FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Friday 12th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

1. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is :

(1) 9/7 (2) 7/5

(3) 27/5 (4) 20/7

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{7}{9\times16}\right)$

$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$= R\left(\frac{5}{4\times9}\right)$

$\frac{\lambda_1}{\lambda_2} = \frac{\frac{5}{36}}{\frac{7}{9\times16}}$

$= \frac{20}{7}$

2. One kg of water, at 20ºC, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to :

[Specific heat of water = 4200 J/kg ºC), Latent heat of water = 2260 kJ/kg]

(1) 3 minutes (2) 22 minutes

(3) 10 minutes (4) 16 minutes

Sol. $Q = P \times t$

$Q = mc\Delta T + mL$

$P = \frac{V_{rms}^2}{R}$

$4200 \times 80 + 2260 \times 10^3 = \frac{(200)^2}{20} \times t$

$t = 1298$ sec

$t \simeq 22$ min

3. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (ν) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then ν is equal to :

(1) 332 ms^{-1} (2) 379 ms^{-1}

(3) 384 ms^{-1} (4) 338 ms^{-1}

Sol. $\nu = 2f(l_2 - l_1)$

$\nu = 2 \times 480 \times (70 - 30) \times 10^{-2}$

$\nu = 960 \times 40 \times 10^{-2}$

$\nu = 38400 \times 10^{-2}$ m/s

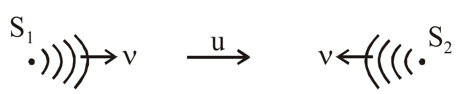
$\nu = 384$ m/s

4. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:

(1) 2.5 m/s (2) 15.0 m/s

(3) 5.5 m/s (4) 10.0 m/s

Sol. f = 660 Hz, ν = 330 m/s



$f_1 = f\left(\frac{\nu - u}{\nu}\right)$

$$f_2 = f\left(\frac{\nu+u}{\nu}\right)$$

$$f_2 - f_1 = \frac{f}{\nu}[\nu + u - (\nu - u)]$$

$$10 = f_2 - f_1 = \frac{f}{\nu}[2u]$$

$u = 2.5$ m/s

5. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$(assume that the particle is at origin at $t = 0$).

(1) $\frac{b^2\tau}{4}$ (2) $\frac{b^2\tau}{2}$

(3) $b^2 \tau$ (4) $\frac{b^2\tau}{\sqrt{2}}$

Sol. $v = b\sqrt{x}$

$$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}$$

$$a = \frac{bv}{2\sqrt{x}}$$

$$a = \frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}}$$

$$\frac{dv}{dt} = a = \frac{b^2}{2}$$

$$v = \frac{b^2}{2}\tau$$

6. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65Å). The de-Broglie wavelength of this electron is :

(1) 12.9 Å (2) 3.5 Å

(3) 9.7 Å (4) 6.6 Å

Sol. $2\pi r_n = n\lambda_n$

$$\lambda_3 = \frac{2\pi(4.65\times10^{-10})}{3}$$

$\lambda_3 = 9.7$ Å

7. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to $V(V = GI_0)$ by connecting a series resistance R_V to it. Then,

(1) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

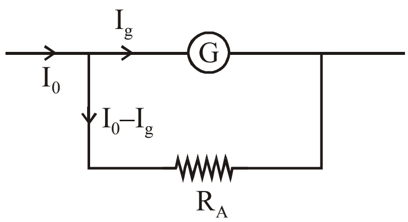
(2) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(3) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(4) $R_A R_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

Sol. When galvanometer is used as an ammeter shunt is used in parallel with galvanometer.

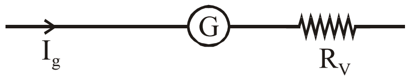
$\therefore$



$\therefore$ $I_g G = (I_0 - I_g)R_A$

$$\therefore\ R_A = \left(\frac{I_g}{I_0 - I_g}\right)G$$

When galvanometer is used as a voltmeter, resistance is used in series with galvanometer.



$I_g(G + R_V) = V = GI_0$ (given $V = GI_0$)

$\therefore\ R_V = \dfrac{(I_0 - I_g)G}{I_g}$

$\therefore\ R_A R_V = G^2\ \ \&\ \ \dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

8. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to :

(1) $n_0\alpha^{1/4}$ (2) $n_0\alpha^{-3}$

(3) $n_0\alpha^{-3/4}$ (4) $\sqrt{n_0}\,\alpha^{1/2}$

Sol. Given number density of molecules of gas as a function of r is

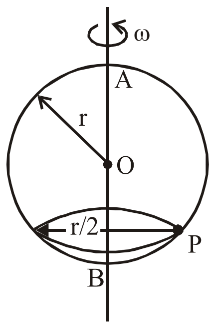
$$n(r) = n_0 e^{-\alpha r^4}$$

$\therefore$ Total number of molecule $= \displaystyle\int_0^\infty n(r)dV$

$$= \int_0^\infty n_0 e^{-\alpha r^4} 4\pi r^2\, dr$$

$\therefore$ Number of molecules is proportional to $n_0\alpha^{-3/4}$

9. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to :



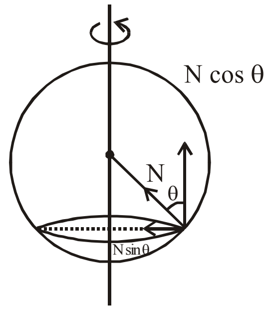
(1) $\left(g\sqrt{3}\right)/r$

(2) $\dfrac{\sqrt{3}\,g}{2r}$

(3) $2g/r$

(4) $2g/\left(r\sqrt{3}\right)$

Sol.



$N\sin\theta = m\dfrac{r}{2}\omega^2$ ......(1)

$N\cos\theta = mg$ ....(2)

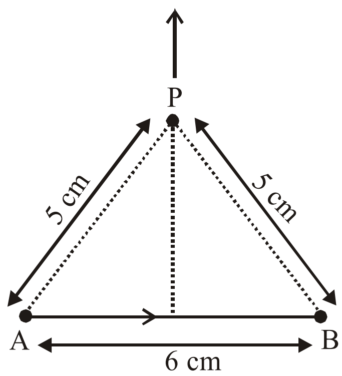
$\tan\theta = \dfrac{r\omega^2}{2g}$

$\dfrac{r}{2\dfrac{\sqrt{3}\,r}{2}} = \dfrac{r\omega^2}{2g}$

$\omega^2 = \dfrac{2g}{\sqrt{3}\,r}$

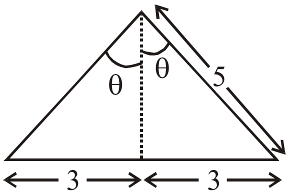
10. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure)

($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})



(1) 3.0×10^{-5} T (2) 2.5×10^{-5} T

(3) 2.0×10^{-5} T (4) 1.5×10^{-5} T

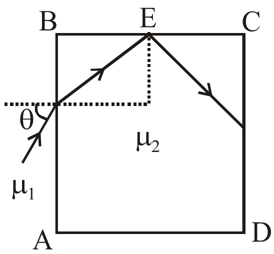
Sol. 

$$B = \frac{\mu_0 I}{4\pi d} \; 2 \sin\theta$$

$d = 4$ cm

$$\sin\theta = \frac{3}{5}$$

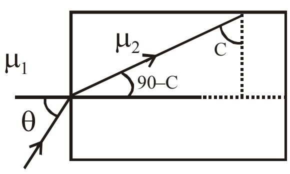
11. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



The θ must satisfy :

(1) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (2) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

(3) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (4) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

Sol. 

$$\sin c = \frac{\mu_1}{\mu_2}$$

$$\mu_1 \sin\theta = \mu_2 \sin(90^\circ - C)$$

$$\sin\theta = \frac{\mu_2\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$$

$$\theta = \sin^{-1}\sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}}$$

For TIR

$$\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$$

12. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

(1) $a = \frac{3R}{2^{1/4}}$ (2) $a = R/\sqrt{3}$

(3) $a = 8^{-1/4}R$ (4) $a = 2^{-1/4}\,R$

Sol. $$E\,4\pi a^2 = \frac{\int_0^a kr\,4\pi r^2\,dr}{\varepsilon_0}$$

$$E = \frac{k\,4\pi a^4}{4\times 4\pi\varepsilon_0}$$

$$2Q = \int_0^R kr\,4\pi r^2\,dr$$

$$k = \frac{2Q}{\pi R^4}$$

$$QE = \frac{1}{4\pi\varepsilon_0}\,\frac{QQ}{(2a)^2}$$

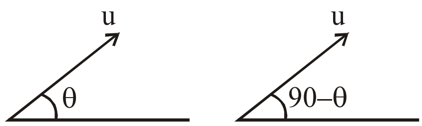
$R = a8^{1/4}$

13. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ?

(1) $R^2 = 2\,h_1h_2$ (2) $R^2 = 16h_1h_2$

(3) $R^2 = 4\,h_1h_2$ (4) $R^2 = h_1h_2$

Sol.



For same range angle of projection will be θ & 90 – θ

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$h_1 = \frac{u^2\sin^2\theta}{g}$$

$$h_2 = \frac{u^2\sin^2(90-\theta)}{g}$$

$$\frac{R^2}{h_1h_2} = 16$$

14. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be :

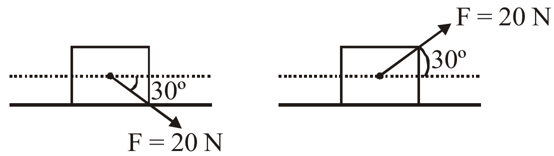
(1) $\frac{1}{n^2}$ (2) n^2 (3) $\frac{1}{n}$ (4) n

Sol. $k_1 = \frac{C}{\ell_1}$

$$k_2 = \frac{C}{\ell_2}$$

$$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C}\ell_2 = \frac{\ell_2}{n\,\ell_2} = \frac{1}{n}$$

15. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30º with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be : ($g = 10\ ms^{-2}$)

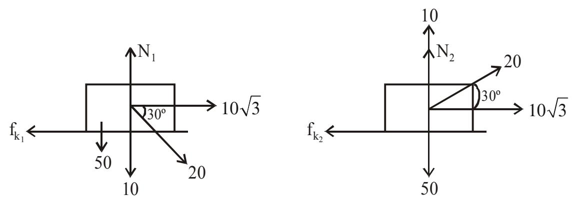


(A) (B)

(1) $0\ ms^{-2}$ (2) $0.8\ ms^{-2}$

(3) $0.4\ ms^{-2}$ (4) $3.2\ ms^{-2}$

Sol.

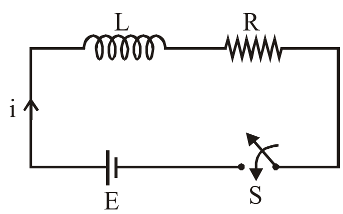


$N_1 = 60$ $N_2 = 40$

$$a_1 = \frac{10\sqrt{3} - 0.2\times 60}{5} \qquad a_2 = \frac{10\sqrt{3} - 0.2\times 40}{5}$$

$a_1 - a_2 = 0.8$

16. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and t = $\frac{L}{R}$ is :



(1) $\frac{EL}{7.3R^2}$ (2) $\frac{EL}{2.7R^2}$

(3) $\frac{7.3EL}{R^2}$ (4) $\frac{2.7EL}{R^2}$

Sol. $q = \int I\,dt$

$$q = \int_0^{L/R} \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right] dt$$

$$q = \frac{EL}{R^2}\frac{1}{e}$$

$$q = \frac{EL}{2.7R^2}$$

17. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :

(1) 16.00 (2) 1.80

(3) 5.33 (4) 10.67

Sol. Since unpolarised light falls on $P_1 \Rightarrow$ intensity of light transmitted from $P_1 = \frac{I_0}{2}$

Pass axis of P_2 will be at an angle of 30° with P_1

$\therefore$ Intensity of light transmitted from

$$P_2 = \frac{I_0}{2}\cos^2 30° = \frac{3I_0}{8}$$

Pass axis of P_3 is at an angle of 60° with P_2

$\therefore$ Intensity of light transmitted from

$$P_3 = \frac{3I_0}{8}\cos^2 60° = \frac{3I_0}{32}$$

$$\therefore \left(\frac{I_0}{I}\right) = \frac{32}{3} = 10.67$$

18. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 1 : 8

(3) 8 : 1 (4) 3 : 8

Sol. $N_A = N_{OA}e^{-\lambda t} = \frac{N_{OA}}{2^{t/t_{1/2}}} = \frac{N_{OA}}{2^6}$

$\therefore$ Number of nuclei decayed

$$= N_{OA} - \frac{N_{OA}}{2^6} = \frac{63N_{OA}}{64}$$

$$N_B = N_{OB}e^{-\lambda t} = \frac{N_{OB}}{2^{t/t_{1/2}}} = \frac{N_{OB}}{2^3}$$

$\therefore$ Number of nuclei decayed

$$= N_{OB} - \frac{N_{OB}}{2^3} = \frac{7N_{OB}}{8}$$

Since $N_{OA} = N_{OB}$

$\therefore$ Ratio of decayed numbers of nuclei

$$A \;\&\; B = \frac{63N_{OA} \times 8}{64 \times 7N_{OB}} = \frac{9}{8}$$

19. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals :

(1) 1/27 (2) 1/9

(3) 27 (4) 9

Sol. We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g \Rightarrow v_T \propto r^2$$

since mass of the sphere will be same

$$\therefore \rho\frac{4}{3}\pi R^3 = 27.\frac{4}{3}\pi r^3\rho \Rightarrow r = \frac{R}{3}$$

$$\therefore \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$$

20. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(1) $\frac{R}{3}$ (2) $\frac{R}{2}$ (3) $\frac{R}{4}$ (4) $\frac{R}{9}$

Sol. Since mass of the object remains same

$\therefore$ Weight of object will be proportional to 'g' (acceleration due to gravity)

Given

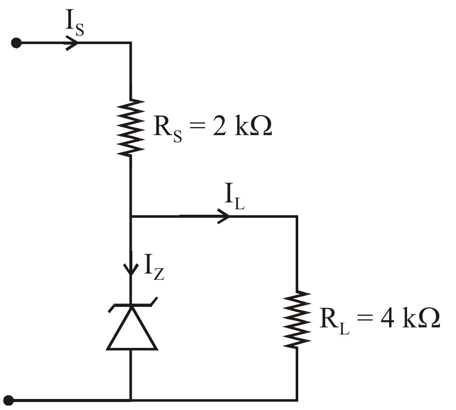
$$\frac{W_{earth}}{W_{planet}} = \frac{9}{4} = \frac{g_{earth}}{g_{planet}}$$

Also, $g_{surface} = \frac{GM}{R^2}$ (M is mass planet, G is universal gravitational constant, R is radius of planet)

$$\therefore \frac{9}{4} = \frac{GM_{earth}}{GM_{planet}} \frac{R^2_{planet}}{R^2_{earth}} = \frac{M_{earth}}{M_{planet}} \times \frac{R^2_{Planet}}{R^2_{earth}} = 9\frac{R^2_{Planet}}{R^2_{earth}}$$

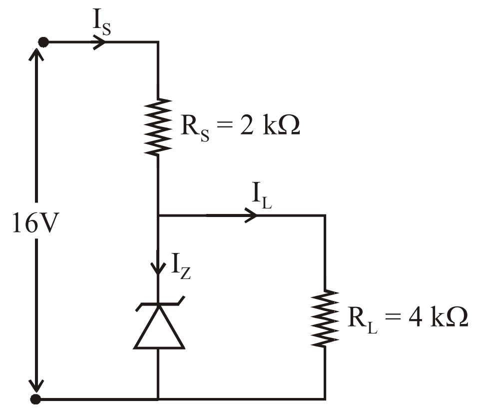
$$\therefore R_{planet} = \frac{R_{earth}}{2} = \frac{R}{2}$$

21. Figure shown a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current ?



(1) 2.5 mA (2) 3.5 mA
(3) 7.5 mA (4) 1.5 mA

Sol. Maximum current will flow from zener if input voltage is maximum.



When zener diode works in breakdown state, voltage across the zener will remain same.

$\therefore$ $V_{across\ 4k\Omega} = 6V$

$\therefore$ Current through $4K\Omega = \frac{6}{4000}A = \frac{6}{4}mA$

Since input voltage = 16V

$\therefore$ Potential difference across $2K\Omega$ = 10V

$\therefore$ Current through $2k\Omega = \frac{10}{2000} = 5mA$

$\therefore$ Current through zener diode

$= (I_S - I_L) = 3.5$ mA

22. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62ºC, its efficiency is doubled. The temperatures of the source and the sink are, respectively

(1) 124ºC, 62ºC (2) 37ºC, 99ºC
(3) 62ºC, 124ºC (4) 99ºC, 37ºC

Sol. Efficiency of Carnot engine = $1 - \frac{T_{sink}}{T_{source}}$

Given,

$$\frac{1}{6} = 1 - \frac{T_{sink}}{T_{source}} \Rightarrow \frac{T_{sink}}{T_{source}} = \frac{5}{6} \quad .....(1)$$

Also,

$$\frac{2}{6} = 1 - \frac{T_{sink} - 62}{T_{source}} \Rightarrow \frac{62}{T_{source}} = \frac{1}{6} \quad .....(2)$$

$\therefore$ $T_{source} = 372$ K = 99°C

Also, $T_{sink} = \frac{5}{6} \times 372 = 310$ K = 37°C

(**Note :-** Temperature of source is more than temperature of sink)

23. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?

(1) 35 J (2) 40 J

(3) 25 J (4) 30 J

Sol. For a diatomic gas, $C_p = \frac{7}{2}R$

Since gas undergoes isobaric process

$\Rightarrow$ $\Delta Q = nC_P\Delta T$

Also, $\Delta W = nR\Delta T = 10J(\text{given})$

$\therefore$ $\Delta Q = n\frac{7}{2}R\Delta T = \frac{7}{2}(nR\Delta T) = 35$ J

24. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound ? [Given reference intensity of sound as $10^{-12}W/m^2$]

(1) 10 cm (2) 30 cm

(3) 40 cm (4) 20 cm

Sol. Loudness of sound is given by

$$dB = 10\log\frac{I}{I_0}\left(\begin{array}{c}\text{I is intensity of sound}\\ I_0 \text{ is reference intensity of sound}\end{array}\right)$$

$$\therefore\ 120 = 10\log\left(\frac{I}{I_0}\right)$$

$\Rightarrow$ $I = 1\ W/m^2$

Also $I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$

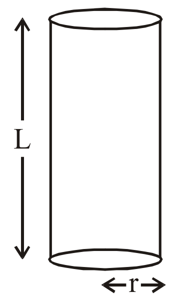
$$\therefore\ r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}}\ m = 0.399\ m \approx 40\ cm$$

25. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equals to :

(1) $F/(3\pi r^2YT)$ (2) $3F/(\pi r^2YT)$

(3) $6F/(\pi r^2YT)$ (4) $9F/(\pi r^2YT)$

Sol.



$\because$ Length of cylinder remains unchanged

so $\left(\frac{F}{A}\right)_{Compressive} = \left(\frac{F}{A}\right)_{Thermal}$

$$\frac{F}{\pi r^2} = Y\alpha T$$

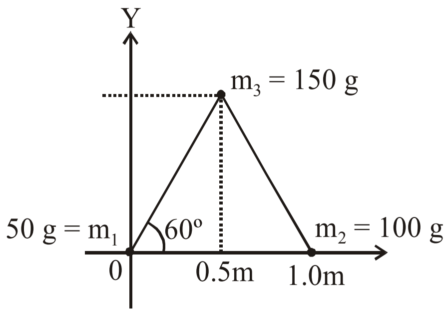
(α is linear coefficient of expansion)

$$\therefore\ \alpha = \frac{F}{YT\pi r^2}$$

$\therefore$ The coefficient of volume expansion $\gamma = 3\alpha$

$$\therefore\ \gamma = 3\frac{F}{YT\pi r^2}$$

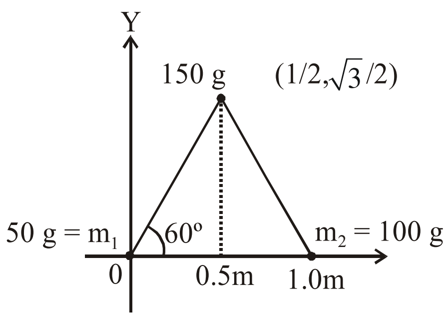
26. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :



(1) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ (2) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

(3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (4) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

Sol.



The co-ordinates of the centre of mass

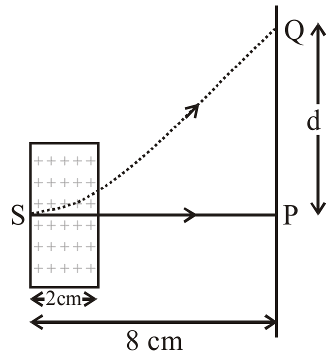
$$\vec{r}_{cm} = \frac{0 + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$\therefore$ Co-ordinate $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$ m

27. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} T)\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is :

(electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)



(1) 12.87 cm (2) 1.22 cm

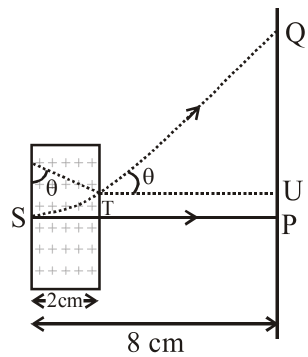
(3) 11.65 cm (4) 2.25 cm

Sol. $R = \frac{mv}{qB}$

$$= \frac{\sqrt{2m(K.E.)}}{qB}$$

$$R = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times (100 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$$

R = 2.248 cm



$\sin\theta = \dfrac{2}{2.248}$

$\tan\theta = \dfrac{QU}{TU}$

$\dfrac{2}{1.026} = \dfrac{QU}{6}$

QU = 11.69

PU = R(1 – cos θ)

= 1.22

d = QU + PU

28. A plane electromagnetic wave having a frequency ν = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave ?

(1) $\vec{B} = 2\times10^{7}\sin(0.5\times10^{3}z + 1.5\times10^{11}t)\hat{i}$

(2) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^{2}x + 0.5\times10^{11}t)\hat{j}$

(3) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^{3}z - 1.5\times10^{11}t)\hat{i}$

(4) $\vec{B} = 60\sin(0.5\times10^{3}x + 1.5\times10^{11}t)\hat{k}$

Sol. Magnetic field when electromagnetic wave propagates in +z direction

$B = B_0 \sin(kz - \omega t)$

where

$B_0 = \dfrac{60}{3\times10^{8}} = 2\times10^{-7}$

$k = \dfrac{2\pi}{\lambda} = 0.5\times10^{3}$

$\omega = 2\pi f = 1.5\times10^{11}$

29. In an amplitude modulator circuit, the carrier wave is given by,

C(t) = 4 sin (20000 πt) while modulating signal is given by, m(t) = 2 sin (200 πt). The values of modulation index and lower side band frequency are :

(1) 0.5 and 9 kHz (2) 0.5 and 10 kHz

(3) 0.3 and 9 kHz (4) 0.4 and 10 kHz

Sol. Modulation index is given by

$m = \dfrac{A_m}{A_c} = \dfrac{2}{4} = 0.5$

& (a) carrier wave frequency is given by

$= 2\pi f_c = 2\times10^{4}\pi$

$f_c = 10$ kHz

(b) modulating wave frequency (f_m)

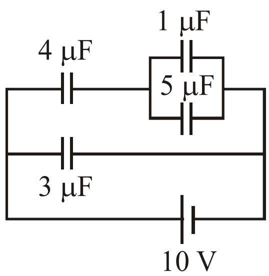
$2\pi f_m = 2000\pi$

$\Rightarrow f_m = 1$ kHz

lower side band frequency $\Rightarrow f_c - f_m$

$\Rightarrow$ 10 kHz – 1 kHz = 9 kHz

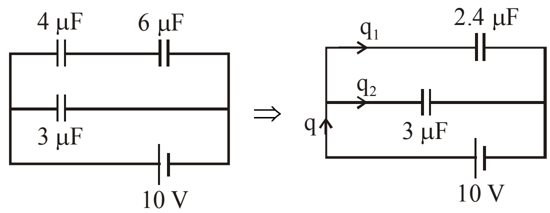
30. In the given circuit, the charge on 4 μF capacitor will be :



(1) 5.4 μC (2) 24 μC

(3) 13.4 μC (4) 9.6 μC

Sol.



So total charge flow = q = 5.4 μF × 10V

= 54 μC

The charge will be distributed in the ratio of capacitance

$\Rightarrow \dfrac{q_1}{q_2} = \dfrac{2.4}{3} = \dfrac{4}{5}$

∴ 9X = 54 μC ⇒ X = 6 μC

∴ charge on 4 μF capacitor

will be = 4X = 4 × 6 μC

= 24 μC

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Friday 12th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

1. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is :

(1) 6.23×10^{-11} M (2) 1.84×10^{-9} M

(3) $\frac{2.49}{1.84} \times 10^{-9}$ M (4) 2.49×10^{-10} M

Sol. $K_{sp} = 4\,(s)^3$

$= 4 \times (1.84 \times 10^{-5})^3$

$Cd(OH)_2 \rightleftharpoons Cd^{+2} + 2OH^-$

S' S' $(10^{-2} + S') \simeq 10^{-2}$

$S' \times (10^{-2})^2 = 4 \times (1.84 \times 10^{-5})^3$

$S' = 4 \times (1.84)^3 \times 10^{-11}$

$(S') = 2.491 \times 10^{-10}$ M

2. The correct statement is :

(1) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate

(2) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2

(3) pig iron is obtained from cast iron

(4) the Hall-Heroult process is used for the production of aluminium and iron

Sol. (1) During leaching when bauxite is treated with concentrated NaOH, then sodium aluminate and sodium silicate is formed in the soluble form, whereas Fe_2O_3 is precipitated

(2) The blistered appearance of copper during the metallurgical process is due to the evolution of SO_2.

(3) Cast iron is obtained from pig iron.

(4) Hall-Heroult process is used for production of only aluminium.

3. Which one of the following is likely to give a precipitate with $AgNO_3$ solution ?

(1) $(CH_3)_3CCl$ (2) $CHCl_3$

(3) $CH_2=CH-Cl$ (4) CCl_4

Sol. $CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Cl + AgNO_3 \longrightarrow (CH_3)_3C^{\oplus} + \underset{\text{(white ppt)}}{AgCl_{(s)}\downarrow}$

Reason :- Due to most stable carbocation formation tert-butyl chloride given the ppt immediately

4. The compound used in the treatment of lead poisoning is :

(1) EDTA (2) Cis-platin

(3) D-penicillamine (4) desferrioxime B

Sol. (1) EDTA (ethylene diamine tetra acetate) is used for lead poisoning

(2) Cis platin is used as a anti cancer drug

(3) D-penicillamine is used for copper poisoning

(4) desferrioxime B is used for iron poisoning

5. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27ºC. The osmotic pressure of the solution is :

(R = 0.08206 L atm K^{-1} mol^{-1})

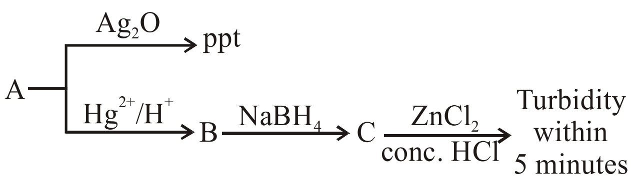
(1) 4.92 atm (2) 1.64 atm

(3) 2.46 atm (4) 8.2 atm

Sol. $\Pi = \dfrac{\left(\dfrac{0.6}{60} + \dfrac{1.8}{180}\right)}{0.1} \times 0.08206 \times 300$

$\Pi = 4.9236$ atm

6. Consider the following reactions :

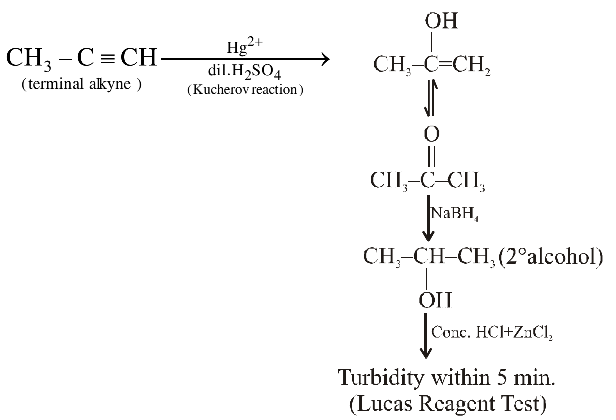


'A' is :

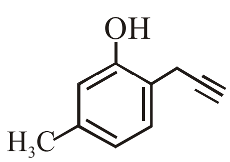
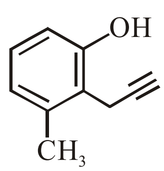
(1) $CH\equiv CH$ (2) $CH_3-C\equiv CH$

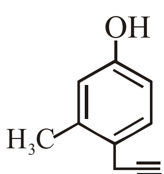
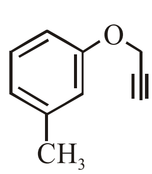
(3) $CH_2=CH_2$ (4) $CH_3-C\equiv C-CH_3$

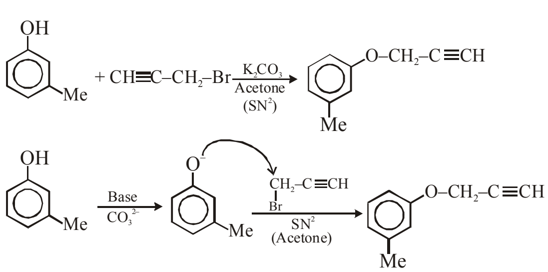
Sol. $\underset{\text{(terminal alkyne)}}{CH_3-C\equiv CH} \xrightarrow[\Delta]{Ag_2O} \underset{\text{(ppt)}}{CH_3-C\equiv C-Ag\downarrow}$



7. What will be the major product when m-cresol is reacted with propargyl bromide ($HC\equiv C-CH_2Br$) in present of K_2CO_3 in acetone

(1)  (2) 

(3)  (4) 

Sol. 

8. The INCORRECT match in the following is :

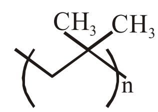
(1) $\Delta G^o < 0$, $K < 1$ (2) $\Delta G^o = 0$, $K = 1$

(3) $\Delta G^o > 0$, $K < 1$ (4) $\Delta G^o < 0$, $K > 1$

Sol. $\Delta G^o = -RT \ln k$

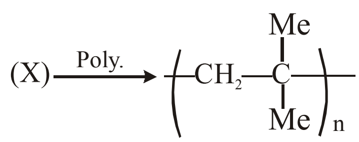
if $K < 1 \Rightarrow \Delta G^o > 0$

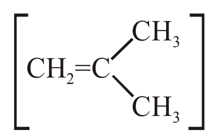
9. The correct name of the following polymer is:



(1) Polyisoprene (2) Polytert-butylene

(3) Polyisobutane (4) Polyisobutylene

Sol.  Polyisobutylene

As per the given structure of the polymer the monomer is –  Isobutylene

10. Among the following, the energy of 2s orbital is lowest in :

(1) K (2) Na (3) Li (4) H

Sol. In 'K', 2s orbital feel maximum attraction from nucleus (So having less energy) due to more Z_{eff}.

11. The primary pollutant that leads to photochemical smog is :

(1) sulphur dioxide (2) acrolein

(3) ozone (4) nitrogen oxides

Sol. Nitrogen oxides and hydrocarbons (unburnt fuel) are primary pollutant that leads to photochemical smog.

12. An ' Assertion' and a 'Reason' are given below. Choose the correct answer from the following options.

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held π-electrons, the cleavage is difficult becuase of strong bonding.

(1) Both (A) and (R) are wrong statements
(2) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)
(3) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A)
(4) (A) is a correct statement but (R) is a wrong statement.

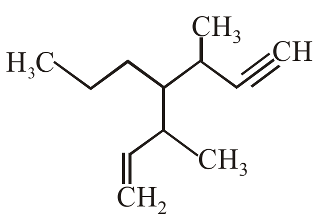
Sol. Vinyl halide $CH_2=CH-Cl$ do not undergo SN reaction
This is due to formation of highly unstable carbocation ($CH_2=\overset{\oplus}{C}H$) ; which cannot be delocalised by the π-electron, also C–Cl has double bond character because of resonance
Hence statement (2) is wrong.

13. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are :
(en=ethane-1,2-diamine)

(1) 3 and 3 (2) 6 and 6
(3) 5 and 6 (4) 5 and 3

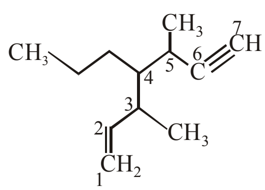
Sol. en and $C_2O_4^{2-}$ are bidentate ligand. So coordination number of $[Co(Cl)(en)_2]Cl$ is 5 and $K_3[Al(C_2O_4)_3]$ is 6.

14. The IUPAC name of the following compound is :



(1) 3,5-dimethyl-4-propylhept-6-en-1-yne
(2) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne
(3) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene
(4) 3,5-dimethyl-4-propylhept-1-en-6-yne

Sol.



3,5-Dimethyl-4-propylhept-1-en-6-yne
Longest carbon chain, including multiple bonds, and numbering starts from double bond.

15. Among the following, the INCORRECT statement about colloids is :

(1) They can scatter light
(2) They are larger than small molecules and have high molar mass
(3) The range of diameters of colloidal particles is between 1 and 1000 nm
(4) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration

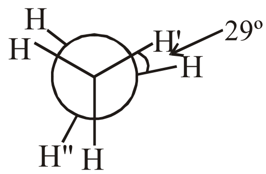
Sol. Colligative properties of colloidal solution are smaller than true solution

16. In comparison to boron, berylium has :

(1) lesser nuclear charge and greater first ionisation enthalpy
(2) lesser nuclear charge and lesser first ionisation enthalpy
(3) greater nuclear charge and greater first ionisation enthalpy
(4) greater nuclear charge and lesser first ionisation enthalpy

Sol. In case of 'Be' electron remove from '2s' orbital while in case of 'B' electron remove from '2p' orbital. '2s' orbital have greater penetration effect then '2p' orbitals. So 'Be' having more I.E. then 'B'

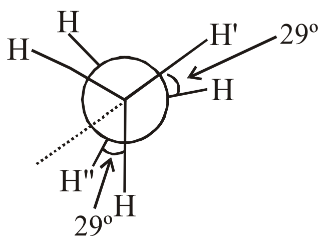
17. In the following skew conformation of ethane, H'–C–C–H" dihedral angle is :



(1) 120° (2) 58°

(3) 149° (4) 151°

Sol.



H'–C–C–H"

Hence angle between

H'—C–C–H" is

(120° + 29°) = 149°

18. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is :

(1) 2.083×10^{-3} mol L^{-1} min^{-1}

(2) 4.167×10^{-3} mol L^{-1} min^{-1}

(3) 8.333×10^{-3} mol L^{-1} min^{-1}

(4) 1.667×10^{-2} mol L^{-1} min^{-1}

Sol. $2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$

t=0 3.0M

t=30 2.75 M

$$\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{0.25}{30}$$

$$\frac{1}{2}\times\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\times\frac{\Delta[NO_2]}{\Delta t}$$

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{0.25}{30}\times 2 = 1.66 \times 10^{-2} \text{ M/min}$$

19. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z, respectively.

(1) K_2MnO_4, $KMnO_4$ and SO_2

(2) K_2MnO_4, $KMnO_4$ and Cl_2

(3) K_3MnO_4, K_2MnO_4 and Cl_2

(4) $KMnO_4$, K_2MnO_4 and Cl_2

Sol. $\underset{(X)}{2KMnO_4} \xrightarrow[\Delta]{513K} \underset{(Y)}{K_2MnO_4} + MnO_2 + O_{2(g)}$

$MnO_2 + 4NaCl + 4H_2SO_4 \rightarrow MnCl_2 + 4NaHSO_4 + 2H_2O + Cl_{2(g)}$

(Z)

pungent gas

20. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains.

(1) 20g of carbon and 5 g of hydrogen

(2) 24g of carbon and 1 g of hydrogen

(3) 18g of carbon and 7 g of hydrogen

(4) 22g of carbon and 3 g of hydrogen

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$\left(\frac{25}{M}\right)$ $\quad x\times\frac{25}{M} = 2$ $\quad \frac{y}{2}\times\frac{25}{M} = 0.5$

C $\quad x\times\frac{25}{M} = 2$

H $\quad y\times\frac{25}{M} = 1$

$C_{2y}H_y \equiv$ 24y gm C + y gm H

or

24 : 1 ratio by mass

21. Which of the given statements is INCORRECT about glycogen ?

(1) It is a straight chain polymer similar to amylose

(2) Only α-linkages are present in the molecule

(3) It is present in animal cells

(4) It is present in some yeast and fungi

Sol. Glycogen is an animal starch.
It consists of α-amylose and amylopectin.
Amylopectin is branched chain polysaccharide
Hence statement (1) is incorrect.

22. The C–C bond length is maximum in

(1) graphite (2) C_{70}

(3) diamond (4) C_{60}

Sol. In diamond C–C bond have only σ bond character while in case of graphite and fullerene (C_{60} and C_{70}) C–C bond contain double bond character. That's why diamond having maximum C–C bond length.

23. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively, are :

(1) $Ca(HCO_3)_2$ and CaO

(2) $Mg(HCO_3)_2$ and $MgCO_3$

(3) $Mg(HCO_3)_2$ and $Mg(OH)_2$

(4) $Ca(HCO_3)_2$ and $Ca(OH)_2$

Sol. Temporary hardness is due to soluble $Mg(HCO_3)_2$ and $Ca(HCO_3)_2$

$Mg(HCO_3)_2 \xrightarrow{\text{Boil}} Mg(OH)_2 \downarrow + 2CO_2 \uparrow$

$Ca(HCO_3)_2 \xrightarrow{\text{Boil}} CaCO_3 \downarrow + H_2O + CO_2 \uparrow$

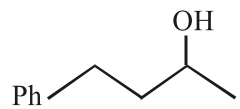
24. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively :

(1) 1 : 2 : 4 (2) 8 : 1 : 6

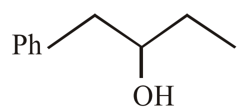
(3) 4 : 2 : 1 (4) 4 : 2 : 3

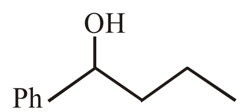
Sol. SC : BCC : FCC
1 : 2 : 4

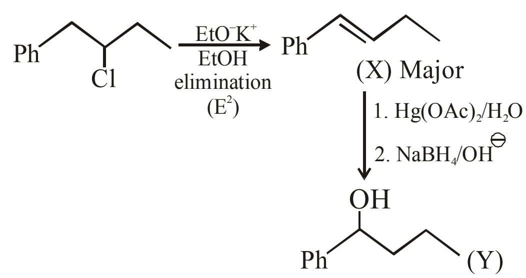
25. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is :

(1) 

(2) 

(3) 

(4) 

Sol. 

26. In which one of the following equilibria, $K_p \neq K_c$?

(1) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

(2) $2\ HI(g) \rightleftharpoons H_2(g) + I_2(g)$

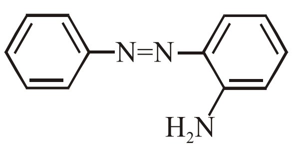
(3) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

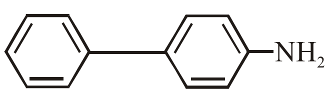
(4) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

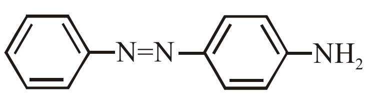
Sol. if $\Delta n_g \neq 0$

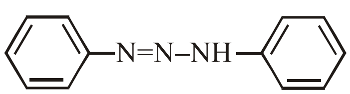
$Kp \neq Kc$

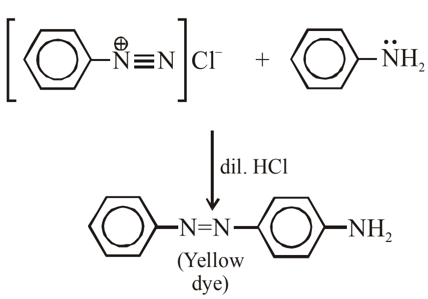
27. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

(1) 

(2) 

(3) 

(4) 

Sol. 

28. The decreasing order of electrical conductivity of the following aqueous solutions is :

0.1 M Formic acid (A),

0.1 M Acetic acid (B)

0.1 M Benzoic acid (C)

(1) C > B > A (2) A > B > C

(3) A > C > B (4) C > A > B

Sol. Order of acidic strength

A > C > B

Acidic strength $\uparrow \Rightarrow$ degree of ionization $\uparrow$

29. The INCORRECT statement is :

(1) Lithium is least reactive with water among the alkali metals.

(2) LiCl crystallises from aqueous solution as $LiCl.2H_2O$.

(3) Lithium is the strongest reducing agent among the alkali metals.

(4) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.

Sol. (4) $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2(g) + \frac{1}{2}O_2(g)$

30. The pair that has similar atomic radii is :

(1) Sc and Ni (2) Ti and HF

(3) Mo and W (4) Mn and Re

Sol. Mo and W has nearly similar atomic radius due to lanthanoid contraction.

FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Friday 12th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

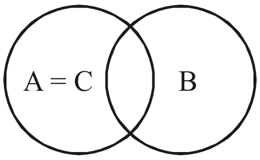
(1) If $(A - C) \subseteq B$, then $A \subseteq B$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A - B) \subseteq C$, then $A \subseteq C$

(4) $B \cap C \neq \phi$

Sol.



for $A = C$, $A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option 1 is **NOT** true

Let $x \in (C\ x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C \text{ or } x \in A)$ and $(x \in C \text{ or } x \in B)$

$\Rightarrow x \in C \text{ or } x \in (A \cap B)$

$\Rightarrow x \in C \text{ or } x \in C \quad (\text{as } A \cup B \subseteq C)$

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C \quad (1)$

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B) \quad (2)$

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option 2 is true

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C \quad (\text{as } A - B \subseteq C)$

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C \quad (\text{as } A \cap B \subseteq C)$

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

as $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

$\Rightarrow$ Option 4 is true.

2. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^{\beta})$, then the ordered pair (A, β) is equal to:

(1) (420, 18) (2) (380, 19)

(3) (380, 18) (4) (420, 19)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

Diff. w.r.t. x

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + {}^nC_2\,(2x) + ...+ {}^nC_n\,n(x)^{n-1}$

Multiply by x both side

$\Rightarrow nx(1+x)^{n-1} = {}^nC_1\,x + {}^nC_2\,(2x^2) +....+ {}^nC_n(n\,x^n)$

Diff w.r.t. x

$\Rightarrow n\,[(1+x)^{n-1} + (n-1)x\,(1+x)^{n-2}]$

$= {}^nC_1 + {}^nC_2\,2^2x + ...\ {}^nC_n\,(n^2)x^{n-1}$

Put x = 1 and n = 20

$\Rightarrow {}^{20}C_1 + 2^2\,{}^{20}C_2 + 3^2\,{}^{20}C_3 + ... + (20)^2\,{}^{20}C_{20}$

$= 20 \times 2^{18}\,[2 + 19] = 420\,(2^{18}) = A(2^{\beta})$

3. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is :

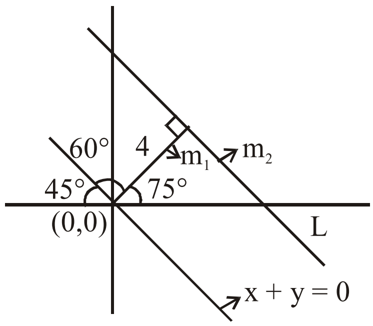
(1) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

(2) $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

Sol.



$m_1 = \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$m_2 = \frac{-1}{m_1} = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}$

$\Rightarrow y = m_2x + C$

$\Rightarrow y = \frac{-(\sqrt{3}-1)x}{\sqrt{3}+1} + C \Rightarrow L$

Distance from origin = 4

$$\therefore \frac{C}{\sqrt{1+\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2}}} = 4$$

$$\Rightarrow \frac{(\sqrt{3}+1)C}{\sqrt{8}} = 4$$

$$\Rightarrow C = \frac{8\sqrt{2}}{(\sqrt{3}+1)}$$

Hence

$$\Rightarrow (\sqrt{3}-1)y + (\sqrt{3}+1)x = 8\sqrt{2}$$

4. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is :}$$

(1) $\frac{7\pi}{24}$ (2) $\frac{\pi}{18}$ (3) $\frac{\pi}{9}$ (4) $\frac{7\pi}{36}$

Sol. $R_1 \to R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$\Rightarrow (1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$

$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$$6\theta = \frac{2\pi}{3} \Rightarrow \boxed{\theta = \frac{\pi}{9}}$$

5. If [x] denotes the greatest integer $\leq$ x, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$

(1) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(3) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(4) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Sol. $[\sin\theta]x + [-\cos\theta]y = 0$ and $[\cos\theta]x + y = 0$ for infinite many solution

$$\begin{vmatrix} [\sin\theta] & [-\cos\theta] \\ [\cos\theta] & 1 \end{vmatrix} = 0$$

ie $[\sin\theta] = -[\cos\theta][\cot\theta]$ (1)

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \frac{1}{2}\right)$

$-\cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

when $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$$-\cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$$

$$\cot\theta \in \left(\sqrt{3}, \infty\right)$$

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ then equation (i) satisfied there fore infinite many solution.

when $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ then equation (i) not satisfied there fore infinite unique solution.

6. $\lim\limits_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 - 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is :

(1) 3 (2) 2 (3) 6 (4) 1

Sol. Rationalize

$$\lim_{x\to 0} \frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x - x + 1}\right)}{x^2+2\sin x+1-\sin^2 x + x - 1}$$

$$\lim_{x\to 0} \frac{(x+2\sin x)(2)}{x^2+2\sin x - \sin^2 x + x}$$

$\frac{0}{0}$ form using L' hospital

$$\Rightarrow \lim_{x\to 0} \frac{(1+2\cos x)\times 2}{2x+2\cos x - 2\sin x\cos x + 1}$$

$$\Rightarrow \frac{2\times 3}{(2+1)} = 2$$

7. If a_1, a_2, a_3,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 200 (2) 280

(3) 120 (4) 150

Sol. $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40 \quad \Rightarrow \boxed{a + 7d = \frac{40}{3}}$

$$S_{15} = \frac{15}{2}(2a + 14d) = 15(a + 7d)$$

$$S_{15} = 15 \times \frac{40}{3} \Rightarrow 200 \quad \boxed{S_{15} = 200}$$

8. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{3}$ (4) 3

Sol. perpendicular vector to the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Eq. of plane

$-3(x - 1) + 3(y - 1) + 3z = 0$

$\Rightarrow x - y - z = 0$

$$d_{(2,1,4)} = \frac{|2-1-4|}{\sqrt{1^2+1^2+1^2}} = \sqrt{3}$$

9. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) $\beta\gamma$ (2) 0 (3) $\alpha\gamma$ (4) $\alpha\beta$

Sol. $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t$, $\gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ (1)

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t \,.\, \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

(from equation 1)

10. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) 36 (2) – 108 (3) – 72 (4) – 36

Sol. $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6-\frac{1}{81}\cdot x^8\left(2x^2-\frac{3}{x^2}\right)^6$

its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r}-\frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

for term independent of x, r for Ist expression is 3 and r for second expression is 5

∴ term independent of x = – 36

11. Let α ∈ R and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set S = {α : $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar}

(1) is singleton

(2) Contains exactly two numbers only one of which is positive

(3) Contains exactly two positive numbers

(4) is empty

Sol. $\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -4\\ \alpha & -2 & 3\end{vmatrix}=0$

$\Rightarrow 3\alpha^2+18=0$

$\Rightarrow \alpha\in\phi$

12. A value of α such that $\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$ is :

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) – 2

Sol. $\int_{\alpha}^{\alpha+1}\frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)}dx=\left(\ell n|x+\alpha|-\ell n|x+\alpha+1|\right)_{\alpha}^{\alpha+1}$

$=\ell n\left|\frac{2\alpha+1}{2\alpha+2}\times\frac{2\alpha+1}{2\alpha}\right|=\ell n\frac{9}{8}$

$\Rightarrow \alpha=-2,1$

13. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3},1\right)$ (2) $\left(\frac{1}{3},2\right)$

(3) $\left(1,\frac{7}{3}\right)$ (4) $\left(\frac{1}{3},\frac{5}{3}\right)$

Sol. Let B(α,β) and C(γ,δ)

$\frac{\alpha+1}{2}=-1\Rightarrow\alpha=-3$

$\frac{\beta+2}{2}=1\Rightarrow\beta=0$

$\Rightarrow B(-3,0)$

Now $\frac{\gamma+1}{2}=2\Rightarrow\gamma=3$

$\frac{\delta+2}{2}=3\Rightarrow\delta=4$

⇒ C(3,4)

⇒ centroid of triangle is $G\left(\frac{1}{3},2\right)$

14. Let α ∈ (0, π/2) be fixed. If the integral $\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx=$ A(x) cos2α + B(x) sin2α + C, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) x – α and $\log_e|\cos(x-\alpha)|$

(2) x + α and $\log_e|\sin(x-\alpha)|$

(3) x – α and $\log_e|\sin(x-\alpha)|$

(4) x + α and $\log_e|\sin(x+\alpha)|$

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let x – α = t

$\Rightarrow \int \frac{\sin(t+2\alpha)}{\sin t}dt = \int \cos 2\alpha dt + \int \cot(t)\sin 2\alpha dt$

$= t.\cos 2\alpha + \ell n|\sin t|.\sin 2\alpha + C$

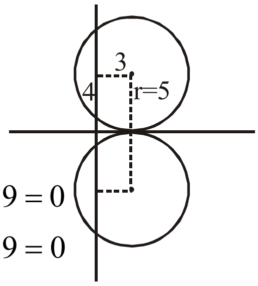
= (x – α) cos2α + ln|sin(x – α)|.sin2α + C

15. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (3, 10) (2) (2,3) (3) (1,5) (4) (3,5)

Sol. Equaiton of circles are

$\begin{cases}(x-3)^2+(y-5)^2=25\\(x-3)^2+(y+5)^2=25\end{cases}$



$\Rightarrow \begin{cases}x^2+y^2-6x-10y+9=0\\x^2+y^2-6x+10y+9=0\end{cases}$

16. For and initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (4) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

Sol. Let X be random varibale which denotes number of problems that candidate is unbale to solve

$\because\ p=\frac{1}{5}$ and X < 2

$\Rightarrow P(X<2)=P(X=0)+P(X=1)$

$=\left(\frac{4}{5}\right)^{50}+{}^{50}C_1.\left(\frac{1}{5}\right).\left(\frac{4}{5}\right)^{49}$

17. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 1 (4) 2

Sol. $f(x)=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$

$=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=\tan^{-1}\left(\tan\left(x-\frac{\pi}{4}\right)\right)$

$\because\ x-\frac{\pi}{4}\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

$\therefore\ f(x)=x-\frac{\pi}{4}$

$\Rightarrow$ its derivative w.r.t. $\frac{x}{2}$ is $\frac{1}{1/2}=2$

18. Let S be the set of all α ∈ R such that the equation, cos2x + αsinx = 2α – 7 has a solution. Then S is equal to :

(1) [2, 6] (2) [3,7] (3) R (4) [1,4]

Sol. cos2x + αsinx = 2α – 7

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\sin^2 x-\frac{\alpha}{2}\sin x+\alpha-4=0$

$\Rightarrow \sin x=2$ (rejected) or $\sin x=\frac{\alpha-4}{2}$

$\Rightarrow\left|\frac{\alpha-4}{2}\right|\le 1$

$\Rightarrow \alpha\in[2,6]$

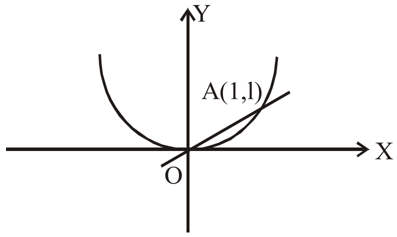
19. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(-\frac{5}{2},-1\right)$ (2) $\left(-\frac{5}{2},1\right)$

(3) $\left(\frac{5}{2},-1\right)$ (4) $\left(\frac{5}{2},1\right)$

Sol. Put $x - 2 = X$ & $y + 1 = Y$

$\therefore$ given curve becomes $Y = X^2$ and $Y = X$



tangent at origin is X-axis

and tangent at A(1,1) is $Y + 1 = 2X$

$\therefore$ there intersection is $\left(\frac{1}{2},0\right)$

$\therefore\ x-2=\frac{1}{2}$ & $y+1=0$

therefore $x=\frac{5}{2}, y=-1$

20. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 25 (2) 28 (3) 27 (4) 24

Sol. ${}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$

$n^2 + 3n = 700$

$\therefore\ n = 25$

21. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is :

(1) $x + y + 4 = 0$ (2) $x - 2y + 16 = 0$

(3) $2x - y + 2 = 0$ (4) $x - y + 4 = 0$

Sol. tangent to the parabola $y^2 = 16x$ is $y = mx + \frac{4}{m}$

solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

condition of common tangent is $D = 0$

$\therefore\ m^3 = 1$

$\Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$

22. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n}=2i-1$ for some natural number n. Then :

(1) $n = 20$ and $\text{Re}(z) = -10$

(2) $n = 20$ and $\text{Re}(z) = 10$

(3) $n = 40$ and $\text{Re}(z) = -10$

(4) $n = 40$ and $\text{Re}(z) = 10$

Sol. Put $z = x + 10i$

$\therefore\ 2(x + 10i) - n = (2i - 1) \cdot [2(x+10i) + n]$

compare real and imginary coefficients

$x = -10,\ n = 40$

23. The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0\ (x \neq 0)$ is :

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$

(2) $y^2 + 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$

(4) $y^2 - 2x^2 + cx^3 = 0$

Sol. $xy\frac{dy}{dx}-y^2+x^3=0$

put $y^2=k \Rightarrow y\frac{dy}{dx}=\frac{1}{2}\frac{dk}{dx}$

$\therefore$ given differential equation becomes

$\frac{dk}{dx}+k\left(-\frac{2}{x}\right)=-2x^2$

I.F. $= e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

$\therefore$ solution is $k \cdot \frac{1}{x^2} = \int -2x^2 \cdot \frac{1}{x^2} dx + \lambda$

$y^2 + 2x^3 = \lambda x^2$

take $\lambda = -c$ (integration constant)

24. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) 2 gain (2) $\frac{1}{2}$ loss (3) $\frac{1}{4}$ loss (4) $\frac{1}{2}$ gain

Sol. win Rs.15 $\rightarrow$ number of cases = 6

win Rs.12 $\rightarrow$ number of cases = 4

loss Rs.6 $\rightarrow$ number of cases = 26

p(expected gain/loss) = $15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36} = -\frac{1}{2}$

25. Let $f(x) = 5 - |x-2|$ and $g(x) = |x+1|$, $x \in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then

$\lim\limits_{x \to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to :

(1) 1/2 (2) –3/2 (3) 3/2 (4) –1/2

Sol. Maxima of f(x) occured at x = 2 i.e. $\alpha = 2$

Minima of g(x) occured at x = –1 i.e. $\beta = -1$

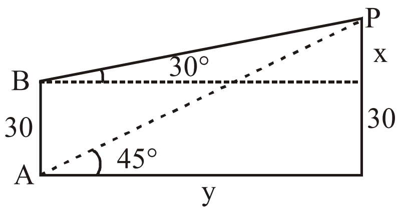
$\therefore \lim\limits_{x \to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \frac{1}{2}$

26. The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:

(1) $15(3-\sqrt{3})$ (2) $15(3+\sqrt{3})$

(3) $15(1+\sqrt{3})$ (4) $15(5-\sqrt{3})$

Sol.



$\tan 45° = 1 = \frac{x+30}{y} \Rightarrow x + 30 = y$ (i)

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{x}{y} \Rightarrow x = \frac{y}{\sqrt{3}}$ (ii)

from (i) and (ii) $y = 15(3+\sqrt{3})$

27. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to :

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $q \Rightarrow \sim p$ (4) $p \wedge q$

Sol. $\sim(p \rightarrow (\sim q)) = \sim(\sim p \vee \sim q)$

$= p \wedge q$

28. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

(1) (2,4,1) (2) (2, – 4, 1)

(3) (1, 4, – 1) (4) (1, – 4, 1)

Sol. equation of bisector of angle

$\frac{2x - y + 2z - 4}{3} = \pm \frac{x + 2y + 2z - 2}{3}$

(+) gives $x - 3y = 2$

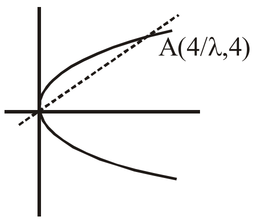
(–) gives $3x + y + 4z = 6$

therefore option (ii) satisfy

29. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 24 (2) 48 (3) $4\sqrt{3}$ (4) $2\sqrt{6}$

Sol.



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}} \left(\sqrt{4\lambda x} - \lambda x\right) dx$$

$$\Rightarrow \lambda = 24$$

30. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ?

(1) $(1, 2\sqrt{2})$

(2) $(2, \sqrt{2})$

(3) $(2, 2\sqrt{2})$

(4) $(\sqrt{2}, 2)$

Sol. given that be = 2 and a = 2

(here a < b)

$\because\ a^2 = b^2(1 - e^2)$

$\therefore\ b^2 = 8$

$\therefore$ equation of ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$