AIEEE 2003
PHYSICS & CHEMISTRY

1. A particle of mass M and charge Q moving with velocity $\vec{v}$ describe a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is

(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) Zero (c) $BQ2\pi R$ (d) $BQ\,v2\pi R$

2. A particle of charge -16×10^{-18} coulomb moving with velocity $10ms^{-1}$ along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude $10^4 V/m$ is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

(a) $10^3 Wb/m^2$ (b) $10^5 Wb/m^2$ (c) $10^{16} Wb/m^2$ (d) $10^{-3} Wb/m^2$

3. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

4. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60^0. The torque needed to maintain the needle in this position will be

(a) $\sqrt{3}W$ (b) W (c) $\frac{\sqrt{3}}{2}W$ (d) 2W

5. The magnetic lines of force inside a bar magnet

(a) are from north-pole to south-pole of the magnet

(b) do not exist

(c) depend upon the area of cross-section of the bar magnet

(d) are from south-pole to north-pole of the magnet

6. Curie temperature is the temperature above which

(a) a ferromagnetic material becomes paramagnetic (b) a paramagnetic material becomes diamagnetic

(c) a ferromagnetic material becomes diamagnetic (d) a paramagnetic material becomes ferromagnetic

7. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of $5m/s^2$, the reading of the spring balance will be

(a) 24 N (b) 74 N (c) 15 N (d) 49 N

8. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f of a battery whose internal resistance is $0.5\,\Omega$. If the balance point is obtained at $l = 30$ cm from the positive end, the e.m.f. of the battery is

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{(100-0.5)}$ (c) $\frac{30(E-0.5i)}{100}$, where i is the current in the potentiometer wire (d) $\frac{30E}{100}$

9. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(a) each of these decreases (b) copper strip increases and that of germanium decreases

(c) copper strip decreases and that of germanium increases (d) each of these increases

10. Consider telecommunication through optical fibres. Which of the following statements is **not** true?

(a) Optical fibres can be of graded refractive index

(b) Optical fibres are subjective to electromagnetic interference from outside

(c) Optical fibres have extremely low transmission loss

(d) Optical fibres may have homogeneous core with a suitable cladding.

11. The thermo e.m.f. of a thermo-couple is $25\,\mu V/{}^0C$ at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermo couple. The smallest temperature difference that can be detected by this sytem is

(a) 16^0C (b) 12^0C (c) 8^0C (d) 20^0C

12. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is

(a) 0.180 g (b) 0.141 g (c) 0.126 g (d) 0.242 g

13. Dimension of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meaning, are

(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

14. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32\, I_X$ (b) $I_Y = 16\, I_X$ (c) $I_Y = I_X$ (d) $I_Y = 64\, I_X$

15. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hours (b) 80 hours (c) 40 hours (d) 20 hours

16. A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is

(a) $\frac{L}{4}$ (b) 2L (c) 4 L (d) $\frac{L}{2}$

17. Which of the following radiations has the least wavelength?

(a) γ-rays (b) β-rays (c) α-rays (d) X-rays

18. When a U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed 'u', the recoil speed of the residual nucleus is

(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$ (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

19. Two spherical bodies of mass M and 5M & radii R & 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) 2.5 R (b) 4.5 R (c) 7.5 R (d) 1.5 R

20. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the

(a) crystal structure (b) variation of the number of charge carriers with temperature

(c) type of bonding (d) variation of scattering mechanism with temperature

21. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is

(a) 12 m (b) 18 m (c) 24 m (D) 6 m

22. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30^0 with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

$[g = 10m/s^2, \sin30^0 = \frac{1}{2}, \cos30^0 = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m (c) 2.60m (d) 8.66m

23. An ammeter reads up to 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A the value of the required shunt is

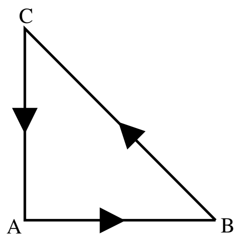
(a) 0.03 Ω (b) 0.3 Ω (c) 0.9 Ω (d) 0.09 Ω

24. The physical quantities not having same dimensions are

(a) torque and work (b) momentum and Planck's constant

(c) stress and Young's modulus (d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

25. Three forces start acting simultaneously on a particle moving with velocity, $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC. The particle will now move with velocity

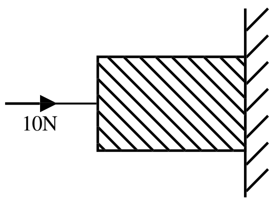


(a) less than $\vec{v}$ (b) greater than $\vec{v}$ (c) $|v|$ in the direction of the largest force BC (d) $\vec{v}$, remaining unchanged

26. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$ (c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$

27. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The co-efficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N (b) 50 N (c) 100 N (d) 2 N

28. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is

(a) 0.02 (b) 0.03 (c) 0.04 (d) 0.01

29. Consider the following two statements:

(A) Linear momentum of a system of particles is zero

(B) Kinetic energy of a system of particles is zero

Then (a) A does not imply B and B does not imply A

(b) A implies B but B does not imply A

(c) A does not imply B but B implies A (d) A implies B and B implies A

30. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils (c) the materials of the wires of the coils.
(d) the currents in the two coils

31. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is
(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$ (c) P (d) $\frac{PM}{M+m}$

32. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is
(a) Both the scales read M kg each (b) The scale of the lower one reads M kg and of the upper one zero
(c) The reading of the two scales can be anything but the sum of the reading will be M kg
(d) Both the scales read M/2 kg each

33. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J

34. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45^0 with the vertical, the escape velocity will be
(a) $11\sqrt{2}$ km/s (b) 22 km/s (c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km/s

35. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is
(a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

36. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of
(a) second law of thermodynamics (b) conservation of momentum
(c) conservation of momentum (d) first law of thermodynamics

37. Two particles A and B of equal masses are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is
(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1}{k_2}$

38. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%

39. The displacement y of a wave travelling in the x-direction is given by $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$ metres where x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1}, is
(a) 300 (b) 600 (c) 1200 (d) 200

40. When the current changes from +2A to -2A in 0.05 second, an e.m.f. of 8V is induced in a coil. The coefficient of self-induction of the coil is

(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H

41. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

42. The core of any transformer is laminated so as to

(a) reduce the energy loss due to eddy currents (b) make it light weight

(c) make it robust and strong (d) increase the secondary voltage

43. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$ (b) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$

(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$ (d) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$

44. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is

(a) 0.4 ln 2 (b) 0.2 ln 2 (c) 0.1 ln 2 (d) 0.8 ln 2

45. A nucleus with Z = 92 emits the following in a sequence:

$\alpha, \beta^-, \beta^- \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. Then Z of the resulting nucleus is

(a) 76 (b) 78 (c) 82 (d) 74

46. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photo electrons (of mass m) coming out are respectively v_1 and v_2, then

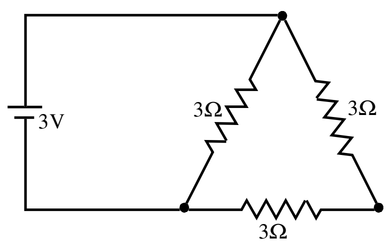
(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$

(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

47. Which of the following cannot be emitted by radioactive substances during their decay?

(a) Protons (b) Neutrinoes (c) Helium nuclei (d) Electrons

48. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be



(a) 1 A (b) 1.5 A (c) 2 A (d) 1/3 A

49. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

(a) decreases (b) remains unchanged (c) becomes infinite (d) increases

50. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is

(a) -4 (b) 4 (c) $4\sqrt{2}$ (d) 8

51. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P a distance $\frac{R}{2}$ from the centre of the shell is

(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$ (c) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_0 R}$

52. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is

(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule (c) 4×10^{-10} joule (d) 32×10^{-32} joule

53. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$ (c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

54. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is

(a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$

55. Which of the following parameters does not characterize the thermodynamic state of matter?

(a) temperature (b) Pressure (c) Work (b) Volume

56. A Carnot engine takes 3×10^6 cal. of heat from a reservoir at 627^0C, and gives it to a sink at 27^0C. The work done by the engine is

(a) 4.2×10^6 J (b) 8.4×10^6 J (c) 16.8×10^6 J (d) Zero

57. A spring of spring constant 5×10^3N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 12.50 N-m (b) 18.75 N-m (c) 25.00 N-m (d) 6.25 N-m

58. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is

(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz

59. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(a) 256 + 2 Hz (b) 256 - 2 Hz (c) 256 - 5 Hz (d) 256 + 5 Hz

60. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true?

(a) K.E. is maximum when x = 0 (b) T.E is zero when x = 0

(c) K.E is maximum when x is maximum (d) P.E. is maximum when x = 0

61. In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$ given that the repulsive potential energy between the two nuclei is $\sim7.7\times10^{-14}$ J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's Constant $k = 1.38\times10^{-23}$ J/K]

(a) 10^7 K (b) 10^5 K (c) 10^3 K (d) 10^9 K

62. Which of the following atoms has the lowest ionization potential?

(a) ${}^{14}_{7}N$ (b) ${}^{133}_{55}Cs$ (c) ${}^{40}_{18}Ar$ (d) ${}^{16}_{8}O$

63. The wavelengths involved in the spectrum of deuterium $\left({}^{2}_{1}D\right)$ are slightly different from that of hydrogen spectrum, because

(a) the size of the two nuclei are different (b) the nuclear forces are different in the two cases

(c) the masses of the two nuclei are different

(d) the attraction between the electron and the nucleus is different in the two cases

64. In the middle of the depletion layer of a reverse biased p-n junction, the

(a) electric field is zero (b) potential is maximum

(c) electric field is maximum (d) potential is zero

65. If the binding energy of the electron in a hydrogen atom is 13.6eV, the energy required to remove the electron from the first excited state of Li^{++} is

(a) 30.6eV (b) 13.6 eV (c) 3.4 eV (d) 122.4 eV

66. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

67. A rocket with a lift-off mass 3.5×10^{4} kg is blasted upwards with an initial acceleration of $10 m/s^{2}$. Then the initial thrust of the blast is

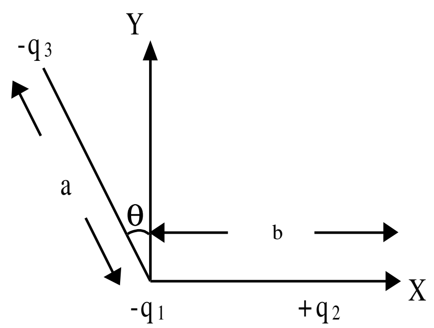
(a) 3.5×10^{5} N (b) 7.0×10^{5} N (c) 14.0×10^{5} N (d) 1.75×10^{5} N

68. To demonstrate the phenomenon of interference, we require two sources which emit radiation

(a) of nearly the same frequency (b) of the same frequency

(c) of different wavelengths (d) of the same frequency and having a definite phase relationship

69. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(a) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta$ (c) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\sin\theta$

70. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be

(a) 750 watt (b) 500 watt (c) 250 watt (d) 1000 watt

71. The image formed by an objective of a compound microscope is

(a) virtual and diminished (b) real and diminished (c) real and enlarged (d) virtual and enlarged

72. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by

(a) Rayleigh Jeans law (b) Planck's law of radiation

(c) Stefan's law of radiation (d) Wien's law

73. To get three images of a single object, one should have two plane mirrors at an angle of

(a) 60^0 (b) 90^0 (c) 120^0 (d) 30^0

74. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to

(a) two (b) three (c) four (d) one

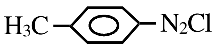
75. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be

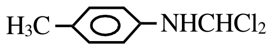
(a) 200% (b) 100% (c) 50% (d) 300%

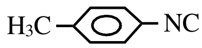
76. Which of the following could act as apropellant for rockets?

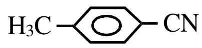
(a) Liquid oxygen + liquid argon (b) Liquid hydrogen + liquid oxygen
(c) Liquid nitrogen + liquid oxygen (d) Liquid hydrogen + liquid nitrogen

77. The reaction of chloroform with alcoholic KOH and p-toluidine forms

(a) 

(b) 

(c) 

(d) 

78. Nylon threads are made of

(a) polyester polymer (b) polyamide polymer (c) polyethylene polymer (d) polyvinyl polymer

79. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2$ NH is

(a) $(CH_3)_2NH < NH_3 < CH_3NH_2$ (b) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(c) $CH_3NH_2 < (CH_3)_2NH < NH_3$ (d) $CH_3NH_2 < NH_3 < (CH_3)_2NH$

80. Bottles containing C_6H_5l and $C_6H_5CH_2I$ lost their original labels. They were labelled A and B for testing A and B were separately taken in test tubes and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance B gave a yellow precipitate. Which one of the following statements is true for this experiment?

(a) A and $C_6H_5CH_2I$ (b) B and C_6H_5I
(c) Addition of HNO_3 was unnecessary (d) A was C_6H_5I

81. The internal energy change when a system goes from state A to B is 40 kJ/mole. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy?

(a)> 40 kJ (b) < 40kJ (c) Zero (d) 40 kJ

82. If at 298 K the bond energies of C-H, C-C, C = C and H-H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction $H_2C = CH_2(g) + H_2(g) \rightarrow H_3C - CH_3(g)$ at 298 K will be

(a) -250 kJ (b) + 125 kJ (c) -125 kJ (d) + 250 kJ

83. The radionucleide $^{234}_{90}Th$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionucleide are

(a) 94 and 230 (b) 90 and 230 (c) 92 and 230 (d) 92 and 234

84. The half-life of a radioactive isotope is three hours. If the initial mass of the isotope were 256 g, the mass of it remaining undecayed after 18 hours would be

(a) 8.0 g (b) 12.0 g (c) 16.0 g (d) 4.0 g

85. If liquids A and B form an ideal solution

(a) the entropy of mixing is zero (b) the free energy of mixing is zero
(c) the free energy as well as the entropy of mixing are each zero (d) the enthalpy of mixing is zero

86. The radius of La^{3+} (Atomic number of La = 57) is 1.06Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71) ?

(a) 1.40Å (b) 1.06Å (c) 0.85Å (d) 1.60Å

87. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it?

(a) In acidic solutions protons coordinate with ammonia molecules forming NH^+_4 ions and NH_3 molecules are not available
(b) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali
(c) Copper hydroxide is an amphoteric substance
(d) In acidic solutions hydration protects copper ions.

88. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of the complex is

(a) $[Co(NH_3)_3Cl_3]. 2NH_3$ (b) $[Co(NH_3)_4Cl_2] Cl. NH_3$ (c) $[Co(NH_3)_4Cl]Cl_2 . NH_3$ (d) $[Co(NH_3)_5Cl] Cl_2$

89 In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is

(a) 0 (b) +1 (c) +2 (d) -1

90. In curing cement plasters water is sprinkled from time to time. This helps in

(a) developing interlocking needle-like crystals of hydrated silicates

(b) hydrating sand and gravel mixed with cement

(c) converting sand into silicic acid (d) keeping it cool

91. Which one of the following statements is not true?

(a) pH + pOH = 14 for all aqueous solutions (b) The pH of 1×10^{-8} M HCI is 8

(c) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode

(d) The conjugate base of $H_2PO_4^-$ is HPO^{2-}_4

92. On mixing a certain alkane with chlorine and irradiating it with ultravioletlight, it forms only one monochloroalkane. This alkane could be

(a) pentane (b) isopentane (c) neopentane (d) propane

93. Butene-1 may be converted to butane by reaction with

(a) Sn - HCI (b) Zn - Hg (c) Pd/H_2 (d) Zn - HCI

94. What may be expected to happen when phosphine gas is mixed with chlorine gas?

(a) PCI_3 and HCI are formed and the mixture warms up

(b) PCI_5 and HCI are formed and the mixture cools down

(c) $PH_3.Cl_2$ is formed with warming up (d) The mixture only cools down

95. The number of d-electrons retained in Fe^{2+} (At.no.of Fe = 26) ion is

(a) 4 (b) 5 (c) 6 (d) 3

96. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that

(a) oxygen in air reacts with the emitted HCI gas to form a cloud of chlorine gas

(b) strong affinity of HCI gas for miosture in air results in forming of droplets of liquid solution which appears like a cloudy smoke.

(c) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards it self. This moisture forms droplets of water and hence the cloud.

(d) concentrated hydrochloric acid emits strongly smelling HCI gas all the time.

97. An ether is more volatile than an alcohol having the same molecular formula. This is due to

(a) alcohols having resonance structures (b) inter-molecular hydrogen bonding in ethers

(c) inter-molecular hydrogen bonding in alcohols (d) dipolar character of ethers

98. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite

(a) is an allotropic form of diamond (b) has molecules of variable molecular masses like polymers

(c) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds

(d) is a non-crystalline substance

99. According to the Periodic Law of elements, the variation in properties of elements is related to their

(a) nuclear masses (b) atomic numbers (c) nuclear neutron-proton number ratios (d) atomic masses

100. Which one of the following statements is correct?

(a) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl

(b) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution

(c) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$.

(d) Manganese salts give a violet borax bead test in the reducing flame

101. Glass is a

(a) super-cooled liquid (b) gel (c) polymeric mixture (d) micro-crystalline solid

102. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)}.\frac{h}{2\pi}$. This momentum for an s-electron will be given by

(a) zero (b) $\frac{h}{2\pi}$ (c) $\sqrt{2}.\frac{h}{2\pi}$ (d) $+\frac{1}{2}.\frac{h}{2\pi}$

103. How many unit cells are present in a cubeshaped ideal crystal of NaCl of mass 1.00 g?
[Atomic masses: Na = 23, Cl = 35.5]

(a) 5.14×10^{21} unit cells (b) 1.28×10^{21} unit cells

(c) 1.71×10^{21} unit cells (d) 2.57×10^{21} unit cells

104. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it?

(a) The C = O bond is weaker than the C-O bond

(b) The anion $HCOO^-$ has two resonating structures

(c) The anion is obtained by removal of a proton from the acid molecule

(d) Electronic orbitals of carbon atom are hybridised

105. Which one of the following characteristics is not correct for physical adsorption?

(a) Adsorption increases with incresae in temperature

(b) Adsorption is spontaneous (c) Both enthalpy and entropy of adsorption are negative

(d) Adsorption on solids is reversible

106. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25^0C. The equilibrium constant of the reaction at 25^0C will be

(a) 29.5×10^{-2} (b) 10 (c) 1×10^{10} (d) 1×10^{-10}

107. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria

(a) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$ (b) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$ (c) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$ (d) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$

108. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product number will be

(a) 4×10^{-10} (b) 1×10^{-15} (c) 1×10^{-10} (d) 4×10^{-15}

109. What volume of hydrogen gas, at 273 K and 1 atm, pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reducti on of boron trichloride by hydrogen?

(a) 67.2 L (b) 44.8 L (c) 22.4 L (d) 89.6 L

110. For the reaction equilibrium $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is

(a) 3×10^{-1} mol L^{-1} (b) 3×10^{-3} mol L^{-1} (c) 3×10^{3} mol L^{-1} (d) 3.3×10^{2} mol L^{-1}

111. Consider the reaction equilibrium $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$; $\Delta H^0 = -198$ kJ. On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is

(a) increasing temperature as well as pressure (b) lowering the temperature and increasing the pressure

(c) any value of temperature and pressure (d) lowering of temperature as well as pressure

112. Which one of the following is an amphoteric oxide?

(a) Na_2O (b) SO_2 (c) B_2O_3 (d) ZnO

113. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is

(a) HgI_2 (b) HgO (c) Pb_3O_4 (d) $(NH_4)_2Cr_2O_7$

114. Standard reduction electrode potentials of three metals A,B&C are respectively +0.5 V, -3.0 V & -1.2 V. The reducing, powers of these metals are

(a) A > B > C (c) C > B > A (c) A > C > B (d) B > C > A

115. Which one of the following substances has the highest proton affinity?

(a) H_2S (b) NH_3 (c) PH_3 (d) H_2O

116. In a 0.2 molal aqueous solution of a weak acid HX the degree of ionization is 0.3. Taking k_f for water as 1.85, the freezing point of the solution will be nearest to

(a) -0.360^0C (b) -0.260^0C (c) $+0.480^0C$ (d) -0.480^0C

117. When during electrolysis of a solution of $AgNO_3$ 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be

(a) 10.8 g (b) 21.6 g (c) 108 g (d) 1.08 g

118. For the redox reaction $Zn(s) + Cu^{2+}(0.1\ M) \rightarrow Zn^{2+}(1M) + Cu(s)$ taking place in a cell, E^0_{cell} is 1.10 volt. E_{cell} for the cell will be $\left(2.303\dfrac{RT}{F} = 0.0591\right)$

(a) 1.80 volt (b) 1.07 volt (c) 0.82 volt (d) 2.14 volt

119. In respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct?

(a) A is adsorption factor (b) E_a is energy of activation

(c) R is Rydberg's constant (d) k is equilibrium constant

120. A reduction in atomic size with increase in atomic number is a characteristic of element of

(A) d-block (b) f-block (c) radioactive series (d) high atomic masses

121. The IUPAC name of $CH_3COCH(CH_3)_2$ is

(a) 2-methyl-3-butanone (b) 4-methylisopropyl ketone (c) 3-methyl-2-butanone (d) Isopropylmethyl ketone

122. When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be

(a) $CH_2 = CH - CH_2OH$ (b) $CH_3 - CH_2 - CH_2OH$

(c) $CH_3 - CH_2 - CHO$ (d) $CH_3 - CH_2 - COOH$

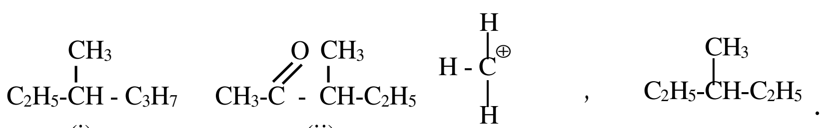
123. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels

(a) in a wavy path (b) in a straight line path (c) with an accelerated velocity (d) in a circular path

124. The general formula $C_nH_{2n}O_2$ could be for open chain

(a) carboxylic acids (b) diols (c) dialdehydes (d) deketones

125. Among the following four structures I to IV.



(i) (ii) (iii) (iv) . It is true that

(a) only I and II are chiral compounds (b) only III i a chiral compound

(c) only II and IV are chiral compounds (d) all four are chiral compounds

126. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?

(a) $Cr_2O^{2-}_7$ and H_2O are formed (b) CrO^{2-}_4 is reduced to +3 state of Cr

(c) CrO^{2-}_4 is oxidized to +7 state of Cr (d) Cr^{3+} and $Cr_2O^{2-}_7$ are formed

127. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be

(a) tin (b) sodium (c) magnesium (d) mercury

128. The substance not likely to contain $CaCO_3$ is

(a) calcined gypsum (b) sea shells (c) dolomite (d) a marble statue

129. Complete hydrolysis of cellulose gives

(a) D-ribose (b) D-glucose (c) L-glucose (d) D-fructose

130. Which one of the following nitrates will leave behind a metal on strong heating?

(a) Copper nitrate (b) Manganese nitrate (c) Silver nitrate (d) Ferric nitrate

131. During dehydration of alcohols to alkenes by heating with conc. H_2SO_4 the initiation step is

(a) formation of carbocation (b) elimination of water

(c) formation of an ester (d) protonation of alcohol molecule

132. The solubilities of carbonates decrease down the magnesium group due to a decrease in

(a) hydration energies of cations (b) inter-ionic attraction

(c) entropy of solution formation (d) lattice energies of solids

133. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value

(a) slightly higher than that when the thunderstorm is not there

(b) uninfluenced by occurence of thunderstorm

(c) which depends on the amount of dust in air

(d) slightly lower than that of rain water without thunderstorm

134. The reason for double helical structure of DNA is operation of

(a) dipole-dipole interaction (b) hydrogen bonding (c) electrostatic attractions (d) van der Waals' forces

135. 25 ml of a solution of barrium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a litre value of 35 ml. The molarity of barium hydroxide solution was

(a) 0.14 (b) 0.28 (c) 0.35 (d) 0.07

136. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is

(a) $-\Delta G = RT\, ln\, K_c$ (b) $\Delta G^0 = RT\, ln\, K_c$ (c) $-\Delta G^0 = RT\, ln\, K_c$ (d) $\Delta G = RT\, ln\, K_c$

137. The rate law for a reaction between the substances A and B is given by Rate $= k[A]^n\ [B]^m$ On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as

(a) $(m + n)$ (b) $(n - m)$ (c) $2^{(n-m)}$ (d) $\frac{1}{2^{(m+n)}}$

138. Ethyl isocyanide on hydrolysis in acidic medium generates

(a) propanoic acid and ammonium salt (b) ethanoic acid and ammonium salt

(c) methylamine salt and ethanoic acid (d) ethylamine salt and methanoic acid

139. The enthalpy change for a reaction does not depend upon

(a) use of different reactants for the same product (b) the nature of intermediate reaction steps

(c) the differences in initial or final temperatures of involved substances

(d) the physical states of reactants and products

140. A pressure cooker reduces cooking time for food because

(a) boiling point of water involved in cooking is increased

(b) the higher pressure inside the cooker crushes the food material

(c) cooking involves chemical changes helped by a rise in temperature

(d) heat is more evenly distributed in the cooking space

141. For the reaction system: $2NO(g) + O_2(g) \rightarrow 2NO_2(g)$ volume is suddenly reduce to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO, the rate of reaction will

(a) diminish to one-eighth of its initial value

(b) increase to eight times of its initial value

(c) increase to four times of its initial value

(d) diminish to one-fourth of its initial value

142. Several blocks of magnesium are fixed to the bottom of a ship to

(a) make the ship lighter

(b) prevent action of water and salt

(c) prevent puncturing by under-sea rocks

(d) keep away the sharks

143. Which one of the following pairs of molecules will have permanent dipole moments for both members?

(a) NO_2 and CO_2 (b) NO_2 and O_3 (c) SiF_4 and CO_2 (d) SiF_4 and NO_2

144. Which one of the following groupings represents a collection of isoelectronic species? (At. nos,: 55, Br:35)

(a) N^{3-}, F^-, Na^+ (b) Be, Al^{3+}, Cl^- (c) Ca^{2+}, Cs^+, Br (d) Na^+, Ca^{2+}, Mg^{2+}

145. Which one of the following processes will produce hard water?

(a) Saturation of water with $MgCO_3$

(b) Saturation of water with $CaSO_4$

(c) Addition of Na_2SO_4 to water

(d) Saturation of water with $CaCO_3$

146. Which one of the following compounds has the smallest bond angle in its molecule?

(a) OH_2 (b) SH_2 (c) NH_3 (d) SO_2

147. The pair of species having identical shapes for molecules of both species is

(a) XeF_2, CO_2 (b) BF_3, PCl_3

(c) PF_5, IF_5 (d) CF_4, SF_4

148. The atomic numbers of vanadium (V), Chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionization enthalpy?

(a) Cr (b) Mn (c) Fe (d) V

149. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen

(a) $5 \rightarrow 2$ (b) $4 \rightarrow 1$ (c) $2 \rightarrow 5$ (d) $3 \rightarrow 2$

150. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately

(a) 10^{-31} metres

(b) 10^{-16} metres

(c) 10^{-25} metres

(d) 10^{-33} metres Planck's constant, $h = 6.63 \times 10^{-34}$ Js.

AIEEE 2003
MATHEMATICS

1. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx=F(k)-F(1)$ then one of the possible values of k, is

(a) 64 (b) 15 (c) 16 (d) 63

2. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then median of the new set

(a) remains the same as that of the original set (b) is increased by 2

(c) is decreased by 2 (d) is two times the original median

3. $\lim\limits_{n\to\infty}\frac{1+2^4+3^4+...n^4}{n^5}-\lim\limits_{n\to\infty}\frac{1+2^3+3^3+...n^3}{n^5}$

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

4. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(a) $t_2=t_1+\frac{2}{t_1}$ (b) $t_2=-t_1-\frac{2}{t_1}$ (c) $t_2=-t_1+\frac{2}{t_1}$ (d) $t_2=t_1-\frac{2}{t_1}$

5. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then

(a) $r>2$ (b) $2<r<8$ (c) $r<2$ (d) $r=2$.

6. The degree and order of the differential equation of the family of all parabolas whose axis is X-axis, are respectively.

(a) 2, 3 (b) 2, 1 (c) 1, 2 (d) 3, 2

7. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

(a) 9 (b) 1 (c) 5 (d) 7

8. If $f(y)=e^y$, $g(y)=y$; $y>0$ and $F(t)=\int_0^t f(t-y)g(y)$, then

(a) $F(t)=te^{-t}$ (b) $F(t)=1-te^{-t}(1+t)$ (c) $F(t)=e^t-(1+t)$ (d) $F(t)=te^t$.

9. The function $f(x)=\log\left(x+\sqrt{x^2+1}\right)$, is

(a) neither an even nor an odd function (b) an even function

(c) an odd function (d) a periodic function

10. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c},\frac{b}{a}$ and $\frac{c}{b}$ are in

(a) Arithmetic - Geometric Progression (b) Arithmetic Progression

(c) Geometric Progression (d) Harmonic Progression

11. If the system of linear equations

$x+2ay+az=0$ $x+3by+bz=0$ $x+4cy+cz=0$

has a non-zero solution, then a, b, c

(a) satisfy $a+2b+3c=0$ (b) are in A.P. (c) are in G.P. (d) are in H.P.

12. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$ (b) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$ (d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$.

13. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2pxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(a) $pq = -1$ (b) $p = q$ (c) $p = -q$ (d) $pq = 1$

14. Locus of a centriod of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is

(a) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$ (b) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

(c) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$ (d) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

15. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x} = k$, the value of k is

(a) $-\frac{2}{3}$ (b) 0 (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

16. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead, the force $\vec{P}$ are turned through an angle α, then the moment of couple becomes

(a) $\vec{H}\sin\alpha - \vec{G}\cos\alpha$ (b) $\vec{G}\sin\alpha - \vec{H}\cos\alpha$ (c) $\vec{H}\sin\alpha + \vec{G}\cos\alpha$ (d) $\vec{G}\sin\alpha + \vec{H}\cos\alpha$

17. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is

(a) 2 : 3 : 1 (b) 3 : 1 : 1 (c) 2 : 3 : 2 (d) 1 : 2 : 3

18. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then P (X = 1) is

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

19. If $f(x) = x^n$, then the value of $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots\ldots\ldots\ldots\ldots\frac{(-1)^n f^n(1)}{n!}$ is

(a) 1 (b) 2^n (c) $2^n - 1$ (d) 0

20. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to

(a) 3 (b) 0 (c) 1 (d) 2

21. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ to the point $5\hat{i}+4\hat{j}-\hat{k}$. The total work done by the forces is

(a) 50 units (b) 20 units (c) 30 units (d) 40 units

22. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

23. The area of the region bounded by the curves $y=|x-1|$ and $y=3-|x|$ is

(a) 6 sq. units (b) 2 sq. units (c) 3 sq. units (d) 4 sq. units

24. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(a) 39 (b) 26 (c) $11\frac{4}{13}$ (d) 13

25. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b' z = c'y + d'$ will be perpendicular, if and only if

(a) $aa' + cc' + 1 = 0$ (b) $aa' + bb' + cc' + 1 = 0$

(c) $aa' + bb' + cc' = 0$ (d) $(a + a')(b + b') + (c + c') = 0$

26. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{1} = \frac{z-5}{1}$ are coplanar if

(a) k = 3 or -2 (b) k = 0 or -1 (c) k = 1 or -1 (d) k = 0 or -3

27. If $f(a + b - x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (c) $\frac{a+b}{2}\int_a^b f(x)dx$ (d) $\frac{b-a}{2}\int_a^b f(x)dx$

28. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by

(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$ (c) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

29. Two stones are projected from the top of a cliff h metres high, with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected at an angle θ to the horizontal then the θ equals

(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$ (c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

30. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) ω^2 (b) 0 (c) 1 (d) ω

31. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is

(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$ (c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$

32. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(a) are vertices of a triangle (b) lie on a straight line (c) lie on an ellipse (d) lie on a circle

33. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $\text{Arg}(z) - \text{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to

(a) -i (b) 1 (c) -1 (d) i.

34. Let Z_1 and Z_2 be two roots of the equation $x^2 + aZ + b = 0$ being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then

(a) $a^2 = 4b$ (b) $a^2 = b$ (c) $a^2 = 2b$ (d) $a^2 = 3b$

35. The solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$, is

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$ (b) $(x-2)=ke^{2\tan^{-1}y}$ (c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$ (d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

36. Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g(x) be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)\,dx$, is

(a) $e+\frac{e^2}{2}+\frac{5}{2}$ (b) $e-\frac{e^2}{2}-\frac{5}{2}$ (c) $e+\frac{e^2}{2}-\frac{3}{2}$ (d) $e-\frac{e^2}{2}-\frac{3}{2}$

37. The lines 2x - 3y = 5 and 3x - 4y = 7 are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y = 62$ (b) $x^2 + y^2 + 2x - 2y = 62$

(c) $x^2 + y^2 + 2x - 2y = 47$ (d) $x^2 + y^2 - 2x + 2y = 47$

38. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{x-1}{4}$ and $P(C)=\frac{1-2x}{4}$. The set of possible values of x are in the interval.

(a) [0, 1] (b) $\left[\frac{1}{3},\frac{1}{2}\right]$ (c) $\left[\frac{1}{3},\frac{2}{3}\right]$ (d) $\left[\frac{1}{3},\frac{13}{3}\right]$

39. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

40. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^3 + (3a - 1)x + 2 = 0$ is twice as large as the other is

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

41. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is

(a) 6th term (b) 7th term (c) 5th term (d) 8th term

42. The number of integral terms in the expansion of $\left(\sqrt{3}+8\sqrt{5}\right)^{256}$ is

(a) 35 (b) 32 (c) 33 (d) 34

43. If ^nC_r denotes the number of combination of n things taken r at a time, then the expression $^nC_{r+1} + {}^nC_{r-1} + 2x\,{}^nC_r$ equals

(a) $^{n+1}C_{r+1}$ (b) $^{n+2}C_r$ (c) $^{n+2}C_{r+1}$ (d) $^{n+1}C_r$

44. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time.

(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$ (c) $\frac{f\cos\alpha}{u}$ (d) u sin α.

45. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot.

(a) 80 m (b) 20 m (c) 40 m (d) 60 m

46. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is

(a) $\frac{64}{3}$ (b) $\frac{8}{3}$ (c) $\frac{16}{3}$ (d) $\frac{32}{3}$

47. If in a triangle ABC $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c

(a) satisfy a+ b = c (b) are in A.P. (c) are in G.P. (d) are in H.P.

48. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2|\vec{c}|$ then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to

(a) 1 (b) 0 (c) -7 (d) 7

49. The value of the integral $I=\int_0^1 x(1-x)^n\,dx$ is

(a) $\frac{1}{n+1}+\frac{1}{n+2}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1}-\frac{1}{n+2}$

50. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 0 (b) 3 (c) 2 (d) 1

51. The radius of the circle in which the sphere
$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(a) 4 (b) 1 (c) 2 (d) 3

52. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(-1, 1, 2). Then the angle between the faces OAB and ABC will be

(a) 90^0 (b) $\cos^{-1}\left(\frac{19}{35}\right)$ (c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^0

53. Let f(a) = g(a) = k and their nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+f(a)}{g(x)-f(x)}=4$ then the value of k is

(a) 0 (b) 4 (c) 2 (d) 1

54. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x^3]}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

55. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of 'c' is

(a) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$ (b) $\frac{1}{2}a_2^2+b_2^2-a_1^2-b_1^2$

(c) $a_1^2-a_2^2+b_1^2-b_2^2$ (d) $\frac{1}{2}\left(a_1^2+a_2^2+b_1^2+b_2^2\right)$

56. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, (a, b, b^2) and (a, c, c^2) are non-coplanar, then the product abc equals

(a) 0 (b) 2 (c) -1 (d) 1

57. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

(a) 3 (b) 2 (c) 4 (d) 1

58. If the function $f(x) = 2x^2 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) $\frac{1}{2}$ (b) 3 (c) 1 (d) 2

59. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0 \quad , & x = 0 \end{cases}$ then f(x) is

(a) discontinuous every where (b) continuous as well as differentiable for all x

(c) continuous for all x but not differentiable at x = 0 (d) neither differentiable nor continuous at x = 0

60. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (b) $(0, 2)$ (c) $(-1, 0) \cup (0, 2)$ (d) $(1, 2) \cup (2, \infty)$

61. If f: R $\rightarrow$R satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7n(n+1)}{2}$ (b) $\frac{7n}{2}$ (c) $\frac{7(n+1)}{2}$ (d) 7n+(n+1)

62. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) -2 (b) 2 (c) 1 (d) -1

63. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

(a) H.P (b) A.G.P (c) A.P (d) G.P.

64. In an experiment with 15 observations on x, the following results were available: $\Sigma x^2 = 2830,\ \Sigma x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is

(a) 8.33 (b) 78.00 (c) 188.66 (d) 177.33

65. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(a) 346 (b) 140 (c) 196 (d) 280

66. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A_2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = 2ab, \beta = a^2 + b^2$ (b) $\alpha = a_2 + b_2, \beta = ab$ (c) $\alpha = a^2 + b^2,\ \beta = 2ab$ (d) $\alpha = a^2 + b^2,\ \beta = a^2 - b^2$

67. The number of ways in which 6 men and 5 women can dine at a found table if no two women are to sit together is given by

(a) 7×5 (b) 6×5 (c) 30 (d) 5×4

68. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a

(a) parallelogram but not a rhombus (b) square (c) rhombus (d) rectangle

69. If $\vec{u},\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0 (c) $\vec{u}.\vec{v}\times\vec{w}$ (d) $\vec{u}.\vec{w}\times\vec{v}$

70. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for

(a) $|a|\geq\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}<|a|<\frac{1}{\sqrt{2}}$ (c) all real values of a (d) $|a|<\frac{1}{2}$

71. Two system of rectangular axes have the same origin. If a plane cuts them at distances a,b,c and a',b',c' from the origin then

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$ (b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$ (d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

72. If $\left(\frac{1+i}{1-i}\right)^x=1$ then

(a) x = 2n+1, where n is any positive integer (b) x = 4n, where n is any positive integer

(c) x = 2n, where n is any positive integer (d) x = 4n+1, where n is any positive integer

73. A function f from the set of natural numbers to integers defined by $f(n)=\begin{cases}\frac{n-1}{2}, & \text{when n is odd}\\ \frac{n}{2}, & \text{when n is even}\end{cases}$ is

(a) neither one-one nor onto (b) one-one but not onto

(c) onto but not one-one (d) one-one and onto both.

74. Let f(x) be a polynomial function of second degree. If f(1) = f(-1) and a, b, c are in A.P, then $f'(a)$, $f'(c)$ are in

(a) Arithmetic-Geometric Progression (b) A.P. (c) G.P. (d) H.P.

75. The sum of the series $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$..................... up to ∞ is equal to

(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$ (c) $\log_e 2-1$ (d) $\log_e 2$

1. Force is $\perp^r$ to displacement $\Rightarrow$ the work done is zero

2. Since there is no deviation in the path of the charged particle, so net force due to presence of electric and magnetic field must be zero $\Rightarrow vB = qE \Rightarrow B = \frac{E}{V} = \frac{10^4}{10} = 10^3\,Wb/m^2$

3. $T \propto \sqrt{l}$

$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}; \quad I \propto L^2$ $\quad \left[\Rightarrow \frac{I_1}{I_2} = \frac{L_1^2}{L_2^2} = \frac{(2L_2)^2}{L_2^2} = 4\right] \Rightarrow \frac{T_1}{T_2} = \sqrt{4} = 2$

$\Rightarrow T_2 = \frac{T_1}{2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{2}$

4. $\tau = (H)\tan 60^0 \qquad = W.\sqrt{3}$

7. Mass $= \frac{49}{9.8} = 5kg$. When lift is moving downward, apparent weight = 5(9.8 - 5) = 5×4.8 = 24 N

8. Potential $\propto$ R

R $\propto$ length $\Rightarrow$ Potential difference $\propto l$

11. $\Delta T = \frac{40}{25\times10^{-6}\times10^{-5}} \Rightarrow \Delta T = 16^0 C$

13. $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = C \Rightarrow \frac{1}{\mu_0\varepsilon_0} = C^2 \Rightarrow \left[\frac{1}{\mu_0\varepsilon_0}\right] = [C]^2$

$[C] = LT^{-1}$ or $[C]^2 = L^2T^{-2}$

14. $I = \frac{1}{2} m\,R^2$ or $M \propto t \propto R^2$

For disc X, $I_x = \frac{1}{2}(m)(R)^2 = \frac{1}{2}(\pi r^2 t).(R)^2$

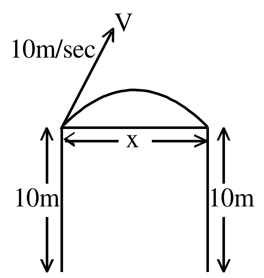
For disc Y, $I_y = \frac{1}{2}[\pi(4R)^2.t/4][4R]^2$

$\Rightarrow \frac{I_x}{I_y} = \frac{1}{(4)^3} \Rightarrow I_y = 64\,I_x$

15. $T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \left(\frac{T_1}{T_2}\right) = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2}$ $\quad \Rightarrow \frac{T_2}{T_1} = (4)^{3/2} = 8$

$\Rightarrow T_2 = 8\times T_1 = 8\times 5 = 40 = 40$ hours



16. Angular momentum $\propto \frac{1}{\text{Angular frequency}} \propto$ Kinetic energy $\Rightarrow \vec{L} = \frac{K.E.}{w}$

$$\frac{L_1}{L_2} = \left(\frac{K.E_1}{w_1}\right) \times \frac{w_2}{KE_2} = 4 \quad \Rightarrow L_2 = \frac{L}{4}$$

17. $\xrightarrow[\text{RMIVUXGE}]{\lambda \text{ Decreasing}}$

R $\rightarrow$ Radio waves ; M $\rightarrow$ Micro waves; I $\rightarrow$ Infra red rays; V $\rightarrow$ Visible rays; U $\rightarrow$ Ultraviolet rays; X $\rightarrow$ X rays; G $\rightarrow \gamma$ rays; C $\rightarrow$ Cosmic rays

$\Rightarrow$ γ rays has least wavelength

18. Applying the principle of conservation of linear momentum

$$(4)(u) = (v)(238) \Rightarrow v = \frac{4u}{238}$$

19. Distance between the surface of the spherical bodies = 12R - R - 2R = 9R

Force $\propto$ Mass, Acceleration $\propto$ Mass, Distance $\propto$ Acceleration

$$\Rightarrow \frac{a_1}{a_2} = \frac{M}{SM} = \frac{1}{5} \Rightarrow \frac{S_1}{S_2} = \frac{1}{5} \Rightarrow S_2 = 5S_1$$

$$S_1 + S_2 = 9 \Rightarrow 6S_1 = 9 \Rightarrow S_1 = \frac{9}{6} = 1.5, \qquad S_2 = 1.5 \times 5 = 7.5$$

Note: Maximum distance will be travelled by smaller bodies due to the greater acceleration caused by the same gravitational force

21. Energy = Work done by force (F)

$$\Rightarrow \frac{1}{2}m.(50)^2 = (F)(6) \qquad \Rightarrow F = \frac{2500m}{2 \times 6}$$

For $v = 100 km/hr$ $\frac{1}{2}.m(100)^2 = (F)(S)$

$$\Rightarrow \frac{1}{2}m(100)^2 = \left(\frac{2500m}{2 \times 6}\right)S$$

$$\Rightarrow S = \frac{100 \times 100 \times 6 \times 2}{2500 \times 2} = 24\,m$$

22. From, the question if the horizontal distance is none other than the horizontal range on the level of the roof of building

$$\text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30)}{g} = \frac{10 \times 10 \times \sqrt{3}}{2 \times 10} = 8.66$$

24. $[\text{momentum}] = [M][L][T^{-1}] = [MLT^{-1}]$

$$(\text{Planck's Constant}) = \frac{E}{u} = \frac{[M][LT^{-1}]^2}{T^{-1}} = ML^2T^{-1}$$

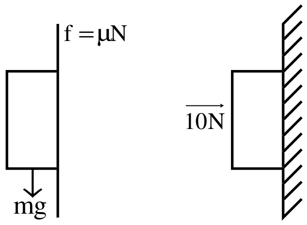
25. According to triangle law of forces, the resultant force is zero.

In presence of zero external force, there is no change in velocity

26. According to Gauss's Law

$\int (E.dA) = q_0 / \varepsilon_0 \quad \Rightarrow q = \varepsilon_0 (\phi_2 - \phi_1)$

$[\text{since}\,\phi = \int E.dA]$

27. 

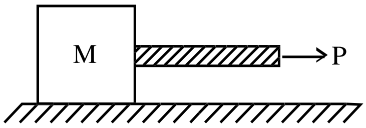
$f = mg \Rightarrow \mu N = W \Rightarrow \mu.10 = W$

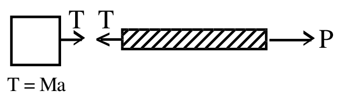
$\Rightarrow 0.2 \times 10 = W \qquad \therefore W = 2N$

28. $a = \mu g = \dfrac{6}{10}$ $\qquad$ [using $v = u + at$]

$\Rightarrow \mu = \dfrac{6}{10 \times g} = \dfrac{6}{10 \times 10} = 0.06$

31. Since the displacement for both block and rope is same so, the acceleration must be same for both





.... (i)

$\Rightarrow p = (m + M)a \qquad \Rightarrow a = \dfrac{P}{m + M}$

$T = M.a = \dfrac{PM}{m + M}$

33. Elastic energy $= \dfrac{1}{2} \times F \times x$

$F = 200$ N, $x = 1$ mm $= 10^{-3}$ m $\qquad \therefore E = \dfrac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1J$

34. Escape velocity of a body is independent of the angle of projection. Hence, changing the angle of projection is not going to effect the magnitude of escape velocity

35. $T = 2\pi\sqrt{\dfrac{M}{K}}$ .... (i)

$\dfrac{5T}{3} = 2\pi\sqrt{\dfrac{M+m}{K}}$ .... (ii)

Dividing equation (ii) by equation (i), $\dfrac{5}{3} = \sqrt{\dfrac{M+m}{M}}$. Squaring both the sides

$\dfrac{25}{9} = \dfrac{M+m}{M} = 1 + \dfrac{m}{M} \Rightarrow \dfrac{m}{M} = \dfrac{25}{9} - 1 = \dfrac{16}{9}$

36. External amount of work must be done in order to flow heat from lower temperature to higher temperature. This is according to second law of thermodynamics.

37. $V_{max} = \omega A = m\omega^2 I = k$

$\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$ $\qquad \Rightarrow \omega \propto \sqrt{K}$ or $\frac{\omega_1}{\omega_2} = \sqrt{\frac{k_1}{k_2}}$

$V_A \max = V_B \max$ $\qquad \Rightarrow \left(\sqrt{\frac{k_1}{m}}\right)(A_1) = \left(\sqrt{\frac{k_2}{m}}\right)(A_2) \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

38. $T = 2\pi\sqrt{\frac{l}{g}}$; $\log T = \log(2\pi) + \frac{1}{2}\log\left(\frac{l}{g}\right)$ $\qquad \Rightarrow \log T = \log(2\pi) + \frac{1}{2}\log(l) - \frac{1}{2}\log(g)$

Differentiating

$\frac{\Delta T}{T} = 0 + \frac{1}{2} \times \frac{\Delta l}{1} - 0 \;\Rightarrow\; \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times \frac{\Delta l}{1} \times 100$ $\qquad = \frac{1}{2} \times 21 = 10.5 \approx 10\%$

Note: In this method, the % error obtained is an approximate value on the higher side. Exact value is less than the obtained one.

39. $y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$. Comparing it with standard equation

$y = A\sin(vt - kx)$; $\qquad v = 600 m/s$

40. $e = -L\frac{dI}{dt} \Rightarrow 8 = (L)\frac{2-(-2)}{0.05} \Rightarrow L = 0.1H$

41. $q = \frac{Q}{\sqrt{2}}$

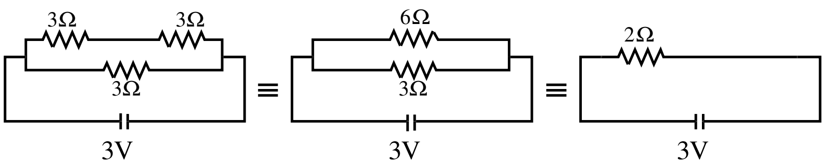
*44. $K = \frac{1}{f}\ln\left(\frac{N}{N}\right) \Rightarrow K = \frac{1}{5}\ln\left(\frac{5000}{1250}\right)$

$\frac{1}{5}\ln(4) = \frac{2}{5}\ln 2 = 0.4\ln 2$

45. No. of α particles emitted = 8, $\qquad$ No. of β^- particles emitted = 4, $\qquad$ No.of β^+ particles emitted = 2

$z = 92 - 2 \times 8 + 4 - 2 = 78$

48. $I = \frac{3}{2} = 1.5A$



50. $x = 4(\cos\pi t + \sin\pi t) = 4[\sin\left(\frac{\pi}{2} - \pi t\right) + \sin\pi t] = 4\left[2 \times \sin\left(\frac{\pi t - \frac{\pi}{2} - \pi t}{2}\right)\cos\left(\frac{\pi t - \frac{\pi}{2} + \pi t}{2}\right)\right]$

$$= 8\left[\sin\frac{\pi}{4}.\cos\left(-\frac{\pi}{4}+\pi t\right)\right]$$

$$= \frac{8}{\sqrt{2}}.\cos\left[\pi t-\frac{\pi}{4}\right] = 4\sqrt{2}\cos\left[\pi t-\frac{\pi}{4}\right]$$

Comparing it with standard equation

X = A cos (wt - Kx) $\Rightarrow A = 4\sqrt{2}$

51. Potential due to spherical shell, $v_1 = \frac{q}{4\pi\varepsilon_0 R}$. Potential difference due to charge at the centre

$$V_2 = \frac{2Q}{4\pi\varepsilon_0 r}; V = V_1 + V_2 = \frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$$

52. Work done $= \frac{1}{2}\frac{q^2}{c} = \frac{(8\times10^{-18})^2}{2\times100\times10^{-8}} = 32\times10^{-32}$ J

53. $V_x = \frac{dx}{dt} = 3\alpha t^2,\ V_y = \frac{dy}{dt} = 3\beta t^2$

$$\vec{v} = \sqrt{V_{x^2} + V_{y^2}} = 3t^2\sqrt{\alpha^2+\beta^2}$$

54. $P \propto T^3 \quad \left(\frac{P_1}{P_2}\right) = \left(\frac{T_1}{T_2}\right)^3$

Comparing it with standard eq. $\frac{C_p}{C_v} = \gamma = \frac{3}{2}$

56. $\eta = \frac{(627+273)-(273+27)}{627+273}$

$$= \frac{900-300}{900} = \frac{600}{900} = \frac{2}{3}$$

work = $(\eta)\times$ Heat

$$= \frac{2}{3}\times3\times10^6\times4.2\,\text{J} = 8.4\times10^6\ \text{J}$$

57. Required work done

$$= \frac{1}{2}K(x_2^2 - x_1^2) = \frac{1}{2}\times5\times10^3[10^2-5^2]\times10^{-4}$$

$$= \frac{1}{2}\times5\times75\times10^3\times10^{-4} \quad = 18.75$$

58. $n = \frac{1}{2l}\sqrt{\frac{T}{\mu}};\ l = 1m$

T = 10 Kg wt. = 10 $\times$ 10 = 100 N

μ = 9.8 g/m = 9.8 $\times 10^{-3}$ kg/m n = 50 hz

66. Power = F. V

$$F = m\left(\frac{dV}{dt}\right) \Rightarrow m.v.\frac{dV}{dt} = \text{constant} = C$$

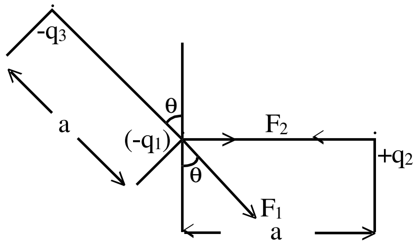
$$\Rightarrow \frac{dV}{dt} = \frac{C}{m} = k \Rightarrow vdv = kdt \qquad \Rightarrow \int v\,dv = \int k\,dt \Rightarrow \frac{V^2}{2} = kt + c$$

$$\Rightarrow v \propto (t)^{1/2} \qquad \frac{ds}{dt} = c.t^{1/2}$$

$$\Rightarrow \int ds = \int (c.t^{1/2})dt \Rightarrow S = C.\frac{2}{3}t^{3/2} \qquad \Rightarrow S = \frac{c.t^{3/2}}{3/2} \Rightarrow s \propto t^{3/2}$$

67. Thrust = Mass $\times$ Acceleration = $3.5 \times 10^4 \times 10 = 3.5 \times 10^5$ N

69. The force body diagram



$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{a^2}; \qquad F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{b^2}$$

$$F_X = F_1 \sin\theta + F_2 = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_3}{a^2}\sin\theta + \frac{q_1}{b_2}\right] \Rightarrow F_x \propto \left(\frac{q_3}{a^2}\sin\theta + \frac{q_2}{b^2}\right)$$

70. $p = \frac{V^2}{R}$ or $R = \frac{(220)^2}{1000}$

Power consumed = $\frac{V^2}{R} = \frac{110 \times 110}{220 \times 220} \times 1000 = 250$ watt

73. According to Image formula

$$n = \frac{360}{\theta} - 1 \Rightarrow 3 = \frac{360}{\theta} - 1$$

$$\Rightarrow \frac{360}{\theta} = 4 \Rightarrow \theta = \frac{360}{4} = 90$$

74. $\frac{dH}{dt} \propto (\theta_2 - \theta_1) = (\Delta\theta)^n \Rightarrow n = 1$

75. $L_1 = 2l$ or $(\pi r^2 l) = (\pi r_2^2)(2l)$

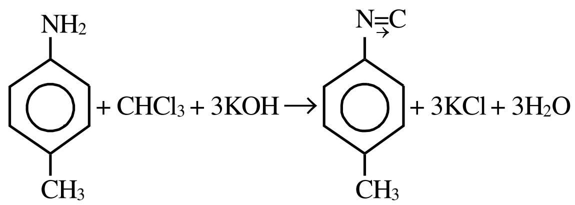
$$\Rightarrow r_2 = \frac{r}{\sqrt{2}}; \qquad R = \rho\frac{l}{\pi r^2}$$

$$R_{new} = (\rho)\frac{2l}{(\pi)\left(r/\sqrt{2}\right)^2} = \frac{(\rho)4l}{(\pi)r^2} = 4 \times R$$

$\therefore \Delta R = 4R - R = 3R$

$$\frac{\Delta R}{R}\% = \frac{3R}{R} \times 100 = 300\%$$

76. Liquid hydrogen and liquid oxygen are used as excellent fuel for rockets. $H_2(l)$ has low mass and high enthalpy of combustion whereas oxygen is a strong supporter of combustion

77. 

78. Nylon is a polyamide polyer

79. More is the no . of + I groups attached to N atom greater is the basic character.

80. C_6H_5I will not respond to silver nitrate test because C-I bond has a partial double bond character.

81. For a cyclic process the net change in the internal energy is zero because the change in internal energy does not depend on the path.

82. $CH_2 = CH_2\ (g) + H_2(g) \rightarrow CH_3 - CH_3$

$$\Delta H = 1(C=C) + 4(C-H) + 1(H-H) - 1(C-C) - 6(C-H) = 1(C=C) + 1(H-H) - 1(C-C) - 2(C-H)$$

$$= 615 + 435 - 347 - 2\times 414 = 1050 - 1175 = -125\,kJ.$$

83. ${}^{234}_{90}Th \xrightarrow{-\beta} {}^{234}_{91}X \xrightarrow{-\beta} {}^{234}_{92}Th \xrightarrow{-\alpha} {}^{230}_{90}Th$

84. $t_{1/2} = 3$ hrs. Initial mass $(C_0) = 256$ g

$$\therefore C_n = \frac{C_0}{2^n} = \frac{256}{(2)^6} = \frac{256}{64} = 4g$$

86. $\Omega \propto \dfrac{1}{z}$

$$\frac{\Omega_1}{\Omega_2} = \frac{z_2}{z_1} \Rightarrow \frac{1.06}{\Omega_2} = \frac{71}{57} \Rightarrow \Omega_2 = 0.85 Å$$

88. $Co(NH_3)_5\ Cl_3 [Co(NH_3)_5Cl]^{+2} + 2Cl^-$

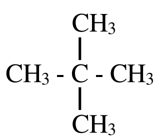
$\therefore$ Structure is $[Co(NH_3)_5\ Cl]\ Cl_2$.

89. $4\ (+1) + x + (-1) \times 4 = 0 \qquad \Rightarrow 4 + x - 4 = 0$

$x = 0$

91. An acidic solution cannot have a pH > 7.

92. In neopentane all the H atoms are same (1^0).



94. $PH_3 + 4Cl_2 \rightarrow PCl_5 + 3HCI$

95. $Fe^{+2} =\ 3d^6\ .\ 4s^0$

96. $4HCI + O_2 \rightarrow 2Cl_2 + 2H_2O$

Cloud of white fumes

99. The properties of elements change with a change in atomic number.

100. Ammonia can dissolve ppt. of Agcl only due to formation of complex as given below:

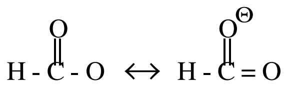
$AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]\ Cl$

101. Glass is a transparent or translucent super cooled liquid.

102. For s-electron, $l = 0$ $\therefore$ angular momentum = zero

103. Number of formulas in cube shaped crystals = $\frac{1.0}{58.5} \times 6.02 \times 10^{23}$ since in NaCl type of structure 4 formula units form a cell

$\therefore$ unit cells = $\frac{1.0 \times 6.02 \times 10^{23}}{58.5 \times 4}$ = 2.57×10^{21} unit cells.

104. 

105. As adsorption is an exothermic process.

$\therefore$ Rise in temperature will decrease adsorption.

106. The equilibrium constant is related to the standard emf of cell by the expression

$$\log K = E^0_{cell} \times \frac{n}{0.059} = 0.295 \times \frac{2}{0.059}$$

$$\log K = \frac{590}{59} = 10 \text{ or } K = 1 \times 10^{-10}$$

107. For spontaneous reaction, dS > 0 an ΔG and dG should be negative i.e. < 0

108. $[A] = 1.0 \times 10^{-5}$, $[B] = [1.0 \times 10^{-5}]$

$K_{sp} = [2.B]^2 [A] = [2 \times 10^{-5}]^2 [1.0 \times 10^{-5}] = 4 \times 10^{-15}$

109. No. of moles of boron = $\frac{21.6}{10.8} = 2$

for BCl_3

$\therefore$ 1 mole of Boron = 3 mole of Cl

$\therefore$ 2 mole of Boron = 6 mole of Cl

$H_2 + Cl_2 \rightarrow 2HCl$

$\Rightarrow$ 3 moles of Hydrogen is required

$= 3 \times 22.4 = 67.2$ Litre

110. $K_C = \frac{[NO_2]^2}{[N_2O_4]} = \frac{[1.2 \times 10^{-2}]^2}{[4.8 \times 10^{-2}]} = 3 \times 10^{-3}$ mol/L

111. Due to exothermicity of reaction low or optimum temperature will be required. Since 3 moles are changing to 2 moles.

$\therefore$ High pressure will be required.

113. $\underset{\text{(insoluble)}}{HgI_2} + KI \longrightarrow \underset{\text{(soluble)}}{K_2HgI_4}$

On heating HgI_2 decomposes as $HgI_2 \rightleftharpoons Hg + I_2$

117. No. of moles of silver = $\frac{9650}{96500} = \frac{1}{10}$ moles

$\therefore$ Mass of silver deposited = $\frac{1}{10} \times 108 = 10.8\,g$

118. $E_{cell} = E^0_{cell} + \frac{0.059}{n} \log \frac{[Cu^{+2}]}{[Zn^{+2}]}$

$= 1.10 + \frac{0.059}{2} \log[0.1] = 1.10 - 0.0295 = 1.07\,V$

120. f-block elements show a regular decrease in atomic size due to lanthanide/actinide contraction.

122. $LiAl^{H}_{4}$ can reduce COOH group and not the double bond

$CH_2 = CH - COOH \xrightarrow{LAH} CH_2{=}CH{-}CH_2OH$

123. According to kinetic theory the gas molecules travel in a straight line path but show haphazard motion due to collisions.

125. A chiral object or structure has four different groups attached to the carbocation.

126. $Cr_2O_7^{2-} + OH^- \rightarrow 2CrO_4^{2-} + H^+$. The above equilibrium shifts to L.H.S. on addition of acid.

127. It is because mercury exists as liquid at room temperature.

128. Gypsum is $CaSO_4.2H_2O$

129. $(C_6H_{10}O_5)n + nH_2O \xrightarrow{H^+} \underset{D-Glucose}{nC_6H_{12}O_6}$

130. $AgNO_3 \rightarrow Ag + NO_2 + \frac{1}{2}O_2$

131. $CH_3CH_2OH + H^+ \xrightarrow{\text{step 1}} \underset{\text{Protonated alcohol}}{CH_3CH_2-\overset{\oplus}{\underset{H}{\underset{|}{O}}}-H}$

132. The solubility is governed by $\Delta H_{solution}$ i.e. $\Delta H_{solution} = \Delta H_{lattice} - \Delta H_{Hydration}$.

Due to increase in size the magnitude of hydration energy decresaes and hence the solubility.

133. The rain water after thunderstorm contains dissolved acid and therefore the pH is less than rain water without thunderstorm.

135. $Ba(OH)_2 + 2HCI \rightarrow BaCl_2 + 2H_2O$

Applying Molarity equation, $\frac{M_1V_1}{\underset{(Ba(OH)_2)}{1}} = \frac{M_2V_2}{\underset{(HCI)}{2}}$ or $25 \times M_1 = \frac{0.1 \times 35}{2}$ $\therefore M_1 = \frac{0.1 \times 35}{2 \times 25} = \frac{0.7}{10} = 0.07$

137. $Rate_1 = k\,[A]^n\,[B]^m$; $Rate_2 = k[2A]^n\,[½B]^m$

$\therefore \frac{Rate_2}{Rate_1} = \frac{k[2A]^n\,[½B]^m}{k[A]^n\,[B]^m} = [2]^n\,[½]^m = 2^n.2^{-m} = 2^{n-m}$

138. $CH_3CH_2N \overset{\rightarrow}{\rightarrow} C + H_2O \xrightarrow{H^+} CH_3CH_2NH_2 + HCOOH$. Therefore it gives only one mono chloroalkane.

140. On increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the b.p. of water increases.

141. $r = k[O_2]\,[NO]^2$. When the volume is reduced to 1/2, The conc. will double.

$\therefore$ New rate $= k[2O_2]\,[2NO]^2 = 8\,k\,[O^2]\,[NO]^2$. The new rate increases by eight times.

142. Magnesium provides cathodic protection and prevent rusting or corrosion.

143. Both NO_2 and O_3 have angular shape and hence will have net dipole moment.

144. N^{3-}, F^- and Na^+ contain 10 electrons each.

145. Permanent hardness of water is due to chlorides and sulphates of calciumand magnesium.

146. In H_2S, due to low electronegativity of sulphur the L.P. - L.P. repulsion is more than B.P. - B.P. repulsion and hence the bond angle is 92^0.

147. Both XeF_2 and and CO_2 have a linear structure.

148. Electronic configuration of Cr is

3d						4s
↑	↑	↑	↑	↑		↑

So due to half filled orbital I.P. is high of Cr.

149. The lines falling in the visible region comprise Balmer series. Hence the third line would be $n_1 = 2$, $n_2 = 5$ i.e. $5 \rightarrow 2$.

150. $\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{60 \times 10^{-3} \times 10} = 10^{-33} m$

AIEEE 2003
MATHEMATICS SOLUTION

1. $\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x}$ or $\int_1^4 \frac{3}{x}e^{\sin x^3}dx=\int_1^4 \frac{3x^2}{x^3}e^{\sin x^3}dx$

   Let $x^3 = t$, $3x^2dx = dt$

   when $x = 1$, $t = 1$ & $x - 4$, $t = 64$

   $$F(t)=\int_1^{64}\frac{e^{\sin t}}{t}dt=\int_1^{64}F(t)\,dt=F(64)-F(1)$$

   K = 64.

2. $n = 9$ then median term $=\left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term. Last four observations are increased by 2. The median is 5th observation which is remaining unchanged. $\therefore$ There will be no change in median.

3. $$\lim_{n\to\infty}\left\{\left(\frac{1}{n}\right)^4+\left(\frac{2}{n}\right)^4+\left(\frac{3}{n}\right)^4+\ldots\ldots\ldots\ldots\left(\frac{n}{n}\right)^4\right\}-\lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{n^4}+\frac{2^3}{n^4}+\ldots\ldots\ldots\frac{n^3}{n^4}\right\}$$

   $$\int_0^1 (x)^4dx-0=\left[\frac{x^5}{5}\right]_0^1=\frac{1}{5}$$

4. Fundamental theorem (fact) $t_2=-t_1-\frac{2}{t_1}$

5. $|r_1-r_2|=C_1C_2$ for intersection

   $\Rightarrow r - 3 < 5 \Rightarrow r < 8$ .... (1)

   and $r_1 + r_2 > C_1C_2$, $r+3 > 5 \Rightarrow r = 2$ ....(2)

   From (1) and (2), $2 < r < 8$.

6. $y^2 = 4a(x - h)$, $2yy_1 = 4a \Rightarrow yy_1 = 2a \Rightarrow y_1^2 + yy_1 = 0$

   Degree = 1, order = 2,

7. $$\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$$

   $$a=\sqrt{\frac{144}{25}},\ b=\sqrt{\frac{81}{25}},\ e=\sqrt{1+\frac{81}{144}}=\frac{15}{12}=\frac{5}{4}$$

   Foci = (3, 0), focus of ellipse = (3, 0) $\Rightarrow e=\frac{3}{4}$

   $$b^2=16\left(1-\frac{9}{16}\right)=7$$

8. $$F(t)=\int_0^t f(t-y)g(y)dy$$

$$= \int_0^t e^{t-y} y dy = e^t \int_0^t e^{-y} y dy$$

$$= e^t \left[-ye^{-y} - e^{-y}\right]_0^t = -e^t \left[ye^{-y} + e^{-y}\right]_0^t$$

$$= -e^t \left[te^{-t} + e^{-t} - 0 - 1\right] = e^t \left[\frac{t+1-e^t}{e^t}\right] = e^t - (1+t)$$

9. $f(x) = \log\left(x + \sqrt{x^2+1}\right)$

$f(-x) = -\log\left(x + \sqrt{x^2+1}\right)$

$f(-x) = -f(x)$, i.e., $f(x)$ is an odd function.

10. $ax^2 + bx + c = 0,\ \alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

As for given condition, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = -\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} - \frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ $\quad \therefore \frac{a}{b}, \frac{b}{a}, \&\ \frac{c}{b}$ are in H.P.

11. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad C_2 \to C_2 - 2C_3$

$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$

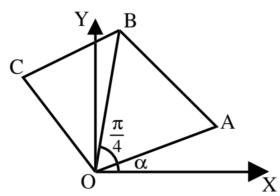
b (c - b) - (b - a) (2c - b) = 0

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore$ a, b, c are in Harmonic Progression.

12. Co-ordinates of A = ($a\cos\alpha$, $a\sin\alpha$)

Equation of OB, $y = \tan\left(\frac{\pi}{4} + \alpha\right)x$



CA $\perp$ r to OB $\quad\therefore$ slope of $CA=-\cot\left(\frac{\pi}{4}+2\right)$

Equation of CA $y-a\sin\alpha=-\cot\left(\frac{\pi}{4}+2\right)(x-a\cos\alpha)$.

$y(\sin\alpha+\cos\alpha)+x(\cos\alpha-\sin\alpha)=a$

13. Equation of bisector of both pair of straight lines,

$px^2 + 2xy - py^2 = 0$ ... (1)

$qx^2 + 2xy - qy^2 = 0$ .... (2)

From (1) and (2). $\frac{q}{1}=\frac{2}{-2p}=\frac{-q}{-1}\Rightarrow pq=-1$.

14. $x=\frac{\cos t+b\sin t+1}{3}\Rightarrow a\cos t+b\sin t=3x-1$

$y=\frac{a\sin t-b\cos t}{3}\Rightarrow a\sin t-b\sin t=3y$

Squaring & adding, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

15. $\lim\limits_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=K$ (by L'Hospital rule)

$\lim\limits_{x\to 0}\frac{\frac{1}{3+x}-\frac{-1}{3-x}}{1}=K \quad \therefore \frac{2}{3}=K$

16. $\vec{a}=\vec{r}\times\vec{p};\ |\vec{a}|=rp\sin\theta$

$|\vec{H}|=rp\cos\theta \quad \left\lfloor\because \sin(90^0+\theta)=\cos\theta\right\rfloor$

$G = rp\sin\theta$ .... (1)

$H = rp\cos\theta$ .... (2)

$x = rp\sin(\theta+\alpha)$ .... (3)

From (1), (2) & (3),

$x=\vec{a}\cos\alpha+\vec{H}\sin\alpha$

17. $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)

On (1) + (2), $5R^2 = 2P^2 + 2Q^2$ .... (4)

On (3)$\times$2+(2), $12R^2 = 3P^2 + 6Q^2$ .... (5)

$2P^2 + 2Q^2 - 5R^2 = 0$ .... (6)

$3P^2 + 6Q^2 - 12R^2 = 0$ .... (7)

$$\frac{P^2}{-24+30}=\frac{Q^2}{24-15}=\frac{R^2}{12-6}$$

$\frac{P^2}{6}=\frac{Q^2}{9}=\frac{R^2}{6}$ or $P^2:Q^2:R^2 = 2:3:2$

18. $\left.\begin{array}{l} np=4 \\ npq=2 \end{array}\right\} \Rightarrow q=\frac{1}{2}.p=\frac{1}{2}, n=8$

$$p(X=1)={}^{8}C_{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{7}=8.\frac{1}{2^{8}}=\frac{1}{2^{5}}=\frac{1}{32}$$

19. $f(x)=x^{n} \Rightarrow f(1)=1$

$f'(x)=nx^{n-1} \Rightarrow f'(1)=n$

$f''(x)=n(n-1)x^{n-2} \Rightarrow f''(1)=n(n-1)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; f^{n}(x)=n! \Rightarrow f^{n}(1)=n!$

$$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+\ldots\ldots\ldots+(-1)^{n}\frac{n!}{n!}$$

$$={}^{n}C_{0}-{}^{n}C_{1}+{}^{n}C_{2}-{}^{n}C_{3}+\ldots\ldots\ldots\ldots+(-1)^{n}\,{}^{n}C_{n}=0$$

20. Since $\vec{n}$ is perpendicular $\vec{u}$ and $\vec{v}$, $\vec{n}=\vec{u}\times\vec{v}$

$$\hat{n}=\frac{\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$$

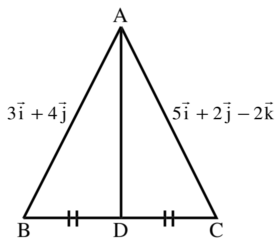
$$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$$

21. $\vec{F}+\vec{F}_{1}+\vec{F}_{2}=7i+2j-4k$

$\vec{d}=$ P.V of $\vec{B}-$ P.V of $\vec{A}=4i+2j-2k$

$W=\vec{F}.\vec{d}=28+4+8=40$ unit

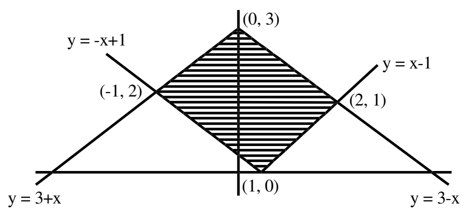
22.



$$\text{P.V of } \overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}$$

$$=4i-j+4k \text{ or } |\overrightarrow{AD}|=\sqrt{16+16+1}=\sqrt{33}$$

23.



$$A=\int_{-1}^{0}\{(3+x)-(-x+1)\}dx+\int_{0}^{1}\{(3-x)-(-x+1)\}dx+\int_{1}^{2}\{(3-x)-(-x-1)\}dx$$

$$=\int_{-1}^{0}(2+2x)dx+\int_{0}^{1}2dx+\int_{1}^{2}(4-2x)dx$$

$$=\left[2x-x^2\right]_{-1}^{0}+\left[2x\right]_{0}^{1}+\left[4x-x^2\right]_{1}^{2}$$

= 0 - (-2+1) + (2 - 0) + (8 - 4) - (4 - 1)

= 1 + 2 + 4 - 3 = 4 sq. units

24. Shortest distance = perpendicular distance $=\left|\dfrac{-2\times12+4\times1+3\times3-327}{\sqrt{144+9+16}}\right|=26$

$\therefore$ Shortest distance

$=26-\sqrt{4+1+15+9}=26-13=13\quad[\because 26-r]$

25. $\dfrac{x-b}{a}=\dfrac{y}{1}=\dfrac{3-d}{c};\dfrac{x-b'}{a'}=\dfrac{y}{1}=\dfrac{3-d'}{c'}$

For perpendicular $aa'+1+cc'=0$

26. $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0\Rightarrow\begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix}=0$$

$k^2+3k^2=0\Rightarrow k(k+3)=0$ or $k=0$ or -3

27. $I=\int_a^b xf(x)\,dx=\int_a^b (a+b-x)f(a+b-x)\,dx$

$$=(a+b)\int_a^b f(a+b-x)\,dx-\int_a^b xf(a+b-x)\,dx$$

$$=(a+b)\int_a^b f(a+b-x)\,dx-\int_a^b xf(x)\,dx$$

$$2I=(a+b)\int_a^b f(x)\,dx$$

$$I=\frac{(a+b)}{2}\int_a^b f(x)\,dx;\ I=\frac{(a+b)}{2}\int_a^b f(a+b-x)\,dx$$

28. Portion OA, OB corresponds to motion with acceleration ‘f’ and retardation ‘r’ respectively.

Area of $\Delta OAB=S$ and $OB=t$. Let $OL=t_1$,

$LB = t_2$ and $AL = v$, $S = \frac{1}{2}OB.AL = \frac{1}{2}t.v;\ v = \frac{2S}{t}$

Also, $f = \frac{v}{t_1},\ t_1 = \frac{v}{f} = \frac{2s}{tf}$ and $r = \frac{v}{t_2}, t_2 = \frac{v}{r} = \frac{2s}{tr}; t = t_1 + t_2 = \frac{2s}{tf} + \frac{2s}{tr}$

$t = \left(\frac{1}{f} + \frac{1}{r}\right)\frac{2s}{t} \quad \Rightarrow t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$

29. $R = u\sqrt{\frac{2h}{g}} = (u\cos\theta)\times t$

$t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$ .... (1)

Now, $h = (-u\sin\theta)t + \frac{1}{2}gt^2$

Substituting 't' from (1),

$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$ $\qquad h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \quad \therefore \tan\theta = u\sqrt{\frac{2}{hg}}$

30. Applying $R_1 \to R_1 + R_2 + R_3$

As, $1 + \omega^n + \omega^{2n} = 0; \quad \therefore \Delta = 0$

31. $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r};\ \sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$r + R = \frac{a}{2}\left[\cot\frac{\pi}{n} + \operatorname{cosec}\frac{\pi}{n}\right] \Rightarrow r + R = \frac{a}{2}.\cot\left(\frac{\pi}{2n}\right)$

32. Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right); (x, y) \& (xr,\ yr)$. Above coordinates satisfy the relation $y = mx$ Therefore lies on the straight line.

33. $|z\omega| = 1$ ..... (1)

As, $\operatorname{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$ therefore $\frac{z}{\omega} = i$

$\therefore \left|\frac{z}{\omega}\right| = 1$ .... (2)

From (1) & (2), $|z| = |\omega| = 1$ and $\frac{z}{\omega} + \frac{\bar{z}}{\omega} = 0; z\bar{\omega} + \bar{z}\omega = 0$

$\bar{z}\omega = -z\bar{\omega} = \frac{-z}{\omega}.\bar{\omega}.\omega;\ \bar{z}\omega = -i|\omega|^2 = -i$

34. $z^2 + az + b = 0$; $z_1 + z_2 = -a$ & $z_1z_2 = b$

$0, z, z_2$ form an equilateral Δ

$\therefore\ 0^2 + z_1^2 + z_2^2 = 0.z_1 + z_1.z_2 + z_2.0$

(for equation Δ, $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$)

$z_1^2 + z_2^2 = z_1z_2$ or $(z_1 + z_2)^2 = 3z_1z_2$

$\therefore\ a^2 = 3b.$

35. $(1+y^2)+(x-e^{\tan^{-1}y})\dfrac{dy}{dx}=0$

$(1+y^2)\dfrac{dx}{dy}+x=e^{\tan^{-1}y} \Rightarrow \dfrac{dx}{dy}+\dfrac{x}{(1+y^2)}=\dfrac{e^{\tan^{-1}y}}{(1+y^2)}$

I.F. = $e^{\int\frac{1}{(1+y^2)}dy} = e\tan^{-1}y$

$x(e^{\tan^{-1}y})=\int\dfrac{e^{\tan^{-1}y}}{1+y}\ e^{\tan^{-1}y}dy$

$x(e^{\tan^{-1}y})=\dfrac{e^{2\tan^{-1}y}}{2}+C \quad \therefore 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

36. Let $f(x) = e^x$

$\therefore \int_0^1 f(x)g(x)dx=\int_0^1 e^x(x^2-e^x)dx$

$=\int_0^1 x^2e^xdx-\int_0^1 e^{2x}dx$

$=\left[x^2e^x\right]_0^1-2\left[xe^x-e^x\right]_0^1-\frac{1}{2}\left[e^{2x}\right]_0^1$

$=e-\left[\frac{e^2}{2}-\frac{1}{2}\right]-2[e-e+1]=e-\frac{e^2}{2}-\frac{3}{2}$

37. $\pi r^2 = 154 \Rightarrow r = 7$

For centre on solving equation

$2x - 3y = 5$ & $3x - 4y = 7$ or $x = 1, y = 1$ centre = $(1, -1)$

Equation of circle, $(x - 1)^2 + (y + 1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

38. $P(A)=\dfrac{3x+1}{3},\ P(B)=\dfrac{1-x}{4},\ P(C)=\dfrac{1-2x}{2}$

These are mutually exclusive

$0\le\dfrac{3x+1}{3}\le 1,\quad 0\le\dfrac{1-x}{4}\le 1 \text{ and } 0\le\dfrac{1-2x}{2}\le 1$

$-1\le 3x\le 2,\ -3\le x\le 1 \text{ and } -1\le 2x\le 1$

$$-\frac{1}{3} \le x \le \frac{2}{3}, -3 \le x \le 1, \text{ and } -\frac{1}{2} \le x \le \frac{1}{2}$$

Also $0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13 \Rightarrow \frac{1}{3} \le x \le \frac{13}{3}$$

$$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le x \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$$

$$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$$

39. $n(S) = {}^5C_2$; $n(E) = {}^2C_1 + {}^2C_1$

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$$

40. $3\alpha = \frac{1-3a}{a^2 - 5a + 3}$ & $2\alpha^2 = \frac{2}{a^2 - 5a + 3}$

$$2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right] = \frac{2}{a^2-5a+3}$$

$$\frac{(1-3a)^2}{(a^2-5a+3)} = 9 \text{ or } 9a^2 - 6a + 1$$

$$= 9a^2 - 45a + 27 \quad \text{or} \quad 39a = 26 \quad \text{or } a = \frac{2}{3}$$

41. $T_{r+1} = \frac{n(n-1)(n-2)..........(n-r+1)}{r!}(x)^r$

For first negative term, $n - r + 1 < 0$ or $r > \frac{32}{5}$

$\therefore$ $r = 7$. Therefore, first negative term is T_8.

42. $T_{r+1} = {}^{256}C_r(\sqrt{3})^{256-r}(\sqrt[8]{5})^r = {}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer. As $0 \le r \le 256$ $\therefore$ $r = 0, 8, 16, 24, ...256$

For above values of r, $\left(\frac{256-r}{2}\right)$ is also an integer.

43. After t; velocity = $f \times t$

$$V_{BA} = \vec{f}\,t + (-\vec{u}) = \sqrt{f^2t^2 + u^2 - 2fut\cos\alpha}$$

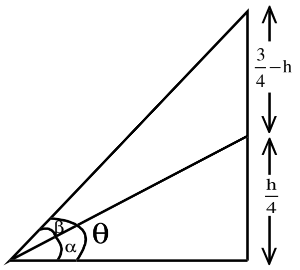
For max. and min.

$$\frac{d}{dt}\left(V_{BA}^2\right) = 2f^2 t - 2fu\cos\alpha = 0 \quad \text{or} \quad t = \frac{u\cos\alpha}{f}$$

Therefore, total no. of values of $r = 33$.

44. Using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r = {}^nC_{r+1} + \underbrace{{}^nC_{r-1} + {}^nC_r}_{} + {}^nC_r = {}^nC_{r+1} + {}^{n+1}C_r + {}^nC_r$

$${}^{n+1}C_{r+1} + {}^{n+1}C_r \Rightarrow {}^{n+2}C_{r+1}$$

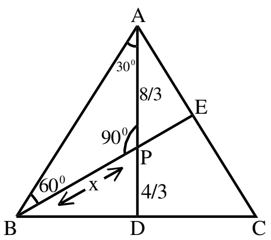
45. 

$\theta = \alpha + \beta$ or $\beta = \theta - \alpha$

$$\tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha} \text{ or } \frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} . \frac{h}{160}}$$

$h^2 - 200\,h + 6400 = 0$, $h = 40$ or 160 metre

Therefore possible height $= 40$ metre

46. 

$$\tan 60^0 = \frac{8/3}{x} \text{ or } x = \frac{8}{3\sqrt{3}}$$

Area of $\Delta ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$ $\quad \therefore$ Area of $\Delta ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$

47. If $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$

$a[\cos C + 1] + c[\cos A + 1] = 3b$

$(a + c) + (a\cos C + c\cos B) = 3b$

$a + c + b = 3b$ or $a + c = 2b$ or a, b, c are in A.P.

48. $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$$

$$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-1-4-9}{2} = -7$$

49. $I = \int_0^1 x(1-x)^n\,dx$

$$-I = \int_0^1 -x(1-x)^n\,dx = \int_0^1 (1-x-1)(1-x)^n\,dx$$

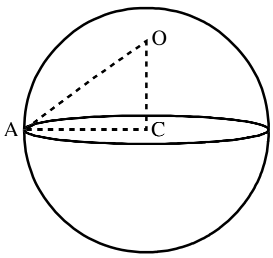
$$=\int_0^1 (1-x)^{n+1}\,dx-\int_0^1 (1-x)^n\,dx$$

$$=\left[\frac{(1-x)^{n+2}}{-(n+2)}\right]_0^1-\left[\frac{(1-x)^{n+1}}{-(n+1)}\right]_0^1=\frac{1}{n+2}-\frac{1}{n+1}$$

$$I=\frac{1}{n+1}-\frac{1}{n+2}$$

50. $$\lim_{x\to 0}\frac{\frac{d}{dx}\int_0^{x^2}\sec^2 t\,dt}{\frac{d}{dx}(x\sin x)}=\lim_{x\to 0}\frac{\sec^2 x^2.2x}{\sin x+x\cos x}\text{ (by L' Hospital rule)}$$

$$\lim_{x\to 0}\frac{2\sec^2 x^2}{\left(\frac{\sin x}{x}+\cos x\right)}=\frac{2\times 1}{1+1}=1$$

51. 

centre of sphere = (-1, 1, 2)

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC=d=\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right|=\frac{12}{3}=4.$$

$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9 \Rightarrow AC = 3$

52. Vector perpendicular to the face OAB

$$=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix} i & j & k\\ 1 & 2 & 1\\ 2 & 1 & 3\end{vmatrix}=5i-j-3k$$

Vector perpendicular to the face ABC

$$=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} i & j & k\\ 1 & -1 & 2\\ -2 & -1 & 1\end{vmatrix}=i-5j-3k$$

Angle between the faces $=$ Angle between their normals

$$\cos\theta=\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|=\frac{19}{35}\text{ or }\theta=\cos^{-1}\left(\frac{19}{35}\right)$$

53. $\lim_{x\to a} \frac{k9(x)-kf(x)}{9(k)-f(x)} = 4$ (By L'Hospital rule)

$\lim_{x\to a} k\frac{9'(x)-f'(x)}{9'(x)-f'(x)} = 4$ or $k = 4$.

54. $\lim_{x\to \frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x = \frac{\pi}{2}+y;\ y \to 0$

$= \lim_{y\to 0} \frac{-\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3} = \lim_{y\to 0} \frac{-\tan\frac{y}{2}2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}$

$= \lim_{y\to 0} \frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2 = \frac{1}{32}$

55. $(h-a_1)^2+(k-b_1)^2 = (h-a_2)^2+(k-b_2)^2$

$(a_1-a_2)x+(b_1-b_2)y+\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)=0$

$C = \frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$

56. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$

(a - b) (b - c) (c - a) + abc(a - b) (b - c) (c - a) = 0

(abc + 1) [(a - b) (b - c) (c - a)] = 0

As $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$ (given condition) $\qquad \therefore$ abc = -1

57. $x^2-3|x|+2=0$ or $(|x|-2)(|x|-1)=0$

$|x|=1,2$ or $x=\pm1,\pm2$ or $\therefore$ No. of solution = 4

58. $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2;\ f''(x) = 12x - 18a$

For max. or min. $6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$

x = a or x = 2a, at x = a max.and at x = 2a min.

$p^2 = q$

$a^2 = 2a \Rightarrow a = 2$ or $a = 0$

but a > 0, therefore, a = 2.

59. $f(0)=0;\ f(x)=xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}$

R.H.L. $\lim\limits_{h\to 0}(0+h)e^{-2/h}=\lim\limits_{h\to 0}\dfrac{h}{e^{2/h}}=0$

L.H.L $\lim\limits_{h\to 0}(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}=0$

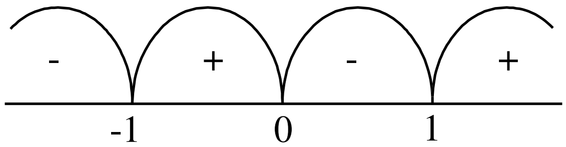
Therefore, f(x) is continuous

R.H.D. $\lim\limits_{h\to 0}\dfrac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{h}=0$

L.H.D. $\lim\limits_{h\to 0}\dfrac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{-h}=1$

Therefore, L.H.D. $\neq$ R.H.D.

f(x) is not differentiable at $x=0$.

60. 

$f(x)=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$

$4-x^2\neq 0; x^3-x>0; x\neq\pm\sqrt{4}$

$\therefore D=(-1,0)\cup(1,\infty)-\{\sqrt{4}\}$

$D=(-1,0)\cup(1,2)\cup(2,\infty)$.

61. $f(x+y)=f(x)+f(y)$. Let $f(\alpha)=m\alpha$

$f(1)=7;\ \therefore\ m=7, f(x)=7x$

$\sum\limits_{r=1}^{n}f(r)=7\sum\limits_{1}^{n}r=\dfrac{7n(n+1)}{2}$

62. $y=x+\dfrac{1}{x}$ or $\dfrac{dy}{dx}=1-\dfrac{1}{x^2}$

For max. or min., $1-\dfrac{1}{x^2}=0\Rightarrow x=\pm 1$

$\dfrac{d^2y}{dx^2}=\dfrac{2}{x^3}\Rightarrow\left(\dfrac{d^2y}{dx^2}\right)_{x=2}=2(+\text{ve minima})$

Therefore $x=1$

63. Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$R_1=\dfrac{u^2}{g(1+\sin\beta)}$ and $R_2=\dfrac{u^2}{g(1-\sin\beta)}$

$\dfrac{1}{R_1}+\dfrac{1}{R_2}=\dfrac{2g}{u^2}$ or $\dfrac{1}{R_1}+\dfrac{1}{R_2}=\dfrac{2}{R}\left[\because R=\dfrac{u^2}{g}\right]$

Therefore, R_1, R, R_2 are in H.P.

64. $\Sigma x=170,\ \Sigma x^2=2830$ increase in $\Sigma x=10$, then

$\Sigma x'=170+10=180$

Increase in $\Sigma x^2=900-400=500$ then

$\Sigma x'^2=2830+500=3330$

$$\text{Variance}=\frac{1}{n}\Sigma x'^2-\left(\frac{1}{n}\Sigma x'\right)^2$$

$$=\frac{1}{15}\times 3330-\left(\frac{1}{15}\times 180\right)^2=222-144=78.$$

65. As for given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining 8 question = ${}^5C_4\times{}^8C_6$ = 140 choices.

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions = ${}^5C_5\times{}^8C_5$ = 56 choices.

Therefore, total number of choices = 140 + 56 = 196.

66. $$A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix}=\begin{bmatrix}a & b\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ b & a\end{bmatrix}$$

$\alpha=a^2+b^2;\ \beta=2ab$

67. No.of ways in which 6mm can be arranged at a round table = (6 - 1)!

Now women can be arranged in 6! ways.

Total number of ways = $6!\times 5!$

68. No option satisfied wrong.

A = (7, -4, 7), B = (1, -6, 10), C = (-1, -3, 4) and D = (5, -1, 5)

$$AB=\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}=\sqrt{36+4+9}=7$$

Similarly BC = 7, CD = $\sqrt{41}$, DA = $\sqrt{17}$

69. $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$$(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})=\frac{\vec{u}.(\vec{u}\times\vec{v})}{0}$$

$$-\frac{\vec{u}.(\vec{u}\times\vec{w})}{0}+\vec{u}.(\vec{v}\times\vec{w})+\frac{\vec{v}.(\vec{u}\times\vec{v})}{0}-\vec{v}.(\vec{u}\times\vec{w})$$

$$+\frac{\vec{v}.(\vec{v}\times\vec{w})}{0}-\vec{w}.(\vec{u}\times\vec{v})+\frac{\vec{w}.(\vec{u}\times\vec{w})}{0}-\frac{\vec{w}.(\vec{u}\times\vec{w})}{0}=\vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$$

$$=[\vec{u}\vec{v}\vec{w}]+[\vec{v}\vec{w}\vec{u}]-[\vec{w}\vec{u}\vec{v}]=\vec{u}.(\vec{v}\times\vec{w})$$

70. $\sin^{-1}x=2\sin^{-1}a$

$$-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2};\quad\therefore -\frac{\pi}{2}\le 2\sin^{-1}a\le\frac{\pi}{2}\qquad -\frac{\pi}{4}\le\sin^{-1}a\le\frac{\pi}{4}\text{ or }\frac{-1}{\sqrt{2}}\le a\le\frac{1}{\sqrt{2}}$$

$\therefore |a|\le\frac{1}{\sqrt{2}}$ (As $\frac{1}{\sqrt{2}}>\frac{1}{2}$). Out of given four option no one is absolutely correct but (c) could be taken into consideration. $\rightarrow |a|\le\frac{1}{\sqrt{2}}$ is correct, if $a<\frac{1}{\sqrt{2}}$ is taken as correct then it domain satisfy for $a=\frac{1}{\sqrt{3}}$ but equation is satisfied. $\frac{1}{\sqrt{2}}>\frac{1}{\sqrt{3}}>\frac{1}{2}$

71. Eq. of planes be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ & $\frac{x}{a_1}+\frac{y}{b_1}+\frac{z}{c_1}=1$ ($\perp$r distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a_1^2}+\frac{1}{b_1^2}+\frac{1}{c_1^2}}}\right|$$

$$\therefore \Sigma\frac{1}{a^2}-\Sigma\frac{1}{a_1^2}=0$$

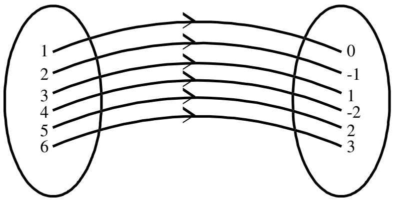
72. $\left(\frac{1+i}{1-i}\right)^x=1 \Rightarrow \left[\frac{(1+i)}{1-i^2}\right]^x=1$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x=1 \Rightarrow (i)^x=1;\ \therefore x=4n;\ n\in 1^+$$

73. $f: N \to 1$

f(1) = 0, f(2) = -1, f(3) = -1, f(4) = -2,

f(5) = 2, and f(6) = -3 so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

74. $f(x) = ax^2 + bx + c$

$f(1) = f(-1) \Rightarrow a + b + c = a - b + c$ or $b = 0$

$\therefore\ f(x) = ax^2 + c$ or $f'(x) = 2ax$

Now $f'(a)$; $f'(b)$; and $f'(c)$ are 2a(a); 2a(b); 2a(c). If a, b, c are in A.P. then $f'(a)$; $f'(b)$ and $f'(c)$ are also in A.P.

75. $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$ .... ∞

Let $T_n=\frac{1}{n(n+1)}=\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$S = T_1 - T_2 + T_3 - T_4 + T_5$ .... ∞

$$=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{5}\right) \ldots\ldots\ldots\ldots$$

$$=1-2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}\ldots\ldots\ldots\ldots\infty\right]$$

$$=1-2[-\log(1+1)+1]=2\log 2-1=\log\left(\frac{4}{e}\right)$$

AIEEE 2003 KEY

Physics And Chemistry		37.	C	75.	D	113.	A	Mathematics		38.	D
		38.	D	76.	B	114.	d	1.	A	39.	A
1.	B	39.	B	77.	C	115.	B	2.	A	40.	B
2.	A	40.	D	78.	B	116.	D	3.	A	41.	D
3.	B	41.	C	79.	B	117.	A	4.	B	42.	C
4.	A	42.	A	80.	D	118.	B	5.	B	43.	C
5.	D	43.	D	81.	C	119.	B	6.	D	44.	A
6.	A	44.	A	82.	C	120.	B	7.	D	45.	A
7.	A	45.	B	83.	B	121.	C	8.	C	46.	NONE
8.	D	46.	A	84.	D	122.	A	9.	C	47.	B
9.	C	47.	A	85.	D	123.	B	10.	D	48.	C
10.	B	48.	B	86.	C	124.	A	11.	D	49.	D
11.	A	49.	B	87.	A	125.	A	12.	A	50.	D
12.	C	50.	C	88.	D	126.	A	13.	A	51.	D
13.	C	51.	C	89.	A	127.	D	14.	C	52.	B
14.	D	52.	D	90.	A	128.	A	15.	D	53.	B
15.	C	53.	B	91.	B	129.	B	16.	B	54.	D
16.	A	54.	D	92.	C	130.	C	17.	C	55.	B
17.	A	55.	C	93.	C	131.	D	18.	B	56.	C
18.	A	56.	B	94.	B	132.	A	19.	D	57.	C
19.	C	57.	B	95.	C	133.	D	20.	A	58.	D
20.	B	58.	A	96.	A	134.	B	21.	D	59.	C
21.	C	59.	A	97.	C	135.	D	22.	D	60.	A
22.	D	60.	A	98.	C	136.	C	23.	D	61.	A
23.	D	61.	D	99.	B	137.	C	24.	D	62.	C
24.	B	62.	B	100.	A	138.	D	25.	A	63.	A
25.	D	63.	C	101.	A	139.	B	26.	D	64.	B
26.	A	64.	C	102.	A	140.	A	27.	A	65.	C
27.	D	65.	B	103.	D	141.	B	28.	A	66.	C
28.	NONE	66.	B	104.	B	142.	B	29.	A	67.	A
29.	C	67.	A	105.	A	143.	B	30.	B	68.	C
30.	B	68.	D	106.	D	144.	A	31.	D	69.	C
31.	D	69.	B	107.	A	145.	B	32.	B	70.	C
32.	A	70.	C	108.	D	146.	B	33.	A	71.	A
33.	D	71.	C	109.	A	147.	A	34.	D	72.	B
34.	C	72.	D	110.	B	148.	A	35.	C	73.	D
35.	C	73.	B	111.	B	149.	A	36..	D	74.	B
36.	A	74.	D	112.	D	150.	D	37.	D	75.	A