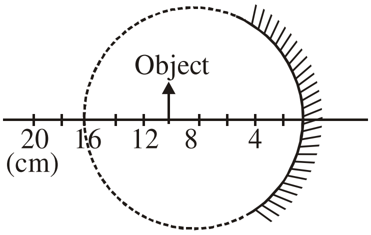
FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. 

A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)

(1) Inverted, real and magnified

(2) Erect, virtual and magnified

(3) Erect, virtual and unmagnified

(4) Inverted, real and unmagnified

Sol. $f = \frac{-8}{2} = -4\text{cm}$

$u = -10$ cm

$v = ?$

as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \left(\frac{1}{-10}\right) = \frac{1}{-4}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{4}$

$\frac{1}{v} = \frac{4-10}{40}$

$v = \frac{40}{-6}$

$v = \frac{-20}{3}$

$m = \frac{-v}{u}$

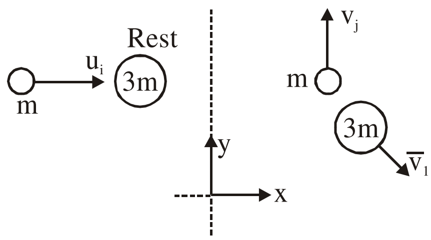
$m = \frac{-\left(-\frac{20}{3}\right)}{-10} \Rightarrow m = \frac{-2}{3}$

or image will be real, inverted and unmagnified.

2. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

(1) $v = \sqrt{\frac{2}{3}}u$ (2) $v = \frac{1}{\sqrt{6}}u$

(3) $v = \frac{u}{\sqrt{3}}$ (4) $v = \frac{u}{\sqrt{2}}$

Sol. 

From momentum conservation

$\vec{P}_i = \vec{P}_f$

$m(ui) + 3m(0) = mvj + 3m\,\bar{v}_1$

$mui - mvj = 3m\,\bar{v}_1$

$\bar{v}_1 = \frac{ui - vj}{3}$

or $|v_1| = \frac{\sqrt{u^2 + v^2}}{3}$

or $v_1^2 = \frac{u^2 + v^2}{9}$ ....(1)

As collision is perfectely elastic hence

$k_i = k_j$

$\frac{1}{2}mu^2 + \frac{1}{2}3m0^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_1^2$

$\Rightarrow u^2 = v^2 + 3v_1^2$

$$u^2 = v^2 + 3\frac{(u^2+v^2)}{9}$$

$\Rightarrow 3u^2 = 3v^2 + u^2 + v^2$

$\Rightarrow 2u^2 = 4v^2$

$$v = \frac{u}{\sqrt{2}}$$

3. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to: (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C

(1) 12 cm (2) 4 cm

(3) 5 cm (4) 2 cm

Sol. Pitch = $\frac{2\pi m}{qB} v\cos\theta$

Pitch = $\frac{2(3.14)(1.67\times10^{-27})\times4\times10^5\times\cos 60}{(1.69\times10^{-19})(0.3)}$

Pitch = 0.04m = 4 cm

4. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity $\rho_C > \rho_T > \rho_M$ and ρ_A respectively. Then:

(1) $\rho_A > \rho_T > \rho_C$ (2) $\rho_C > \rho_A > \rho_T$

(3) $\rho_A > \rho_M > \rho_C$ (4) $\rho_M > \rho_A > \rho_C$

Sol. $\rho_M > \rho_A > \rho_C$

5. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?

(1) T : Large retentivity, small coercivity

(2) P : Small retentivity, large coercivity

(3) T : Large retentivity, large coercivity

(4) P : Large retentivity, large coercivity

Sol. As for permanent magnet large retentivity and large coercivity required

6. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is : (VSD is vernier scale division)

(1) 3.21 cm (2) 2.99 cm

(3) 3.2 cm (4) 3.07 cm

Sol. Least count = 1 mm or 0.01 cm

Zero error = 0 + 0.01 × 7 = 0.07 cm

Reading = 3.1 + (0.01 × 4) – 0.07

= 3.1 + 0.04 – 0.07

= 3.1 – 0.03

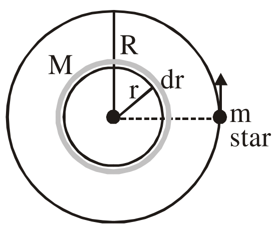
= 3.07 cm

7. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :

(1) $T \propto R$ (2) $T^2 \propto \frac{1}{R^3}$

(3) $T^2 \propto R$ (4) $T^2 \propto R^3$

Sol.



dm = ρdv

$$dm = \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

dm = 4πkrdr

$$M = \int_0^R dm = \int_0^R 4\pi krdr$$

$$M = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

M = 2πk(R^2 – 0)
M = 2πkR^2
for circular motion gravitational force will provide required centripital force or

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G(2\pi kR^2)m}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{2\pi GkR}$$

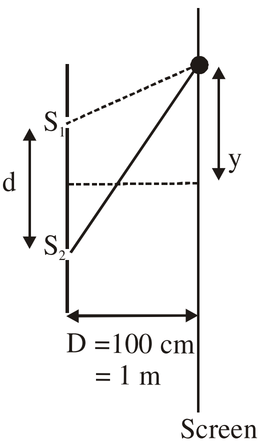
Time period $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi R}{\sqrt{2\pi GkR}} \propto \sqrt{R}$$

or $T^2 \propto R$

8. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :
(1) 1.27 μm (2) 2 nm
(3) 2.87 nm (4) 2.05 μm

Sol.

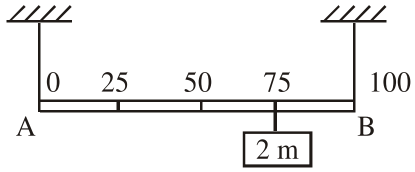


$$y = \frac{nD\lambda}{d}$$

$$n = \frac{yd}{D\lambda} = \frac{1.27\times10^{-3}\times10^{-3}}{1\times632.8\times10^{-9}} = 2$$

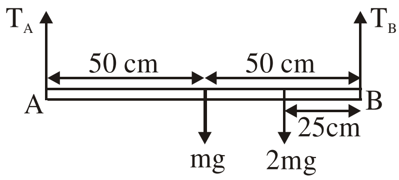
Path difference Δx = nλ
= 2 × 632.8 nm
= 1265.6 nm
= 1.27 μm

9.



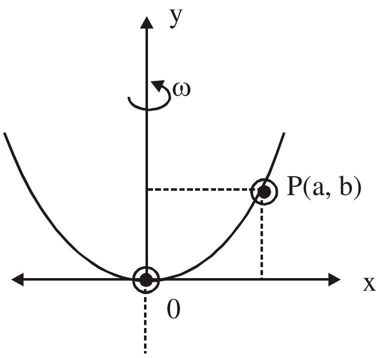
Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :
(1) 2 mg (2) 0.5 mg
(3) 0.75 mg (4) 1 mg

Sol.



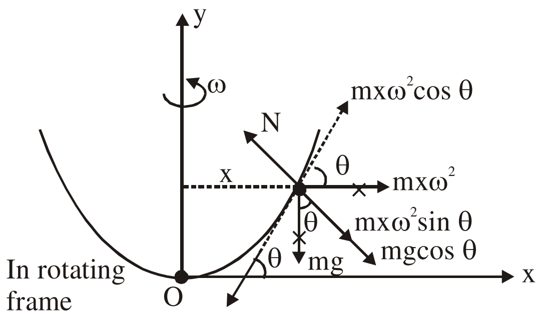
τ_B = 0 (torque about point B is zero)
(T_A) × 100 – (mg) × 50 – (2mg) × 25 = 0
100 T_A = 100 mg
T_A = 1 mg

10. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola y = $4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) :



(1) $\sqrt{\frac{2gC}{ab}}$ (2) $2\sqrt{2gC}$ (3) $\sqrt{\frac{2g}{C}}$ (4) $2\sqrt{gC}$

Sol.



$mx\omega^2\cos\theta = mg\sin\theta$

$x\omega^2 = g\tan\theta$

$$x\omega^2 = g.\frac{dy}{dx}$$

$x\omega^2 = g.(8cx)$

$\omega^2 = 8\,gc$

$$\omega = 2\sqrt{2gc}$$

11. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/ m³ in vacuum. The amplitude of the magnetic field of the wave is close to

$(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\frac{Nm^2}{C^2}$ and speed of light $= 3 \times 10^8$ ms⁻¹) :

(1) 180 nT (2) 160 nT

(3) 150 nT (4) 190 nT

Sol. Energy density $\frac{dU}{dV} = \frac{B_0^2}{2\mu_0}$

$$1.02\times10^{-8} = \frac{B_0^2}{2\times4\pi\times10^{-7}}$$

$B_0^2 = (1.02\times10^{-8})\times(8\pi\times10^{-7})$

$B_0 = 16 \times 10^{-8}$ T = 160 nT

12. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, N = 6.023×10^{26} per kilo mole and 1 eV = 1.6×10^{-19} J. The power output of the reactor is close to :

(1) 125 MW (2) 60 MW

(3) 35 MW (4) 54 MW

Sol. Number of uranium atoms in 2kg

$$= \frac{2\times6.023\times10^{26}}{235}$$

energy from one atom is 200×10^6 e.v. hence total energy from 2 kg uranium

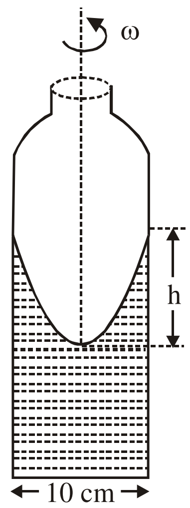
$$= \frac{2\times6.023\times10^{26}}{235}\times200\times10^6\times1.6\times10^{-19}\,J$$

2 kg uranium is used in 30 days hence this energy is recieved in 30 days hence energy recived per second or power is

$$\text{Power} = \frac{2\times6.023\times10^{26}\times200\times10^6\times1.6\times10^{-19}}{235\times30\times24\times3600}$$

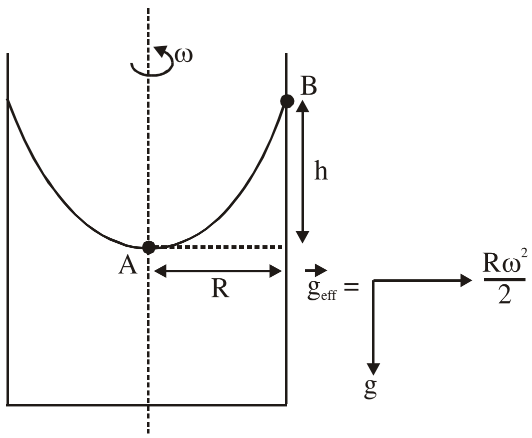
Power = 63.2×10^6 watt or 63.2 Mega Watt

13. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s⁻¹. The difference in the height, h(in cm) of liquid at the centre of vessel and at the side will be:



(1) $\frac{25\omega^2}{2g}$ (2) $\frac{2\omega^2}{5g}$ (3) $\frac{5\omega^2}{2g}$ (4) $\frac{2\omega^2}{25g}$

Sol.



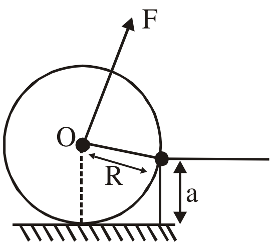
Applying pressure equation from A to B

$$P_0 + \rho.\frac{R\omega^2}{2}.R - \rho gh = P_0$$

$$\frac{\rho R^2\omega^2}{2} = \rho gh$$

$$h = \frac{R^2\omega^2}{2g} = (5)^2\frac{\omega^2}{2g} = \frac{25}{2}\frac{\omega^2}{g}$$

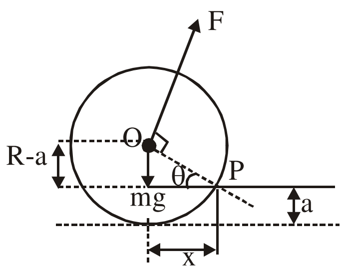
14. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :



(1) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

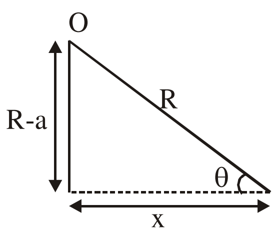
(3) $Mg\frac{a}{R}$ (4) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

Sol.



$(\tau)_P = 0$

F.R. – mgx = 0



$$x = \sqrt{R^2-(R-a)^2}$$

$$F = mg\frac{x}{R}$$

$$F = mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

= minimum value of force to pull

15. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(1) 11 (2) 15

(3) 20 (4) 13

Sol. $u = \frac{f_1n_1RT}{2} + \frac{f_2n_2RT}{2}$

$$u = \frac{5}{2}\times 3RT + \frac{3\times 5RT}{2} = 15RT$$

16. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be :

(1) $FA^{-1}V^0$ (2) FA^2V^{-1}

(3) FA^2V^{-3} (4) FA^2V^{-2}

Sol. $Y = F^xA^yV^z$

$M^1L^{-1}T^{-2} = [MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$

$M^1L^1T^{-2} = [M]^x[L]^{x+2y+z}[T]^{-2x-z}$

comparing power of ML and T

x = 1...(1)

x + 2y + z = –1 ....(2)

–2x – z = –2 ...(3)

after solving

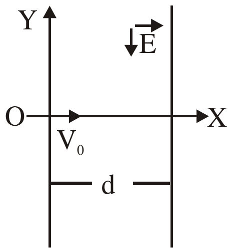
x = 1

y = –1

z = 0

$Y = FA^{-1}V^0$

17. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is :



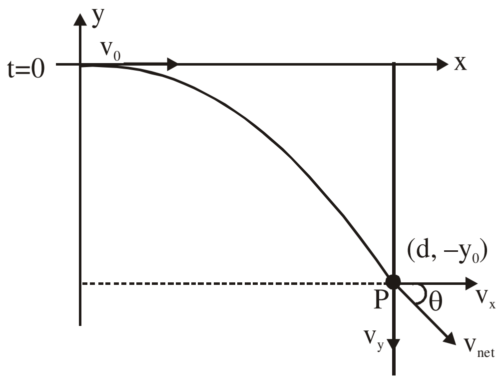
(1) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$

(2) $y = \frac{qEd}{mV_0^2}(x - d)$

(3) $y = \frac{qEd}{mV_0^2}x$

(4) $y = \frac{qEd^2}{mV_0^2}x$

Sol.



Let particle have charge q and mass 'm'

Solve for (q,m) mathematically

$F_x = 0$, $a_x = 0$, $(v)_x$ = constant

time taken to reach at 'P' = $\frac{d}{v_0} = t_0$ (let) ...(1)

(Along –y), $y_0 = 0 + \frac{1}{2}.\frac{qE}{m}.t_0^2$ ....(2)

$v_x = v_0$

$v = u + at$ (along -ve 'y')

speed $v_{y0} = \frac{qE}{m}.t_0$

$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m.v_0}, (t_0 = \frac{d}{v_0})$

$\tan\theta = \frac{qEd}{m.v_0^2}$

$\boxed{\text{slope} = \frac{-qEd}{mv_0^2}}$

Now we have to find eq^n of straight line whose slope is $\frac{-qEd}{mv_0^2}$ and it pass through point → (d, $-y_0$)

Because after x > d

No electric field ⇒ $F_{net} = 0$, $\vec{v}$ = const.

$y = mx + c$, $\left\{\begin{matrix} m = \frac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix}\right\}$

$-y_0 = \frac{-qEd}{mv_0^2}.d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$

Put the value

$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$

$y_0 = \frac{1}{2}.\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$

$y = \frac{-qEd}{mv_0^2}x + \frac{1}{2}\frac{qEd^2}{mv_0^2}$

$\boxed{y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)}$

18. An amplitude modulated wave is represented by the expression $v_m = 5(1 + 0.6 \cos 6280t) \sin(211 \times 10^4 t)$ volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively :

(1) 5V, 8V
(2) $\frac{3}{2}$V, 5V
(3) $\frac{5}{2}$V, 8V
(4) 3V, 5V

(Close Option is 3 Amax. = 8, Amin. = 2)

Sol. $V_m = 5(1+0.6 \cos 6280t) \sin (2\pi \times 10^4 t)$

$V_m = [5+3\cos 6820t] \sin (2\pi \times 10^4 t)$

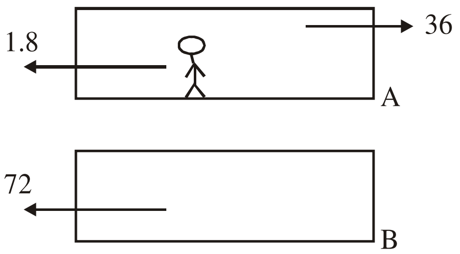
$V_{max.} = 5 + 3 = 8$

$V_{min.} = 5 - 3 = 2$

19. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hr. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 30.5 ms^{-1}
(2) 29.5 ms^{-1}
(3) 31.5 ms^{-1}
(4) 28.5 ms^{-1}

Sol.



Velocity of man with respect to ground

$\vec{V}_{m/g} = \vec{V}_{m/A} + \vec{V}_A = -1.8 + 36$

Velocity of man w.r.t. B

$\vec{V}_{m/B} = \vec{V}_m - \vec{V}_B$

$= -1.8 + 36 - (-72)$

$= 106.2$ km/hr

$= 29.5$ m/s

20. Two identical strings X and Z made of same material have tension T_X and T_Z in them. It their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is :

(1) 0.44
(2) 1.5
(3) 2.25
(4) 1.25

Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

For identical string l and μ will be same

$f \propto \sqrt{T}$

$\frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$

$\frac{T_x}{T_y} = \frac{9}{4} = 2.25$

21. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$J then value of X to the nearest integer is________.

Sol. $u_i = \frac{1}{2} \times 5 \times 10^{-6} (220)^2$

Final common potential

$v = \frac{220 \times 5 + 0 \times 2.5}{5 + 2.5} = 220 \times \frac{2}{3}$

$u_f = \frac{1}{2}(5+2.5) \times 10^{-6}\left(220 \times \frac{2}{3}\right)^2$

$\Delta u = u_f - u_i$

$\Delta u = -403.33 \times 10^{-4}$

$\Rightarrow -403.33 \times 10^{-4} = \frac{X}{100}$

$X = -4.03$

or magnitude or value of X is approximate 4

22. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabaticaly to l/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ________.

Sol. Diatomic :

$f = 5$

$\gamma = 7/5$

$T_i = T = 273 + 20 = 293$ K

$V_i = V$

$V_f = V/10$

Adiabatic $\quad TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T.V^{7/5-1} = T_2\left(\frac{V}{10}\right)^{7/5-1}$$

$\Rightarrow T_2 = T.\ 10^{2/5}$

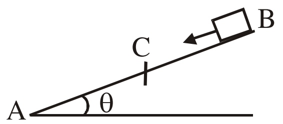
$$\Delta U = \frac{nfR(T_2 - T_1)}{2} = \frac{5\times5\times\frac{25}{3}\times(T.10^{2/5} - T)}{2}$$

$$= \frac{25\times25\times}{6}T(10^{2/5} - 1)$$

$$= \frac{625\times293\times(10^{2/5}-1)}{6}$$

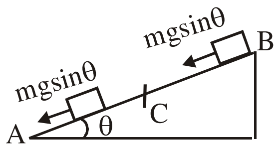
$$= 4.033\times10^3 \approx 4\text{kJ}$$

23.



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by $\mu = k\tan\theta$). The value of k is ________.

Sol.



Apply work energy theorem

$mg\sin\theta\ (AC + 2AC) - \mu mg\cos\theta AC = 0$

$\mu = 3\tan\theta$

24. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in μV) in the coil will be close to the integer ________.

Sol. $r = 0.1$ m $\quad \frac{T}{2} = 0.2$ sec

$B = 3\times10^{-5}$ m $\qquad T = 0.4$ sec

At any time

flux $\phi = BA\cos\omega t$

$$|emf| = \left|\frac{d\phi}{dt}\right| = |BA\omega\ \sin\omega t|$$

$$(emf)_{max} = BA\omega = BA\ \frac{2\pi}{T}$$

$$= \frac{3\times10^{-5}\times\pi\times(0.1)^2\times2\pi}{0.4}$$

$$= \frac{6\pi^2}{4}\times10^{-6} \qquad \left(\begin{matrix}\pi^2 \simeq 10\\ \text{take}\end{matrix}\right)$$

$= 15\times10^{-6}$

$= 15$ μV

25. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is nλ then value of n will be _______.

Sol. $\frac{hc}{\lambda}=\frac{hc}{\lambda_0}+eV$ .....(i)

$\frac{hc}{3\lambda}=\frac{hc}{\lambda_0}+\frac{e\cdot V}{4}$ .....(ii)

(multiply by 4)

$\frac{4hc}{3\lambda}=\frac{4hc}{\lambda_0}+eV$ ....(iii)

From (i) & (iii)

$\frac{hc}{\lambda}-\frac{hc}{\lambda_0}=\frac{4hc}{3\lambda}-\frac{4hc}{\lambda_0}$

$-\frac{hc}{3\lambda}=-\frac{3hc}{\lambda_0}$

$\boxed{9\lambda=\lambda_0}$

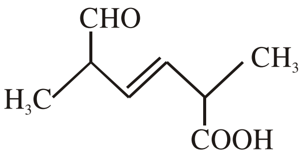
n = 9

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

CHEMISTRY TEST PAPER WITH ANSWER & SOLUTION

1. The IUPAC name for the following compound is:



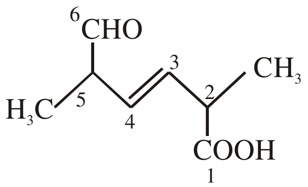
(1) 2, 5-dimethyl-6-carboxy-hex-3-enal

(2) 6-formyl-2-methyl-hex-3-enoic acid

(3) 2, 5-dimethyl-5-carboxy-hex-3-enal

(4) 2, 5-dimethyl-6-oxo-hex-3-enoic acid

Soll.

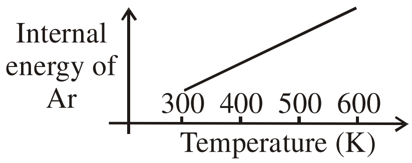


IUPAC name

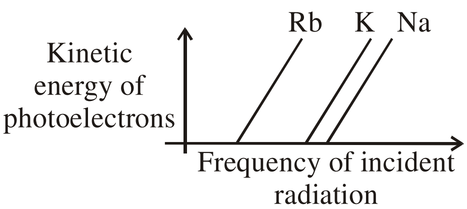
2, 5-dimethyl-6-oxo-hex-3-enoic acid

2. The figure that is not a direct manifestation of the quantum nature of atoms is :

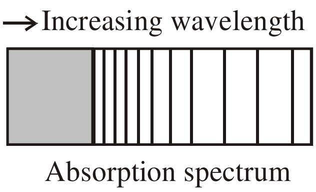
(1)



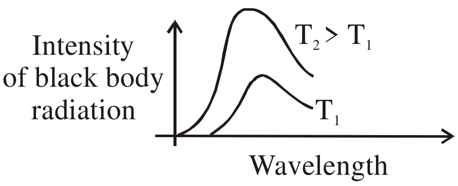
(2)



(3)



(4)



Sol. Photoelectric effect (option 2), atomic spectrum (option 3) and Black body radiations (option 4) may be explained by quantum theory.

As on increasing temperature, all the values of internal energy becomes possible, it is not directly explained from quantum theory.

3. For the following Assertion and Reason, the correct option is

Assertion (A) : When Cu (II) and sulphide ions are mixed, they react together extremely quickly to give a solid.

Reason (R) : The equilibrium constant of $Cu^{2+}(aq) + S^{2-}(aq) \rightleftharpoons CuS(s)$ is high because the solubility product is low.

(1) Both (A) and (R) are true and (R) is the explanation for (A)

(2) Both (A) and (R) are false

(3) (A) is false and (R) is true

(4) Both (A) and (R) are true but (R) is not the explanation for (A)

Sol. Slow or fast process is kinetic parameter but extent less or more is thermodynamic parameter.

4. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is :

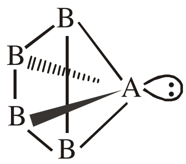
(1) Square pyramidal

(2) Tetrahedral

(3) Square planar

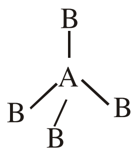
(4) Rectangular planar

Sol. (1) If AB_4 molecule is a square pyramidal then it has one lone pair and their structure should be



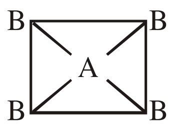
and it should be polar because dipole moment of lone pair of 'A' never be cancelled by others.

(2) If AB_4 molecule is a tetrahedral then it has no lone pair and their structure should be



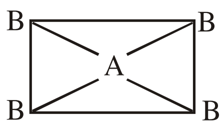
and it should be non polar due to perfect symmetry.

(3) If AB_4 molecule is a square planar then



it should be non polar because vector sum of dipole moment is zero.

(4) If AB_4 molecule is a rectangular planar then



it should be non polar because vector sum of dipole moment is zero.

5. On heating compound (A) gives a gas (B) which is constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be:

(1) $(NH_4)_2Cr_2O_7$

(2) $Pb(NO_3)_2$

(3) NaN_3

(4) NH_4NO_2

Sol. $\underset{\text{Compound}}{(A)} \xrightarrow{\Delta} \underset{\text{Gas}}{(B)} \xrightarrow[\text{catalyst}]{+H_2} \underset{\text{Basic gas}}{(C)}$ (Haber's process)

Basic gas (C) must be ammonia (NH_3).
It means (B) gas should be N_2 which is formed by heating of compound (A).

(1) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2\uparrow + Cr_2O_3 + 4H_2O\uparrow$

(2) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2\uparrow + \frac{1}{2}O_2\uparrow$

(3) $2NaN_3 \xrightarrow{\Delta} 2Na + 3N_2\uparrow$

(4) $NH_4NO_2 \xrightarrow{\Delta} N_2\uparrow + 2H_2O\uparrow$

So, (A) should not be $Pb(NO_3)_2$

6. In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is

(1) Electronegativity

(2) Electron gain enthalpy

(3) Ionization enthalpy

(4) Atomic radius

Sol. In general across a period atomic radius decreases while ionisation enthalpy, electron gain enthalpy and electronegativity increases because effective nuclear charge (Z_{eff}) increases.

7. The statement that is not true about ozone is :

(1) in the stratosphere, it forms a protective shield against UV radiation.

(2) it is a toxic gas and its reaction with NO gives NO_2.

(3) in the atmosphere, it is depleted by CFCs.

(4) in the stratophere, CFCs release chlorine free radicals ($\dot{Cl}$) which reacts with O_3 to give chlorine dioxide radicals.

Sol. In the stratosphere, CFCs release chlorine free radical $(\dot{Cl})$

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{Cl}(g) + \dot{C}F_2Cl(g)$$

which react with O_3 to give chlorine oxide $(Cl\dot{O})$ radical not chlorine dioxide $(\dot{Cl}O_2)$ radical.

$$\dot{Cl}(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$$

8. The metal mainly used in devising photoelectric cells is:

(1) Na (2) Rb

(3) Li (4) Cs

Sol. Cs used in photoelectric cell as it has least ionisation energy.

9. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements :

(I) both the complexes can be high spin

(II) Ni(II) complex can very rarely be low spin.

(III) with strong field ligands, Mn(II) complexes can be low spin.

(IV) aqueous solution of Mn(II) ions is yellow in color.

The **correct** statements are :

(1) (I), (III) and (IV) only

(2) (II), (III) and (IV) only

(3) (I), (II) and (III) only

(4) (I) and (II) only

Sol. (I) Under weak field ligand, octahedral Mn(II) and tetrahedral Ni(II) both the complexes are high spin complex.

(II) Tetrahedral Ni(II) complex can very rarely be low spin because square planar (under strong ligand) complexes of Ni(II) are low spin complexes.

(III) With strong field ligands Mn (II) complexes can be low spin because they have less number of unpaired electron (unpaired electron = 1)

While with weak field ligands Mn(II) complexes can be high spin because they have more number of unpaired electron (unpaired electron = 5)

(IV) Aqueous solution of Mn(II) ions is pink in colour.

10. Consider that a d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is :

(1) tetrahedral and $-1.6\ \Delta_t + 1P$

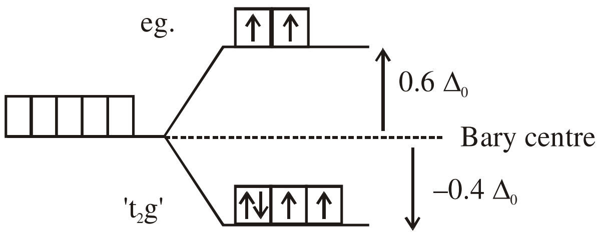
(2) tetrahedral and $-0.6\ \Delta_t$

(3) octahedral and $-1.6\ \Delta_0$

(4) octahedral and $-2.4\ \Delta_0 + 2P$

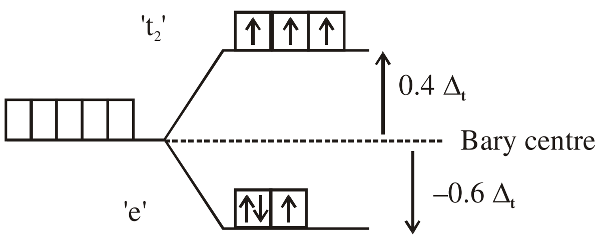
Sol. If spin only magnetic moment of the complex is 4.90 BM, it means number of unpaired electrons should be 4.

(A) In octahedral complex : $[M(H_2O)_6]^{2+}$
d^6



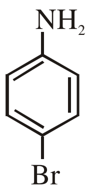
C.F.S.E. = $(-0.4\ \Delta_0) \times 4 + (+0.6\ \Delta_0) \times 2 + 0 \times P$
$= -0.4\ \Delta_0$

(B) In tetrahedral complex : $[M(H_2O)_4]^{2+}$
d^6

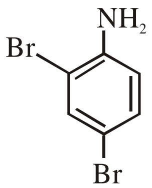


C.F.S.E. = $(-0.6\ \Delta_t) \times 3 + (+0.4\ \Delta_t) \times 3 + 0 \times P$
$= -0.6\ \Delta_t$

11. In Carius method of estimation of halogen, 0.172g of an organic compound showed presence of 0.08g of bromine. Which of these is the **correct** structure of the compound :

(1) 

(2) H_3C-CH_2-Br

(3) 

(4) H_3C-Br

Sol. In Carius method

mass of organic compound = 0.172 gm

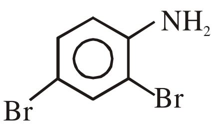
mass of Bromine = 0.08 gm

Hence % of Bromine = $\frac{0.08}{0.172}\times 100$

= 46.51%

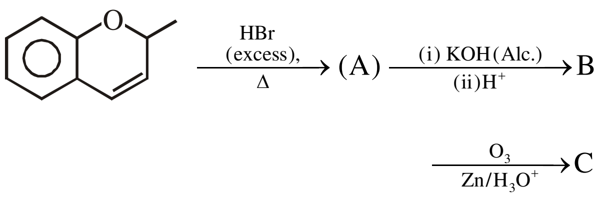
(1)  C_6H_6NBr $\left[\%Br = \frac{80}{172}\times 100\right] = 46.51\%$

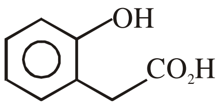
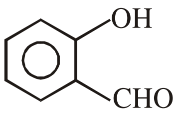
(2) CH_3CH_2Br C_2H_5Br $\%Br = \frac{80}{109}\times 100 = 73.33\%$

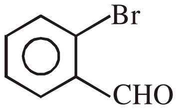
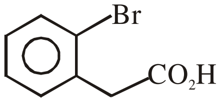
(3)  $C_6H_5NBr_2$

(4) CH_3Br

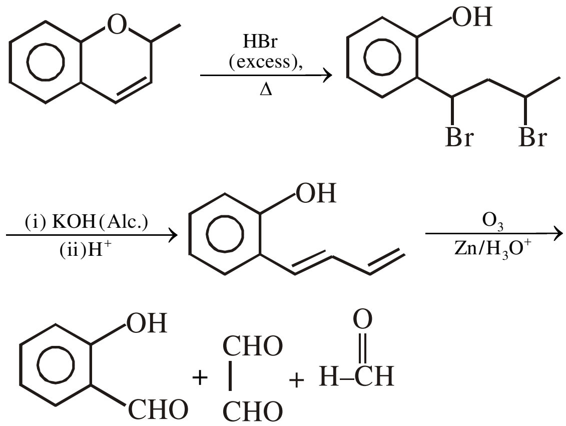
12. The major aromatic product C in the following reaction sequence will be :



(1)  (2) 

(3)  (4) 

Sol.



13. An open beaker of water in equilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules :

(1) leaves the vapour increases

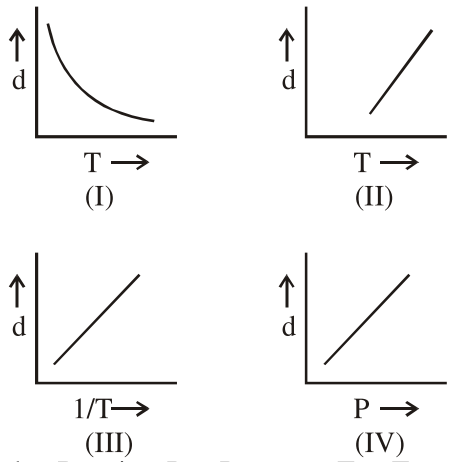
(2) leaves the solution increases

(3) leaves the solution decreases

(4) leaves the vapour decreases

Sol. With addition of solute in solvent, surface area for vapourisation decreases causes lowering in vapour pressure

14. Which one of the following graphs is **not correct** for ideal gas ?



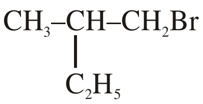
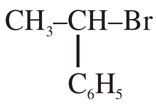
d = Density, P = Pressure, T = Temperature

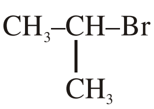
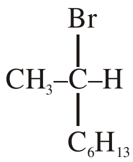
(1) II (2) III

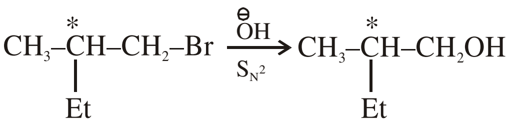
(3) I (4) IV

Sol. PM = dRT $\Rightarrow$ $d \propto \frac{1}{T}$

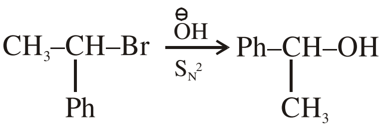
15. Which of the following compounds will show retention in configuration on nucleophic substitution by OH^- ion ?

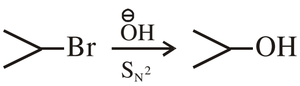
(1)  (2) 

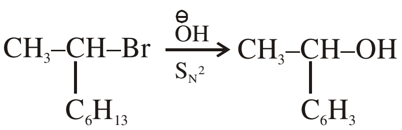
(3)  (4) 

Sol. (1) 

* (NCERT)

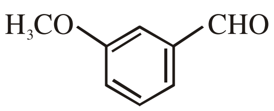
(2) 

(3) 

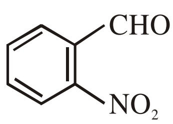
(4) 

As language given, we have to go with option (1) as stereochemistry of chiral centre is not distortet.

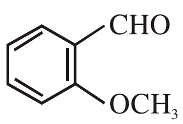
16. The increasing order of the following compounds towards HCN addition is :



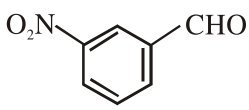
(i)



(ii)



(iii)



(iv)

(1) (iii) < (iv) < (ii) < (i)

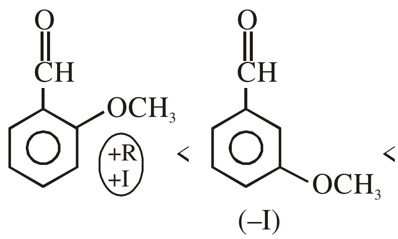
(2) (iii) < (iv) < (i) < (ii)

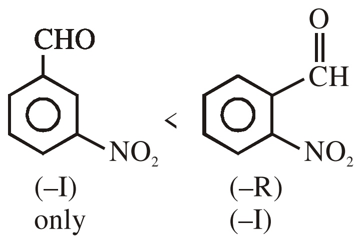
(3) (iii) < (i) < (iv) < (ii)

(4) (i) < (iii) < (iv) < (ii)

Sol. Increasing order of reactivity towards HCN addition

Greater the electrophilicity on $-\overset{\overset{O}{\|}}{C}-$ group greater the reactivity in nucleophilic addition.





(iii) < (i) < (iv) < (ii)

17. While titrating dilute HCl solution with aqueous NaOH, which of the following will **not** be required?

(1) Clamp and phenolphthalein

(2) Pipette and distilled water

(3) Burette and porcelain tile

(4) Bunsen burner and measuring cylinder

Sol. Lab manual

18. Consider the following reactions :

(i) Glucose + ROH $\xrightarrow{\text{dry HCl}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{\text{x eq.of}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2} A \xrightarrow[(CH_3CO)_2O]{y\ eq.of}$ acetyl derivative

(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{z\ eq.of}$ acetyl derivative

'x', 'y' and 'z' in these reactions are respectively.

(1) 5, 6, & 5 (2) 4, 5 & 5

(3) 5, 4 & 5 (4) 4, 6 & 5

Sol.

(i) Glucose + dry HCl $\xrightarrow{ROH}$ Acetal $\xrightarrow[(CH_3CO)_2O]{x\ Eq}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2} A \xrightarrow[(CH_3CO)_2O]{y\ Eq}$ Acetyl derivative

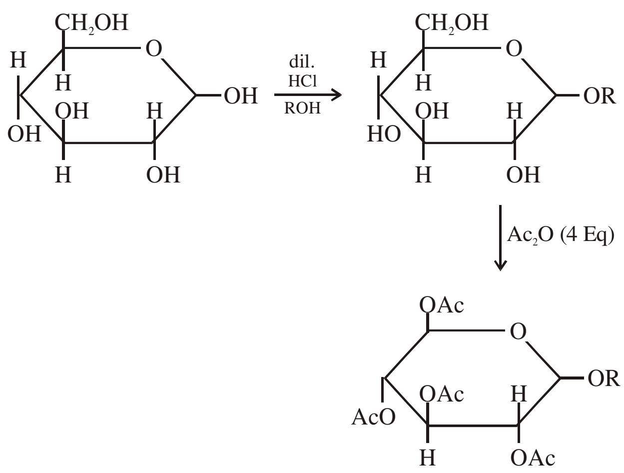
(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{z\ Eq}$ Acetyl derivative

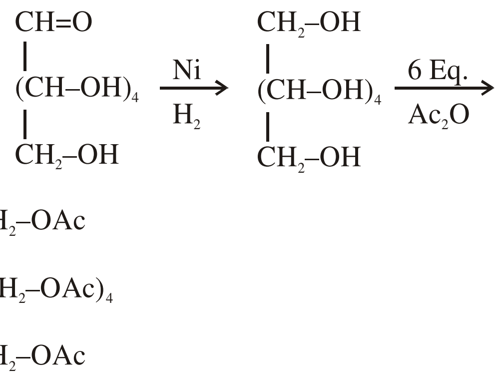
due to presence of –OH group in Glucose the reaction is

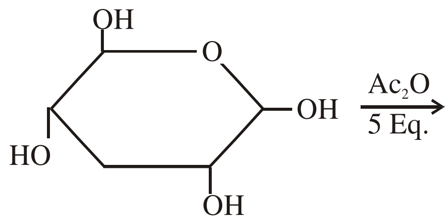
$$R{-}OH + CH_3{-}\overset{O}{\overset{\|}{C}}{-}O{-}\overset{O}{\overset{\|}{C}}{-}CH_3 \rightarrow R{-}O{-}\overset{O}{\overset{\|}{C}}{-}CH_3$$

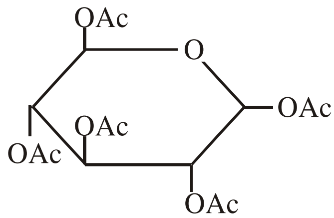
Acetyl derivative

so for (i)

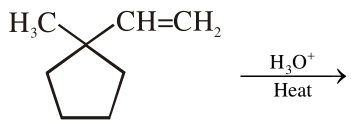


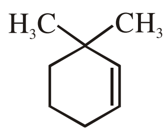
(ii) 

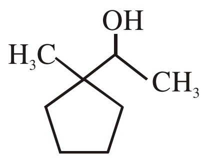
(iii) 

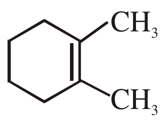
Glucose pentacetase 

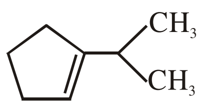
19. The major product in the following reaction is :

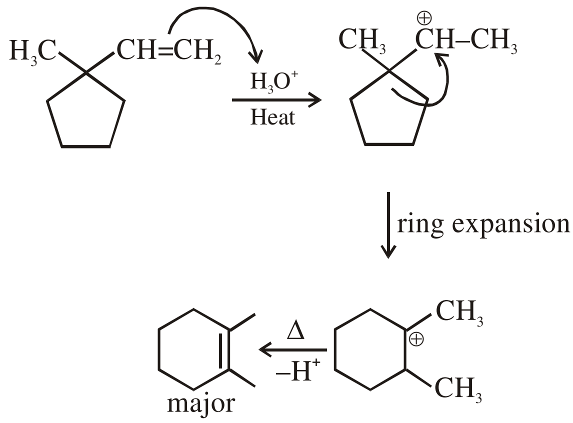


(1) 

(2) 

(3) 

(4) 

Sol. 

20. Which of the following is used for the preparation of colloids ?

(1) Ostwald Process (2) Van Arkel Method
(3) Bredig's Arc Method (4) Mond Process

Sol. Bredig's Arc method is used to from metal colloids.

21. The Gibbs energy change (in J) for the given reaction at $[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298K is:
$Cu(s) + Sn^{2+}(aq.) \rightarrow Cu^{2+}(aq.) + Sn(s)$;

$(E^0_{Sn^{2+}|Sn} = -0.16V,\ E^0_{Cu^{2+}|Cu} = 0.34V,$

Take F = 96500 C mol^{-1})

Sol. $\Delta G = \Delta G^o + RT \ln \left[\frac{Sn^{+2}}{Cu^{+2}}\right]$

$= -2 \times 96500\ [(-0.16) - 0.34] + RT \ln \left(\frac{1}{1}\right)$

= 96500 J

22. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between $\log \frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is : (Given log 3 = 0.4771)

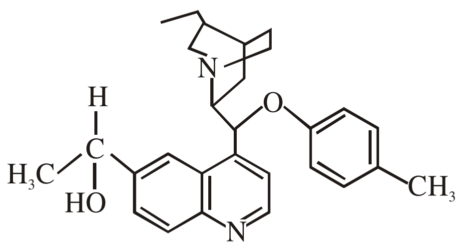
Sol. $\frac{x}{m} = k\,p^x$ ...(1)

$\Rightarrow \underbrace{\log \frac{x}{m}}_{y} = \underbrace{\log k}_{c} + \underbrace{x}_{m} \underbrace{\log p}_{x}$

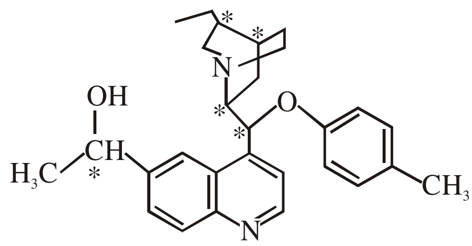
Given c = log k = 0.4771 or k = 3
slope x = 2

put in eq. (1) $\frac{x}{m} = 3 \times (4)^2 \Rightarrow 48$

23. The number of chiral carbons present in the molecule given below is _____ .



Sol. No. of chiral centres



24. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x,y and z is ____.

$Na_4[Fe(CN)_5NOS)]$ (A) $Na_4[FeO_4]$ (B) $[Fe_2(CO)_9]$ (C)

Sol. (A) $Na_4[\overset{x}{Fe}(CN)_5(NOS)]$

$(+1)4 + x + (-1)5 + (-1)\ 1 = 0$

$\boxed{x = +2}$

(B) $Na_4[\overset{y}{Fe}O_4]$

$(+1)4 + y + (-2)4 = 0$

$\boxed{y = +4}$

(C) $[\overset{z}{Fe_2}(CO)_9]$

$2z + 0 \times 9 = 0$

$\boxed{z = 0}$

so (x + y + z) = +2 + 4 + 0
= 6

25. The internal energy change (in J) when 90g of water undergoes complete evaporation at 100ºC is ____ .

(Given : ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1})

Sol. $H_2O(\ell) \rightleftharpoons H_2O(g)$ 90 gm of H_2O

$\Delta H = \Delta U + \Delta n_g RT$ $\Rightarrow$ 5 moles of H_2O

5×41000 J $= \Delta U + 1 \times 8.314 \times 373 \times 5$

ΔU = 189494.39 Joule

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

1. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series
$(x+y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3)+.....$

(1) $\dfrac{x+y-xy}{(1-x)(1-y)}$ (2) $\dfrac{x+y-xy}{(1+x)(1+y)}$

(3) $\dfrac{x+y+xy}{(1+x)(1+y)}$ (4) $\dfrac{x+y+xy}{(1-x)(1-y)}$

Sol. $|x| < 1$, $|y| < 1$, $x \neq y$
$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3)$ + ...
By multiplying and dividing $x - y$:

$$\frac{(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+......}{x-y}$$

$$=\frac{(x^2+x^3+x^4+......)-(y^2+y^3+y^4+......)}{x-y}$$

$$=\frac{\dfrac{x^2}{1-x}-\dfrac{y^2}{1-y}}{x-y}$$

$$=\frac{(x^2-y^2)-xy(x-y)}{(1-x)(1-y)(x-y)}$$

$$=\boxed{\frac{x+y-xy}{(1-x)(1-y)}}$$

2. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to :

(1) 176 (2) 336
(3) 352 (4) 84

Sol. Let t_{r+1} denotes

r + 1th term of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$

$$t_{r+1} = {}^{10}C_r \alpha^{10-r}(x)^{\frac{10-r}{9}}.\beta^r x^{-\frac{r}{6}}$$

$$= {}^{10}C_r\, \alpha^{10-r}\, \beta^r\, (x)^{\frac{10-r}{9}-\frac{r}{6}}$$

If t_{r+1} is independent of x

$$\frac{10-r}{9}-\frac{r}{6}=0 \quad \Rightarrow \quad r = 4$$

maximum value of t_5 is 10 K (given)

$\Rightarrow \quad {}^{10}C_4\, \alpha^6\, \beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\dfrac{\alpha^3}{2}+\dfrac{\alpha^3}{2}+\dfrac{\beta^2}{2}+\dfrac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4 \leq 16}$

So, $10\text{ K} = {}^{10}C_4 16$

$\Rightarrow$ K = 336

3. If a function f(x) defined by

$$f(x)=\begin{cases} ae^x+be^{-x}, & -1\leq x<1 \\ cx^2, & 1\leq x\leq 3 \\ ax^2+2cx, & 3<x\leq 4\end{cases}$$

be continuous for some a, b, c $\in$ R and $f'(0) + f'(2) = e$, then the value of of a is :

(1) $\dfrac{e}{e^2-3e-13}$ (2) $\dfrac{e}{e^2+3e+13}$

(3) $\dfrac{1}{e^2-3e+13}$ (4) $\dfrac{e}{e^2-3e+13}$

Sol. $$f(x)=\begin{cases} ae^x+be^{-x}, & -1\leq x<1 \\ cx^2, & 1\leq x\leq 3 \\ ax^2+2cx, & 3<x\leq 4\end{cases}$$

For continuity at x = 1

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$\Rightarrow$ $\boxed{ae + be^{-1} = c}$ $\Rightarrow$ $\boxed{b = ce - ae^2}$ ...(1)

For continuity at x = 3

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

$\Rightarrow$ 9c = 9a + 6c

$\Rightarrow$ c = 3a ...(2)

$f'(0) + f'(2) = e$

$(ae^x - be^x)_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow$ $\boxed{a - b + 4c = e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow$ $a(e^2 + 13 - 3e) = e$

$$\Rightarrow\ a = \frac{e}{e^2 - 3e + 13}$$

4. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$

(3) $\frac{4}{17}$ (4) $\frac{2}{5}$

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \rightarrow$ where box-II selected

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

{31, 37, 41, 43, 47}

$$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

$$= \frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{3}{4}} = \frac{8}{17}$$

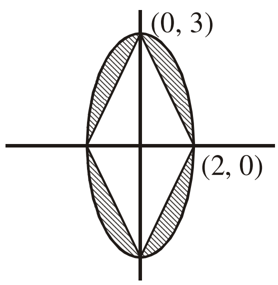
5. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

(1) $3(4 - \pi)$ (2) $6(\pi - 2)$

(3) $3(\pi - 2)$ (4) $6(4 - \pi)$

Sol. $\frac{|x|}{2}+\frac{|y|}{3}=1$

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$



Area of Ellipse = $\pi ab = 6\pi$

Required area,

$= \pi \times 2 \times 3$ – (Area of quadrilateral)

$$= 6\pi - \frac{1}{2}6\times 4$$

$= 6\pi - 12$

$= 6(\pi - 2)$

6. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S

(1) contains more than two elements.

(2) is a singleton.

(3) contains exactly two elements.

(4) is an empty set.

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow$ $-2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow$ $\lambda = 1, -\frac{1}{2}$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$= 2(1 + \lambda)$

whichis not equal to zero for

$\lambda = 1, -\frac{1}{2}$

7. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements :

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$,

where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:

(1) (P) is true and (Q) is false

(2) Both (P) and (Q) are false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

$$\text{So, } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$\Rightarrow$ $tr(A) = 2$

$\Rightarrow$ Q is true

8. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :

(1) If I will catch the train, then I reach the station in time.

(2) If I do not reach the station in time, then I will not catch the train.

(3) If I will not catch the train, then I do not reach the station in time.

(4) If I do not reach the station in time, then I will catch the train.

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$

is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

9. Let $y = y(x)$ be the solution of the differential equation,

$$\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$$. If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to :

(1) (2, 1) (2) $\left(2, \frac{3}{2}\right)$

(3) (1, –1) (4) (1, 1)

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$$

By integrating both sides :

$$\ell n\,|y+1| = -\ell n\,|2+\sin x| + \ell nK$$

$$\Rightarrow y + 1 = \frac{K}{2+\sin x} \qquad (y + 1 > 0)$$

$$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$$

Given $y(0) = 1 \Rightarrow K = 4$

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$$

So, (a, b) = (1, 1)

10. Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to :

(1) –7 (2) 7

(3) 9 (4) –27

Sol. σ^2 = variance

μ = mean

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

$\mu = 17$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17} = 17$$

$\Rightarrow 9a + b = 17$ ....(1)

$\sigma^2 = 216$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17} = 216$$

$$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17} = 216$$

$\Rightarrow a^2 81 - 18 \times 9a^2 + a^2\, 3 \times (35) = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9 \Rightarrow a = 3\ (a > 0)$

$\Rightarrow$ From (1), b = –10

So, a + b = –7

11. If the tangent to the curve y = x + sin y at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then :

(1) b = a
(2) $b = \frac{\pi}{2} + a$
(3) |b – a| = 1
(4) |a+b| = 1

Sol. Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\frac{2 - \frac{3}{2}}{\frac{1}{2} - 0} = 1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a} = 1$

$\frac{dy}{dx} = 1 + \cos y . \frac{dy}{dx}$ (from equation of curve)

$\left.\frac{dy}{dx}\right]_{x=a} = 1 + \cos b . \left.\frac{dy}{dx}\right]_{x=a}$

$\Rightarrow$ cos b = 0

$\Rightarrow$ sin b = ±1

Now, from curve y = x + sin y

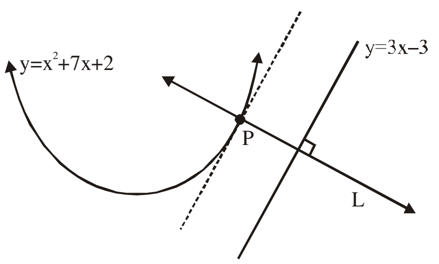
b = a + sin b

$\Rightarrow$ |b – a| = |sin b| = 1

12. Let P(h, k) be a point on the curve y = x^2 + 7x + 2, nearest to the line, y = 3x – 3. Then the equation of the normal to the curve at P is :

(1) x + 3y – 62 = 0
(2) x – 3y – 11 = 0
(3) x – 3y + 22 = 0
(4) x + 3y + 26 = 0

Sol.



Let L be the common normal to parabola y = x^2 + 7x + 2 and line y = 3x – 3

$\Rightarrow$ slope of tangent of y = x^2 + 7x + 2 at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}} = 3$

$\Rightarrow$ 2x + 7 = 3 $\Rightarrow$ x = –2 $\Rightarrow$ y = –8

So P(–2, –8)

Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

$\boxed{\text{Normal} : x + 3y + 26 = 0}$

13. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point :

(1) (0, 6, –2)
(2) (–2, 0, 1)
(3) (0, –6, 2)
(4) (2, 0, –1)

Sol. Two points on the line (L say) $\frac{x}{3} = \frac{y}{2}, z = 1$ are

(0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow$ 2(x – 1) –3(y – 2) –5(z–1) = 0

$\Rightarrow$ 2x – 3y – 5z + 9 = 0

14. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1,2,3...., then :

(1) $5S_6 + 6S_5 = 2S_4$

(2) $5S_6 + 6S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 + 2S_4 = 0$

(4) $6S_6 + 5S_5 = 2S_4$

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ ...(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ ...(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For n = 4

$\boxed{5S_6 + 6S_5 = 2S_4}$

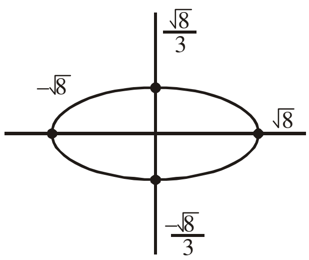
15. If $R = \{(x,y) : x,y \in Z,\ x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

(1) {–2, –1, 1,2} (2) {–1, 0, 1}

(3) {–2, –1, 0, 1, 2} (4) {0, 1}

Sol. $R = \{(x, y) : x, y \in z,\ x^2 + 3y^2 \le 8\}$

For domain of R^{-1}



Collection of all integral of y's

For x = 0, $3y^2 \le 8$

$\Rightarrow y \in \{-1, 0, 1\}$

16. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) $[-3, \infty)$ (2) $(-\infty, 9]$

(3) $(-\infty, -9] \cup [3, \infty)$ (4) $(-\infty, -3] \cup [9, \infty)$

Sol. Let three terms of G.P. are $\frac{a}{r}$, a, ar

product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$$S = \frac{3}{r} + 3r + 3$$

For r > 0

$$\frac{\frac{3}{r} + 3r}{2} \ge \sqrt{3^2} \quad \text{(By AM} \ge \text{GM)}$$

$$\Rightarrow \frac{3}{r} + 3r \ge 6 \quad ...(1)$$

For r < 0 $\frac{3}{r} + 3r \le -6$ ...(2)

From (1) & (2)

$S \in (-\infty - 3] \cup [9, \infty]$

17. A line parallel to the straight line 2x – y = 0 is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to :

(1) 5 (2) 6

(3) 8 (4) 10

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) is

$$\frac{xx_1}{4} - \frac{yy_1}{2} = 1 \quad (T = 0)$$

Slope : $\frac{1}{2}\frac{x_1}{y_1} = 2 \Rightarrow \boxed{x_1 = 4y_1}$ ...(1)

(x_1, y_1) lies on hyperbola

$$\Rightarrow \boxed{\frac{x_1^2}{4}-\frac{y_1^2}{2}=1} \qquad \ldots(2)$$

From (1) & (2)

$$\frac{(4y_1)^2}{4}-\frac{y_1^2}{2}=1 \Rightarrow 4y_1^2-\frac{y_1^2}{2}=1$$

$$\Rightarrow 7y_1^2=2 \Rightarrow \boxed{y_1^2=2/7}$$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$$= (21)y_1^2 = 21 \times \frac{2}{7}=6$$

18. The domain of the function $f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a]\cup[a, \infty)$. Then a is equal to :

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$

(3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$

Sol. $f(x)=\sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain :

$$-1\le\frac{|x|+5}{x^2+1}\le 1$$

Since $|x| + 5$ & $x^2 + 1$ is always positive

So $\frac{|x|+5}{x^2+1}\ge 0\ \forall x\in R$

So for domain :

$$\frac{|x|+5}{x^2+1}\le 1$$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$$\Rightarrow 0\le\left(|x|-\frac{1+\sqrt{17}}{2}\right)\left(|x|-\frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x|\ge\frac{1+\sqrt{17}}{2} \text{ or } |x|\le\frac{1-\sqrt{17}}{2} \quad \text{(Rejected)}$$

$$\Rightarrow x\in\left(-\infty,-\frac{1+\sqrt{17}}{2}\right]\cup\left[\frac{1+\sqrt{17}}{2},\infty\right)$$

So, $a=\frac{1+\sqrt{17}}{2}$

19. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $\frac{1}{2}\left(\sqrt{3}-i\right)$ (2) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

(3) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ (4) $\frac{1}{2}\left(1-i\sqrt{3}\right)$

Sol. The value of $\left(\frac{1+\sin 2\pi/9+i\cos 2\pi/9}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)$

$$=\left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$$

$$=\left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$=\left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$$

$$= \cos\frac{5\pi}{6}+i\sin 5\pi/6$$

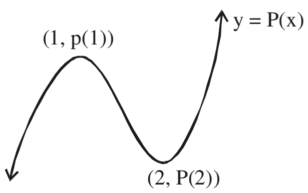
$$= -\frac{\sqrt{3}}{2}+i/2$$

20. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to:

(1) 12 (2) –24

(3) 6 (4) –12

Sol.



Since p(x) has realtive extreme at

x = 1 & 2

so p'(x) = 0 at x = 1 & 2

$\Rightarrow$ p'(x) = A(x – 1) (x – 2)

$\Rightarrow$ p(x) = $\int A(x^2-3x+2)dx$

p(x) = $A\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)+C$ ...(1)

P(1) = 8

From (1)

$$8 = A\left(\frac{1}{3}-\frac{3}{2}+2\right)+C$$

$$\Rightarrow 8 = \frac{5A}{6}+C \Rightarrow \boxed{48 = 5A+6C} \quad ...(3)$$

P(2) = 4

$$\Rightarrow 4 = A\left(\frac{8}{3}-6+4\right)+C$$

$$\Rightarrow 4 = \frac{2A}{3}+C \Rightarrow \boxed{12 = 2A+3C} \quad ...(4)$$

From 3 & 4, C = –12

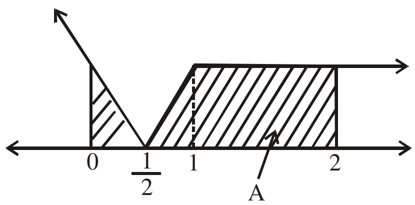
So P(0) = C = $\boxed{-12}$

21. The integral $\int_0^2 ||x-1|-x|dx$ is equal to______.

Sol. $\int_0^2 ||x-1|-x|dx$

Let f(x) ||x – 1|–x|

$$= \begin{cases} 1, & x \geq 1 \\ |1-2x|, & x \leq 1 \end{cases}$$



$$A = \frac{1}{2}+1 = \frac{3}{2}$$

or

$$\int_0^{1/2}(1-2x)dx+\int_{1/2}^1(2x-1)+\int_1^2 1dx$$

$$= \left[x-x^2\right]_0^{\frac{1}{2}}+\left[x^2-x\right]_{1/2}^1+\left[x\right]_1^2$$

$$= \boxed{3/2}$$

22. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$.

Then $|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$ is equal to ________.

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 8$$

$\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$

$\Rightarrow$ $4 - 2(\vec{a}.\vec{b} + \vec{a}.\vec{c}) = 8$

$\Rightarrow$ $\boxed{\vec{a}.\vec{b} + \vec{a}.\vec{c} = -2}$

$|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$

$= |a^2| + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$

$= 10 + 4(\vec{a}.\vec{b} + \vec{a}.\vec{c})$

$= 10 - 8$

$= \boxed{2}$

23. If $\lim\limits_{x \to 1} \dfrac{x + x^2 + x^3 + ... + x^n - n}{x - 1} = 820, (n \in N)$ then the value of n is equal to_______.

Sol. $\lim\limits_{x \to 1} \dfrac{x + x^2 + ... + x^2 - n}{x - 1} = 820$

$\Rightarrow$ $\lim\limits_{x \to 1} \left(\dfrac{x-1}{x-1} + \dfrac{x^2-1}{x-1} + ... \dfrac{x^n - 1}{x-1} \right) = 820$

$\Rightarrow$ $1 + 2 + ... + n = 820$

$\Rightarrow$ $n(n + 1) = 2 \times 820$

$\Rightarrow$ $n(n + 1) = 40 \times 41$

Since $n \in N$, so $\boxed{n = 40}$

24. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ______.

Sol. MOTHER

$1 \to E$

$2 \to H$

$3 \to M$

$4 \to O$

$5 \to R$

$6 \to T$

So position of word MOTHER in dictionary

$2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1$

$= \boxed{309}$

25. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ______.

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$$\Rightarrow \left| \frac{3 \times 1 + 4 \times 2 - k}{\sqrt{3^2 + 4^2}} \right| < 1$$

$\Rightarrow$ $|11 - k| < 5$

$\Rightarrow$ $6 < k < 16$

$\Rightarrow$ $k \in \{7, 8, 9, ... 15\}$ since $k \in I$

Number of K is $\boxed{9}$