

Homework 4

Due: October 8th, 11:59 pm.

Instructions: Please scan or typeset your solutions and upload them as a single pdf file to Canvas. All handwritten parts must be legible. Both, textbook and out-of-textbook problems are mandatory. In each part of OPT1 write down the required probability statement first, and then compute it in R.

Readings: Chapter 5 (Sections 5.1 - 5.5)

Textbook problems:

Section 5.6: 3 (a, b, c, d)

Section 5.6: 7. In addition to computing the probabilities, also express the answers in the standard normal cdf $\Phi(\cdot)$

Section 5.6: 8

Out-of-textbook problems (OTP):

OTP 1. During the pandemic a grocery delivery business started with the policy of delivering orders with a maximum delay of 30 minutes (with respect the desired delivery time picked by customers in their online purchases). Say the delivery delay is preliminary modeled using a *Uniform* $[0, 30]$ -distributed random variable.

a) What is the probability that a (random) order arrives during the last 5 minutes of the 30 minutes time window?

b) Is the probability in (a) the same that the probability that the order arrives in the first 5 minutes (of the 30 minutes time window)?

Say after some months of operation data suggested that a better model for the delivery delay time turned out to be a Normal distribution with mean 18 and standard deviation 6 minutes.

- c) What is the probability that the order arrives after the 25th minute of delay?
- d) What is the probability that the order arrives before the time picked by the customer?
- e) Quantitatively comment on how well the business is doing with respect to its policy.

OTP 2. Draw a graph overlaying the pdfs of $N(0, 1)$, $t(3)$, and $t(10)$, and then **comment** comparatively.

OTP 3. Briefly describe the relationships among the following distributions $N(\mu, \sigma^2)$, $N(0, 1)$, $\chi^2(n)$, $t(\nu)$, and $F(\nu_1, \nu_2)$.

Problems by learning objectives (plus rubric):

Question	Points	Goal: to reinforce...
5.6.3	4 pt	pdf definition, expectation and variance in continuous case
5.6.7	5 pt	Normal distribution manipulation
5.6.8	5 pt	properties of expectation and variance in continuous case
OTP1	5 pt	continuous distributions application
OTP2	4 pt	pdf visualization and description
OTP3	4 pt	distributions derived from the Normal distribution
27 pts.		