JEE(Advanced) – 2014 TEST PAPER WITH SOLUTION

(HELD ON SUNDAY 25th MAY, 2014)

PAPER-1

PART - I : PHYSICS

SECTION–1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

1. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ?

(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in series
(D) 0.5 if wires are in parallel

Ans. (B,D)

Sol. Resistance of heater 1, $R = \frac{4\rho L}{\pi d^2}$

Resistance of heater 2, $R_1 = \frac{R}{4}$, $R_2 = \frac{R}{4}$

Series $R_{net} = \frac{R}{2}$

Power $= 2\frac{V^2}{R}$

$\Rightarrow$ power is twice, hence time is $\frac{1}{2}$

time $= \frac{1}{2}$ of 4 min = 2 min

Parallel $R_{Net} = \frac{R}{8}$

Power = 8 times

time $= \frac{R}{8}$ times $= \frac{R}{8} \times 4$ min = 0.5 sec

2. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :-

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

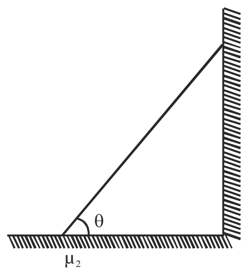
(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

Ans. (A,C,D)

Sol. $\left.\begin{array}{l} \text{At } x = 0 \quad y = 0 \\ \quad x = 3 \quad y \neq 0 \\ \frac{\omega}{k} = 100\ m/s \end{array}\right]$ The equation satisfying all three conditions is correct.

Hence answer ACD

3. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then :-

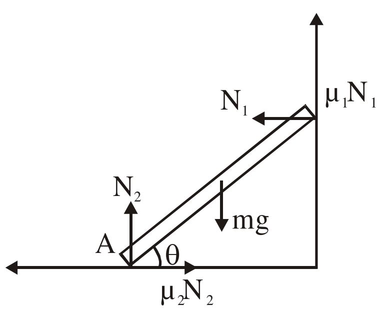


(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$ (B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$ (D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

Ans. (C,D)

Sol.



(i) If $\mu_2 = 0$ equilibrium is not possible for $0 < \theta < 90$

(ii) $N_1 = \mu_2 N_2$ ......(1)

$\mu_1 N_1 + N_2 = mg$ ......(2)

Torque at A

$$N_1\ell \sin\theta + \mu_1 N_1 \ell \cos\theta = mg\frac{\ell}{2}\cos\theta$$

$$N_1 \tan\theta + \mu_1 N_1 = \frac{mg}{2} \qquad ......(3)$$

From (3) $\mu_1 = 0,\ \mu_2 \neq 0$ Ans is D

From (1) and (2) $(\mu_1 \mu_2 + 1)N_2 = mg$ Ans is C

4. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then :-

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3^{rd} maximum of λ_2 overlaps with 5^{th} minimum of λ_1

(D) The angular separation of fringes of λ_1 is greater than λ_2

Ans. (A,B,C)

Sol. $\beta = \frac{\lambda D}{d}$

$\lambda_1 < \lambda_2$

$\beta_2 > \beta_1$

$m_1 \beta_1 = m_2 \beta_2 = y$

$\Rightarrow m_1 > m_2$

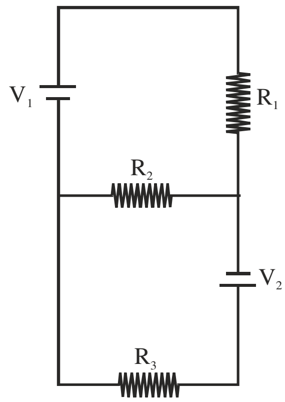
$$\left.\begin{aligned} y_1 &= 3\frac{\lambda_2 D}{d} \\ y_2 &= 4.5\frac{\lambda_1 D}{d} \end{aligned}\right] \qquad \text{Here } y_1 = y_2$$

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

Hence A, B & C are correct choices

PHYSICS

5. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if :-



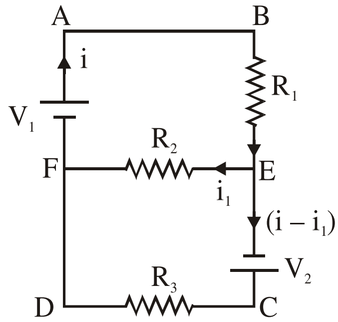
(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Ans. **(A,B,D)**

Sol. Since current through R_2 is zero

Hence $\left[\frac{V_1}{R_1} = \frac{V_2}{R_3}\right]$

The above equation is satisfied by options (A, B, D)



6. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then :-

(A) $Q = 4\sigma\pi r_0^2$
(B) $r_0 = \frac{\lambda}{2\pi\sigma}$
(C) $E_1(r_0/2) = 2E_2(r_0/2)$
(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Ans. **(C)**

Sol. Point charge Line charge Infinite sheet

$$E_1(r_0) = \frac{Q}{4\pi \in_0 r_0^2} \qquad E_2(r_0) = \frac{\lambda}{2\pi \in_0 r_0} \qquad E_3(r_0) = \frac{\sigma}{2 \in_0}$$

Given

$E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\frac{Q}{4\pi \in_0 r_0^2} = \frac{\lambda}{2\pi \in_0 r_0} = \frac{\sigma}{2 \in_0} \qquad ...(i)$$

So, $\frac{Q}{4\pi \in_0 r_0^2} = \frac{\lambda}{2\pi \in_0 r_0} \Rightarrow Q = 2\lambda r_0$

Now, $E_1(r_0/2) = \frac{Q}{4\pi \in_0 \left(\frac{r_0}{2}\right)^2} = \frac{Q}{\pi \in_0 r_0^2} = \frac{2\lambda r_0}{\pi \in_0 r_0^2} = \frac{2\lambda}{\pi \in_0 r_0}$

$$E_2(r_0/2) = \frac{\lambda}{2\pi \in_0 \frac{r_0}{2}} = \frac{\lambda}{\pi \in_0 r_0} = \frac{E_1(r_0/2)}{2} \neq 4\ E_3$$

From equation (i)

$$\frac{Q}{4\pi \in_0 r_0^2} = \frac{\sigma}{2 \in_0} \Rightarrow Q = 2\sigma\pi r_0^2$$

Also from equation (i)

$$\frac{\lambda}{2\pi \in_0 r_0} = \frac{\sigma}{2 \in_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$$

7. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(**Useful information :** $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$ (B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$ (D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Ans. (D)

Sol. $V = \sqrt{\frac{\gamma RT}{m}}$ [m should be taken in Kg]

Neon and argon are monoatomic gas $\Rightarrow \gamma = 1.67$

Nitrogn and oxygen are diatomic gas $\Rightarrow \gamma = 1.4$

$\therefore\ V = \sqrt{\dfrac{1.67 \times RT \times 1000}{m}}$ (m is in grams)

$\therefore$ For Neon, $V = \sqrt{167RT \times \dfrac{10}{20}} = 640 \times \dfrac{7}{10}$ m/s = 448 m/s

For nitrogen, $V = \sqrt{140RT \times \dfrac{10}{28}} = 590 \times \dfrac{3}{5}$ m/s = 354 m/s

For oxygen, $V = \sqrt{140RT \times \dfrac{10}{32}} = 590 \times \dfrac{9}{16}$ m/s = 331.875 m/s

For argon, $V = \sqrt{167RT \times \dfrac{10}{36}} = 640 \times \dfrac{17}{32}$ m/s = 340 m/s.

From the given information, f = 244 Hz

$\left.\dfrac{\lambda}{4}\right|_{max} = 0.350 + 0.005 = 0.355$ m $\qquad \left.\dfrac{\lambda}{4}\right|_{min} = 0.350 - 0.005 = 0.345$ m

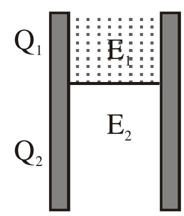
$\therefore\ \lambda_{max} = 1.420$ m $\qquad \lambda_{min} = 1.380$ m

$\therefore\ V_{max} = 244 \times \lambda_{max} = 346.48$ m/sec

$V_{min} = 244 \times \lambda_{min} = 336.72$ m/sec

Only Argon is between them

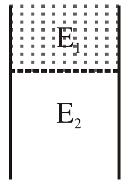
8. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\dfrac{E_1}{E_2} = 1$ $\qquad$ (B) $\dfrac{E_1}{E_2} = \dfrac{1}{K}$ $\qquad$ (C) $\dfrac{Q_1}{Q_2} = \dfrac{3}{K}$ $\qquad$ (D) $\dfrac{C}{C_1} = \dfrac{2+K}{K}$

Ans. (A,D)

Sol.



$C_1 = \dfrac{K\varepsilon_0 (A/3)}{d}$ (With dielectric)

& let $C_2 = \dfrac{\epsilon_0 (2A/3)}{d}$ (without dielectric)

$C = \dfrac{K\varepsilon_0 A/3}{} + \dfrac{\varepsilon_0 2A/3}{} = \dfrac{\varepsilon_0 A/3}{}[K+2]$

$\therefore \ \frac{C}{C_1} = \frac{K+2}{K}$

As potential difference is same and gap is same.

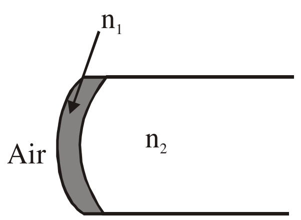
$\therefore \ E_1 = E_2$

$\therefore \ \frac{E_1}{E_2} = 1$

$Q_1 = C_1V, \ Q_2 = C_2V$

$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{2}$

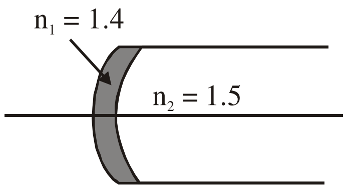
9. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8\ R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4\ R$

Ans. **(A,C)**

Sol. When rays are moving from air to glass,



$$\frac{1.5}{f_1} = \frac{(1.4-1)}{+R} + \frac{(1.5-1.4)}{+R}$$

$$\frac{1.5}{f_1} = \frac{0.4}{R} + \frac{0.1}{R} = \frac{0.5}{R}$$

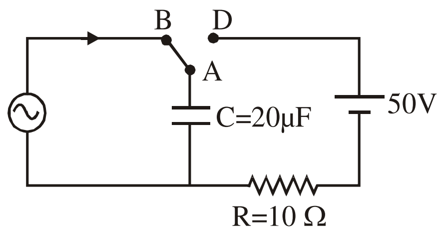
$|f_1| = 3R$

When rays are moving from glass to air,

$$\frac{1}{F_2} = \frac{(1-1.4)}{-R} + \frac{(1.4-1.5)}{-R} = \frac{0.5}{R}$$

$|f_2| = 2R$

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1A$ and $\omega = 500 \text{ rad s}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement (s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C.

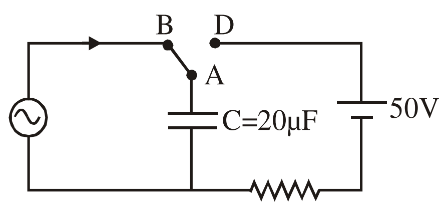
(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10A

(D) $Q = 2 \times 10^{-3}$ C

Ans. **(C,D)**

Sol. Current $I = I_0 \cos(\omega t)$



$$\frac{dq}{dt} = I_0 \cos(\omega t)$$

$$\Rightarrow q = \frac{I_0}{\omega}\sin(\omega t)$$

$$\Rightarrow q = \frac{1}{500}\sin(\omega t)$$

$$\Rightarrow q = (2\times10^{-3})\sin(\omega t)$$

So, maximum charge = 2×10^{-3} C

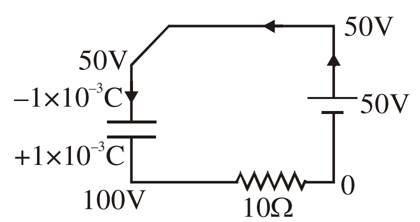
immediately before $t = \frac{7\pi}{6\omega}$

Current in left part just before $t = \frac{7\pi}{6\omega}$

$$I = I_0\cos\left(\omega\times\frac{7\pi}{6\omega}\right) = -\frac{I_0\sqrt{3}}{2}$$

Since current is negative hence current will be anticlockwise.

immediately after $t = \frac{7\pi}{6\omega}$



$$q = (2\times10^{-3})\sin\left(\omega\times\frac{7\pi}{6\omega}\right)$$

Current in 10 Ω resistance,

$$I = \frac{100}{10} = 10A$$

At steady state, potential difference of capaictor is same as of battery,
So, final charge is
$Q_f = C\varepsilon = (20\ \mu F)\ (50\ V) = +\ 1 \times 10^{-3}\ C$
change in charge $= +10^{-3} - (-10^{-3}) = 2 \times 10^{-3}\ C$

SECTION–2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, and value of $\frac{R_1}{R_2}$ is

Ans. 3

Sol. If current is in same direction, then magnetic field at point P will be

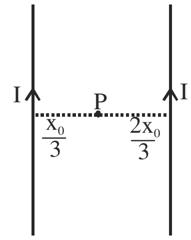
$$B_1 = \frac{\mu_0 I}{2\pi\left(\frac{x_0}{3}\right)} - \frac{\mu_0 I}{2\pi\left(\frac{2x_0}{3}\right)}$$

$$B_1 = \frac{3\mu_0 I}{4\pi x_0}$$

If current is in opposite direction, then magnetic field at point P will be

$$B_2 = \frac{\mu_0 I}{2\pi\left(\frac{x_0}{3}\right)} + \frac{\mu_0 I}{2\pi\left(\frac{2x_0}{3}\right)}$$

$$B_2 = \frac{9\mu_0 I}{4\pi x_0}$$



Radius of curvature, $R = \frac{mu}{qB}$

$$\frac{R_1}{R_2} = \frac{B_2}{B_1} = 3$$

12. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2}m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is.

Sol. Using searle's method young modules is calculated

$$y = \frac{F/A}{\frac{\Delta\ell}{\ell}}$$

$$\frac{dy}{y} = \frac{dF}{F} + \frac{dA}{A} + \frac{d\ell}{\ell} + \frac{d(\Delta\ell)}{\Delta\ell}$$

Only $\Delta\ell$ calculations have error

$$\% \text{ error of } y = \frac{dy}{y} \times 100 = \frac{d\Delta\ell}{\Delta\ell} \times 100 = \frac{1 \times 10^{-5}}{25 \times 10^{-5}} \times 100 = 4\%$$

13. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is.

Ans. 3

Sol. $L = (I)^n(d)^y(f)^z$

$L = (M^1L^0T^{-3})^x(M^1L^{-3})^y(T^{-1})^2$

$L = M^{x+y}L^{-3y}T^{-3x-2}$

$-3y = 1$ $\qquad\qquad$ $x + y = 0$

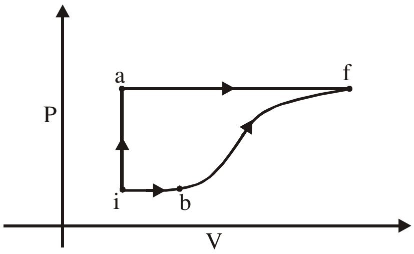
$y = -\frac{1}{3}$ $\qquad\qquad$ $x - \frac{1}{3} = 0$

$x = \frac{1}{3}$

$L = (I)^{1/3}(d)^{-1/3}(f)^2$

$n = 3$

14. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf}/Q_{ib} is.



Ans. 2

Sol. $Q_{iaf} = W_{iaf} + \Delta U_{iaf}$

$500 = 200 + \Delta U_{iaf}$

$\Delta U_{iaf} = 300$

$U_f = 400$

$Q_{ib} = W_{ib} + \Delta U_{ib}$

$= 50 + 100$

$Q_{ib} = 150$

$Q_{bf} = W_{bf} + \Delta U_{bf}$

$= 100 + 200$

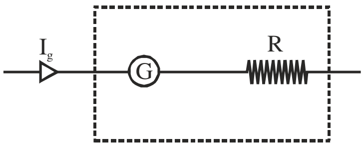
$Q_{bf} = 300$

$\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$ ans

15. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is

Ans. 5

Sol.



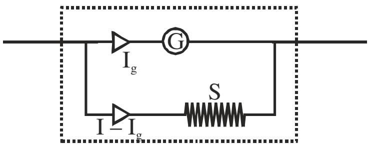
$V = I_g(R + g)$

$30 = 0.006\ (R + g)$

$R + G = 5000 \Rightarrow G = 10\Omega$

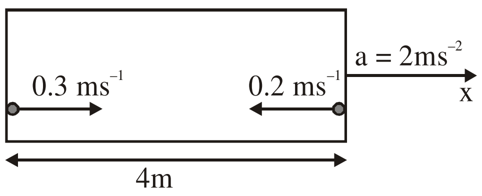
$I_gG = (I - I_g)g$

$(.006)(10) = (1.494)(S)$



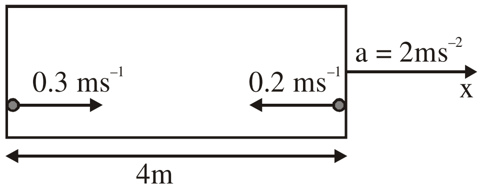
$S = \frac{10}{249} \Rightarrow \therefore n = 5$

16. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Ans. 2 or 8

Sol. Assuming open chamber



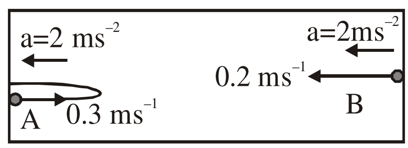
$V_{relative} = 0.5$ m/s
$S_{relative} = 4$m

time = $\frac{4}{0.5} = 8$ m/s

Alternate

Assuming closed chamber

In the frame of chamber :



Maximum displacement of ball A from its left end is $\frac{u_A^2}{2a} = \frac{(0.3)^2}{2(2)} = 0.0225$ m

This is negligible with respect to the length of chamber i.e. 4m. So, the collision will be very close to the left end.

Hence, time taken by ball B to reach left end will be given by

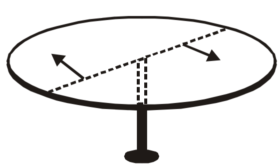
$$S = u_B t + \frac{1}{2}at^2$$

$$4 = (0.2)(t) + \frac{1}{2}(2)(t)^2$$

Solving this, we get

$$t \approx 2 \text{ s}$$

17. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



Ans. 4

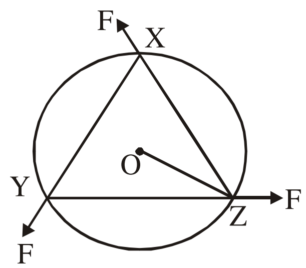
Sol. Since no external torque acts therefore angular momentum remains conserved.

Angular momentum of ball = Angular momentum of plateform

$$0.05 \times 9 \times 0.25 \times 2 = \frac{1}{2} \times 0.45 \times 0.5 \times 0.5 \times \omega$$

$\omega = 4$ rad/s

18. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



Ans. 2

Sol. Angular impulse = change in angular momentum

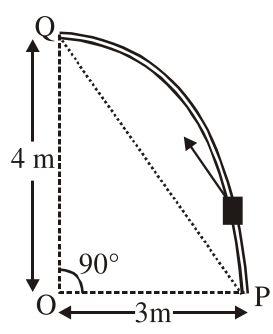
$\tau\Delta t = I\omega$

$3 \times F \times R \sin 30 \times \Delta t = I\omega$

$$3 \times 0.5 \times 0.5 \times \frac{1}{2} \times 1 = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5 \times \omega$$

$\omega = 2$ rad/s

19. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



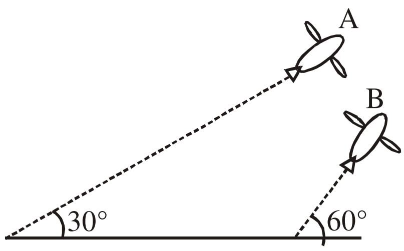
Ans. 5

Sol. $K_f - K_i = W_{all}$

$K_f = W_{ext} + W_{gr}$

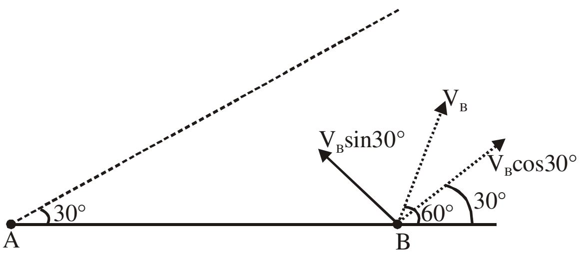
$= 18 \times 5 - 1 \times 10 \times 4 = 50 = 5 \times 10$

20. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



Ans. **5**

Sol.



As observed from A, B moves perpendicular to line of motion of A. It means velocity of B along A is equal to velocity of A

$V_B\cos30 = 100\sqrt{3}$

$V_B = 200$

If A is observer A remains stationary therefore

$$t = \frac{500}{V_B \sin 30} = \frac{500}{100} = 5$$

PART - II : CHEMISTRY

SECTION–1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

21. For the reaction

$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

The correct statement(s) in the balanced equation is / are :

(A) Stoichiometric coefficient of HSO_4^- is 6

(B) Iodide is oxidized

(C) Sulphur is reduced

(D) H_2O is one of the products

Ans. (A, B, D)

Sol. Oxidation half reaction :

$2I^- \rightarrow I_2 + 2e^-$ ...... (1)

Reduction half reaction

$6H^+ + ClO_3^- + 6e^- \rightarrow Cl^- + 3H_2O$ ....(2)

Multiplying equation (1) by 3 and add in (2)

$6I^- + ClO_3^- + 6H^+ \rightarrow Cl^- + 3I_2 + 3H_2O$

$6I^- + ClO_3^- + 6H_2SO_4 \rightarrow Cl^- + 3I_2 + 3H_2O + 6HSO_4^-$

22. The pair(s) of reagents that yield paramagnetic species is / are :

(A) Na and excess of NH_3

(B) K and excess of O_2

(C) Cu and dilute HNO_3

(D) O_2 and 2-ethylanthraquinol

Ans. (A,B,C) / (B,C)

Sol. If ammonia considered as a gas then reaction will be :

(A) $Na + \underset{(\text{excess})}{NH_3} \longrightarrow NaNH_2 + \frac{1}{2}H_2$

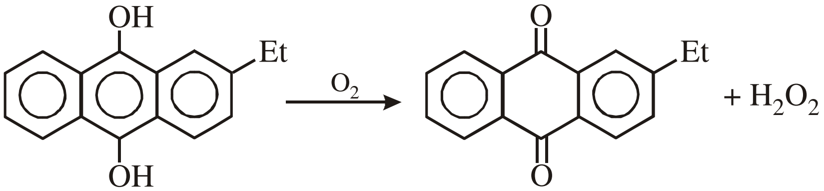
($NaNH_2 + \frac{1}{2}H_2$ are diamagnetic)

If ammonia considered as a liquid then reaction will be

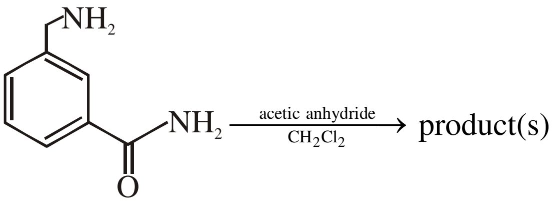
$M + (x+y)NH_3 \rightarrow [M(NH_3)_x]^+ + \underset{\substack{\text{ammoniated } e^- \\ \text{blue colour}}}{[e(NH_3)y]^-}$

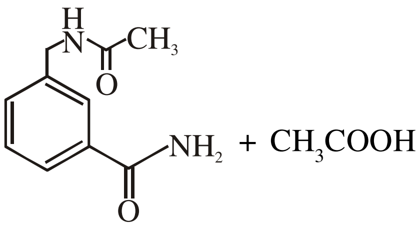
(B) $K + \underset{\text{(excess)}}{O_2} \longrightarrow \underset{\text{Paramagnetic}}{KO_2\ (K^+,\ O_2^-)}$

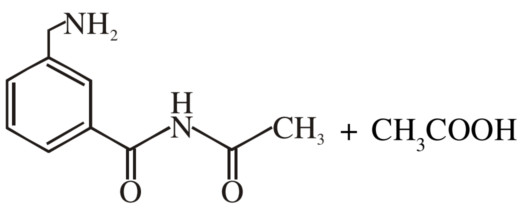
(C) $3Cu + 8HNO_3 \longrightarrow \underset{\text{Paramagnetic}}{3Cu(NO_3)_2} + \underset{\text{Paramagnetic}}{2NO} + 4H_2O$

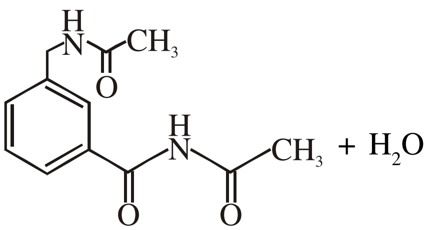
(D) 

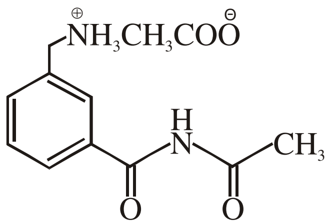
23. In the reaction shown below, the major product(s) formed is / are :



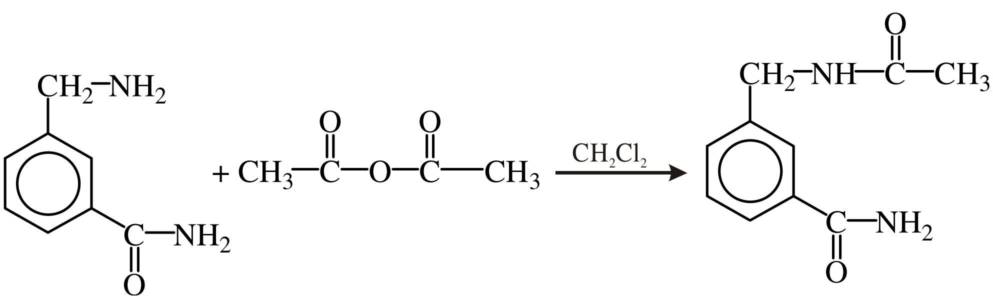
(A) 

(B) 

(C) 

(D) 

Ans. (A)



$-CH_2-NH_2$ is more nucleophilic then $-\underset{\underset{O}{\|}}{C}-NH_2$

24. In a galvanic cell , the salt bridge -

(A) Does not participate chemically in the cell reaction

(B) Stops the diffusion of ions from one electrode to another

(C) Is necessary for the occurence of the cell reaction

(D) Ensures mixing of the two electrolytic solutions

Ans. (A,B)

Note : We feel option (C) is incorrect because in some type of concentration cells, salt bridge is not required. Which can be confirmed from NCERT (XII-Chemistry, Part-1) in Sub section 3.2 Galvanic Cell.

"*The electrolytes of the two half-cells are connected internally through a salt bridge as shown in Fig. 3.1. Sometimes, both the electrodes dip in the same electrolyte solution and in such cases we do not require a salt bridge.* "

25. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are

(A) $CuFeS_2$ (B) CuO (C) Cu_2O (D) $CuSO_4$

Ans. (C)

Sol. $Cu_2S + 2Cu_2O \xrightarrow{\Delta} 6\ Cu + SO_2$

26. Hydrogen bonding plays a central role in the following phenomena

(A) Ice floats in water

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

(C) Formic acid is more acidic than acetic acid

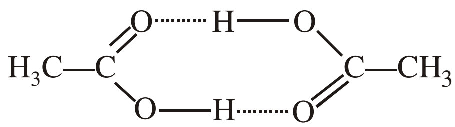
(D) Dimerisation of acetic acid in benzene

Ans. (A,B,D)

Sol. Hint

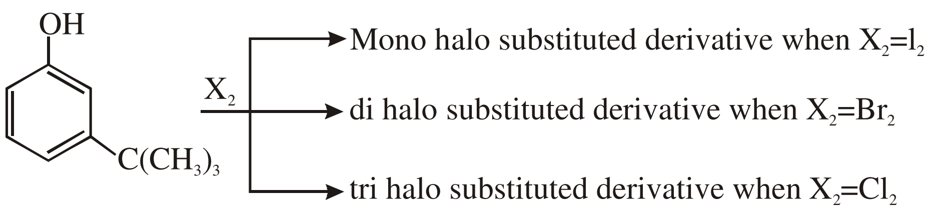
$\Rightarrow$ Ice floats in water due to the low density of ice as compare to water which is due to open cage like structure (formed by intermolecular H-bonding)

$\Rightarrow$ Dimerisation of acetic acid in benzene is due to intermolecular hydrogen bonding



$\Rightarrow$ Basic strength of $RNH_2 > R_3N$ it also explained by hydrogen bonding.

27. The reactivity of compound Z with different halogens under appropriate conditions is given below-



The observed pattern of electrophilic substitution can be explained by -

(A) The steric effect of the halogen

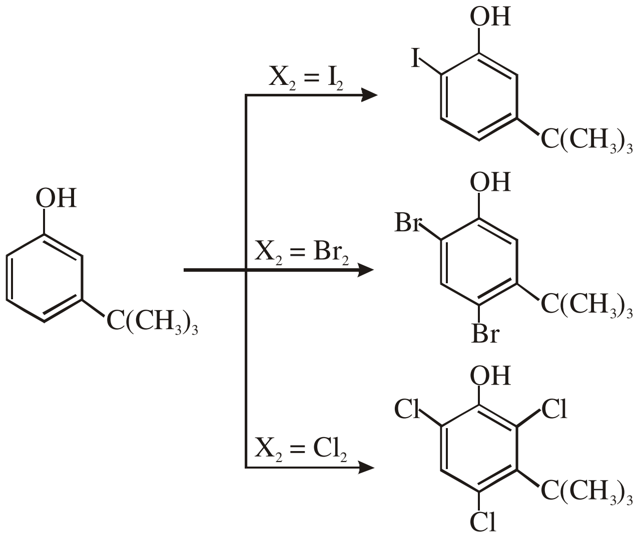
(B) The steric effect of the tert-butyl group

(C) The elctronic effect of the phenolic group

(D) The electronic effect of the turt-butyl group

Sol.

Sol.



Orientation in electrophilic substition reaction is decided by
(A) The steric effect of the halogen
(B) The steric effect of the tert-butyl group
(C) The electronic effect of the phenolic group

28. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are-

(A) *tert*-butanol and 2-methylpropan-2-ol (B) *tert*-butanol and 1, 1-dimethylethan-1-ol

(C) *n*-butanol and butan-1-ol (D) isobutyl alcohol and 2-methylpropan-1-ol

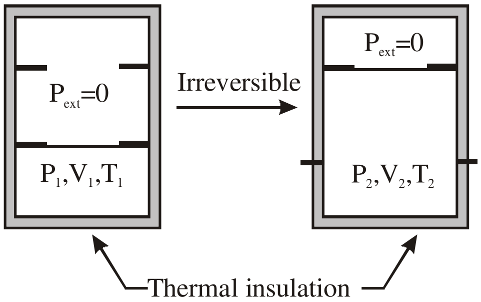
Ans. (A,B,C,D)

Sol. The combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

Formula	**Names**
$CH_3CH_2CH_2CH_2OH$	n-butyl alcohol / n-butanol / butan-1-ol
$CH_3-\underset{\mid \atop CH_3}{CH}-CH_2-OH$	isobutyl alcohol / 2-methyl propan-1-ol
$CH_3-CH_2-\underset{\mid \atop CH_3}{CH}-OH$	Secondary butyl alcohol / butan-2-ol
$CH_3-\overset{CH_3 \atop \mid}{\underset{\mid \atop CH_3}{C}}-OH$	Tertiary butyl alcohol / tert butanol / 2-methyl propan-2-ol / 1,1-dimethyl ethan-1-ol

Reference : National Institute of standards and technology (NIST)

29. An ideal gas in thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irrversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$ (B) $T_2 = T_1$ (C) $P_2V_2 = P_1V_1$ (D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Ans. (A,B,C)

Sol. Process is adiabatic $q = 0$

$P_{ext} = 0$ $w = 0$

$\Delta U = q + w = 0$

The change in internal energy of an ideal gas depends only on temperature therefore $\Delta T = 0$

hence process is isothermal therefore

$T_2 = T_1$ & $P_2V_2 = P_1V_1$

D is incorrect, it is valid for adiabatic reversible process

30. The correct statements(s) for orthoboric acid is/are-

(A) It behaves as a weak acid in water due to self ionization

(B) Acidity of its aqueous solution increses upon addition of ethylene glycol

(C) It has a three dimensional structure due to hydrogen bonding.

(D) It is a weak electrolyte in water

Ans. (D)

Sol. (A) It does not self ionized in water and ionized in water as follows

$$H_3BO_3 + H_2O \rightleftharpoons B(OH)_4^- + H^+$$

(B) Acidity of the aq.solution of boric acid not affected by ethylene glycol

(C) In boric acid due to hydrogen bonding two dimensional sheet structure is formed.

(D) In water the pKa value of H_3BO_3 is 9.25

$$H_3BO_3 + H_2O \rightleftharpoons B(OH)_4^- + H^+$$

SECTION–2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

31. In an atom, the total number of electrons having quantum numbers n = 4, $|m_\ell| = 1$ and $m_s = -\frac{1}{2}$ is

Ans. (6)

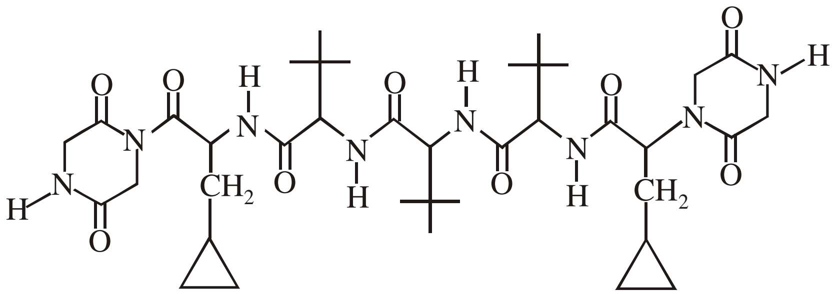
Sol. For n = 4, orbitals are

$\boxed{0}$ $\boxed{-1 \mid 0 \mid +1}$ $\boxed{-2 \mid -1 \mid 0 \mid +1 \mid +2}$ $\boxed{-3 \mid -2 \mid -1 \mid 0 \mid +1 \mid +2 \mid +3}$

Total number of orbitals having $\{|m_\ell| = 1\} = 6$

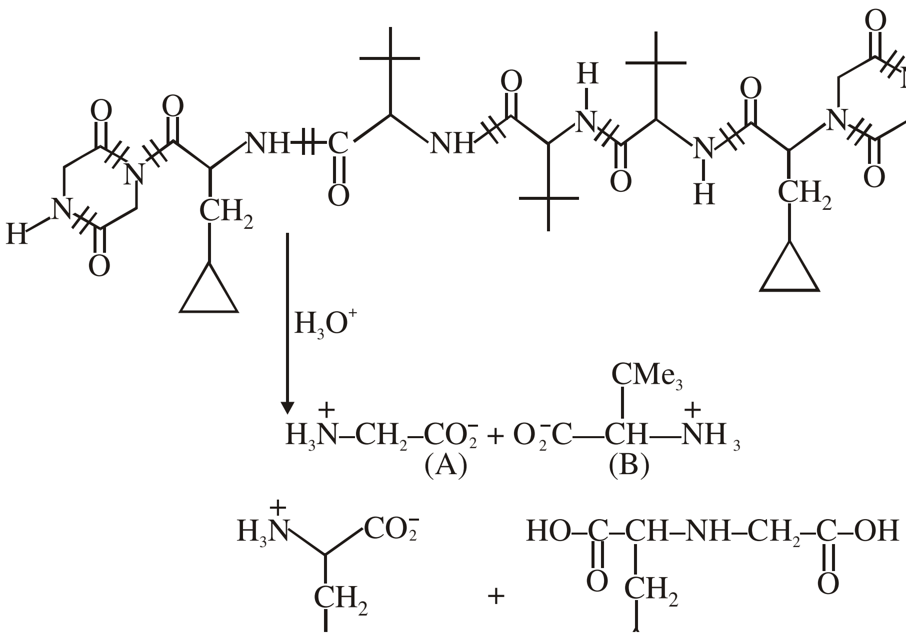
Total number of electrons having $\{|m_\ell| = 1$ and $m_s = -\frac{1}{2}\} = 6$

32. The total number of ***distinct*** ***naturally*** ***occurring amino*** ***acids*** obtained by complete acidic hydrolysis of the peptide shown below is



Ans. (1)

Sol.



33. If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is 1.380×10^{-23} JK^{-1}, then the number of significant digits in the calculated value of the universal gas constant is

Ans. (4)

Sol. Universal gas constant $R = kN_A$

where k= Boltzman constant and N_A = Avogadro number

$\therefore$ R = $1.380 \times 10^{-23} \times 6.023 \times 10^{23}$ J/K-mole

= 8.31174

$\cong 8.312$

So significant figures = 4

34. A compound $\mathbf{H_2X}$ with molar weight of 80 g is dissolved in a solvent having density of 0.4 g mol^{-1}, Assuming no change in volume upon dissolution, the **molality** of a 3.2 molar solution is

Ans. (8)

Sol. Molarity = 3.2 M

Let volume of solution = 1000 ml = volume of solvent

Mass of solvent = 1000 × 0.4 = 400 gm

n_{solute} = 3.2 mole

Molality (m) = $\dfrac{3.2}{400/1000} = 8$

35. MX_2 dissociates into M^{2+} and X^- ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

Ans. (2)

Sol. $MX_2 \longrightarrow M^{2+} + 2X^-$

$$\frac{(\Delta T_f)_{observed}}{(\Delta T_f)_{Theoretical}} = i = 1 + \alpha\ (n-1)$$

$\therefore$ i = 1 + 0.5 (3–1)

i = 2

36. Consider the following list of reagents :

Acidifeid $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2 , Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$.

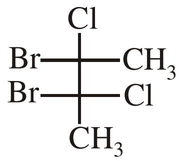
The total number of reagents that can oxidise aqueous iodide to iodine is

Ans. (7)

Sol. Acidified $K_2Cr_2O_7$, $CuSO_4$, H_2O_2, Cl_2, O_3 $FeCl_3$, HNO_3 oxidise aq. iodide to iodine.

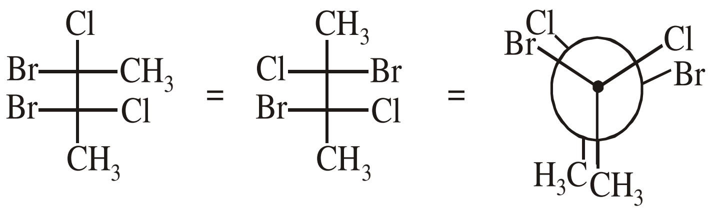
CHEMISTRY

37. The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are)

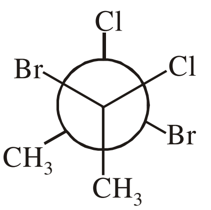


Ans. (3)

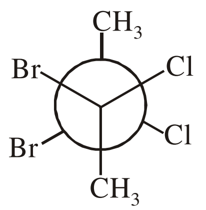
Sol.



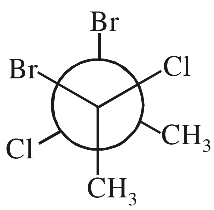
Stable conformer (with $\mu \neq 0$)



(Me-Me) gauche



(Br-Me) gauche



(Cl-Me) gauche

38. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3, and SnS_2 the total number of **BLACK** coloured sulphides is

Ans. (6) / (7)

Sol. PbS, CuS, HgS, Ag_2S, NiS, CoS are black

MnS – dirty pink/Buff

SnS_2 – yellow

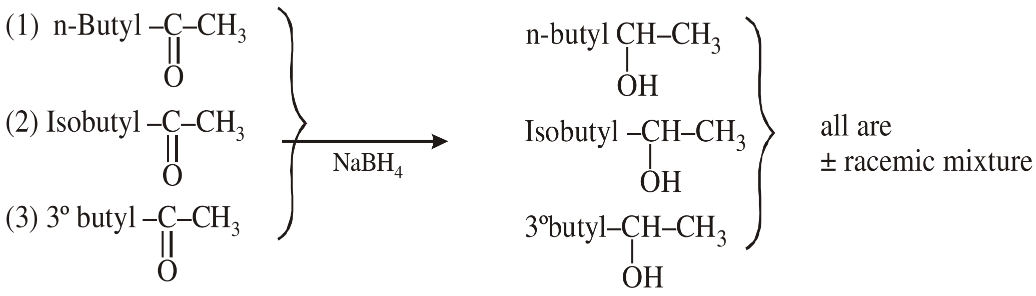
Bi_2S_3 – brown / black (brownish black)

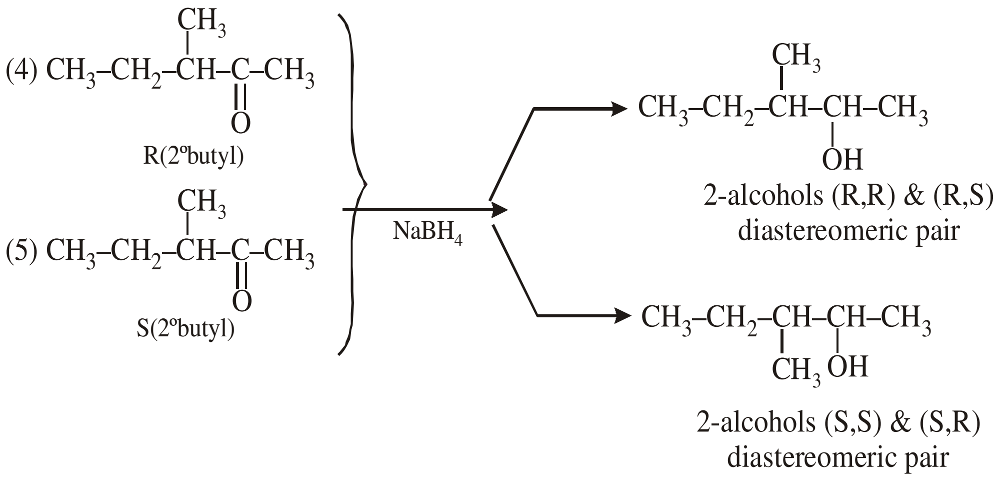
39. Consider all possible isomeric ketones including stereoisomers of MW = 100, All these isomers are independently reacted with $NaBH_4$ (**NOTE :** stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are

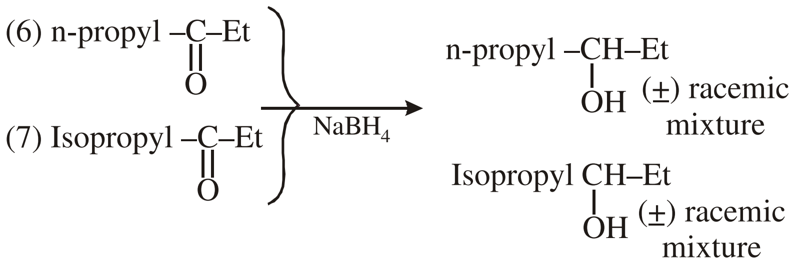
Ans. (5)

Sol. M. wt 100 of ketone

So m. formula = $C_6H_{12}O$







(1 ; 2 ; 3 ; 6 ; 7)

40. A list of species having the formula XZ_4 is given below :

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Ans. (4)

Sol. XeF_4, BrF_4^- , $[Cu(NH_3)_4]^{2+}$, $[PtCl_4]^{2-}$ are square planar

SF_4 – Sea saw

SiF_4 , BF_4^-, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ are tetrahedral

PART - III : MATHEMATICS

SECTION–1 : (One or More Than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

41. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \int_a^x f(t)\,dt & \text{if} \quad a \le x \le b \\ \int_a^b f(t)\,dt & \text{if} \quad x > b \end{cases}.$$

Then

(A) g(x) is continuous but not differentiable at a

(B) g(x) is differentiable on R

(C) g(x) is continuous but not differentiable at b

(D) g(x) is continuous and differentiable at either a or b but not both.

41. Ans. (A,C)

Given that $f : [a, b] \to [1, \infty)$

$$g(x) = \begin{cases} 0 & x < a \\ \int_a^x f(t)dt, & a \le x \le b \\ \int_a^b f(t)dt, & x > b \end{cases}$$

Now $g(a^-) = 0 = g(a^+) = g(a)$

$$g(b^-) = g(b^+) = g(b) = \int_a^b f(t)dt$$

$\Rightarrow$ g is continuous $\forall\ x \in R$

$$\text{Now } g'(x) = \begin{cases} 0 & : \ x < a \\ f(x) & : \ a < x < b \\ 0 & : \ x > b \end{cases}$$

$g'(a^-) = 0$ but $g'(a^+) = f(a) \ge 1$

$\Rightarrow$ g is non differentiable at x = a

and $g'(b^+) = 0$ but $g'(b^-) = f(b) \ge 1$

$\Rightarrow$ g is non differentiable at x = b

42. For every pair of continuous function $f,g : [0, 1] \to R$ such that

$$\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0,1]\},$$

the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

Sol. Ans. (A,D)

f, g $[0,1] \rightarrow R$

we take two cases.

Let f & g attain their common maximum value at P.

$\Rightarrow f(p) = g(p)$ where $p \in [0,1]$

let f & g attain their common maximum value at different points.

$\Rightarrow f(a) = M$ & $g(b) = M$

$\Rightarrow f(a) - g(a) > 0$ & $f(b) - g(b) < 0$

$\Rightarrow f(c) - g(c) = 0$ for some $c \in [0,1]$ as 'f' & 'g' are continuous functions.

$\Rightarrow f(c) - g(c) = 0$ for some $c \in [0,1]$ for all cases. ...(1)

Option (A) $\Rightarrow f^2(c) - g^2(c) + 3\,(f(c) - g(c)) = 0$

which is true from (1)

Option (D) $\Rightarrow f^2(c) - g^2(c) = 0$ which is true from (1)

Now, if we take $f(x) = 1$ & $g(x) = 1\ \forall\ x \in [0,1]$

options (B) & (C) does not hold. Hence option (A) & (D) are correct.

43. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Sol. Ans. (C,D)

Let $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

(A) Given that $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a = b = c = \alpha$(let)

$\Rightarrow M = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix} \Rightarrow |M| = 0 \Rightarrow$ Non-invertible

(B) Given that $[b\ c] = [a\ b] \Rightarrow a = b = c = \alpha$(let)

again $|M| = 0 \Rightarrow$ Non-invertible

(C) As given $M = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix} \Rightarrow |M| = ac \neq 0$ ($\because$ a & c are non zero)

$\Rightarrow$ M is invertible

(D) $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow |M| = ac - b^2 \neq 0$

$\because$ ac is not equal to square of an integer

$\therefore$ M is invertible

44. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$.

If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

(A) $\vec{b} = (\vec{b}.\vec{z})(\vec{z} - \vec{x})$ (B) $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$

Sol. Ans. (A,B,C)

Given that $|\vec{x}|=|\vec{y}|=|\vec{z}|=\sqrt{2}$

and angle between each pair is $\frac{\pi}{3}$

$\therefore \quad \vec{x}.\vec{y}=\vec{y}.\vec{z}=\vec{z}.\vec{x}=\sqrt{2}.\sqrt{2}.\frac{1}{2}=1$

Now $\vec{a}$ is $\perp$ to $\vec{x}$ & $(\vec{y}\times\vec{z})$

Let $\vec{a}=\lambda(\vec{x}\times(\vec{y}\times\vec{z}))$

$=\lambda((\vec{x}.\vec{z})\vec{y}-(\vec{x}.\vec{y})\vec{z})=\lambda(\vec{y}-\vec{z})$

$\vec{a}.\vec{y}=\lambda(\vec{y}.\vec{y}-\vec{y}.\vec{z})=\lambda(2-1)=\lambda$

$\Rightarrow \quad \vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$

Now let $\vec{b}=\mu\left(\vec{y}\times(\vec{z}\times\vec{x})\right)=\mu(\vec{z}-\vec{x})$

$\vec{b}.\vec{z}=\mu(2-1)=\mu$

$\Rightarrow \quad \vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$

Now $\vec{a}.\vec{b}=(\vec{a}.\vec{y})(\vec{y}-\vec{z}).(\vec{b}.\vec{z})(\vec{z}-\vec{x})$

$=(\vec{a}.\vec{y})(\vec{b}.\vec{z})(\vec{y}.\vec{z}-\vec{y}.\vec{x}-\vec{z}.\vec{z}+\vec{z}.\vec{x})$

$=(\vec{a}.\vec{y})(\vec{b}.\vec{z})(1-1-2+1)$

$=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

45. From a point P(λ,λ,λ), perpendiculars PQ and PR are drawn respectively on the lines y = x, z = 1 and y = –x, z = –1. If P is such that ∠QPR is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) –1 (D) $-\sqrt{2}$

Sol. Ans. (C)

Line L_1 given by y = x ; z = 1 can be expressed as

L_1: $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}$

Similarly L_2(y = –x; z = –1) can be expressed as

$L_2:\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}$

Let any point Q (α,α,1) on L_1 and R(β,–β,–1) on L_2

Given that PQ is perpendicular to L_1

$\Rightarrow (\lambda-\alpha).1+(\lambda-\alpha).1+(\lambda-1).0=0 \Rightarrow \lambda=\alpha$

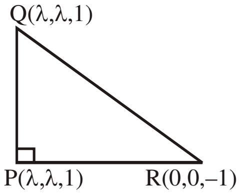
$\therefore$ Q(λ,λ,1)

Similarly PR is perpendicular to L_2

$(\lambda-\beta).1+(\lambda+\beta)(-1)+(\lambda+1).0=0 \Rightarrow \beta=0$

$\therefore$ R(0,0,–1)

Now as given



$\Rightarrow \quad \overrightarrow{PR}.\overrightarrow{PQ}=0$

$0.\lambda+0.\lambda+(\lambda-1)(\lambda+1)=0$

MATHEMATICS

46. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then
(A) determinant of $(M^2 + MN^2)$ is 0
(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is zero matrix
(C) determinant of $(M^2 + MN^2) \geq 1$
(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)$ U equals the zero matrix then U is the zero matrix

Sol. Ans. (A,B)

(A) $(M - N^2)(M + N^2) = \mathbf{O}$ ...(1) $(\because MN^2 = N^2M)$

$\Rightarrow |M - N^2|\,|M + N^2| = 0$

Case I : If $|M + N^2| = 0$

$\therefore |M^2 + MN^2| = 0$

Case II : If $|M + N^2| \neq 0 \Rightarrow M + N^2$ is invertible

from (1)

$(M - N^2)(M + N^2)(M + N^2)^{-1} = \mathbf{O} \Rightarrow M - N^2 = \mathbf{O}$ which is wrong

(B) $(M + N^2)(M - N^2) = \mathbf{O}$

pre-multiply by M

$\Rightarrow (M^2 + MN^2)(M - N^2) = \mathbf{O}$ ...(2)

Let $M - N^2 = U$

$\Rightarrow$ from equation (2) there exist same non zero 'U'

$(M^2 + MN^2)U = \mathbf{O}$

47. Let $f : (0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}.$$

Then
(A) $f(x)$ is monotonically increasing on $[1, \infty)$
(B) $f(x)$ is monotonically decreasing on $[0, 1)$
(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in [0, \infty)$
(D) $f(2^x)$ is an odd function of x on R

Sol. Ans. (A,C,D)

$$f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$f'(x) = 1.\frac{e^{-\left(x+\frac{1}{x}\right)}}{x} - \left(\frac{-1}{x^2}\right)\frac{e^{-\left(\frac{1}{x}+x\right)}}{1/x}$$

$$= \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} + \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} = \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}$$

$\therefore$ $f(x)$ is monotonically increasing on $(0, \infty) \Rightarrow$ A is correct & B is wrong.

Now $f(x) + f\left(\frac{1}{x}\right) = \int_{1/x}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt + \int_{x}^{1/x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$

$= 0 \ \forall \ x \in (0, \infty)$

Now let $g(x) = f(2^x) = \int_{2^{-x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$

$$g(-x) = f(2^{-x}) = \int_{2^x}^{2^{-x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt = -g(x)$$

$\therefore$ $f(2^x)$ is an odd function.

48. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by

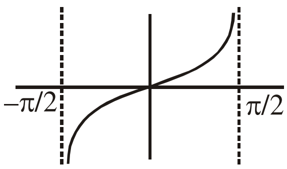
$f(x) = (\log(\sec x + \tan x))^3$ Then :-

(A) $f(x)$ is an odd function
(B) $f(x)$ is a one-one function
(C) $f(x)$ is an onto function
(D) $f(x)$ is an even function

Sol. Ans. (A,B,C)

$$f(x) = (\ell n(\sec x + \tan x))^3$$

$$f'(x) = \frac{3(\ell n(\sec x + \tan x))^2(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)} > 0$$



$f(x)$ is an increasing function

$\lim_{x \to -\frac{\pi}{2}} f(x) \to -\infty$ & $\lim_{x \to \frac{\pi}{2}} f(x) \to \infty$

Range of $f(x)$ is R and onto function

$$f(-x) = (\ell n(\sec x - \tan x))^3 = \left(\ell n\left(\frac{1}{\sec x + \tan x}\right)\right)^3$$

$f(-x) = -(\ell n(\sec x + \tan x))^3$

$f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function.

49. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then :-

(1) radius of S is 8
(B) radius of S is 7
(3) centre of S is (–7, 1) (D) centre is S is (–8, 1)

Sol. Ans. (B,C)

Let circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Put (0,1) $\quad 1 + 2f + c = 0 \quad$ ....(1)

orthogonal with $\quad x^2 + y^2 - 2x - 15 = 0$

$2g(-1) = c - 15 \Rightarrow c = 15 - 2g \quad$ ....(2)

orthogonal with $\quad x^2 + y^2 - 1 = 0$

$c = 1 \quad$ ....(3)

$\Rightarrow g = 7 \ \& \ f = -1$

centre is $(-g, -f) \equiv (-7,1)$

radius $= \sqrt{g^2 + f^2 - c} = \sqrt{49 + 1 - 1} = 7$

MATHEMATICS

50. Let $a \in R$ and let $f : R \to R$ be given by

$$f(x) = x^5 - 5x + a$$

Then

(A) $f(x)$ has three real roots if $a > 4$

(B) $f(x)$ has only one real roots if $a > 4$

(C) $f(x)$ has three real roots if $a < -4$

(D) $f(x)$ has three real roots if $-4 < a < 4$

Sol. Ans. (B,D)

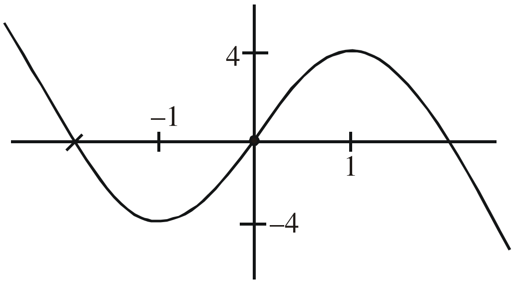
$f(x) = x^5 - 5x + a$

$\therefore\ a = 5x - x^5$

$\therefore$ f(x) has only one real root if

$a > 4$ or $a < -4$

f(x) has three real roots if $-4 < a < 4$



SECTION–2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

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51. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is

Sol. Ans. 8

$(y - x^5)^2 = x(1 + x^2)^2$

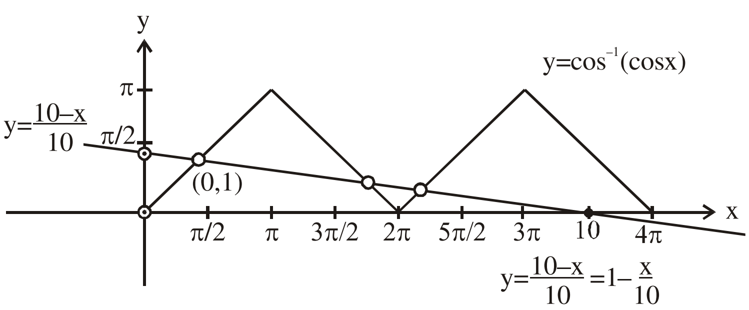
$$\Rightarrow 2\ (y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)^2 + 2x(1 + x^2).2x$$

Put $x = 1,\ y = 3$

$$\frac{dy}{dx} = 8$$

52. Let $f : [0,4\pi] \to [0,\pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0,4\pi]$ satisfying the equation $f(x) = \dfrac{10 - x}{10}$ is

Sol. Ans. 3



from above figure it is clear that $y = \dfrac{10 - x}{10}$ and $y = \cos^{-1}(\cos x)$ intersect at 3 distinct points, so number of solutions = 3

53. The largest value of the non-negative integer a for which $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is

Sol. Ans. 0

$$\lim_{x\to 1}\left\{\frac{\sin(x-1)+a(1-x)}{(x-1)+\sin(x-1)}\right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\lim_{x\to 1}\left\{\frac{\frac{\sin(x-1)}{(x-1)}-a}{1+\frac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow \left(\frac{1-a}{2}\right)^2=\frac{1}{4}$$

$\Rightarrow (a - 1)^2 = 1 \Rightarrow a = 2$ or 0

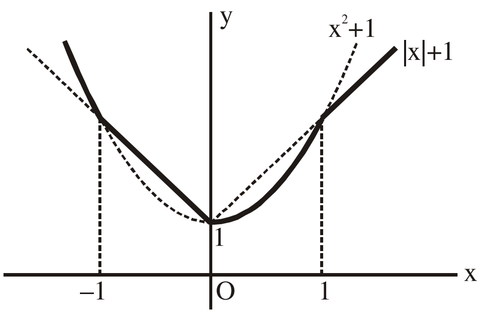
but for $a = 2$ base of above limit approaches $-\frac{1}{2}$ and exponent approaches to 2 and since base cannot be negative hence limit does not exist.

54. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x)=\begin{cases}\max\{f(x),g(x)\} & \text{if} \quad x\le 0,\\ \min\{f(x),g(x)\} & \text{if} \quad x>0.\end{cases}$$

The number of points at which h(x) is not differentiable is

Sol. Ans. 3



h(x) is not differentiable at $x = \pm 1$ & 0

55. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

Sol. Ans. 6

Let P(x,y) is the point in I quad.

Now $2\le\left|\frac{x-y}{\sqrt{2}}\right|+\left|\frac{x+y}{\sqrt{2}}\right|\le 4$

$2\sqrt{2}\le|x-y|+|x+y|\le 4\sqrt{2}$

Case-I : $x \geq y$

$$2\sqrt{2} \leq (x-y)+(x+y) \leq 4\sqrt{2}$$

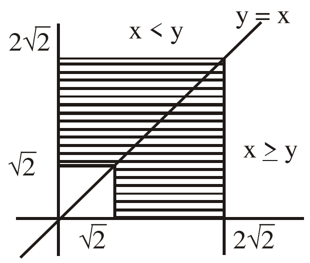
$$\Rightarrow \quad x \in [\sqrt{2}, 2\sqrt{2}]$$

Case-II : $x < y$

$$2\sqrt{2} \leq y-x+(x+y) \leq 4\sqrt{2}$$

$$y \in [\sqrt{2}, 2\sqrt{2}]$$

$$A = \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 6$$



56. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The the number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is

Sol. Ans. 7

as $n_1 \geq 1, n_2 \geq 2, n_3 \geq 3, n_4 \geq 4, n_5 \geq 5$

Let $n_1 - 1 = x_1 \geq 0$, $n_2 - 2 = x_2 \geq 0$ .... $n_5 - 5 = x_5 \geq 0$

$\Rightarrow$ New equation will be

$x_1 + 1 + x_2 + 2 + \ldots\ldots. x_5 + 5 = 20$

$\Rightarrow$ $x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So 7 possible cases will be there.

57. The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\} dx$ is

Sol. Ans. 2

using integration by part

$$\int_0^1 4x^3 \left(\left(1-x^2\right)^5\right)'' dx$$

$$= 4x^3 \left(\left(1-x^2\right)^5\right)' \Big]_0^1 - \int_0^1 12x^2 \left(\left(1-x^2\right)^5\right)' dx$$

$$= -12\left[x^2\left(\left(1-x^2\right)^5\right)\Big|_0^1 - \int_0^1 2x\left(1-x^2\right)^5 dx\right]$$

$$= 12.2.\int_0^1 x(1-x^2)^5 dx$$

Let $1 - x^2 = t \Rightarrow xdx = -\frac{dt}{2}$

$$= 24\int_1^0 t^5\left(-\frac{dt}{2}\right)$$

$$= 12\int_0^1 t^5 dt = 2$$

58. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$.

If $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

Sol. Ans. 4

We know $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$$= \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{5}{4} - \frac{3}{4} = \frac{1}{2}$$

$\therefore \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \frac{1}{\sqrt{2}}$ ...(1)

as given $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$

take dot product with $\vec{a}$

$\Rightarrow \vec{a}.\left(\vec{a}\times\vec{b}\right) + \vec{a}.\left(\vec{b}\times\vec{c}\right) = p\vec{a}^2 + q\vec{b}.\vec{a} + r\vec{c}.\vec{a} \Rightarrow 0 + \frac{1}{\sqrt{2}} = p + \frac{q}{2} + \frac{r}{2}$ ...(2)

Now, take dot product with $\vec{b}$ & $\vec{c}$

$0 = \frac{p}{2} + q + \frac{r}{2}$ ...(3)

& $\frac{1}{\sqrt{2}} = \frac{p}{2} + \frac{q}{2} + r$ ...(4)

equation (2) – equation (4) $\Rightarrow \frac{p}{2} - \frac{r}{2} = 0 \Rightarrow p = r \Rightarrow p + q = 0$ by equation (3)

$\therefore \frac{p^2+2q^2+r^2}{} = \frac{p^2+2p^2+p^2}{} = 4$

59. Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is

Sol **Ans. 4**

Let a,b,c are a, ar, ar^2 where $r \in N$

also $\frac{a+b+c}{3} = b+2$

$\Rightarrow a + ar + ar^2 = 3(ar) + 6$

$\Rightarrow ar^2 - 2ar + a = 6$

$\Rightarrow (r-1)^2 = \frac{6}{a}$

$\because \frac{6}{a}$ must be perfect square & $a \in N$

$\therefore$ a can be 6 only.

$\Rightarrow r - 1 = \pm 1 \Rightarrow r = 2$

& $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7} = 4$ Ans.

60. Let n $\geq$ 2 b an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Sol. **Ans. 5**

Numbr of red line segments = ${}^nC_2 - n$

Number of blue line segments = n

$\therefore {}^nC_2 - n = n$

$\frac{n(n-1)}{2} = 2n \Rightarrow n = 5$ Ans.