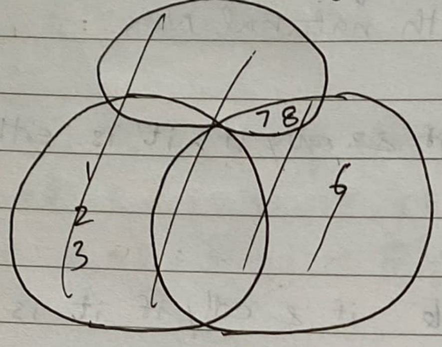
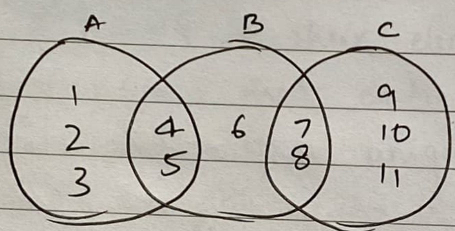
2.5. 1) Unions & Intersections.

$A = \{1, 2, 3, \underline{4}, \underline{5}\}$ $B = \{\underline{4}, \underline{5}, 6, \underline{\underline{7}}, \underline{\underline{8}}\}$ $C = \{\underline{\underline{7}}, \underline{\underline{8}}, 9, 10, 11\}$





(a) $A \cup B \cup C = \{x \in \mathbb{Z} : x \in A \text{ or } x \in B \text{ or } x \in C\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}.$

(b) $A \cap B \cap C = \{x \in \mathbb{Z} : x \in A \text{ and } x \in B \text{ and } x \in C\}.$

$= \phi.$

(c) $\because$ $A \cap B \neq \phi$ & $B \cap C \neq \phi$

$\therefore$ The collection $\{A, B, C\}$ can not be said as mutually exclusive or disjoint (pairwise)

Also by defination "A collection of sets is pairwise disjoint if all pairs ($\{A, B\}, \{B, C\}, \{A, C\}$) have no same elements or are all mutually exclusive.

$A \cap B \neq \phi$, $B \cap C \neq \phi$, $A \cap C = \phi$.

2.5.3) Complements

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 3, 5, 7, 9\}$.
$B = \{1, 2, 3, 5, 7\}$.

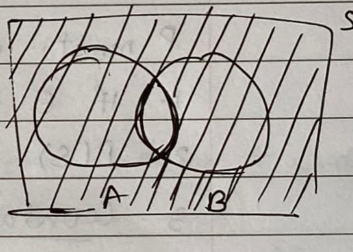
(a) $A^c = \{x : x \notin A\} = \{2, 4, 6, 8, 10\}$.

(b) $B^c = \{x \in \mathbb{Z} : x \notin B\} = \{4, 6, 8, 9, 10\}$.

(c) $(A \cup B)^c$, $A \cup B = \{1, 2, 3, 5, 7, 9\}$.

now, $(A \cup B)^c = \{x : x \notin A \cup B\}$.

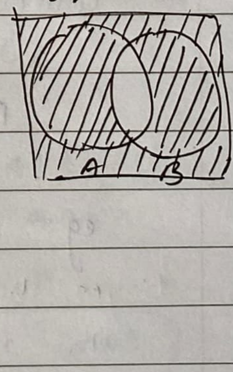
$= \{4, 6, 8, 10\}$.



(d) $(A \cap B)^c = \{x : x \notin (A \cap B)\}$.

$A \cap B = \{1, 3, 5, 7\}$.

or



$\therefore (A \cap B)^c = \{2, 4, 6, 8, 9, 10\}$.

OR

$(A \cup B)^c = S - A \cap B$

$= \{1, \ldots 10\} - \{1, 3, 5, 7\}$.

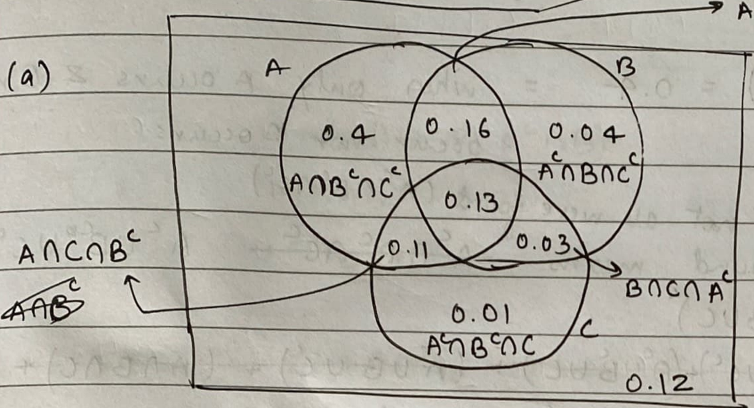
$= \{2, 4, 6, 8, 9, 10\}$.

(d) $P(A \cap B \cap C)^c = 1 - P(A \cap B \cap C)$
$= 1 - 0.13$
$= 0.87$

(e) $P(A^c \cap (B \cup C)) =$ ~~P~~

Manipulation of Event Probablities (discrete).

3.7.1 (a)



Qu = $P(A) = 0.8$
To = $P(B) = 0.36$
Aq = $P(C) = 0.28$
$P(A \cap B) = 0.29$
$P(B \cap C) = 0.24$
S. $P(A \cap C) = 0.16$

$P(A)$ [without overlap] $= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$
= only A occurs $= 0.8 - 0.29 - 0.24 + 0.13$
$= \underline{0.4}$

$P($B without overlap$) = P(B) - P(B \cap A) - P(B \cap C) + P(A \cap B \cap C)$
$= 0.36 - 0.29 - 0.16 + 0.13$
$= \underline{0.04}$.

$P($C without overlap$) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$= 0.28 - 0.16 - 0.24 + 0.13$.
$= \underline{0.01}$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$
$- P(A \cap C) + P(A \cap B \cap C)$.
$= 0.8 + 0.36 + 0.28 - 0.29 - 0.24 - 0.16 + 0.13$
$= 1.44 - 0.56$
$= \underline{0.88}$.

$\therefore P([A \cup B \cup C]^c) = 1 - 0.88$
$= \underline{0.12}$

(b) $P(Qu \cap To \cap Aq^c)$

$P(A \cap B \cap C^c) = 0.16$ from Venn diagram

~~$= P(C^c) \cap P(A \cap B)$~~

$P(A \cap B \cap C^c) =$ Quartz & Tourmaline will be found but not aquamarine

(c) $P(A \cap B^c \cap C^c) = 0.4$ = when only A occurs & B not occur nor C occurs.

(d) None were found means ~~$A^c \cap B^c \cap C^c$~~ $A^c \cup B^c \cup C^c$.

$\neq$ not all were found $(A^c \cap B^c \cap C^c)$

i.e $1-(A \cup B \cup C)^c$

~~$= 1-[(A \cup B^c \cup C^c)+(A^c \cup B^c \cup C)+(A^c \cup B \cup C^c)+(A \cap B \cap C)+$~~

$= 1-P[A \cap B^c \cap C^c + A^c \cap B^c \cap C + A^c \cap B \cap C^c +$

$A \cap B \cap C^c + A \cap B^c \cap C + A^c \cap B \cap C + A \cap B \cap C]$

$= 1-[0.4+0.04+0.01+0.16+0.11+0.03+0.13]$

$= 1-0.88$

$= \underline{0.12}.$

(e) $P(A^c \cap (B \cup C)) =$ ~~$P[A^c \cap ($~~ $=$

~~$= P[A^c \cap (B+C+B \cap C)]$~~

= Not in A and in (B or C)

= By Venn diagram we get = $0.08 = (0.04+0.03+0.01)$

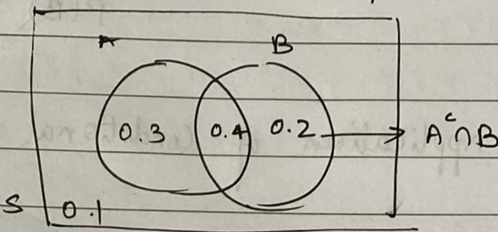
3.7.7 Disjoint events, independent events, conditional probability.

$P(A) = 0.7$.

$P(B) = 0.6$

$P(A^c \cap B) = 0.2$

(a)



$\because P(B) = P(A \cap B) + P(A^c \cap B)$

$0.6 = P(A \cap B) + 0.2$

$\therefore P(A \cap B) = \underline{0.4}$

$\because P(A) = P(A \cap B^c) + P(A \cap B)$

$\therefore P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.7 - 0.4$

$= \underline{0.3}$

$\because P(A \cup B)^c = 1 - P(A \cup B)$

$\therefore P(A \cup B)^c = 1 - 0.9$

$= \underline{0.1}$

(b) A & B can not be disjoint or mutually exclusive because $(A \cap B) \neq \phi$.

(c) $P(A \cap B^c) = \underline{0.3}$ [as calculated above].

$P(A \cup B^c) = (0.3 + 0.4) + 0.1 = \underline{0.8}$.

(d) If A & B are independent then,

$P(A \cap B) = P(A) \cdot P(B)$ must be true

$\Downarrow$

0.4 [as calculated above].

$0.6 \times 0.7 = 0.42$.

$\because P(A \cap B) \neq P(A) \cdot P(B)$ & $P(A)$ or $P(B) \neq 0$

A & B are ~~indepen~~ not independent or dependent.

Also, (c) $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c) = P(A\cap B) + P(A \cap B^c) \cancel{+ P(B\cap A) + P(B \cap A^c)} - P(A \cap B^c) + 1 - P(B)$

$\cancel{= 2P(A\cap B) + P(B \cap A^c)}$

$= P(A \cap B) + 1 - P(B) = 0.4 + 1 - 0.6 = 1 - 0.2 = \underline{\underline{0.8}}$

(e) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.4}{0.6} = \dfrac{4}{6} = \dfrac{2}{3} = \underline{\underline{0.66}}$

3.7.8 Application of Conditional Probablity.

$P(+|D) = 0.62$ $\qquad P(-|D^c) = 0.82$.

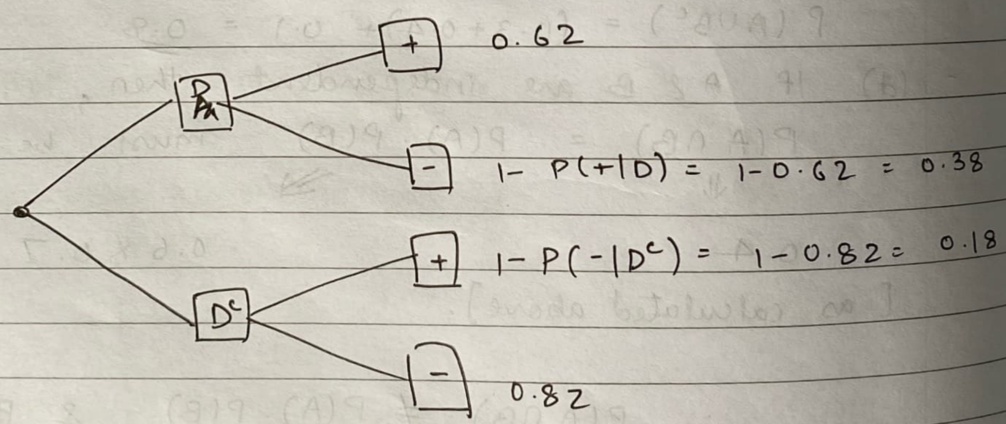
(a) $P(\text{fase positive}) = P(+|D^c) = P(A|D^c)$

$D =$ arterial pressure ≥ 90 mm Hg.
$D^c =$ arterial pressure < 90 mm Hg
$A = + =$ pre-eclampsia predicted.
$A^c = - =$ pre-eclampsia not detected.

idk why I did this when I can use tree

$P(A|D) = \cancel{P(D|A)}\ \dfrac{P(A \cap D)}{P(D)} = \dfrac{P(D|A)\,P(A)}{P(D)} = 0.62$

$P(A^c|D^c) = \dfrac{P(A^c \cap D^c)}{P(D^c)} = \dfrac{P(D^c|A^c)\,P(A^c)}{P(D^c)} = 0.82$.



(a) False positive $= (+|D^c) = 0.18$.
(b) False negative $= (-|D) = 0.38$

(c) adding $P(D) = 0.05$