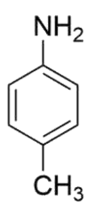
a.



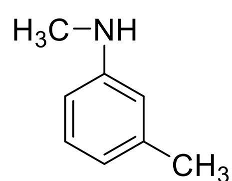
b.



c.

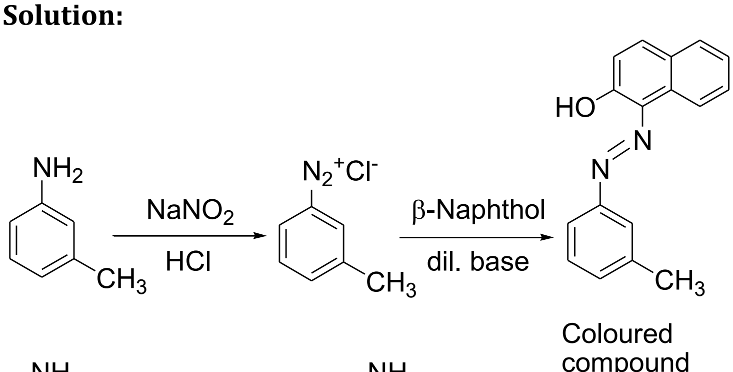


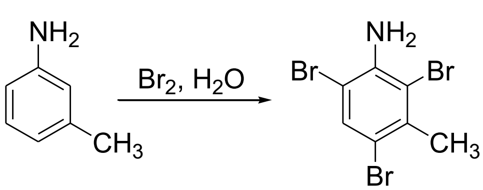
d.



Answer: b

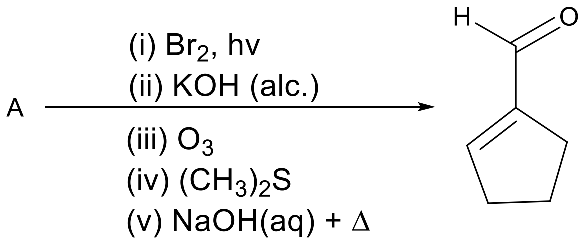
Solution:



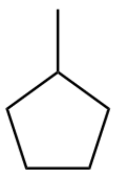


B ($C_7H_6NBr_3$)

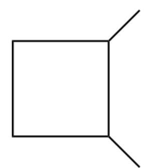
20. In the following reaction A is :



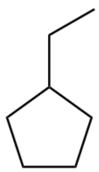
a.



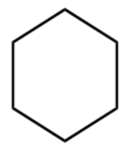
b.



c.

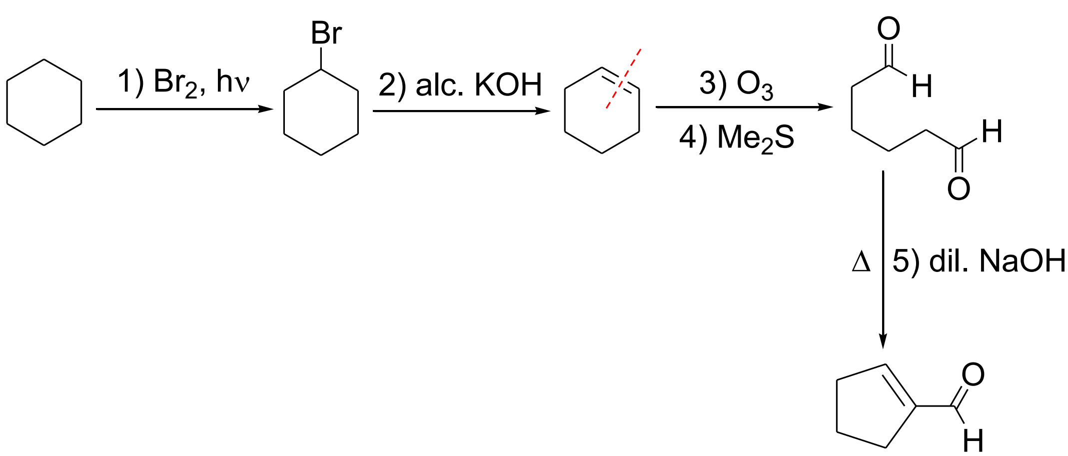


d.



Answer: d

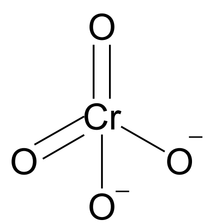
Solution:



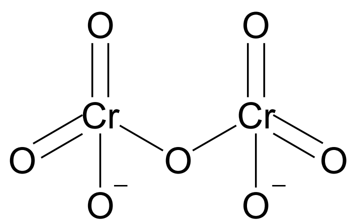
21. The sum of total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is ____

Answer: 12

Solution:



Chromate ion



Dichromate ion

22. A sample of milk splits after 60 min. at 300K and after 40 min at 400K when the population of lactobacillus acidophilus in it doubles . The activation energy (in kJ/mol) for this process is closest to ----- .
(Given, $R = 8.3 \text{ J mol}^{-1}\text{K}^{-1}$), $\ln\left(\frac{2}{3}\right) = 0.4$, $e^{-3} = 4.0$)

Answer: 3.98

Solution:

The generation time can be utilized to get an indication of the rate ratio. Let the amount generated be (x).

$$\text{Rate} = \frac{\text{Amount generated}}{\text{Time taken}}$$

$\text{Rate}_{300\text{ K}} = \frac{(x)}{60}$ $\text{Rate}_{400\text{ K}} = \frac{(x)}{40}$

$$\frac{\text{Rate}_{300\text{K}}}{\text{Rate}_{400\text{K}}} = \frac{40}{60}$$

For the same concentration (which is applicable here), the rate ratio can also be equaled to the ratio of rate constants.

$$\ln\left[\frac{K_{\text{at }400\text{K}}}{K_{\text{at }300\text{K}}}\right] = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln\frac{60}{40} = \frac{E_a}{8.3}\left[\frac{1}{300} - \frac{1}{400}\right]$$

$E_a = 0.4 \text{ x } 8.3 \text{ x } 1200 = 3984 \text{ J/mol} = 3.98 \text{ kJ/mol}$

23. One litre of sea water (d $= 1.03 \frac{g}{cm^3}$) contains 10.3 mg of O_2 gas. Determine the concentration of O_2 in ppm:

Answer: 10.00

Solution:

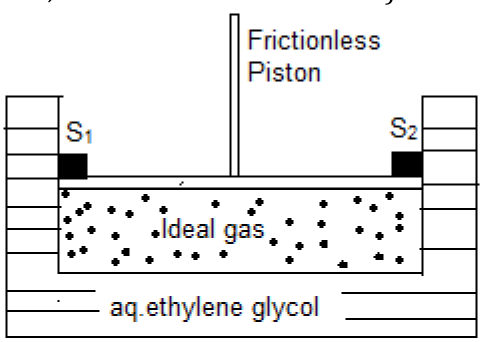
Ppm = $\frac{w_{Solute}}{w_{Solution}}$ x 100

Using the density of the solution and its volume (1L = 1000 mL = 1000 cm^3), the weight of the solution can be calculated.

$W_{solution}$ = 1.03 x 1000 = 1030 g

Thus, ppm = $\frac{10.3 \text{ x } 10^{-3} g}{1030 \text{ g}}$ x 100

24. A cylinder containing an ideal gas (0.1 mol of 1.0 dm^3)
is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at it freezing point. If the stoppers S_1 and S_2 (as shown in the figure) suddenly withdrawn, the volume of the gas in liters after equilibrium is achieved will be ----(Given, K_f (water)=2.0 K kg mol^{-1} ,R = 0.08 dm^3 atm K^{-1} mol^{-1})



Answer: 2.18

Solution:

$K_f = 2$

Molality, 'm' = 0.5

$\Delta T_f = K_f \, . \, m$

$= (0.5 \times 2) = 1$

So, the initial temperature now becomes 272 K. Further using the given value of moles and initial volume of the gas and the calculated initial temperature value, we can find out the initial pressure of the ideal gas contained inside the piston.

$$P_{gas} = \frac{nRT}{V_1}$$
$$= (0.1)(0.08)(272) = 2.176 \text{ atm}$$

Now, on releasing the piston against an external pressure of 1 atm, the gas will expand until the final pressure of the gas, i.e. P_2 becomes equal to 1 atm. During this expansion, since no reaction is happening and the temperature of the gas is not changing as well, the boyle's law relation can be applied.

$P_1V_1 = P_2V_2$

$2.176 \times 1 = 1 \times V_2$

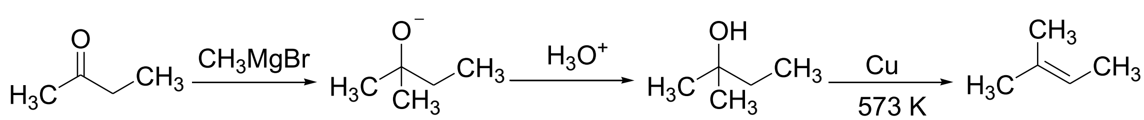
25. Consider the following reactions

$$A \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CH_3MgBr} B \xrightarrow[573\text{ K}]{Cu} \text{2-methyl-2-butene}$$

The mass percentage of carbon in A is:

Answer: 66.67

Solution:



(A) (B)

Compound A is $CH_3(CO)CH_2CH_3$ (C_4H_8O)

The percentage of carbon in compound A by weight is $\frac{w_{Carbon}}{w_{Compound}} = \frac{12 \times 4}{72} = 66.67$

JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. If $A = \{x \in \boldsymbol{R} : |x| < 2\}$ and $B = \{x \in \boldsymbol{R} : |x - 2| \geq 3\}$ then :

a. $A - B = [-1,2]$
b. $B - A = \mathbf{R} - (-2,5)$
c. $A \cup B = \boldsymbol{R} - (2,5)$
d. $A \cap B = (-2,-1)$

Answer: (b)

Solution:

$A = \{x: x \in (-2,2)\}$

$B = \{x: x \in (-\infty,-1] \cup [5,\infty)\}$

$A \cap B = \{x: x \in (-2,-1]\}$

$B - A = \{x: x \in (-\infty,-2] \cup [5,\infty)\}$

$A - B = \{x: x \in (-1,2)\}$

$A \cup B = \{x: x \in (-\infty,2) \cup [5,\infty)\}$

2. If 10 different balls has to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

a. $\frac{965}{2^{10}}$
b. $\frac{945}{2^{10}}$
c. $\frac{945}{2^{11}}$
d. $\frac{965}{2^{11}}$

Answer: (b)

Solution:

Total ways to distribute 10 balls in 4 boxes is $= 4^{10}$

Total ways of placing exactly 2 and 3 balls in any two of these boxes is

$$= {}^{4}C_{2}! \times {}^{10}C_{5} \times \frac{5!}{2!\,3!} \times 2 \times 2^{5}$$

$$P(E) = \frac{945}{2^{10}}$$

3. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta, \theta \in [0,2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

a. $-\frac{3}{8}$

b. $\frac{3}{4}$

c. $\frac{3}{2}$

d. $-\frac{3}{4}$

Answer: (Bonus)

Solution:

$$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2}\, cosec^2\frac{3\theta}{2}\,\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2}\, cosec^2\frac{3\theta}{2}\right)\frac{1}{(2\cos\theta - 2\cos 2\theta)}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{3}{8}$$

None of the above option satisfies the answer.

4. Let f and g be differentiable functions on $\boldsymbol{R}$, such that fog is the identity function. If for some $a, b \in \boldsymbol{R}, g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

a. $\frac{2}{5}$

b. 5

c. 1

d. $\frac{1}{5}$

Answer: (d)

Solution:

$f(g(x)) = x$

$f'(g(x))g'(x) = 1$

Put $x = a$

$$f'(g(a))g'(a) = 1 \;\Rightarrow f'^{(b)} \times 5 = 1 \Rightarrow f'(b) = \frac{1}{5}$$

5. In the expansion of $\left(\frac{x}{\cos\,\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :

a. $16:1$

b. $8:1$

c. $1:8$

d. $1:16$

Answer: (a)

Solution:

$$T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\,\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^{r}$$

For term independent of x,

$$16 - 2r = 0 \Rightarrow r = 8$$

$$T_9 = {}^{16}C_8\left(\frac{1}{\sin\theta\cos\theta}\right)^8 = {}^{16}C_8 2^8\left(\frac{1}{\sin 2\theta}\right)^8$$

$$l_1 = {}^{16}C_8 2^8 \quad \text{at } \theta = \frac{\pi}{4}$$

$$l_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 2^{12} \quad \text{at } \theta = \frac{\pi}{8}$$

$$\frac{l_2}{l_1} = 16:1$$

6. Let $a, b \in \boldsymbol{R}, a \ne 0$, such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the root of this equation, then $\alpha^2 + \beta^2$ is equal to:

a. 24

b. 25

c. 26

d. 28

Answer: (b)

Solution:

$ax^2 - 2bx + 5 = 0$ has both roots as α

$$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

And $\alpha^2 = \frac{5}{a}$

$\Rightarrow b^2 = 5a(a \neq 0)$ ... (1)

$\Rightarrow \alpha + \beta = 2b$ & $\alpha\beta = -10$

$\alpha = \frac{b}{a}$ is also a root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

$\because b^2 = 5a \Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$

$\Rightarrow \alpha^2 = 20, \beta^2 = 5 \Rightarrow \alpha^2 + \beta^2 = 25$

7. Let a function $f: [0,5] \to \boldsymbol{R}$, be continuous, $f(1) = 3$ and $\boldsymbol{F}$ be defined as:

$F(x) = \int_1^x t^2\, g(t)dt$, where $g(t) = \int_1^t f(u)du$ Then for the function $\boldsymbol{F}$, the point $x = 1$ is

a. a point of inflection.

b. a point of local maxima

c. a point of local minima.

d. not a critical point

Answer: (c)

Solution:

$F(x) = x^2 g(x)$

Put $x = 1$

$\Rightarrow F(1) = g(1) = 0$ ... (1)

Now $F''(x) = 2xg(x) + g'(x)x^2$

$F''(1) = 2g(1) + g'(1)$ $\{\because g'(x) = f(x)\}$

$F''(1) = f(1) = 3$ ... (2)

From (1) and (2), $F(x)$ has local minimum at $x = 1$

8. Let $[t]$ denotes the greatest integer $\leq t$ and $\lim_{x \to 0} x \left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2] \sin \pi x$ is discontinuous, when x is equal to

a. $\sqrt{A+1}$

b. $\sqrt{A}$

c. $\sqrt{A+5}$

d. $\sqrt{A+21}$

Answer: (a)

Solution:

$f(x) = [x^2] \sin \pi x$

It is continuous $\forall x \in \mathbf{Z}$ as $\sin \pi x \to 0$ as $\to \mathbf{Z}$.

$f(x)$ is discontinuous at points where $[x^2]$ is discontinuous i.e. $x^2 \in \mathbf{Z}$ with an exception that $f(x)$ is continuous as x is an integer.

$\therefore$ Points of discontinuity for $f(x)$ would be at

$x = \pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}, \dots \dots$

Also, it is given that $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$ (indeterminate form $(0 \times \infty)$)

$\Rightarrow \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow 4 - \lim_{x\to 0}\left\{\frac{4}{x}\right\} = A$

$\Rightarrow A = 4$

$\sqrt{A+5} = 3$

$\sqrt{A+1} = \sqrt{5}$

$\sqrt{A+21} = 5$

$\sqrt{A} = 2$

$\therefore$ Points of discontinuity for $f(x)$ is $x = \sqrt{5}$

9. Let $a - 2b + c = 1$,

If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

a. $f(-50) = 501$

b. $f(-50) = -1$

c. $f(50) = 1$

d. $f(50) = -501$

Answer: (c)

Solution:

Given $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$a - 2b + c = 1$

Applying $R_1 \to R_1 - 2R_2 + R_3$

$f(x) = \begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

Using $a-2b+c=1$

$\therefore f(x)=(x+3)^2-(x+2)(x+4)$

$\Rightarrow f(x)=1$

$\Rightarrow f(50)=1$

$\Rightarrow f(-50)=1$

10. Given: $f(x)=\begin{cases} x\,, & 0\le x<\frac{1}{2} \\ \frac{1}{2}, & x=\frac{1}{2} \\ 1-x\,, & \frac{1}{2}<x\le 1 \end{cases}$

and $g(x)=\left(x-\frac{1}{2}\right)^2, x\in \boldsymbol{R}$. Then the area (in sq. units) of the region bounded by the curves $y=f(x)$ and $y=g(x)$ between the lines $2x=1$ to $2x=\sqrt{3}$ is :

a. $\frac{\sqrt{3}}{4}-\frac{1}{3}$

b. $\frac{1}{3}+\frac{\sqrt{3}}{4}$

c. $\frac{1}{2}+\frac{\sqrt{3}}{4}$

d. $\frac{1}{2}-\frac{\sqrt{3}}{4}$

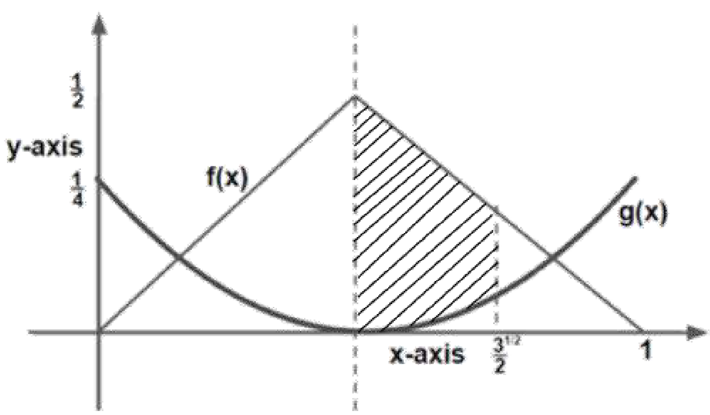
Answer: (a)

Solution:

Given $f(x)=\begin{cases} x & 0<x<\frac{1}{2} \\ \frac{1}{2} & x=\frac{1}{2} \\ 1-x & \frac{1}{2}<x<1 \end{cases}$

$g(x)=\left(x-\frac{1}{2}\right)^2$

The area between $f(x)$ and $g(x)$ from $x=\frac{1}{2}$ to $=\frac{\sqrt{3}}{2}$:



Points of intersection of $f(x)$ and (x) :

$1-x=\left(x-\frac{1}{2}\right)^2$

$\Rightarrow x=\frac{\sqrt{3}}{2}\ ,-\frac{\sqrt{3}}{2}$

Required area $=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}(f(x)-g(x))dx$

$$=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(1-x-\left(x-\frac{1}{2}\right)^2\right)dx$$

$$=x-\frac{x^2}{2}-\frac{1}{3}\left(x-\frac{1}{2}\right)^3\Bigg|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

11. The following system of linear equations

$7x+6y-2z=0$,

$3x+4y+2z=0$

$x-2y-6z=0$, has

a. infinitely many solutions, (x,y,z) satisfying $y=2z$
b. infinitely many solutions (x,y,z) satisfying $x=2z$
c. no solution
d. only the trivial solution

Answer: (b)

Solution:

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$

As the system of equations are Homogeneous $\Rightarrow$ the system is consistent.

$$\Rightarrow\begin{vmatrix}7 & 6 & -2\\ 3 & 4 & 2\\ 1 & -2 & -6\end{vmatrix}=0$$

$\Rightarrow$ Infinite solutions exist (both trivial and non-trivial solutions)

When $y=2z$

Let's take $y=2, z=1$

When $(x,2,1)$is substituted in the system of equations

$\Rightarrow 7x+10=0$,$3x+10=0$, $x-10=0$ (which is not possible)

$\therefore y = 2z \Rightarrow$ Infinitely many solutions does not exist.

For $x = 2z$, lets take $x = 2, z = 1, y = y$

Substitute $(2, y, 1)$in system of equations

$\Rightarrow y = -2$

$\therefore$For each pair of (x, z), we get a value of y.

Therefore, for $x = 2z$ infinitely many solutions exists.

12. If $p \rightarrow (p \ \wedge \sim q)$ is false. Then the truth values of p and q are respectively

a. F, T
b. T, F
c. F, F
d. T, T

Answer: (d)

Solution:

Given $p \rightarrow (p \ \wedge \sim q)$

Truth table:

p	q	$\sim q$	$(p \ \wedge \sim q)$	$p \rightarrow (p \ \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

$p \rightarrow (p \ \wedge \sim q)$ is false when p is true and q is true.

13. The length of minor axis (along y-axis) of an ellipse of the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line $x + 6y = 8$, then its eccentricity is :

a. $\frac{1}{2}\sqrt{\frac{5}{3}}$
b. $\frac{1}{2}\sqrt{\frac{11}{3}}$
c. $\sqrt{\frac{5}{6}}$
d. $\frac{1}{3}\sqrt{\frac{11}{3}}$

Answer: (b)

Solution:

If $2b = \frac{4}{\sqrt{3}}$

$b = \frac{2}{\sqrt{3}}$

Comparing $y = -\frac{x}{6} + \frac{8}{6}$ with $y = mx \pm \sqrt{a^2m^2 + b^2}$

$m = -\frac{1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{11}{12}}$

14. If z be a complex number satisfying $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$, then $|z|$ cannot be:

a. $\sqrt{7}$

b. $\sqrt{\frac{17}{2}}$

c. $\sqrt{10}$

d. $\sqrt{8}$

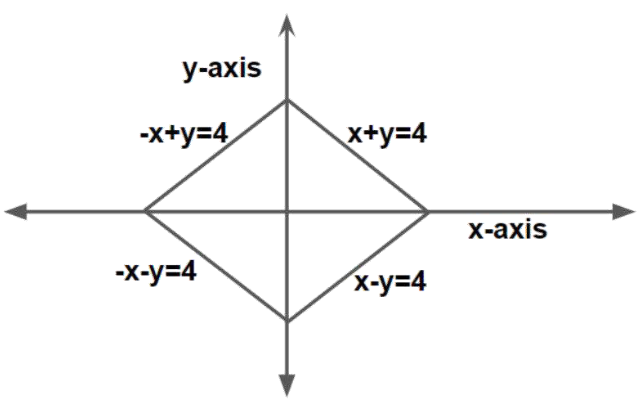
Answer: (a)

Solution:

$|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$

Let $z = x + iy$

$\Rightarrow |x| + |y| = 4$



$\therefore$ z lies on the rhombus.

Maximum value of $|z| = 4$ when $z = 4, -4, 4i, -4i$

Minimum value of $|z| = 2\sqrt{2}$ when $z = 2 \pm 2i,\ \pm 2 + 2i$

$|z| \in [2\sqrt{2}\,, 4]$

$|z| \in [\sqrt{8}\,, \sqrt{16}]$

$|z| \neq \sqrt{7}$

15. If $x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, where $0 < \theta < \frac{\pi}{4}$, then:

a. $y(1+x) = 1$
b. $x(1-y) = 1$
c. $y(1-x) = 1$
d. $x(1+y) = 1$

Answer: (c)

Solution:

$y = 1 + \cos^2\theta + \cos^4\theta + \cdots$

$\Rightarrow y = \dfrac{1}{1-\cos^2\theta} \Rightarrow \dfrac{1}{y} = \sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta - \cdots$

$\Rightarrow x = \dfrac{1}{1-(-\tan^2\theta)} = \cos^2\theta$

$\therefore x + \dfrac{1}{y} = 1 \Rightarrow y(1-x) = 1$

16. If $\dfrac{dy}{dx} = \dfrac{xy}{x^2+y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:

a. $\sqrt{3}e$
b. $\frac{1}{2}\sqrt{3}e$
c. $\sqrt{2}e$
d. $\frac{e}{\sqrt{2}}$

Answer: (a)

Solution:

Let $y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\Rightarrow v + x\dfrac{dv}{dx} = \dfrac{vx^2}{x^2(1+v^2)} = \dfrac{v}{1+v^2}$

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{-v^3}{1+v^2}$

$\Rightarrow \dfrac{1}{x}dx = \left(-\dfrac{1}{v^3} - \dfrac{1}{v}\right)dv$

$\Rightarrow \log x = \frac{1}{2v^2} - \log v + \log c$

$\Rightarrow \log x = \frac{x^2}{2y^2} - \log y + \log x + \log c$

$\log c + \frac{x^2}{2y^2} - \log y = 0$

$y(1) = 1 \Rightarrow \log c + \frac{1}{2} - 0 = 0$

$\log c = -\frac{1}{2}$

$y(x) = e$

$\Rightarrow -\frac{1}{2} + \frac{x^2}{2e^2} - 1 = 0$

$\Rightarrow \frac{x^2}{e^2} = 3$

$\Rightarrow x = \pm\sqrt{3}\mathrm{e}$

17. If one end of focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of tangent to it at B is

a. $x + 2y + 8 = 0$
b. $2x - y - 24 = 0$
c. $x - 2y + 8 = 0$
d. $2x + y - 24 = 0$

Answer: (c)

Solution:

Let PQ be the focal chord of the parabola $y^2 = 8x$

$\Rightarrow P(t_1) = (2t_1^2, 4t_1)$ & $Q(t_2) = (2t_2^2, 4t_2)$

$\Rightarrow t_1 t_2 = -1$

$\because \left(\frac{1}{2}, -2\right)$ is one of the ends of the focal chord of the parabola

Let $\left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$

$\Rightarrow t_2 = -\frac{1}{2}$

$\Rightarrow$ Other end of focal chord will have parameter $t_1 = 2$

$\Rightarrow$ The co-ordinate of the other end of the focal chord will be $(8, 8)$

$\therefore$ The equation of the tangent will be given as $\rightarrow\ 8y = 4(x+8)$

$\Rightarrow 2y - x = 8$

18. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$ then $\sum_{n=1}^{200} a_n$ is equal to:

a. 300
b. 175
c. 225
d. 150

Answer: (d)

Solution:

a_n is a positive term of GP.

Let GP be $a, ar, ar^2, \ldots..$

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots. + a_{201}$

$200 = ar^2 + ar^4 + \ldots\ldots. + ar^{201} \Rightarrow 200 = \frac{ar^2(r^{200}-1)}{r^2-1}$ $\quad \ldots(1)$

Also, $\quad \sum_{n=1}^{100} a_{2n} = 100$

$100 = a_2 + a_4 + \ldots\ \ldots.. + a_{200} \Rightarrow\ 100 = ar + ar^3 + \ \ldots\ldots.. + ar^{199}$

$100 = \frac{ar(r^{200}-1)}{r^2-1}$ $\quad \ldots(2)$

From (1) and (2), $r\ =\ 2$

And $\sum_{n=1}^{100} a_{2n+1} + \sum_{n=1}^{100} a_{2n} = 300$

$\Rightarrow\ a_2 + a_3 + a_4\ \ldots\ \ldots.. + a_{200} + a_{201} = 300$

$\Rightarrow\ ar + ar^2 + ar^3 + \ldots\ldots. + ar^{200} = 300 \Rightarrow r(a + ar + ar^2 + \ldots.. + ar^{199}) = 300$

$\Rightarrow 2(a_1 + a_2 + a_3 + \ \ldots\ldots + a_{200}\ ) = 300$

$\sum_{n=1}^{200} a_n = 150$

19. A random variable X has the following probability distribution:

X	1	2	3	4	5
P(X)	K^2	$2K$	K	$2K$	$5K^2$

Then $P(X > 2)$ is equal to:

a. $\frac{7}{12}$
b. $\frac{23}{36}$
c. $\frac{1}{36}$
d. $\frac{1}{6}$

Answer: (b)

Solution:

We know that $\sum_{X=1}^{5} P(X) = 1$

$\Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow K = -1, \frac{1}{6} \Rightarrow K = \frac{1}{6}$

$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$

$= K + 2K + 5K^2 = \frac{23}{36}$

20. If $\int \frac{d\theta}{\cos^2\theta\ (\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is constant if integration, then the ordered pair $(\lambda,\ f(\theta))$ is equal to:

a. $(-1, 1 - \tan\theta)$

b. $(-1, 1 + \tan\theta)$

c. $(1, 1 + \tan\theta)$

d. $(1, 1 - \tan\theta)$

Answer: (b)

Solution:

Let $I = \int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)}$

$I = \int \frac{\sec^2\theta d\theta}{\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right) + \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$

$I = \int \frac{(1-\tan^2\theta)(\sec^2\theta)d\theta}{(1+\tan\theta)^2}$

Let $\tan\theta = k \Rightarrow \sec^2\theta\ d\theta = \mathrm{d}k$

$I = \int \frac{(1-k^2)}{(1+k)^2}\, dk = \int \frac{(1-k)}{(1+k)}\, dk$

$I = \left(\frac{2}{1+k} - 1\right) dk$

$I = 2\ln|1 + k| - k + c$

$I = 2\ln|1 + \tan\theta| - \tan\theta + c$

Given $I = \lambda\tan\theta + 2\log f(\theta) + c$

$\therefore \lambda = -1, f(\theta) = |1 + tan\theta|$

21. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b}.\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to _______

Answer : (30)

Solution:

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}| \sin\theta$ where θ is the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$

$\theta = \frac{\pi}{2}$ given

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b} \times \vec{c}| = \sqrt{3}|\vec{b}||\vec{c}| \sin\frac{\pi}{3}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2}|\vec{c}|$

Now, $|\vec{b}||\vec{c}| \cos\theta = 10$

$5|\vec{c}|\frac{1}{2} = 10$

$|\vec{c}| = 4$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = 30$

22. If $C_r = {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \cdots + 101 \cdot C_{25} = 2^{25} \cdot k$ then k is equal to_________.

Answer: (51)

Solution:

$S = {}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \cdots + 97\,{}^{25}C_{24} + 101\,{}^{25}C_{25} = 2^{25}k$ (1)

Reverse and apply property ${}^{n}C_r = {}^{n}C_{n-r}$ in all coefficients

$S = 101\,{}^{25}C_0 + 97\,{}^{25}C_1 + \cdots + 5\,{}^{25}C_{24} + {}^{25}C_{25}$ (2)

Adding (1) and (2), we get

$2S = 102[{}^{25}C_0 + {}^{25}C_1 + \cdots + {}^{25}C_{25}]$

$S = 51 \times 2^{25}$

$\Rightarrow k = 51$

23. If the curves $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0, (k > 0)$ touch each other at a point, then the largest value of k is ________.

Answer: (36)

Solution:

Two circles touch each other if $C_1C_2 = |r_1 \pm r_2|$

$\sqrt{k} + 1 = 5$ or $\left|\sqrt{k} - 1\right| = 5$

$\Rightarrow k = 16$ or 36

Maximum value of k is 36

24. The number of terms common to the A.P.'s $3, 7, 11, \dots 407$ and $2, 9, 16, \dots 709$ is ______.

Answer: (14)

Solution:

First common term is 23

Common difference $= \text{LCM}(7, 4) = 28$

$23 + (n-1)28 \leq 407$

$n - 1 \leq 13.71$

$n = 14$

25. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$, $(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______

Answer: (3)

Solution:

We find the point of intersection of the two lines, and the distance of given plane from the two lines is the distance of plane from the point of intersection.

$\therefore (2p - 1,\ 4p + 3,\ 3p - 1) = (2q - 3, 6q - 2,\ \lambda q + 1)$

$p = -\frac{1}{2}$ and $q = \frac{1}{2}$

$\lambda = -7$

Point of intersection is $\left(-2, 1, -\frac{5}{2}\right)$

$$\therefore \frac{k}{\sqrt{633}} = \left|\frac{-46 - 10 + 5 + 48}{\sqrt{633}}\right| \Rightarrow k = 3$$