FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Sunday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

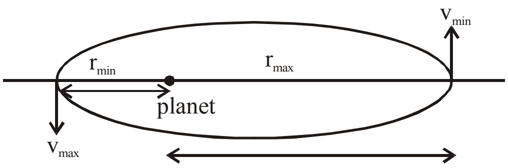
PHYSICS | TEST PAPER WITH ANSWER & SOLUTION

1. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

(1) 1 : 6 (2) 3 : 4

(3) 1 : 3 (4) 1 : 2

Sol.



By angular momentum conservation

$r_{min}v_{max} = r_{max}v_{min}$ ... (i)

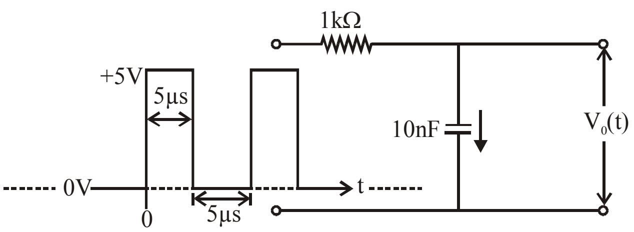
Given $v_{min} = \frac{v_{max}}{6}$

from equation (i)

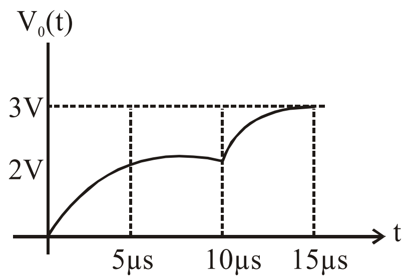
$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

Ans. (1)

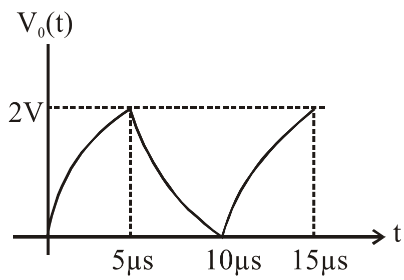
2. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_D(t)$, across the capacitor is correctly depicted by:



(1)



(2)



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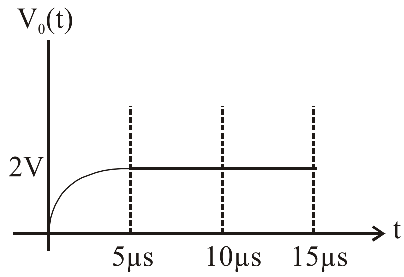

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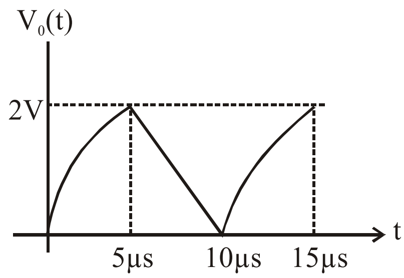
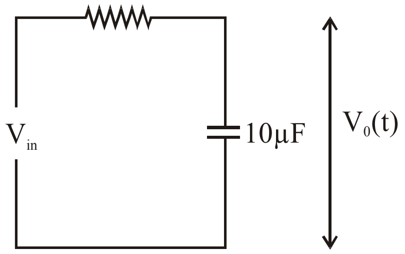


Sol.



$$V_0(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

at t = 5μs

$$V_0(t) = 5\left(1 - e^{-\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)$$

= 5 (1 – $e^{-0.5}$) = 2V

Now V_{in} = 0 means discharging

$$V_0(t) = 2e^{-\frac{t}{RC}} = 2e^{-0.5}$$

= 1.21 V

Now for next 5 μs

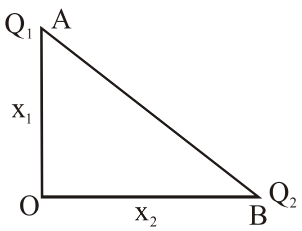
$$V_0(t) = 5 - 3.79e^{-\frac{t}{RC}}$$

after 5 μs again

$V_0(t)$ = 2.79 Volt ≈ 3V

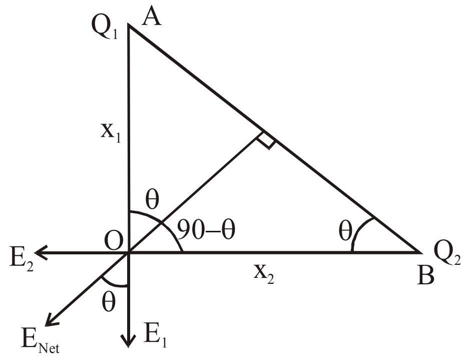
Most approperiate Ans. (1)

3. Charges Q_1 and Q_2 arc at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to :



(1) $\frac{x_2^2}{x_1^2}$ (2) $\frac{x_1^3}{x_2^3}$ (3) $\frac{x_1}{x_2}$ (4) $\frac{x_2}{x_1}$

Sol.



E_2 = electric field due to Q_2

$$= \frac{kQ_2}{x_2^2}$$

$$E_1 = \frac{kQ_1}{x_1^2}$$

From diagram

$$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2}$$

$$\frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$$

$$\frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2}$$

$$\frac{Q_2}{Q_1} = \frac{x_2}{x_1}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

Ans. (3)

4. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively :

(1) Negative, 2 μm
(2) Positive, 10 μm
(3) Positive, 0.1 μm
(4) Positive, 0.1 mm

Sol. Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{no. of division on circular scale}}$$

$$= \frac{0.5}{50}\text{mm} = 1\times10^{-5}\text{m}$$

= 10 μm

Zero error in positive

Ans. (2)

5. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

(1) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (2) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

(3) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (4) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

Sol. An elastic wire can be treated as a spring with

$$k = \frac{YA}{\ell}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{m\ell}}$$

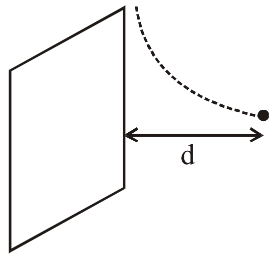
Ans. (1)

6. A particle of charge q and mass m is moving with a velocity $-\upsilon\hat{i}\,(\upsilon \neq 0)$ towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of υ for which the particle will not hit the screen is:

(1) $\frac{qdB_0}{2m}$ (2) $\frac{qdB_0}{m}$

(3) $\frac{2qdB_0}{m}$ (4) $\frac{qdB_0}{3m}$

Sol.



In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'd', particle will not hit the screen.

$$d = \frac{mv}{qB_0}$$

$$v = \frac{qB_0 d}{m}$$

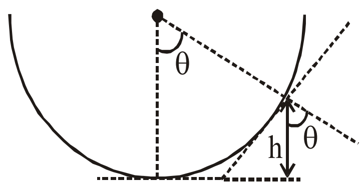
∴ correct option is (2)

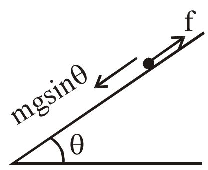
7. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is : ($g = 10ms^{-2}$)

(1) 0.80 m (2) 0.60 m

(3) 0.45 m (4) 0.20 m

Sol.



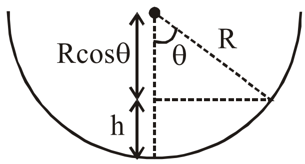


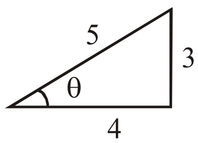
For balancing mgsinθ = f

mgsinθ = μmgcosθ

tanθ = μ

$$\tan\theta = \frac{3}{4}$$





$h = R - R\cos\theta$

$$= R - R\left(\frac{4}{5}\right) = \frac{R}{5}$$

$$h = \frac{R}{5} = 0.2\text{m}$$

$\therefore$ correct option is (4)

8. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:

(1) 10^{-3} (2) 10^{-2}
(3) 10^{-4} (4) 10^{-1}

Sol. R = 0.1 m

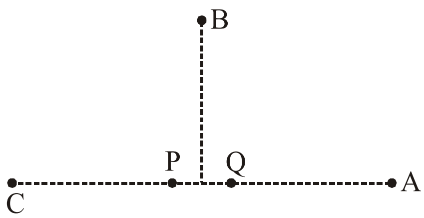
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \text{ rad/sec}$$

$a = \omega^2 R$
$= (0.105)^2 (0.1)$
$= 0.0011$
$= 1.1 \times 10^{-3}$

Average acceleration is of the order of 10^{-3}

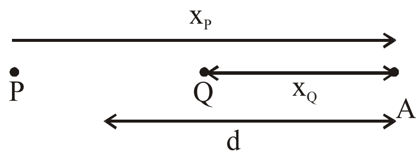
$\therefore$ correct option is (1)

9. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:



(1) 0 : 1 : 2 (2) 4 : 1 : 0
(3) 0 : 1 : 4 (4) 2 : 1 : 0

Sol. For (A)



$x_P - x_Q = (d + 2.5) - (d - 2.5)$
$= 5\text{m}$

$$\Delta\phi \text{ due to path difference} = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5)$$

$$= \frac{\pi}{2}$$

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

Total phase difference at A

$$= \frac{\pi}{2} - \frac{\pi}{2} \text{ (due to P being ahead of Q by 90°)}$$

$= 0$

$$I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$$

$$= I + I + 2\sqrt{I}\sqrt{I}\cos(0)$$

$= 4I$

For C

$x_Q - x_P = 5$ m

$$\Delta\phi \text{ due to path difference} = \frac{2\pi}{\lambda}(\Delta x)$$

$$= \frac{2\pi}{20}(5) = \frac{\pi}{2}$$

$$\text{Total phase difference at C} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta\phi)$$

$$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$$

For B

$x_P - x_Q = 0,$

$$\Delta\phi = \frac{\pi}{2} \text{ (Due to P being ahead of Q by 90°)}$$

$$I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$$

$I_A : I_B : I_C = 4I : 2I : 0$

$= 2 : 1 : 0$

$\therefore$ correct option is (4)

10. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_P is:

(1) $\lambda_e < \lambda_P < \lambda_{He^{++}}$ (2) $\lambda_e < \lambda_{He^{++}} = \lambda_P$

(3) $\lambda_e > \lambda_{He^{++}} > \lambda_P$ (4) $\lambda_e > \lambda_P > \lambda_{He^{++}}$

Sol. $\lambda = \dfrac{h}{P} = \dfrac{h}{\sqrt{2m(KE)}}$

$$\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \lambda = \frac{C}{\sqrt{m}}$$

$$m_{He^{++}} > m_P > m_e$$

$$\therefore \lambda_{He^{++}} < \lambda_P < \lambda_e$$

$\therefore$ correct option is (4)

11. An electron is moving along + x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along + y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

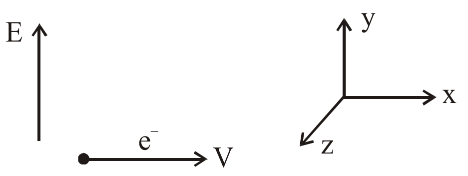
(1) 5×10^{-3} T, along +z direction

(2) 3×10^{-4} T, along –z direction

(3) 3×10^{-4} T, along +z direction

(4) 5×10^{-3} T, along –z direction

Sol.



$\vec{B}$ must be in +z axis.

$\vec{V} = 6 \times 10^6 \hat{i}$

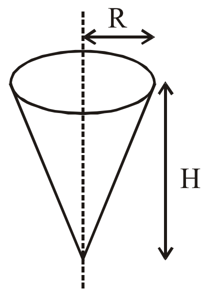
$\vec{E} = 300\hat{j}$ V/cm $= 3 \times 10^4$ V/m

$q\vec{E} + q\vec{V} \times \vec{B} = 0$

$E = VB$

$$B = \frac{E}{V} = \frac{3 \times 10^4}{6 \times 10^6} = 5 \times 10^{-3}\,T$$

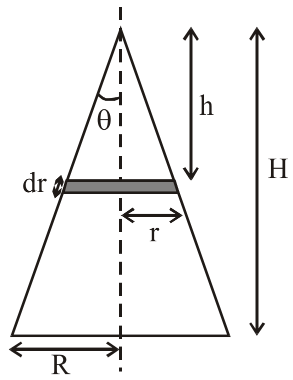
12. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is:



(1) $\dfrac{MR^2}{2}$ (2) $\dfrac{MH^2}{3}$

(3) $\dfrac{MR^2}{3}$ (4) $\dfrac{M(R^2+H^2)}{4}$

Sol.



Area $= \pi R\ell = \pi R\left(\sqrt{H^2+R^2}\right)$

Area of element dA $= 2\pi r d\ell = 2\pi r \dfrac{dh}{\cos\theta}$

mass of element $dm = \dfrac{M}{\pi R\sqrt{H^2+R^2}} \times \dfrac{2\pi r dh}{\cos\theta}$

$$dm = \frac{2Mh\tan\theta dh}{R\sqrt{H^2+R^2}\cos\theta} \quad \text{(here r = h tan}\theta)$$

$$I = \int (dm)r^2 = \int \frac{h^2\tan^2\theta}{\cos\theta}\left(\frac{2m}{R}\frac{h\tan\theta}{\sqrt{R^2+H^2}}\right)dh$$

$$= \frac{2M}{\cos\theta R}\frac{\tan^3\theta}{\sqrt{R^2+H^2}}$$

$$\int_0^H h^3 dh = \frac{MR^2H^4}{2RH^3\sqrt{R^2+H^2}\cos\theta}$$

$$= \frac{MR^2H\sqrt{R^2+H^2}}{2\sqrt{R^2+H^2}\times H}$$

$$= \frac{MR^2}{2}$$

13. An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is :

(1) 0.5 (2) 2

(3) 20 (4) 400

Sol. $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}$

$$= \frac{1}{100}\sqrt{40\times10^3}$$

$$= \frac{200}{100} = 2$$

14. If the potential energy between two molecules is given by $U = \frac{A}{r^6}+\frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

(1) $\left(\frac{B}{A}\right)^{1/6}, 0$ (2) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

(3) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (4) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

Sol. $U = \frac{-A}{r^6}+\frac{B}{r^{12}}$

$$F = -\frac{dU}{dr} = -\left(A\left(-6r^{-7}\right)\right)+B\left(-12r^{-13}\right)$$

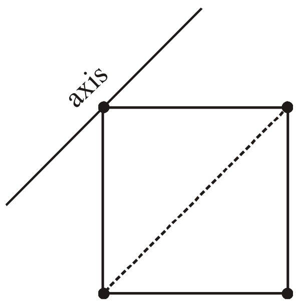
$$0 = \frac{6A}{r^7}-\frac{12B}{r^{13}}$$

$$\frac{6A}{12B} = \frac{1}{r^6} \Rightarrow r = \left(\frac{2B}{A}\right)^{1/6}$$

$$U\left(r=\left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A}+\frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B}+\frac{A^2}{4B} = \frac{-A^2}{4B}$$

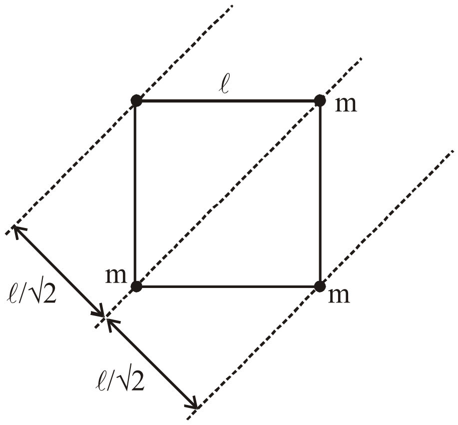
15. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through oneof the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:



(1) $2m\ell^2\omega$ (2) $3m\ell^2\omega$

(3) $m\ell^2\omega$ (4) $4m\ell^2\omega$

Sol.



$$I = m(0)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 \times 2 + m\left(\sqrt{2}\ell\right)^2$$

$$= \frac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$$

Angular momentum L = Iω

$= 3m\ell^2\omega$

16. You are given that Mass of ${}^{7}_{3}Li$ = 7.0160 u,

Mass of ${}^{4}_{2}He = 4.0026$ u

and Mass of ${}^{1}_{1}H$ = 1.0079 u.

When 20 g of ${}^{7}_{3}Li$ is converted into ${}^{4}_{2}He$ by proton capture, the energy liberated, (in kWh), is: [Mass of nudeon = 1 GeV/c^2]

(1) 8×10^6 (2) 1.33×10^6

(3) 6.82×10^5 (4) 4.5×10^5

Sol. ${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow 2\left({}^{4}_{2}He\right)$

$\Delta m \Rightarrow [m_{Li} + m_H] - 2[M_{He}]$

Energy released in 1 reaction $\Rightarrow \Delta mc^2$.

In use of 7.016 u Li energy is Δmc^2

In use of 1gm Li energy is $\dfrac{\Delta mc^2}{m_{Li}}$

In use of 20 gm energy is $\Rightarrow \dfrac{\Delta mc^2}{m_{Li}} \times 20gm$

$$\Rightarrow \frac{[(7.016 + 1.0079) - 2 \times 4.0026]u \times c^2}{7.016 \times 1.6 \times 10^{-24} gm} \times 20gm$$

$$\Rightarrow \left(\frac{0.0187 \times 1.6 \times 10^{-19} \times 10^9}{7.016 \times 1.6 \times 10^{-24} gm} \times 20gm\right) \text{Joule}$$

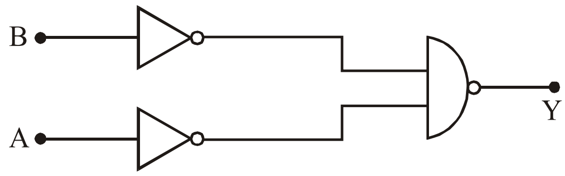
$\Rightarrow 0.05 \times 10^{+14}$ J

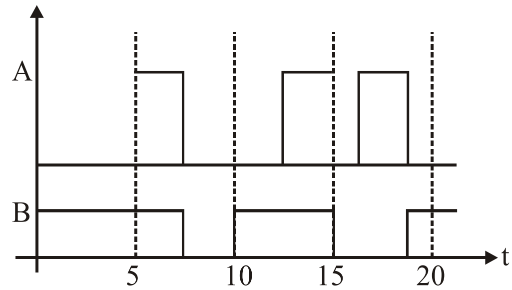
$\Rightarrow 1.4 \times 10^{+6}$ kwh

[1 J $\Rightarrow 2.778 \times 10^{-7}$ kwh]

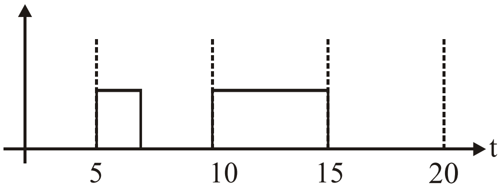
Ans. (2)

17. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.





(1)



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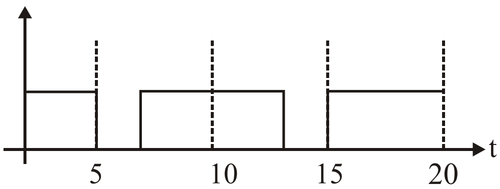

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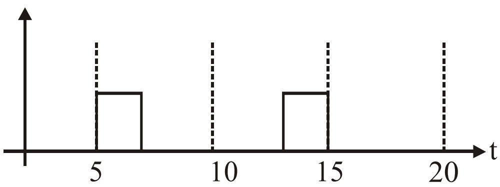

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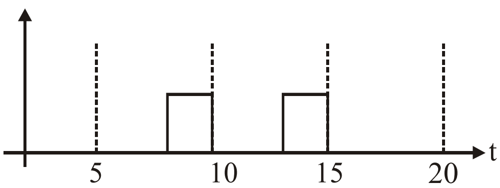
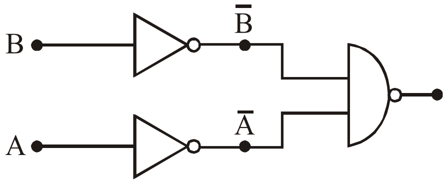
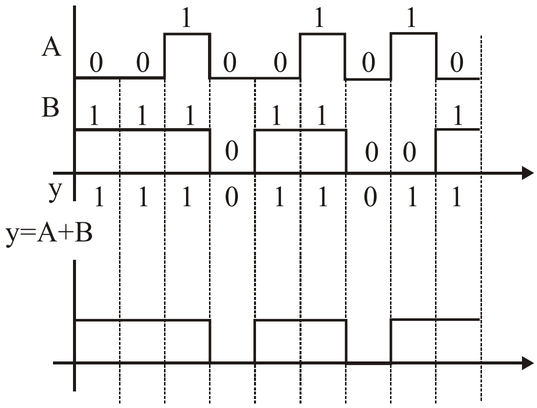


Sol.



$y = \overline{\overline{A}.\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$



18. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_P}{C_v}\right)$ given, respectively, by:

(1) $U=\frac{5}{2}RT$ and $\gamma=\frac{6}{5}$

(2) U = 5RT and $\gamma=\frac{7}{5}$

(3) U = 5RT and $\gamma=\frac{6}{5}$

(4) $U=\frac{5}{2}RT$ and $\gamma=\frac{7}{5}$

Sol. Total degree of freedom = 3 + 2 = 5

$$U=\frac{nfRT}{2}\Rightarrow\frac{5RT}{2}$$

$$\gamma\Rightarrow\frac{C_P}{C_V}\Rightarrow 1+\frac{2}{f}\Rightarrow 1+\frac{2}{5}\Rightarrow\frac{7}{5}$$

Ans. (4)

19. A point like object is placed at a distance of 1m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is :

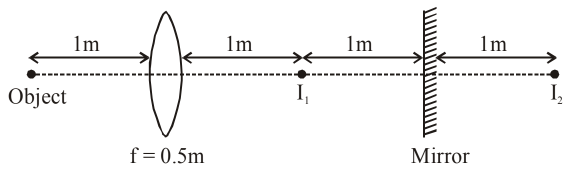
(1) 1 m from the mirror, virtual

(2) 1 m from the mirror, real

(3) 2.6 m from the mirror, real

(4) 2.6 m from the mirror, virtual

Sol.



Object is at 2f. So image will also be at '2f'. (I_1).

Image of I_1 will be 1m behind mirror.

i.e. $\Rightarrow I_2$

Now I_2 will be object for lens.

$\therefore$ u $\Rightarrow$ – 3m

f $\Rightarrow$ +0.5 m

$$\frac{1}{v}\Rightarrow\frac{1}{f}+\frac{1}{u}$$

$$\Rightarrow\frac{1}{+0.5}+\frac{1}{-3}$$

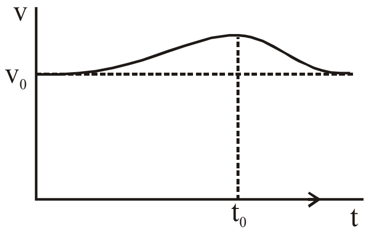
$$v\Rightarrow\frac{3}{5}\Rightarrow 0.6m$$

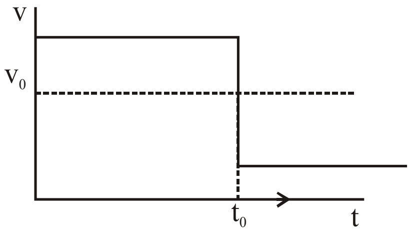
So total distance from mirror $\Rightarrow$ 2 + 0.6

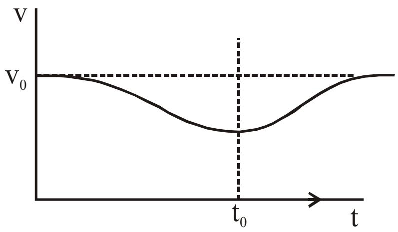
$\Rightarrow$ 2.6 m and real image

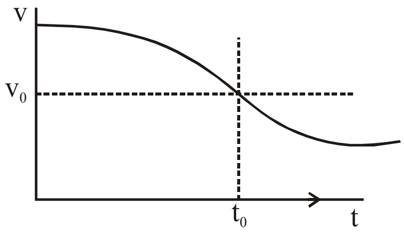
Ans. (3)

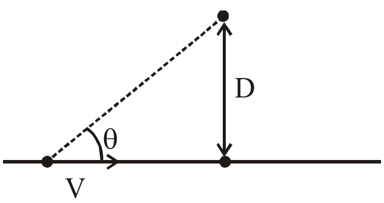
20. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by :
(t_0 represents the instant when the distance between the source and observer is minimum)

(1)


(2)


(3)


(4)


Sol.


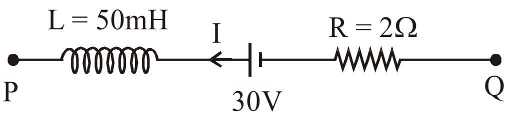
$$f_{observed} \Rightarrow \left(\frac{v_{sound}}{v_{sound} - v\cos\theta}\right) f_0$$

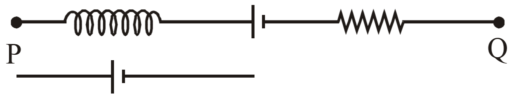
initially θ will be less ⇒ cosθ more

∴ $f_{observed}$ more, then it will decrease.

∴ Ans. (4)

21. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is.



Sol.


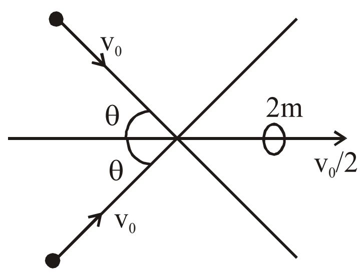
$$\frac{Ldi}{dt} = 5$$

$V_P - 5 - 30 + 2 \times 1 = VQ$

$V_P - V_Q = 33$ volt

Ans. 33.00

22. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is.

Sol.


Momentum conservation along x

$$2mv_0\cos\theta = 2m\frac{v_0}{2}$$

$\cos\theta = \frac{1}{2}$

$\theta = 60$

Angle is $2\theta = 120$

Ans. 120.00

23. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right) W/m^2$. The rms electric field, in units of V/m associated with this source is close to the nearest integer is

($\in_0 = 8.86 \times 10^{-12}$ C^2 Nm^{-2}; $c = 3 \times 10^8$ ms^{-1})

Sol. $I = \in_0 E_{rms}^2 C$

$$E_{rms}^2 = \frac{I}{\in_0 C}$$

$$= \frac{315}{\pi \in_0} \times \frac{1}{C}$$

$$= \frac{4 \times 315}{4\pi \in_0} \times \frac{1}{3 \times 10^8}$$

$$= \frac{4 \times 315 \times 9 \times 10^9}{3 \times 10^8}$$

$E_{rms}^2 = 4 \times 315 \times 30$

$E_{rms} = 2\sqrt{315 \times 30}$

$= 194.42$

Ans. 194.00

24. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .

Sol. $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$$\rho = \frac{6}{\pi} M\ D^{-3}$$

taking log

$$\ell n\rho = \ell n\left(\frac{6}{\pi}\right) + \ell nM - 3\ell mD$$

Differentiates

$$\frac{d\rho}{\rho} = 0 + \frac{dM}{M} - 3\frac{d(D)}{D}$$

for maximum error

$$100 \times \frac{d\rho}{\rho} = \frac{dM}{M} \times 100 + \frac{3dD}{D} \times 100$$

$= 6 + 3 \times 1.5$

$= 10.5\ \%$

$= \frac{1050}{100}\%$ so $x = 1050.00$

25. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is.

Sol. $PV = nRT$

$P_1V_1 = nR\ 250$

$$P_2(2V_1) = \frac{5n}{4} R \times 2000$$

Divide

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5}$$

$$\frac{P_2}{P_1} = 5$$

Ans. 5.00

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Sunday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. The set that contains atomic number of only transition element is -

(1) 21, 32, 53, 64

(2) 21, 25, 42, 72

(3) 9, 17, 34, 38

(4) 37, 42, 50, 64

2. The lanthanoid that does NOT show +4 oxidation state is

(1) Dy

(2) Eu

(3) Ce

(4) Tb

3. The INCORRECT statement is :

(1) bronze is an alloy of copper and tin.

(2) brass is an alloy of copper and nickel

(3) cast iron is used to manufacture wrought iron.

(4) german silver is an alloy of zinc, copper and nickel

4. The correct statement with respect to dinitrogen is :

(1) liquid dinitrogen is not used in cryosurgery.

(2) it can be used as an inert diluent for reactive chemicals.

(3) it can combine with dioxygen at 25°C

(4) N_2 is paramagnetic in nature.

5. A solution of two components containing n_1 moles of the 1st component and n_2 moles of the 2nd component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2nd component, then C_2 can be expressed as :

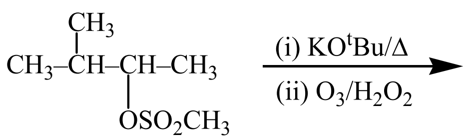
(1) $C_2 = \dfrac{1000x_2}{M_1 + x_2(M_2 - M_1)}$

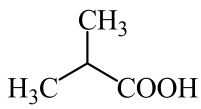
(2) $C_2 = \dfrac{dx_2}{M_2 + x_2(M_2 - M_1)}$

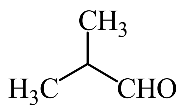
(3) $C_2 = \dfrac{dx_1}{M_2 + x_2(M_2 - M_1)}$

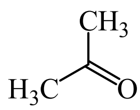
(4) $C_2 = \dfrac{1000dx_2}{M_1 + x_2(M_2 - M_1)}$

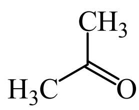
6. The major products of the following reaction are :



(1)  + HCOOH

(2)  + HCHO

(3)  + CH_3CHO

(4)  + CH_3COOH

7. Kraft temperature is the temperature

(1) below which the formation of micelles takes place.

(2) below which the aqueous solution of detergents starts freezing.

(3) above which the formation of micelles takes place.

(4) above which the aqueous solution of detergents starts boiling.

8. Consider the Assertion and Reason given below.

Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R): High density polymers are closely packed and are chemically inert. Choose the correct answer from the following :

(1) (A) is correct but (R) is wrong.

(2) (A) and (R) both are wrong.

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

9. The species that has a spin only magnetic moment of 5.9 BM, is -

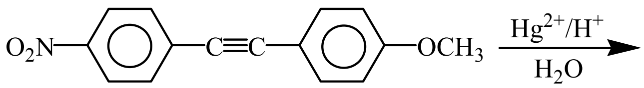
(1) $Ni(CO)_4(T_d)$

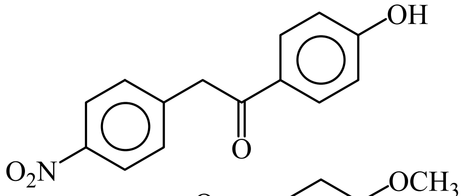
(2) $[MnBr_4]^{2-}(T_d)$

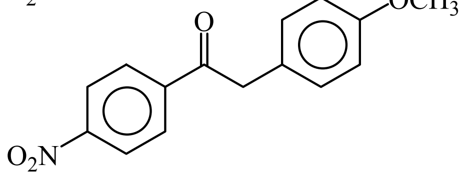
(3) $[NiCl_4]^{2-}(T_d)$

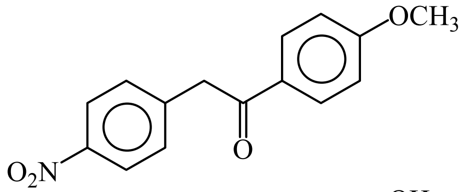
(4) $[Ni(CN)_4]^{2-}$ (square planar)

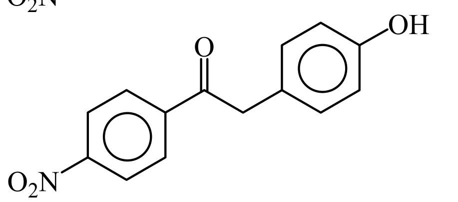
10. The major product obtained from the following reaction is -



(1)


(2)


(3)


(4)


11. For the reaction :

$$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$$

(1) $K_C = K_P(RT)$ (2) $K_C = K_P(RT)^{-1/2}$

(3) $K_C = K_P(RT)^{-3/2}$ (4) $K_C = K_P(RT)^{1/2}$

12. Arrange the following solutions is the decreasing order of pOH :

(A) 0.01 M HC1

(B) 0.01 M NaOH

(C) 0.01 M CH_3COONa

(D) 0.01 M NaCl

(1) (B) > (C) > (D) > (A)

(2) (A) > (C) > (D) > (B)

(3) (B) > (D) > (C) > (A)

(4) (A) > (D) > (Q > (B)

13. The presence of soluble fluoride ion upto 1 ppm concentration in drinking water, is :

(1) harmful to bones

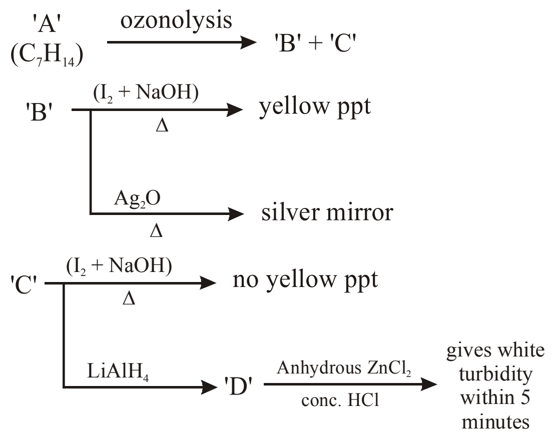
(2) harmful for teeth

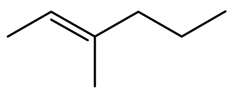
(3) safe for teeth

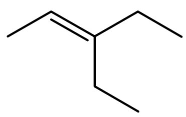
(4) harmful to skin

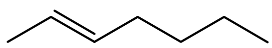
14. Consider the following reactions :

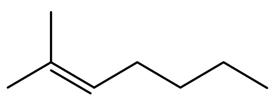
'A' is -



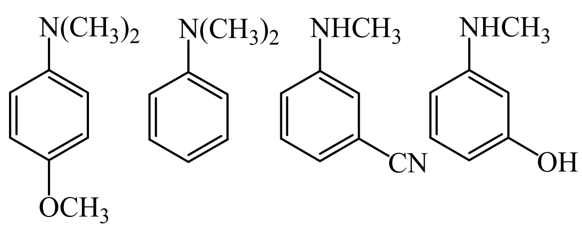
(1) 

(2) 

(3) 

(4) 

15. The increasing order of pK_b values of the following compounds is -



I II III IV

(1) I < II < IV < III

(2) II < IV < III < I

(3) II < I < III < IV

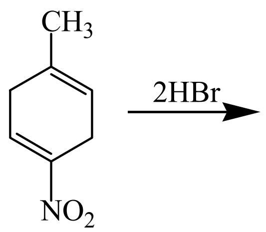
(4) I < II < III < IV

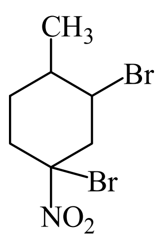
16. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively, are:

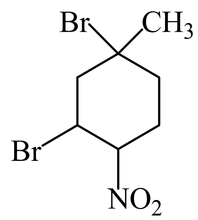
(1) high and high (2) poor and poor

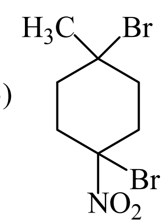
(3) high and poor (4) poor and high

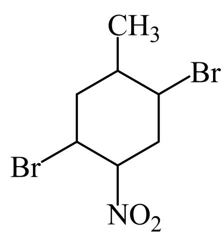
17. The major product of the following reaction is



(1) 

(2) 

(3) 

(4) 

18. The variation of equilibrium constant with temperature is given below :

Temperature	Equilibrium constant
$T_1 = 25°C$	$K_1 = 100$
$T_2 = 100°C$	$K_2 = 100$

The values of $\Delta H°$, $\Delta G°$ at T_1 and $\Delta G°$ at T_2 (in kJ mol^{-1}) respectively, are close to

[Use R = 8.314 $JK^{-1}mol^{-1}$]

(1) 0.64, –5.71 and –14.29

(2) 28.4, –7.14 and –5.71

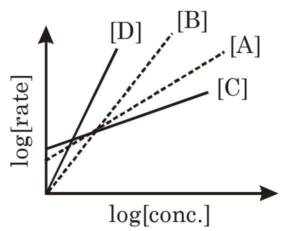
(3) 28.4, –5.71 and –14.29

(4) 0.64, –7.14 and –5.71

19. Consider the following reactions :

$A \rightarrow P1$; $B \rightarrow P2$; $C \rightarrow P3$; $D \rightarrow P4$

The order of the above reactions are a, b, c, and d, respectively. The following graph is obtained when log [rate] vs. log[conc] are plotted:



Among the following , the correct sequence for the order of the reactions is:

(1) $a > b > c > d$
(2) $c > a > b > d$
(3) $d > b > a > c$
(4) $d > a > b > c$

20. Which of the following compound shows geometrical isomerism

(1) 2-methylpent-2-ene
(2) 4-methylpent-l-ene
(3) 4-methylpent-2-ene
(4) 2-methylpent-l-ene

21. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is _______

(Atomic mass, Ag=108, Br = 80 g mol^{-1})

22. The elevation of boiling point of 0.10 m aqueous $CrCl_3.xNH_3$ solution is two times that of 0.05m aqueous $CaCl_2$ solution. The value of x is____.

[Assume 100% ionisation of the complex and $CaCl_2$, coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere]

23. A spherical balloon of radius 3 cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12 cm containing the same amount of gas will be_____$\times 10^{-6}$ bar.

24. The number of $Cl = O$ bonds in perchloric acid is, "_________"

25. Potassium chlorate is prepared by the electrolysis of KCl in basic solution

$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10 g of $KClO_3$ using a current of 2 A is_____.

(Given : F = 96,500 C mol^{-1} molar mass of $KClO_3$=122 $gmol^{-1}$)

FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 06th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

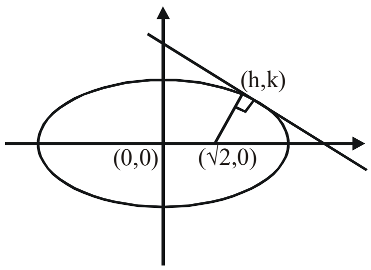
MATHEMATICS — TEST PAPER WITH SOLUTION

1. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $\left(-1,\sqrt{3}\right)$ (2) $\left(-1,\sqrt{2}\right)$

(3) $\left(-2,\sqrt{3}\right)$ (4) (1,2)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given)

a =2 , $b=\sqrt{2}$, $e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus (ae,0) $=\left(\sqrt{2},0\right)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k)

$(k - mh)^2 = 4m^2 + 2$ ...(1)

line perpendicular to tangent will have slope $-\frac{1}{m}$

$y - 0 = -\frac{1}{m}\left(x-\sqrt{2}\right)$

$my=-x+\sqrt{2}$

$(h + mk)^2 = 2$ ...(2)

Add equaiton (1) and (2)

$k^2(1 + m^2) + h^2(1 + m^2) = 4(1 + m^2)$

$h^2 + k^2 = 4$

$x^2 + y^2 = 4$ (Auxilary circle)

$\therefore$ $\left(-1,\sqrt{3}\right)$ lies on the locus.

2. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) 2!3!4! (2) $(3!)^3.(4!)$

(3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

Sol. Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$=\underset{\text{Arrangment of 3 Families}}{3!}\quad\underset{\text{Interval Arrangment of families members}}{3!\times3!\times4!}$

$\therefore$ Probability of same family memebers are together $=\frac{3!\,3!\,3!\,4!}{10!}=\frac{1}{700}$

so option(2) is correct.

3. $\lim_{x\to1}\left(\frac{\int_0^{(x-1)^2} t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

Sol. $\lim\limits_{x\to 1}\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\left(\dfrac{0}{0}\right)$

Apply L Hopital Rule

$$=\lim_{x\to 1}\frac{2(x-1).(x-1)^2\cos(x-1)^4-0}{(x-1).\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$

4. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\dfrac{3^{200}}{8}\right\}$, is equal to

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{3}{8}$ (4) $\frac{7}{8}$

Sol. $\left\{\dfrac{3^{200}}{8}\right\}=\left\{\dfrac{(3^2)^{100}}{8}\right\}$

$$=\left\{\frac{(1+8)^{100}}{8}\right\}$$

$$=\left\{\frac{1+{}^{100}C_1.8+{}^{100}C_2.8^2+\ldots+{}^{100}C_{100}8^{100}}{8}\right\}$$

$$=\left\{\frac{1+8m}{8}\right\}$$

$$=\frac{1}{8}$$

5. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

Now $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & \lambda\end{vmatrix}=0$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

Now $D_1=\begin{vmatrix}2 & 1 & 1\\ 5 & 2 & 3\\ \mu & 3 & 5\end{vmatrix}=0$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

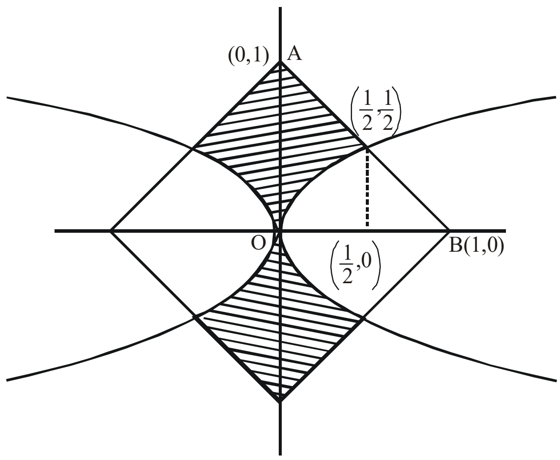
$\mu = 8$

6. The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ is :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{7}{6}$ (4) $\frac{5}{6}$

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$$y = \frac{1}{2} \text{ or } -1$$

Now Area of $\Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}} \int_0^{\frac{1}{2}} \sqrt{x}\, dx = \frac{1}{6}$

Now area of shaded region in first quadrant

= Area of $\Delta OAB - R_1 - R_2$

$$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$

so option (4) is correct.

7. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{15}{101}$ (2) $\frac{5}{101}$ (3) $\frac{5}{33}$ (4) $\frac{10}{99}$

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = ${}^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

$2b \Rightarrow$ even number

$$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$$

so favourable cases = ${}^6C_2 + {}^5C_2$

= 15 + 10 = 25

P(3 numbers are in A.P. $= \frac{25}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$)

8. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

(1) $n\sqrt{a-1}$ (2) $\sqrt{a-1}$

(3) a – 1 (4) $\sqrt{n(a-1)}$

Sol. S.D $= \sqrt{\frac{\sum_{i=1}^{n}(x_i - a)}{n} - \left(\frac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2}$

$$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2}$$

{Given $\sum_{i=1}^{n}(x_i - a) = n$ $\sum_{i=1}^{n}(x_i - a)^2 = na$ }

$= \sqrt{a-1}$

9. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) x + 3 = 0 (2) x + 2y = 0

(3) 2x + 1 = 0 (4) x + 2 = 0

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x+2) + \frac{2}{m'}$

$$y = m'x + 2\left(m' + \frac{1}{m'}\right)$$

since lines intersect at right angles

$\therefore mm' = -1$

Now $y = mx + m + \frac{1}{m}$ ...(1)

$$y = m'x + 2\left(m' + \frac{1}{m'}\right)$$

$$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$$

$$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right) \quad ...(2)$$

From equation (1) and (2)

$$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$$

$$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$$

$\therefore$ x + 3 = 0

10. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$

(3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

Sol. Negation of $\phi \vee (\sim p \wedge q)$

$p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q)$

$= (T) \wedge (p \vee q)$

$= (p \vee q)$

now negation of $(p \vee q)$ is

$\sim (p \vee q) = \sim p \wedge \sim q$

11. If $f(x + y) = f(x) f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Sol. $f(x + y) = f(x).\ f(y)$

$\sum_{x=1}^{\infty} f(x) = 2$ where x,y $\in$ N

$f(1) + f(2) + f(3) + ... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put x = y = 1

$f(2) = f(1 + 1) = f(1).\ f(1) = (f(1))^2$

$f(3) = f(2 + 1) = f(2).\ f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$$\frac{f(1)}{1 - f(1)} = 2$$

$$\Rightarrow f(1) = \frac{2}{3}$$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$$f(4) = \left(\frac{2}{3}\right)^4$$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

12. The general solution of the differential equation

$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$ is :

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

Sol. $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$$\Rightarrow \sqrt{(1+x)^2(1+y^2)}+xy\frac{dy}{dx}=0$$

$$\Rightarrow \sqrt{1+x^2}\sqrt{1+y^2}=-xy\frac{dy}{dx}$$

$$\Rightarrow \int\frac{ydy}{\sqrt{1+y^2}}=-\int\frac{\sqrt{1+x^2}}{x}dx \quad ...(1)$$

Now put $1+x^2=u^2$ and $1+y^2=v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow$ $xdx = udu$ and $ydy = vdv$

substitute these values in equation (1)

$$\int\frac{vdv}{v}=-\int\frac{u^2.du}{u^2-1}$$

$$\Rightarrow \int dv=-\int\frac{u^2-1+1}{u^2-1}du$$

$$\Rightarrow v=-\int\left(1+\frac{1}{u^2-1}\right)du$$

$$\Rightarrow v=-u-\frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right|+c$$

$$\Rightarrow \sqrt{1+y^2}=-\sqrt{1+x^2}+\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$$

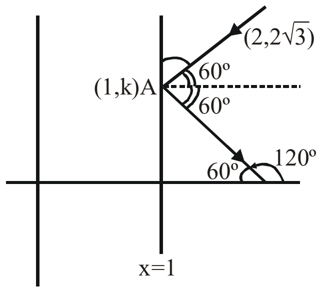
$$\Rightarrow \sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$$

13. A ray of light coming from the point $\left(2,2\sqrt{3}\right)$ is incident at an angle 30° on the line x=l at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (2) $\left(3,-\sqrt{3}\right)$

(3) $\left(4,-\frac{\sqrt{3}}{2}\right)$ (4) $\left(4,-\sqrt{3}\right)$

Sol. For point A



$$\tan 60°=\frac{2\sqrt{3}-k}{2-1}$$

$$\sqrt{3}=2\sqrt{3}-k$$

$$\therefore\ k=\sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$

Now slope of line AB is m_{AB} = tan120°

$m\ m_{AB}=-\sqrt{3}$

Now equation of line AB is

$$y-\sqrt{3}=-\sqrt{3}(x-1)$$

$$\sqrt{3}x+y=2\sqrt{3}$$

Now satisfy options

14. Let a,b,c,d and p be any non zero distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)=0$. Then :

(1) a,c,p are in G.P. (2) a,c,p are in A.P.

(3) a,b,c,d are in G.P. (4) a,b,c,d are in A.P.

Sol. $(a^2+b^2+c^2)p^2+2(ab+bc+cd)p+b^2+c^2+d^2=0$

$\Rightarrow (a^2p^2+2abp+b^2)+(b^2p^2+2bcp+c^2)+(c^2p^2+2cdp+d^2)=0$

$\Rightarrow (ab+b)^2+(bp+c)^2+(cp+d)^2=0$

This is possible only when

$ap+b=0$ and $bp+c=0$ and $cp+d=0$

$$p=-\frac{b}{a}=-\frac{c}{b}=-\frac{d}{c}$$

$$\text{or } \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

$\therefore$ a,b,c,d are in G.P.

15. If $I_1=\int_0^1\left(1-x^{50}\right)^{100}dx$ and $I_2=\int_0^1\left(1-x^{50}\right)^{101}dx$ such that $I_2=\alpha I_1$ then α equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

Sol. $I_1 = \int_0^1 (1-x^{50})^{100}\,dx$ and $I_2 = \int_0^1 (1-x^{50})^{101}\,dx$

and $I_1 = \lambda I_2$

$$I_2 = \int_0^1 (1-x^{50})^{101}\,dx$$

$$I_2 = \int_0^1 (1-x^{50})(1-x^{50})^{100}\,dx$$

$$I_2 = \int_0^1 (1-x^{50})\,dx - \int_0^1 x^{50}.(1-x^{50})^{100}\,dx$$

$$I_2 = I_1 - \int_0^1 \underbrace{x}_{I}.\underbrace{x^{49}.(1-x^{50})^{100}\,dx}_{II}$$

Now apply IBP

$$I_2 = I_1 - \left[x\int x^{49}.(1-x^{50})^{100}\,dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.(1-x^{50})^{100}\,dx\right]$$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1\left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101}dx\right]$$

$$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$$

$$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$$

$$\therefore\ \alpha = \frac{5050}{5051}$$

$$I_2 = \frac{5050}{5051}I_1$$

$\because I_2 = \alpha.I_1$

16. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1,t_2]$ is attained at the point :

(1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$

(3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

Sol. $$\frac{f(t_2)-f(t_1)}{t_2-t_1} = 2at+b$$

$$\frac{a(t_2^2-t_1^2)+b(t_2-t_1)}{t_2-t_1} = 2at+b$$

$\Rightarrow a(t_2 + t_1) + b = 2at + b$

$$\Rightarrow t = \frac{t_1+t_2}{2}$$

17. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality :

(1) $y^2 \ge x + 1$ (2) $y^2 \ge 2(x + 1)$

(3) $y^2 \le x + \frac{1}{2}$ (4) $y^2 \le 2\left(x+\frac{1}{2}\right)$

Sol. $z = x + iy$

$|z| - ke(z) \le 1$

$$\Rightarrow \sqrt{x^2+y^2} - x \le 1$$

$$\Rightarrow \sqrt{x^2+y^2} \le 1+x$$

$$\Rightarrow x^2+y^2 \le 1+2x+x^2$$

$$\Rightarrow y^2 \le 2x+1$$

$$\Rightarrow y^2 \le 2\left(x+\frac{1}{2}\right)$$

18. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

(1) 1 (2) 3

(3) 4 (4) 2

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$, $\alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$$

$$= \frac{64}{(256)^{5/8}}$$

$= 2$

19. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$x=\frac{-2z-4}{3}$ ...(3)

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$\frac{3x+4}{-2}=-3y+1=z$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}}=\frac{y-\frac{1}{3}}{-\frac{1}{3}}=\frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}}=\frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}}=\frac{z-0}{1}$$

$$S.D.=\left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a}=(1,-1,0)$

$\vec{b}=\left(-\frac{4}{3},\frac{1}{3},0\right)$

$\vec{c}=(0,-1,1)$

$\vec{d}=\left(-\frac{2}{3},-\frac{1}{3},1\right)$

$\Rightarrow S.D=\frac{1}{\sqrt{3}}$

20. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix}\cos^2 x & 1+\sin^2 x & \sin 2x\\ 1+\cos^2 x & \sin^2 x & \sin 2x\\ \cos^2 x & \sin^2 x & 1+\sin 2x\end{vmatrix}.$$

Then the ordered pair (m,M) is equal to

(1) (–3,–1) (2) (–4,–1)

(3) (1,3) (4) (–3,3)

Sol. $$\begin{vmatrix}\cos^2 x & 1+\sin^2 x & \sin 2x\\ 1+\cos^2 x & \sin^2 x & \sin 2x\\ \cos^2 x & \sin^2 x & 1+\sin 2x\end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix}-1 & 1 & 0\\ 1 & 0 & -1\\ \cos^2 x & \sin^2 x & 1+\sin 2x\end{vmatrix}$$

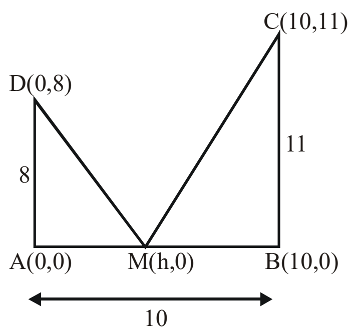
$=-1(\sin^2 x)-1(1+\sin 2x+\cos^2 x)$

$=-\sin 2x-2$

$m = -3$, $M = -1$

21. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

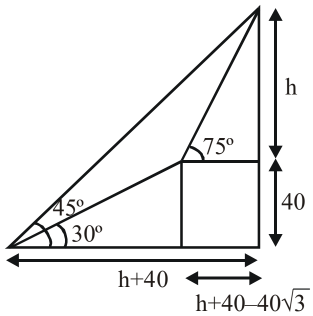
$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

it is minimum if h = 5

22. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is_.

Sol.



$\tan 75^\circ = \dfrac{h}{h+40-40\sqrt{3}}$

$\dfrac{2+\sqrt{3}}{1} = \dfrac{h}{h+40-40\sqrt{3}}$

$\Rightarrow 2h+80-80\sqrt{3}+\sqrt{3}h+40\sqrt{3}-120 = h$

$\Rightarrow h\left(\sqrt{3}+1\right) = 40+40\sqrt{3}$

$\Rightarrow h = 40$

$\therefore$ Height of hill = 40 + 40 = 80m

23. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _.

Sol. $2^m - 2^n = 112$

$m = 7, n = 4$

$(2^7 - 2^4 = 112)$

$m \times n = 7 \times 4 = 28$

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is _.

Sol. $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$

$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right)+\sqrt{2-2\cos\theta}$

$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right)+\sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$= 2\sqrt{3}\left|\cos\dfrac{\theta}{2}\right|+2\left|\sin\dfrac{\theta}{2}\right|$

$\le \sqrt{\left(2\sqrt{3}\right)^2+(2)^2} = 4$

25. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\dfrac{1}{x}\right)+5x^2 & , \quad x<0 \\ 0 & , \quad x=0 \\ x^5\cos\left(\dfrac{1}{x}\right)+\lambda x^2 & , \quad x>0 \end{cases}$$. The value of λ for which $f''(0)$ exists, is _.

Sol. $f(x) = x^5.\sin\dfrac{1}{x}+5x^2$ if $x<0$

$f(x) = 0$ if $x = 0$

$f(x) = x^5.\cos\dfrac{1}{x}+\lambda x^2$ if $x>0$

LHD of $f'(x)$ at x = 0 is 10

RHD of $f'(x)$ at x = 0 is 2λ

if $f''(0)$ exists then

$2\lambda = 10$

$\Rightarrow \lambda = 5$