

**JEE – ADVANCED– PAPER – 2****CODE : 0****Time : 3 Hours.****Maximum Marks : 240****READ THE INSTRUCTIONS CAREFULLY****GENERAL :**

1. The sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet, verify that the booklet contains 32 pages and that all the 60 questions along with the options are legible.

**QUESTION PAPER FORMAT AND MARKING SCHEME :**

8. The question paper has parts : Physics, Chemistry and Mathematics. Each part has three sections.
9. Carefully read the instructions given at the beginning of each section.
10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).  
Marking Scheme : +4 for correct answer and 0 in all other cases.
11. Section 2 contains 8 multiple choice questions with one or more than one correct option.  
Marking Scheme : +4 for correct answer, 0 if not attempted and –2 in all other cases.
12. Section 3 contains 2 paragraph type questions. Each paragraph describes an experiment, a situation or a problem. Two multiple choice questions will be asked based on this paragraph. One or more than one option can be correct.  
Marking scheme : +4 for correct answer, 0 if not attempted and –2 in all other cases.

**OPTICAL RESPONSE SHEET :**

13. The ORS consists of an original (top sheet) and its carbon-less copy (bottom sheet).
14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.
15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.
16. You will be allowed to take away the carbon-less copy at the end of the examination.
17. Do not tamper with or mutilate the ORS.
18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

# PHYSICS

## SECTION : 1 (Maximum Marks : 32)

- This section contains EIGHT questions
- The answer to each question is a SINGLE DIGIT NUMBER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:  
 +4 If the bubble corresponding to the answer is darkened.  
 0 In all other cases.

1. The densities of two solid spheres A and B of the same radii  $R$  vary with radial distance  $r$  as  $\rho_A(r) = k\left(\frac{r}{R}\right)$  and  $\rho_B(r) = k\left(\frac{r}{R}\right)^5$ , respectively, where  $k$  is a constant. The moments of inertia of the individual spheres about axes passing through their centres are  $I_A$  and  $I_B$ , respectively. If  $\frac{I_B}{I_A} = \frac{n}{10}$ , the value of  $n$  is

Ans. 6

Sol: 
$$I_A = \int_0^R \frac{2}{3} \left( \frac{Kr}{R} \cdot 4\pi r^2 dr \right) \cdot r^2 = \frac{8\pi K R^5}{18}$$

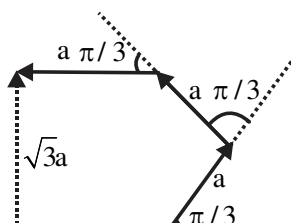
$$I_B = \int_0^R \frac{2}{3} \left( \frac{Kr^5}{R^5} \cdot 4\pi r^2 dr \right) \cdot r^2 = \frac{8\pi K R^5}{30}$$

Now  $\frac{I_B}{I_A} = \frac{18}{30} = \frac{6}{10}$   
 $\therefore n = 6$

2. Four harmonic waves of equal frequencies and equal intensities  $I_0$  have phase angles  $0, \pi/3, 2\pi/3$  and  $\pi$ . When they are superposed, the intensity of the resulting wave is  $nI_0$ . The value of  $n$  is

Ans. 3

Sol: From the given diagram  
 Resultant amplitude is  $\sqrt{3}a$   
 $\therefore$  Resultant intensity  $= 3I_0$



3. For a radioactive material, its activity  $A$  and rate of change of its activity  $R$  are defined as  $A = -\frac{dN}{dt}$  and  $R = -\frac{dA}{dt}$ , where  $N(t)$  is the number of nuclei at time  $t$ . Two radioactive sources  $P$  (mean life  $\tau$ ) and  $Q$  (mean life  $2\tau$ ) have the same activity at  $t = 0$ . Their rates of change of activities at  $t = 2\tau$  are  $R_P$  and  $R_Q$ , respectively. If  $\frac{R_P}{R_Q} = \frac{n}{e}$ , then the value of  $n$  is

Ans. 2

Sol:  $N = N_0 e^{-\lambda t}$

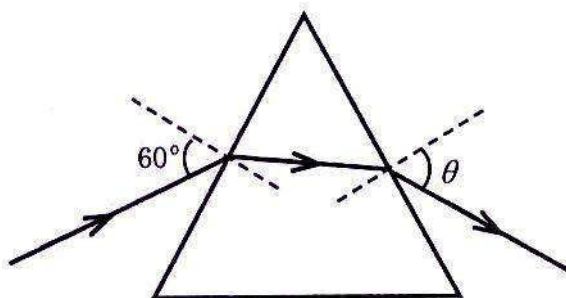
$$\therefore A = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} \text{ \& }$$

$$\text{ \& } R = -\frac{dA}{dt} = \lambda^2 N_0 e^{-\lambda t}$$

$$\text{Now } \frac{R_P}{R_Q} = \frac{\lambda_P^2 (N_0)_P e^{-\lambda_P t}}{\lambda_Q^2 (N_0)_Q e^{-\lambda_Q t}} = \left( \frac{\lambda_P}{\lambda_Q} \right)^2 \times \frac{1}{2} e^{(\lambda_Q - \lambda_P)t} = \frac{2}{e}$$

$$\therefore n = 2$$

4. A monochromatic beam of light is incident at  $60^\circ$  on one face of an equilateral prism of refractive index  $n$  and emerges from the opposite face making an angle  $\theta(n)$  with the normal (see the figure. For  $n = \sqrt{3}$  the value of  $\theta$  is  $60^\circ$  and  $\frac{d\theta}{dn} = m$ . The value of  $m$  is



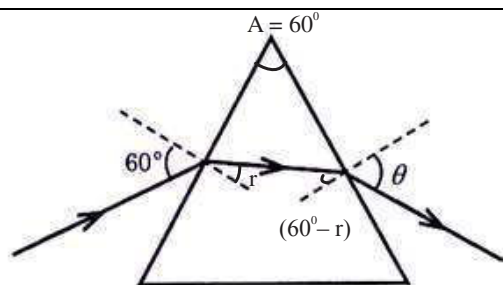
Ans. 2

Sol: Using Snell's law at the

$$\frac{\sin 60^\circ}{\sin r} = n = \frac{\sin \theta}{\sin (60 - r)}$$

$$2n \sin r = \sqrt{3} \quad \dots (1) \quad n \sin (60 - r) = \sin \theta \quad \dots (2)$$

$$\sin r = \frac{\sqrt{3}}{2n}$$



$$\cos r = \frac{\sqrt{4n^2 - 3}}{2n}$$

Putting values of  $\sin r$  &  $\cos r$  in equation (2)

$$n \times \left\{ \frac{\sqrt{3}}{2} \times \frac{\sqrt{4n^2 - 3}}{2n} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2n} \right\} = \sin \theta$$

$$\sqrt{3} \times \sqrt{4n^2 - 3} - \sqrt{3} = 4 \sin \theta$$

differentiate both sides w.r.t.  $n$

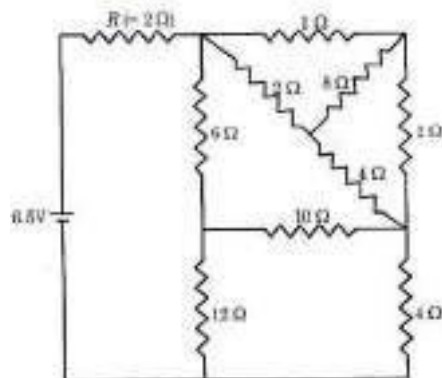
$$\sqrt{3} \times \frac{2n}{2\sqrt{4n^2 - 3}} = 4 \cos \theta \cdot \frac{d\theta}{dn}$$

$$\frac{d\theta}{dn} = \frac{\sqrt{3}n}{\cos \theta \sqrt{4n^2 - 3}}$$

Putting  $n = \sqrt{3}$  &  $\theta = 60^\circ$

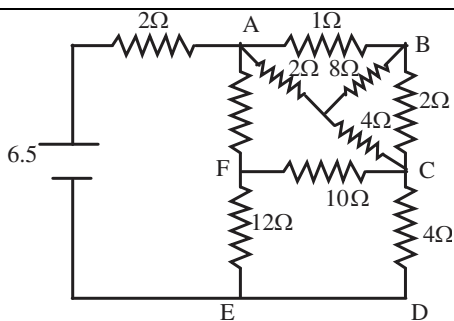
$$\frac{d\theta}{dn} = \frac{\sqrt{3} \times \sqrt{3}}{\frac{1}{2} \sqrt{4 \times 3 - 3}} = \frac{6}{\sqrt{9}} = 2$$

5. In the following circuit, the current through the resistor  $R (= 2 \Omega)$  is  $I$  Amperes. The value of  $I$  is

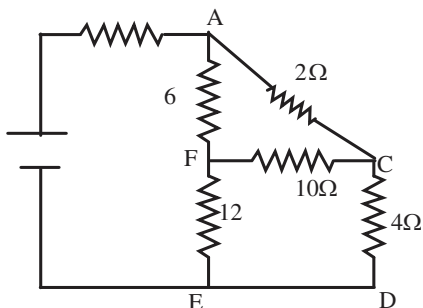


Ans. 1

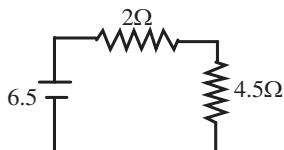
Sol:



∴ ABCA is a balanced wheatstone's bridge of equivalent resistance  $2\ \Omega$ . So



∴ ACDEFA is also a balanced wheatstone's bridge of equivalent resistance  $4.5\ \Omega$ . So



Hence the current is 1 ampere

6. An electron in an excited state of  $\text{Li}^{2+}$  ion has angular momentum  $3h/2\pi$ . The de Broglie wavelength of the electron in this state is  $p\pi a_0$  (where  $a_0$  is the Bohr radius). The value of  $p$  is

**Key. (2)**

Sol. Given  $mvr = \frac{3h}{2\pi}$

$$mv = \frac{3h}{2\pi r}$$

$$\therefore \lambda_{\text{debroglie}} = \frac{h}{mv} = \frac{h \times 2\pi r}{3h} = 2\pi \frac{r}{3} \quad \dots(i)$$

Now radius of the electron in 3<sup>rd</sup> orbit of  $\text{Li}^{2+}$  ion

$$r = \frac{a_0 n^2}{Z} = \frac{a_0 3^2}{3} = 3a_0 \quad \dots(ii)$$

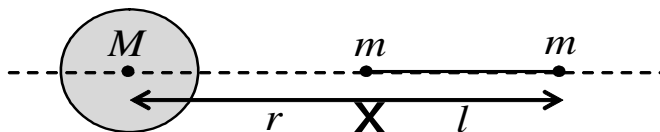
∴ Putting  $r$  from (ii) in (i)

$$\lambda_{\text{debroglie}} = 2\pi \frac{3a_0}{3} = 2\pi a_0$$

$$\therefore p = 2.$$

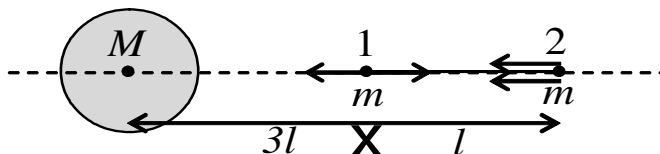
7. A large spherical mass  $M$  is fixed at one position and two identical point masses  $m$  are kept on a line passing through the centre of  $M$  (see figure). The point masses are connected by a rigid massless rod of length  $l$  and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to  $M$  is at a distance  $r = 3l$  from  $M$ , the tension in the rod is zero for  $m = k \left( \frac{M}{288} \right)$ .

The value of  $k$  is



**Key. (7)**

Sol. For Tension in rod to be zero forces on both  $m$ 's should be equal



Force on mass - 1 = Force on mass - 2

$$\Rightarrow \frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = \frac{GMm}{16l^2} + \frac{Gm^2}{l^2}$$

$$\Rightarrow m = \frac{7}{288} M$$

$$\therefore K = 7.$$

8. The energy of a system as a function of time  $t$  is given as  $E(t) = A^2 \exp(-\alpha t)$ , where  $\alpha = 0.2s^{-1}$ . The measurement of  $A$  has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of  $E(t)$  at  $t = 5$  s is

**Key. (4)**

Sol.  $E = A^2 e^{-\alpha t}$

Taking Log on both sides and differentiating we get  $\frac{dE}{E} = 2 \frac{dA}{A} - \alpha dt$

Maximum % error

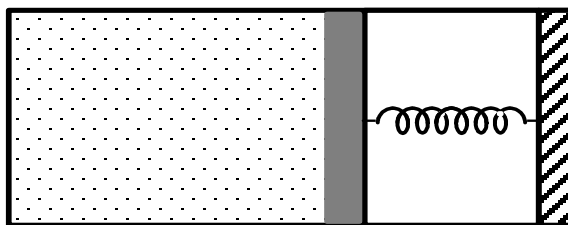
$$\begin{aligned} \frac{dE}{E} \times 100 &= \left[ 2 \frac{dA}{A} \times 100 + \alpha t \frac{dt}{t} \times 100 \right] \\ &= [2 \times 1.25 + 0.2 \times 5 \times 1.5] \\ &= 2.5 + 1.5 \\ &= 4 \% \end{aligned}$$

Ans : 4.

## SECTION : 2 (Maximum Marks : 32)

- This section contains EIGHT questions
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:  
 +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
 0 If one of the bubbles is darkened.  
 –2 In all other cases.

9. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature  $T_1$ , pressure  $P_1$  and volume  $V_1$  and the spring is in its relaxed state. The gas is then heated very slowly to temperature  $T_2$ , pressure  $P_2$  and volume  $V_2$ . During this process the piston moves out by a distance  $x$ . Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



- (A) If  $V_2 = 2V_1$  and  $T_2 = 3T_1$ , then the energy stored in the spring is  $\frac{1}{4}P_1 V_1$
- (B) If  $V_2 = 2V_1$  and  $T_2 = 3T_1$ , then the change in internal energy is  $3P_1 V_1$
- (C) If  $V_2 = 3V_1$  and  $T_2 = 4T_1$ , then the work done by the gas is  $\frac{7}{3}P_1 V_1$
- (D) If  $V_2 = 3V_1$  and  $T_2 = 4T_1$ , then the heat supplied to the gas is  $\frac{17}{6}P_1 V_1$

**Key. (B)**

Sol.  $kx_1 = P_1 A$

$$k(x_1 + x) = P_2 A$$

$$kx = (P_2 - P_1) A$$

$$\frac{1}{2} kx^2 = \frac{1}{2} (P_2 - P_1) A \cdot x$$

$$= \frac{1}{2} \left( \frac{3}{2} P_1 - P_1 \right) (\Delta v)$$

$$= \frac{1}{2} \left( \frac{P_1}{2} \right) (v_1)$$

$$\frac{P_1 v_1}{4}$$

$$\Delta U = \frac{nf}{2} R \Delta T = \frac{3}{2} n R \Delta T$$

$$\Delta W = \frac{1}{2}(kx) \times x = \frac{1}{2}(P_2 - P_1) \times \frac{\Delta V}{A} = \frac{1}{2} P_2 \Delta V$$

$$\frac{P_2}{P_1} = \frac{nRT_2}{nRT_1} \Rightarrow P_2 = \frac{T_2}{T_1} P_1$$

When  $V_2 = 2V_1$  and  $T_2 = 3T_1$

$$\text{Energy stored in the spring} = \frac{1}{2} P_2 \Delta V$$

$$= \frac{1}{2} \times \frac{3T_1}{T_1} \times P_1 \times V_1 = \frac{3}{2} P_1 V_1$$

$$\Delta U = \frac{3}{2} nR \times 2T_1 = 3nRT_1 = 3P_1 V_1$$

When  $V_2 = 3V_1$ ,  $T_2 = 4T_1$

$$\text{Work done} = \frac{1}{2} P_2 \Delta V = \frac{1}{2} \times \frac{T_2}{T_1} P_1 \times 2V_1$$

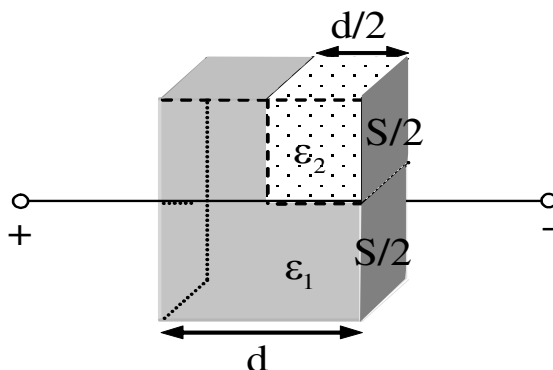
$$= \frac{1}{2} \times \frac{4T_1}{T_1} \times P_1 \times 2V_1 = 4P_1 V_1$$

$$\Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2} nR \times 3T_1 = \frac{9}{2} nRT_1 = \frac{9}{2} P_1 V_1$$

$$\Delta Q = \Delta U + \Delta W = \frac{9}{2} P_1 V_1 + 4P_1 V_1 = \frac{17}{2} P_1 V_1$$

∴ Correct option is (B).

10. A parallel plate capacitor having plates of area  $S$  and plate separation  $d$ , has capacitance  $C_1$  in air. When two dielectrics of different relative permittivities ( $\epsilon_1 = 2$  and  $\epsilon_2 = 4$ ) are introduced between the two plates as shown in the figure, the capacitance becomes  $C_2$ . The ratio  $\frac{C_2}{C_1}$  is



(A) 6/5

(B) 5/3

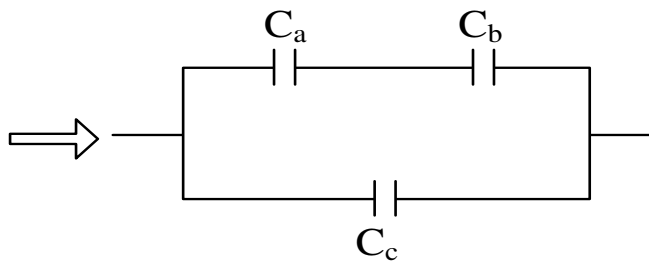
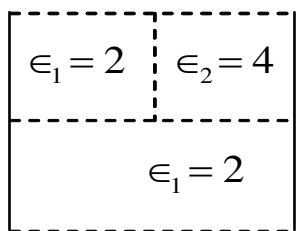
(C) 7/5

(D) 7/3

Key. (D)

Capacitance of the capacitor is  $C_1 = \frac{\epsilon_0 A}{d}$





After the insertion of di-electrics

$$\text{Where } C_a = \frac{2\epsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{2\epsilon_0 A}{d}$$

$$C_b = \frac{4\epsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{4\epsilon_0 A}{d}$$

$$C_c = \frac{2\epsilon_0 \frac{A}{2}}{d} = \frac{\epsilon_0 A}{d}$$

$$\text{So, } C_2 = (C_a \text{ } \square \text{ } C_b) \text{ } \square \text{ } C_c$$

$$= \frac{C_a C_b}{C_a + C_b} + C_c$$

$$= \frac{7\epsilon_0 A}{3d}$$

$$\therefore \frac{C_2}{C_1} = \frac{7}{3}$$

$\therefore$  (D).

11. A spherical body of radius  $R$  consists of a fluid of constant density and is in equilibrium under its own gravity. If  $P(r)$  is the pressure at  $r$  ( $r < R$ ), then the correct option (s) is (are)

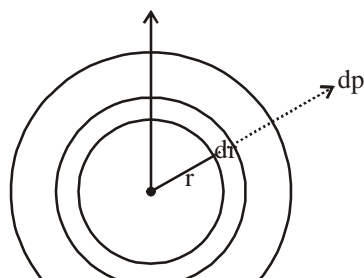
- (A)  $P(r=0) = 0$  (B)  $\frac{P(r=3R/4)}{P(r=2R/3)} = \frac{63}{80}$   
 (C)  $\frac{P(r=3R/5)}{P(r=2R/5)} = \frac{16}{21}$  (D)  $\frac{P(r=R/2)}{P(r=R/3)} = \frac{20}{27}$

**Ans. (B, C)**

$$\text{Sol: } \int dp = \int_r^R \frac{G \cdot m \, dm}{r^2 \cdot 4\pi r^2}$$

$$P = G \int \frac{p \times \frac{4}{3} \pi r^3 \times 4\pi r^2 dr}{r^2 \cdot 4\pi r^2}$$

$$P = GP \frac{4}{3} \pi \int_r^R \frac{r^5}{r^4} dr$$



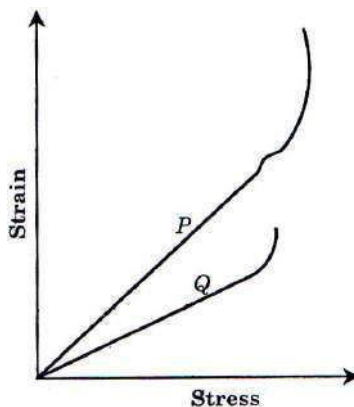
of  $r$ )

$$P\left(r = \frac{3R}{5}\right) = G\rho \frac{4}{3}\pi \left[ R^2 - \frac{9R^2}{25} \right] = \frac{16R^2}{21R^2} = \frac{16}{21}$$

$$P\left(r = \frac{2R}{5}\right) = G\rho \frac{4}{3}\pi \left[ R^2 - \frac{4R^2}{25} \right]$$

$$\frac{P_r\left(\frac{3R}{4}\right)}{P_r\left(\frac{2R}{3}\right)} = \left[ \frac{R^2 - \frac{9R}{16}}{R^2 - \frac{4R^2}{9}} \right] = \frac{7R^2 \times 9}{16 \times 5R^2} = \frac{63}{80}$$

12. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement (s) is (are)



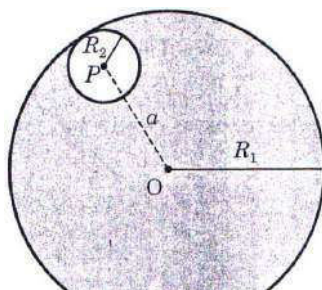
- (A) P has more tensile strength than Q  
 (B) P is more ductile than Q  
 (C) P is more brittle than Q  
 (D) The Young's modulus of P is more than that of Q

Ans. (A, B)

- Sol: (A) Correct from graph plastic range of P is more than Q  
 (B) Correct from graph plastic range of P is more than Q  
 (C) Q is more brittle because plastic range of P is more than Q  
 (D) Young's modulus 'Y' =  $\frac{\text{stress}}{\text{strain}}$

So, Y (Young's modulus) of Q is more than P

13. Consider a uniform spherical charge distribution of radius  $R_1$  centred at the origin O. In this distribution, a spherical cavity of radius  $R_2$ , centred at P with distance  $OP = a = R_1 - R_2$  (see figure) is made. If the electric field inside the cavity at position  $\vec{r}$  is  $\vec{E}(\vec{r})$ , then the correct statement (s) is (are)



- (A)  $\vec{E}$  is uniform, its magnitude is independent of  $R_2$  but its direction depends on  $\vec{r}$   
 (B)  $\vec{E}$  is uniform, its magnitude depends on  $R_2$  and its direction depends on  $\vec{r}$   
 (C)  $\vec{E}$  is uniform, its magnitude is independent of  $a$  but its direction depends on  $\vec{a}$   
 (D)  $\vec{E}$  is uniform and both its magnitude and direction depends on  $\vec{a}$

Ans. (D)

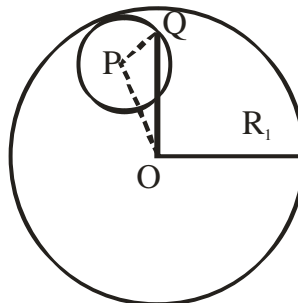
Sol: Field at Q due to whole sphere  $\vec{E}_1 = \frac{\rho \vec{OQ}}{3\epsilon_0}$

Field at Q due to removed sphere  $\vec{E}_2 = \frac{\rho \vec{PQ}}{3\epsilon_0}$

Hence Net field at O

$$\begin{aligned}\vec{E} &= \vec{E}_1 - \vec{E}_2 \\ &= \frac{\rho}{3\epsilon_0} (\vec{OQ} - \vec{PQ}) \\ &= \frac{\rho}{3\epsilon_0} \vec{OP}\end{aligned}$$

$$\boxed{\vec{E} = \frac{\rho}{3\epsilon_0} \vec{a}}$$



14. In terms of potential difference  $V$ , electric current  $I$ , permittivity  $\epsilon_0$ , permeability  $\mu_0$  and speed of light  $c$ , the dimensionally correct equation(s) is (are)

- (A)  $\mu_0 I^2 = \epsilon_0 V^2$  (B)  $\epsilon_0 I^2 = \mu_0 V$   
 (C)  $I = \epsilon_0 c V$  (D)  $\mu_0 c I = \epsilon_0 V$

Ans. (A,C)

Sol: Use these formula to find  $\mu_0$ ,  $\epsilon_0$  &  $V$

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \frac{I_1 I_2}{r} \quad \text{for } \mu_0 \Rightarrow \mu_0 = [MLA^{-2}T^{-2}]$$

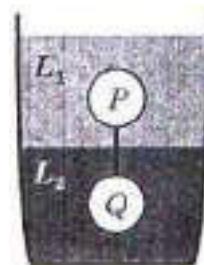
$$F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{for } \epsilon_0 \Rightarrow \epsilon_0 = [M^{-1}L^{-3}A^2T^4]$$

$$V = \frac{W}{q} \Rightarrow V = [ML^2 A^{-1} T^{-3}]$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Substituting above data with all the choices we get (C) is correct. Rest all are incorrect

15. Two spheres P and Q of equal radii have densities  $\rho_1$  and  $\rho_2$  respectively. The spheres are connected by a massless string and placed in liquid  $L_1$  and  $L_2$  of densities  $\sigma_1$  and  $\sigma_2$  and viscosities  $\eta_1$  and  $\eta_2$ , respectively. They float in equilibrium with the sphere P and  $L_1$  and sphere Q in  $L_2$  and the string being taut (see figure). If sphere P alone in  $L_2$  has terminal velocity  $\vec{V}_P$  and Q alone in  $L_1$  has terminal velocity  $\vec{V}_Q$ , then



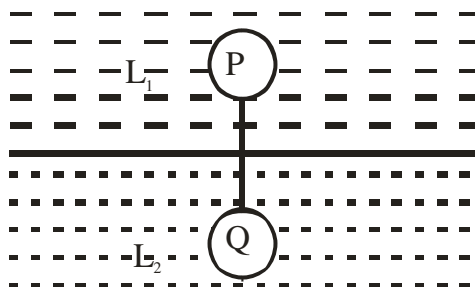
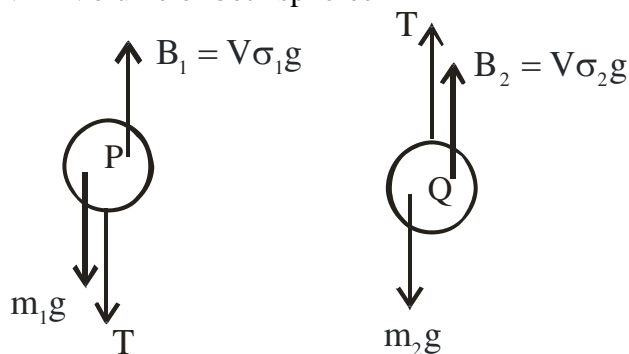
- (A)  $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$  (B)  $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$

(C)  $\vec{V}_P \cdot \vec{V}_Q > 0$

(D)  $\vec{V}_P \cdot \vec{V}_Q < 0$

Ans. (A, D)

Sol.


 $V = \text{Volume of both spheres}$ 


$$V\sigma_1 g = V\rho_1 g + T \quad \dots(i)$$

$$- \quad V\rho_2 g = V\sigma_2 g + T \quad \dots(ii)$$

$$\frac{V(\sigma_1 - \rho_2)g = V(\rho_1 - \sigma_2)g}{\Rightarrow (\sigma_1 - \rho_2) = (\rho_1 - \sigma_2)} \quad \dots(iii)$$

$$\vec{V}_P = \frac{2}{9} \frac{r^2 (\rho_1 - \sigma_2)}{\eta_2} g \quad \dots(iv)$$

$$\vec{V}_Q = \frac{2}{9} \frac{r^2 (\rho_2 - \sigma_1)}{\eta_1} g \quad \dots(v)$$

$$\frac{\vec{V}_P}{\vec{V}_Q} = \frac{(\rho_1 - \sigma_2) \eta_1}{(\rho_2 - \sigma_1) \eta_2}$$

$$\frac{\vec{V}_P}{\vec{V}_Q} = (-1) \left( \frac{\eta_1}{\eta_2} \right) \Rightarrow \vec{V}_P \text{ \& \; } \vec{V}_Q \text{ are opposite}$$

$$\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2} \quad \vec{V}_P \cdot \vec{V}_Q < 0$$

16. A fission reaction is given by  ${}_{92}^{236}\text{U} \rightarrow {}_{54}^{140}\text{Xe} + {}_{38}^{94}\text{Sr} + x + y$ ; where  $x$  and  $y$  are two particles.

Considering  ${}_{92}^{236}\text{U}$  to be at rest, the kinetic energies of the products are denoted by

$K_{\text{Xe}}$ ,  $K_{\text{Sr}}$ ,  $K_x$  (2 MeV) and  $K_y$  (2 MeV), respectively. Let the binding energies per nucleon

of  ${}_{92}^{236}\text{U}$ ,  ${}_{54}^{140}\text{Xe}$  and  ${}_{38}^{94}\text{Sr}$  be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering

- (A)  $x = n, y = n, K_{Sr} = 129 \text{ MeV}, K_{Xe} = 86 \text{ MeV}$
- (B)  $x = p, y = e^-, K_{Sr} = 129 \text{ MeV}, K_{Xe} = 86 \text{ MeV}$
- (C)  $x = p, y = n, K_{Sr} = 129 \text{ MeV}, K_{Xe} = 86 \text{ MeV}$
- (D)  $x = n, y = n, K_{Sr} = 86 \text{ MeV}, K_{Xe} = 129 \text{ MeV}$

**KEY: (A)**

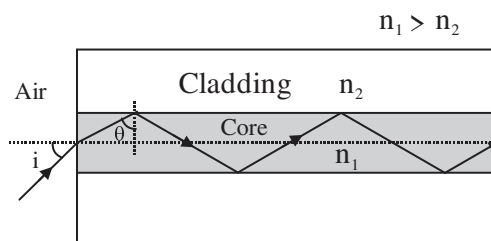
**SOL:**

**SECTION : 3 (Maximum Marks : 16)**

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:  
**+4** If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
**0** If one of the bubbles is darkened.  
**-2** In all other cases.

**Paragraph – 1**

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index  $n_1$  surrounded by a medium of lower refractive index  $n_2$ . The light guidance in the structure takes place due to successive total internal reflections at the interface of the media  $n_1$  and  $n_2$  as shown in the figure. All rays with the angle of incidence  $i$  less than a particular values  $i_m$  are confined in the medium of refractive index  $n_1$ . The numerical aperture (NA) of the structure is defined as  $\sin i_m$ .



- 17. For two structures namely  $S_1$  with  $n_1 = \sqrt{45}/4$  and  $n_2 = 3/2$ , and  $S_2$  with  $n_1 = 8/5$  and  $n_2 = 7/5$  and taking the refractive index of water to be  $4/3$  and that of air to be 1, the correct option(s) is (are)**
- (A) NA of  $S_1$  immersed in water is the same as that of  $S_2$  immersed in a liquid of refractive index  $\frac{16}{3\sqrt{15}}$
- (B) NA of  $S_1$  immersed in liquid of refractive index  $\frac{6}{\sqrt{15}}$  is the same as that of  $S_2$  immersed in water

(C) NA of  $S_1$  placed in air is the same as that of  $S_2$  immersed in liquid of refractive index

$\frac{4}{\sqrt{15}}$  (D) NA of  $S_1$  placed in air is the same as that of  $S_2$  placed in water.

**KEY: (A,C)**

SOL:  $n \sin i_m = n_1 \sin r_m$

$$r_m = 90 - \theta$$

$$\text{So, } \sin i_m = \frac{n_1}{n} \times \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

Put the value and check the answer

example

for  $S_1$

$$\sin i_m = \frac{\frac{\sqrt{45}}{4}}{\frac{4}{3}} \sqrt{1 - \frac{\frac{9}{4}}{\frac{45}{16}}} = \frac{9}{16}$$

For  $S_2$

$$\sin i_m = \frac{\frac{8}{5}}{\frac{16}{3\sqrt{15}}} \sqrt{1 - \frac{\frac{49}{25}}{\frac{64}{25}}} = \frac{9}{16}$$

18. If two structures of same cross-sectional area, but different numerical apertures  $NA_1$  and  $NA_2$  ( $NA_2 < NA_1$ ) are joined longitudinally, the numerical aperture of the combined structure is

(A)  $\frac{NA_1 NA_2}{NA_1 + NA_2}$       (B)  $NA_1 + NA_2$       (C)  $NA_1$       (D)  $NA_2$

**KEY: (D)**

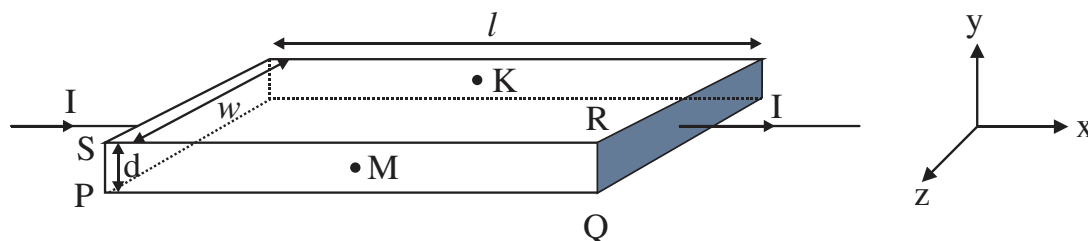
SOL: conceptual

Paragraph -2

In a thin rectangular metallic strip a constant current  $I$  flows along the positive  $x$ -direction, as shown in the figure. The length, width and thickness of the strip are  $l$ ,  $w$  and  $d$ , respectively.

A uniform magnetic field  $\vec{B}$  is supplied on the strip along the positive  $y$ -direction. Due to this, the charge carriers experience a net deflection along the  $z$ -direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the  $z$ -direction is thus

electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are  $w_1$  and  $w_2$  and thickness are  $d_1$  and  $d_2$  respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure).  $V_1$  and  $V_2$  are the potential differences between K and M in strips 1 and 2 respectively. Then, for a given current  $I$  flowing through them in a given magnetic field strength  $B$ , the correct statement(s) is (are)

- (A) If  $w_1 = w_2$  and  $d_1 = 2d_2$ , then  $V_2 = 2V_1$   
 (B) If  $w_1 = w_2$  and  $d_1 = 2d_2$ , then  $V_2 = V_1$   
 (C) If  $w_1 = 2w_2$  and  $d_1 = d_2$ , then  $V_2 = 2V_1$   
 (D) If  $w_1 = 2w_2$  and  $d_1 = d_2$ , then  $V_2 = V_1$

**KEY: (A,D)**

$$\text{SOL: } V_d B = E \Rightarrow \frac{I}{ne(Wd)} B = \frac{V}{W}$$

$$\Rightarrow V \propto \frac{1}{d}$$

$\therefore$  independent of 'w'

Also  $Vd = \text{constant}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{d_1}{d_2}$$

$\therefore$  correct options are (A) and (D)

20. Consider two different metallic strips (1 and 2) of same dimensions (length  $l$ , width  $w$  and thickness  $d$ ) with carrier densities  $n_1$  and  $n_2$  respectively. Strips 1 is placed in magnetic field  $B_1$  and strips 2 is placed in magnetic field  $B_2$ , both along positive y-directions. Then  $V_1$  and  $V_2$  are the potential differences developed between K and M in strips 1 and 2 respectively. Assuming that the current  $I$  is the same for both the strips, the correct option(s) is (are)

- (A) If  $B_1 = B_2$  and  $n_1 = 2n_2$ , then  $V_2 = 2V_1$   
 (B) If  $B_1 = B_2$  and  $n_1 = 2n_2$ , then  $V_2 = V_1$

(C) If  $B_1 = B_2$  and  $n_1 = 2n_2$ , then  $V_2 = 0.5V_1$

(D) If  $B_1 = 2B_2$  and  $n_1 = n_2$ , then  $V_2 = V_1$

**KEY: (A,C)**

SOL: For same dimension and current

$$V \propto \frac{B}{n} \quad \text{and} \Rightarrow \frac{nV}{B} = \text{constant}$$

$$B_1 = B_2$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{V_2}{V_1}$$

$\therefore$  option (A) correct

For same 'n'

$$\frac{V}{B} = \text{constant}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{B_1}{B_2}$$

$\therefore$  option (C) is also correct

$\therefore$  correct options are (A) & (C)

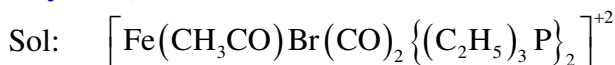


**PART II: CHEMISTRY****SECTION : 1 (Maximum Marks : 32)**

- This section contains EIGHT questions
- The answer to each question is a SINGLE DIGIT NUMBER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:  
+4 If the bubble corresponding to the answer is darkened.  
0 In all other cases.

21. In the complex acetylbromidodicarbonylbis (triethylphosphine) iron (II), the number of Fe–C bond(s) is

**Key: (3)**



Out of the given donors, only CO donates through carbon.

Hence, number of Fe – Carbon bonds = 2

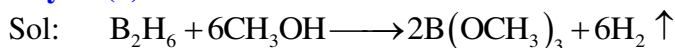
22. Among the complex ions,  $\{\text{Co}(\text{NH}_2.\text{CH}_2.\text{CH}_2.\text{NH}_2)_2\text{Cl}_2\}^+$ ,  $\{\text{CrCl}_2(\text{C}_2\text{O}_4)_2\}^{3-}$ ,  $[\text{Fe}(\text{H}_2\text{O})_4(\text{OH}_2)]^+$ ,  $[\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$ ,  $[\text{Co}(\text{NH}_2.\text{CH}_2.\text{CH}_2.\text{NH}_2)_2(\text{NH}_3)\text{Cl}]^{2+}$  and  $[\text{Co}(\text{NH}_3)_4(\text{H}_2\text{O})\text{Cl}]^{2+}$ , the number of complex ion(s) that show(s) cis-trans isomerism is

**Key: (6)**

Sol: All six complexes will exhibit Cis-trans isomerism.

23. Three moles of  $\text{B}_2\text{H}_6$  are completely reacted with methanol. The number of moles of boron containing product formed is

**Key: (6)**



One mole  $\text{B}_2\text{H}_6$  when completely reacted with methanol produces 2 moles of Boron containing product.

$\therefore$  3 moles of  $\text{B}_2\text{H}_6$  produce 6 moles of such product.

24. The molar conductivity of a solution of a weak acid HX (0.01 M) is 10 times smaller than the molar conductivity of a solution of a weak acid HY (0.10 M). If  $\lambda_{\text{X}}^0 = \lambda_{\text{Y}}^0$ , the difference in their  $\text{pK}_\text{a}$  values,  $\text{pK}_\text{a}(\text{HX}) - \text{pK}_\text{a}(\text{HY})$ , is (consider degree of ionization of both acids to be  $\ll 1$ )

**Key: (3)**

Assume HX = I

Sol:

Assume HY = II

$$\alpha_{\text{I}} = \frac{\Lambda_{\text{I}}}{\Lambda_{\text{o}}}$$

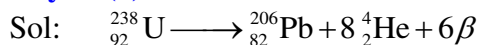
$$\alpha_{\text{II}} = \frac{\Lambda_{\text{II}}}{\Lambda_{\text{o}}}$$

$$\frac{\alpha_{\text{I}}}{\alpha_{\text{II}}} = \frac{\Lambda_{\text{I}}}{\Lambda_{\text{II}}} = \frac{1}{10}$$

$$\frac{\text{K}_{\text{aI}}}{\text{K}_{\text{aII}}} = \frac{\text{C}_{\text{I}}\alpha_{\text{I}}^2}{\text{C}_{\text{II}}\alpha_{\text{II}}^2} = \frac{0.01}{0.1} \left( \frac{1}{10} \right)^2 = 10^{-3}$$

25. A closed vessel with rigid walls contains 1 mol of  ${}^{238}_{92}\text{U}$  and 1 mol of air at 298 K. Considering complete decay of  ${}^{238}_{92}\text{U}$  to  ${}^{206}_{82}\text{Pb}$ , the ratio of the final pressure to the initial pressure of the system at 298 K is

**Key:** (9)



1 mol Uranium  $\rightarrow$  238 gives 8 mols He.

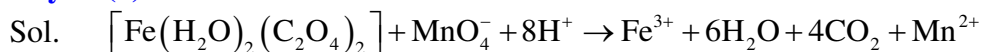
$\therefore$  Number of moles of gases = 8 + 1 = 9

Ratio of final pressure to initial pressure

$$\frac{P_f}{P_i} = \frac{n_f}{n_i} = \frac{9}{1} = 9$$

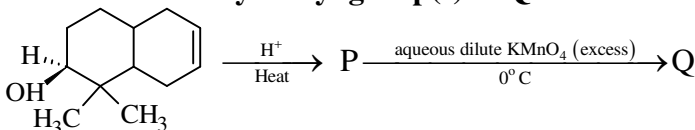
26. In dilute aqueous  $\text{H}_2\text{SO}_4$ , the complex diaquodioxalatoferrate (II) is oxidized by  $\text{MnO}_4^-$ . For this reaction, the ratio of the rate of change of  $[\text{H}^+]$  to the rate of change of  $[\text{MnO}_4^-]$  is

**Key:** (8)



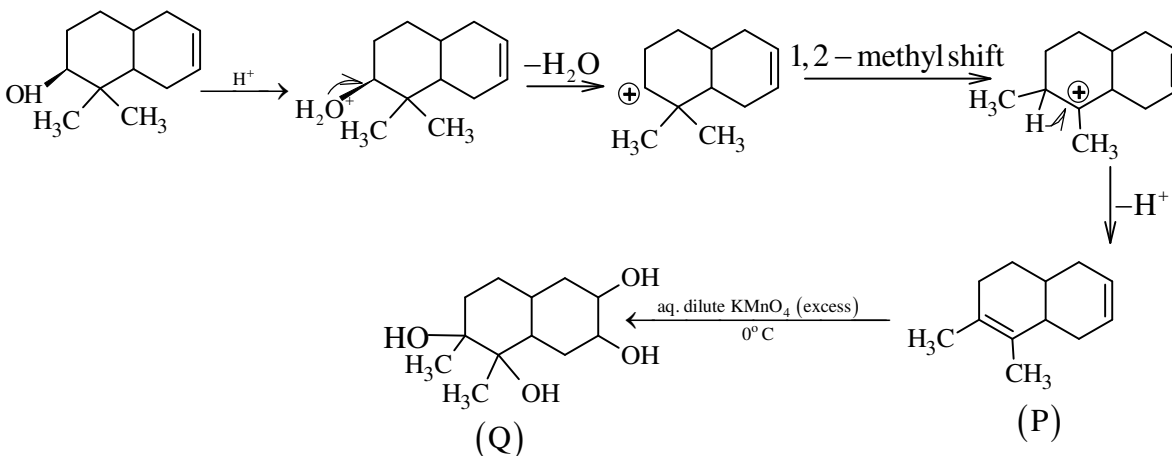
$$\therefore \frac{\text{Rate of change of } (\text{H}^+)}{\text{Rate of change } (\text{MnO}_4^-)} = \frac{8}{1} = 8$$

27. The number of hydroxyl group(s) in Q is

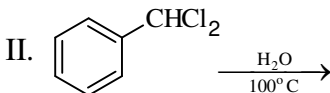
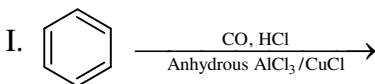


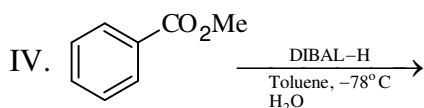
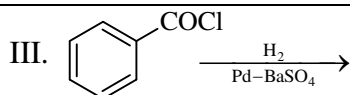
**Key:** (4)

Sol.

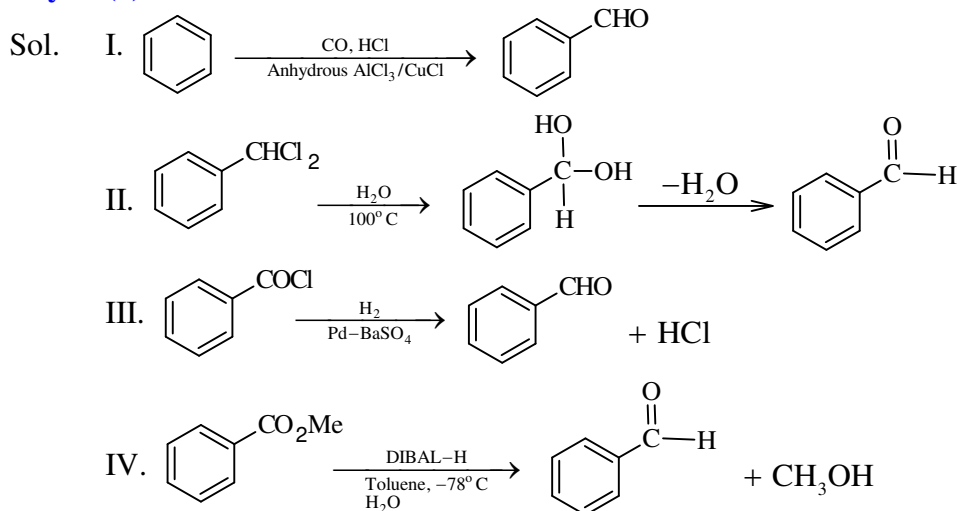


28. Among the following, the number of reaction(s) that produce(s) benzaldehyde is





**Key: (4)**



### SECTION : 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:  
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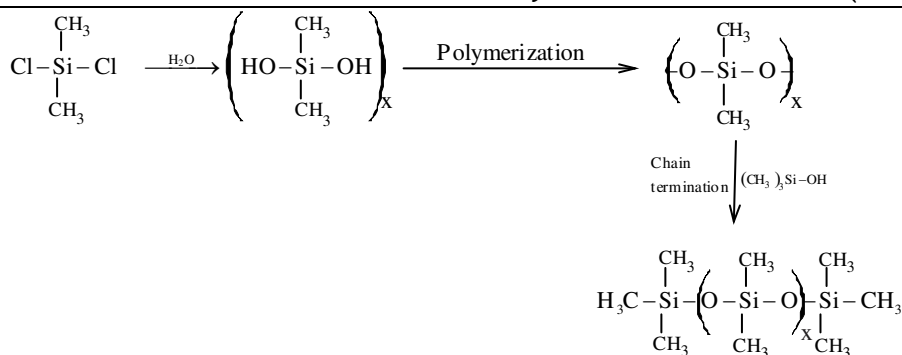
29. When  $O_2$  is adsorbed on a metallic surface, electron transfer occurs from the metal to  $O_2$ . The **TRUE** statement(s) regarding this adsorption is(are)
- (A)  $O_2$  is physisorbed (B) Heat is released  
 (C) occupancy of  $\pi_{2p}^*$  of  $O_2$  is increased (D) bond length of  $O_2$  is increased

**Key: (B), (C), (D)**

Sol. When electron is transferred from metal to  $O_2$  then the bond is ionic, so, we consider this as chemical adsorption. In adsorption,  $\Delta H = -ve$ , electron goes to  $\pi_{2p}^*$  of  $O_2$ , so, bond length increases.

30. Under hydrolytic conditions, the compounds used for preparation of linear polymer and for chain termination, respectively, are
- (A)  $CH_3SiCl_3$  and  $Si(CH_3)_4$  (B)  $(CH_3)_2SiCl_2$  and  $(CH_3)_3SiCl$   
 (C)  $(CH_3)_2SiCl_2$  and  $CH_3SiCl_3$  (D)  $SiCl_4$  and  $(CH_3)_3SiCl$

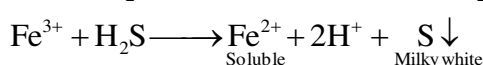
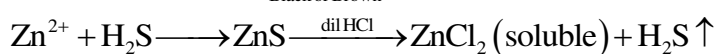
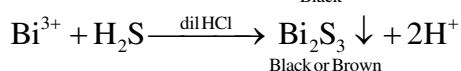
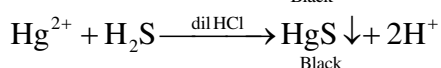
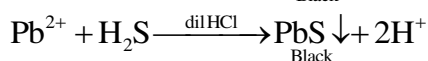
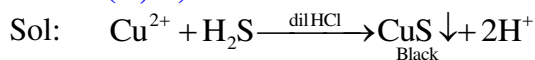
**Key: (B)**



31. The pair(s) of ions where BOTH the ions are precipitated upon passing  $\text{H}_2\text{S}$  gas in presence of dilute  $\text{HCl}$ , is (are)

- (A)  $\text{Ba}^{2+}$ ,  $\text{Zn}^{2+}$   
 (B)  $\text{Bi}^{3+}$ ,  $\text{Fe}^{3+}$   
 (C)  $\text{Cu}^{2+}$ ,  $\text{Pb}^{2+}$   
 (D)  $\text{Hg}^{2+}$ ,  $\text{Bi}^{3+}$

Ans. (C, D)



32. The correct statement(s) regarding, (i)  $\text{HClO}$ , (ii)  $\text{HClO}_2$ , (iii)  $\text{HClO}_3$  and (iv)  $\text{HClO}_4$ , is (are)

- (A) The number of  $\text{Cl}=\text{O}$  bonds in (ii) and (iii) together is two  
 (B) The number of lone pairs of electrons on  $\text{Cl}$  in (ii) and (iii) together is three  
 (C) The hybridization of  $\text{Cl}$  in (iv) is  $\text{sp}^3$   
 (D) Amongst (i) to (iv), the strongest acid is (i)

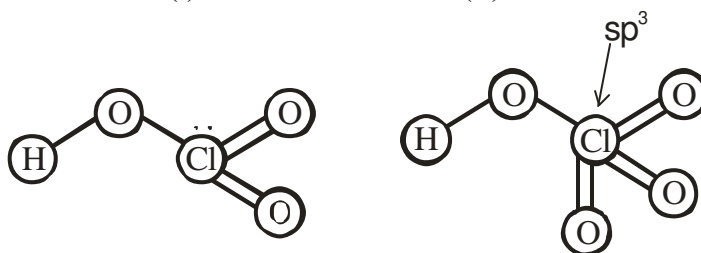
Ans. (B, C)

Sol: Sum of lone pair of electron in (2) and (3) = 3

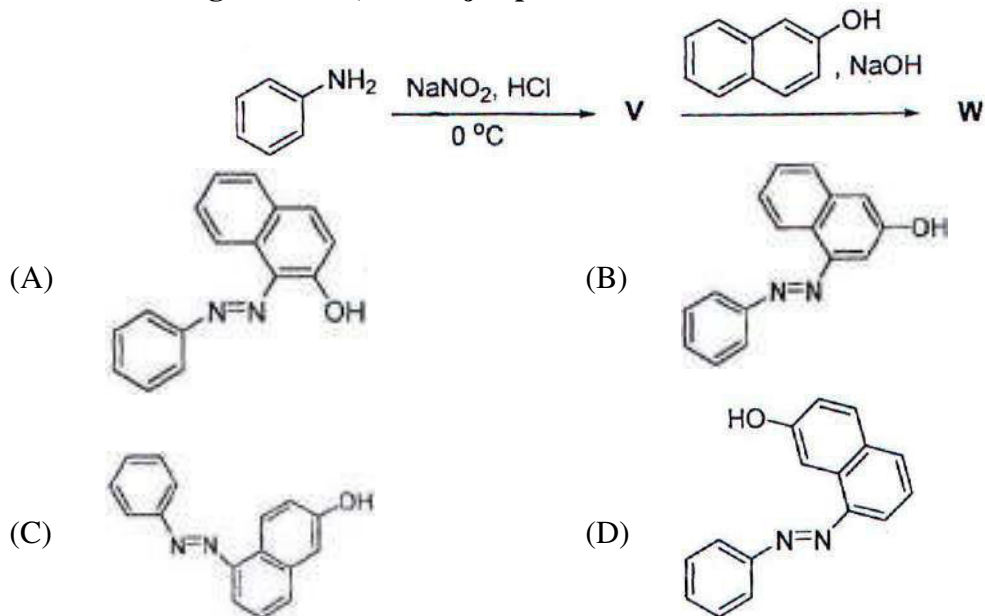


(i)

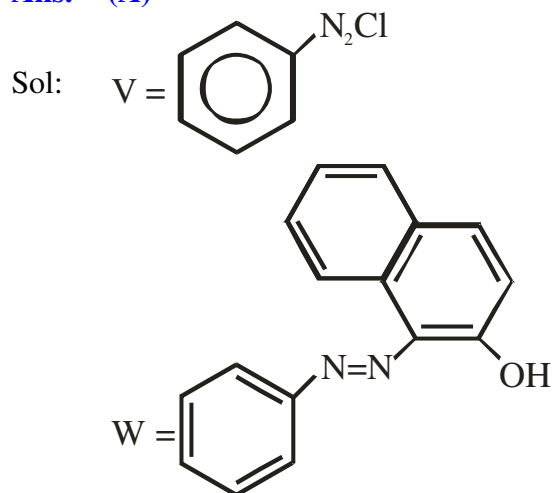
(ii)



33. In the following reactions, the major product W is

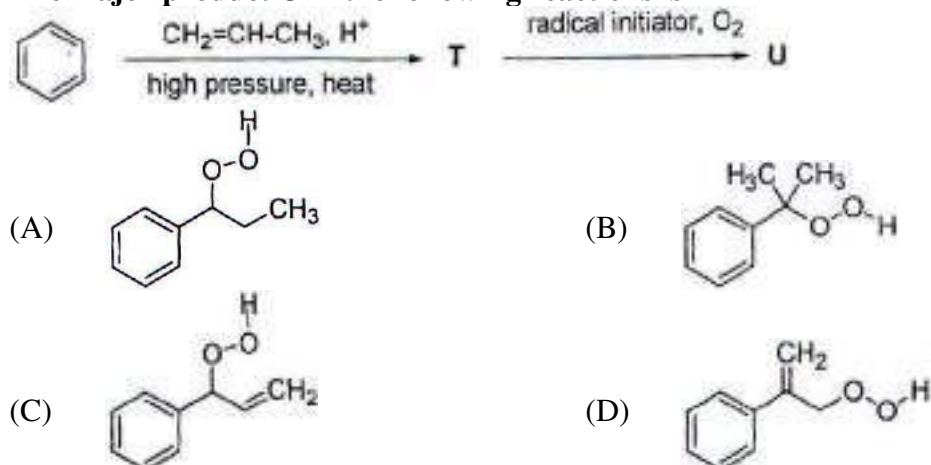


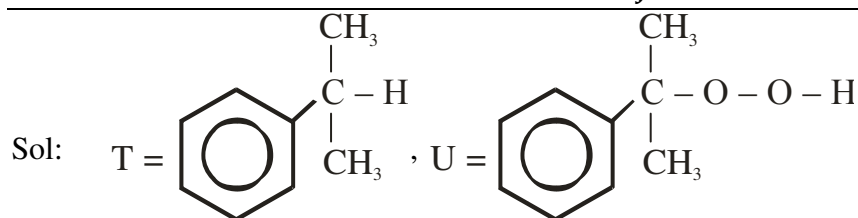
Ans. (A)



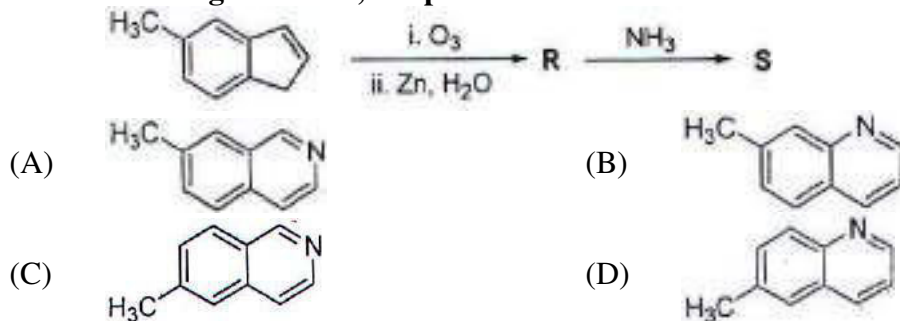
$\beta$ -naphthol favours  $E^+$  attack at ortho position due to greater stability of  $\sigma$ -complex.

34. The major product U in the following reactions is

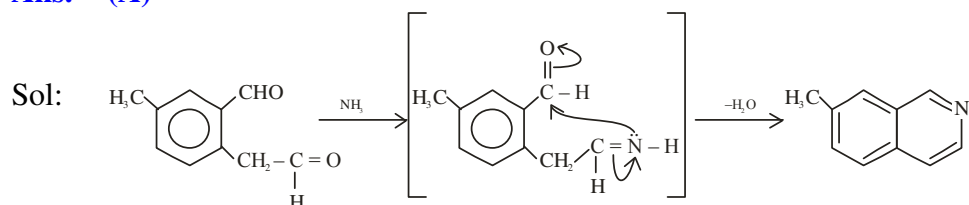




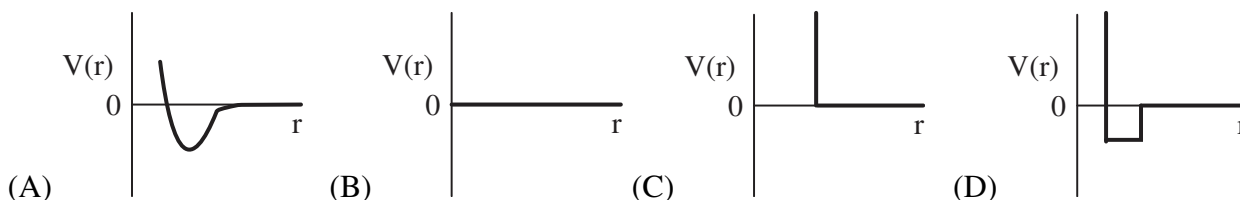
35. In the following reactions, the product S is



Ans. (A)



36. One mole of monatomic real gas satisfies the equation  $p(V - b) = RT$  where  $b$  is a constant. The relationship of interatomic potential  $V(r)$  and interatomic distance  $r$  for the gas is given by



KEY: (C)

SOL: Since the equation suggest that  $a = 0$ , hence intermolecular force of attraction is zero. In such a situation when molecules come close to each other there will be no decrease in potential energy. When the distance between the molecules is equal to the sum of their radii, the force of repulsion becomes too strong and potential energy shows a steep rise.

Hence the answer is (C)

**SECTION : 3 (Maximum Marks : 16)**

- This section contains TWO paragraphs
- Based on each paragraph, there will be TWO questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:  
 +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
 0 If one of the bubbles is darkened.  
 -2 In all other cases.

**PARAGRAPH – I**

When 100 mL of 1.0 M HCl was mixed with 100 mL of 1.0 M NaOH in an insulated beaker at constant pressure, a temperature increase of  $5.7^{\circ}\text{C}$  was measured for the beaker and its contents (Expt. 1). Because the enthalpy of neutralization of a strong acid with a strong base is a constant ( $-57.0 \text{ kJ mol}^{-1}$ ), this experiment could be used to measure the calorimeter constant. In a second experiment (Experiment 2) 100 mL of 2.0 M acetic acid ( $K_a = 2.0 \times 10^{-5}$ ) was mixed with 100 mL of 1.0 M NaOH (under identical conditions of Expt. 1) where a temperature rise of  $5.6^{\circ}\text{C}$  was measured.

(Consider heat capacity of all solutions as  $4.2 \text{ J g}^{-1} \text{ K}^{-1}$  and density of solutions as  $1.0 \text{ g mL}^{-3}$ )

37. Enthalpy of dissociation ( in  $\text{kJ mol}^{-1}$ ) of acetic acid obtained from the Expt. 2 is

- (A) 1.0                                      (B) 10.0                                      (C) 24.5                                      (D) 51.4

**KEY: (A)**

SOL: Standard enthalpy of neutralization =  $-57 \text{ kJ/mol}$

Amount of acid used = 0.1 mole

Amount of base used = 0.1 mole

Amount of heat released on neutralization =  $5.7 \text{ kJ}$

Change in temperature =  $5.7^{\circ}\text{C}$

Let the calorimeter constant be  $x$

Now,  $5700 \text{ J} = 200 \times 4.2 \times 5.7 + x \times 5.7$

$$x = 160 \text{ J}$$

For acetic acid and NaOH.

Mole of acetic acid neutralized = 0.1

Energy released in neutralization =  $200 \times 5.6 \times 4.2 + 160 \times 5.6 = 5600 \text{ J}$

Energy released per mole =  $\frac{5600}{0.1} = 56000 \text{ J} = 56 \text{ kJ}$

Enthalpy of ionization of acetic acid =  $1 \text{ kJ}$ .

Hence answer is (A)

38. The pH of the solution after Expt. 2 is

(A) 2.8

(B) 4.7

(C) 5.0

(D) 7.0

**KEY: (B)**

SOL: No. of moles of acetic acid =  $\frac{100 \times 2}{1000} = 0.2$

No. of moles of NaOH =  $\frac{100 \times 1}{1000} = 0.1$

Conc. of remaining acetic acid =  $\frac{0.1}{200} \times 1000 = 0.5$

Conc. of  $\text{CH}_3\text{COONa} = \frac{0.1}{200} \times 1000 = 0.5$

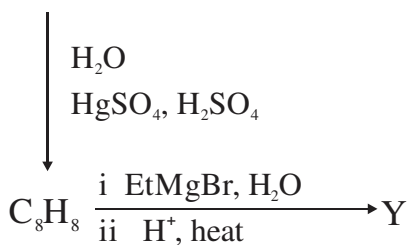
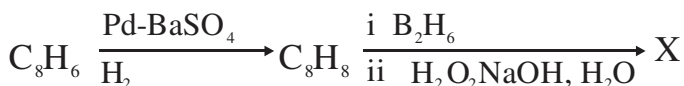
The solution is a buffer, hence

$$\text{pH} = \text{pK}_a + \log \frac{[\text{salt}]}{[\text{acid}]} = -\log [2 \times 10^{-5}] + \log \left[ \frac{0.5}{0.5} \right]$$

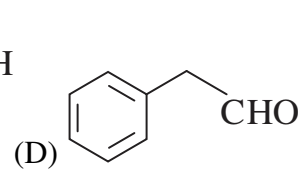
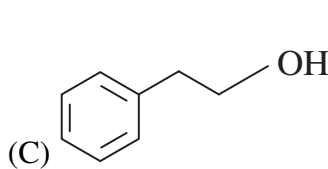
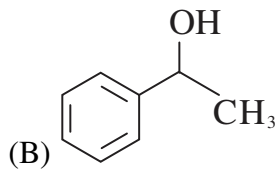
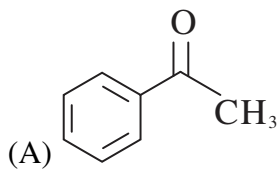
$\text{pH} = 4.7$

**PARAGRAPH – 2**

In the following reactions

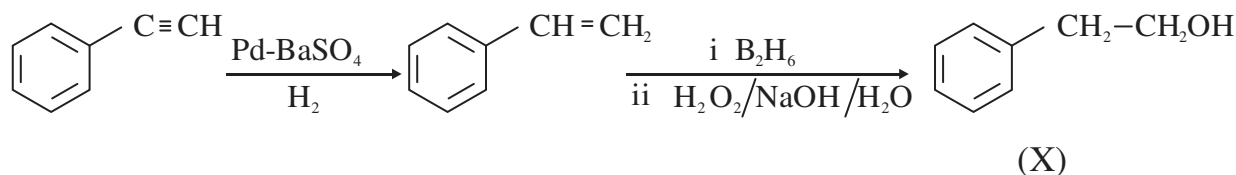


39. Compound X is



**KEY: (C)**

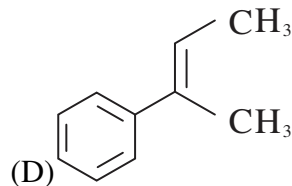
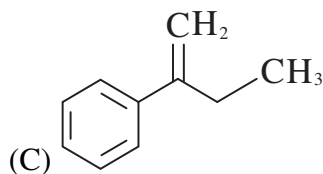
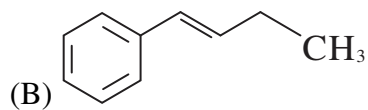
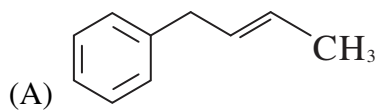
SOL:



Hence Answer is (C)

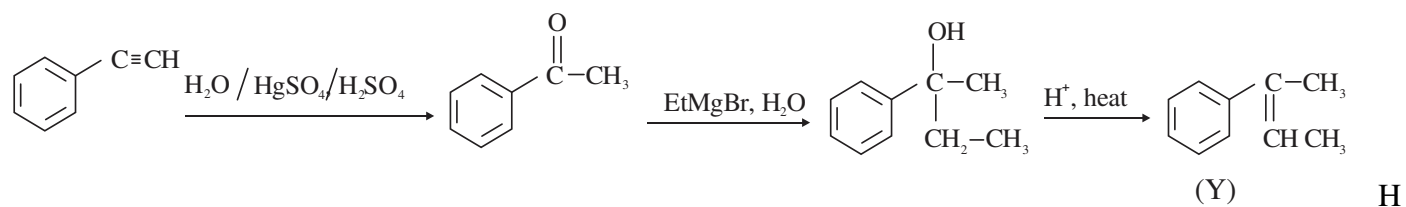


40. The major compound Y is



**KEY: (D)**

SOL:



ence Answer is (D)

**PART III: MATHEMATICS****SECTION : 1 (Maximum Marks : 32)**

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT NUMBER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:  
+4 If the bubble corresponding to the answer is darkened.  
0 In all other cases.

41. The coefficient of  $x^9$  in the expansion of  $(1+x)(1+x^2)(1+x^3) \dots (1+x^{100})$  is

Ans. 8

Sol:  $(1+x)(1+x^2)(1+x^3) \dots (1+x^{100})$

$$(1-x^{16})(1+x^3)(1+x^5)(1+x^6)(1+x^7)(1+x^9)(1-x)^{-1}$$

$$(1-x^{16})(1-x)^{-1}(1+x^3)(1+x^5)(1+x^6)(1+x^7)(1+x^9)$$

Now coefficient of  $x^9$  in

$$(1-x^{16})(1+x+x^2+x^3+x^4+\dots\infty)(1+x^3)(1+x^5)(1+x^6)(1+x^7)(1+x^9)$$

Coefficient of  $x^9$  in

$$(1-x^{16})(1+x+x^2+\dots\infty)(1+x^3+x^5+x^6+x^7+x^8+2x^9)$$

$$= 8$$

42. Suppose that the foci of the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  are  $(f_1, 0)$  and  $(f_2, 0)$  where  $f_1 > 0$  and  $f_2 < 0$ . Let  $P_1$  and  $P_2$  be two parabolas with a common vertex at  $(0, 0)$  and with foci at  $(f_1, 0)$  and  $(2f_2, 0)$ , respectively. Let  $T_1$  be a tangent to  $P_1$  which passes through  $(2f_2, 0)$  and  $T_2$  be a tangent to  $P_2$  which passes through  $(f_1, 0)$ . If  $m_1$  is the slope of  $T_1$  and  $m_2$  is the slope of  $T_2$ , then

the value of  $\left(\frac{1}{m_1^2} + m_2^2\right)$  is

Ans. 4

Sol:  $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$= 1 - \frac{5}{9}$$

$$= \frac{4}{9}$$

$$e = \frac{2}{3}$$

$$(f_1, 0) = (ae, 0) = (2, 0)$$

$$(f_2, 0) = (-ae, 0) = (-2, 0)$$

$P_1$  parabola is  $y^2 = 8x$

$P_2$  parabola is  $y^2 = -16x$

Equation of tangent to  $P_1$

$$T_1 \text{ is } y = m_1x + \frac{a}{m_1} \Rightarrow y = m_1x + \frac{2}{m_1}$$

$T_1$  passes through  $(-4, 0)$

$$0 = -4m_1 + \frac{2}{m_1}$$

$$2m_1 = \frac{1}{m_1} \Rightarrow m_1^2 = \frac{1}{2}$$

$$T_2 \text{ is } y = m_2x + \frac{a}{m_2} \Rightarrow y = m_2x + \frac{-4}{m_2}$$

$$\text{Passes through } (2, 0) \Rightarrow 0 = 2m_2 - \frac{4}{m_2} \Rightarrow m_2^2 = 2$$

$$\text{Now, } \left( \frac{1}{m_1^2} + m_2^2 \right) = 2 + 2 = 4$$

**43. Let  $m$  and  $n$  be two positive integers greater than 1. If**

$$\lim_{\alpha \rightarrow 0} \left( \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left( \frac{e}{2} \right)$$

Then the value of  $\frac{m}{n}$  is

**Ans. (2)**

$$\begin{aligned} \text{Sol } \lim_{\alpha \rightarrow 0} \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} &= \lim_{\alpha \rightarrow 0} \frac{e(e^{\cos \alpha^n - 1} - 1)}{(\cos \alpha^n - 1) \cdot (\alpha^m)} (\cos \alpha^n - 1) \\ &= \lim_{\alpha \rightarrow 0} \frac{e(e^{\cos \alpha^n - 1} - 1)}{\cos \alpha^n - 1} \times \frac{-2 \sin^2 \frac{\alpha^n}{2}}{\alpha^m} \\ &= \lim_{\alpha \rightarrow 0} \frac{e(e^{\cos \alpha^n - 1} - 1)}{(\cos \alpha^n - 1)} \times -2 \cdot \left( \frac{\sin \frac{\alpha^n}{2}}{\frac{\alpha^n}{2}} \right)^2 \frac{\alpha^{2n}}{4 \cdot \alpha^m} = \frac{-e}{2} \end{aligned}$$

$$\text{When } 2n = m \Rightarrow \frac{m}{n} = 2$$

44. If  $\alpha = \int_0^1 \left( e^{9x+3\tan^{-1}x} \right) \left( \frac{12+9x^2}{1+x^2} \right) dx$

Where  $\tan^{-1} x$  takes only principal values, then the value of  $\left( \log_e |1+\alpha| - \frac{3\pi}{4} \right)$  is

Ans. 9

Sol:  $\alpha = \int_0^1 e^{9x+3\tan^{-1}x} \left( 9 + \frac{3}{1+x^2} \right) dx$

Let  $9x + 3\tan^{-1}x = t$

$\Rightarrow \left( 9 + \frac{3}{1+x^2} \right) dx = dt$

$\alpha = \int_0^{9+\frac{3\pi}{4}} e^t dt = e^{9+\frac{3\pi}{4}} - 1$

$\alpha + 1 = e^{9+\frac{3\pi}{4}}$

$\log_e (1+\alpha) = 9 + \frac{3\pi}{4}$

$\log_e (1+\alpha) - \frac{3\pi}{4} = 9$

45. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be a continuous odd function, which vanishes exactly at one point and  $f(1) = \frac{1}{2}$ .

Suppose that  $F(x) = \int_{-1}^x f(t) dt$  for all  $x \in [-1, 2]$  and  $G(x) = \int_{-1}^x t |f(f(t))| dt$  for all  $x \in [-1, 2]$ . If

$\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$ , then the value of  $f\left(\frac{1}{2}\right)$  is

Ans. (7)

Sol.  $\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$

$\Rightarrow \lim_{x \rightarrow 1} \frac{F'(x)}{G'(x)} = \frac{1}{14} \quad (F(1) = G(1) = 0)$

$\Rightarrow \lim_{x \rightarrow 1} \frac{f(x)}{x |f(f(x))|} = \frac{1}{14}$

$\Rightarrow \lim_{x \rightarrow 1} \frac{f(x)}{x (f(f(x)))} = \frac{1}{14}$

$\Rightarrow \frac{f(1)}{f(f(1))} = \frac{1}{14}$

$\Rightarrow f\left(\frac{1}{2}\right) = 7 \quad \left( f(1) = \frac{1}{2} \right)$

- \*46. Suppose that  $\vec{p}, \vec{q}$  and  $\vec{r}$  are three non-coplanar vectors in  $\mathbb{R}^3$ . Let the components of a vector  $\vec{s}$  along  $\vec{p}, \vec{q}$  and  $\vec{r}$  be 4, 3 and 5, respectively. If the components of this vector  $\vec{s}$  along  $(-\vec{p} + \vec{q} + \vec{r}), (\vec{p} - \vec{q} + \vec{r})$  and  $(-\vec{p} - \vec{q} + \vec{r})$  are  $x, y$  and  $z$ , respectively, then the value of  $2x + y + z$  is

Ans. (9/DEL)

Sol.  $\vec{s} = 4\vec{p} + 3\vec{q} + 5\vec{r}$

Also,  $\vec{s} = x(-\vec{p} + \vec{q} + \vec{r}) + y(\vec{p} - \vec{q} + \vec{r}) + z(-\vec{p} - \vec{q} + \vec{r})$

Hence,

$$-x + y - z = 4$$

$$x - y - z = 3$$

$$x + y + z = 5$$

Solving above equations,  $x = 4$  and  $y + z = 1$

Hence  $2x + y + z = 9$

47. For any integer  $k$ , let  $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$ , where  $i = \sqrt{-1}$ . The value of the expression

$$\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^3 |\alpha_{4k-1} - \alpha_{4k-2}|} \text{ is}$$

Ans. (4)

Sol.  $\therefore \alpha_k = \cos\left(\frac{2k\pi}{7}\right) + i \sin\left(\frac{2k\pi}{7}\right)$

$$\therefore \frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^3 |\alpha_{4k-1} - \alpha_{4k-2}|} = \frac{12 \times 2 \sin \frac{\pi}{14}}{3 \times 2 \sin \frac{\pi}{14}} = 4$$

[ $\because$  length of each side of a regular polygon is  $2 \sin(\pi/n)$ ].

48. Suppose that all the terms of an arithmetic progression (A. P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A. P. is

Key: (9)

Sol. According to question,  $\frac{\frac{7}{2}[2a + 6d]}{\frac{11}{2}[2a + 10d]} = \frac{6}{11}$

$$\frac{7a + 21d}{a + 5d} = 6 \Rightarrow a = 9d$$

$$130 < T_7 < 140$$

$$130 < a + 6d < 140$$

$$130 < 15d < 140$$

$$120 < 15d < 140$$

**SECTION : 2 (Maximum Marks : 40)**

- This section contains TEN questions
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:  
 +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
 0 If one of the bubbles is darkened.  
 –2 In all other cases.

49. Let  $f(x) = 7 \tan^8 x + 7 \tan^6 x - 3 \tan^4 x - 3 \tan^2 x$  for all  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . Then the correct expression(s) is(are)

(A)  $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

(B)  $\int_0^{\pi/4} f(x) dx = 0$

(C)  $\int_0^{\pi/4} x f(x) dx = \frac{1}{6}$

(D)  $\int_0^{\pi/4} f(x) dx = 1$

**Key:** (A), (B)

Sol.  $f(x) = 7 \tan^8 x + 7 \tan^6 x - 3 \tan^4 x - 3 \tan^2 x$

$$f(x) = 7 \tan^6 x (\sec^2 x) - 3 \tan^2 x (\sec^2 x) = (7 \tan^6 x - 3 \tan^2 x) \sec^2 x$$

$$\int_0^{\pi/4} f(x) dx = \int_0^1 (7z^6 - 3z^2) dz = 0$$

$$\int_0^{\pi/4} x (7 \tan^6 x - 3 \tan^2 x) \sec^2 x dx = \left[ x [\tan^7 x - \tan^3 x] \right]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x) dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x (\tan^2 x - 1) \sec^2 x dx = - \left[ \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} \right]_0^{\pi/4} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$$

50. Let  $f, g : [-1, 2] \rightarrow \mathbb{R}$  be continuous functions which are twice differentiable on the interval  $(-1, 2)$ . Let the values of  $f$  and  $g$  at the points  $-1, 0$  and  $2$  be as given in the following table.

	$x = -1$	$x = 0$	$x = 2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals  $(-1, 0)$  and  $(0, 2)$  the function  $(f - 3g)''$  never vanishes. Then the correct statement(s) is(are)

(A)  $f'(x) - 3g'(x) = 0$  has exactly three solutions in  $(-1, 0) \cup (0, 2)$

(B)  $f'(x) - 3g'(x) = 0$  has exactly one solution in  $(-1, 0)$

(C)  $f'(x) - 3g'(x) = 0$  has exactly one solution in  $(0, 2)$

(D)  $f'(x) - 3g'(x) = 0$  has exactly two solutions in  $(-1, 0)$  and exactly two solutions in  $(0, 2)$

Sol. Let,  $h(x) = f(x) - 3g(x)$

Since,  $h(x)$  is differentiable and  $h(-1) = h(0) = h(2)$

$\therefore h'(x) = f'(x) - 3g'(x) = 0$  must have at least one root in  $(-1, 0)$  and  $(0, 2)$  each.

By using Rolle's theorem,  $h''(x) = 0$  must have at least one root in  $(-1, 2)$ .

so, therefore,  $h''(x) = 0$  at  $x = 0$

Hence,  $h'(x) = 0$  must have exactly one root in  $(-1, 0)$  and other root in  $(0, 2)$

**51. The option(s) with the values of  $a$  and  $L$  that satisfy the following equation is (are)**

$$\frac{\int_0^{4\pi} e^t (\sin^6 at + \cos^4 at) dt}{\int_0^{\pi} e^t (\sin^6 at + \cos^4 at) dt} = L ?$$

(A)  $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

(B)  $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

(C)  $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

(D)  $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

**Ans. (A, C)**

Sol:  $I_1 = \int_0^{4\pi} e^t (\sin^6 at + \cos^4 at) dt$

$$= \int_0^{4\pi} e^t (\sin^4 at (1 - \cos^2 at) + \cos^4 at) dt$$

$$= \int_0^{4\pi} e^t [1 - 2\sin^2 at \cos^2 at - \sin^4 at \cos^2 at] dt$$

$$= \int_0^{4\pi} e^t [1 - \sin^2 at \cos^2 at (2 + \sin^2 at)] dt$$

$$= \int_0^{4\pi} e^t \left[ 1 - \frac{(\sin 2at)^2}{4} \left( 2 + \frac{1 - \cos 2at}{2} \right) \right] dt$$

$$= \int_0^{4\pi} e^t dt - \frac{1}{2} \int_0^{4\pi} e^t \sin^2 2at dt - \frac{1}{8} \int_0^{4\pi} e^t (\sin 2at)^2 (1 - \cos 2at) dt = \frac{11}{16} (e^{4\pi} - 1)$$

$$I_2 = \int_0^{\pi} e^t (\sin^6 at + \cos^4 at) dt$$

$$= \left( \int_0^{\pi} e^t dt \right) \frac{11}{16} \quad (\text{as same way as above})$$

$$= (e^{\pi} - 1) \frac{11}{16}$$

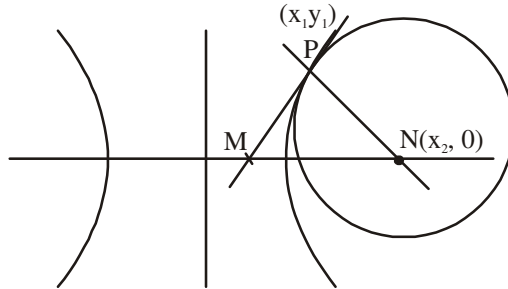
$$\therefore \frac{I_1}{I_2} = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$$

52. Consider the hyperbola  $H : x^2 - y^2 = 1$  and a circle  $S$  with centre  $N(x_2, 0)$ . Suppose that  $H$  and  $S$  touch each other at a point  $P(x_1, y_1)$  with  $x_1 > 1$  and  $y_1 > 0$ . The common tangent to  $H$  and  $S$  at  $P$  intersects the  $x$ -axis at point  $M$ . If  $(l, m)$  is the centroid of the triangle  $\triangle PMN$  then the correct expression (s) is (are)

- (A)  $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$  for  $x_1 > 1$   
 (B)  $\frac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$  for  $x_1 > 1$   
 (C)  $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$  for  $x_1 > 1$   
 (D)  $\frac{dm}{dy_1} = \frac{1}{3}$  for  $y_1 > 0$

**Ans. (A, B, D)**

**Sol:** Tangent at  $P(x_1, y_1)$  is  $xx_1 - yy_1 = 1$



$$M\left(\frac{1}{x_1}, 0\right)$$

Now normal  $y - y_1 = -\frac{y_1}{x_1}(x - x_1)$

Passes through  $N(x_2, 0)$

Hence  $0 - y_1 = -\frac{y_1}{x_1}(x_2 - x_1)$

$\Rightarrow x_2 = 2x_1$

For centroid  $l = \frac{x_1 + \frac{1}{x_1} + 2x_1}{3}$ ,  $m = \frac{y_1}{3} = \frac{\sqrt{x_1^2 - 1}}{3}$

$\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ ,  $\frac{dm}{dy_1} = \frac{1}{3}$ ,  $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$



53. Let  $E_1$  and  $E_2$  be two ellipses whose centers are at the origin. The major axes of  $E_1$  and  $E_2$  lie along the x-axis and the y-axis, respectively. Let  $S$  be the circle  $x^2 + (y-1)^2 = 2$ . The straight line  $x + y = 3$  touches the curves  $S$ ,  $E_1$  and  $E_2$  at  $P$ ,  $Q$  and  $R$ , respectively. Suppose that  $PQ = PR = \frac{2\sqrt{2}}{3}$ . If  $e_1$  and  $e_2$  are the eccentricities of  $E_1$  and  $E_2$ , respectively, then the correct expression (s) is (are)

(A)  $e_1^2 + e_2^2 = \frac{43}{40}$

(B)  $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(C)  $|e_1^2 - e_2^2| = \frac{5}{8}$

(D)  $e_1 e_2 = \frac{\sqrt{3}}{4}$

**Key. (A, B)**

Sol.  $P \equiv (1, 2) ; Q \equiv \left( \frac{a^2}{3}, \frac{b^2}{3} \right) ; R \equiv \left( \frac{A^2}{3}, \frac{B^2}{3} \right)$

Also  $a^2 + b^2 = 9 = A^2 + B^2$

$$PQ = \frac{2\sqrt{2}}{3}$$

$$PQ^2 = \frac{8}{9}$$

$$\Rightarrow \left( \frac{a^2}{3} - 1 \right)^2 + \left( \frac{9 - a^2}{3} - 2 \right)^2 = \frac{8}{9}$$

or  $(a^2 - 3)^2 + (3 - a^2)^2 = 8$

$$(a^2 - 3)^2 = 4$$

$$a^2 - 3 = \pm 2$$

$$a^2 = 5, 1 \Rightarrow a^2 = 5, A^2 = 1$$

$$e_1^2 = 1 - \frac{b^2}{a^2} = \left( 1 - \frac{9-5}{5} \right) = \frac{1}{5}$$

$$e_2^2 = 1 - \frac{A^2}{B^2} = \left( 1 - \frac{1}{8} \right) = \frac{7}{8}$$

$$e_1^2 + e_2^2 = \frac{1}{5} + \frac{7}{8} = \frac{8+35}{40} = \frac{43}{40}$$

$$e_1^2 e_2^2 = \frac{7}{40}$$

$$e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$$

$$|e_1^2 - e_2^2| = \left| 1 - \frac{7}{8} \right| = \frac{1}{8}$$

54. If  $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$  and  $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$ , where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

- (A)  $\cos\beta > 0$  (B)  $\sin\beta < 0$   
 (C)  $\cos(\alpha + \beta) > 0$  (D)  $\cos\alpha < 0$

**Key. (B, C, D)**

Sol.  $\alpha = 3\sin^{-1}\frac{6}{11} > 90^\circ$   
 $\beta = 3\cos^{-1}\frac{4}{9} > 180^\circ$   
 $\alpha + \beta > 270^\circ$   
 $\cos\alpha < 0$   
 $\cos\beta < 0$   
 $\sin\beta < 0$   
 $\cos(\alpha + \beta) > 0$

55. Let S be the set of all non zero real numbers  $\alpha$  such that the quadratic equation  $ax^2 - x + \alpha = 0$  has two distinct real roots  $x_1$  and  $x_2$  satisfying the inequality  $|x_1 - x_2| < 1$ . Which of the following intervals is (are) a subset(s) of S?

- (A)  $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$  (B)  $\left(-\frac{1}{\sqrt{5}}, 0\right)$   
 (C)  $\left(0, \frac{1}{\sqrt{5}}\right)$  (D)  $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

**Key. (A, D)**

Sol.  $|x_1 - x_2| < 1$   
 $0 < (x_1 - x_2)^2 < 1$   
 $0 < (x_1 + x_2)^2 - 4x_1x_2 < 1$   
 $0 < \frac{1}{\alpha^2} - 4 < 1$   
 $\frac{1}{5} < \alpha^2 < \frac{1}{4}$   
 $\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

56. Let  $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$  for all  $x \in \mathbb{R}$  with  $f\left(\frac{1}{2}\right) = 0$ . If  $m \leq \int_{1/2}^1 f(x) dx \leq M$ , then the possible values of m and M are

- (A)  $m = 13, M = 24$  (B)  $m = \frac{1}{4}, M = \frac{1}{2}$   
 (C)  $m = -11, M = 0$  (D)  $m = 1, M = 12$

**Key (D)**

$$\begin{aligned} \frac{1}{2} &\leq x \leq 1 \\ \Rightarrow 8 &\leq f'(x) \leq 96 \\ \Rightarrow \int_{1/2}^x 8 dx &\leq \int_{1/2}^x f'(x) dx \leq \int_{1/2}^x 96 dx \\ 8x - 4 &\leq f(x) - f\left(\frac{1}{2}\right) \leq 96x - 48 \\ 8x - 4 &\leq f(x) \leq 96x - 48 \\ \int_{1/2}^1 (8x - 4) dx &\leq \int_{1/2}^1 f(x) dx \leq \int_{1/2}^1 (96x - 48) dx \\ \left[4x^2 - 4x\right]_{1/2}^1 &\leq \int_{1/2}^1 f(x) dx \leq \left[48x^2 - 48x\right]_{1/2}^1 \\ 1 &\leq \int_{1/2}^1 f(x) dx \leq 12 \\ m = 1, M &= 12 \end{aligned}$$

**SECTION : 3 (Maximum Marks : 16)**

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme: +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
0 If one of the bubbles is darkened.  
-2 In all other cases.

**PARAGRAPH 1**

Let  $F: \square \rightarrow \square$  be a thrice differentiable function. Suppose that  $F(1) = 0$ ,  $F(3) = -4$  and  $F'(x) < 0$  for all  $x \in (1/2, 3)$ . Let  $f(x) = xF(x)$  for all  $x \in \square$ .

**57. The correct statement(s) is (are)**

- (A)  $f'(1) < 0$  (B)  $f(2) < 0$   
(C)  $f'(x) \neq 0$  for any  $x \in (1, 3)$  (D)  $f'(x) = 0$  for some  $x \in (1, 3)$

**Key. (A, B, C)**

**Sol.**  $f'(x) = F(x) + xF'(x)$

$$\begin{aligned} f'(1) &= F(1) + F'(1) \\ &= 0 + (-ive) = -ive \end{aligned}$$

$$f(2) = 2F(2) < 0 \quad \text{as } F(x) \text{ decreasing for } x > 1$$

$$f'(x) = F(x) + xF'(x) < 0 \quad \forall x \in (1, 3)$$

58. If  $\int_1^3 x^2 F'(x) dx = -12$  and  $\int_1^3 x^3 F''(x) dx = 40$ , then the correct expression(s) is (are)

- (A)  $9f'(3) + f'(1) - 32 = 0$  (B)  $\int_1^3 f(x) dx = 12$   
 (C)  $9f'(3) - f'(1) + 32 = 0$  (D)  $\int_1^3 f(x) dx = -12$

**Key. (C, D)**

Sol.  $\int_1^3 x^2 F'(x) dx = -12$   
 $\left[ x^2 F(x) \right]_1^3 - \int_1^3 2xF(x) dx = -12$   
 $9F(3) - F(1) - 2 \int_1^3 f(x) dx = -12$   
 $-36 - 2 \int_1^3 f(x) dx = -12 \Rightarrow \int_1^3 f(x) dx = -12$   
 $\int_1^3 x^3 F''(x) dx = 40$   
 $\left[ x^3 F'(x) \right]_1^3 - \int_1^3 3x^2 F'(x) dx = 40$   
 $x^2 (xF'(x)) \Big|_1^3 - 3 \int_1^3 x^2 F'(x) dx = 40$   
 $9(f'(3) - F(3)) - 1(f'(1) - F(1)) - 3(-12) = 40$   
 $\Rightarrow 9f'(3) - f'(1) + 32 = 0$

### PARAGRAPH 2

Let  $n_1$  and  $n_2$  be the number of red and black balls, respectively, in box I. Let  $n_3$  and  $n_4$  be the number of red and black balls, respectively, in box II.

59. One of the two boxes I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is  $\frac{1}{3}$ , then the correct option(s) with the possible values of  $n_1, n_2, n_3$  and  $n_4$  is(are)

- (A)  $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$  (B)  $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$   
 (C)  $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$  (D)  $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

**Key. (A, B)**

Sol. 

$n_1$	red
$n_2$	black

$n_3$	red
$n_4$	black

Box I

Box II

$$P = \frac{\frac{1}{2} \frac{n_3}{n_3 + n_4}}{\frac{1}{2} \frac{n_1}{n_1 + n_2} + \frac{1}{2} \frac{n_3}{n_3 + n_4}} = \frac{1}{3}$$

$$\frac{n_3}{(n_1 + n_2)(n_3 + n_4) + n_3} = \frac{1}{3}$$

$$3n_3(n_3 + n_4) = n_1(n_3 + n_4) + n_3$$

is satisfied by (A), (B)

60. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is  $\frac{1}{3}$ , then the correct option (s) with the possible

values of  $n_1$  and  $n_2$  is (are)

(A)  $n_1 = 4$  and  $n_2 = 6$

(B)  $n_1 = 2$  and  $n_2 = 3$

(A)  $n_1 = 10$  and  $n_2 = 20$

(D)  $n_1 = 3$  and  $n_2 = 6$

**Key (C, D)**

Sol. 
$$\frac{n_1}{n_1 + n_2} \cdot \frac{n_1 - 1}{n_1 + n_2 - 1} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

$$3[n_1(n_1 + n_2 - 1)] = (n_1 + n_2)(n_1 + n_2 - 1)$$

$$3n_1 = n_1 + n_2$$

$$2n_1 = n_2$$