JEE Main 2020 Paper

Date of Exam: **7th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

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1. A box weighs 196 N on a spring balance at the North Pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ m/s^2$ at the North Pole and radius of the Earth $= 6400\ km$)

   a. $194.32\ N$
   b. $194.66\ N$
   c. $195.32\ N$
   d. $195.66\ N$

   Solution: (c)
   Weight of the object at the pole, $W = mg = 196\ N$
   Mass of the object, $m = \frac{W}{g} = \frac{196}{10} = 19.6\ kg$
   Weight of object at the equator (W') = Weight at pole − Centrifugal acceleration

   $$W' = mg - m\omega^2 R$$

   $$W' = 196 - (19.6)\left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3 = 195.33\ N$$

2. In a building, there are 15 bulbs of $45\ W$, 15 bulbs of $100\ W$, 15 small fans of $10\ W$ and 2 heaters of $1\ kW$. The voltage of electric main is $220\ V$. The minimum fuse capacity (rated value) of the building will be approximately

   a. $10\ A$
   b. $20\ A$
   c. $25\ A$
   d. $15\ A$

   Solution: (b)
   Total power consumption of the house (P) = Number of appliances × Power rating of each appliance

   $$P = (15 \times 45) + (15 \times 100) + (15 \times 10) + (2 \times 1000) = 4325\ W$$

   So, minimum fuse current $I = \frac{Total\ power\ consumption}{Voltage\ supply} = \frac{4325}{220} A = 19.66\ A$

3. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecules changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas, then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by

a. $\frac{1}{2}$

b. $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

c. $\left(\frac{1}{2}\right)^{\gamma}$

d. 2

Solution: (challenge question)

Relaxation time (τ) dependence on volume and temperature can be given by $\tau \propto \frac{V}{\sqrt{T}}$

Also, for an adiabatic process,

$$T \propto \frac{1}{V^{\gamma-1}}$$

$$\Rightarrow \tau \propto V^{\frac{1+\gamma}{2}}$$

Thus,

$$\frac{\tau_2}{\tau_1} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}}$$

$$\frac{\tau_2}{\tau_1} = (2)^{\frac{1+\gamma}{2}}$$

4. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45^0 with the vertical. Then F equals (Take $g = 10\ m/s^2$ and rope to be massless)

a. 100 N

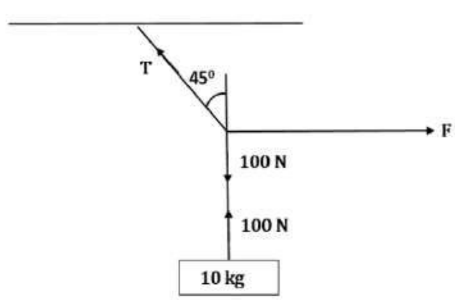
b. 90 N

c. 75 N

d. 70 N

Solution: (a)

Equating the vertical and horizontal components of the forces acting at point



$$\frac{T}{\sqrt{2}} = 100$$
$$\frac{T}{\sqrt{2}} = F$$
$$F = 100\ N$$

5. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is

a. $2\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$
b. $2\pi a^4\left(\frac{aA}{4} + \frac{B}{5}\right)$
c. $\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$
d. $2\pi a^4\left(\frac{A}{4} + \frac{B}{5}\right)$

Solution: (a)

$$\sigma = A + Br$$
$$\int dm = \int (A + Br) 2\pi r dr$$
$$I = \int dm\, r^2$$
$$= \int_0^a (A + Br) 2\pi r^3\, dr$$
$$= 2\pi\left(A\frac{a^4}{4} + B\frac{a^5}{5}\right)$$
$$= 2\pi a^4\left(\frac{A}{4} + \frac{Ba}{5}\right)$$

6. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is the temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2 if both the engines perform equal amount of work?

a. $T = \frac{2T_1T_2}{T_1+T_2}$
b. $T = \frac{T_1+T_2}{2}$
c. $T = 0$
d. $T = \sqrt{T_1T_2}$

Solution: (b)

Heat input to 1^{st} engine= Q_H

Heat rejected from 1^{st} engine=Q

Heat rejected from 2^{nd} engine= Q_L

Work done by 1^{st} engine = Work done by 2^{nd} engine

$$Q_H - Q = Q - Q_L$$

$$2Q = Q_H + Q_L$$

$$2 = \frac{T_1}{T} + \frac{T_2}{T}$$

$$T = \frac{T_1 + T_2}{2}$$

7. The activity of a radioactive substance falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half-life is close to

a. 66 min
b. 62 min
c. 52 min
d. 72 min

Solution: (b)

Using the half-life equation,

$$\ln\frac{A_0}{A_t} = \lambda t$$

At half-life, $t = t_{\frac{1}{2}}$ and $A_t = \frac{A_0}{2}$

$\Rightarrow \ln 2 = \lambda\, t_{\frac{1}{2}}$ ------------- (1)

Also given

$\ln\frac{700}{500} = \lambda\,(30)$ ------------- (2)

Dividing the equations,

$$\frac{\ln 2}{\ln\left(\frac{7}{5}\right)} = \frac{t_{\frac{1}{2}}}{30}$$

$\Rightarrow t_{\frac{1}{2}} = 61.8$ minutes

8. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is

a. 5.9 mm
b. 3.9 mm
c. 6.9 mm
d. 4.9 mm

Solution: (a)

$$\beta = \lambda \frac{D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}}$$
$$= 5.9\ mm$$

9. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of minimum and maximum velocities of fluid in this pipe is

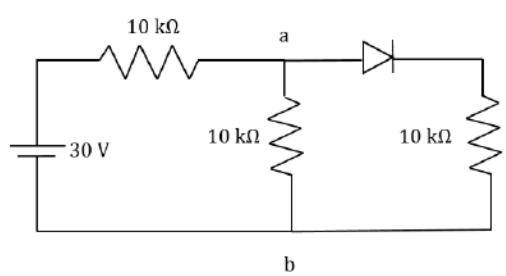
a. $\sqrt{\frac{3}{2}}$
b. $\frac{9}{16}$
c. $\frac{3}{4}$
d. $\frac{3}{4}$

Solution: (b)
Given,
Maximum diameter of pipe $= 6.4\ cm$
Minimum diameter of pipe $= 4.8\ cm$
Using equation of continuity

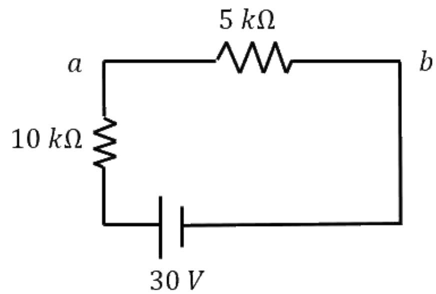
$$A_1V_1 = A_2V_2$$
$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

10. In the figure, potential difference between a and b is

a. 0 V
b. 15 V
c. 10 V
d. 5 V



Solution:
(c)
Diode is in forward bias, so it will behave as simple wire. So, the circuit effectively becomes



$V_{ab} = \frac{30}{5+10} \times 5 \; = \; 10\,V$

11. A particle of mass m and charge q has an initial velocity $\vec{v} = v_o\,\hat{\jmath}$. If an electric field $\vec{E} = E_0\,\hat{\imath}$ and magnetic field $\vec{B} = B_0\,\hat{\imath}$ act on the particle, its speed will double after a time

a. $\frac{\sqrt{3}mv_o}{qE_0}$

b. $\frac{\sqrt{2}mv_o}{qE_0}$

c. $\frac{3mv_o}{qE_0}$

d. $\frac{2mv_o}{2qE_0}$

Solution: (a)

Magnetic field can only change direction of speed as it cannot do any work

As $\vec{v} = v_o\,\hat{\jmath}$ (magnitude of velocity does not change in y–z plane)

$$(2v_o)^2 = v_o^2 + v_x^2$$

$$v_x = \sqrt{3}v_o$$

$$\therefore\ \sqrt{3}v_0 = 0 + \frac{qE_o}{m}t \Rightarrow t = \frac{mv_o\sqrt{3}}{qE_o}$$

12. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one receded with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $v_0 = 1400\,Hz$ and the velocity of sound in air is $350\,m/s$. The speed of each tuning fork is close to

a. $\frac{1}{4}\,m/s$

b. $1\,m/s$

c. $\frac{1}{2}\,m/s$

d. $\frac{1}{8}\,m/s$

Solution: (a)

$$f_0\left(\frac{C}{C-V}\right) - f_0\left(\frac{C}{C+V}\right) = 2$$

$$V = \frac{1}{4}\,m/s$$

13. An electron (of mass m) and a photon have the same energy E in the range of few eV. The ratio of the de Broglie wavelength associated with the electron and the wavelength of the photon is. ($c =$ speed of light in vacuum)

a. $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

b. $\frac{1}{c}\left(\frac{2E}{m}\right)^{\frac{1}{2}}$

c. $c(2mE)^{\frac{1}{2}}$

d. $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

Solution: (d)

λ_d for electron $= \frac{h}{\sqrt{2mE}}$

λ_p for photon $= \frac{hc}{E}$

Ratio $= \frac{h}{\sqrt{2mE}}\frac{E}{hc} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

14. A planar loop of wire rotates in a uniform magnetic field. Initially at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane, then the magnitude of induced emf will be maximum and minimum, respectively at

a. 2.5 sec and 5 sec

b. 5 sec and 7.5 sec

c. 2.5 sec and 7.5 sec

d. 5 sec and 10 sec

Solution: (a)

$\omega = \frac{2\pi}{T} = \frac{\pi}{5}$

When $\omega t = \frac{\pi}{2}$

Then φ_{flux} will be minimum

$\therefore$ e will be maximum

$t = \frac{\frac{\pi}{2}}{\frac{\pi}{5}} = 2.5\ sec$

When $\omega t = \pi$

Then φ_{flux} will be maximum

$\therefore e$ will be minimum

$t = \frac{\pi}{\frac{\pi}{5}} = 5\ sec$

15. The electric field of a plane electromagnetic wave is given by $\vec{E}(t) = E_0\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}\cos(kz + \omega t)$. At $t = 0$, a positively charged particle is at the point $(x, y, z) = (0, 0, \pi/k)$. If its instantaneous velocity at $t = 0$ is $v_0\,\hat{k}$, the force acting on it due to the wave is

a. zero

b. antiparallel to $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

c. parallel to $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

d. parallel to $\hat{k}$

Solution: (b)

Force due to electric field is in direction $-\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Because at $t = 0, E = -\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}E_0$

Force due to magnetic field is in direction $q(\vec{v} \times \vec{B})$ and $\vec{v} \parallel \hat{k}$

$\therefore$ It is parallel to $\vec{E}$

$\therefore$ Net force is antiparallel to $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$.

16. A thin lens made of glass (refractive index $= 1.5$) of focal length $f = 16\,cm$ is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_l, then the ratio f_l/f is closest to the integer

a. 9

b. 17

c. 1

d. 5

Solution:

(a)

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{f_a}{f_l} = \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{\left(\frac{\mu_g}{\mu_a} - 1\right)} = \frac{\left(\frac{1.50}{1.42} - 1\right)}{\left(\frac{1.50}{1} - 1\right)} = \frac{0.08}{(1.42)(0.5)}$$

$$\frac{f_l}{f_a} = \frac{(1.42)(0.5)}{0.08} = 8.875 = 9$$

17. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being $68\,kg$. The mass of the elevator itself is $920\,kg$ and it moves with a constant speed of $3\,m/s$. The frictional force opposing the motion is $6000\,N$. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\,m/s^2$) must be at least

a. 66000 W

b. 63360 W

c. 48000 W

d. 56300 W

Solution: (a)

Net force on motor will be

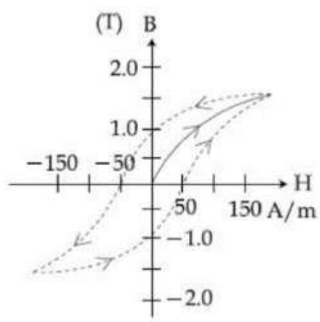
$$F_m = [920 + 68(10)]g + 6000$$
$$F_m = 22000\ N$$

So, required power for motor

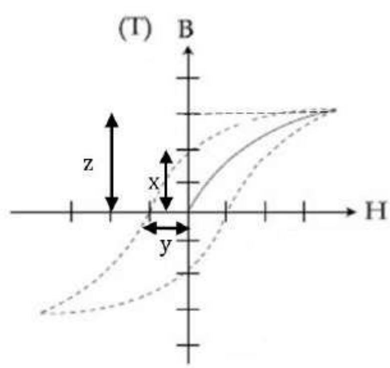
$$P_m = \overrightarrow{F_m}.\vec{v}$$
$$= 22000 \times 3$$
$$= 66000\ W$$

18. The figure gives experimentally measured B vs H variation in a ferromagnetic material. The retentivity, coercivity and saturation, respectively, of the material are

a. 1.5 T, 50 A/m, 1 T
b. 1 T, 50 A/m, 1.5 T
c. 1.5 T, 50 A/m, 1 T
d. 150 A/m, 1 T, 1.5 T



Solution: (b)



x = retentivity
y = coercivity
z = saturation magnetization

19. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and $t = 40$ s is close to (take $e^2 = 7.389$)

a. 1.06
b. 1.46
c. 1.15
d. 0.84

Solution: (a)

$$i = i_o\left(1 - e^{\frac{-t}{L/R}}\right)$$
$$= \frac{20}{5}\left(1 - e^{\frac{-t}{0.01/5}}\right)$$
$$= 4(1 - e^{-500t})$$
$$i_\infty = 4$$
$$i_{40} = 4(1 - e^{-500\times 40}) = 4\left(1 - \frac{1}{(e^2)^{10000}}\right) = 4\left(1 - \frac{1}{7.389^{10000}}\right)$$
$$\frac{i_\infty}{i_{40}} \approx 1 \text{ (Slightly greater than one)}$$

20. The dimension of $\frac{B^2}{2\mu_o}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is

a. $ML^{-1}T^{-2}$
b. ML^2T^{-2}
c. MLT^{-2}
d. ML^2T^{-1}

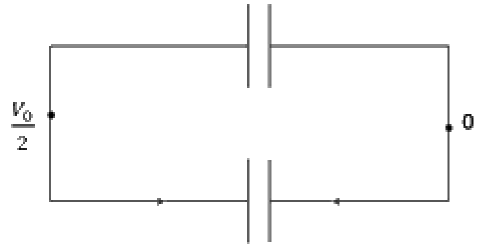
Solution: (a)

Energy density in magnetic field $= \frac{B^2}{2\mu_o}$

$$= \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^{\mathbf{3}}} = \frac{MLT^{-2}.L}{L^3} = ML^{-1}T^{-2}$$

21. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) ______.

Solution: (6)



$V_0 = 20\,V$

Initial potential energy $U_i = \frac{1}{2}CV_0^2$

After connecting identical capacitor in parallel, voltage across each capacitor will be $\frac{V_0}{2}$. Then, final potential energy $U_f = 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right]$

Heat loss $= U_i - U_f$

$$= \frac{CV_0^2}{2} - \frac{CV_0^2}{4} = \frac{CV_0^2}{4} = \frac{60\times10^{-12}\times20^2}{4} = 6\times10^{-9} = 6\text{ nJ}$$

22. M grams of steam at 100^oC is mixed with $200\,g$ of ice at its melting point in a thermally insulated container. If it produces liquid water at 40^oC [heat of vaporization of water is $540\,cal/g$ and heat of fusion of ice is $80\,cal/g$], the value of M is ______.

Solution: (40)

Here, heat absorbed by ice $= m_{ice}\,L_f + m_{ice}C_w(40-0)$

Heat released by steam $= m_{steam}\,L_v + m_{steam}C_w(100-40)$

Heat absorbed = heat released

$m_{ice}\,L_f + m_{ice}C_w(40-0) = m_{steam}\,L_v + m_{steam}C_w(100-40)$

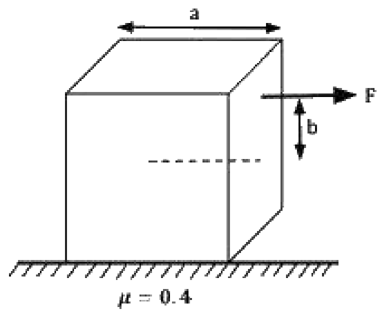
$\Rightarrow 200\times 80\,cal/g + 200\times 1\,cal/g/°C\times(40-0)$

$= m\times 540\,cal/g + 540\times 1\,cal/g/°C\times(100-40)$

$\Rightarrow 200\,[80+(40)1] = m[540+(60)1]$

$m = 40\,g$

23. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum value of $100\times\frac{b}{a}$ for the box not to topple before moving is _______.



Solution: (50)

F balances kinetic friction so that the block can move

So, $F = \mu mg$

For no toppling, the net torque about bottom right edge should be zero

i.e. $$F\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$$

$$\mu mg\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$$

$$\mu\frac{a}{2} + \mu b \le \frac{a}{2}$$

$$0.2a + 0.4b \le 0.5a$$

$$0.4b \le 0.3a$$

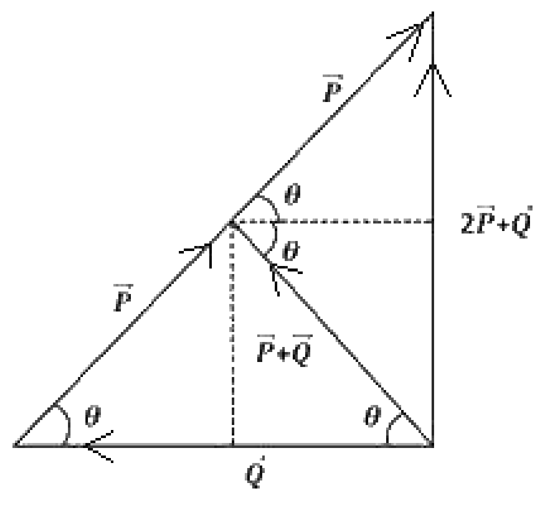
$$b \le \frac{3}{4}a$$

But, maximum value of b can only be $0.5a$

∴ Maximum value of $100\frac{b}{a}$ is 50.

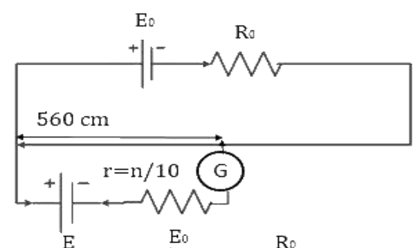
24. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $\overrightarrow{2P}$ and $\vec{Q}$ will make with $\vec{Q}$ is ________

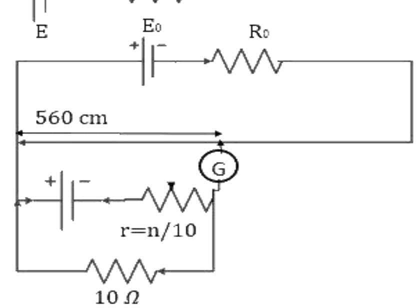
Solution: (90°)



25. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}$ Ω, the value of N is ______

Solution :(12)





Let the emf of cell is ε internal resistance is $'r'$ and potential gradient is x.

$$\varepsilon = 560\, x \qquad (1)$$

After connecting the resistor

$$\frac{\varepsilon \times 10}{10 + r} = 500x \quad (2)$$

From (1) and (2)

$$\frac{560 \times 10}{10 + r} = 500\, s$$

$$56 = 540 + 5r$$

$$r = \frac{6}{5} = 1.2\, \Omega$$

$$n = 12$$

JEE Main 2020 Paper

Date: 7th January 2020

Time: 02:30 PM – 05:30 PM

Subject: Chemistry

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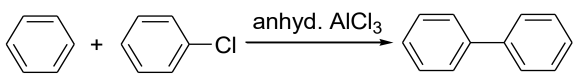
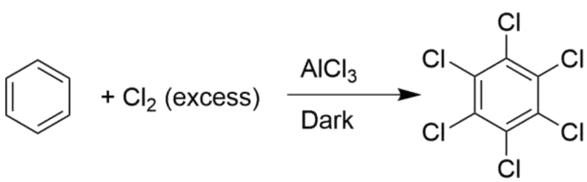
1. Consider the following reactions:

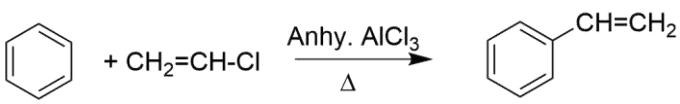
A.



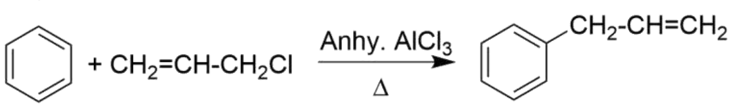
B.



C.



D.



Which of these reactions are possible?

a. A and D
b. B and D
c. B, C and D
d. A and B

Answer: b

Solution:

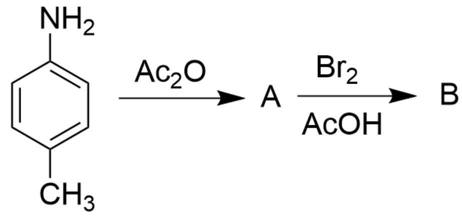
In aryl halides, due to the partial double bond character generated by chlorine, the aryl cation is not formed.

Vinyl halides do not give Friedel-Crafts reaction, because the intermediate that is generated (vinyl cation) is not stable.

$H_2C=\overset{+}{C}H$

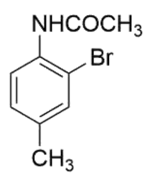
vinyl
cation

2. In the following reaction sequence,

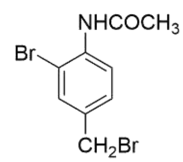


The major product B is:

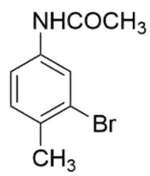
a.



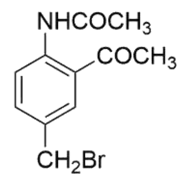
b.



c.



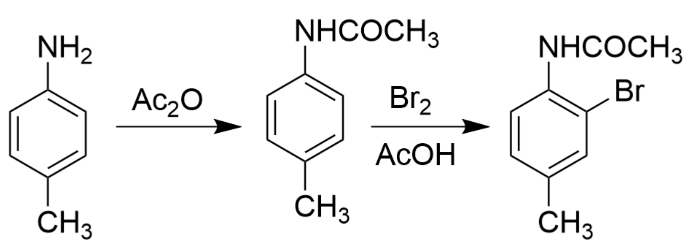
d.



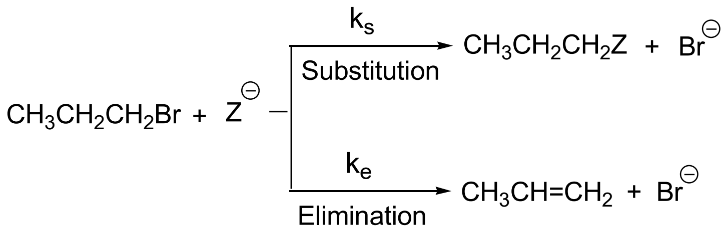
Answer: a

Solution:

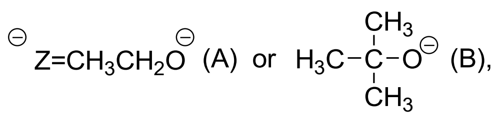
During trisubstitution, the acetanilide group attached to the benzene ring is more electron donating than the methyl group attached, owing to +M effect, and therefore, the incoming electrophile would prefer ortho w.r.t the acetanilide group.



3. For the following reactions,



Where,



k_s and k_e, are, respectively, the rate constants for substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct option is ______.

a. $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$
b. $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$
c. $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$
d. $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

Answer: c

The base $CH_3CH_2O^-$ favours substitution reaction over elimination reaction and thus $k_e(A) < k_s(A)$.

Since the bulkier base tertiary butoxide (B) favours elimination reaction over substitution reaction, $k_e(B)$ is higher. Since $\mu = \frac{k_s}{k_e}$, μ_A will be greater as it is having lower k_e value.

Solution:

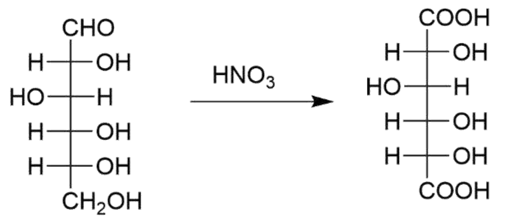
4. Which of the following statements is correct?

a. Gluconic acid can form cyclic (acetal/hemiacetal) structure
b. Gluconic acid is a dicarboxylic acid
c. Gluconic acid is obtained by oxidation of glucose with HNO_3
d. Gluconic acid is a partial oxidation product of glucose

Answer: d

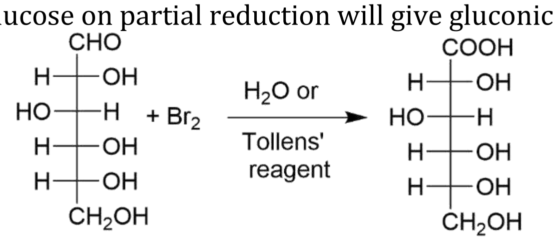
Solution:

The gluconic acid formed is a monocarboxylic acid which is formed during the partial oxidation of glucose. Glucose on reaction with HNO_3 will give glucaric acid:



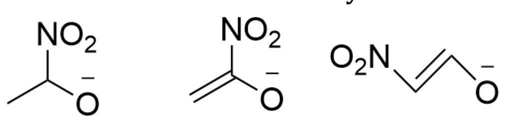
Glucose Glucaric acid

Glucose on partial reduction will give gluconic acid:



Glucose Gluconic acid

5. The correct order of stability for the following alkoxides is:



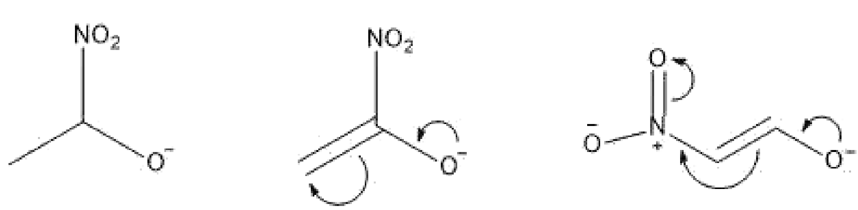
(A) (B) (C)

a. (C) > (A) > (B)
b. (C) > (B) > (A)
c. (B) > (A) > (C)
d. (B) > (C) > (A)

Answer: b

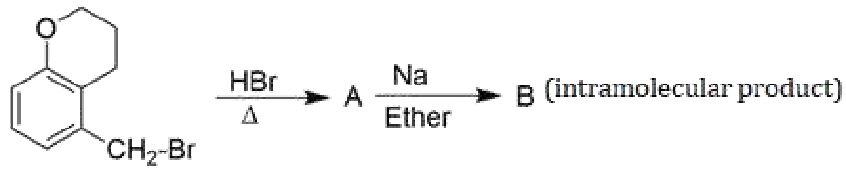
Solution:

Higher the delocalization of the negative charge, more will be the stability of the anion.

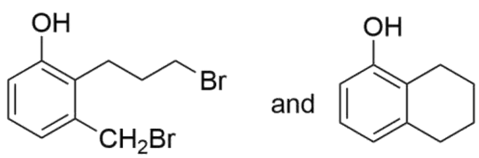


(A) The negative charge is stabilized only through –I effect exhibited by the – NO_2 group.
(B) The negative charge is stabilized by the delocalization of the double bond and the –I effect exhibited by the – NO_2 group.
(C) The negative charge is stabilized by extended conjugation.

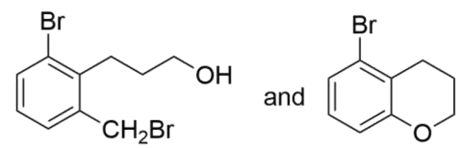
6. In the following reaction sequence, structures of A and B, respectively will be:



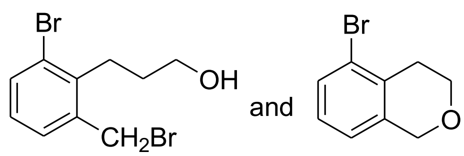
a.



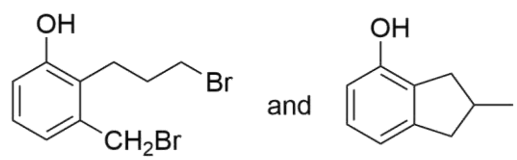
b.



c.

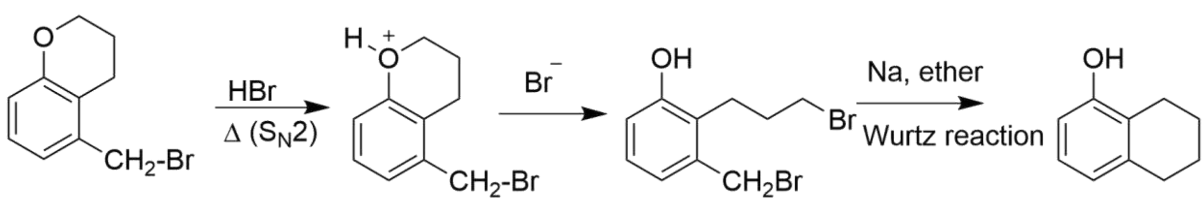


d.



Answer: a

Solution:



7. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide, (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane : ethyl acetate (20:80), the sequence of obtained compounds is:
   a. (B), (A) and (C)
   b. (C), (A) and (B)
   c. (B), (C) and (A)
   d. (A), (B) and (C)

Answer: b

Solution:

Since the column is eluted with the solvent having more proportion of ethyl acetate, the more polar compound will come out first.

The order of polarity in the given compounds is ; acetophenone > benzanilide > aniline.

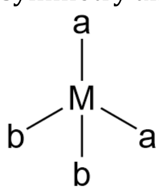
(Dipolemoment (μ) of acetophenone=3.05 D, benzanilide=2.71 D and aniline=1.59 D).

8. The number of possible optical isomers for the complexes $[MA_2B_2]$ with sp^3 or dsp^2 hybridized metal atom, respectively, is:
   Note: A and B are unidentate neutral and unidentate monoanionic ligands, respectively.
   a. 0 and1
   b. 0 and 0
   c. 2 and 2
   d. 0 and 2

Answer: b

Solution:

Case 1: If M is sp^3 hybridized, the geometry will be tetrahedral. There will be a plane of symmetry and thus it does not show optical activity.



Case 2: If M is dsp^2 hybridized, the geometry will be square planar. Due to the presence of a plane of symmetry, it does not show optical activity.

9. The bond order and magnetic characteristics of CN^- are:
   a. 3, paramagnetic
   b. 2.5, diamagnetic
   c. 3, diamagnetic
   d. 2.5, paramagnetic

Answer: c

Solution:

CN^- is a 14 electron system. The bond order and magnetism can be predicted using MOT.

The MOT electronic configuration of CN^- is:

$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\pi_{2p_x}^2\pi_{2p_y}^2\sigma_{2p_z}^2$

Bond order $= \frac{1}{2} \times (N_{bonding} - N_{antibonding}) = 3$

As CN^- does not have any unpaired electrons, and hence it is diamagnetic.

10. The equation that is incorrect is:
   a. $\Lambda_m^0 NaBr - \Lambda_m^0 NaI = \Lambda_m^0 KBr - \Lambda_m^0 NaBr$
   b. $\Lambda_m^0 NaBr - \Lambda_m^0 NaCl = \Lambda_m^0 KBr - \Lambda_m^0 KCl$
   c. $\Lambda_m^0 KCl - \Lambda_m^0 NaCl = \Lambda_m^0 KBr - \Lambda_m^0 NaBr$
   d. $\Lambda_m^0 H_2O = \Lambda_m^0 HCl + \Lambda_m^0 NaOH - \Lambda_m^0 NaBr$

Answer: a

Solution:

$$\Lambda_m^0 NaI - \Lambda_m^0 NaBr = \Lambda_m^0 NaBr - \Lambda_m^0 KBr$$

$$[\lambda_m^0 Na^+ + \lambda_m^0 I^-] - [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] = [\lambda_m^0 Na^+ + \lambda_m^0 Br^-] - [\lambda_m^0 K^+ + \lambda_m^0 Br^-]$$

$$\lambda_m^0 I^- - \lambda_m^0 Br^- \neq \lambda_m^0 Na^+ - \lambda_m^0 K^+$$

11. In the following reactions, product (A) and (B), respectively, are:
$NaOH + Cl_2 \rightarrow (A) + \text{side products}$
(hot & conc.)
$Ca(OH)_2 + Cl_2 \rightarrow (B) + \text{side products}$
(dry)
   a. $NaClO_3$ and $Ca(ClO_3)_2$
   b. $NaOCl$ and $Ca(ClO_3)_2$
   c. $NaOCl$ and $Ca(OCl)_2$
   d. $NaClO_3$ and $Ca(OCl)_2$

Answer: d

Solution:

$$6NaOH + 3Cl_2 \rightarrow 5NaCl + NaClO_3 + 3H_2O$$
$$2Ca(OH)_2 + Cl_2 \rightarrow Ca(OCl)_2 + CaCl_2 + H_2O$$

12. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non-volatile solute are together sealed in a container. Over time:
   a. the volume of the solution and the solvent does not change
   b. the volume of the solution increases and the volume of the solvent decreases
   c. the volume of the solution decreases and the volume of the solvent increases
   d. the volume of the solution does not change and the volume of the solvent decreases

Answer: b

Solution:

Consider beaker I contains the solvent and beaker 2 contains the solution. Let the vapour pressure of the beaker I be P^o and the vapour pressure of beaker II be P^s. According to Raoult's law, the vapour pressure of the solvent (P^o) is greater than the vapour pressure of the solution (P^s)

($P^o > P^s$)

Due to a higher vapour pressure, the solvent flows into the solution. So volume of beaker II would increase.

In a closed beaker, both the liquids on attaining equilibrium with the vapour phase will end up having the same vapour pressure. Beaker II attains equilibrium at a lower vapour pressure and so in its case, condensation will occur so as to negate the increased vapour pressure from beaker I, which results in an increase in its volume.

On the contrary, since particles are condensing from the vapour phase in beaker II, the vapour pressure will decrease. Since beaker I at equilibrium attains a higher vapour pressure evaporation will be favoured more so as to compensate for the decreased vapour pressure, as mentioned in the previous statement.

13. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is:

a. vapour phase refining
b. distillation
c. liquation
d. zone refining

Answer: c

Solution:

Liquation is the process of refining a metal with a low melting point containing impurities of high melting point

14. Among statements I-IV, the correct ones are:

I. Decomposition of hydrogen peroxide gives dioxygen
II. Like hydrogen peroxide, compounds, such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberate dioxygen.
III. 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.
IV. Hydrogen peroxide is used for the manufacture of sodium perborate

a. I,II, III and IV
b. I, II and III only
c. I, III and IV only
d. I and III only

Answer: a

Solution:

Decomposition of H_2O_2 : $2H_2O_2(l) \rightarrow O_2(g) + 2H_2O(l)$

Industrially, H_2O_2 is prepared by the auto-oxidation of 2-alklylanthraquinols.

$2KClO_3 \xrightarrow{150-300℃} 2KCl + 3O_2$

$2Pb(NO_3)_2 \xrightarrow{200-470℃} 2PbO + 4NO_2 + O_2$

$2NaNO_3 \rightarrow 2NaNO_2 + O_2$

Synthesis of sodium perborate:

$Na_2B_4O_7 + 2NaOH + 4H_2O_2 \rightarrow 2NaBO_3 + 5H_2O$

15. The redox reaction among the following is:
    a. formation of ozone from atmospheric oxygen in the presence of sunlight
    b. reaction of H_2SO_4 with NaOH
    c. combination of dinitrogen with dioxygen at 2000 K
    d. Reaction of $[Co(H_2O)_6]Cl_3$ withAgNO$_3$

Answer: c

Solution:

$N_2 + O_2 \xrightarrow{2000\ K} 2NO$: The oxidation state of N changes from 0 to +2, and the oxidation state of O changes from 0 to -2

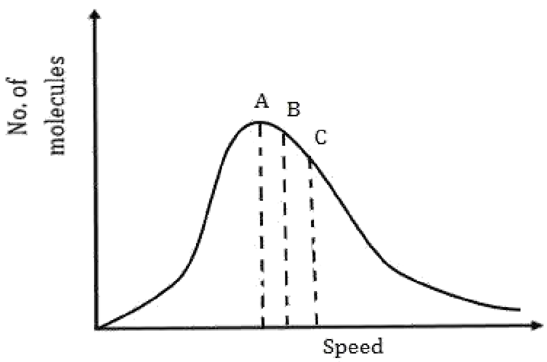
In all the remaining reactions, there is no change in oxidation states of the elements participating in the reaction.

$3O_2 \rightarrow 2O_3$

$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$ (Neutralisation reaction)

$3\ AgNO_3 + [Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6](NO_3)_3 + 3\ AgCl$ (Double displacement)

16. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}); most probable speed (V_{mp}); average speed (V_{av})

a. $A = V_{mp}, B = V_{av}, C = V_{rms}$

b. $A = V_{mp}, B = V_{rms}, C = V_{av}$

c. $A = V_{av}, B = V_{rms}, C = V_{mp}$

d. $A = V_{rms}, B = V_{mp}, C = V_{av}$

Answer: a

Solution:

$C_{RMS} = \sqrt{\frac{3RT}{M}}$

$C_{Average} = \sqrt{\frac{8RT}{\pi M}}$

$C_{MPS} = \sqrt{\frac{2RT}{M}}$

$\sqrt{3} > \sqrt{\frac{8}{\pi}} > \sqrt{2}$

$C_{RMS} > C_{Average} > C_{MP}$

17. For the reaction,
$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$
The observed rate expression is, rate = $k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is:
a. $k_b[N_2][H_2O]^2$
b. $k_b[N_2][H_2O]$
c. $k_b[N_2][H_2O]^2/[H_2]$
d. $k_b[N_2][H_2O]^2/[NO]$

Answer: c

Solution:

The reverse reaction is:

$2H_2(g) + 2NO(g) \rightleftharpoons N_2(g) + 2H_2O(g)$

The equilibrium constant, $K_c = \frac{k_f}{k_b} = \frac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$

At equilibrium, rate of forward reaction=rate of backward reaction

i.e. , $k_f[NO]^2[H_2] = k_b \frac{[N_2][H_2O]^2}{[H_2]}$

Hence, rate of reverse reaction=$k_b \frac{[N_2][H_2O]^2}{[H_2]}$

18. Within each pair of elements F & Cl, S & Se and Li & Na, respectively, the elements that release more energy upon an electron gain are:
    a. Cl, Se and Na
    b. Cl, S and Li
    c. F, S and Li
    d. F, Se and Na

Answer: b

Solution:

Element	First Electron gain enthalpy(kJ/mol)
Li	-60
Na	-53
F	-320
S	-200
Cl	-340
Se	-195

Despite F being more electronegative than Cl, due to the small size of F, Cl would have a more negative value of electron gain enthalpy because of inter-electronic repulsions.

As we go down the group, the negative electron gain enthalpy decreases.

19. Among the following statements A-D, the incorrect ones are:
    A. Octahedral Co(III) complexes with strong field ligands have high magnetic moments
    B. When $\Delta_o < P$, the d- electron configuration of Co(III) in an octahedral complex is t_{2g}^4, e_g^2 .
    C. Wavelength of light absorbed by $[Co(en)_3]^{3+}$is lower than that of $[CoF_6]^{3-}$.
    D. If the $\Delta_o$for an octahedral complex of Co(III) is 18000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be16000 cm^{-1} .

    a. B and C only
    c. A and D only
    b. A and B only
    d. C and D only

Answer: c

Solution:

Co^{3+} has d^6 electronic configuration. In the presence of strong field ligand, $\Delta_o > P$. Thus the splitting occurs as: t_{2g}^6, e_g^0; so the magnetic moment is zero.

According to the spectrochemical series, en is a stronger ligand than F and therefore promotes pairing. This implies that the Δ_o of en is more than the Δ_o of F.

$$\Delta_o = \frac{hc}{\lambda_{abs}}$$

$$\Delta_t = \frac{4}{9}\Delta_o = 8000\ cm^{-1}$$

20. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by:
    a. 200 mL of 0.2 N HCl
    b. 200 mL of 0.4 N HCl
    c. 100 mL of 0.1 N HCl
    d. 100 mL of 0.2 N HCl

Answer: d

Solution:

Moles of urea= $\left(\frac{0.6}{60}\right)$= 0.01

$$NH_2CONH_2 + 2NaOH \rightarrow Na_2CO_3 + 2NH_3$$

0.01 0.02

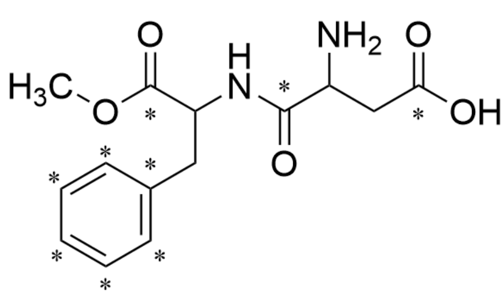
0.02 moles of NH_3 reacts with 0.02 moles of HCl.

Moles of HCl in option a= $0.2 \times \frac{100}{1000} = 0.02$

21. Number of sp^2 hybrid carbon atoms present in aspartame is ____.

Answer: 9

Solution:



The marked carbons are sp^2 hybridised.

22. 3 grams of acetic acid is added to250 mL of 0.1 M HCl and the solution is made up to 500 mL. To 20 mL of this solution $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of this solution is_____.
(Given: log 3 = 0.4771, pK_a of acetic acid = 4.74, molar mass of acetic acid = 60 g/mole).

Answer: 5.22

Solution:

mmole of acetic acid in 20 mL = 2
mmole of HCl in 20 mL = 1
mmole of NaOH = 2.5

HCl	$+ NaOH$	$\rightarrow NaCl$	$+ H_2O$
1	2.5	-	-
-	1.5	1	1

CH_3COOH	$+$ NaOH (remaining)	$\longrightarrow CH_3COONa$	+ water
2	1.5	-	-
0.5	0	1.5	

$$pH = pK_a + \log\frac{1.5}{0.5} = 4.74 + \log 3 = 4.74 + 0.48 = 5.22$$

23. The flocculation value of HCl for As_2S_3 sol is 30 mmolL[-1]. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 mL required for the above purpose is ____.

Answer: 0.3675 g

Solution:

For 1L sol 30 mmol of HCl is required
For 1L sol 15 mmol of H_2SO_4 is required
For 250 mL of sol,
$\frac{15}{4} \times 98 \times 10^{-3}$ g of H_2SO_4= 0.3675 g

24. Consider the following reactions :
$NaCl + K_2Cr_2O_7 + H_2SO_4 \rightarrow$ (A) + side products
(A) + NaOH $\rightarrow$ (B) + side products
(B) + H_2SO_4(dil.) + $H_2O_2 \rightarrow$ (C) + side products

The sum of the total number of atoms in one molecule of (A), (B) & (C) is ______. .

Answer: 18

Solution:

$$NaCl \xrightarrow{K_2Cr_2O_7/Conc.H_2SO_4} CrO_2Cl_2 \xrightarrow{NaOH} Na_2CrO_4 + NaCl$$

$$Na_2CrO_4 \xrightarrow{Dil.H_2SO_4} Na_2Cr_2O_7 \xrightarrow{Dil.H_2O_2} CrO_5$$

$(A) = CrO_2Cl_2, (B) = Na_2CrO_4$ and $(C) = CrO_5$

25. The standard heat of formation ($\Delta_f H_{298}°$) of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are -1560, -393.5 and -286 kJ/mol, respectively, is______.

Answer: -192.5 kJ/mol

Solution:

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)\ \ \Delta H_c° = -286$ kJ/mol……………………………….(1)

$H_2(g) + 0.5O_2(g) \rightarrow H_2O(l)\ \ \Delta H_c° = -393.5$ kJ/mol ……………………….......(2)

$C_2H_6(g) + 3.5O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)\ \Delta H_c° = -1560$ kJ/mol …. (3)

$2C_{(graphite)} + 3H_2(g) \rightarrow C_2H_6(g)$; $\Delta H_f° = ?$

By inverting (3) and multiplying (1) by 2 and (2) by 3 and adding, we get,

$2 \times (-286) + 3 \times (-393.5) - (-1560) = -192.5$ kJ/mol

JEE Main 2020 Paper

Date of Exam: **7th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. If $3x+4y=12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$, for some $a \in R$ then the distance between the foci of the ellipse is :

   a. $2\sqrt{5}$
   b. $2\sqrt{7}$
   c. $2\sqrt{2}$
   d. 4

   Answer: (b)
   Solution:
   $3x+4y=12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$

   Equation of tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ is $y=mx+\sqrt{a^2m^2+9}$

   Now, $3x+4y=12\sqrt{2} \Rightarrow y=-\frac{3}{4}x+3\sqrt{2}$

   $\Rightarrow m=-\frac{3}{4}$ and $\sqrt{a^2m^2+9}=3\sqrt{2}$

   $\Rightarrow a^2\left(-\frac{3}{4}\right)^2+9=18$

   $\Rightarrow a^2 \times \frac{9}{16}=9$

   $\Rightarrow a^2=16 \Rightarrow a=4$

   $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

   Distance between foci is $2ae=2\times 4\times\frac{\sqrt{7}}{4}=2\sqrt{7}$

2. Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$ then $A \subseteq C$ " is :

   a. If $A \subseteq C$, then $B \subset A$ or $D \subset B$
   b. If $A \nsubseteq C$, then $A \subseteq B$ and $B \subseteq D$
   c. If $A \nsubseteq C$, then $A \nsubseteq B$ and $B \subseteq D$
   d. If $A \nsubseteq C$, then $A \nsubseteq B$ or $B \nsubseteq D$

   Answer: (d)
   Solution:
   Given statements: $A \subseteq B$ and $B \subseteq D$
   Let $A \subseteq B$ be p
   $B \subseteq D$ be q
   $A \subseteq C$ be r

Modified statement: $(p \wedge q) \Rightarrow r$
Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$
$\sim r \Rightarrow (\sim p \vee \sim q)$
$\therefore$ $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

3. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$ is :

a. 420
b. 330
c. 210
d. 120

Answer: (b)
Solution:

Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$

Applying sum of terms of G.P. = $\dfrac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{\left(1-\frac{x}{1+x}\right)} = (1+x)^{11} - x^{11}$

Coefficient of $x^7 \Longrightarrow {}^{11}C_7 = 330$

4. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

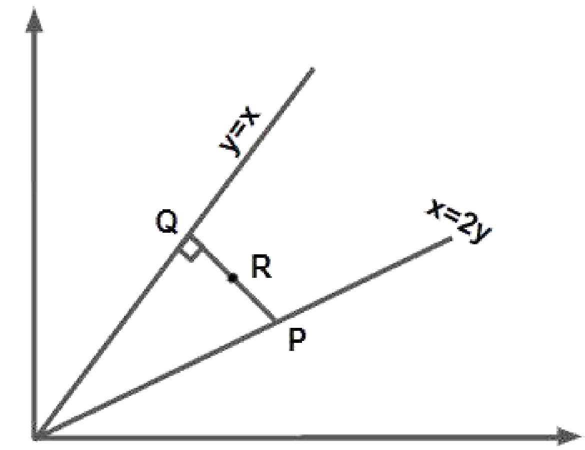
a. $\frac{17}{2}$
b. 4
c. $\frac{17}{4}$
d. $\frac{17}{8}$

Answer: (d)
Solution:
$P(\text{machine being faulty}) = p = \frac{1}{4}$
$\therefore q = \frac{3}{4}$
P(at most two machines being faulty) = P(zero machine being faulty)+P(one machine being faulty)+P(two machines being faulty)
$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$
$= q^5 + 5pq^4 + 10p^2q^3$
$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4}\left(\frac{3}{4}\right)^4 + 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$
$= \left(\frac{3}{4}\right)^3\left[\frac{9}{16} + \frac{15}{16} + \frac{10}{16}\right]$
$= \left(\frac{3}{4}\right)^3 \times \frac{34}{16} = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$
$\therefore k = \frac{17}{8}$

5. The locus of mid points of the perpendiculars drawn from points on the line $x = 2y$ to the line $x = y$ is :

a. $2x - 3y = 0$
b. $3x - 2y = 0$
c. $5x - 7y = 0$
d. $7x - 5y = 0$

Answer: (c)
Solution:



Let R be the midpoint of PQ
PQ is perpendicular on line $y = x$
$\therefore$ Equation of the line PQ can be written as $y = -x + c$
$y = -x + c$ intersects $y = x$ at Q: $\left(\frac{c}{2}, \frac{c}{2}\right)$
$y = -x + c$ intersects $x = 2y$ at P: $\left(\frac{2c}{3}, \frac{c}{3}\right)$
$\therefore$ Midpoint R: $\left(\frac{7c}{12}, \frac{5c}{12}\right)$
Locus of R : $x = \frac{7c}{12}$, $y = \frac{5c}{12}$
$\Rightarrow 5x - 7y = 0$

6. The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|} \, dx = 5$, is :

a. $\log_e 2$
b. $\log_e \sqrt{2}$
c. $\log_e \left(\frac{4}{3}\right)$
d. $\log_e \left(\frac{3}{2}\right)$

Answer: (a)

Solution:

$$4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$$

$$\Rightarrow 4\alpha\left[\int_{-1}^{0} e^{-\alpha|x|}dx + \int_{0}^{2} e^{-\alpha|x|}dx\right] = 5$$

$$\Rightarrow 4\alpha\left[\int_{-1}^{0} e^{\alpha x}\,dx + \int_{0}^{2} e^{-\alpha x}\,dx\right] = 5$$

$$\Rightarrow 4\alpha\left[\left(\frac{1-e^{-\alpha}}{\alpha}\right) + \left(\frac{e^{-2\alpha}-1}{-\alpha}\right)\right] = 5$$

$\Rightarrow 4[1 - e^{-2\alpha} - e^{-\alpha} + 1] = 5$

Let $e^{-\alpha} = t$

$\Rightarrow\ 4t^2 + 4t - 3 = 0$

$\Rightarrow\ t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \log_e 2$

7. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 +$ ….. is (102)m, then m is equal to :

a. 10
b. 25
c. 5
d. 20

Answer: (d)
Solution:

$S = \underline{3+4} + \underline{8+9} + 13 + 14 + \ldots\ldots$ 40 terms

$S = 7 + 17 + 27 + 37 + \cdots \ldots\ldots .20$ terms

$$S = \frac{20}{2}[14 + (19)10] = 20 \times 102$$

$\therefore m = 20$

8. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0,2\pi]$, is a real number, then the argument of $\sin\theta + i\cos\theta$ is :

a. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
b. $-\tan^{-1}\left(\frac{3}{4}\right)$
c. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
d. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: (a)
Solution:

Let $z = \frac{3+i\sin\theta}{4-i\cos\theta} \times \frac{4+i\cos\theta}{4+i\cos\theta}$

$$= \frac{12 - \sin\theta\cos\theta + i(4\sin\theta + 3\cos\theta)}{16 + \cos^2\theta}$$

z is real.

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\Rightarrow \tan\theta = -\frac{3}{4}$ [θ lies in 2^{nd} quadrant]

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

9. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

a. $\frac{1}{9}$

b. $\frac{1}{81}$

c. $\frac{1}{3}$

d. 3

Answer: (c)
Solution:
$b_{ij} = (3)^{(i+j-2)} a_{ji}$

$$B = \begin{bmatrix} 3^0 a_{11} & 3a_{21} & 3^2 a_{31} \\ 3a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{21} & 3^2 a_{31} \\ 3a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{vmatrix}$$

Taking 3^2 common each from C_3 and R_3

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{21} & a_{31} \\ 3a_{12} & 3^2 a_{22} & 3a_{32} \\ a_{13} & 3a_{23} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 and R_2

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

Given $|B| = 81$

$\Rightarrow 81 = 81(9)|A| \Rightarrow |A| = \frac{1}{9}$

10. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0} \left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

a. $f(1) - 4f(-1) = 4$
b. $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.
c. f is an odd function.
d. $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

Answer: (d)
Solution:
Given $\lim_{x \to 0} \left(2 + \frac{f(x)}{x^3}\right) = 4$

$\lim_{x \to 0} \frac{f(x)}{x^3} = 2$

$\lim\limits_{x\to 0}\frac{f(x)}{x^3}$ Limit exists and it is finite

$\therefore f(x) = ax^5 + bx^4 + cx^3$

$\Rightarrow \lim\limits_{x\to 0}(ax^2 + bx + c) = 2$

$c = 2$

Also $f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$f'(1) = 5a + 4b + 6 = 0$

$f'(-1) = 5a - 4b + 6 = 0$

$b = 0, \quad a = -\frac{6}{5}$

$f(x) = -\frac{6}{5}x^5 + 2x^3 \quad \Rightarrow f(x)$ is odd

$f'(x) = -6x^4 + 6x^2$

$f''(x) = -24x^3 + 12x$

$(f''(1) < 0) \qquad (f''(-1) > 0)$

At $x = -1$ there is local minima and at $x = 1$ there is local maxima.

And $f(1) - 4f(-1) = 4$

11. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

a. 4
b. 6
c. 2
d. 3

Answer: (a)

Solution:

Using ${}^{36}C_{r+1} = \frac{36}{r+1} \times {}^{35}C_r$

$\frac{36}{r+1} \times {}^{35}C_r \times (k^2 - 3) = {}^{35}C_r \times 6$

$k^2 - 3 = \frac{r+1}{6}$

$k^2 = \frac{r+1}{6} + 3$

$k \in I$

$r \rightarrow$ Non-negative integer $0 \leq r \leq 35$

$r = 5 \Rightarrow k = \pm 2$

$r = 35 \Rightarrow k = \pm 3$

No. of ordered pairs $(r, k) = 4$

12. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to :

a. 171
b. $\frac{511}{3}$
c. -171
d. -513

Answer: (c)
Solution:
$a_1 + a_2 = 4 \Rightarrow a + ar = 4 \Rightarrow a(1+r) = 4$
$a_3 + a_4 = 16 \Rightarrow ar^2 + ar^3 = 16 \Rightarrow ar^2(1+r) = 16 \Rightarrow 4r^2 = 16$
$\Rightarrow r = \pm 2$
If $r = 2, a = \frac{4}{3}$ which is not possible as $a_1 < 0$
If $r = -2, a = -4$
$\sum_{i=1}^{9} a_i = \frac{a(r^9-1)}{r-1} = \frac{(-4)[(-2)^9-1]}{-3} = \frac{4}{3}(-512-1) = 4(-171)$
$\lambda = -171$

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair $(\lambda, \vec{d})$ is equal to :

a. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$
b. $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$
c. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$
d. $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

Answer: (c)
Solution:
Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\quad \lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$
$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \overrightarrow{|0|}^2$
$\overrightarrow{|a|}^2 + \overrightarrow{|b|}^2 + \overrightarrow{|c|}^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$
$\lambda = -\frac{3}{2}$
Also $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$
$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \left(-\vec{a} - \vec{b}\right) + \left(-\vec{a} - \vec{b}\right) \times \vec{a}$
$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$
$\Rightarrow \vec{d} = 3\left(\vec{a} \times \vec{b}\right) = 3\vec{a} \times \vec{b}$

14. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$
This curve intersects the $x -$ axis at a point whose abscissa is :

a. $2 + e$
b. 2
c. $2 - e$
d. $-e$

Answer: (c)
Solution:
$$(y^2 - x)\frac{dy}{dx} = 1$$
$$\frac{dx}{dy} + x = y^2$$
$xe^y = \int y^2 e^y dy$
$x = y^2 - 2y + 2 + ce^{-y}$
Given $y(0) = 1$

$\Rightarrow c = -e$

$\therefore$ Solution is $x = y^2 - 2y + 2 - e^{-y+1}$

$\therefore$ The value of x where the curve cuts the x − axis will be at $x = 2 - e$

15. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to :

a. $\frac{2\pi}{3}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{3} + \frac{1}{6}$

d. $\frac{\pi}{9}$

Answer: (b)

Solution:

$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0,2\pi)$

$\Rightarrow 2\mathrm{cosec}^2\theta - 2 - 5\mathrm{cosec}\,\theta + 4 = 0$

$\Rightarrow 2\mathrm{cosec}^2\theta - 4\mathrm{cosec}\,\theta - \mathrm{cosec}\,\theta + 2 = 0$

$\Rightarrow \mathrm{cosec}\,\theta = 2$ or $\frac{1}{2}$ (Not possible)

As $\theta \in [0,2\pi)$,

$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1+\cos 6\theta)}{2} \, d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + \frac{\sin 6\theta}{12}\Bigg|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$

16. Let α and β are the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k, k \geq 1$ then which one of the following statements is not true?

a. $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
b. $p_5 = 11$
c. $p_5 = p_2 \cdot p_3$
d. $p_3 = p_5 - p_4$

Answer: (c)

Solution:

Given α, β are the roots of $x^2 - x - 1 = 0$

$\Rightarrow \alpha + \beta = 1$ & $\alpha\beta = -1$

$\Rightarrow \alpha^2 = \alpha + 1$ & $\beta^2 = \beta + 1$

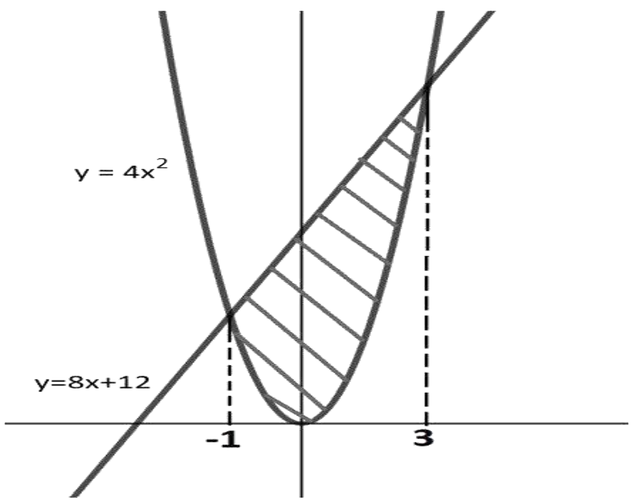
$p_k = \alpha^{k-2}\alpha^2 + \beta^{k-2}\beta^2$

$p_k = \alpha^{k-2}(\alpha + 1) + \beta^{k-2}(\beta + 1)$
$p_k = \alpha^{k-1} + \beta^{k-1} + \alpha^{k-2} + \beta^{k-2}$
$\Rightarrow p_k = p_{k-1} + p_{k-2}$
$\Rightarrow p_3 = p_2 + p_1 = 4$
$p_4 = p_3 + p_2 = 7$
$p_5 = p_4 + p_3 = 11$
$\therefore p_5 \neq p_2 \cdot p_3$ & $p_1 + p_2 + p_3 + p_4 + p_5 = 26$
& $p_3 = p_5 - p_4$

17. The area (in sq. units) of the region $\{(x, y) \epsilon R | 4x^2 \leq y \leq 8x + 12\}$ is :

a. $\frac{125}{3}$
b. $\frac{128}{3}$
c. $\frac{124}{3}$
d. $\frac{127}{3}$

Answer: (b)
Solution:



For point of intersection,
$4x^2 = 8x + 12$
$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x = 3, -1$
Area bounded is given by
$A = \int_{-1}^{3} (8x + 12 - 4x^2) dx$
$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$
$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$
$A = 44 - \frac{4}{3} = \frac{128}{3}$

18. The value of c in Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, where $x \in [0,1]$ is :

a. $\frac{4-\sqrt{7}}{3}$

b. $\frac{2}{3}$

c. $\frac{\sqrt{7}-2}{3}$

d. $\frac{4-\sqrt{5}}{3}$

Answer: (a)
Solution:
LMVT is applicable on $f(x)$ in $[0,1]$, therefore it is continuous and differentiable in $[0,1]$
Now, $f(0) = 11, f(1) = 16$
$f'(x) = 3x^2 - 8x + 8$
$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$
$\Rightarrow 3c^2 - 8c + 8 = 5$
$\Rightarrow 3c^2 - 8c + 3 = 0$
$\Rightarrow c = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$
As $c \in (0,1)$
We get, $c = \frac{4-\sqrt{7}}{3}$

19. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to :

a. $-\frac{\sqrt{5}}{2}$

b. $\frac{\sqrt{5}}{2}$

c. $-\frac{\sqrt{5}}{4}$

d. $\frac{2}{\sqrt{5}}$

Answer: (a)
Solution:
$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$
Differentiating w.r.t. x on both the sides, we get:
$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$
$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$
$\Rightarrow y'\left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$
Putting $x = \frac{1}{2}, y = -\frac{1}{4}$
$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}}\right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2}+\frac{1}{2\sqrt{15}}\right]=-\frac{1}{4\sqrt{3}}-\frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{45}+1}{2\sqrt{15}}\right]=-\frac{1+\sqrt{45}}{4\sqrt{3}}$

$\Rightarrow y'=-\frac{\sqrt{5}}{2}$

20. Let the tangents drawn from the origin to the circle, $x^2+y^2-8x-4y+16=0$ touch it at the points A and B. The $(AB)^2$ is equal to :

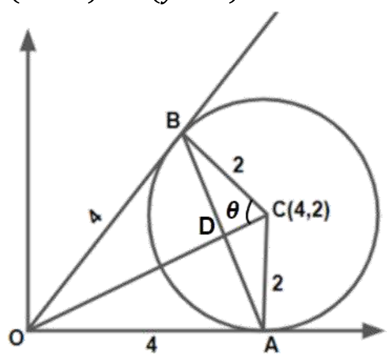
a. $\frac{32}{5}$

b. $\frac{64}{5}$

c. $\frac{52}{5}$

d. $\frac{56}{5}$

Answer: (b)
Solution:
$x^2+y^2-8x-4y+16=0$
$(x-4)^2+(y-2)^2=4 \Rightarrow$ Centre $(4,2)$, radius $=2$



$OA=4=OB$
In ΔOBC
$tan\,\theta=\frac{4}{2}=2 \Rightarrow sin\,\theta=\frac{2}{\sqrt{5}}$
In ΔBDC
$sin\,\theta=\frac{BD}{2} \Rightarrow BD=\frac{4}{\sqrt{5}}$
Length of chord of contact $(AB)=\frac{8}{\sqrt{5}}$

Alternative
(l) length of tangent = 4 and (r) radius =2
$\Rightarrow$Length of chord of contact = $\frac{2lr}{\sqrt{(l^2+r^2)}}$
Square of length of chord of contact = $\frac{64}{5}$

21. If system of linear equations
$x+y+z=6$
$x+2y+3z=10$
$3x+2y+\lambda z=\mu$
has more than two solutions, then $\mu-\lambda^2$ is equal to _______.

Answer: (13)
Solution:
The system of equations has more than 2 solutions
$\therefore\ D=D_3=0$

$$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 3 & 2 & \lambda\end{vmatrix}=0\ \Rightarrow 2\lambda-6-\lambda+9+2-6=0$$

$\Rightarrow \lambda=1$

$$\begin{vmatrix}1 & 1 & 6\\ 1 & 2 & 10\\ 3 & 2 & \mu\end{vmatrix}=0 \Rightarrow 2\mu-20-\mu+30-24=0$$

$\Rightarrow \mu=14$
So, $\mu-\lambda^2=13$

22. If the foot of perpendicular drawn from the point $(1,0,3)$ on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to _________.

Answer: (4)
Solution:
Given points $P(1,0,3)$ and $Q\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

Direction ratios of line L: $\left(\alpha-\frac{5}{3},7-\frac{7}{3},1-\frac{17}{3}\right)$

$=\left(\frac{3\alpha-5}{3},\frac{14}{3},-\frac{14}{3}\right)$

Direction ratios of PQ: $\left(-\frac{2}{3},-\frac{7}{3},-\frac{8}{3}\right)$

As line L is perpendicular to PQ

So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right)+\left(\frac{14}{3}\right)\left(-\frac{7}{3}\right)+\left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right)=0$

$\Rightarrow -6\alpha+10-98+112=0 \Rightarrow 6\alpha=24\ \Rightarrow \alpha=4$

23. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by

$$f(x)=\begin{cases}\left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) & when \quad x\neq 0\\ k\ , & when \quad x=0\end{cases}$$

is continuous, the k is equal to _______.

Answer: (5)
Solution:

As $f(x)$ is continuous

$\Rightarrow \lim_{x\to 0} f(x) \quad = f(0) = k$

$\Rightarrow \lim_{x\to 0} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) = k$

$$\Rightarrow \lim_{x\to 0} \frac{3\log(1+3x)}{3x} + \lim_{x\to 0} \frac{(-2)\log(1-2x)}{(-2x)} = k$$

$\Rightarrow 3 + 2 = k \ \Rightarrow k = 5$

24. If the mean and variance of eight numbers $3, 7, 9, 12, 13, 20, x$ and y be 10 and 25 respectively then xy is equal to _______.

Answer: (54)
Solution:

Mean $= 10 \Rightarrow \frac{64+x+y}{8} = 10$

$\Rightarrow x + y = 16$

Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow 25 = \frac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} - 100$

$\Rightarrow 1000 = 852 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 148$

$\Rightarrow (x+y)^2 - 2xy = 148$

$\Rightarrow 256 - 2xy = 148$

So, $xy = 54$

25. Let $X = \{n \in \boldsymbol{N}: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of 2$\}$ and $B = \{n \in X: n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _______.

Answer: (29)
Solution:

$A = \{$x: x is multiple of 2$\} = \{2,4,6,8, \dots\}$

$B = \{$x: x is multiple of 7$\} = \{7,14,21, \dots\}$

$X = \{x : 1 \le x \le 50, x \in \mathbf{N}\}$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50.

P = {x: x is a multiple of 2 less than or equal to 50}

Q = {x: x is a multiple of 7 less than or equal to 50}

$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$= 25 + 7 - 3$

$= 29$