4.5.1). $X(S) = \{1, 3, 4, 6\}$. $P(X=1) = P(X=6) = 0.1$
$P(X=3) = P(X=4) = 0.4$.

(c) $EX. = \sum_{x=x_i}^{x_n} x_i P(X=x_i) = \sum x f(x)$

$= (1)P(X=1) + (3)P(X=3) + (4)(P(X=4) + (6)P(X=6)$
$= (1)(0.1) + 3(0.4) + 4(0.4) + 6(0.1)$
$= 0.1 + 1.2 + 1.6 + 0.6$
$EX = \cancel{4}\ \boxed{3.5}$

(d) $\text{Var}\,X = EX^2 - (EX)^2$ [$\because X$ is a disc r.v].

x	$f(x)$	$x f(x)$	x^2	$x^2 f(x)$
1	0.1	0.1	1	0.1
3	0.4	1.2	9	3.6
4	0.4	1.6	16	6.4
6	0.1	0.6	36	3.6.
		$\Sigma = 3.5$ [EX].		$\Sigma = 13.7$ [EX^2]

$\therefore \text{Var}\,X = \cancel{(3.5)^2 - 13.7}$
$= 13.7 - (3.5)^2$
$= \boxed{1.45}$

(e) $S.D = \sqrt{\text{Var}\,X} = \boxed{\sqrt{1.45}} = 1.2$

4.5.2) die is tossed. , $P(X = x) = (7 - x)/20$. $x = 1, 2, 3, 4, 5$.
$P(X = 6) = 0$.

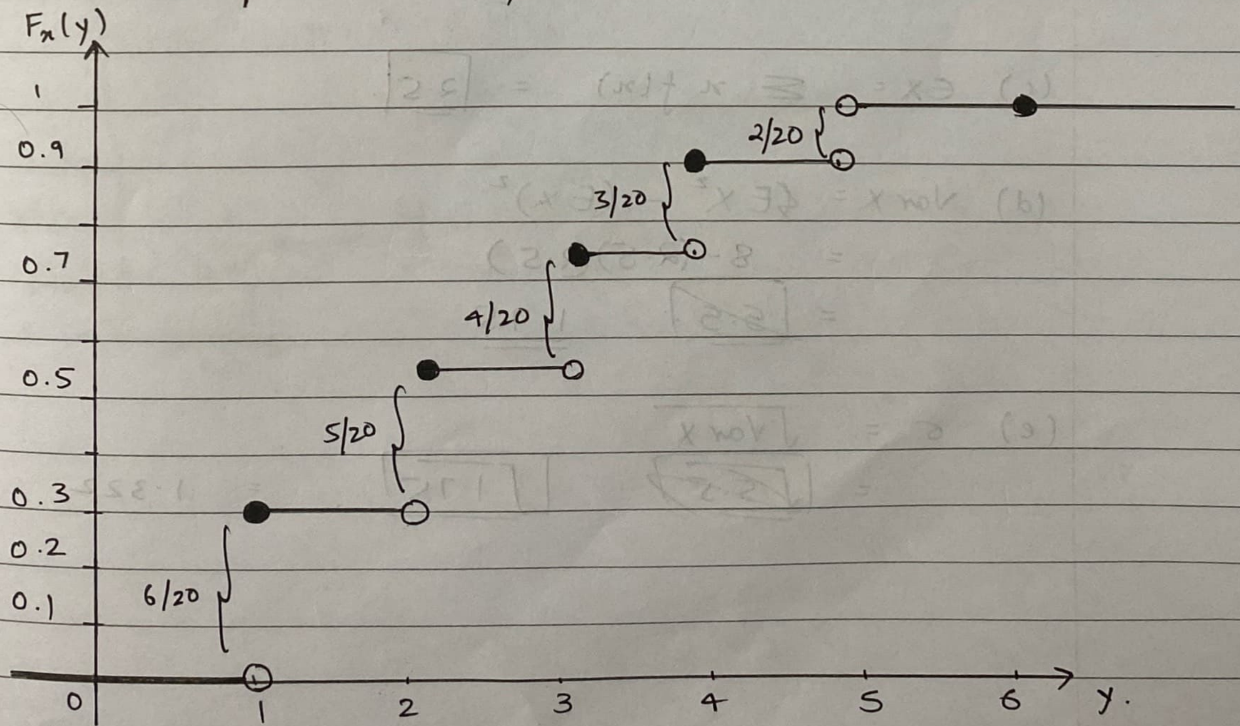
(a) PMF.

$f(1) = P(X = 1) = 6/20$
$f(2) = P(X = 2) = 5/20$
$f(3) = P(X = 3) = 4/20$
$f(4) = P(X = 4) = 3/20$
$f(5) = P(X = 5) = 2/20$
$f(6) = P(X = 6) = 0$

(b) CDF.

$\cancel{f(0) = 0 \quad \because 0 \notin X(S)}$.

$$F_x(y) = \begin{bmatrix} 0 : & -\infty < y < 1 \\ 6/20 : & 1 \le y < 2 \\ 11/20 : & 2 \le y < 3 \\ 15/20 : & 3 \le y < 4 \\ 18/20 : & 4 \le y < 5 \\ 20/20 : & 5 \le y < 6 \\ 20/20 : & 6 \le y < \infty \end{bmatrix} = \begin{matrix} 0 \\ 0.3 \\ 0.55 \\ 0.75 \\ 0.9 \\ 1 \\ 1 \end{matrix}$$



x	$f(x)$	x^2	$x f(x)$	$x^2 \cancel{f(x)}$
1	0.3	1	1	
2	~~0.55~~ 0.25	4	8	
3	~~0.75~~ 0.2	9	27	
4	~~0.9~~ 0.15	16	64	
5	+ 0.1	25	125	
~~6~~	0	36.	0	

~~x $f(x)$~~

x	x^2	$f(x)$	$x f(x)$	$x^2 f(x)$
1	1	0.3	0.3	0.3
2	4	0.25	0.5	1~~2~~
3	9	0.2	0.6	1.8
4	16	0.15	0.6	2.4
5	25	0.1	0.5	2.5
6.	36	0	0	0
			$\Sigma = 2.5$	$\Sigma = 8.$
			EX	EX^2

(c) $EX = \sum x f(x) = \boxed{2.5}$.

(d) $\text{Var } X = EX^2 - (EX)^2$

$= 8 - (2.5)(2.5)$

$= \cancel{\boxed{5.5}}$ $\underline{1.75}$

(e) $\sigma = \sqrt{\text{Var } X}$

$= \cancel{\boxed{\sqrt{5.5}}}$ $\boxed{\sqrt{1.75}}$. $= 1.322$

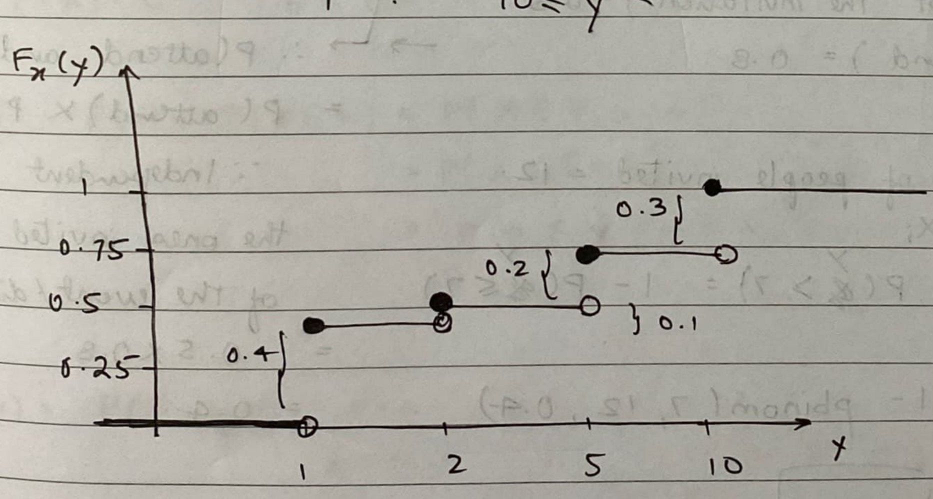
4.5.3) Sample Space $= S = \{1, 1, 1, 1, 2, 5, 5, 10, 10, 10\}$.

Let X be R.V where a ticket is drawn
$\therefore$ X can take values either $\{1, 2, 5, 10\} = X(S)$
now, let $f(x)$ denote the p.m.f.

(a)
$$f(1) = P(X=1) = 4/10 = 0.4$$
$$f(2) = P(X=2) = 1/10 = 0.1$$
$$f(5) = P(X=5) = 2/10 = 0.2$$
$$f(10) = P(X=10) = 3/10 = 0.3.$$

(b) for cdf, $F_x(y)$ will be.

$$F_x(y) = \begin{cases} 0 & : -\infty < y < 1 \\ 0.4 & : 1 \leq y < 2 \\ 0.5 & : 2 \leq y < 5 \\ 0.7 & : 5 \leq y < 10 \\ 1 & : 10 \leq y < \infty \end{cases}$$



x	x^2	$f(x)$	$x f(x)$	$x^2 f(x)$
1	1	0.4	0.4	0.4
2	4	0.1	0.2	0.4
5	25	0.2	1	5
10	100	0.3	3	30
			$EX = 4.6$	$EX^2 = 35.8$

(c) $EX = \sum x f(x) = 4.6$.

(d) $\text{Var } X = EX^2 - (EX)^2 = 35.8 - (4.6)^2$

$= 35.8 - 21.16$

$= \boxed{14.64}$.

(e) $\sigma = \sqrt{\text{Var } X} = \sqrt{14.64} \approx 3.826$

4.5.10) total guests (accomodate) = $N = 7$.

P(accept the invitation) = 0.5.

P(attend) = 0.8

$\rightarrow$ ∴ P(attend and accept)

= P(attend) × P(accept)

∵ Independent & considering the ones invited only know of the event / dinner party

$= 0.5 \times 0.8$

$= 0.4$.

n = no of people invited = 12.

$Y = \sum_{i=1}^{12} X_i$

To find: $P(Y > 7) = 1 - P(Y \le 7)$

= 1 - pbinom(7, 12, 0.4)

$= \boxed{0.0573}$.

Here. event for each person attending is either a success or failure i.e Bernoulli & all the bernoulli adds up to create a Binomial distribution, $X_i \sim$ Bernoulli $(p = 0.4)$

$Y \sim$ Binomial $(12, 0.4)$

11) $p(\text{not attended}) = 0.1$ } assumption.

$p(\text{attended}) = 1 - 0.1 = 0.9$

Total rooms = 100.

reservations accepted = 110

P(more than 100 statisticians will arrive) = ?

Let each person attending JSM be an Bernoulli event.

i.e success = a statistician will attend JSM.

failure = a statistician will not attend JSM.

Let $E_1, E_2, E_3, \dots E_n$ be mutually independent Bernoulli trials

& $Y = \sum_{i=1}^{n} E_i$, where Y follows Binomial Dist.

$\cancel{P(Y) =} P(Y = x) = {}^nC_x \, p^x q^{n-x}$

actually 4% failed to claim, so.

for Bernoulli events $P(\text{success})$ or attending. $= \frac{100-4}{100} = \frac{96}{100} = 0.96$.

$P(Y > 100) = 1 - P(Y \leq 100)$.

$= 1 - \text{pbinom}(110 = \text{size}, y = 100, p = 0.96)$

$= 0.987$

15) Total students = 1500.

P(Heads) = 0.3. $\therefore$ P(T) = 0.7

each student spins penny 89 times.
let spinning a penny be a Bernoulli trial with success = ~~#~~ Heads

and no of heads encountered while tossing the coin 89 times will be represented by random variable x,
where X = no of Heads.

$X \sim \text{Binomial}(\cancel{0.3}\ 89, 0.3)$.

To find, mike will encounter at least 1 student who observes no more than 2 heads.

$= \cancel{P(Y \leq 2)}.\quad P(Y \geq 1)$

$= 1 - P(Y=0)$.

$= 1 - \cancel{p}\ \text{dbinom}(0, 1500, p)$. — I

Here 'p' is unknown. & $p = \cancel{P(X \geq 2)}\ P(X \leq 2)$

$p = \text{pbinom}(2, 89, 0.3)$

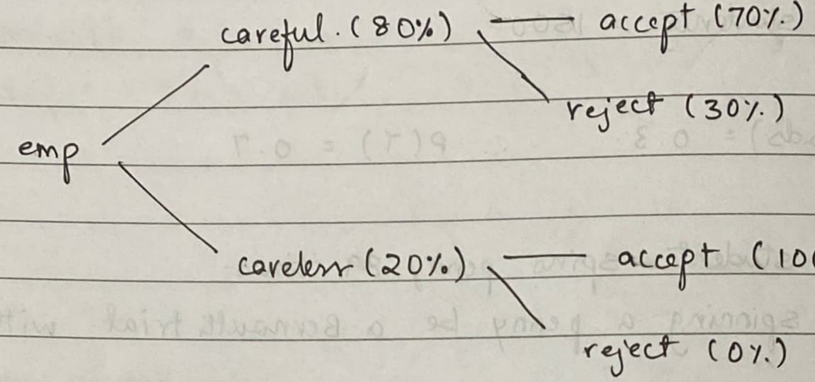
$p = 1.24\,e^{-11} \approx 0$. — II

putting p in I, we get

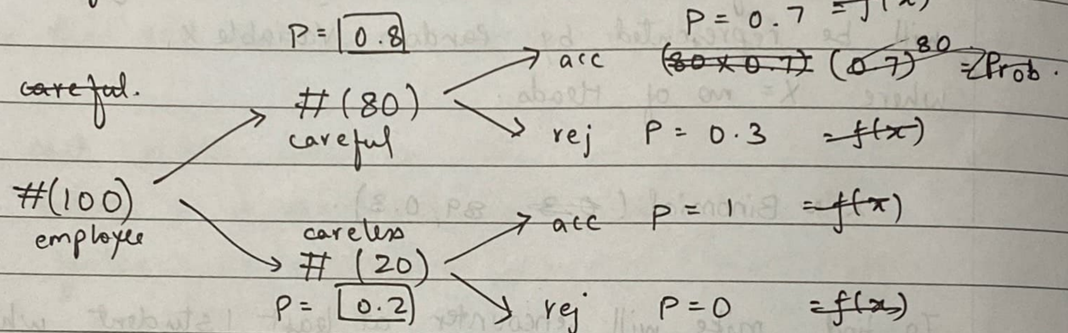
$1 - \text{dbinom}(0, 1500, 1.24e^{-11})$.

$= 1.86\,e^{-8}$.

OTP)



(a) # Emp = 100.



$X = R.V = \#\text{Employees}$

$EX = \Sigma\, x f(x)$ [Considering each Bernoulli trials 1].

$= (80 \times 0.7) + (20 \times 1)$

$= 56 + 20$

$= \underline{76}$

$X = RV = \#$ Cookies accepted.

(b) ~~∵ employee's prob is asked, consider.~~ employee's Probability

Exp no of cookies $= \Sigma\, x f(x). = x_1(0.8) + x_2(0.2)$

$= (100 \times 0.7)\,0.8 + (100 \times 1)\,0.2$

$= 70(0.8) + \cancel{30 \times 0.2}\ 100(0.2)$

$= \boxed{76}$.

(c) $P(\text{Careless} \mid \text{rejected}) = \dfrac{P(\text{careless} \cap \text{rejected})}{P(\text{rejected})}$

$= \boxed{0}$

(d) $P(\cancel{\text{accepted} |} \text{careful}) = \dfrac{P(\text{accepted} \cap \cancel{\text{careful}})}{P(\text{careful})}$

$= \cancel{\dfrac{0.7 \times 0.8}{0}}$

$P(\text{careful} \mid \text{accepted}) = \dfrac{P(\text{careful} \cap \text{accepted})}{P(\text{accepted})}$

$= \dfrac{0.8 \times 0.7}{(0.8 \times 0.7) + (0.2 \times 1)} = \dfrac{0.56}{0.56 + 0.2}$

$= \dfrac{0.56}{0.76}$

$= \underline{0.736}$